EVALUATION OF THE PERFORMANCE OF CUSTOMER SERVICE REPRESENTATIVES IN A CALL CENTER USING DEA/ NETWORK MODEL/ FUZZY SETS

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(ABSTRACT)

Data Envelopment Analysis (DEA) is a linear programming technique that has been used extensively in the literature to measure relative efficiency. One of the main attributes of DEA is that it can model multiple inputs and multiple outputs in the model. In this research work, attributes pertaining to service quality have been modeled using the Network model

The primary research is the augmentation of the existing Network model to include input/ output variables that are imprecise from a measurement point of view. These variables are qualitative assessments that have a linguistic representation/ interpretation. A very good example of this variable would be " Pleasantness". Given the fact that there are different evaluators, there is a certain degree of impreciseness associated with the representation of each of these qualitative variables. This imprecision is captured using the fuzzy sets. The triangular membership functions were used to describe the membership functions. So a unique network model that captured fuzzy variables was created.

The second main research contribution is that this is the first attempt of capturing service quality and efficiency of customer service representatives. The generic model that was created was used to evaluate the performance of the customer service representatives in a major airline. The results that were obtained, was shared with the decision makers at the airline for validation. The results that were obtained from the model also helped us validate the model with the other existing models.

One of the main advantages of using the DEA/ Network/ Fuzzy model was that the imprecision involved in measuring the customer service representatives were accounted for. This enabled the decision maker in making the right decisions and not penalizing a

customer service representative for imprecision in the data. Graphical Interpretations were also provided for the results that were obtained from the analysis.

To

My advisor Dr. Konstantinos P.Triantis

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I would like to thank my advisor Dr.Triantis, who has been my primary motivator in this whole process. I want to thank him wholeheartedly for backing me all the way to the very end. He has led me to think critically and has refined my thought process to a great extend.

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CHAPTER 1 - INTRODUCTION AND SCOPE OF THE RESEARCH

This chapter describes the research problem, motivation, research objectives, and methodology for this research study. In Chapter 2, the research literature pertinent to this research is documented. In Chapter 3, the unique methodology that has been developed, and the generic model is explained. In Chapter 4, the results obtained, when the generic model is applied to the specific research problem under consideration is explained. In Chapter 5, the conclusions and the areas for future research are explained.

1.1 INTRODUCTION

Numerous changes have occurred in the airline industry in the last few years mostly motivated by the entry of smaller regional airlines — a fact that in turn has increased the competition. The regional airlines have achieved spectacular growth due to their commitment of improving efficiency, and quality of service (Young, Cunningham and Lee, 1994). As a response to these pressures most of the big commercial airlines want to minimize the resources used as much as possible, while increasing their service level to attract more customers.

1.2 PROBLEM STATEMENT

Commercial airlines focus their individual sales strategy in multiple distribution channels. Two of the main channels are phone sales through customer representatives grouped to work in call centers strategically located throughout the country, and internet sales.

The number of customer service representatives (CSR) working at call centers is generally high due to the high volume of calls. Thus, one of their most important needs is to measure the efficiency of their large call center workforce. Hence, airline companies need to devise a valid mechanism to evaluate the performance of their CSRs.

At the present time CSR's performance is tracked over multiple quantitative and qualitative attributes. An example of a quantitative variable would be “Average talk time”. This is the average time that the CSR spend with the customer providing information on the ticket, etc. This variable would be modeled as an input variable as it is a resource that is consumed in the production process. An example of an output variable would be the number of paper tickets that were sold by the CSR. This would be modeled as an output variable as it is an output to the whole process. In the production process the amount of output needs to be increased.

Some of these quality attributes are of an imprecise, qualitative nature. The impreciseness is present in some of the input and the output variable because of the method by which it is measured. For example an input variable like pleasantness is measured based on the perception of the evaluator. So based on the evaluator the value of the variable might vary. So there is presence of impreciseness in the variables.

Moreover the qualitative factors like “pleasantness” cannot be combined in a straightforward manner with the quantitative indicators such as sales. Thus, call centers' management does not possess a single consolidated index that would be an indicator of their overall efficiency. Due to the lack of this measure they find it very difficult to track performance over time. It is also very difficult to compare the customer representatives with each other.

Hence, there is a lack of a formal methodology to rank/compare/evaluate customer service representatives. Hence the decision makers are unable to make valid recommendations for the improvement of the performance of the CSRs.

1.3 MOTIVATION

Service quality is a very important factor in determining customer loyalty and customer satisfaction. Kayanama and Black (2000) assert the importance of service quality in improving customer satisfaction and loyalty in business settings. Over the past few years, great importance has been given to the level of service associated with successful enterprises.

A perplexing issue confronting commercial airlines is to measure the quality of their service (Young, Cunningham and Lee, 1994) in a valid way. It is expected that by measuring their efficiency and service quality, airlines will be able to improve their quality of service.

This research will examine efficiency and service quality issues in the commercial airline industry and will attempt to devise a linear programming model to evaluate customer representatives' efficiency considering service quality indicators that are not always quantitative or crisp. A modification of Data Envelopment Analysis (DEA) (Charnes, Cooper, Rhodes, 1978) to address this problem will be attempted.

In the real world a lot of the inputs and outputs that are collected are not precise in nature. Fuzzy sets have been one of the methods used to model the uncertainty associated with the input and the output variables. The use of fuzzy techniques in DEA has been limited. The incorporation of imprecise or "fuzzy" attributes into efficiency measures has been attempted in the DEA literature and is an ongoing research field (e.g. Kabnurkar, 2001; Sheth, 1999; Sengupta, 1992, Triantis and Girod, 1998).

1.4 RESEARCH OBJECTIVES

This research effort has multiple objectives associated with it. The first objective is to come up with an overall efficiency evaluation, which also accounts for service quality using Data Envelopment Analysis (DEA) and the network model (Färe and Grosskopf, 1996).

- The second objective is to capture the uncertainty in measuring quality attributes for some input and output variables by fuzzy sets.
- The third objective is to implement the DEA/Network/fuzzy model to the data collected in a call center at a major airline carrier.

- The fourth objective of the research is to suggest improvements with respect to the allocation of input resources and to focus on the unique insights that the model provides.

- The final objective is to validate the model by comparing the fuzzy model with the non-fuzzy model. The Network model could also be compared with the Non-network model. The model will also be validated with the decision makers at the airline company.

1.5 RESEARCH CONTRIBUTIONS

- The primary research is the augmentation of the existing Network model to include input/ output variables that are imprecise from a measurement point of view. These variables are qualitative assessments that have a linguistic representation/ interpretation. Given the fact that there are different evaluators, there is a certain degree of impreciseness associated with the representation of each of these qualitative variables. This imprecision is captured using the fuzzy sets.
- The second main research contribution is that this is the first attempt of capturing service quality and efficiency of customer service representatives.

1.6 METHODOLOGY

This section briefly describes the three major fields on which this study will be grounded, namely Data Envelopment Analysis, particularly network models, service quality theory, and fuzzy set theory.

1.6.1 Service Quality

One of the major objectives of this research effort is to incorporate service quality into a performance measurement and evaluation model. Service quality is a perceived human judgment, where the customer compares their expectations with the services, which they have received. The scale SERVQUAL, developed by Parasuraman, Zeithaml and Berry (1988) is being used extensively by both academics and in the industrial world. The constructs in SERVQUAL are used as a guiding framework in this research to incorporate service quality. The input variables that pertain to the service quality attributes are directly mapped to the constructs defined in SERVQUAL.

1.6.2 Data Envelopment Analysis

Data Envelopment Analysis, DEA, was conceived by Charnes *et al* in 1978 and has evolved a lot over the years. Though DEA was created in 1978, many of its core concepts originated in the seminal paper by Farrell in 1957.

DEA is a linear programming technique that is used to measure the relative efficiency of production units. One of the most prominent features of DEA is its ability to handle multiple inputs and outputs. The variables that are used in DEA could be measured in different scales. The other advantage of DEA is that it itself assigns weights to the various inputs and outputs and does not need the decision maker to specify the weights. Since there are multiple inputs and outputs in the problem of evaluating call center's performance, DEA lends itself to the present research.

To incorporate service quality into the DEA formulation, service quality could be modeled in such a way that it is a sub process of the main production process. The network model that was introduced by Färe (1991) is used. One of the main features of the Network Model is that there are multiple nodes and the presence of intermediate outputs. Service quality can be represented as one of the nodes.

In the real world, most of the production processes consist of multiple sub processes where an output from one process is used as an input to another. This output, which is produced and consumed within the production process, is called an intermediate product. Färe (1991) merged the idea of efficiency measurement and graph theory, in which the intermediate inputs can be analyzed. The network model approach is flexible as multiple processes can be represented in the model and has been applied to many complicated production processes. Färe *et al.* (1995) integrated DEA and the network model in analyzing the efficiency of dairy farms to account for intermediate products. Lõthgren and Tambour (1999) applied the network model, to incorporate customer satisfaction as a sub process to the main production process. They proposed a model that estimates measures of efficiency and productivity considering both the qualitative and quantitative aspects. Thus by merging DEA and the network model, they could model multiple input, output variables and processes. A similar approach will be taken in the present study.

1.6.3 Fuzzy Set Theory

In the research problem pertinent to this study, there is the presence of certain quality attributes that are imprecise in nature. These quality attributes are imprecise due to the subjectivity present in human judgment. One of the methods to handle imprecision is to model uncertainty by applying chance constrained programming. Carlsson and Korhonen (1986) indicated that chance constrained programming models are often very complex in structure, and so are rarely used in practice. They are built on the assumption that some of the parameters used in describing the model are random variables, which are difficult to determine exactly. Girod (1996) and Triantis and Girod (1998) are of the opinion that sensitivity analysis, shadow prices, and parametric programming are not suitable for the overall analysis of imprecision in the parameters found in DEA models.

Sengupta (1992) feels that stochastic methods are not suited for DEA due to the following reasons:

- The assumption of any specific error distribution is not very realistic and the normal distribution is not appropriate here due to the nonnegativity restrictions on the input-output space.

- The sample sizes used in DEA are usually very small.

Another method to deal with uncertainty is to apply the theory of fuzzy sets, which proposes a conceptual and theoretical framework for handling complexity, imprecision and vagueness. The theory of fuzzy sets is a flexible and powerful approach to deal with the overall analysis of the effect of uncertainty. Therefore, fuzzy sets can be used to model the fuzzy quality attributes in a DEA framework.

Sengupta (1992) explored the use of fuzzy-set theoretic measures in the context of DEA. The objective of the paper was to explore the use of fuzzy measures and mathematical programs in the DEA models, where the data set provided vague and imprecise knowledge. The DEA model that he proposed has a fuzzy objective function and fuzzy constraints. He treats the input and output co-efficients as crisp. In this research it is proposed that certain input and output co-efficient can be treated as fuzzy.

The fuzzy sets in this research problem were described using triangular membership functions. In this research work a base was created from which the modeler could convert linguistic words to mathematical representations. For example pleasantness could be measured over three options "pleasant", "unpleasant" and "most pleasant". A customer service representative was measured over these three factors and then using the fuzzy rules, an unpleasantness index was created. A similar methodology was used to measure the "satisfaction" with the particular service.

One of the most important terms in this research is "the level of satisfaction". The "level of satisfaction" corresponds to the membership value. So by changing the "level of satisfaction", the efficiency variation of the CSR could be studied. This would enable the decision maker to identify the CSR that are not sensitive to the variation in the fuzzy variables.

Zimmerman (1996) suggested a lot of modifications to the classic linear program so that it could be used in a fuzzy environment. Further, he has classified his fuzzy Linear Programming (LP) into the following two types.

- Symmetric Fuzzy LP where the objective function and the constraints are fuzzy.
- Unsymmetrical Fuzzy LP where the constraints are fuzzy but the objective function is crisp.

In this research we use the Unsymmetrical Fuzzy LP, integrated with the network model.

1.7 RESEARCH SITE

In order to demonstrate the applicability of this research, data collected from a call center at a major airline will be used. The number of customer service representatives in the call center is fifty. Multiple evaluators were used to measure the quality variables. The quantitative factors were collected from an internal database, which is maintained by the airline company.

1.8 RESULTS AND CONCLUSIONS

The primary result from this research work is that a generic Fuzzy / DEA/ Network model was created that would take into account the service quality variables. This model also has the capability to include linguistic variables. The model also accounts for the impreciseness present in the input/output variables, because of the way it is measured.

This model was then utilized to identify the inefficient CSRs were identified and improvements were suggested. The peer members for the CSRs were also identified. One of the primary advantages of identifying the peer members is that the inefficient CSRs can be grouped with their more efficient CSRs. The grouping the CSRs in like groups will aid the company in training them more effectively.

The CSRs that were most affected by the fuzzy variables were also identified. This will aid the decision maker in making the right decisions when evaluating the performance of the CSR. The change in the efficiency score when the membership function of the fuzzy sets is varied can be viewed graphically in the graphs to help the decision maker make better decisions. In addition, the helps in identifying the borderline DMUs. These are the DMUs that move from the inefficient set to the efficient set when the constraints are relaxed. The model that was created was them validated using the statistical tests. In the first scenario, the DEA/ Network Model is compared with the BCC model. It was statistically validated using the paired two-sample t-tests that there was a difference among the results. In the next case, the fuzzy model is compared with the crisp model. Using the paired two-sample t-tests, it was found that the two models varied statistically.

CHAPTER 2 LITERATURE REVIEW

This chapter deals with all the literature related to this research. It dwells on the various kinds of efficiency, and discusses the evolution of Data Envelopment Analysis (DEA). The attributes of service quality and the related work are touched upon. The uses of the Network model in various applications are described. Finally the basics of fuzzy sets and the related applicability of fuzzy sets in DEA techniques are described.

The Literature review has been divided into the following sections:

2.1 Definitions of efficiency
2.2 Introduction to Data Envelopment Analysis
2.3 Discussion on service quality
2.4 Evolution of the network model and its applications
2.5 Introduction to fuzzy sets and application of fuzzy techniques in DEA.

2.1 DEFINITION OF EFFICIENCY

The commonly used measure of efficiency is output divided by input.

$$Efficiency(E) = \frac{Output}{Input} \qquad (2.1)$$

For long time efficiency only captured labor productivity (Farrell, 1957). An example of this kind of measure would be "output per worker hour" or "output per worker employed". These measures are known as "partial productivity measures" as they capture only the labor aspect of the input. They fail to capture other measures. The problem with these measures is that an increase in output due to an increment in capital may be attributed to labor. So the measure of efficiency needs to capture multiple inputs and multiple outputs. Thus, Farrell (1957) proposed a measure for technical efficiency (TE) as

$$TechnicalEfficiency(TE) = \frac{AggregateOutputMeasure}{AggregateInputMeasure} \qquad (2.2)$$

To understand the concept of technical efficiency better, we need to understand the concept of a production function.

2.1.1 Production Function

According to Farrell (1957), there are two ways of coming up with the production function. The two kinds of production functions are the theoretical production function and the empirical production function.

a) Theoretical production function

In the theoretical production function, the outputs produced are represented as a function of the inputs. It is represented by a series of mathematical relationships describing the physics of how the inputs are transformed into outputs. Though this method may be ideal, it is very difficult to specify a theoretical function for a complicated process.

b) Empirical production function

In this method the efficiency of the firm is compared with the efficiency of the best performing firm. Farrell (1957) then estimates the efficient production function from the observations of the inputs and the outputs. Further, each firm can be represented on an

isoquant diagram so that the firms will represent a scatter diagram as seen in Figure 2.1. SS′ represents the isoquant production function.

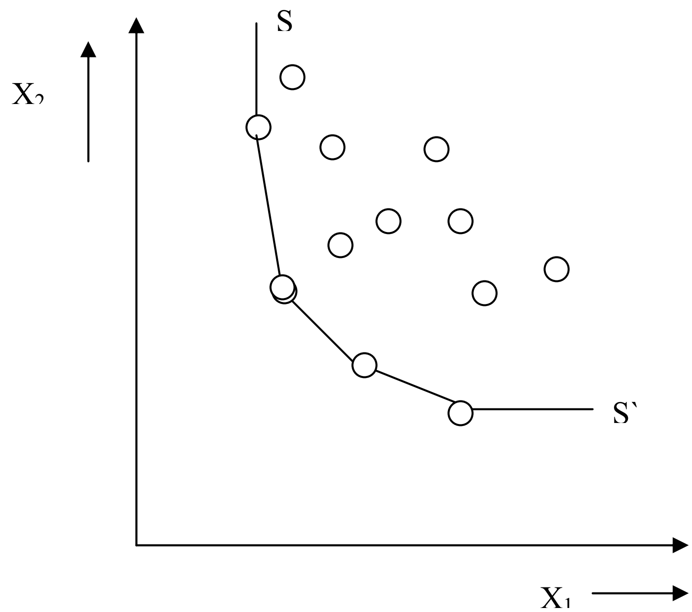


Figure 2.1 Piecewise Linear Isoquant

The isoquant is convex in origin, and what this assumption essentially means is that if two points are attainable in practice, then so is any point representing the weighted average of them. In this proposed research we will be using the empirical method to determine the efficiency of the firms.

2.1.2 Input Reducing and Output Increasing Measures

In the input reducing measure, the inputs are decreased while the outputs are held constant. The output increasing measure captures the proportion by which the output can be increased when the inputs are constant. The concept of technical efficiency with respect to the input decreasing and the output increasing measures can be explained as follows.

2.1.2.1 Input Reducing Technical efficiency

Farrell (1957) explains the concept of technical efficiency (TE) using Figure 2.2. For illustrative purposes, consider a firm employing two factors of production to produce a single product, under constant returns to scale. It is also assumed that the production function is SS` known.

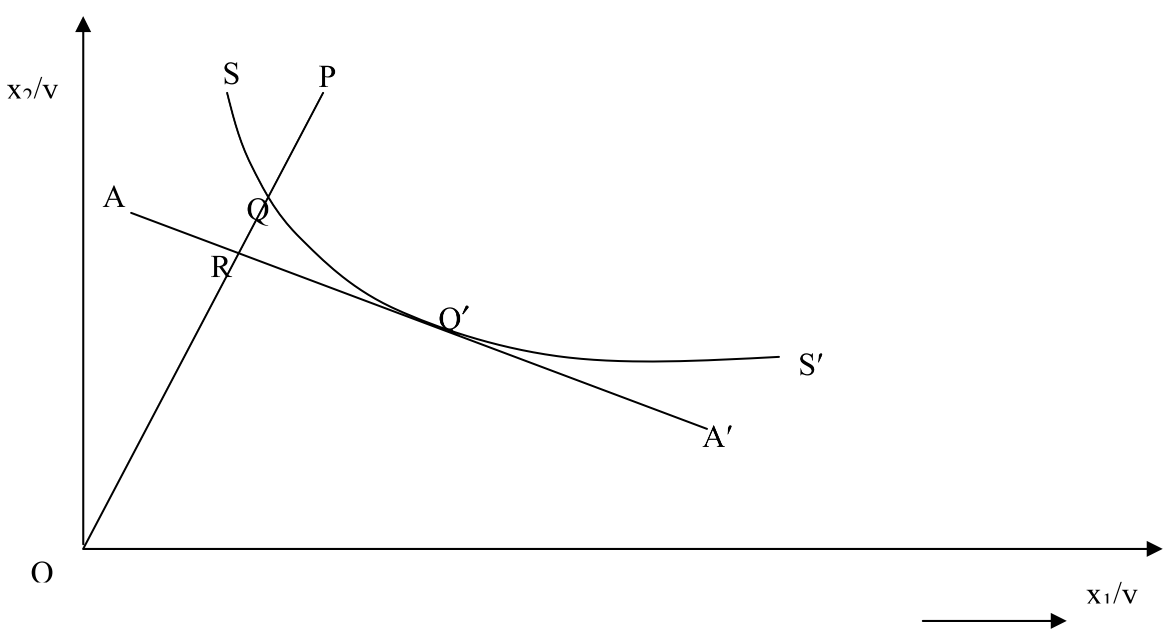


Figure 2.2 Input Orientation

Let the two inputs that the firm produces be x_1 and x_2. It produces a single output y. SS′ is an isoquant, which represents all the combinations of the two factors that an efficient firm might produce. In Figure 2.2, the point P represents the input of the two factors, per unit of output, that the firm is observed to use.

