AN APPROACH TO CONSIDERING UNCERTAINTY IN DEVELOPING LONG-TERM, LEAST-COST WOOD PROCUREMENT POLICIES

by

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I. INTRODUCTION

Operations research techniques were initially applied to the solution of forestry problems in the early 1960's. Since then, foresters and researchers in related fields have attacked certain forestry problems with most of the available operations research techniques. However, this previous research has been conducted almost entirely under the condition of assumed certainty--that is, the operations research models employed have been deterministic. The certainty assumption is as unrealistic in forestry as it is in almost all management functions; because forestry decisions, like many others, are made using incomplete information about complex and uncertain events. There is obviously a need for methods to cope with uncertainty--not for ways to avoid uncertainty. This study attempts to consider uncertainty by introducing variation into the right-hand side vector of a linear programming problem. The problem used to illustrate this consideration of uncertainty is one previously solved as a deterministic linear program (Thompson, et al., In press). The result of considering the uncertainty, which is intrinsic to the specific decision making process, is a model which more nearly reflects the real world situation.

Linear programming was first applied in forestry by Coutu and Ellertsen (1960). Several recent articles have dealt with the application of linear programming to forest management planning and decision making (e.g., Curtis, 1962; Kidd, et al., 1966; and Thompson, et al., In press). As indicated, this previous work was deterministic. Also, it

was largely concerned with proving that linear programming was applicable to forestry problems.

An early forestry attempt to consider uncertainty was made by Dowdle (1962). Although he was a leader in this effort, he did not develop a functional analytic technique. Flora (1964) considered the influence of uncertainty in forestry decision making but lacked a consistent approach in his attempt to treat uncertainty as risk.

Although the final effects of risk and uncertainty are much the same, there is a great difference in their origin. Uncertainty is a condition in which the probability distribution of the outcomes of a decision are unknown and can only be estimated by subjective means. The term subjective refers to one's degree of rational belief. This concept of uncertainty is subject to argument because there are those, like Hadley (1967: 1), who believe that "uncertainty is so fundamental that it is difficult to explain what it means in terms of more fundamental ideas".

As opposed to uncertainty, risk is a condition in which the probability distribution of the outcomes of a decision can be determined by objective means. Thompson (1968) has reviewed the theoretical differences between risk and uncertainty and developed the argument for systematically considering uncertainty in forestry decision making.

The lack of distinction between risk and uncertainty has been a major stumbling block for most forestry research concerned with uncertainty reduction. This is because of the general failure to recognize that uncertainty, by definition, can be described only with a subjective probability distribution. Two recent forestry references have recommended using such distributions. Thompson (1968) has suggested a possible approach for

incorporating such distributions, and Bentley and Kaiser (1967) used subjective probabilities in developing survival rates to be used in applying decision tree techniques. Bentley and Kaiser based their subjective evaluations on technical information, economic analysis, external expert opinion, and management judgment.

Objectives

The primary objective of this study was to consider uncertainty by introducing variation into a least-cost wood procurement problem that has previously been solved as a deterministic linear program. The secondary objective was to evaluate and interpret any differences between the deterministic solution and the uncertainty solutions with respect to the implementation of wood procurement policies.

II. BASIC PROCUREMENT THEORY

The problem which will be used to illustrate the consideration of uncertainty deals with minimizing the cost of raw material procurement for an integrated forest products firm. Generally, wood procurement decision-making has been characterized by decision rules of thumb rather than by ordered bodies of theory. If the sophisticated analytic techniques of operations research are to be applied to the wood procurement process, the process must be described in specific terms. The general wood procurement problem can be described as a multi-item, multi-source procurement system.[1/] The problem exists as a result of a demand stimulus on available supply sources. The wood procurement manager is faced with three problems: when to procure, how much to procure, and from what source to procure.

The Theoretical Procurement System

The theoretical procurement system may be described as follows: a stock of certain items is maintained in inventory to meet demand for the items, whenever the number of items on hand and on order falls to a predetermined level (procurement level), action is initiated to procure a replenishment quantity (procurement quantity) from one of several possible sources. The objective, then, is to "determine the procurement level, the procurement quantity, and the procurement source in the light of the relevant costs and properties of demand and procurement lead time, so that the sum of all costs associated with the procurement and inventory system will be minimized (Fabrycky and Torgerson, 1966: 257)."

1/ The arguments in this chapter closely follow those in Fabrycky and Torgerson, Chapter 10 (1966).

Procurement systems can be designed to fit four general procurement situations. This study will consider the multi-item, multi-source system which is at the apex in the hierarchy of procurement and inventory systems (Fabrycky and Banks, 1967: 14). The lower order systems, single-item, single source; multi-item, single source; single-item; multi-source, will not be considered because they do not describe the wood procurement problem.

In most procurement systems (especially the multi-item, multi-source system) the supply of raw material originates from a combination of four alternative types of supply. Each alternative has certain characteristics which influence or detract from the capabilities of each source and which give each alternative distinct capabilities. The two major alternatives are (1) the purchase alternative and (2) the intrafirm transfer; the minor alternatives are (1) the manufacture alternative and (2) the re-manufacture alternative.

The combination of sources chosen to fulfill the procurement quantity depends upon several source dependent parameters (parameters whose values depend upon the source chosen). The most important parameter in determining the choice of type of sources is the item cost of supply from each. The item cost should not be the sole criteria used to determine the types of supply chosen. In many cases, a trade-off relationship between the item cost of one source and the lead time (time interval between ordering additional items and their delivery) and/or procurement cost (costs associated with the procurement process) of another source which would make the latter source of supply more attractive may exist. The procurement quantity is made up of the particular alternative types of supply which result in an economic advantage to the firm as a whole.

The Decision Environment

The procurement system discussed in the previous section includes parameters that are both dependent upon and independent of the source. Two source dependent parameters were mentioned (item cost and procurement cost), but only one source independent parameter was briefly considered--demand. Demand "is the primary stimulus on the procurement system and the justification for its existence (Fabrycky and Torgerson, 1966: 261)." The simplest system considering this demand pattern may be classified as deterministic.

"The primary objective of the procurement system is to meet demand at a minimum cost (Fabrycky and Torgerson, 1966: 261)." This requires the assignment of appropriate values (when, how much, and from what source to procure) to construct and manipulate an effectiveness function of the form:[2/]

$$E = F\ (X_i,\ Y_j) \qquad (1)$$

where:

E = measure of effectiveness sought (e.g., minimize total system cost)

X_i = policy parameters under management control (when, how, what source)

Y_j = source dependent and independent parameters.

This is the model for the general procurement system. Stochastic variation can be introduced into either the X_i or the Y_j parameters. The

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2/ The source of equation (1) is Fabrycky and Torgerson (1966: 263).

result is that the solutions of a stochastic approach will represent a distribution which approximates the distribution of the stochastic parameters.

The Wood Procurement System

The general procurement model which was discussed in the previous section is applicable to all procurement situations (with some modifications required). The wood procurement system is best described as a multi-item, multi-source system because wood is a non-homogeneous raw material and may be procured in several forms and from several alternative types of sources. Wood can come in any one of a number of different forms (e.g., roundwood, chips, slabs, etc.). It is a multi-source system because generally two alternative types of sources are considered: the purchase alternative and the intrafirm transfer alternative. These two alternatives are considered because firms may obtain their supply from sources on the open market and company owned sources. The wood procurement system is unique in one aspect; the sources of supply considered under the intrafirm transfer alternative could also be considered under the manufacture alternative depending upon the organization of the company. The more important source dependent and independent parameters that are considered in the wood procurement system are item cost, procurement cost, and demand.

III. GENERAL MODEL

The basic mathematical model for linear programming is:

Maximize (or minimize) $Z = c_j x_j$

Subject to:

$$a_{ij} x_j = b_i$$

$$x_j \geq 0$$

where:

c_j = cost (or value) of alternative x_j

x_j = alternatives

a_{ij} = matrix of coefficients; $i = 1 \ldots m$; $j = 1 \ldots n$

b_i = right hand side vector.

The difference between deterministic and stochastic linear programming is determined by the method in which the values used in the model are derived. In the deterministic approach, the model assumes certainty; that is, c_j, a_{ij}, and b_i are considered single-valued. The model builder is assumed to possess perfect knowledge of all events in the present and future. However, in the stochastic model, c_j, a_{ij}, and/or b_i are multi-valued and are represented by random variables.

The model developed in this study is termed a partial stochastic linear program. It is called a partial model because uncertainty was considered by introducing variation into only one element of the model. This partial model is stochastic in the general sense, as explained by Matheny. If a value for the stochastic resource vector (a random right hand side vector) is chosen from the appropriate distribution and the linear program solved, the result will be a solution vector for each right

hand side. When this process is repeated a large number of times, the values of the optimal solution vectors will form their own distributions. "Hence, . . . the optimal solution vectors are indeed a function of the stochastic resource vector and are themselves stochastic (Matheny, 1967: 25)."

It should be pointed out that a model is stochastic only in the sense that it is formulated as a stochastic model. Once the problem is implemented with the linear programming algorithm, the problem becomes an expected value model.

The probability distributions from which the random variables are drawn in the stochastic approach are very important. If the distributions are objectively determined, the decision is made under risk. If there is no objective basis for the distribution, the decision is made under uncertainty (Thompson, 1968: 156). To include the consideration of uncertainty in decision making, a manager must be able to express some degree of rational belief in the occurrence of the relevant events (Hadley, 1967: 1). Such probabilities are referred to as personal or subjective. In statistical decision theory, subjective probabilities play at least as an important role as those probabilities which are based upon historical fact. As long as the method by which the subjective probabilities are developed is logically consistent, the method is acceptable to modern statistical decision theory (Christ, 1966: 6).