Point Q represents an efficient unit when compared to P. Q produces the same output as P using only a fraction OQ/OP as much of each factor. Thus, the fraction OQ/OP is defined as the technical efficiency of P. This ratio has the properties that a measure of efficiency requires. It takes a maximum value of 100% for an efficient firm, and will become indefinitely small for an inefficient firm. Hence,

$$TE_P = OQ/OP \quad (2.3)$$

AA′ represents the slope of the price of the two factors, whereas Q′ is the cost minimizing point of production and not Q. Comparing the two points Q and Q′, it can be seen that the cost at the point Q is a fraction of the cost at Q′. Thus, the allocative efficiency (AE) of the firm operating at P is defined to the ratio

$$AE_P = OR/OQ \quad (2.4)$$

In case of the firm being perfectly efficient, technically and price wise then the ratio OR/OP could be a representation of the overall economic efficiency (EE) of the firm. Overall efficiency is the product of the technical and the allocative efficiencies. Thus,

$$EE_P = OR/OP \quad (2.5)$$

Farrell accounted for both technical and price optimization in his paper.

2.2.1.2 Output Increasing Technical Efficiency

Let us consider a firm where the production involves two outputs $y_1$and y_2 and a single input x. The input is held fixed at a constant level, and we represent the production possibility curve in two dimensions. ZZ` is the production possibility curve and the point A corresponds to an inefficient firm.

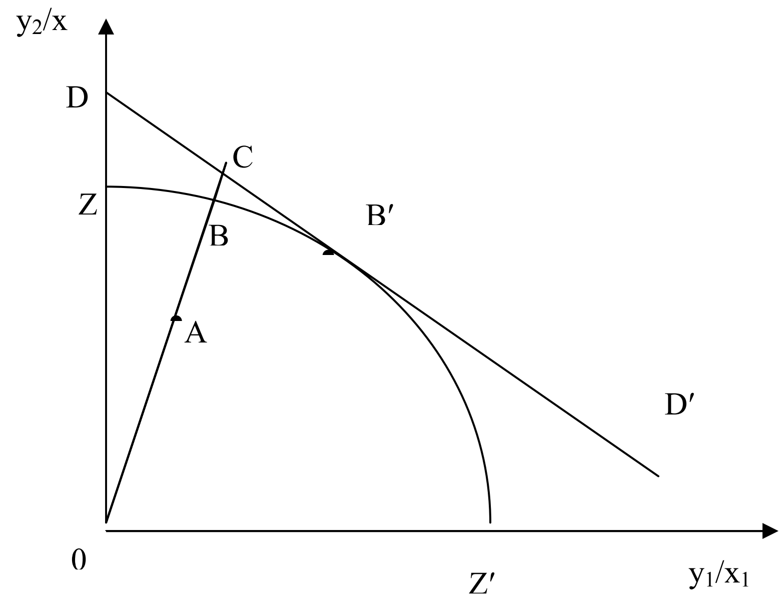


Figure 2.3 Output-Orientation

The firm B that lies on the curve is efficient when compared to A. The technical efficiency of the firm A is given by the ratio OA/OB.

$$TE_A = OA/OB \quad (2.6)$$

Using the price information we can draw the isorevenue line DD′ and the allocative efficiency of the firm A is given by the ratio OB/OC.

$$AE_A = OB/OC \quad (2.7)$$

The overall economic efficiency (EE) of the firm is the product of the allocative efficiency and the technical efficiency and hence is represented by OA/OC.

$$EE_A = OA/OC \quad (2.8)$$

One of the points regarding technical efficiency is that it is measured along a ray from the origin to the observed production point. Thus, these measures hold the relative proportions of the inputs and the outputs. One of the advantages of this measure is that it is unit invariant. This means that for example if a unit of measurement is changed from person hours to person years, it does not change the measure of efficiency.

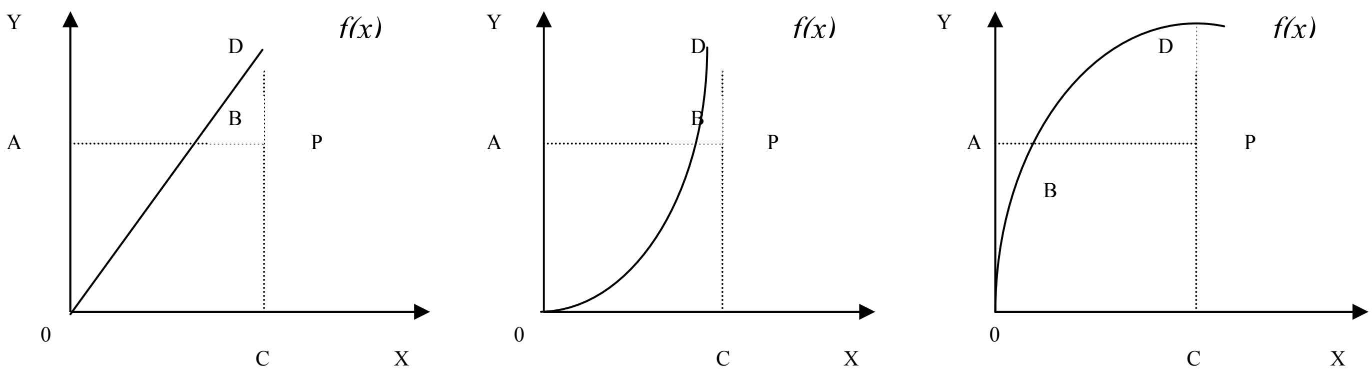
2.1.2.3 Returns to Scale

In the literature, there are two kinds of returns of scale

- Constant returns to scale
- Variable returns to scale

A firm exhibits constant returns to scale when a proportional increase in the inputs leads to a proportional increase in the outputs. A firm possesses variable returns to scale when variation (increase or decrease) in the inputs leads to a non-proportional change (increase or decrease) in the outputs.

Let us consider a firm that has a single input x, and a single output y. Consider the firm under three cases, the first having constant returns to scale (CRS), the second having increasing returns to scale (IRS) and the third having decreasing returns of scale (DRS) technology. The firm P is inefficient in all the cases as shown in Figure 2.4.



(a) CRS (b) IRS (c) DRS

Figure 2.4 Returns to Scale

The Farrell input oriented measure of TE would be equal to the ratio AB/AP while the output-oriented measure of TE would be represented by CP/CD.

In Figure 2.4 (a), the function $f(x)$ is a straight line and has a single slope. For every unit increase in the input or the output, the output would increase by the same proportion. The triangles OAB and DCO are similar.

By the law of similar triangles:

$$AB/OC = OA/CD \qquad (2.9)$$

$$AB/AP = CP/CD \tag{2.10}$$

So under the constant returns to scale, the input reducing measures, and the output reducing measures, have the same technical efficiency. In Figure 2.4 (b), the slope of $f(x)$ is increasing. So in this case, for every increase in the input there would be a more than proportional increase in the outputs. In the third case as seen in Figure 2.4 (c), $f(x)$ is represented by a firm that has a slope that decreases all the time. So for an increase in the inputs, there would be a less than proportional increase in the outputs. It is also clearly seen that the technical efficiency for the input and output measures for IRS and DRS is not the same.

It is seen that the distances AB or CD is not the same in the two cases. In case of the increasing returns to scale the input reducing efficiency is higher than the output increasing efficiency. In case of the decreasing returns to scale the output increasing efficiency is higher than the input reducing case.

2.2 INTRODUCTION TO DATA ENVELOPMENT ANALYSIS (DEA)

Data Envelopment Analysis (DEA) is a linear programming approach to productivity measurement. One of the most important characteristics of DEA is that it has the capacity to handle multiple inputs and multiple outputs. The production frontiers are constructed from production data. In DEA, the organization under study is called the Decision Making Unit (DMU). A DMU is the entity that is responsible for converting inputs into outputs and whose performance is going to be evaluated. An example of a DMU would be hospitals, schools, public libraries and so forth.

Then, the efficiency of each DMU is computed relative to the other DMUs in the sample space. One of other advantages of this technique is that the price data is not required for the construction of the production frontiers. Typically, prices have been used as weights in productivity analysis. Another aspect of DEA is that the inputs and the outputs of DEA need not have the same units and can have different scales. DEA could be employed to identify the inefficiencies in both the inputs as well as the outputs. There have been various models that have been proposed over the years and let us dwell on the basic models that are important to this research effort.

2.2.1 CCR Model

The first model proposed in the DEA research is that of Charnes, Cooper and Rhodes in 1978. The efficiency is calculated with respect to the other DMUs and so is called relative efficiency. The DMUs are those entities that are being measured and in most cases will have multiple inputs and outputs. The other important factor is that the model itself decides the weights for the inputs and the outputs.

In words, Charnes *et al* (1978) describe their proposed measure of efficiency of any DMU obtained as the maximum of a ratio of weighted outputs to weighted inputs subject to the condition that the similar ratios for every DMU be less than or equal to unity. The CCR model can be represented mathematically as follows:

$$Max\ h_0 = \frac{\sum_{r=1}^{S} u_r y_{rj}^0}{\sum_{i=1}^{m} v_i x_{ij}^0}$$

Subject to:

$$\frac{\sum_{r=1}^{S} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \quad j = 1, 2, \ldots, n \qquad (2.10)$$

where $u_r, v_i \geq \varepsilon$ for each r and i

r = 1,…, s outputs and i =1,…, m inputs

The superscript "o" refers to the DMU that is being evaluated. In the above equations, n represents the number of DMUs. The x_{ij} represents input i for DMU j and y_{rj} represents output r for DMU j. u_r and v_i are the variable weights to be determined by the solution of the problem. s represents the number of outputs, and m represents the number of inputs. ε is a very small number. The efficiency of one member of this reference set of j=1,.., n

DMUs is to be rated relative to the remaining DMUs including itself. All the x_{ij} and y_{rj} values will be observed and depending on the weights that are assigned, the DMUs will be deemed efficient or inefficient. Thus, by using this model we can assess the efficiency of the resource utilization and output production of the various DMUs.

The above model is a fractional formulation and has to be changed to a linear program. One of the problems with this solution is that there are infinite numbers of solutions associated with it. To resolve this problem the denominator is equated to a value of 1.

$$Max\ h_0 = \sum_{r=1}^{s} \mathrm{u}_\mathrm{r} y_{rj}^{\ 0}$$

Subject to:

$$\sum_{i=1}^{m} \mathrm{v}_\mathrm{i} x_{ij} \quad = 1$$

$$\sum_{r=1}^{s} \mathrm{u}_\mathrm{r} y_{rj} \ - \sum_{i=1}^{m} \mathrm{v}_\mathrm{i} x_{ij} \ \leq 0 \ \ j = 1, 2, 3, \ldots, n \qquad (2.11)$$

$$u_r \geq \varepsilon \quad r = 1, \ldots, s$$

$$v_i \geq \varepsilon \quad i = 1, \ldots, m$$

The number of variables that this model possesses is $m+ s$ and the number of constraints is $1+n+m+s$. The dual then would possess $1+n+m+s$ variables and $s+m$ constraints. Usually the number of DMUs will be usually higher than $m+s$. So, computationally the primal would be more difficult to solve than the dual. The dual form is also known as the envelopment form and can be represented by

$$Min\,\theta_0 - \varepsilon \sum_{r=1}^{s} \mathrm{s}_\mathrm{r}^{+} - \varepsilon \sum_{i=1}^{m} s_i^{-} \qquad \text{for DMU}_0$$

Subject to:

$$x_{ij0}\,\theta_0 = \sum_{i=1}^{n} \mathrm{x}_\mathrm{ij} \lambda_j + s_i^{-} \qquad i = 1, 2, 3, \ldots, m$$

$$\sum_{j=1}^{n} y_{rj}\lambda_j = y_{rj_0} + s_r^+ \qquad r = 1, 2, 3, \ldots, s \qquad (2.12)$$

$\lambda_j \geq 0$ $\qquad$ $j = 1, 2, 3, \ldots, n$

$s_r^+ \geq 0$ $\qquad$ $r = 1, 2, 3, \ldots, s$

$s_i^- \geq 0$ $\qquad$ $i = 1, 2, 3, \ldots, m$

θ_0 $\qquad$ unconstrained

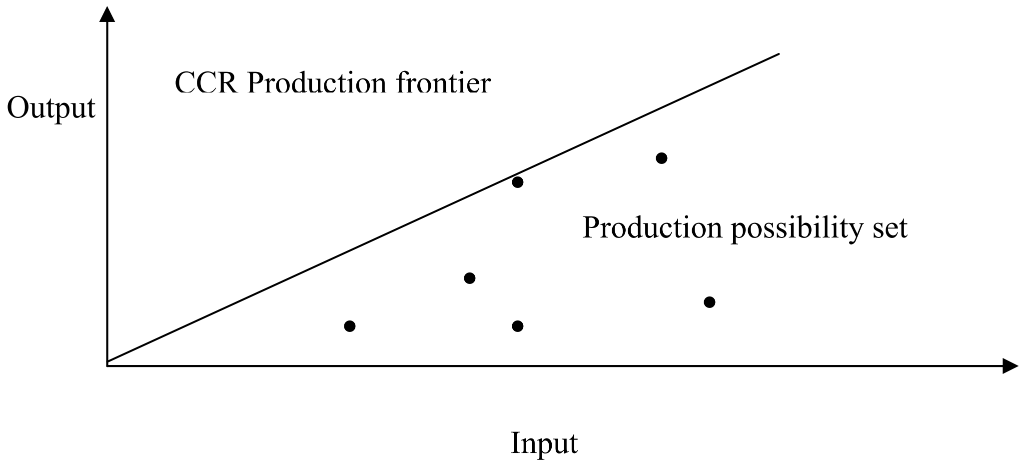
2.2.2 BCC Model

In the CCR model constant returns to scale are assumed. This is appropriate only when the firms under consideration are working under optimal conditions. Many factors like imperfect competition, constraints on finance, etc may cause the firm not to be operating at the optimal scale. In the CCR model the firms could be compared with larger or smaller firms. Banker, Charnes and Cooper (1984) then extended the CCR model.

They created models that enabled them to separate technical and scale efficiencies. They included a new constraint (2.13) that made it possible to determine whether the operations were conducted in the regions of increasing, constant or decreasing returns to scale. This constraint essentially ensures that an inefficient firm is only "benchmarked" against firm of a similar size.

$$\sum_{i=1}^{n} \lambda_j = 1 \qquad (2.13)$$

The difference between the BCC model and the CCR model is illustrated in Figure 2.5.



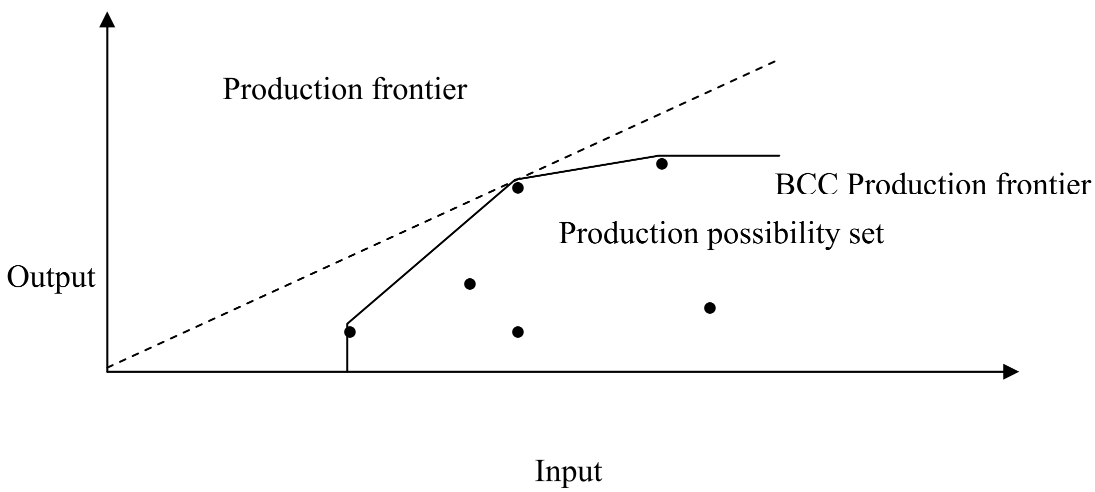


Figure 2.5 CCR and BCC Production Frontiers

The BCC approach forms a convex hull of intersecting planes that envelops the data points more tightly than the CRS conical hull, thus provides technical efficiency scores which are greater or equal to that provided by the CCR model.

2.3 DISCUSSION OF SERVICE QUALITY

The very intense competition in the service industries has led these firms to seek methods by which they could differentiate themselves and attract more customers. Service quality is one of the primary tools that is used to attract more customers.

In traditional industries, like manufacturing plants, the customer satisfaction may be dependent on the physical attributes of the product (e.g. defects). In the service industry customer satisfaction is dependent on the perception and expectation of the customer (Parasuraman, Zeithaml, and Berry 1985). In the last 20 years, sectors like banking, telecommunication and passenger airlines have been trying to attract customers through increased service quality.

Parasuraman *et al.* (1988) defined service quality as the degree of discrepancy between the customer's normative expectations for the service and their perceptions of the service performance. As service quality is perceived judgment, measuring service quality poses a lot of difficulty.

A lot of researchers have used the SERVQUAL construct to measure the service quality in various industries. For instance, Clifford *et al.* (1994) measured the service quality perceptions of the current and the potential customers. Kayanma and Black (2000) used the framework of SERVQUAL to capture the service quality of Online Travel Agencies.

2.3.1 SERVQUAL

This scale was developed by Parasuraman, Zeithaml and Berry (1988) and was then refined by them in the following years. SERVQUAL has a 22-item scale for measuring service quality along five dimensions including: reliability, responsiveness, assurance, empathy, and tangibles.

Reliability: (Ability to perform the promised service dependably and accurately)

1. Providing services as promised.
2. Dependability in handling customer's service problems.
3. Performing services right the first time.
4. Providing services at the promised time.
5. Maintaining error-free records.