This study assumed that all variation about a source of supply's mean value could be described by the normal distribution. The use of one distribution would allow the manager to predict the shape of the stochastic solution vector. If several types of distributions were used to describe

the random variation about the supply source's mean values then the manager could predict very little as to the shape of the stochastic solution vectors.

The following example will illustrate the use of subjective probabilities in decision making and introduce the linear programming model to be applied in the next chapter. Thompson, et al. (In press) contained the following hypothetical example of deterministic linear programming over time.

A forest products firm's wood processing plant requires 4,000 cords of wood during a fixed time period (T). The firm obtains its wood from two suppliers, supplier A and supplier B. The firm divides the period (T) into four equal-length sub-periods (t_1, t_2, t_3, and t_4). Prior to the beginning of time (T), each supplier is asked to estimate the wood volume he can furnish in each sub-period. The firm then places four separate wood orders with each supplier, one for each sub-period. From experience the firm knows that one-half of the wood which could be furnished but is not ordered in a sub-period can be furnished in the subsequent sub-period. This "carried over" wood may be ordered at the cost prevailing in the subsequent sub-period.

For a given period (T) the suppliers estimate that they can furnish the following volumes of wood at the indicated costs.

Sub-period	Supplier	Volume (cords)	Cost per cord (dollars)
t_1	A	600	19
t_1	B	700	20
t_2	A	400	18
t_2	B	800	22
t_3	A	700	20
t_3	B	500	18
t_4	A	700	20
t_4	B	300	18

The firm must decide the volume of wood to order from each supplier in each sub-period. The firm's objective is to minimize total procurement cost over the entire period (T), while satisfying the plant's requirement of 1,000 cords per sub-period. Let A_1 be the volume ordered from supplier A in sub-period t_1, B_1 be the volume ordered from supplier B in sub-period t_1, A_2 be the volume ordered from supplier A in sub-period t_2, etc.

The objective function is: Minimize total cost: $19A_1 + 20B_1 + 18A_2 + 22B_2 + 20A_3 + 18B_3 + 20A_4 + 18B_4$.

The constraining equations are derived as follows: The wood requirement for each sub-period is 1,000 cords. Therefore, $A_1 + B_1 = 1{,}000$; $A_2 + B_2 = 1{,}000$; $A_3 + B_3 = 1{,}000$; $A_4 + B_4 = 1{,}000$.

Supplier A can furnish 600 cords in sub-period t_1. Thus, $A_1 \leq 600$.

Supplier B can furnish 700 cords in sub-period t_1. Thus, $B_1 \leq 700$.

During sub-period t_2, supplier A can furnish 400 cords. However, 300 additional cords could be furnished, if no wood is ordered from supplier A in sub-period t_1. Therefore, the wood availability equation for supplier

A in sub-period t_2 is: $.5A_1 + A_2 = 700$.[3/] By similar reasoning the rest of the wood availability equations are:

$$.5B_1 + B_2 \leq 1{,}150$$
$$.5A_2 + A_3 \leq 900$$
$$.5B_2 + B_3 \leq 900$$
$$.5A_3 + A_4 \leq 1{,}050$$
$$.5B_3 + B_4 \leq 550$$

The resulting linear programming over time matrix is:[4/]

Wood Order Alternatives

A_1	B_1	A_2	B_2	A_3	B_3	A_4	B_4		b_i
1	1							=	1,000
1								≤	600
	1							≤	700
		1	1					=	1,000
.5		1						≤	700
	.5		1					≤	1,150
				1	1			=	1,000
		.5		1				≤	900
			.5		1			≤	900
						1	1	=	1,000
				.5		1		≤	1,050
					.5		1	≤	550

3/ Note, when all 600 cords which supplier A can furnish in sub-period t_1 are ordered for sub-period t_1, this equation becomes $A_2 = 400$.

4/ The necessary slack and artificial variables are not included; when they are added, the matrix will have 12 rows and 20 columns.

The rectangles formed by the solid lines in the matrix indicate the linkages between suppliers and sub-periods. These linkages create a single linear programming over time problem rather than four separate linear programming problems.

Analysis of the above simple problem illustrates the usefulness of linear programming over time in obtaining wood procurement schedules. If the firm's woodlands department considers only one sub-period at a time, it will follow a different wood ordering schedule than if linear programming over time is used to consider the four sub-periods simultaneously (Table 1). Following the linear programming over time wood ordering schedule leads to a cost reduction of approximately 12 cents per cord.

Whenever alternatives can be scheduled in more than one sub-period of a fixed time period, linear programming over time may be the most efficient technique for obtaining an optimum schedule.

The example is deterministic because c_j, a_{ij}, and b_i are single-valued. Uncertainty can be considered by introducing variation into any one or all of the above elements. In the current study, variation is introduced only into the right hand side vector, B. Therefore, the resulting model is properly termed a partial stochastic linear program.

In order to illustrate the consideration of uncertainty, the values in the deterministic right hand side for the preceding example were considered mean expected values and assigned the following associated standard errors.

	Mean expected value	Standard error
	cords	
Wood requirement, t = 1.01 [5/]	1,000	0.0
Available supply A	600	1.5
Available supply B	700	2.6
Wood requirement, t = 2	1,000	0.0
Available supply A	700	121.7
Available supply B	1,150	299.8
Wood requirement, t = 3	1,000	0.0
Available supply A	900	247.9
Available supply B	900	371.9
Wood requirement, t = 4	1,000	0.0
Available supply A	1,050	365.0
Available supply B	550	286.8

A FORTRAN IV program to generate alternative right hand sides from the distribution formed by the means and standard deviations was prepared (Appendix 2a), and ten right hand sides were generated.

The total minimum costs of procuring wood for time period (T) is:

Using linear programming over time	\$77,725
Using partial stochastic linear programming	\$77,750 ± \$455.60

For this example, only seven alternative right hand sides were generated. As more right hand sides are included, the stochastic mean will approach the deterministic value.[6/] From the decision making

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5/ In this first time period t = 1.01 because the logarithm of 1.00 is zero and this would have caused difficulty in calculating the standard deviation. (The standard deviation calculation is explained in Appendix 2a).

6/ Assuming the right hand sides are generated from a normal distribution.

Table 1. Wood ordering schedule, by supplier and sub-period, with and without linear programming over time

	Wood ordering schedule			
	With linear programming over time		Without linear programming over time	
Sub-period	Supplier A	Supplier B	Supplier A	Supplier B
	cords			
t_1	300.0	700.0	600	400
t_2	550.0	450.0	400	600
t_3	325.0	675.0	500	500
t_4	787.5	212.5	700	300
TOTAL COST (dollars)	77,725		78,200	

standpoint the important aspect is the additional information available to the manager. In this example, the manager has a feel for the range of wood procurement costs rather than a single value with no estimate of variation. In the example, the manager has the information that the wood procurement cost for period (T) will fall between $77,394.40 and $78,205.60 with 68 percent probability. Additional managerial use of such information will be discussed in more detail after an actual application of the model has been presented.

IV. APPLICATION OF THE MODEL

The study by Thompson, et al., (In press) was a case study of an integrated forest products firm in which a least-cost wood procurement schedule was developed using deterministic linear programming over time. The firm's manufactured product dictated the limits within which roundwood, chips, slabs, and shavings could be combined to form an acceptable raw material mix. The firm's timbershed was located within a 50-mile radius of the mill and was divided into five procurement zones based upon distance from the mill. The objective of the program was to minimize the present value (using a six percent discount rate) of all wood procurement costs over a twenty-year period which was divided into ten, two-year subperiods. A managerial constraint required that at the end of the first twenty year period the firm must own enough standing timber to provide at least fifty percent of its requirement for a second twenty year period. The linear programming matrix contained 274 constraints (rows) and 425 variables (columns).[7/]

To achieve the objectives of the present study, the above problem was reformulated as a partial stochastic linear program. The general model developed in the preceding chapter, in which the right hand side vector was stochastic and the objective function and technological matrix are deterministic, was used in the application.

The additional data needed to solve the partial stochastic program were quantitative estimates of the uncertainty associated with each of

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7/ For a complete discussion of the actual operation of the firm, the design of the case study, and the choice of alternatives, constraints, assumptions, and costs see Thompson, et al. (In press).

the elements in the original right hand side vector. The elements of the right hand side vector represent, for the most part, future wood requirements and future wood availability by species and zone. Such data are necessarily estimates. In the deterministic study, the cooperating firm's wood procurement personnel were asked to furnish point estimates of these elements based upon their experience and judgment. For the present study, the original point estimates were considered mean expected values and the cooperating firm's personnel were asked to estimate the upper and lower limit associated with each mean. The estimated means, estimated limits, the standard deviations calculated from the estimates, and the FORTRAN IV program to generate alternative right hand side vectors are given in Appendix 2b. Required modifications in the original linear program are also discussed in Appendix 2b.

Results

The results of the partial stochastic linear program indicated that differences do exist between the stochastic and deterministic approaches. However, the wood procurement policies that were suggested are largely the same in both programs. The magnitude of the differences were due to the amount of variation considered in the stochastic approach.

The partial stochastic approach was applied by testing the same procurement policy with four variations of the resources available to the firm. These four different variations differed in two aspects. First, two sets of multipliers, which are explained and given in Appendix 3, were used: one set for runs 1 and 2 (a run consisted of 25 linear programming solutions--each determined from a different resource situation); and one set of multipliers for runs 3 and 4. The runs also differed in the

Table 2. Summary of the results from the stochastic and deterministic approach

Approach	Mean	Standard Deviation
Deterministic	$15,404,916	————
Stochastic		
Run 1	$15,430,916	$ 39,887.83
Run 2	$15,465,382	$ 16,725.19
Run 3	$15,608,996	$157,398.71
Run 4	No feasible solutions	

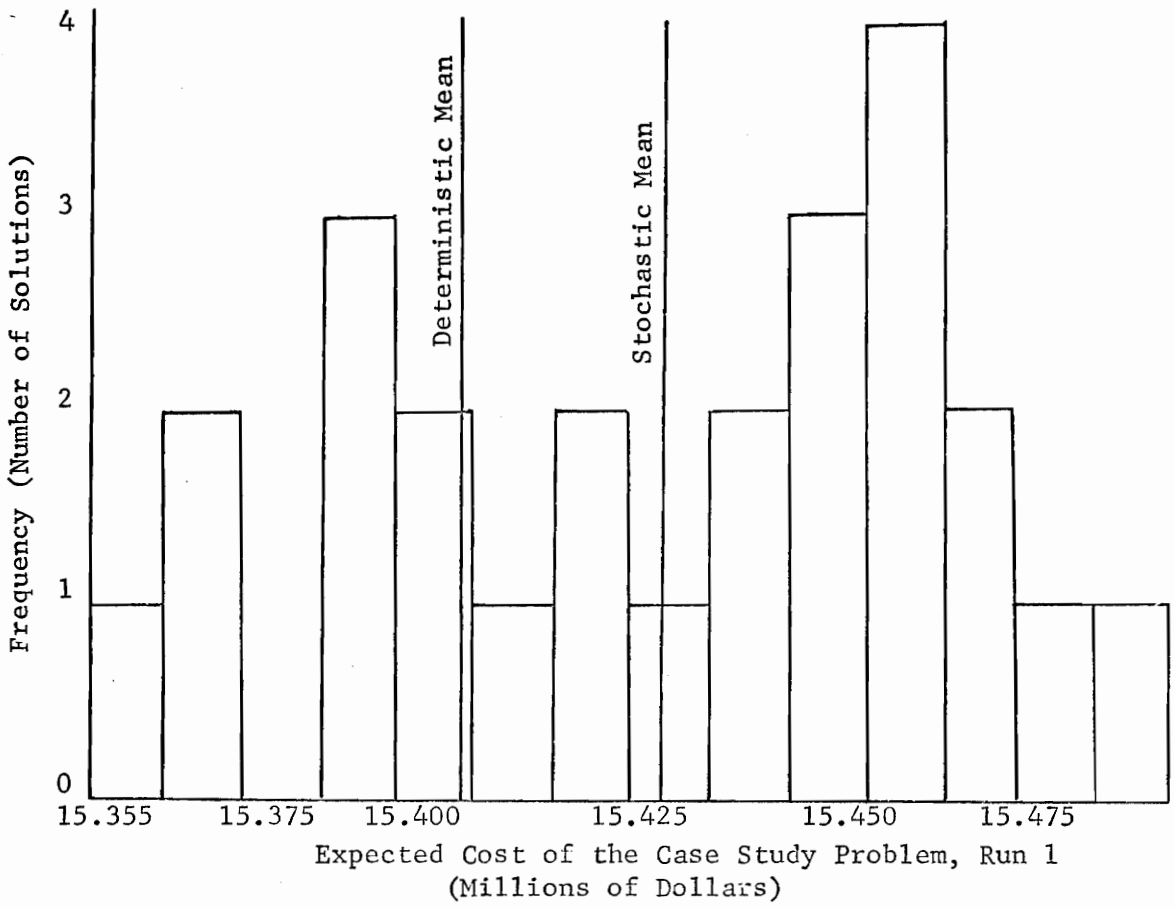

Figure 1. Histogram Showing the Frequency Distribution Between Class Intervals of the First Stochastic Solution Vector

proportion of the population of numbers being used to generate the resource situation. Runs 1 and 3 considered approximately 68 percent of the population of possible numbers and runs 2 and 4 considered 95 percent.

The results of the four stochastic runs and the results of the deterministic approach are given in Table 2. A comparison of the results of run 1 and the deterministic mean is given in Figure 1. The histogram represents the proportional numbers of optimal stochastic solutions that fell into $10,000 intervals.

There were no feasible linear programming solutions for run 4. The variation was so great over the ten time periods that continuity was lost within the system and the solutions were unbounded. The standard deviation of run 1 is larger than that of run 2. Logically it should be the other way around; however, the range of answers for run 2 was five times as great as that for run 1.

Discussion

Most of the discussion will concentrate on the initial sub-period because this sub-period indicates the procurement policy the firm should implement initially to minimize procurement costs for the twenty-year period. The discussion will concentrate on run 1 because this run most nearly reflects the cooperating firm's estimates.

The most important finding was that the procurement policies suggested by both the deterministic and stochastic approaches were much the same. At first, this would seem to be alarming. It would seem that the consideration of uncertainty would change the suggested procurement policies. The reason that little difference existed between these two approaches was that they both used the same objective function. Thus, the linear programming

algorithm chose the same alternative sources. The only difference was the amount of the resource available.

The question might be asked: "Why were the expected mean values for the stochastic approach higher than the mean value for the deterministic approach?" The reason for this was that the variation created when considering uncertainty could be sufficiently large so as to result in a non-feasible solution. Two more sources were added to prevent this. One of these sources, soft hardwood from beyond zone 5 (shipped by rail), substitutes for pine roundwood from zone 5 and chips. This substitution was not totally unexpected. Thompson, et al. (In press) recommend that soft hardwood be procured from beyond zone 5 based upon the deterministic results.

The Kolmogorov-Smirnov (K-S) test was applied to run 1 to determine which distribution it fit best. The results showed that this distribution (Figure 1) could be described as normal (see Appendix 5 for the details and results of the test). However, the distribution in Figure 1 was bimodal, and if this was a normal distribution as the K-S test suggests, then 25 solutions was not enough to clearly reflect the central tendency expected in a normal distribution.

V. DISCUSSION

This study emphasized the merit of including the consideration of uncertainty in wood procurement decision making. A previously solved deterministic linear program, which minimized wood procurement cost was reformulated and solved as a partial stochastic linear program in order to illustrate the inclusion of uncertainty (the details on how this program was implemented are in Appendix 2).

The procurement policies suggested by the results of both the deterministic and stochastic approaches were much the same. The question inevitably arises regarding the value of constructing a stochastic model, if it yields solutions which are analogous to the ones suggested by a simpler deterministic model. The answer is that the value of a stochastic model lies in the added insight into the decision making process.

A primary value of the stochastic approach is the increased ability to clarify the complex decision making process and, therefore, give the manager a better understanding of the problem. The "optimality analysis" approach in decision making requires the manager to take all alternatives into account and determine which comes closest to meeting his objective. It would seem, intuitively, that an obvious optimal solution to each problem exists (Baumol, 1965: 4). However, the maze of alternatives often leads to the undoing of traditional management. Bell (1967: 1) described the problem:

> "Without a method of systematically simplifying the process, and without an analytical procedure for dealing with selected factors, management often becomes overwhelmed by the complexities of a process, satisfied to 'muddle through' to use 'rules of thumb', or 'hunches'."

This traditional approach, although it considers a great number of factors, may use very few in the decision making process--and may use these poorly. The linear programming algorithm is a superior technique because it is designed to systematically simplify a number of alternatives to obtain an optimal answer. The main advantage of linear programming in a wood procurement system is not so much the answer it provides but the thought process required to set it up. To use linear programming to determine a wood procurement policy, the decision maker first must identify the relationships in his timbershed which underlie his available sources of supply. In stochastic linear programming, the decision maker must specify not only expected values, but also the variation in these values.

Another advantage of stochastic decision models is in establishing trade-offs between expected cost and variation. With deterministic models, this type comparison is not possible. To demonstrate this concept of trade-offs, a fifth run was made. The problem was reformulated to force 24,000 cords of chips into solution each sub-period. Chips are a relatively expensive source of supply, but are relatively less certain than some alternative sources. Except for the chip purchase requirement, run 5 was the same as run 3; no chip purchases were included in the solutions for run 3. The results of runs 3 and 5 were:

	Run 3	Run 5
Mean Expected Cost	$15,608,996.00	$16,481,222.50
Standard Deviation	$ 157,398.71	$ 150,200.69

Using the above information, the decision maker must decide whether to adopt the policy of requiring chip purchases (run 5). If the chip purchase policy is adopted, the mean expected cost will increase, but the standard deviation of the mean will decrease. In the example used, it is

quite obvious that the policy represented by run 5 should not be adopted. At two standard deviations above the lower mean and two standard deviations below the higher mean, the two distributions do not overlap. Therefore, the probability of reducing total cost by forcing chips into solution is extremely small. However, in a case where there was considerable overlap between distributions, the decision maker would have to decide between the lower mean expected cost of one policy and the lower variation of the alternative policy.

One of the major limitations of deterministic linear programming is that the user has no knowledge of how sensitive his model is to random variation. There is an agendum which will increment certain elements along a fixed range (parametric programming). Other than this agendum, the decision maker makes his decisions based upon a single solution vector representing only a particular resource situation.

The results of the stochastic approach provide several optimal solution vectors--each of which represents a different resource situation. The final answer to a stochastic linear programming problem is the expected mean of the several optimal solution vectors and the associated standard deviation and range of solutions. This approach allows the decision maker to judge the sensitivity of a solution vector in order to give him some degree of confidence with which the answers may be accepted. Using a stochastic approach the decision maker can judge either a single procurement policy against different resource situations (runs 1, 2, and 3) or he can test several alternative procurement policies against the same resource situations (runs 3, and 5).

Another ramification of this approach is that the decision maker can judge which variables are most affected by random variation (Appendix 4).

This permits the decision maker to analyze and change the variables so as to reduce the amount of uncertainty associated with his procurement policy.