Responsiveness: (Willingness to help customers and provide prompt service)

6. Keeping customers informed about when the services will be performed.

7. Prompt service to customers.
8. Willingness to help the customers.
9. Readiness to respond to customer's requests.

Assurance: (Knowledge and courtesy of employees and their ability to inspire trust and confidence)

10. Employees who instill confidence in others.
11. Making customers feel safe in their transactions.
12. Employees who are consistently courteous.
13. Employee who have the knowledge to answer customer questions.

Empathy:

14. Giving customer's individual attention.
15. Employees who deal with the customers in a caring fashion.
16. Having the customer's best interest at heart.
17. Employees who understand the needs of their customers.
18. Convenient business hours.

Tangibles: (Physical facilities, equipment and appearance of personnel)

19. Modern equipment
20. Visually appealing facilities.
21. Employees who have a neat, professional appearance.
22. Visually appealing materials associated with the service.

In this research, it is proposed that the constructs of service quality may be used as a guiding framework to incorporate service quality into the model. The constructs reliability, empathy, assurance and responsiveness were used in this research work. These constructs were used as a guiding tool to decide on the appropriate iinput and the output variables to model service quality.

2.4 EVOLUTION OF DEA/ NETWORK MODEL AND ITS APPLICATIONS

The network model is a technique that enables us to map a transformation process where multiple sub processes are associated with it. By merging DEA and network theory, it was possible to model multiple processes with multiple inputs and outputs. The model allowed the inclusion of intermediate products (outputs). The incorporation of multiple sub processes allowed the decision maker to mimic reality in a better way. The next four sub divisions illustrate the application of the Network model to various scenarios. Färe (1991) proposed the network model and modeled the intermediate output. Färe and Grosskopf (1996) then extended on this concept and came up with a model in which the inputs need not be allocated efficiently between the nodes. Färe and Whittaker (1995) also implemented the network model to model the production process of a dairy farm. Lõthgren and Tambour (1998) modeled productivity and customer satisfaction using the Network model. In the next sections, all of these papers are explained briefly touching on the most important points.

2.4.1 Measuring Farrell Efficiency for a firm with Intermediate Inputs – Färe (1991)

Farrell (1957) published the path breaking paper on efficiency measurement. One of the assumptions is that the technology towards which efficiency is measured does not allow for the evaluation of the impact of intermediate inputs. Färe (1991) introduced the concept of the intermediate inputs that could be incorporated into the model. Färe (1991) integrated the Network Model with DEA to provide with a framework that could account for multiple sub processes.

In Färe's model, the production process consists of three sub processes 1, 2 and 3 and can be seen in Figure 2.6. To these sub processes, a node for collecting exogenous inputs 0 and a node for collecting final outputs 4 was added. The network model is depicted in

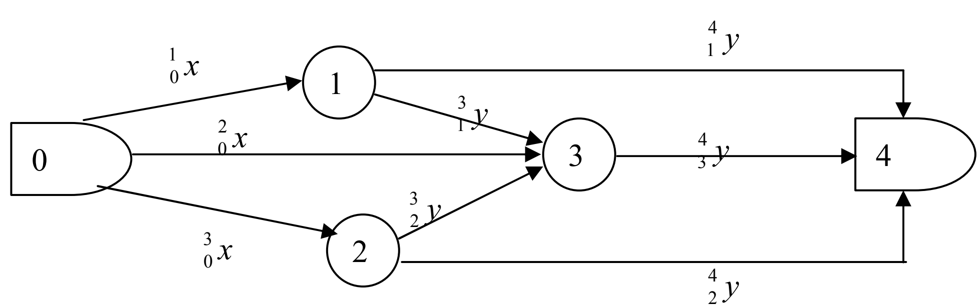


Figure 2.6 Färe's DEA/Network Model

The number of exogenous inputs are given by $({}^{1}_{0}x, {}^{2}_{0}x, {}^{3}_{0}x)$ where ${}^{i}_{0}x$, i = 1, …, 3 denotes the number of exogenous inputs that is used in the sub processes. The exogenous outputs are given by $({}^{4}_{1}y, {}^{4}_{2}y, {}^{4}_{3}y)$. The outputs ${}^{3}_{1}y$ and ${}^{4}_{1}y$ are the outputs from the processes 1 and 2 and they then become the intermediate inputs to sub process 3.

In Node 1 let us consider that there are k = 1, …, K^{1} observations of inputs ${}^{1}_{0}x^{k} = ({}^{1}_{0}x_{k1}, \ldots, {}^{1}_{0}x_{kn}, \ldots, {}^{1}_{0}x_{kN})$, where n=1,….,N denotes the number of inputs and suppose there are k= 1,…..K^{1} observations of outputs $({}^{3}_{1}y + {}^{4}_{1}y)^{k} = ({}^{3}_{1}y_{k1} + {}^{4}_{1}y_{k1}, \ldots, {}^{3}_{1}y_{km} + {}^{4}_{1}y_{km}, \ldots, {}^{3}_{1}y_{km^{1}} + {}^{4}_{1}y_{km^{1}})$ where M = 1,…, M^{1} denotes the number of outputs from sub process 1. Farrell (1957) had suggested two measures of efficiency, one input based and the other output based. Färe (1991) discussed the network model with reference to the output-based measure. The variables z_k, μ_k, and λ_k are the intensity variables.

Therefore the reference technology for node 1 can be written as

(a) $\sum_{k=1}^{k1} z_k ({}^{3}_{1}y_{km} + {}^{4}_{1}y_{km}) \geq {}^{3}_{1}y_m + {}^{4}_{1}y_m$, m = 1,….$M^{1}$,

(b) $\sum_{k=1}^{k1} z_k \, {}^{1}_{0}x_{kn} \leq {}^{1}_{0}x_n$, n = 1,…,N

(c) $z_k \geq 0$, k= 1,….,K^{1} (2.14)

The left-hand side of the constraint 2.14 a, represents the ideal hypothetical output and the right hand side of the constraint refer to the output of the specific DMU. This constraint means that the amount of output produced by the hypothetical DMU should always be greater or equal to that of the DMU that is being currently evaluated. Similarly the left hand side of the constraint 2.14 represents the hypothetical input and the right hand side of the constraint refers to the input that is being currently measured. This constraint means that the amount of input that is produced by the hypothetical DMU should always be lesser or equal to that of the input that is being measured.

It is assumed that the technology is exhibiting constant returns to scale. From Figure 2.6 it can be seen that the sub process 2 is the same as 1, and so we can start formulating sub process 3.

(d) $\sum_{k=1}^{k3} \lambda_k \, {}_3^4y_{km} \geq {}_3^4y_m$, m=1, …, M^3

(e) $\sum_{k=1}^{k3} \lambda_k \, {}_0^3x_{kn} \leq {}_0^3x_n$, n=1, ..., N

(f) $\sum_{k=1}^{k3} \lambda_k \, {}_1^3y_{km} \leq {}_1^3y_m$, m=1, …, M^1

(g) $\sum_{k=1}^{k3} \lambda_k \, {}_2^3y_{km} \leq {}_2^3y_m$, m =1, ..., M^2

(h) $\lambda_k \geq 0$, k= 1,…., K^3 (2.15)

The important equations are in 2.15 part (f) and (g), where the intermediate inputs are included. These inputs, which may be also considers as final outputs are produced in the production process and used (partly) considered as inputs

We can calculate the output increasing technical efficiency by solving each observation k'

θ : Output technical efficiency

$F_0(x^{k'}, {}_1^4y^{k'}, {}_2^4y^{k'}, {}_3^4y^{k'}) = Max\theta$

Subject to:

(a) $\sum_{k=1}^{k} \lambda_k \, {}_3^4y_{km} \geq \theta \, {}_3^4y_{k'm}$, m=1,…, M^3

(b) $\sum_{k=1}^{k} \lambda_k \, {}_0^3x_{kn} \leq {}_0^3x_n$, n=1,…., N

(c) $\sum_{k=1}^{k} \lambda_k \, {}_1^3y_{km} \leq {}_1^3y_m$, m=1,…, M^1

(d) $\sum_{k=1}^{k} \lambda_k \, {}_2^3y_{km} \leq {}_2^3y_m$, m =1,….., M^2

(e) $\lambda_k \geq 0$, k= 1,…., K

(f) $\sum_{k=1}^{k} \mu_k ({}^{3}_{2}y_{km} + {}^{4}_{2}y_{km}) \geq {}^{3}_{2}y_m + \theta {}^{4}_{2}y_{k'm}$, $m = 1,\ldots.M^2$

(g) $\sum_{k=1}^{k} \mu_k \, {}^{2}_{0}x_{kn} \leq {}^{2}_{0}x_n$, n=1,…., N

(h) $\mu_k \geq 0$, k= 1,…., K

(i) $\sum_{k=1}^{k} z_k ({}^{3}_{1}y_{km} + {}^{4}_{1}y_{km}) \geq {}^{3}_{1}y_m + \theta {}^{4}_{1}y_{k'm}$, m=1,….$M^1$,

(j) $\sum_{k=1}^{k} z_k \, {}^{1}_{0}x_{kn} \leq {}^{1}_{0}x_n$, n = 1,…,N

(k) $z_k \geq 0$, k= 1,…., K

(l) ${}^{1}_{0}x_n + {}^{2}_{0}x_n + {}^{3}_{0}x_n \leq x_{k'n}$, n= 1,…,N (2.16)

The value θ occurs in the constraints 2.16a, 2.16i, and 2.16f. They occur only with the final outputs $({}^{4}_{1}y, {}^{4}_{2}y, {}^{4}_{3}y)$ do not occur with the intermediate outputs.

2.4.2 Productivity and Intermediate Products: A Frontier Approach –Färe and Grosskopf (1996)

Färe and Grasskopf (1996) proposed some modifications to the model that Färe initially proposed in 1991. They introduced the frontier model with intermediate products that do not require that inputs be efficiently allocated among the sectors. In this paper, a network activity model that explicitly recognizes the concept that inputs could be produced and consumed within the production technology is recognized. The model consists of two production units that are interconnected into a network to form a production technology. Some of the outputs in the first node are used as input in the second node.

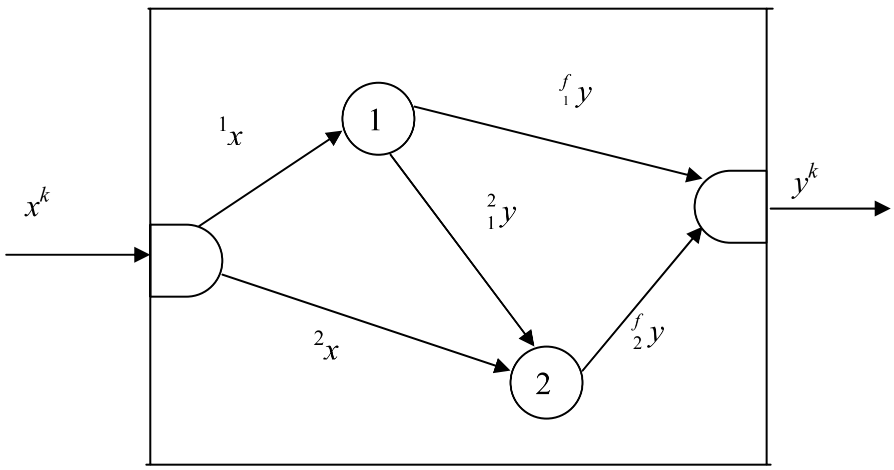


Figure 2.7 Intermediate Product Network Model

It is assumed that there are k=1, …, K observations of exogenous inputs $x^k \in R_+^N$ and final outputs $y^k \in R_+^M$.

In Figure 2.7, the exogenous inputs in the process *i*, *i*= 1, 2 by ^{i}x, is represented as

$$x^k \geq {}^{1}x + {}^{2}x \tag{2.17}$$

Similarly the total output is the sum of the output of node 1 and node 2. The final output is given by

$$y^k = {}^{f}_{1}y + {}^{f}_{2}y \tag{2.18}$$

The equations 2.17 and 2.18 are the unique constraints that were added in to the model. But in this model there is an additional output for node 1 used as an intermediate input for node 2, which are denoted by ${}^{2}_{1}y$. Thus the total output from node 1 is

$${}^{2}_{1}y + {}^{f}_{1}y \tag{2.19}$$

The formulation is similar to that of Färe (1991), which was described earlier in 2.4.1. The other main difference between the earlier formulations is that the network model has only two processes. The network model in 2.4.1 had one main process and two sub processes.

2.4.3 An Intermediate Input Model of Dairy Production Using Complex Survey Data – Färe and Whittaker (1995)

Färe *et al* (1995) modeled the crop and dairy production using the network model. He illustrated how efficiency can be measured when the process has been decomposed into constituent sub processes. In Figure 2.7, the triangle that is labeled 0 at the bottom of the diagram indicates a source of the inputs. The two diamonds 1 and 2 represents the two production sub processes and 3 represent the junction where the outputs are collected.

In Figure 2.8 ${}^{1}_{0}x$ represents the flow of the inputs from the source 0 to the production process 1. The intermediate production is represented by ${}^{2}_{1}y$, which is the output for the process 1 to the process 2. The model then is converted to a linear form similar to the formulation done by Färe (1991). One of the modifications in this model is that the inputs are modeled in such a way as to account for variable returns to scale.

The production process was decomposed into two sub processes; crop production and milk production. As explained earlier in the mathematical formulation there are three categories of inputs for this analysis, inputs that are used in the dairy production, inputs that are used in the crop production and inputs that are allocated between the two production processes. Some of the types of Inputs/Outputs are as follows:

Inputs (crops): (Land (acres), Fertilizer ($) etc.

Inputs (dairy): Purchase hay ($), Veterinary ($) etc.
Inputs (common to Crops and Dairy): Labor ($), Fuel ($), Equipment Lease ($) etc.
Final Outputs: Hay (tons), and milk (pounds).
Intermediate input: Hay (tons).

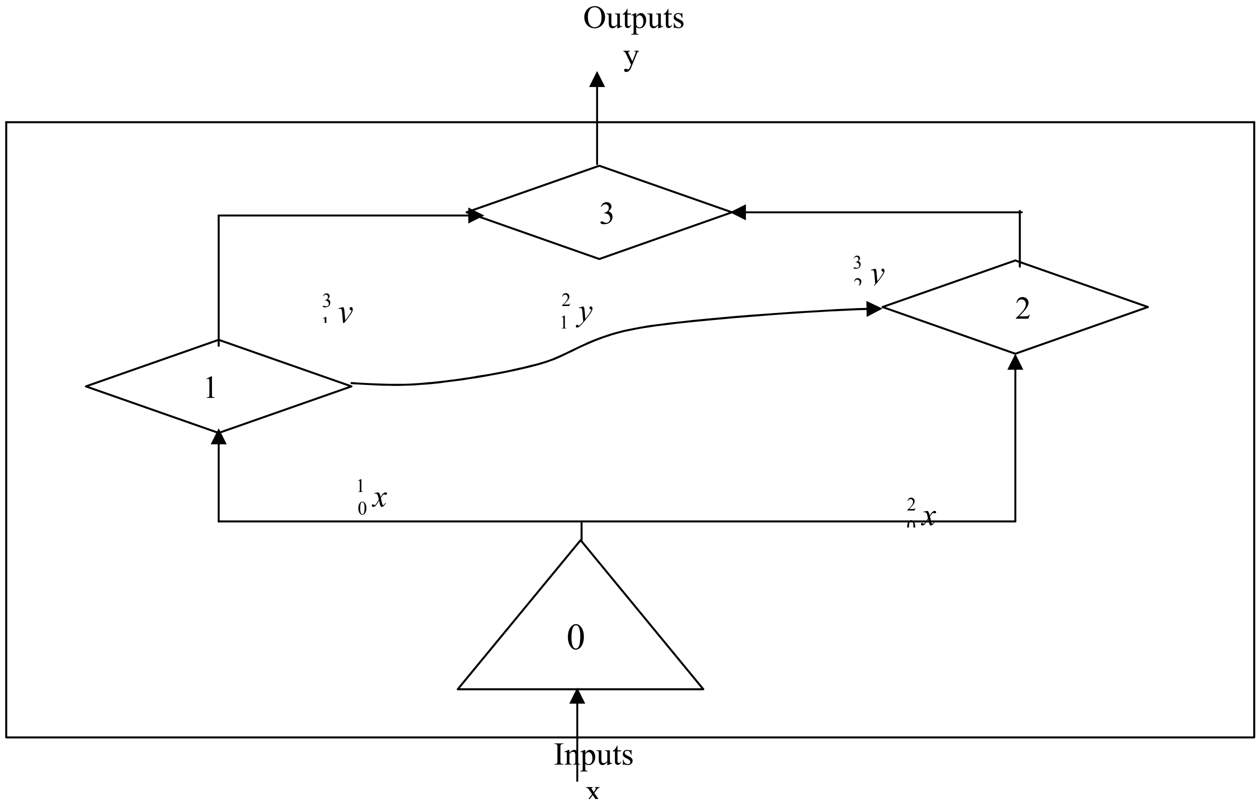


Figure 2.8 Network Model Depicting Dairy Production

The concept of the intermediate product can be explained using this concept. The hay that was manufactured by one sub process was part of the final output as well as it was internally used by dairy production. Thus hay was treated as an intermediate input.

The author is of the opinion that intermediate production method is a better representation of reality.

2.4.4 Productivity and Customer Satisfaction in Swedish Pharmacies: A DEA Network Model – Lõthgren and Tambour (1998)

In this paper the authors present a network model in which they have incorporated customer satisfaction into efficiency and productivity measures. As seen in Figure 2.9, there are two nodes. One represents the production node and another represents the consumption node. The network model facilitates the allocation of the resources between the customer oriented activities (x^c) and traditional production (x^p). In this model the (y^c), subvector of the outputs represents characteristic and quality attributes and is treated as an intermediate input in a consumption technology. The quality assessments (q) as well as the outputs (y^p) are considered as final exogenous production from the consumption and production node, respectively.

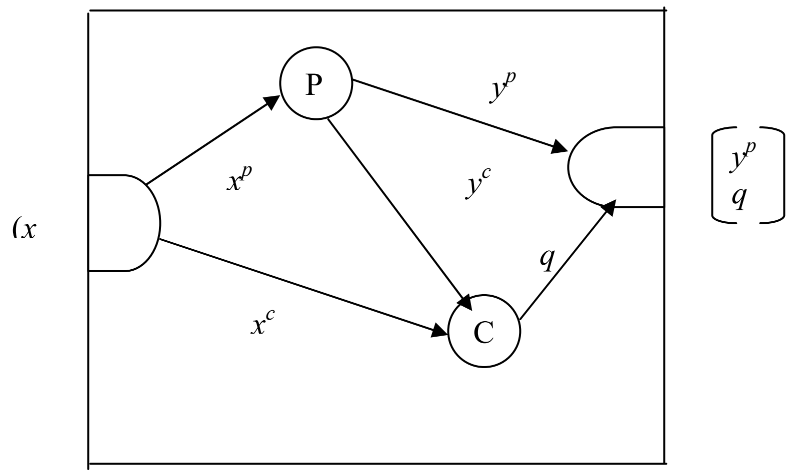


Figure 2.9 Modeling of Productivity and Customer Satisfaction using Network Model

The two inputs are given by x_1 = LABOR and x_2 = COSTO (the value of the other inputs). The data includes a decomposition of the total labor hours for each pharmacy in terms of hours that is devoted to production (x_1^p) and to customer activities (x_1^c) in the customer model. The second input dimension, $x = (x_2^p)$ = COSTO is entirely allocated to the P node.