If a partial stochastic linear programming model has more decision making utility than a deterministic model, then a complete, or more complete, stochastic model is even better, etc. Indeed, the process of model building is a never-ending one and management is sure to ask: how far should we go? In economic terms, the problem is one of determining the point where the opportunity cost of model building reaches zero.

There are two principles which may be suggested as rough guides in determining the extent to which model building may be carried out. The first principle is a direct application of the marginal cost-marginal revenue concept from the theory of the firm. This principle states that model building will continue as long as the expected marginal cost of implementing an improved model does not exceed the expected marginal gain to be derived from it. This principle is an acceptable principle except that a manager can seldom tell the marginal cost of implementing an improved model in advance (Morris, 1964: 94); nor, can the manager tell what additional gain will be derived from the new model. Morris (1964: 95) developed a mathematical function that would perform this marginal analysis considering the probability density functions of the present worth of the next opportunity discovered, and the cost of implementing an improved model.

The second principle stems directly from the Simon-March Hypothesis. This principle states that model building is to be continued only until a model is constructed which fulfills the requirements of management. This

approach requires a determination of the characteristics of an acceptable model in advance. The point at which a model would become acceptable is called the "aspiration level". The establishment of the "aspiration level" is the key to this principle. Ideally, the "aspiration level" ought to be set so that the marginal cost of implementing an improved model, "if the level is raised, is just equal to the marginal gain from the model thus obtained (Morris, 1964: 96)."

Implications for Future Study

The partial stochastic model is an advancement of any method that is presently published for wood procurement. However, it is by no means meant to be the ultimate in wood procurement model building. Rather, it is a step toward an ideal model. The ideal model is one whose optimal answer coincides with the real world situation. There always will be some gap or lag between the ideal and the actual situation; and because of this no model builder should feel that his model is completely satisfactory.

A logical step in wood procurement model building would be to develop the present model to include stochastic elements in both the objective function and the right hand side vectors. This approach would be limited to short-run planning because the technical coefficients are deterministic. In an expanded partial stochastic model, serious consideration should be given to either building the subtle relationships between price and supply availability into the model or constructing a utility function for the objective function. This utility function could represent or measure a trade-off between cost or return and the amount of variation associated with that cost or return.

A subsequent phase in the movement toward an ideal wood procurement model should allow for more flexible inclusion of uncertainty. This

phase should be a truly dynamic system and it should be a non-optimizing system. It will need to be a dynamic model so it can consider the relationship of time in changes in the supply situation. It should not be an "optimizing system as this approach requires a more sophisticated model than foresters are able to provide (Gould and O'regan, 1965: 3)." At the present the only problem-solving technique that has this dual capacity is simulation. Simulation allows tremendous flexibility. Its capabilities and flexibility are limited only by the imagination of the programmer.

Future studies should be undertaken to delineate the wood procurement system. Particular care should be taken to define the following parameters: lead times of suppliers, procurement period, procurement cycle, shortage cost, holding cost, and the replenishment rate. These future studies should include an attempt to define the probability density function of available supply. In this study the normal distribution was used because there was no empirical data to support any other. Empirical studies should also be undertaken to compare subjective estimates with empirical results.

VI. SUMMARY

The objectives of this study were twofold: primarily to consider uncertainty by introducing variation into a least-cost wood procurement problem that had previously been solved as a deterministic linear program; secondly, to evaluate and interpret any differences between the deterministic solution and the uncertainty solutions with respect to the implementation of wood procurement policies.

The first objective was accomplished by using three programs: two FORTRAN programs and the standard IBM Linear Programming System III. The first program was designed to treat uncertainty as risk and to generate right hand sides based on the normal distribution. The second FORTRAN program was designed to match the output of the first program to the input requirements of the IBM Linear Programming System III. The last program was a general linear program formulated for cost minimization. All programs were implemented with data from a case study conducted by Thompson, et al. (In press).

Decision makers will find the partial stochastic model of value for three reasons. First, it requires the decision maker to have a fuller understanding of his problem than did previous models. This understanding requires the decision maker to evaluate the process under consideration in greater detail. Secondly, the model more nearly approximates reality than does the deterministic approach. A vector of solutions can reflect the sensitivity of an expected mean; while a single-value (deterministic) model gives the decision maker no idea of how sensitive it is. Lastly, a partial stochastic approach permits the decision maker to develop trade-off relationships between uncertainty and expected cost.

Future studies should give serious attention to the shortcomings of this study. Namely: first, a failure to recognize those subtle dependent relationships between types of available supply and those relationships linking the time periods. This presents a question of the trade-off relationships between the various types of supply over each time period. Second, a failure of the model to recognize the effect of competition among certain types of supply in each zone. In economic terms this could be described as assuming that the cross elasticities between types of supplies are zero. Third, a skewed distribution might better represent the actual situation. The use of a skewed distribution would allow the model builder to consider a higher probability of certain events occurring on one side of the mean than on the other. This would imply that the supply of a certain type of wood would most likely be less than the expected mean or vice versa.

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Appendix 1. Variable Names and the Mean Value Used in the Right Hand Side Generating Program for the Case Study

Variable Name	Code 1/	Mean Value
Bare land available for purchase	kBLP	7,500 acres
Available land with timber, zone 1	kLTZ1	1,000 acres
Available land with timber, zone 2	kLTZ2	3,000 acres
Available land with timber, zone 3	kLTZ3	5,000 acres
Available land with timber, zone 4	kLTZ4	7,000 acres
Available land with timber, zone 5	kLTZ5	9,000 acres
Available pine, zone 1	kPZ1	3,000 cords
Available pine, zone 2	kPZ2	15,000 cords
Available pine, zone 3	kPZ3	12,500 cords
Available pine, zone 4	kPZ4	30,000 cords
Available pine, zone 5	kPZ5	17,500 cords
Available soft hardwood, zone 1	kHDZ1	1,250 cords
Available soft hardwood, zone 2	kHDZ2	7,000 cords
Available soft hardwood, zone 3	kHDZ3	11,000 cords
Available soft hardwood, zone 4	kHDZ4	25,000 cords
Available soft hardwood, zone 5	kHDZ5	25,000 cords
Available chips	kCHIPS	15,000 cord equivalents
Available slabs	kSLAB	30,000 cord equivalents
Available shavings	kSHAV	15,000 cord equivalents
Pine plant	kPLPN	6,000 acres

1/ Where k is sub-period 1, 2, . . . 10.

Appendix 2. Development of the Right Hand Side Generators for the Example and the Case Study

Uncertainty was considered by generating a number of right hand side vectors and then obtaining a linear programming solution for each random resource vector. This appendix will describe in detail how the generator was constructed using FORTRAN IV, the underlying assumptions, and how the variation, which reflected uncertainty, was developed.

Since the case study involved a 20-year planning period, the first assumption required was one regarding the affect of time upon uncertainty. A logarithmic approximation was used to relate time and uncertainty of the form.

$$y = \log (t) \qquad (2)$$

where:

y = uncertainty or variation which reflects uncertainty

t = time

The logarithmic function was used because it represents a functional relationship based upon the assumption that uncertainty increases with time but at a decreasing rate. The slope of the logarithmic function will never become negative because the slope will only decrease to a certain point and then become asymptotic to a line drawn parallel to the expected mean. The graphical relationship implied in equation (3) is expressed as the curved line in Figure 2; and the logarithmic approximation used to determine that trace of uncertainty (hereafter referred to as boundaries) is:

$$B = (\bar{x})\ (c)\ (\log(t))$$

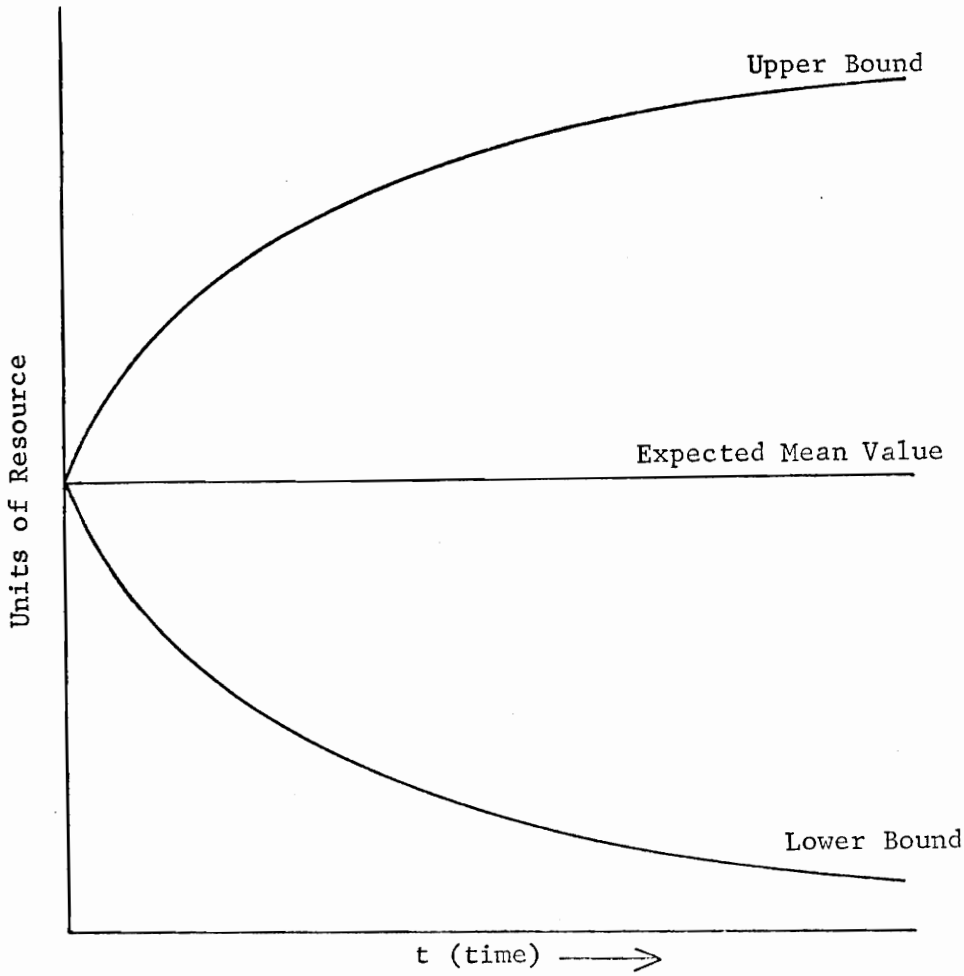

Figure 2. Assumed Relationship Between the Bounds and the Expected Mean Value of a Resource

where:

B = bound

$\bar{x}$ = mean

c = constant which reflects the manager's subjective evaluation of the uncertainty associated with the particular variable

t = time in years.

The other assumption was that the mean of each supplier can be given as:

$$\bar{x} = \frac{\text{upper bound} + \text{lower bound}}{2}$$

This equation implies nothing more than that the bounds are equi-distant from the mean. The actual mean values were the point estimates from the previously solved deterministic study. This assumption could be contrary to reality in that a case could be had where the bounds are not equi-distant from the mean. The assumption was adopted because of the ease of handling the problem with no *a priori* reason to do otherwise.

The theory that formed the basis of the program to generate right hand sides was that for each source of wood supply a normal distribution could be constructed with a mean equal to the deterministic point estimate and a standard deviation calculated from the bounds. The normal distribution constructed hopefully represents the decision maker's estimate of the uncertainty associated with each source of supply.[2/]

2/ This is not to say that the program is limited to the normal distribution.

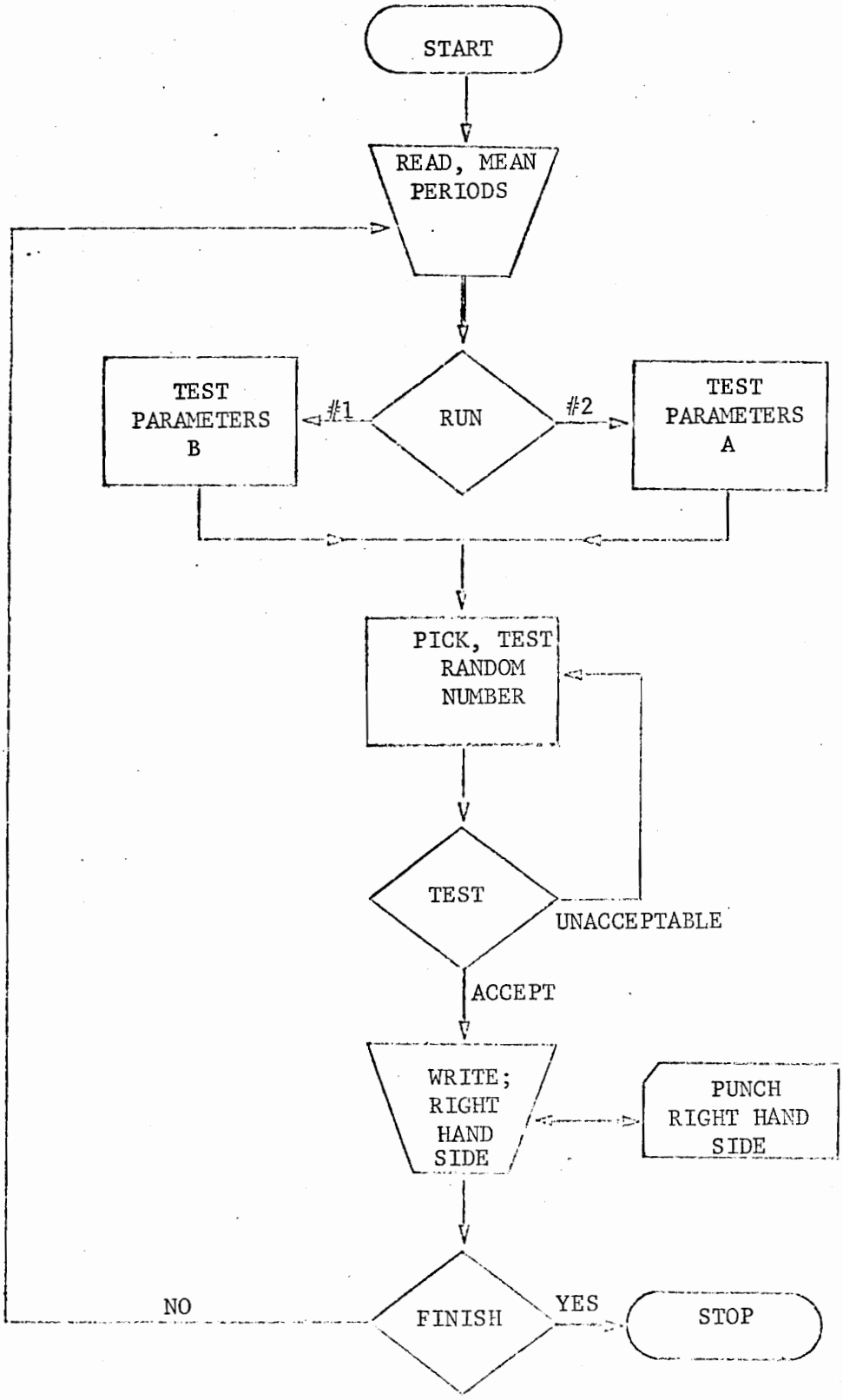

Figure 3. Flow Diagram of the FORTRAN Program Used in the Example Linear Program.

Once the right hand side vectors had been selected randomly they were incorporated with the rest of the linear program model and executed. Each right hand side yielded a distinct and optimal answer.

The actual programming techniques for generating right hand side vectors can best be explained by referring to the hypothetical example. Referring to the flow diagram (Figure 3) of the FORTRAN program, there are three major parts to the program: calculation of the bounds, calculation of the standard deviations, and calculation of the values for each right hand side.

The general bounds equation has already been explained in the preceding section; however, the way in which the program would differentiate between the amounts of uncertainty associated with different sources of supply has not been explained. It is possible to change the bounds as calculated by equation (3) by multiplying the log (time) by some constant (same as multiplying the mean). The normal bound equation (equation (3)) assumes a multiplier of one. To represent an increase or decrease in uncertainty, the bounds can be expanded by increasing or decreasing (c). For the example program the normal equation was used to calculate supplier A's bounds; while a multiplier of 1.5 was used to determine supplier B's bounds. This would imply that the manager was half again as uncertain of supplier B as of supplier A.

The standard deviations were calculated using the formula for the standard deviation of a uniform distribution discussed both in Naylor (1966: 78) and Hemmerle (1967: 32). The equation for this method will not be derived here but is derived in both Naylor and Hemmerle. The equation is:

$$s = \frac{\text{(upper bound - lower bound)}}{\sqrt{12.0}}$$

where:

s = standard deviation

upper bound = the manager's subjective evaluation of the upper limit which the variable could assume.

lower bound = the manager's subjective evaluation of the lower limit which the variable could assume.

This equation is based upon the uniform (rectangular) distribution with a mean equal to equation (4).

Random variation was introduced into the program by drawing random numbers from a random number generator and checking them against a range of plus or minus one standard deviation from the mean of each supplier. Then the random numbers were converted to the standard normal distribution using equation (4).

The single standard deviation represents an appropriate measure of the variation associated with each mean. The random number generator used was a functional sub-program (RNOR). This random number generator is based upon the standard normal distribution whose mean is zero and standard deviation is one. The equation used, in a general sense, to transform any variable to the standard normal distribution is:

$$z = \frac{y - m}{s_x}$$

where:

z = transfered variable

y = random variant

m = mean of population x

s_x = standard deviation of population x.

However, if m and s_x are known then y can be determined from RNOR:

$$y = s_x \text{ (RNOR)} + m \qquad (7)$$

The relationship expressed in equation (7) is used to convert the random number, the mean value of a supplier, and the standard deviation of the supplier to the standard normal population. At this point an immediate discrepancy is apparent (or should be). The standard deviations were calculated using equation (5) which is based upon the uniform (rectangular) distribution; while the random numbers were picked from a normal distribution which was truncated by the standard deviations. This may seem highly unorthodox. Upon closer inspection, however, the only effect of using non-homogeneous distributions is that the standard deviations are slightly over-estimated.

The output of the right side generating program for the example was punched cards in the required format for use in the RHSVEC section of the linear programming algorithm.