Then by using DEA, the distance functions are estimated. This will help identify the optimal allocation of the production resources. Also the optimal level of the characteristics and attributes subvector (y^c) can be identified.

2.5 INTRODUCTION TO FUZZY SETS AND APPLICATION OF FUZZY TECHNIQUES IN DEA

From early times uncertainty was viewed, as undesirable in science and the general objective was to avoid it. Then there was the emergence of statistical methods and probability theory to model this uncertainty. In certain unique cases statistical methods may not be the best tool to use. This led to the emergence of Fuzzy sets. A very good example of the use of fuzzy sets would be the capturing the vagueness of the linguistic terms (Klir, Clair and Yuan, 1997).

2.5.1 Definition of a Fuzzy Set

Zadeh (1965) introduced the concept of the fuzzy set. It is a set, which has a boundary that is not precise or sharp. Each fuzzy set, A is defined in terms of a relevant universal set, X, by a membership function. This membership function assigns to each element *x* of *X* a number, A(x), in the closed interval [0, 1] that characterizes the degree of membership of *x* in *A*. Membership functions are thus functions of the form

$$A : X \to [0,1] \tag{2.20}$$

Contrary to classical crisp sets a fuzzy set does not have sharp boundaries. For example the concept "warm" includes a warm range of temperatures. We can represent WARM as a set with several readings, such as $65^0, 55^0, 95^0$ and 75^0. It could be definitely said that 95^0 definitely belongs to the set WARM. Now the temperature reading 65^0 would not appear to all of us to belong to the set. It is a borderline temperature reading. For some people it would be warm and for some people it is a bit chilly. So this observation belongs to the fuzzy set to a particular degree. In this case we define the fuzzy set by assigning to each temperature a number between 0 and 1, which indicate the degree or grade of membership in the set. The assignment of o to a particular temperature means that the temperature definitely does not belong to the set. The assignment of the value 1 means that the temperature does belong to this set.

The assignment of the grade of membership in the fuzzy set WARM to each considered temperature is called a membership grade function of this fuzzy set. An example of this function can be seen in Figure 2.10.

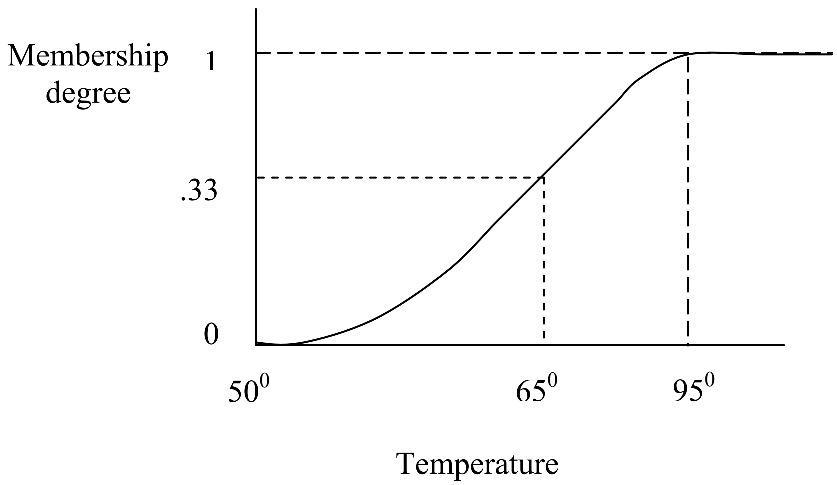


Figure 2.10 Possible Membership Function of Fuzzy Set WARM

In Figure 2.10, the temperature 95^0 has a membership value of 1, 50^0 has a membership value of 0 and 65^0 has a membership value of .33.

2.5.2 Types of Membership Functions

Each fuzzy set is represented by a unique membership function. The most commonly used membership functions are the triangular, trapezoidal and the bell shaped membership function.

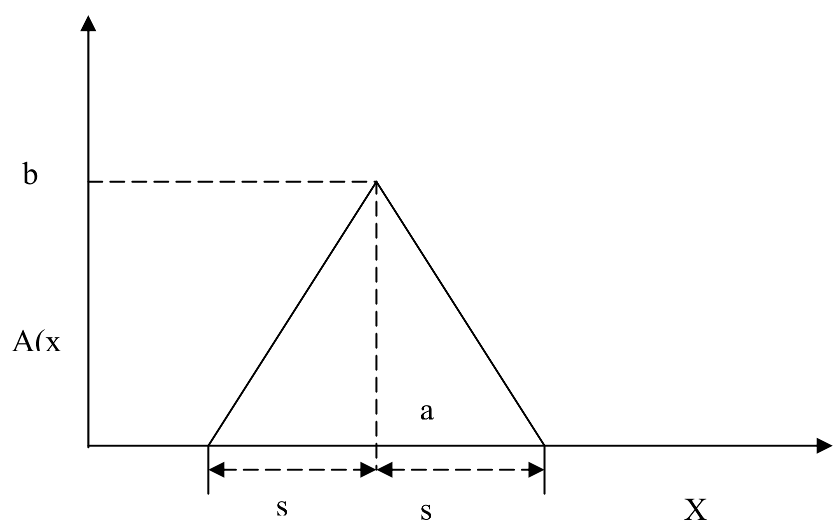


Figure 2.11 Triangular Membership Function

In Figure 2.10, A (x) represents the triangular membership function. This triangular membership function is characterized by the three parameters a, b, and s.

$$A(x) = \begin{cases} b(1 - \frac{|x-a|}{s}) & \text{when } a - s \le x \le a + s \\ 0 & \text{otherwise} \end{cases} \quad (2.21)$$

Here b is the maximum value that membership function can posses and s is the value by which the value 'a' can vary. So when the value of x is equal to a, then membership value is b. It can be seen from the membership function that it will have values between o and b as the membership function is varied.

The other important type of membership function is trapezoidal shaped seen in Figure 2.11.

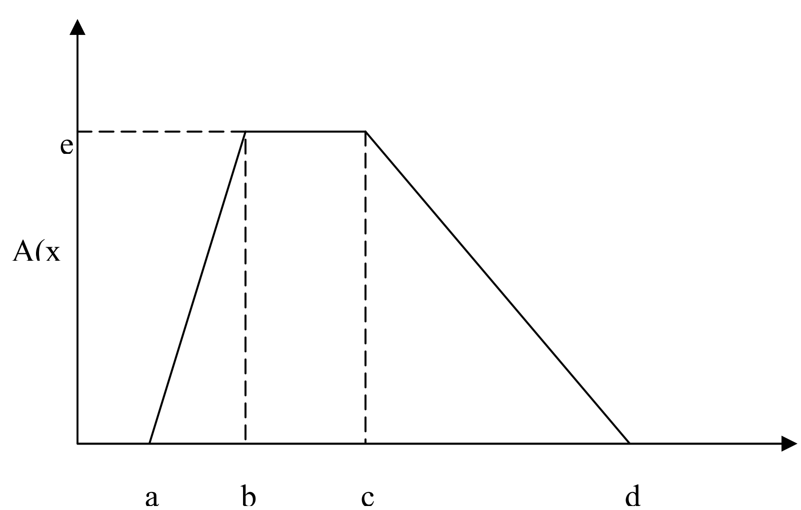


Figure 2.12 Trapezoidal Membership Function

This function is characterized by the five parameters a, b, c, d and e in the generic form.

$$A(x) = \begin{cases} \dfrac{(a-x)e}{(a-b)} & \text{when } a \le x \le b \\ e & \text{when } b \le x \le c \\ \dfrac{(d-x)e}{(d-c)} & \text{when } c \le x \le d \\ 0 & \text{otherwise} \end{cases} \quad (2.22)$$

Here e id the maximum value that the membership function A(x) can possess. The value of the membership function will vary between the values of o and e as the membership function is varied.

2.5.3 Properties of Fuzzy Sets

In this section some of the standard fuzzy operations are described.

2.5.3.1 Standard Fuzzy Complement

Given a membership a defined on a universal set X, its complement $\overline{A}$ is another fuzzy set on A that inverts, in some sense, the degrees of membership associated with A. While for each $x \in X$, $A(x)$ expresses the degree to which x belongs to a, $\overline{A}(x)$ expresses the degree to which then does not belong to A.

$$\overline{A}(x) = 1 - A(x) \quad (2.23)$$

2.5.3.2 Standard Fuzzy Union

Let us consider a universal set X and two fuzzy sets A and B defined on X. Then the standard fuzzy union of A and B, denoted by $A \cup B$, is defined by the membership functions via the formula

$$(AUB)(x) = \max[A(x), B(x)] \quad (2.24)$$

2.5.3.3 Standard Fuzzy Intersection

Consider two fuzzy sets A and B defined on X. The standard fuzzy intersection denoted by $A \cap B$, is defined by the membership functions via the formula

$$(A \cap B)(x) = \min[A(x), B(x)] \quad (2.25)$$

2.5.3.4 $\alpha-$cuts of Fuzzy Sets

One of the most important concepts is that of an α - cut and a strong α -cut. Given a fuzzy set A that is defined on X and any number $\alpha \in [0, 1]$, the α - cut ${}^{\alpha}A$ and a strong α -cut, ${}^{\alpha+1}A$, are represented by the crisp sets:

$$ {}^{\alpha}A = \{x / a(x) \geq \alpha\} \tag{2.26} $$

$$ {}^{\alpha+1}A = \{x / a(x) > \alpha\} \tag{2.27} $$

2.5.4 Fuzzy Decision Making

Let us assume that there is a fuzzy goal $\tilde{G}$ and a fuzzy constraint $\tilde{C}$ in a space of alternatives X. Then $\tilde{G}$ and $\tilde{C}$ combine to form a decision, $\tilde{D}$, which is the fuzzy set resulting from the intersection of $\tilde{G}$ and $\tilde{C}$. In symbols, $\tilde{D} = \tilde{G} \cap \tilde{C}$, and correspondingly,

$$ \mu_{\tilde{D}} = \min\{\mu_{\tilde{G}}, \mu_{\tilde{C}}\} $$

Let us assume that we have n goals $\tilde{G}_1,...,\tilde{G}_n$ and m constraints $\tilde{C}_1,...,\tilde{C}_m$. Then the resultant decision is the intersection of the given goals $\tilde{G}_1,...,\tilde{G}_n$ and the given constraints $\tilde{C}_1,...,\tilde{C}_m$.

$$ \tilde{D} = \tilde{G}_1 \cap \tilde{G}_2 \cap .... \cap \tilde{G}_n \cap \tilde{C}_1 \cap \tilde{C}_2 \cap .... \cap \tilde{C}_m $$

and correspondingly

$$ \begin{aligned} \mu_{\tilde{D}} &= \min\{\mu_{\tilde{G}1}, \mu_{\tilde{G}2},....., \mu_{\tilde{G}n}, \mu_{\tilde{C}1}, \mu_{\tilde{C}2},..., \mu_{\tilde{C}m}\} \\ &= \min\{\mu_{\tilde{G}i}, \mu_{\tilde{C}j}\} \\ &= \min\{\mu_i\} \end{aligned} \tag{2.28} $$

2.5.5 Fuzzy Linear Programming

In our proposed research we would be dealing with Fuzzy DEA and so we need to identify fuzzy linear programming techniques that were used in the literature.

Fuzzy Linear Programming can occur due to various reasons, which can be listed as follows

- The decision maker may not want to strictly maximize or minimize the objective function. The goal might be just to set aspiration levels for the objective function that might not be crisp in nature.
- The second case is when the constraints might be vague in various ways. The ≤ may not be necessarily strictly met. There may be cases in which slight variations of these constraints may be acceptable and these variations represent aspiration levels. In certain cases the coefficients in the constraints might possess a fuzzy character.

2.5.5.1 Symmetric Fuzzy *LP*

In a Symmetrical Fuzzy LP (Zimmerman, 1996), the objective function and the constraints are fuzzy. In this model we assume that the decision maker can establish an aspiration level, z, for the value of the objective function that he or she wants to achieve and the constraints are modeled as fuzzy sets. Our fuzzy LP set then becomes:

$$c^T x \, \tilde{\geq} \, z$$
$$Ax \, \tilde{\leq} \, b$$
$$X \, \tilde{\leq} \, 0 \qquad (2.29)$$

In the above equations $\tilde{\leq}$ represents the fuzzified version of ≤ and has the linguistic meaning "essentially smaller than or equal to." The sign $\tilde{\geq}$ represents the fuzzified version of≥ and has the linguistic expression "essentially greater than or equal to". The objective function is written as a minimizing goal in order to consider *z* as an upper bound.

We can now see that the model is fully symmetrical with respect to the objective function and the constraints and we can make it obvious by substituting $\begin{pmatrix} -c \\ A \end{pmatrix} = B$ and $\begin{pmatrix} -z \\ b \end{pmatrix} = d$.

The model then becomes

$$Bx \, \tilde{\leq} \, d$$
$$x \, \tilde{\geq} \, 0 \qquad (2.30)$$

Let us assume that the number of constraints present is m. The m +1 rows in the model will be represented by a fuzzy set, the membership of which are $\mu_i(x)$.

The membership function of the fuzzy set "decision" is given by

$$\mu_{\tilde{D}}(x) = \min_i \{\mu_i(x)\} \qquad (2.31)$$

It represents the degree to which x fulfills the fuzzy inequality $B_i x < d_i$ (where B_i represents the ith row of B).

Assuming that the decision maker is interested in a fuzzy set, but in a crisp "optimal" solution, the "maximizing solution" is suggested to the equation 2.31.

$$\max_{x \geq 0} \min_i \{\mu_i(x)\} = \max \mu_{\tilde{D}}(x)$$

Now the membership function $\mu_i(x)$ needs to be specified.

$$\mu_i(x) = \begin{cases} 1 & \text{if } B_i x \leq d_i \\ \in [0,1] & \text{if } d_i < B_i x \leq d_i + p_i \quad i = 1,\ldots,m+1 \\ 0 & \text{if } B_i x > d_i + p_i \end{cases} \qquad (2.32)$$

The membership function $\mu_i(x)$ is equal to 0 if the constraints are strongly violated and 1 if they are very well satisfied. The membership function will be increasing from 0 to 1 and we assume that in this case that it is linearly increasing over a tolerance level p_i.

$$\mu_i(x) = \begin{cases} 1 & \text{if } B_i x \leq d_i \\ 1 - \dfrac{B_i x - d_i}{p_i} & \text{if } d_i < B_i x \leq d_i + p_i \quad i = 1, \ldots, m+1 \\ 0 & \text{if } B_i x > d_i + p_i \end{cases} \qquad (2.33)$$

The values of p_i are subjectively chosen constants of the constraints and the objective function.

The solution to the problem is given by

$$\max_{x \geq 0} \min_{i} \left(1 - \frac{B_i x - d_i}{\mu_i} \right) \tag{2.34}$$

Introducing one new variable λ we arrive at the equation.

Maximize λ

Such that

$$\lambda p_i + B_i x \leq d_i + p_i \quad i = 1, \ldots, m+1$$
$$x \geq 0 \tag{2.35}$$

The intuition behind the concept is that when λ is 1 then the constraint is not violated at all and is a precise constraint. When the value of λ is 0 then the constraint is a fuzzy constraint, as it is fully violated. The variable λ represents the degree to which the constraints are fulfilled and x_0 is the maximizing solution of the model.

2.5.5.2 Unsymmetrical Fuzzy LP

According to Zimmerman (1996), a model in which the objective function is crisp and the constraints are all partially or fully fuzzy is no longer symmetrical. The objective function defines the decision space in a fuzzy way, and the former induces an order of the decision alternatives. The main problem is to determine the extremum of a crisp function over a fuzzy domain.

Generally, an extremum (maximum or minimum) of a crisp function f over a given domain D is attained at a precise point x_0. If the function f happens to be the objective function of the decision model, possibly constrained by a set of other functions, then the point x_0 at which the function attains the optimum is called the optimal decision. There is a unique relationship between the extremum of the objective function and the notion of the optimal decision of a decision model.

When there is the presence of fuzziness, the concept changes. According to Bellman and Zadeh (1970, p. 150), the "optimal decision" is often considered to be the crisp set D_m, that contains those elements of the fuzzy set "decision" attaining the maximum degree of membership. Zadeh (1972) provides the following definition for the maximizing set:

Let f be a real valued-function in X. Let f be bounded from below by inf (f) and from above by sup(f). The fuzzy set $\widetilde{M} = \{(x, \mu_{\widetilde{M}}(x)\}, x \in X$

$$\mu_{\widetilde{M}}(x) = \frac{f(x) - \inf(x)}{\sup(f) - \inf(f)} \tag{2.36}$$

In the equation (2.33), sup stands for supremum (upper bound or maximum) and inf stands for infrenum (lower bound or minimum).

Werner (1984) provided a similar kind of definition. Let f: X $\longrightarrow$ R^1 be the objective function, $\widetilde{R}$ = feasible region, S ($\widetilde{R}$) = support of f and R_1=1-α cut of $\widetilde{R}$ for α=1. The membership function of the goal given solution space $\widetilde{R}$ is then defined as

$$G(x) = \begin{cases} 0 & \text{if } f(x) \le \sup\limits_{R_1} f \\ \dfrac{f(x) - \sup\limits_{R_1} f}{\sup\limits_{S(\widetilde{R})} f - \sup\limits_{R_1} f} & \text{if } \sup\limits_{R_1} f < \sup\limits_{S(\widetilde{R})} f \\ 1 & \text{if } \sup\limits_{S(\widetilde{R})} f \le f(x) \end{cases} \tag{2.37}$$

Maximize $f(x) = c^T x$

Such that

$Ax \,\widetilde{\le}\, b$

$Dx \le b'$

$x \,\widetilde{\ge}\, 0$ (2.38)

In the above formulation the first constraint is imprecise and the second constraint is a precise constraint. So compared to the earlier formulation the new constraint that has been added is $Dx \le b'$.

The membership functions of the fuzzy sets representing the fuzzy constraints be represented by

$$\mu_i(x) = \begin{cases} 1 & \text{if } A_i x \le b_i \\ \dfrac{b_i + p_i - A_i x}{p_i} & \text{if } b_i < A_i x < b_i + p_i \text{ , i = 1,…, m+1} \\ 0 & \text{if } A_i x > b_i + p_i \end{cases} \quad (2.39)$$

The membership function of the objective function is obtained by solving these two LPs

Maximize $f(x) = c^T x$ (2.40)

such that

$Ax < b + p$

$Dx \le b'$

$x \ge 0$

This yields $\sup_{R_1} f = (c^T x)_{opt} = f_1$

Maximize $f(x) = c^T x$

such that

$Ax < b$

$Dx \le b'$

$x \ge 0$ (2.41)

This yields $\sup_{S(\tilde{R})} f = (c^T x)_{opt} = f_0$

The membership function of the objective function is therefore

$$\mu_G(x) = \begin{cases} 1 & \text{if } f_0 < c^T x \\ \dfrac{c^T x - f_1}{f_0 - f_1} & \text{if } f_1 < c^T x < f_0 \\ 0 & \text{if } c^T x < f_1 \end{cases} \quad (2.42)$$

Now the symmetry has been achieved between the constraints and the objective function.

The equivalent model is given by

Maximize λ

Such that

$$\lambda(f_0 - f_1) - c^T x \leq -f_1$$

$$\lambda_p + A_x < b + p$$

$$Dx \leq b^{'}$$

$$\lambda \leq 1$$

$$\lambda, \text{x} \geq 0 \qquad (2.43)$$

λ is an indicator of the degree to which the constraints were fulfilled.

2.6 FUZZY DEA

Sengupta (1992) was the first person, who proposed a fuzzy DEA model. He fuzzified the objective function and the constraints of a CCR model. In 1998, Triantis and Girod came up with a model that would account for the imprecision in the input and the output values. Sheth (1999) provided a fuzzy version of Athanasopoolos (1995) GoDEA model and Kabnurkar (2001) modeled the uncertainty related to the weighted bounds.

2.6.1 A Mathematically Programming Approach for Measuring Technical Efficiency in a Fuzzy Environment - Triantis and Girod (1998)

This paper introduced an approach, where the technical efficiency can be measured in a fuzzy environment. In a lot of situations the input consumption and the output production data cannot be accurately collected. The authors view the inputs and the outputs to vary between two bounds, the risk free and the impossible bounds. The risk free bound can be interpreted as one representing a production function that is realistically implementable. The impossible bound is representing a scenario where the production scenario is non implementable. A decision maker who is truly knowledgeable about the production technology should be able to make this decision. Once the bounds are decided they can be incorporated into a membership function. The membership function represents the degree to which the production scenario is realistic or possible.

In this paper three assumptions:

- Input membership functions are monotonically linearly increasing;
- Output membership functions are monotonically linearly decreasing; and
- Both the functions will be equal to zero, if the input or output bound is impossible, and will be equal to one if it is risk-free.

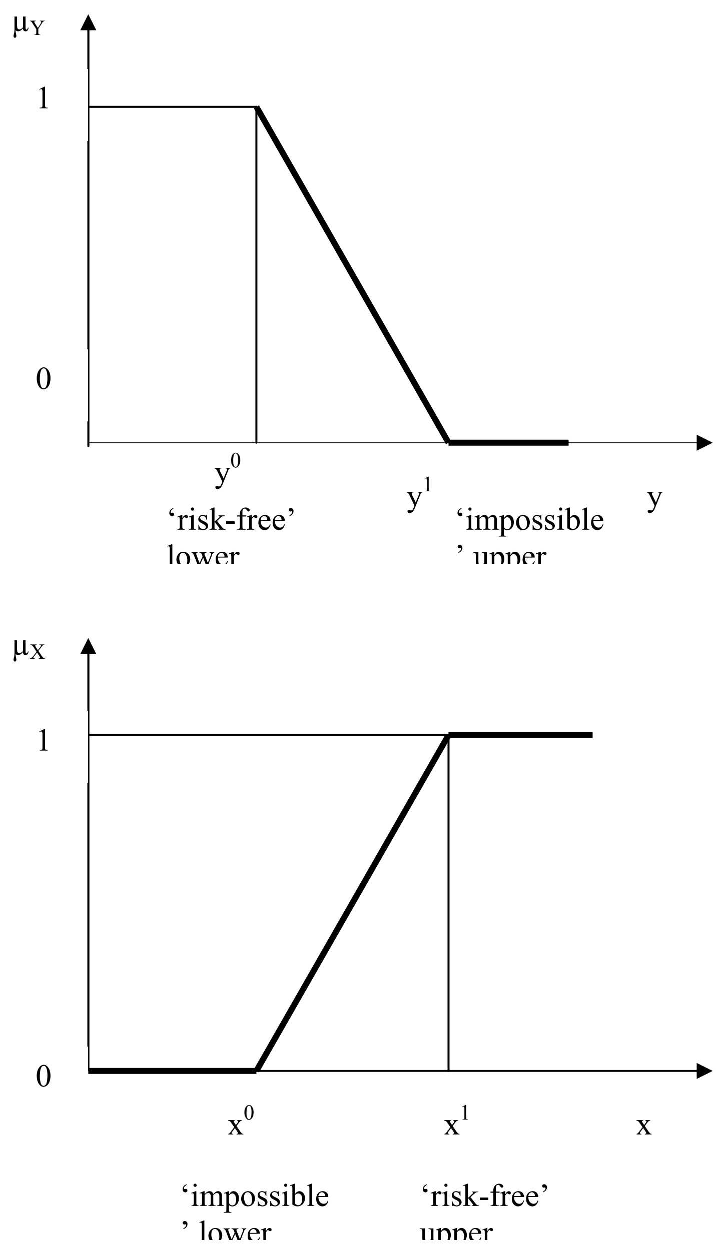


Figure 2.13 Input and Output Linear Membership Functions

Consider a set of N decision making units (DMUs) converting *I* fuzzy inputs into *J* fuzzy outputs. Let X^0 and X^1 denote the *I X N* matrices representing the impossible and the risk free bounds. The possible linear membership function associated with x_{ih}, which is the i^{th} fuzzy input for the h^{th} DMU is given by the equation (2.41).

$$\mu_X(x_{i,h}) = \frac{x^0_{i,h} - x_{i,h}}{x^0_{i,h} - x^1_{i,h}} \quad i = \{1, 2, \ldots, I\} \quad h = \{1, 2, \ldots, N\} \tag{2.44}$$

Similarly Y^0 and Y^1 denote *JXN* matrices representing the risk free and impossible bounds respectively of all the fuzzy output variables for each DMU. A possible linear membership function associated with y_{jh} which is the j^{th} output for the h^{th} DMU given by:

$$\mu_Y(y_{i,h}) = \frac{y_{j,h} - y^1_{j,h}}{y^0_{j,h} - y^1_{i,h}} \quad j = \{1, 2, \ldots, J\} \quad h = \{1, 2, \ldots, N\} \tag{2.45}$$

The decision maker can express both $x_{i,h}$ and $y_{j,h}$ in relation to the respective risk-free and impossible bounds and the fuzzy parameter μ_x or μ_y, i.e.,

$$x_{i,h} = x^0_{i,h} - (x^0_{i,h} - x^1_{i,h})\mu_X \tag{2.46}$$

$$y_{j,h} = (y^0_{j,h} - y^1_{j,h})\mu_Y + y^1_{j,h} \tag{2.47}$$

Then the original Charnes, Cooper and Rhodes model (1978) can be modeled as follows: To find the vectors u and v for DMU *p*, such that,

$$Max \frac{\sum_{j=1}^{j=J} v_j((y^0_{j,p} - y^1_{j,p})\mu_Y + y^1_{j,p})}{\sum_{i=1}^{i=I} u_i(x^0_{i,p} - (x^0_{i,p} - x^1_{i,p})\mu_X)}$$

Subject to:

$$\frac{\sum_{j=1}^{j=J} v_j((y^0_{j,h} - y^1_{j,h})\mu_Y + y^1_{j,h})}{\sum_{i=1}^{i=I} u_i(x^0_{i,h} - (x^0_{i,h} - x^1_{i,h})\mu_X)} \le 1 \quad h = \{1, \ldots, N\} \tag{2.48}$$

$u_i \ge 0 \quad i = \{1, \ldots, I\}$

$v_j \ge 0 \quad j = \{1, \ldots, J\}$

This parametric fractional linear program can be easily converted into a linear form to determine, for example, the input-minimizing weights: Find the vectors *u* and *v* such that,

$$Max\sum_{j=1}^{j=J} v_j((y^0_{j,p} - y^1_{j,p})\mu_Y + y^1_{j,p})$$

subject to:

$$\sum_{i=1}^{i=I} u_i(x^0_{i,h} - (x^0_{i,h} - x^1_{i,h})\mu_X) = 1.0 \text{, for each h} = \{1, \ldots, N\}$$

$$\sum_{j=1}^{j=J} v_j((y^0_{j,h} - y^1_{j,h})\mu_Y + y^1_{j,h}) - \sum_{i=1}^{i=I} u_i(x^0_{i,h} - (x^0_{i,h} - x^1_{i,h})\mu_X) \le 0,$$

for each h = {1, …, N} (2.49)

$u_i \ge 0$ i = {1, …, I}

$v_j \ge 0$ j = {1, …, J}

From Carlsson and Korhonen (1986), the decision maker knows for the model that the fuzzy decision is reached when $\mu_x = \mu_y = \mu = \min(\mu_x, \mu_y)$. Now the model can be written as:

To Find the vectors *u* and *v* such that,

$$Max\sum_{j=1}^{j=J} v_j((y^0_{j,p} - y^1_{j,p})\mu + y^1_{j,p})$$

Subject to:

$$\sum_{i=1}^{i=I} (x^0_{i,h} - (x^0_{i,h} - x^1_{i,h})\mu) = 1.0 \text{, for each h} = \{1, \ldots, N\}$$

$$\sum_{j=1}^{j=J} v_j((y^0_{j,h} - y^1_{j,h})\mu + y^1_{j,h}) - \sum_{i=1}^{i=I} u_i(x^0_{i,h} - (x^0_{i,h} - x^1_{i,h})\mu) \le 0 \text{, for each h} = \{1, \ldots, N\} \ldots (2.50)$$

$u_i \ge 0$ i = {1, …, I}

$v_j \ge 0$ j = {1, …, J}

The dual form can be written as follows: Find θ_p and γ such that,

$Min\theta_p$

Subject to:

$$\theta_p((x^0_{i,p}-(x^0_{i,p}-x^1_{i,p})\mu)-\sum_{h=1}^{N}\gamma_h((x^0_{i,h}-(x^0_{i,h}-x^1_{i,h})\mu)\geq 0, \ 1=\{1, \ldots, I\}$$

$$\sum_{h=1}^{N}\gamma_h((y^0_{j,h}-y^1_{j,h})\mu+y^1_{j,h})\geq(y^0_{j,p}-y^1_{j,p})\mu+y^1_{j,p}, \ j=\{1, \ldots, J\} \quad (2.51)$$

$$\theta_p,\gamma_h\geq 0, \ h=\{1, \ldots, N\}$$

Further the fuzzy version of DEA's Banker, Charnes and Cooper (1984) model can be written as: Find θ_p and the weight vector γ such that:

$$T\,E_{FBCC}(X_p,Y_p)=Min\theta_P$$

Subject to:

$$[X^0_P-(X^0_P-X^1_P)\mu]\theta_P-[X^0-(X^0-X^1)\mu]\gamma\geq 0$$

$$[(Y^0-Y^1)\mu+Y^1]\gamma\geq[(Y^0_P-Y^1_P)\mu+Y^1_P]$$

$$\sum_{h=1}^{N}\gamma_h=1 \quad (2.52)$$

θ_P and $\gamma\geq 0$

CHAPTER 3 METHODOLOGY

In Chapter 2, we explained various models in which DEA and Network theory were used. We also viewed the models that were used to model uncertainty in conjunction with Linear Programming techniques.

In this chapter, we will develop a network model incorporating service quality into efficiency measurement. We will also develop fuzzy models for capturing the uncertainty when measuring the qualitative variables. This chapter is divided into three sections. In section 3.1, we develop the model in which service quality is modeled as a sub process using the network model. In section 3.2, the imprecision of the qualitative variables is modeled using fuzzy sets. In the final section, the theoretical model is then applied to the data collected at the call center for a major US Airline.

3.1 THE DEA/ NETWORK MODEL

After Färe (1991) introduced the network model, various modifications to the model has been proposed and the model has been applied in various applications. As we discussed in section 2.2, the network model has been used in various applications. In this research, we modified the DEA network model introduced by Färe (1991) to model service quality in an efficiency framework.

3.1.1 Structure of the network model

We use a network model that consists of multiple nodes as shown in Figure 3.1. Node 0 represents a process that distributes all the inputs, and Node 3 represents the process that receives all the outputs. The network model under consideration will aid modeling of a sub process. By using this specific kind of network model, the intermediate products are accounted for.

The model consists of two production units that are interconnected into a network to form the production technology. Node 1 and Node 2 represent two different production units where inputs are transformed into outputs. The Node 1 represents the main production, and Node 2 represents the production process pertaining to service quality only. Node 2 is a sub process as it impacts the performance of Node 1. The output from Node 2 is used as an input in Node 1.

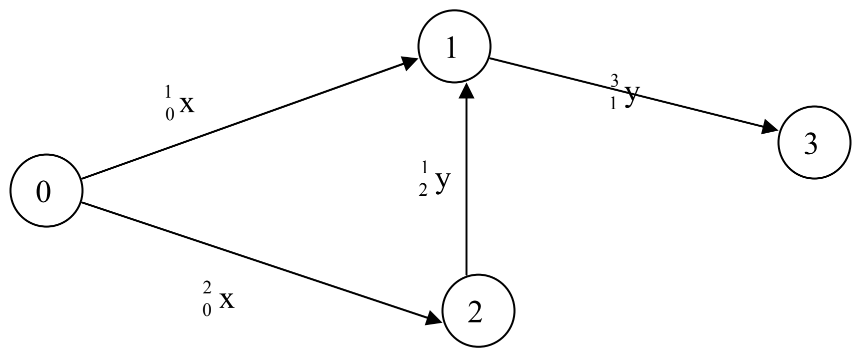


Figure 3.1 The Network Model

The connecting arrows between the nodes show the path of the inputs and the output through the network. The input is represented by a vector x, and the output by a vector y. The left hand subscript is the origin and the left hand superscript is the destination of the vector. For example, ${}_{0}^{1}x$ represents the flow of inputs from Node 0 to Node 1.

Node 0 distributes two kinds of inputs, ${}_{0}^{1}x$ and ${}_{0}^{2}x$, to Nodes 1 and 2 respectively. The input ${}_{0}^{1}x$ represents the input to the main production Node 1 and the input ${}_{0}^{2}x$ represents the input to Node 2 that represents service quality. Node 2 has an output ${}_{2}^{1}y$, which then acts as the input to Node 1. Node 1 has an output ${}_{1}^{3}y$, which is the final output of the overall process.

Input ${}_{0}^{2}x$ represents the inputs responsible service quality. It is proposed in this research that the constructs in the SERVQUAL model (Parasuraman, Zeithaml and Berry, 1988) may be used as a guiding framework to choose the input variables for the appropriate application. Various dimensions of the SERVQUAL model have been explained in section 2.3.

Let us assume that there are k = 1, …,K observations of inputs and outputs. Then n = 1,…, N will represent the number of inputs and m = 1,…, M represents the number of outputs.

Consider Node 2, assuming that the production technology is exhibiting variable returns to scale. According to Färe (1991), the reference technology for sub process 2 can be written as a piecewise linear model.

K: number of DMUs

z_k, a_k: intensity variables

x_{kn} : Amount of input *n* used by DMU *k*

y_{km} : Amount of output *m* produced by DMU *k*

N1, N2: Number of inputs

M1, M2, M3: Number of outputs

x_{nt} : Amount of input *n* used by the DMU that is being evaluated

y_{mt} : Amount of output *m* produced by the DMU being evaluated

The variables N1, N2, M1, M2 and M3 are not named to represent any particular node. They represent the number of inputs and outputs and the suffixes are used to distinguish amongst each other.

(a) $\sum_{k=1}^{K} z_k \, {}_0^2x_{kn} \leq {}_0^2x_{nt}$, n = 1,….,N1

(b) $\sum_{k=1}^{K} z_k \, {}_2^1y_{km} \geq {}_2^1y_{mt}$, m = 1,….,M1

(c) $\sum_{k=1}^{K} z_k = 1$

(d) $z_k \geq 0$, k= 1,….,K (3.1)

In the first constraint 3.1(a), the right hand side refers to the amount of input produced by the DMU under consideration. The left hand side of the constraint is the amount of input produced by a hypothetical DMU that is created by a convex combination of the DMUs under consideration. This constraint means that the amount of input produced by the hypothetical DMU should always be lesser than or equal to the amount of input produced by the DMU under consideration. In the second constraint 3.1(b), the right hand side of the constraint refers to the amount of output that is produced by the DMU under consideration. The left hand side of the constraint refers to the amount of output produced by the hypothetical DMU. This constraint means that the amount of output produced by the hypothetical DMU would always be greater than or equal to the amount of output produced by the DMU under consideration. The third constraint 3.1(c) assumes that the technology is exhibiting variable returns to scale.

Similarly consider Node 1, the reference technology may be written as

(e) $\sum_{k=1}^{K} a_k \, {}^1_0x_{kn} \leq {}^1_0x_{nt}$, n = 1,….,N2

(f) $\sum_{k=1}^{K} a_k \, {}^1_2y_{km} \leq {}^1_2y_{mt}$, m = 1,….,M2

(g) $\sum_{k=1}^{K} a_k \, {}^3_1y_{km} \geq {}^3_1y_{mt}$, m = 1,….,M3

(h) $\sum_{k=1}^{K} a_k = 1$

(i) $a_k \geq 0$, k= 1,….,K (3.1)

The novelty in expression is 3.1 (f), where intermediate inputs are included. These inputs are produced and consumed within the production process. The output from Node 2 is used as an input in this node. The variable ${}^1_2y$ is treated as an input in this constraint. In the next section the objective function will be included into the model.

3.1.2 The Farrell Technical Efficiency

Farrell (1957) suggested two measures of efficiency, one is input based and another is output based. The first takes outputs as given and measures potential input waste and its value. The output-based approach takes inputs as given and measure lost outputs and its value. Here we discuss only the input based method. The technical efficiency can be calculated by:

Min θ

Subject to

(a) $\sum_{k=1}^{K} z_k \, {}^{2}_{0}x_{kn} \le \theta \, {}^{2}_{0}x_{nt}$, n = 1,….,N1

(b) $\sum_{k=1}^{K} z_k \, {}^{1}_{2}y_{km} \ge {}^{1}_{2}y_{mt}$, m = 1,….,M1

(c) $\sum_{k=1}^{K} z_k = 1$

(d) $z_k \ge 0$, k= 1,….,K

(e) $\sum_{k=1}^{K} a_k \, {}^{1}_{0}x_{kn} \le \theta \, {}^{1}_{0}x_{nt}$, n = 1,….,N2

(f) $\sum_{k=1}^{K} a_k \, {}^{1}_{2}y_{km} \le {}^{1}_{2}y_{mt}$, m = 1,….,M2

(g) $\sum_{k=1}^{K} a_k \, {}^{3}_{1}y_{km} \le {}^{3}_{1}y_{mt}$, m = 1,….,M3

(h) $\sum_{k=1}^{K} a_k = 1$

(i) $a_k \ge 0$, k= 1,….,K (3.2)

In the above equations θ is a measure of the input reducing technical efficiency. It is present in the equations 3.2(a) and 3.2(e). The reason that θ is presented only in these constraints is because we want to decrease the input resources used. The above model is a crisp model and in the next section we shall discuss the fuzzification of the DEA/Network model.

3.2 FUZZY MODEL FOR THE DEA/NETWORK MODEL

In the previous section, the model was created in which the service quality was incorporated. In this section, we develop the fuzzy model to account for the uncertainty associated with qualitative variables.