Appendix 2a. Program to Generate Right Hand Side Values for the Example Problem.

```
C       THIS PROGRAM PICKS RANDOM RIGHT HAND SIDES FOR
C       THE EXAMPLE LINEAR PROGRAM
        DIMENSION W(100),STDY(10),UPBND(10),BOBND(10),
1BND(10),U(100),VEC(10,100),SUBTR(10),PLUS(10)
        K=0
C       READ IN THE TIME PERIODS AND THE MEANS FOR EACH
C       SUPPLIER
        INTEGER H
        H=1
      5 READ(5,10) (W(I),I=1,4)
        READ(5,20) (U(M),M=1,4)
     10 FORMAT(4F4.2)
     20 FORMAT(4F6.0)
        K=K+1
C       CALCULATE THE BOUNDS FOR EACH SUPPLIER
        DO 35 J=1,4
        IF(K-2) 30,25,25
     25 BND(J)=U(J)*1.5*(ALOG10(W(J)))
        GO TO 35
     30 BND(J)=U(J)*(ALOG10(W(J)))
     35 CONTINUE
        DO 40 N=1,4
        UPBND(N)=U(N)+BND(N)
        BOBND(N)=U(N)-BND(N)
     40 CONTINUE
C       CALCULATE THE STANDARD DEVIATIONS
        DO 50 L=1,4
        STDY(L)=(UPBND(L)-BOBND(L))/(SQRT(12.0))
        WRITE(6,200) STDY(L)
    200 FORMAT(5X,'STDY=',F8.2)
     50 CONTINUE
        DO 60 N1=1,4
C       DETERMINE THE UPPER AND LOWER SIGMA LIMITS
        DO 60 L1=1,10
        PLUS(N1)=U(N1)+STDY(N1)
        SUBTR(N1)=U(N1)-STDY(N1)
C       PICK AND TEST THE RANDOM NUMBER
     55 Y=(RNDK(H)*STDY(N1))+U(N1)
        H=2
        IF(Y.GE.PLUS(N1).OR.Y.LE.SUBTR(N1)) GO TO 55
        VEC(N1,L1)=Y
     60 CONTINUE
C       THE REST OF THE PROGRAM WRITES AND PUNCHES OUT
C       THE RIGHT HAND SIDES
```

```
      DO 90 I1=1,10
      DO 90 I3=1,4
      I2=K
      WRITE(6,150) I1,I2,I3,VEC(I3,I1)
  150 FORMAT(6X,4HRHS ,I2,1HS,2I2,1X,F8.2)
   90 CONTINUE
      IF(K-2) 5,100,100
  100 STOP
      END
      FUNCTION RNDR(K)
      IF(K.GE.2) GO TO 52
      IX=123456789
   52 S=1.0
      AM=0.0
      A=0.0
      DO 51 I=1,12
      IY=IX*65539
      IF (IY) 2,3,3
    2 IY=IY+2147483647+1
    3 YFL=IY
      YFL=YFL*.4656613E-9
      IX=IY
   51 A=A+YFL
      RNDR=(A-6.0)*S+AM
      RETURN
      END
```

Appendix 2b. Program to Generate Right Hand Side Values for the Case Study.

```
      COMMON VEC,U,STDY
      DIMENSION W(10),BND(20,10),STDY(20,10),PLUS(20,10),
     1SUBT(20,10),VEC(25,25,25),U(20,10),R(25),MULT(20)
      REAL MULT
      INTEGER H
      H=1
   10 FORMAT(10F4.2)
   11 FORMAT(10F8.2)
   12 FORMAT(13F6.2/7F6.2)
   13 FORMAT(5X,10F10.2)
  151 FORMAT(6X,3HRHS ,I3,I2,4HPCAP,1X,5H2000.)
  153 FORMAT(6X,3HRHS ,I3,I2,4HMINP,2X,4H720.)
  154 FORMAT(6X,3HRHS ,I3,I2,4HMAXO,2X,4H360.)
  155 FORMAT(6X,3HRHS ,I3,I2,4HMAXS,2X,4H408.)
  156 FORMAT(6X,3HRHS ,I3,I2,4HMAXC,2X,4H480.)
  157 FORMAT(6X,3HRHS ,I3,I2,4HMXSH,2X,4H240.)
  158 FORMAT(6X,3HRHS ,I3,I2,4HMSCS,2X,4H648.)
  166 FORMAT(6X,3HRHS ,I3,1X,I2,3HPZ1,F8.2)
  167 FORMAT(6X,3HRHS ,I3,1X,I2,3HPZ2,F8.2)
  168 FORMAT(6X,3HRHS ,I3,1X,I2,3HPZ3,F8.2)
  169 FORMAT(6X,3HRHS ,I3,1X,I2,3HPZ4,F8.2)
  170 FORMAT(6X,3HRHS ,I3,1X,I2,3HPZ5,F8.2)
  171 FORMAT(6X,3HRHS ,I3,I2,4HHDZ1,F8.2)
  172 FORMAT(6X,3HRHS ,I3,I2,4HHDZ2,F8.2)
  173 FORMAT(6X,3HRHS ,I3,I2,4HHDZ3,F8.2)
  174 FORMAT(6X,3HRHS ,I3,I2,4HHDZ4,F8.2)
  175 FORMAT(6X,3HRHS ,I3,I2,4HHDZ5,F8.2)
  176 FORMAT(6X,3HRHS ,I3,I2,4HSLAB,F8.2)
  177 FORMAT(6X,3HRHS ,I3,I2,4HCHIP,F8.2)
  178 FORMAT(6X,3HRHS ,I3,I2,4HSHAV,F8.2)
  159 FORMAT(6X,3HRHS ,I3,1X,I2,3HBLP,F8.2)
  160 FORMAT(6X,3HRHS ,I3,I2,4HLTZ1,F8.2)
  161 FORMAT(6X,3HRHS ,I3,I2,4HLTZ2,F8.2)
  162 FORMAT(6X,3HRHS ,I3,I2,4HLTZ3,F8.2)
  163 FORMAT(6X,3HRHS ,I3,I2,4HLTZ4,F8.2)
  164 FORMAT(6X,3HRHS ,I3,I2,4HLTZ5,F8.2)
  165 FORMAT(6X,3HRHS ,I3,I2,4HPLPN,F8.2)
      READ(5,10) (W(I),I=1,10)
      READ(5,12) (MULT(J),J=1,20)
      DO 1 J=1,20
      READ(5,11) (U(J,M),M=1,10)
      WRITE(6,13) (U(J,M),M=1,10)
    1 CONTINUE
      DO 4 J=1,20
```

```
      DO 4 K=1,10
      BND(J,K)=U(J,K)*MULT(J)*(ALOG10(W(K)))
      STDY(J,K)=(2.*BND(J,K))/SQRT(12.0)
    4 CONTINUE
      DO 2 N=1,25
      DO 2 J=1,20
      DO 2 K=1,10
      PLUS(J,K)=U(J,K)+STDY(J,K)
      SUBT(J,K)=U(J,K)-STDY(J,K)
    3 R(N)=(RNGR(H)*STDY(J,K))+U(J,K)
      H=2
      IF(R(N).GE.PLUS(J,K)) GO TO 3
      IF(R(N).LE.SUBT(J,K)) GO TO 3
      IF(R(N).LE.0.) R(N)=.01
      VEC(N,J,K)=R(N)
    2 CONTINUE
      CALL TABLE
      DO 5 N=1,25
      DO 5 K=1,10
      WRITE(6,151) N,K
      WRITE(6,153) N,K
      WRITE(6,154) N,K
      WRITE(6,155) N,K
      WRITE(6,156) N,K
      WRITE(6,157) N,K
      WRITE(6,158) N,K
      WRITE(6,165) N,K,VEC(7,N,K)
      WRITE(6,166) N,K,VEC(8,N,K)
      WRITE(6,167) N,K,VEC(9,N,K)
      WRITE(6,168) N,K,VEC(10,N,K)
      WRITE(6,169) N,K,VEC(11,N,K)
      WRITE(6,171) N,K,VEC(13,N,K)
      WRITE(6,172) N,K,VEC(14,N,K)
      WRITE(6,173) N,K,VEC(15,N,K)
      WRITE(6,174) N,K,VEC(16,N,K)
      WRITE(6,175) N,K,VEC(17,N,K)
      WRITE(6,176) N,K,VEC(18,N,K)
      WRITE(6,177) N,K,VEC(19,N,K)
      WRITE(6,178) N,K,VEC(20,N,K)
      WRITE(6,159) N,K,VEC(1,N,K)
      WRITE(6,160) N,K,VEC(2,N,K)
      WRITE(6,161) N,K,VEC(3,N,K)
      WRITE(6,162) N,K,VEC(4,N,K)
      WRITE(6,163) N,K,VEC(5,N,K)
      WRITE(6,164) N,K,VEC(6,N,K)
    5 CONTINUE

      OUTPUT FORMS AVAILABLE OTHER THAN PRINTED

      WRITE(7) IS FOR CARD OUTPUT
```

```
WRITE(7,151) N,K
WRITE(7,153) N,K
WRITE(7,154) N,K
WRITE(7,155) N,K
WRITE(7,156) N,K
WRITE(7,157) N,K
WRITE(7,158) N,K
WRITE(7,165) N,K,VEC(7,N,K)
WRITE(7,166) N,K,VEC(8,N,K)
WRITE(7,167) N,K,VEC(9,N,K)
WRITE(7,168) N,K,VEC(10,N,K)
WRITE(7,169) N,K,VEC(11,N,K)
WRITE(7,170) N,K,VEC(12,N,K)
WRITE(7,171) N,K,VEC(13,N,K)
WRITE(7,172) N,K,VEC(14,N,K)
WRITE(7,173) N,K,VEC(15,N,K)
WRITE(7,174) N,K,VEC(16,N,K)
WRITE(7,175) N,K,VEC(17,N,K)
WRITE(7,176) N,K,VEC(18,N,K)
WRITE(7,177) N,K,VEC(19,N,K)
WRITE(7,178) N,K,VEC(20,N,K)
WRITE(7,159) N,K,VEC(1,N,K)
WRITE(7,160) N,K,VEC(2,N,K)
WRITE(7,161) N,K,VEC(3,N,K)
WRITE(7,162) N,K,VEC(4,N,K)
WRITE(7,163) N,K,VEC(5,N,K)
WRITE(7,164) N,K,VEC(6,N,K)

WRITE(1) IS FOR CARD IMAGES ON TAPE

WRITE(1,151) N,K
WRITE(1,153) N,K
WRITE(1,154) N,K
WRITE(1,155) N,K
WRITE(1,156) N,K
WRITE(1,157) N,K
WRITE(1,158) N,K
WRITE(1,165) N,K,VEC(7,N,K)
WRITE(1,166) N,K,VEC(8,N,K)
WRITE(1,167) N,K,VEC(9,N,K)
WRITE(1,168) N,K,VEC(10,N,K)
WRITE(6,170) N,K,VEC(12,N,K)
WRITE(1,170) N,K,VEC(12,N,K)
WRITE(1,171) N,K,VEC(13,N,K)
WRITE(1,172) N,K,VEC(14,N,K)
WRITE(1,173) N,K,VEC(15,N,K)
WRITE(1,174) N,K,VEC(16,N,K)
WRITE(1,175) N,K,VEC(17,N,K)
WRITE(1,176) N,K,VEC(18,N,K)
```

```
      WRITE(1,177) N,K,VEC(19,N,K)
      WRITE(1,178) N,K,VEC(20,N,K)
      WRITE(1,159) N,K,VEC(1,N,K)
      WRITE(1,160) N,K,VEC(2,N,K)
      WRITE(1,161) N,K,VEC(3,N,K)
      WRITE(1,162) N,K,VEC(4,N,K)
      WRITE(1,163) N,K,VEC(5,N,K)
      WRITE(1,164) N,K,VEC(6,N,K)
      WRITE(1,169) N,K,VEC(11,N,K)
      STOP
      END

      SUBROUTINE TABLE
      DIMENSION BSQ(20,10,25),SD(20,10),SBSQ(20,10),
     1XBAR(20,10),AA(20,10)
      COMMON VEC(25,25,25),U(20,10),STDY(20,10)
      DATA AA/200*0.0/,SBSQ/200*0.0/
      DO 32 J=1,20
      DO 32 K=1,10
      DO 30 N=1,25
      AA(J,K)=AA(J,K)+VEC(N,J,K)
   30 CONTINUE
      XBAR(J,K)=AA(J,K)/25.
      DO 31 N=1,25
      BSQ(J,K,N)=    (VEC(N,J,K)-XBAR(J,K))**2
      SBSQ(J,K)=SBSQ(J,K)+BSQ(J,K,N)
   31 CONTINUE
      SD(J,K)=SBSQ(J,K)/25.
   32 CONTINUE
      WRITE(6,40)
   40 FORMAT(1H1)
      WRITE(6,41)
   41 FORMAT(43X,'ALL TABLE VALUES WILL BE SCALED B,
     1'Y A FACTOR OF 100',/)
      WRITE(6,42)
   42 FORMAT(40X,'READ IN',4X,'CALCULATED',4X,'ACTUAL',4X,
     1'ACTUAL')
      WRITE(6,43)
   43 FORMAT(40X,'MEAN',7X,'DEVIATION',5X,'MEAN',6X,
     1'DEVIATION',/)
      DO 34 J=1,20
      WRITE(6,46) J
   46 FORMAT(50X,'SOURCE NUMBER',I2,/)
      DO 33 K=1,10
      WRITE(6,44) U(J,K),STDY(J,K),XBAR(J,K),SD(J,K)
   44 FORMAT(40X,F10.3,F10.3,4X,F10.3,F10.3)
   33 CONTINUE
      WRITE(6,45)
   45 FORMAT(1H0)
   34 CONTINUE
```

```
      RETURN
      END

      FUNCTION RNOR(K)
      IF(K.GE.2) GO TO 52
      IX=123456789
   52 S=1.0
      AM=0.0
      A=0.0
      DO 51 I=1,12
      IY=IX*65539
      IF (IY) 2,3,3
    2 IY=IY+2147483647+1
    3 YFL=IY
      YFL=YFL*.4656613E-9
      IX=IY
   51 A=A+YFL
      RNOR=(A-6.0)*S+AM
      RETURN
```