3.2.1 Identification of the Uncertainty

The variables associated with the model were found to be both quantitative and qualitative in nature. Certain quality attributes are subjective in nature. This could be best illustrated with an example. Consider a qualitative attribute like "pleasantness". When measuring a qualitative variable like pleasantness, there would be variability in this measure. Different people might rate the same person in a different way. Fuzzy set modeling is a powerful technique that could be employed to model the uncertainty. To represent these qualitative factors, changes need to be done in the network model. It will affect the Linear Program that was developed in section 3.1.2.

3.2.2 DEA/ Network Model

To model the fuzzy variables, let us introduce two new vectors p and q, representing the fuzzy input variable and the fuzzy output variable respectively. Assume that the fuzzy variables exist at the entering and exiting of Node 2. The vectors that represent the fuzzy variables are p and q, which represent the input and the output fuzzy variables respectively. Because of these changes, the number of constraints will increase by three.

p_{kn} : Amount of fuzzy input n used by DMU k

q_{km} : Amount of fuzzy output m produced by DMU k

So the constraints in the network model can be now represented as

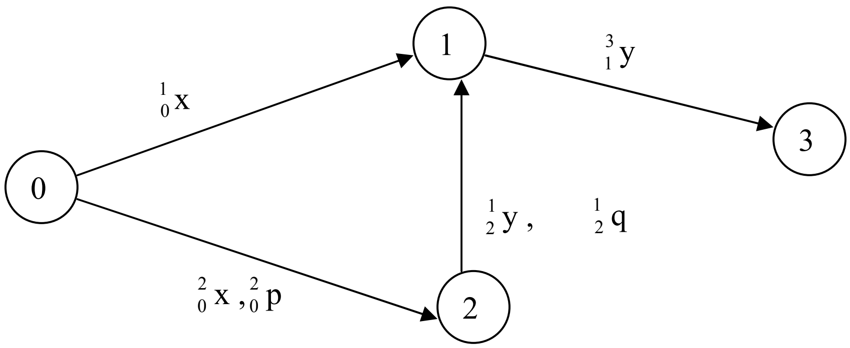


Figure 3.2 Fuzzy Network Model

Min θ

Subject to

(a) $\sum_{k=1}^{K} z_k \, {}_{0}^{2}x_{kn} \leq \theta \, {}_{0}^{2}x_{nt}$, n = 1,…,N1

(b) $\sum_{k=1}^{K} z_k \, {}_{0}^{2}p_{kn} \leq \theta \, {}_{0}^{2}p_{nt}$, n = 1,…,N2

(c) $\sum_{k=1}^{K} z_k \, {}_{2}^{1}y_{km} \geq {}_{2}^{1}y_{mt}$, m = 1,…,M1

(d) $\sum_{k=1}^{K} z_k \, {}_{2}^{1}q_{km} \geq {}_{2}^{1}q_{mt}$, m = 1,…,M2

(e) $\sum_{k=1}^{K} z_k = 1$

(f) $z_k \geq 0$, k= 1,…,K

(g) $\sum_{k=1}^{K} a_k \, {}_{0}^{1}x_{kn} \leq \theta \, {}_{0}^{1}x_{nt}$, n = 1,…,N3

(h) $\sum_{k=1}^{K} a_k \, {}_{2}^{1}y_{km} \leq {}_{2}^{1}y_{mt}$, m = 1,…,M3

(i) $\sum_{k=1}^{K} a_k \, {}_{2}^{1}q_{km} \leq {}_{2}^{1}q_{mt}$, m = 1,...,M4

(j) $\sum_{k=1}^{K} a_k \, {}_1^3 y_{km} \geq {}_1^3 y_{mt}$, m = 1,…,M5

(k) $\sum_{k=1}^{K} a_k = 1$

(l) $a_k \geq 0$, k= 1,….,K (3.3)

It is proposed in this research that fuzzy sets be employed to model these fuzzy variables into the model. When these fuzzy sets are introduced into the model, then the resulting Linear Programming Model turns out to be a Fuzzy Linear Programming Model.

In this research, we use the Zimmerman (1996) model, where the goals and constraints are represented by fuzzy sets. A fuzzy set is a class of objects with a continuum of grades of membership. A fuzzy set is characterized by a membership function, which assigns to each object a grade of membership ranging between 0 and 1. The fuzzy sets are represented by membership functions and the triangular membership function is used to represent the fuzzy sets. A triangular membership function could be used to model most practical scenarios.

3.2.3 The Triangular Membership Function

Yuan and Klir (1995) proposed the use of the triangular function. To completely describe the membership functions we need to specify the following:

- the most desirable value gets a membership value of 1;
- the least desirable value one on either side of the most desirable value is assigned grade of 0.

3.2.4 Fuzzy Linear Programming

According to Zimmerman (1996), fuzzy models in which aspiration levels can be specified for the objective function are called symmetrical models. Symmetrical models are easy to solve because the membership function of the fuzzy set “decision” is the intersection of the fuzzy sets of the objective function and the constraints.

In our model the objective function remains crisp, and the constraints are fuzzy and so it is an unsymmetrical fuzzy model. Initially, we will account for the fuzziness in the constraints and then we will deal with the objective function.

3.2.4.1 Fuzziness in the Constraint.

The fuzzy variable is represented by the triangular membership function and the fuzzy variable is present in the equation (3.5) part (b), (d), and (i). To illustrate the concept, let us fuzzify the part (b) alone at this juncture.

$$\sum_{k=1}^{K} z_k \, {}^{2}_{0} p_{kn} \leq \theta \, {}^{2}_{0} p_n \text{ , n = 1,..., N2} \qquad (3.4)$$

In this constraint there is fuzziness in both sides of the constraint only. This concept can be better explained with an example. Let us consider a variable denoting "unpleasantness". This variable refers to the degree to which a person is unpleasant. Let us assume that there are many people being measured, to determine if they are pleasant. There are multiple evaluators doing this. Now due to human subjectivity, the evaluators will perceive different people differently. This can be modeled using the fuzzy sets.

This variable can be represented by a membership function $\mu(p)$ of unpleasantness as shown in Figure 3.3.

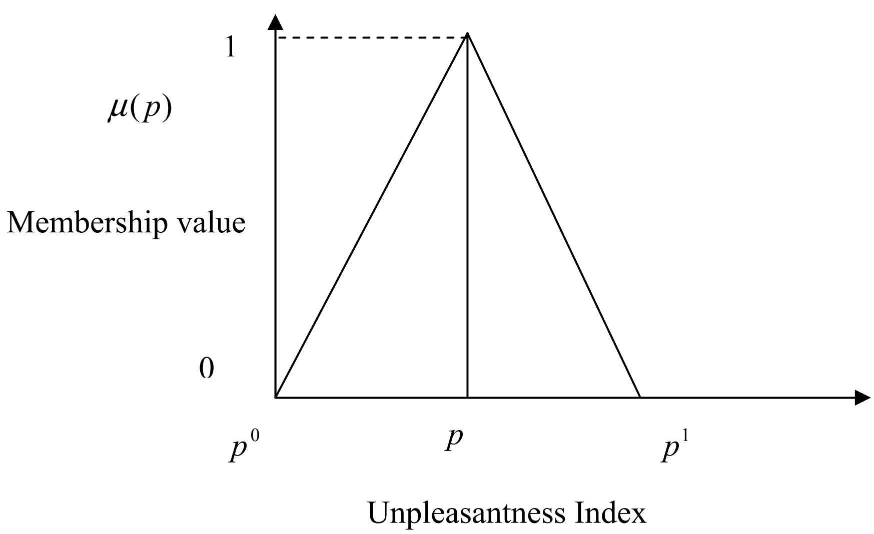


Figure 3.3 Membership Function of Unpleasantness

The parameters p^0, p and p^1 characterize the membership function. The degree of unpleasantness increases as we move on the x-axis from p^0 to p^1. The membership value is on the y-axis. The variable p belongs to this set with a membership value of 1, and the values p^0 and p^1 has a membership value of 0. The variable p is the precise value and p^0 and p^1 are the two extreme values.

In the DEA framework "Unpleasantness Index" would be an input variable and the objective would be to decrease this variable. So in this research, the scenarios in which there could be potential improvements in the efficiency of the person are to be studied. Consequently, only one side of the triangular membership function will be used.

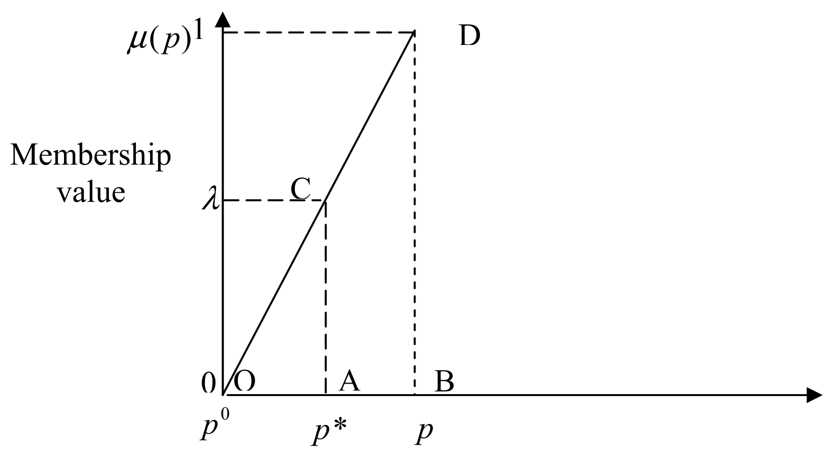


Figure 3.4 Membership Function of Unpleasantness

The term λ that is present in the above figure can be defined as the "level of satisfaction". When the value of λ is 1, then the level of satisfaction is the maximum and when the value of λ is 0, the level of satisfaction is minimum. In Figure 3.4, p^* represents the value of the input variable when the level of satisfaction varies between 0 and 1.

Let us consider the two triangles ΔOBD and ΔOAC that are similar.

$$\frac{BD}{CA} = \frac{OB}{OA}$$

$$\frac{1}{\lambda} = \frac{p - p^0}{p^* - p^0}$$

$$p^* = p^0 + \lambda(p - p^0)$$

So the value of $p^* = p_{nt}^0 + \lambda(p_{nt} - p_{nt}^0)$

As the value of λ changes from 0 to 1, the value of p^* varies between the extreme values p and p^0.

So the right hand side of the constraint in equation (3.4) will become

$$\theta\lambda(p_{nt}^0 + \lambda(p_{nt} - p_{nt}^0)) \qquad (3.5)$$

Because of the way that DEA is formulated, there will be changes on the left hand side of the constraint also.

$$-z_t({}_0^2p_{nt} - {}_0^2p_{nt}^*) + \sum_{k=1}^{K} z_k \, {}_0^2p_{kn} \qquad (3.6)$$

The new term in the constraint takes into account the change in the variable that takes on the right hand side of the constraint.

$$-z_t({}_0^2p_{nt} - ({}_0^2p_{nt}^0 + \lambda({}_0^2p_{nt} - {}_0^2p_{nt}^0)) + \sum_{k=1}^{K} z_k \, {}_0^2p_{kn}$$

$$-z_t(1+\lambda)({}_0^2p_{nt} - {}_0^2p_{nt}^0) + \sum_{k=1}^{K} z_k \, {}_0^2p_{kn} \le \theta({}_0^2p_{nt}^0 + \lambda({}_0^2p_{nt} - {}_0^2p_{nt}^0)), \text{ n = 1,…, N2} \qquad (3.7)$$

Here when the membership value is varied between 0 and 1, then the input variable for the DMU being measured alone changes. All the other DMUs have the precise input variable. So, when the membership value is 0, it could represent a scenario, which is not possible to obtain practically.

Let us consider the constraint 3.3 (d), which represents the output variable

$$\sum_{k=1}^{K} z_k \, {}_2^1q_{km} \ge {}_2^1q_m \text{, m = 1,..., M2}$$

"Satisfaction Index" represents the satisfaction with a particular service. It could be represented by the triangular membership function $\mu(q)$ as seen in Figure 3.5.

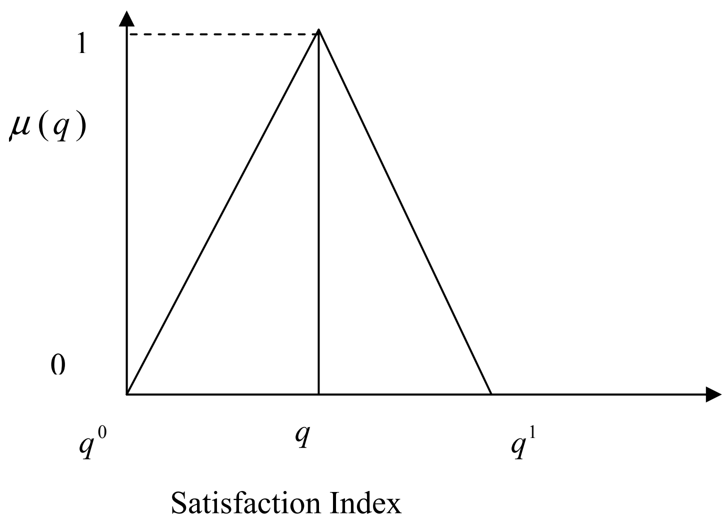


Figure 3.5 Membership Function of Satisfaction

The parameters q^0, q and q^1 characterize the membership function. The degree of unpleasantness increases as we move on the x-axis from q^0 to q^1. The membership value is on the y-axis. The variable *p* belongs to this set with a membership value of 1, and the values q^0 and q^1 has a membership value of 0. The variable *q* is the precise value and q^0 and q^1 are the two extreme values.

The satisfaction index is also a subjective with people. Some people might be more satisfied than the others. In this research we want to study the scenario where a person might be unfairly penalized because of the perception of a person. So, we want to study the scenarios where a person could have his overall efficiency improved.

For an output variable we consider only the right hand side of the triangular membership function.

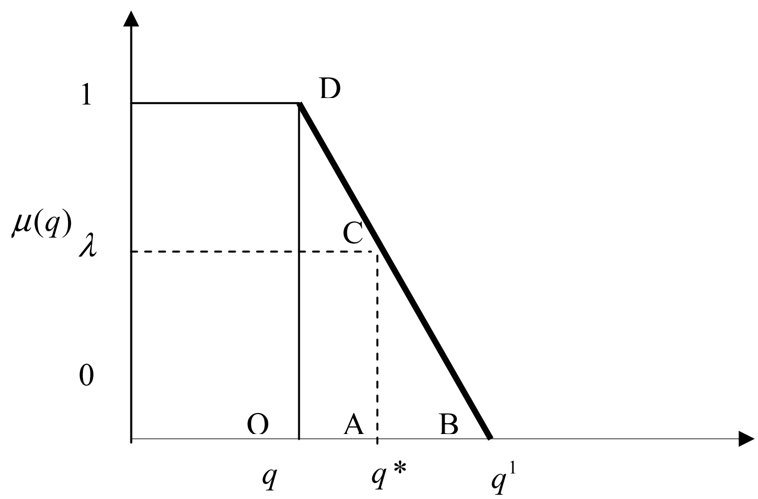


Figure 3.6 Membership Function of Satisfaction

As was previously defined the term λ that is present in the above figure can be defined as the "level of satisfaction". When the value of λ is 1, then the level of satisfaction is the maximum and when the value of λ is 0, the level of satisfaction is minimum

In Figure 3.5 q^* represents the value of the input variable when the level of satisfaction varies between 0 and 1.

Let us consider the two triangles ΔOBD and ΔABC that are similar.

$$\frac{OD}{CA} = \frac{OB}{AB}$$

$$\frac{1}{\lambda} = \frac{q^1 - q}{q^1 - q*}$$

$$q* = q^1 - \lambda(q^1 - q)$$

So the right hand side of the constraint will look like

$${}_2^1q_{mt}^1 - \lambda({}_2^1q_{mt}^1 - {}_2^1q_{mt}) \qquad (3.8)$$

Because of the change in the constraint on the right hand side, there will be a change on the left hand side of the constraint also.

$$a_t({}_2^1q_{mt}^{*} - {}_2^1q_{mt}) + \sum_{k=1}^{K} a_k\, {}_2^1q_{km} \quad (3.9)$$

The change in the output variable because of its fuzzy nature is only for that specific DMU that is being evaluated. It does not impact the output variables of the other DMUs.

$$a_t((({}_2^1q_{mt}^1 - \lambda({}_2^1q_{mt}^1 - {}_2^1q_{mt}) - {}_2^1q_{mt})) + \sum_{k=1}^{K} a_k\, {}_2^1q_{km} \geq {}_2^1q_{mt}^1 - \lambda({}_2^1q_{mt}^1 - {}_2^1q_{mt})$$

$$a_t(1-\lambda)({}_2^1q_{mt}^1 - {}_2^1q_{mt}) + \sum_{k=1}^{K} a_k\, {}_2^1q_{km} \geq ({}_2^1q_{mt}^1 - \lambda({}_2^1q_{mt}^1 - {}_2^1q_{mt})) \quad (3.10)$$

3.2.4.2 Fuzziness in the Objective Function

In this method the crisp objective function can be represented as “maximizing set” (Zadeh, 1972). The maximizing set is constructed by determining the extremum (upper and lower bounds) of the crisp function over the fuzzy domain. For our model we can use Werners’ (1984) definition to determine the maximizing set of the objective function. Once the maximizing set is determined, we can determine the crisp “maximization solution” of the decision.

Werners (1984) definition of "maximizing set" is

Let f: X $\longrightarrow$ R^1 be the objective function, $\tilde{R}$ = the feasible region, S ($\tilde{R}$) = the support of $\tilde{R}$ and R_1=1-α cut of $\tilde{R}$ for α=1. The membership function of the goal given solution space $\tilde{R}$ is then defined as

$$G(x) = \begin{cases} 0 & \text{if } f(x) \leq \sup_{R_1} f \\ \dfrac{f(x) - \sup_{R_1} f}{\sup_{S(\tilde{R})} f - \sup_{R_1} f} & \text{if } \sup_{R_1} f < f(x) < \sup_{S(\tilde{R})} f \\ 1 & \text{if } \sup_{S(\tilde{R})} f \leq f(x) \end{cases} \quad (3.11)$$

The maximizing set for the objective function can be determined by determining two values of the objective function by solving two LPs. The two LPs have the same set of fuzzy constraints but satisfied to different degrees.

Determination of f_1

f_1 is the supremum of f (the objective function) over R_1**.**

Min θ

Subject to:

(a) $\sum_{k=1}^{k} z_k\,{}^2_0x_{kn} \le \theta\,{}^2_0x_{nt}$, n = 1, …, N1

(b) $-z_t(1+\lambda)({}^2_0p_{nt} - {}^2_0p^0_{nt}) + \sum_{k=1}^{K} z_k\,{}^2_0p_{kn} \le \theta({}^2_0p^0_{nt} + \lambda({}^2_0p_{nt} - {}^2_0p^0_{nt}))$, n = 1, ..., N2

(c) $\sum_{k=1}^{K} z_k\,{}^1_2y_{km} \ge {}^1_2y_{mt}$, m = 1, ..., M1

(d) $z_t(1-\lambda)({}^1_2q^1_{mt} - {}^1_2q_{mt}) + \sum_{k=1}^{K} z_k\,{}^1_2q_{km} \ge ({}^1_2q^1_{mt} - \lambda({}^1_2q^1_{mt} - {}^1_2q_{mt}))$, m = 1, ..., M2

(e) $\sum_{k=1}^{K} z_k = 1$

(f) $z_k \ge 0$, k= 1,…, K

(g) $\sum_{k=1}^{K} a_k\,{}^1_0x_{kn} \le \theta\,{}^1_0x_{nt}$, n = 1,….,N3

(h) $\sum_{k=1}^{K} a_k\,{}^1_2y_{km} \le {}^1_2y_{mt}$, m = 1,….,M3

(i) $-a_t(1+\lambda)({}^2_0q_{nt} - {}^2_0q^0_{nt}) + \sum_{k=1}^{K} a_k\,{}^2_0q_{kn} \le \theta({}^2_0q^0_{nt} + \lambda({}^2_0q_{nt} - {}^2_0q^0_{nt}))$, m = 1,….,M4

(j) $\sum_{k=1}^{K} a_k\,{}^3_1y_{km} \ge {}^3_1y_{mt}$, m = 1,….,M5

(k) $\sum_{k=1}^{K} a_k = 1$

(l) $a_k \ge 0$, k= 1,….,K

(m) $\lambda = 1$ (3.12)

The supermum of f over this region is:

$$\sup_{\tilde{R}_1} f = f_1$$

Determination of f_0

f_o is the supermum of over S ($\widetilde{R}$), which is the support of the fuzzy region is given by

Min θ

Subject to:

(a) $\sum_{k=1}^{k} z_k\, {}_0^2x_{kn} \leq \theta\, {}_0^2x_{nt}$, n = 1, …, N1

(b) $-z_t(1+\lambda)({}_0^2p_{nt} - {}_0^2p_{nt}^0) + \sum_{k=1}^{K} z_k\, {}_0^2p_{kn} \leq \theta({}_0^2p_{nt}^0 + \lambda({}_0^2p_{nt} - {}_0^2p_{nt}^0))$, n = 1, ..., N2

(c) $\sum_{k=1}^{K} z_k\, {}_2^1y_{km} \geq {}_2^1y_{mt}$, m = 1, ..., M1

(d) $z_t(1-\lambda)({}_2^1q_{mt}^1 - {}_2^1q_{mt}) + \sum_{k=1}^{K} z_k\, {}_2^1q_{km} \geq ({}_2^1q_{mt}^1 - \lambda({}_2^1q_{mt}^1 - {}_2^1q_{mt}))$, m = 1, ..., M2,

(e) $\sum_{k=1}^{K} z_k = 1$

(f) $z_k \geq 0$, k= 1,., K

(g) $\sum_{k=1}^{K} a_k\, {}_0^1x_{kn} \leq \theta\, {}_0^1x_{nt}$, n = 1,…, N3

(h) $\sum_{k=1}^{K} a_k\, {}_2^1y_{km} \leq {}_2^1y_{mt}$, m = 1,…, M3

(i) $-a_t(1+\lambda)({}_0^2q_{nt} - {}_0^2q_{nt}^0) + \sum_{k=1}^{K} a_k\, {}_0^2q_{kn} \leq \theta({}_0^2q_{nt}^0 + \lambda({}_0^2q_{nt} - {}_0^2q_{nt}^0))$, m = 1,…, M4

(j) $\sum_{k=1}^{K} a_k\, {}_1^3y_{km} \geq {}_1^3y_{mt}$, m = 1,…, M5

(k) $\sum_{k=1}^{K} a_k = 1$

(l) $a_k \geq 0$, k= 1,..., K

(m) $\lambda = 0$ (3.13)

The supermum of f over this region is:

$$\sup_{S(\widetilde{R})} f = f_0$$

3.2.5 Membership function of the Objective function

The following figure graphically depicts the variation of the membership function $\mu(\theta)$ between 0 and 1, as the objective function varies between f_o and f_1.

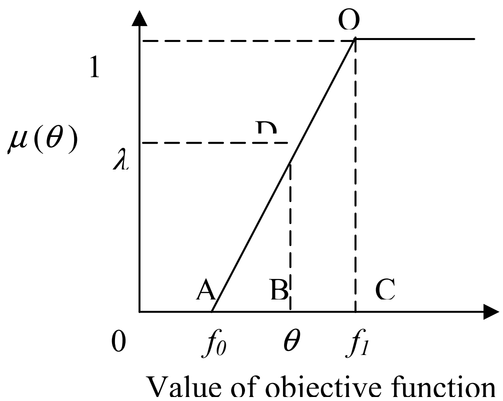


Figure 3.7 Membership Function of the Goal

In the figure 3.5, λ represents the level of satisfaction and it will vary between the values of 0 and 1.

Consider ΔABD and ΔACO. By the law of similarity,

$$\frac{AB}{BD} = \frac{AC}{CO}$$

$$\frac{\theta - f_0}{\lambda} = \frac{f_1 - f_0}{1}$$

$$\theta = f_0 + \lambda(f_1 - f_0)$$

$$\lambda(f_1 - f_0) - \theta \leq -f_0 \qquad (3.14)$$

3.2.6 Equivalent Crisp formulation

Thus the final crisp formulation is as seen below:

Max λ

(a) $\lambda(f_1 - f_0) - \theta \leq -f_0$

(b) $\sum_{k=1}^{k} z_k {}_0^2 x_{kn} \leq \theta_0^2 x_{nt}$, n = 1, …, N1

(c) $-z_t(1+\lambda)({}_0^2p_{nt}-{}_0^2p_{nt}^0)+\sum_{k=1}^{K} z_k\,{}_0^2p_{kn} \le \theta({}_0^2p_{nt}^0+\lambda({}_0^2p_{nt}-{}_0^2p_{nt}^0))$, n = 1, ..., N2

(d) $\sum_{k=1}^{K} z_k\,{}_2^1y_{km} \ge {}_2^1y_{mt}$, m = 1, ..., M1

(e) $z_t(1-\lambda)({}_2^1q_{mt}^1-{}_2^1q_{mt})+\sum_{k=1}^{K} z_k\,{}_2^1q_{km} \ge ({}_2^1q_{mt}^1-\lambda({}_2^1q_{mt}^1-{}_2^1q_{mt}))$, m = 1, ..., M2

(f) $\sum_{k=1}^{K} z_k = 1$

(g) $z_k \ge 0$, k= 1,..., K

(h) $\sum_{k=1}^{K} a_k\,{}_0^1x_{kn} \le \theta\,{}_0^1x_{nt}$, n = 1,…, N3

(i) $\sum_{k=1}^{K} a_k\,{}_2^1y_{km} \le {}_2^1y_{mt}$, m = 1,., M3

(j) $-a_t(1+\lambda)({}_0^2q_{nt}-{}_0^2q_{nt}^0)+\sum_{k=1}^{K} a_k\,{}_0^2q_{kn} \le \theta({}_0^2q_{nt}^0+\lambda({}_0^2q_{nt}-{}_0^2q_{nt}^0))$, m = 1,…, M4

(k) $\sum_{k=1}^{K} a_k\,{}_1^3y_{km} \ge {}_1^3y_{mt}$, m = 1,., M5

(l) $\sum_{k=1}^{K} a_k = 1$

(m) $a_k \ge 0$, k= 1,…, K (3.15)

The variable λ is the level of satisfaction. If the value of λ=1, then the level of satisfaction is maximum. If the value of λ=0, then the level of satisfaction is minimum. Since it is a quadratic programming model, we can use the parametric algorithm from Sakawa (1984) to solve the model. The parametric algorithm is used to find the maximum feasible value of λ in an efficient method. The parametric equation can be described as follows.

1. Set λ=0 and check the feasibility of the equation.
2. If the problem is feasible, go to 3. Otherwise STOP.
3. Check λ=1. Check the feasibility.
4. If the problem is feasible, that is the solution - STOP. Otherwise, go to5.
5. Set $\lambda_{\max}$=1, $\lambda_{\mathbf{min}}$=0.

6. If $\lambda_{\max}$ - $\lambda_{\mathbf{min}}$ $< \varepsilon$, STOP, otherwise go to7.
7. Set λ**=(**$\lambda_{\max}$ +1 $\lambda_{\mathbf{min}}$**)/2**
8. **C**heck the feasibility for λ
9. If the problem is infeasible, set $\lambda_{\max} = \lambda$ go to 6. Otherwise set $\lambda_{\mathbf{min}}$**=** λ and go to 6.

The solution of the parametric algorithm gives the optimal value of the membership function, which is an indication to the degree to which the constraints are satisfied.

CHAPTER 4 APPLICATION RESULTS AND DISCUSSION

This chapter deals with the illustration of the model and a discussion of the results. We illustrate the model by applying the data that was collected at a major Airline. The data is also analyzed using the model that was created. The network model is then compared to the non-network model and then is validated with statistical models.

4.1 DESCRIPTION OF THE PROBLEM

In this section the research problem is described. The input variables and the output variables are provided.

4.1.1 Focus of the study

The focus of the study is the customer service representative (CSR) in a call center at a major Airline. The CSR is the DMU in this research work. The call center is the first point of customer contact. Whenever a customer wants to travel, he or she will call the call center to make a booking.

There are a total of 50 call center representatives (CSRs) who are well trained to provide the best service to the customer. Due to the high labor cost, the performance of the CSR is tracked, so that they can be very effective. They are measured over multiple variables, both quantitative and qualitative. The airline does not have a consolidated index by which they could track the performance of the CSR. So in this research DEA and Fuzzy techniques are proposed to evaluate the performance of the CSR's.

4.1.2 Fuzzy DEA Network Model

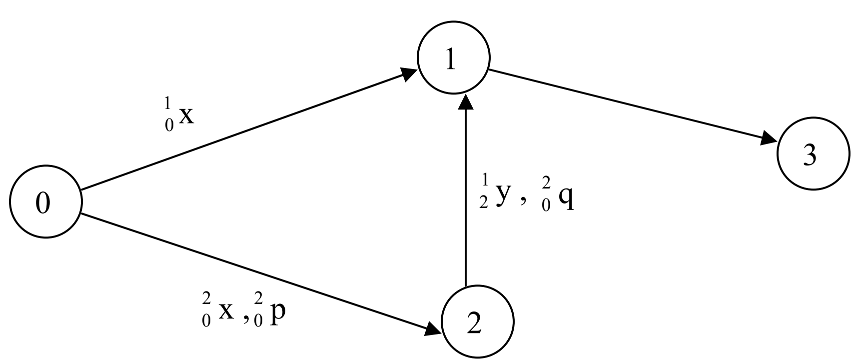


Figure 4.1 Fuzzy Network Model

As explained earlier in Chapter 3, the network model consists of multiple nodes. Node 0 represents a process that distributes all the inputs, and Node 3 represents the process that receives all the outputs. The network model under consideration will aid modeling of a sub process. The Node 1 represents the main production, and Node 2 represents the production process pertaining to service quality only. Node 2 is a sub process as it impacts the performance of Node 1. The output from Node 2 is used as an input in Node 1. The various input and the output variables that are used in this model can be explained as follows.

4.1.3 Input and Output Variables

All the data had been collected for a period of three months starting from Jan 2002 to March 2002. The quantitative variables were collected from the internal database that was available at US Airways. Quality review persons, who evaluate the CSR on qualitative data, measured the qualitative variables.

Various variables that are measured can be explained as follows:

a) Average talk time (ATT)
It is the average talk time measured in seconds of the CSR over a period of three months. During this time the CSR provides information to the customer on the availability of seats, cost of the ticket, and also books the reservation. The objective of the company is to decrease the talk time as much as possible. Since this is a resource that needs to be decreased, it is an input variable.

b) Average work time (AWT)
It is the average time in seconds spent by the CSR on the computer, working when not talking with the customer. The CSR has to update information on the computer system regarding the transaction that occurred with the customer. This would also be an input variable and the objective would be to minimize this variable.

c) Number of paper tickets sold (PTKT)

It is the average number of paper tickets sold per hundred calls that the CSR receives. This variable is an output variable and the objective is to maximize the number of paper tickets sold.

d) Number of electronic tickets sold (ETKT)
It is the average number of electronic tickets sold per 100 calls that the CSR receives. The objective is to maximize the number of electronics tickets sold and it is also an output measure.

e) Measure of unpleasantness (UP)
When there is an interaction with the customer, a Quality Review Person (QRP) monitors the telephone call and gives a review. The QRP collects qualitative data like "Measure of unpleasantness" and "Measure of Satisfaction". The variables denote service quality and falls under the construct "Assurance" defined by Parasuraman, Zeithmal and Berry (1988). A CSR who is pleasant will appear courteous and instill confidence in the customer.

The "Measure of unpleasantness" is obtained from linguistic variables. In this specific case there are three choices in which a CSR can fall under. The CSR can fall under: "pleasant" and "unpleasant" and "most pleasant". The QRP determines whether the CSR is unpleasant or not. There is subjectivity involved in perceiving the non-pleasantness of the CSR and so there is imprecision involved. To model this imprecision we use fuzzy sets as can be seen in fig (4.1). In this call center there are multiple QRPs. So there is a possibility that a CSR might get a different score based on the QRP that monitors the call.

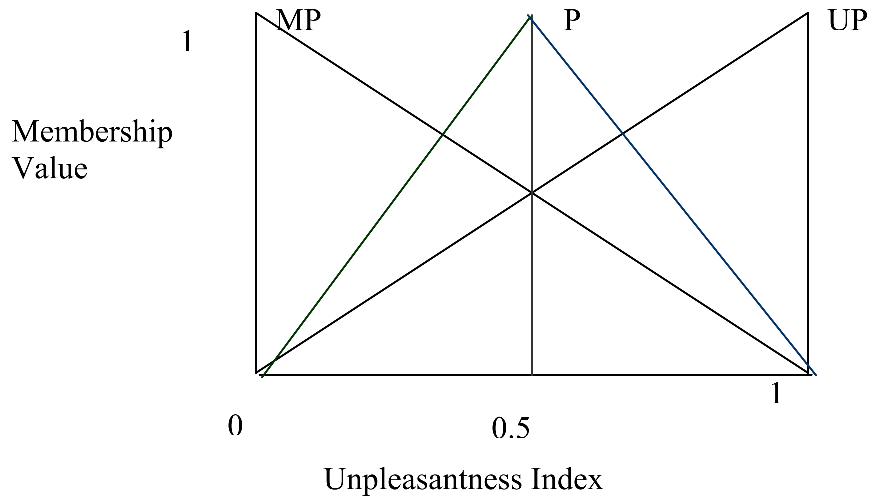


Figure 4.2 Fuzzy Sets Representing Pleasant, Unpleasant and Most Pleasant Membership Functions

It was decided that the uncertainty associated with the perception of service quality could be modeled using three triangular membership functions. In Figure 4.1, the three membership functions “Pleasant membership function” (P), “Unpleasant membership function” (UP) and “Most Pleasant membership function” (MP) can be seen. Now the “Most Pleasant membership” is represented by (0,0,1), “Pleasant membership” is represented by (0, .5,1) and “Unpleasant membership” is represented by (0,1,1).

We can illustrate this concept with an example. Let us assume that a particular customer representative is measured two times pleasant, two times unpleasant and two times most pleasant. Then the value of the variable by using Fuzzy Arithmetic rules is:

2P + 2UP +2MP = 2(0,0,1) +2 (0,1,1) +2(0,0.5,1)

= (0,0,2) + (0,2,2) +(0,1,2)

= (0,3,6)

Average = (0,3,6)/6

= (0, .5,1)

So this CSR will have an index value of 0.5 with a membership value of 1.

The resultant triangular membership will be in the form in Figure 4.2.

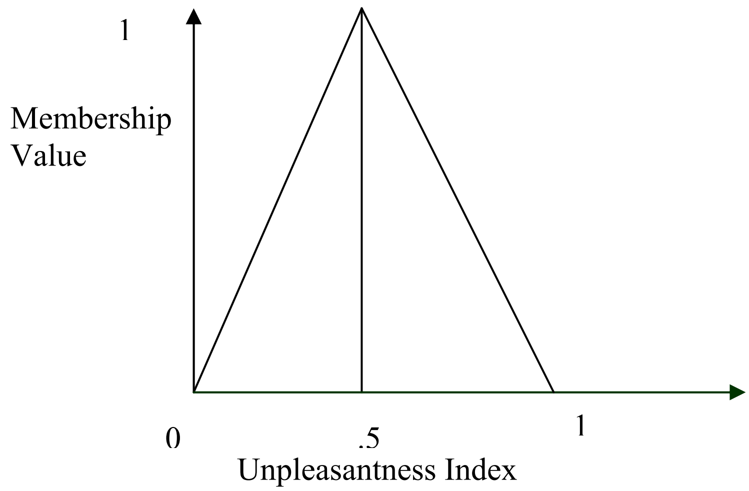


Figure 4.3 Membership Function of Unpleasantness

This CSR has a value of 0.5 with a grade of membership of 1. The CSR has an index value of 0, with a membership value of 0 and an index value of 1 with a membership value of 0.

f) Measure of Satisfaction with Service (S)

The quality review person also measures this variable. This variable is also modeled by using the fuzzy set. The quality review person could either be "satisfied", "dissatisfied" or "most satisfied" with the service.

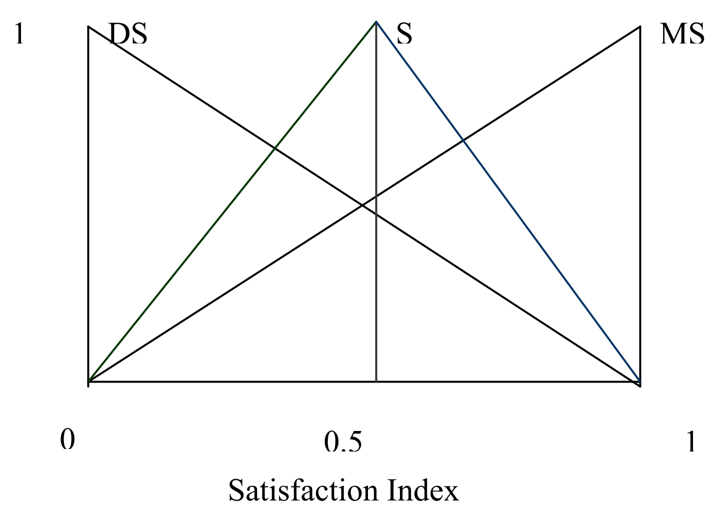


Figure 4.4 Membership Function for Satisfaction

In Figure 4.3, there is the presence of three membership functions “satisfied membership function” (S), “most satisfied membership function” (MS) and “dissatisfied membership function” (DS). The “Dissatisfied membership function” is represented by (0,0,1), “Satisfied membership function” is represented by (0, .5,1) and “Most satisfied membership” is represented by (0,1,1). The membership function for the satisfaction index is created similarly as the membership function for the unpleasantness index. This index is modeled as an intermediate product in this research. It as treated as an output variable, as the value of the index is based on multiple input variables.