Appendix 2c. Program Used to Compile the Output of the Generating Program with the Linear Program System.

```
$FILE          'FTC01.',U01,*,BLOCK=15,REEL,HOLD,LRL=14,
$ETC           RCT=1,EOR=REORX,ERR=RERRX.,TYPE3,EOF=S.JXIT
$FILE          'FTC00.',U00,*,MOUNT,BLOCK=15,REEL,HOLD,
$ETC           LRL=14,RCT=1,EOR=REORX.,ERR=RERRX.,TYPE3,
$ETC           EOF=S.JXIT
C      THIS PROGRAM COMPILES A S.SIN TAPE FROM BOTH CARDS
C      AND CARD IMAGES ON TAPE FOR LP40 SYSTEM AND IS THE
C      SECOND PROGRAM IN THE SYSTEM OF THREE PROGRAMS
C      CARD IMAGES ON TAPE FOR LP40 SYSTEM AND IS THE SECOND
C      PROGRAM IN THE SYSTEM OF THREE PROGRAMS
       REWIND 0
       DIMENSION A(80)
       INTEGER A
       INTEGER Z,D
       DATA Z/1HZ/,D/1HS/
       REWIND 0
       REWIND 1
       CALL CHKPT(30)
       DO 1 I=1,15000
       READ(5,100) A
       IF(I.EQ.1) A(1)=D
       IF(I.EQ.2) A(1)=D
       IF(I.EQ.3) A(1)=D
       IF(I.EQ.4) A(1)=D
  100  FORMAT(80A1)
       IF(A(1).EQ.Z) GO TO 2
       WRITE(1,100) A
    1  CONTINUE
    2  DO 3 I=1,6750
       READ(0,100) A
       WRITE(1,100) A
    3  CONTINUE
       DO 4 I=1,1000
       READ(5,100) A
       IF(A(1).EQ.Z) GO TO 5
       WRITE(1,100) A
    4  CONTINUE
    5  WRITE(1,200)
  200  FORMAT('$IBSYS')
       STOP
       END
```

Appendix 3. The Multipliers (c)[1/] Used in the Four Stochastic Runs and the Assumptions They Were Based on for the Case Study Problem

The bounds used in the main study were based upon the cooperating firm's judgment. The bounds seemed reasonable because they satisfied the following a priori relationships:

1. The uncertainty associated with a given area's roundwood supply increases with time. That is, the deviation about the expected mean supply will be smaller next year than for ten years in the future.
2. Stumpage owned by the firm is the least uncertain form of raw material supply.
3. The supply of open-market roundwood is more uncertain than the supply of open-market chips, slabs, and shavings.
4. An unlimited supply of both pine and soft hardwood is available by rail shipments.

The multipliers used in the main study were estimated using information supplied by the cooperating firm, subjective evaluations, and the relationships listed above. These multipliers are in the following table.

Variable	Runs 1 and 2	Runs 3 and 4
BLP	1.10	1.10
LTZ1	.10	.50
LTZ2	.15	.55

1/ In the equation $B = (\bar{x})\ (c)\ (\log(t))$

Appendix 3. The Multipliers (c) 1/ Used in the Four Stochastic Runs and the Assumptions They Were Based on for the Case Study Problem. (Cont'd.)

Variable	Runs 1 and 2	Runs 3 and 4
LTZ3	.20	.50
LTZ4	.25	.55
LTZ5	.30	.50
PLPN	.10	.50
PZ1	.40	.80
PZ2	.60	.90
PZ3	.80	1.00
PZ4	1.00	1.10
PZ5	1.20	1.20
HDZ1	.40	.80
HDZ2	.60	.90
HDZ3	.80	1.00
HDZ4	1.00	1.10
HDZ5	1.20	1.20
SLAB	.20	.40
CHIP	.20	.40
SHAV	.20	.40

APPENDIX 4. SUMMARY OF DATA USED IN THE PARTIAL STOCHASTIC PROGRAM(RUN#1)

FIRM'S ESTIMATES OF MEAN*	STANDARD DEVIATION OF ESTIMATES	MEAN OF DATA	STANDARD DEVATION OF DATA
	BLP		
75.00	14.338	72.292	7.542
75.00	28.677	71.846	13.927
75.00	37.064	79.915	20.819
75.00	43.015	74.644	25.167
75.00	47.631	82.213	24.514
75.00	51.403	72.726	26.621
75.00	54.592	84.890	31.376
75.00	57.354	78.568	32.690
75.00	59.790	75.176	34.311
75.00	61.970	77.550	38.150
	LTZ1		
10.00	0.174	10.029	0.091
10.00	0.348	10.017	0.142
10.00	0.449	10.019	0.238
10.00	0.521	10.090	0.308
10.00	0.577	9.891	0.351
10.00	0.623	10.004	0.327
10.00	0.662	10.081	0.298
10.00	0.695	10.125	0.331
10.00	0.725	9.899	0.388
10.00	0.751	9.950	0.346
	LTZ2		
30.00	0.782	30.085	0.471
30.00	1.564	30.127	0.797
30.00	2.022	29.625	0.876
30.00	2.346	29.921	1.372
30.00	2.598	29.684	1.377
30.00	2.804	29.877	1.466
30.00	2.978	29.554	1.518
30.00	3.128	29.987	1.548
30.00	3.261	29.289	1.631
30.00	3.380	30.723	1.705

* ALL VALUES ARE SCALED BY A FACTOR OF 100

APPENDIX 4 (CONTINUED)

FIRM'S ESTIMATES OF MEAN	STANDARD DEVIATION OF ESTIMATES	MEAN OF DATA	STANDARD DEVATION OF DATA
	LTZ3		
50.00	1.738	49.761	1.022
50.00	3.476	50.221	1.682
50.00	4.493	49.759	2.306
50.00	5.214	49.463	2.602
50.00	5.773	50.117	3.573
50.00	6.231	49.553	3.667
50.00	6.617	50.421	3.644
50.00	6.952	50.037	3.461
50.00	7.247	50.432	3.691
50.00	7.511	51.135	3.935
	LTZ4		
70.00	3.041	70.028	1.492
70.00	6.083	69.237	3.054
70.00	7.862	70.374	4.676
70.00	9.124	70.670	4.534
70.00	10.104	69.602	5.949
70.00	10.904	71.753	5.202
70.00	11.580	71.662	6.304
70.00	12.166	68.879	6.450
70.00	12.683	68.070	6.282
70.00	13.145	70.982	7.475
	LTZ5		
90.00	4.693	90.707	2.758
90.00	9.385	89.863	5.269
90.00	12.130	92.629	7.594
90.00	14.073	90.300	8.256
90.00	15.588	84.999	7.499
90.00	16.823	92.575	8.674
90.00	17.866	91.140	8.428
90.00	18.770	86.948	10.018
90.00	19.560	89.757	10.599
90.00	20.281	91.067	11.140

APPENDIX 4 (CONTINUED)

FIRM'S ESTIMATES OF MEAN	STANDARD DEVIATION OF ESTIMATES	MEAN OF DATA	STANDARD DEVATION OF DATA
	PZ1		
30.00	2.086	29.927	1.108
30.00	4.171	29.996	2.317
30.00	5.391	29.566	2.951
30.00	6.257	30.711	3.697
30.00	6.928	29.354	3.673
30.00	7.477	29.080	3.610
30.00	7.941	28.804	3.892
30.00	8.342	29.371	5.420
30.00	8.697	29.321	4.046
30.00	9.014	27.854	4.536
	PZ2		
150.00	15.642	150.141	7.091
150.00	31.284	151.193	14.681
150.00	40.434	143.632	17.103
150.00	46.926	151.374	26.594
150.00	51.962	148.466	26.290
150.00	56.076	154.073	27.820
150.00	59.555	123.746	25.589
150.00	62.568	148.947	25.969
150.00	65.226	144.809	35.649
150.00	67.603	152.150	33.291
	PZ3		
125.00	46.247	123.217	23.594
125.00	54.987	120.231	23.664
125.00	60.021	113.634	27.569
125.00	63.627	124.730	35.489
125.00	66.425	137.355	51.492
125.00	68.711	126.248	42.206
125.00	70.643	130.188	48.906
125.00	72.317	133.971	35.903
125.00	73.794	117.946	46.001
125.00	75.115	135.271	38.752

APPENDIX 4 (CONTINUED)

FIRM'S ESTIMATES OF MEAN	STANDARD DEVIATION OF ESTIMATES	MEAN OF DATA	STANDARD DEVATION OF DATA
	PZ4		
300.00	138.742	299.656	75.723
300.00	164.812	298.103	93.519
300.00	180.062	314.338	98.170
300.00	190.882	295.688	114.441
300.00	199.275	260.821	109.022
300.00	206.132	305.847	96.531
300.00	211.930	348.777	124.263
300.00	216.952	276.557	120.369
300.00	221.382	293.931	117.762
300.00	225.345	340.479	138.209
	PZ5		
175.00	97.120	179.310	53.912
175.00	115.369	181.538	55.287
175.00	126.044	179.162	69.608
175.00	133.618	167.859	56.139
175.00	139.492	177.288	62.332
175.00	144.292	202.641	84.622
175.00	148.351	185.204	73.210
175.00	151.867	208.690	67.016
175.00	154.967	170.880	91.364
175.00	157.741	219.435	85.848
	HDZ1		
12.50	2.312	12.574	1.104
12.50	2.747	12.741	1.444
12.50	3.001	12.725	1.647
12.50	3.181	13.222	1.715
12.50	3.321	12.333	1.666
12.50	3.436	12.710	1.579
12.50	3.532	12.615	2.179
12.50	3.616	12.411	2.121
12.50	3.690	12.199	1.891
12.50	3.756	12.500	2.146

APPENDIX 4 (CONTINUED)

FIRM'S ESTIMATES OF MEAN	STANDARD DEVIATION OF ESTIMATES	MEAN OF DATA	STANDARD DEVATION OF DATA
	HDZ2		
70.00	19.424	71.442	7.182
70.00	23.074	70.251	11.790
70.00	25.209	64.801	13.394
70.00	26.724	71.076	12.598
70.00	27.893	70.162	12.016
70.00	28.058	70.364	14.951
70.00	29.670	69.734	14.254
70.00	30.373	71.421	20.228
70.00	30.993	70.469	16.890
70.00	31.548	64.227	16.817
	HDZ3		
110.00	40.693	107.803	22.555
110.00	48.345	102.588	24.261
110.00	52.818	106.464	28.145
110.00	55.992	104.599	29.992
110.00	58.454	115.532	34.720
110.00	60.465	114.230	30.912
110.00	62.166	107.169	36.578
110.00	63.639	112.323	37.644
110.00	64.939	123.576	31.542
110.00	66.101	123.454	31.405
	HDZ4		
250.00	40.693	252.881	62.224
250.00	48.345	241.506	71.486
250.00	52.818	260.353	78.422
250.00	55.992	265.542	94.504
250.00	58.454	211.617	78.562
250.00	60.465	227.378	80.454
250.00	62.166	252.459	87.514
250.00	63.639	249.810	72.574
250.00	64.939	233.242	107.333
250.00	66.101	242.065	102.800

APPENDIX 4 (CONTINUED)

FIRM'S ESTIMATES OF MEAN	STANDARD DEVIATION OF	MEAN OF DATA	STANDARD DEVATION OF DATA
	HDZ5		
250.00	138.742	267.862	83.928
250.00	164.812	249.517	93.609
250.00	180.062	253.115	88.540
250.00	190.882	266.139	109.692
250.00	199.275	262.303	109.577
250.00	206.132	226.701	108.771
250.00	211.930	264.686	120.811
250.00	216.952	256.835	100.459
250.00	221.382	259.461	128.900
250.00	225.345	290.393	133.235
	SLAB		
300.00	23.124	299.261	12.167
300.00	27.469	299.392	15.077
300.00	30.010	297.360	14.996
300.00	31.814	297.919	14.270
300.00	33.212	300.490	17.585
300.00	34.355	304.920	21.906
300.00	35.322	295.634	19.608
300.00	36.159	299.096	24.073
300.00	36.897	300.725	18.786
300.00	37.557	299.764	22.568
	CHIP		
150.00	13.874	152.419	6.900
150.00	16.481	154.169	8.789
150.00	18.006	145.372	9.225
150.00	19.080	150.850	10.758
150.00	19.927	147.975	10.013
150.00	20.613	144.394	10.364
150.00	21.193	151.841	11.824
150.00	21.695	140.471	12.725
150.00	22.138	150.551	13.991
150.00	22.534	154.462	13.150

APPENDIX 4 (CONTINUED)

FIRM'S ESTIMATES OF MEAN	STANDARD DEVIATION OF ESTIMATES	MEAN OF DATA	STANDARD DEVATION OF DATA
	SHAV		
120.00	11.099	122.012	6.370
120.00	13.185	120.339	7.406
120.00	14.405	121.909	9.089
120.00	15.271	119.001	8.744
120.00	15.942	119.695	8.455
120.00	16.491	117.963	8.041
120.00	16.954	119.569	8.542
120.00	17.356	119.136	8.640
120.00	17.711	119.325	10.519
120.00	18.028	117.847	10.320
	PLPN		
60.000	1.043	604.576	0.495
120.000	4.171	604.576	2.310
180.000	8.087	604.576	4.183
240.000	12.514	604.576	6.790
300.000	17.320	604.576	7.120
360.000	22.430	604.576	12.137
420.000	27.792	604.576	16.163
480.000	33.370	604.576	18.474
540.000	39.135	604.576	20.649
600.000	45.069	604.576	22.400

Appendix 5. The Kolmogorov-Smirnov Test for the Goodness of Fit of the Normal Distribution on the Results of the First Run, First Time Period.

x	Relative Frequency	Actual Frequency (S_x)	Theoretical Probability (F_x)	$F_x - S_x$
15,360	1	.0400	.03515	.0048
15,370	2	.1200	.06178	.0582
15,380	0	.1200	.10027	.0197
15,390	3	.2400	.15386	.0861
15,400	2	.3200	.22363	.0964
15,410	1	.3600	.30854	.0515
15,420	2	.4400	.40517	.0348
15,430	1	.4800	.51197	.0320
15,440	2	.5600	.61409	.0541
15,450	3	.6800	.70884	.0288
15,460	4	.8400	.79103	.0490
15,470	2	.9200	.85769	.0623
15,480	1	.9600	.90824	.0518
15,490	1	1.0000	.94520	.0548

$\bar{x}$ = 15,429 s = 38.2

DMAX $F(x) - S_x$ = .1052 N = 25

The critical value for a 95% level of significance is .27 (Ostle, 1963).

VITA

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Richard W Haynes

AN APPROACH TO CONSIDERING UNCERTAINTY IN DEVELOPING LONG-TERM, LEAST-COST WOOD PROCUREMENT POLICIES

by

Richard Walter Haynes

ABSTRACT

An approach was developed to consider the uncertainty which is intrinsic to forestry decisions. The approach was termed a partial stochastic linear program because uncertainty was considered by introducing variation into one element of the linear programming model (the right hand side). To implement this approach, subjective evaluations were made, regarding the amount of uncertainty associated with the values in question.

This approach was applied to a wood procurement problem which had been previously solved as deterministic. The previous problem was a case study of an integrated forest products firm with the objective of minimizing the present value of wood procurement over a 20-year study using linear programming. The management of this firm was required to make subjective estimates of the variation associated with each available source of supply. The orginal case study was then reformulated as a partial stochastic linear program.

The solutions of the partial stochastic approach were compared to the deterministic solution. This comparison showed the procurement policies suggested by both approaches were much the same. However, the stochastic approach differed in that management could obtain information about the sensitivity of a policy or a source and establish trade-off relationships between the cost of one policy and the uncertainty of

another policy. The questions of the extent of model building and the implications for future study in this area are also considered.