Then this variable becomes an input to the main production process. In this case it is treated as an input. In the DEA framework the input variables need to be reduced. So to use the Satisfaction Index effectively as an input variable, the index is subtracted from 1. So essentially in this case it becomes the “ Dissatisfaction index”. When the “Dissatisfaction Index” is reduced the degree of satisfaction will increase.

g) Percentage of Times when the Cheapest Fare was not Quoted (CFNQ)

This variable is also captures service quality and it falls under the construct “Reliability” defined by Parasuraman, Zeithmal and Berry (1988). The CSR is supposed to quote the cheapest fare the customer every time when the customer calls the airline. This variable denotes the number of times that the cheapest fare was not given to the customer and is measured as percentage. This variable will be modeled as an input and it has to be decreased for the service quality to increase. The CSR was monitored a total of six times in the quarter to obtain this measure and this was also modeled as an input measure.

h) Percentage of Times when the Fare Rules was not Explained (FRNE)

This variable is also captures service quality and it falls under the construct “Responsiveness” defined by Parasuraman, Zeithmal and Berry (1988). The CSR is expected to explain the fare rules to the customer each time a fare is quoted. The CSR needs to inform the customer that every time when the customer makes a change there would be a cost associated with it and that the ticket may or may not be refundable.

A refundable ticket is that ticket in which the cost of the ticket would be returned if the customer did not use the ticket. A non-refundable ticket is that ticket, where the customer looses the money if the customer does not use the ticket. This variable also is modeled as an input and the objective is to decrease it as much as possible. Similar to CFNQ the CSR was monitored a total of six times to obtain this measure and it is modeled as an input measure.

4.1.4 Summary of the Inputs and the outputs

Inputs to Node 1

${}_0^1x$: Average talk time (ATT)

${}_0^1x$: Average work time (AWT)

Output from Node 1

${}_1^3y$: Number of paper tickets sold (PTKT)

${}_1^3y$: Number of electronic tickets sold (ETKT)

Inputs to Node 2

${}_0^2x$: Percentage of times when the cheapest fare were not quoted (CFNQ)

${}_0^2x$: Percentage of times when the fare rules was not explained (FRNQ)

${}_0^2p$: Measure of unpleasantness (UP)

Output from Node 2 (Intermediate product)

${}_1^2q$: Measure of satisfaction with service (S)

4.2 NON-NETWORK MODEL

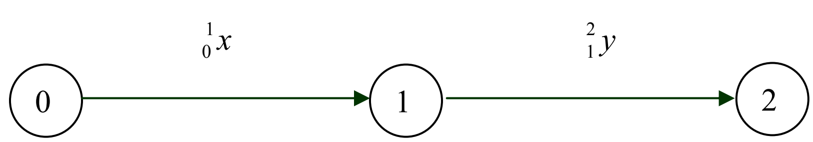


Figure 4.5 The Non Network Model

Node 0 represents a process that distributes all the inputs, and Node 2 represents the process that receives all the outputs. The Node 1 represents the main production process where all the inputs are converted into outputs.

To model the Non Network Model the BCC Model that was explained in Chapter 2 was implemented. It could also be viewed as a sub model of the Network Model. The notations that are used in the model are:

K: number of DMUs

z_k: intensity variables

x_{kn}: Amount of input n used by DMU k

y_{km}: Amount of output m produced by DMU k

N1: Number of inputs

M1: Number of outputs

x_{nt}: Amount of input n used by the DMU t that is being evaluated

y_{mt}: Amount of output m produced by the DMU t being evaluated

Min θ

Subject to

(a) $\sum_{k=1}^{K} z_k \, {}_0^1x_{kn} \leq \theta \, {}_0^1x_{nt}$, n = 1,….,N1

(b) $\sum_{k=1}^{K} z_k \, {}_2^1y_{km} \geq {}_2^1y_{mt}$, m = 1,….,M1

(c) $\sum_{k=1}^{K} z_k = 1$

(d) $z_k \geq 0$, k= 1,….,K

The inputs and the outputs that were explained earlier are modeled as follows.

Inputs to Node 1

${}_0^1x$: Average talk time (ATT)

${}_0^1x$: Average work time (AWT)

${}_0^1x$: Percentage of times when the cheapest fare were not quoted (CFNQ)

${}_0^1x$: Percentage of times when the fare rules was not explained (FRNQ)

${}_{0}^{1}x$: Measure of unpleasantness (UP)

Output from Node 1
${}_{1}^{2}y$: Number of paper tickets sold (PTKT)

${}_{1}^{2}y$: Number of electronic tickets sold (ETKT)

${}_{1}^{2}y$: Measure of satisfaction with service (S)

4.3 IMPLEMENTATION AND RESULTS

In this section the models are implemented and the results obtained are discussed.

4.3.1 Data Set

The data set that was collected at the call center is listed below. The data set collected has five inputs, one intermediate product and two outputs. The data set was collected for the first quarter of 2002 in the time period from January to March 2002. The origin of the data is an internal database system maintained at the airline.

	Type of Variable	Units	Average	SD	MIN	MAX
Electronic tickets	Output	Percentage	12.86	2.9	5.58	20.27
Paper tickets	Output	Percentage	2.89	0.77	1.33	5.22
Average talk time	Input	Second/call	260.56	59.67	162	440
Average work time	Input	Second/call	25.44	13.42	4	64
Fare rules not explained	Input	Percentage	0.28	0.16	0	0.67
Cheapest fare not quoted	Input	Percentage	0.31	0.14	0	0.83
Measure of unpleasantness	Input	Index	0.56	0.26	0.17	0.92

Table 4.1 Descriptive Statistics of the Data Set

4.3.2 Modeling tool used

The software that is used for the programming and the running of the models is Excel Standard Solver and Excel Premium Solver Plus V3.5 as an add-in to Excel 97 on a

Windows 2000 platform. The codes and the formulation of the linear programs are tabulated in Appendix 1 of the document.

4.3.3 Results from the Network Model

The results from the network and the non-network models (BCC) are tabulated in Table 4.2, which shows the efficiency scores. It is seen that in the network model the number of DMUs seemed efficient are fewer when compared to the non-network model.

	Network Model	Non Network Model
DMU 1	1.00	1.00
DMU 2	0.71	1.00
DMU 3	1.00	1.00
DMU 4	0.92	0.94
DMU 5	0.69	1.00
DMU 6	0.65	1.00
DMU 7	0.82	1.00
DMU 8	1.00	1.00
DMU 9	0.90	1.00
DMU 10	1.00	1.00
DMU 11	0.75	1.00
DMU 12	0.77	0.98
DMU 13	1.00	1.00
DMU 14	0.62	1.00
DMU 15	1.00	1.00
DMU 16	0.99	1.00
DMU 17	0.74	0.77
DMU 18	1.00	1.00
DMU 19	0.73	0.87
DMU 20	0.84	0.84
DMU 21	0.78	0.79
DMU 22	0.81	0.92
DMU 23	1.00	1.00
DMU 24	0.85	0.89
DMU 25	1.00	1.00
DMU 26	0.94	0.94
DMU 27	0.80	1.00
DMU 28	1.00	1.00
DMU 29	1.00	1.00
DMU 30	0.69	0.80
DMU 31	0.99	1.00
DMU 32	0.82	0.96

DMU 33	1.00	1.00
DMU 34	0.79	1.00
DMU 35	0.86	1.00
DMU 36	0.82	0.86
DMU 37	0.68	0.88
DMU 38	1.00	1.00
DMU 39	0.86	0.94
DMU 40	0.62	0.73
DMU 41	0.70	1.00
DMU 42	1.00	1.00
DMU 43	0.97	1.00
DMU 44	1.00	1.00
DMU 45	0.75	0.95
DMU 46	0.85	0.93
DMU 47	0.82	0.95
DMU 48	1.00	1.00
DMU 49	0.69	0.81
DMU 50	1.00	1.00

Table 4.2 Comparisons of the efficiency scores between the Network and the Non-Network Models

Out of a total of 50 DMUs, 17 DMUs were found efficient in the network model and 30 DMUs were found efficient in the non-network model (BCC). The descriptive statistics for the two models are tabulated in Table 4.3. It is also seen that the mean of the Non Network model is higher than the Non Network model (BCC). It was also found that the efficiency score of the DMUs in the network model is always lower or equal to the efficiency score of the non-network (BCC) model.

	Network Model	Non-Network Model
Mean	0.86	0.96
Median	0.86	1.00
Standard deviation	0.13	0.07
Variance	0.02	0.01
Maximum	1.00	1.00
Minimum	0.62	0.73
Count	50.00	50.00

Table 4.3 Descriptive Statistics of the Network and the Non-Network (BCC) Model

To check if the results obtained from the Network model are significantly different from the Non Network (BCC) model, the paired two-sample t-test was used. The results of the test can be seen in Table 4.4.

	Non Network (BCC) Model Efficiency	Network Model Efficiency
Mean	0.96	0.86
Variance	0.01	0.02
Observations	50.00	50.00
Pearson Correlation	0.52	
Hypothesized Mean Difference	0.00	
Df	49.00	
t Stat	5.89	
P(T<=t) one-tail	0.00	
t Critical one-tail	1.68	

Table 4.4 Results of Paired Two-Sample t-tests comparing the Efficiency Scores of the Network Model and the Non Network Model.

In the above case the null hypothesis is rejected and the alternate hypothesis that there is a significant difference between the mean scores of the Network model and the Non Network model is accepted. In Figures 4.6(a) and 4.6(b) the comparisons between the efficiency scores of the Network model and the Non-network (BCC) model are seen. It can be seen in the graphs that the efficiency score of the network model is always equal to or lower than that of the non-network (BCC) model.

It is also seen that that there are more DMUs that are efficient in the non-network (BCC) model than in the network model. In the Non-Network model 60% of the DMUs are efficient and only 34% of the DMUs are efficient in the Network model. In DEA when there is the present of large number of inputs and outputs then more DMUs are deemed efficient. In certain cases when the majority of the DMUs are deemed efficient then whole model becomes invalid, as the number of inefficient DMUs become less. The Network model takes care of that effect and the number of DMUs that are deemed efficient are lesser compared to the Non Network (BCC) model.

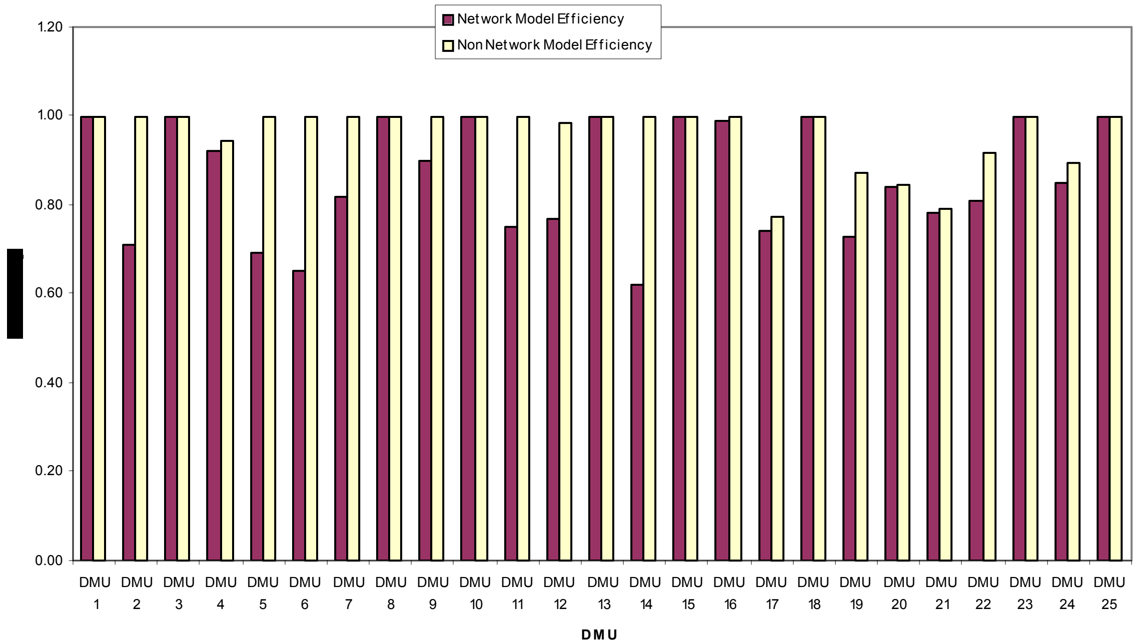


Figure 4.6a Comparison of the efficiency scores between the network and the non-network model

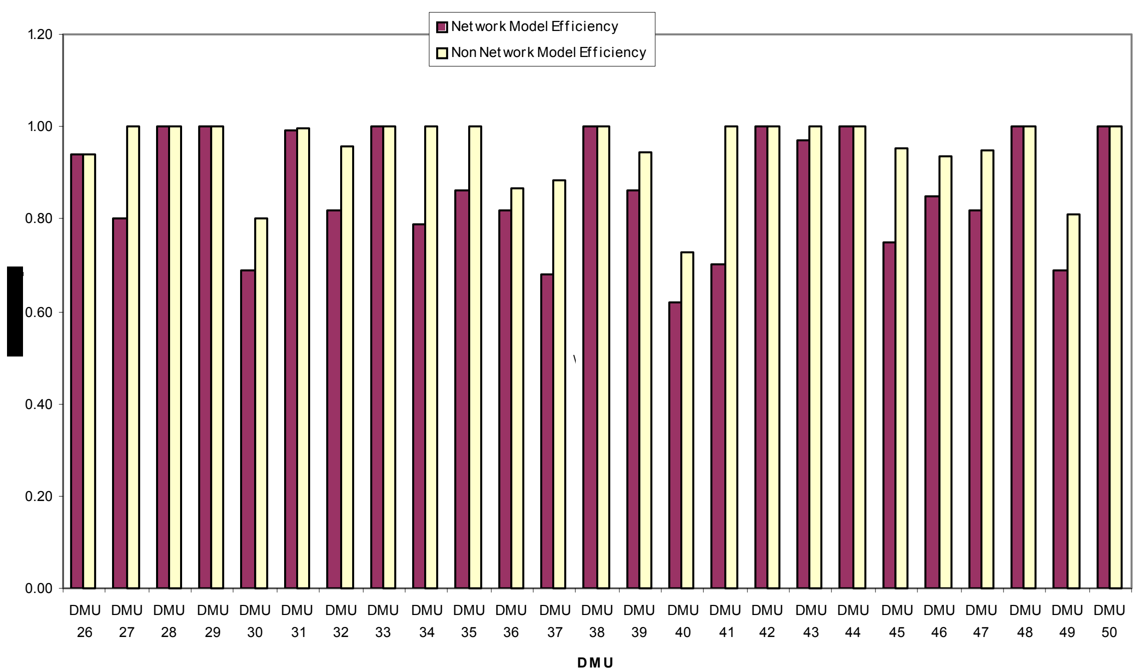


Figure 4.6b Comparison of the efficiency scores between the network and the non-network (BCC) model

4.3.4 Results of the Fuzzy Network Model

In this section the Fuzzy DEA Network Model that was explained in the section 3.2.4 is used to evaluate the CSRs. The results were compared with that of the the DEA Network Model. In the table below the efficiency scores of the two models are tabulated.

DMU	DEA Network Model (Efficiency)	Fuzzy DEA Network Model (Efficiency)	Level of Satisfaction
DMU 1	1.00	1	1
DMU 2	0.71	1	0.2
DMU 3	1.00	1	1
DMU 4	0.92	1	0.2
DMU 5	0.69	1	0.5
DMU 6	0.65	1	0.2
DMU 7	0.82	1	0.5
DMU 8	1.00	1	1
DMU 9	0.90	1	0.5
DMU 10	1.00	1	1
DMU 11	0.75	1	0.5
DMU 12	0.77	1	0.5
DMU 13	1.00	1	1
DMU 14	0.62	1	0.2
DMU 15	1.00	1	1
DMU 16	0.99	1	0.4
DMU 17	0.74	1	0.2
DMU 18	1.00	1	1
DMU 19	0.73	1	0.2
DMU 20	0.84	1	0.3
DMU 21	0.78	1	0.3
DMU 22	0.81	1	0.2
DMU 23	1.00	1	1
DMU 24	0.85	1	0.3
DMU 25	1.00	1	1
DMU 26	0.94	1	0.2
DMU 27	0.80	1	0.1
DMU 28	1.00	1	1
DMU 29	1.00	1	1
DMU 30	0.69	1	0.2
DMU 31	0.99	1	0.2
DMU 32	0.82	1	0.5
DMU 33	1.00	1	1
DMU 34	0.79	1	0.5
DMU 35	0.86	1	0.5
DMU 36	0.82	1	0.5

DMU 37	0.68	1	1
DMU 38	1.00	1	1
DMU 39	0.86	1	0.2
DMU 40	0.62	1	0.2
DMU 41	0.70	1	0.2
DMU 42	1.00	1	1
DMU 43	0.97	1	0.2
DMU 44	1.00	1	1
DMU 45	0.75	1	0.6
DMU 46	0.85	1	0.2
DMU 47	0.82	1	0.5
DMU 48	1.00	1	1
DMU 49	0.69	1	0.1
DMU 50	1.00	1	1

Table 4.5 Efficiency Score of the Network model and the Fuzzy Network Model

The third column indicates the membership value. It can also be defined as the degree to which the constraints are not fulfilled. A membership function of 1 indicates that the constraints are fully satisfied and a membership function of 0 indicates that the constraints are not satisfied at all.

To illustrate the insight that the membership function adds let us consider three DMUs 19 and 45.

DMU	(Network Model) Efficiency	Fuzzy Network Model(Fuzzy)	Membership Value
1	1.00	1.00	1
45	0.75	1.00	0.6
40	0.69	1	0.1

Table 4.6 Comparisons of efficiency scores of the network and fuzzy network models

In the case of DMU 1 the efficiency value is 1 in the fuzzy model and 1 in the Network model. The DMU has an efficiency of 1 with level of satisfaction of 1. In the case of the DMU 45, the efficiency score is 0.75 in the Network Model and 1.00 in the fuzzy Network model. It has an efficiency score of 1.00 with a level of satisfaction of 0.6. But in the case of the DMU 40 the efficiency score changes from 0.69 to 1.00 when the level of satisfaction is 0.1. So comparing DMU 45 and 40, it is seen that the DMU 45 is more sensitive to the fuzzy variables than the DMU 40. This will aid the decision maker in giving training/compensation to the CSR. The decision maker could be more lenient with a CSR who has a level of satisfaction 0.6, compared to a level of satisfaction 0.1.

In this data set, all the DMUs are found to be sensitive to the change in membership function and are listed in Table 4.7.

Membership Value	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
DMU 1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
DMU 2	0.71	0.71	0.71	0.71	0.71	0.72	0.73	0.95	1.00	1.00	1.00
DMU 3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
DMU 4	0.92	0.92	0.92	0.92	0.92	0.92	0.93	0.93	1.00	1.00	1.00
DMU 5	0.69	0.75	0.75	0.75	0.84	1.00	1.00	1.00	1.00	1.00	1.00
DMU 6	0.65	0.66	0.68	0.70	0.74	0.79	0.84	0.91	0.98	1.00	1.00
DMU 7	0.82	0.85	0.88	0.92	0.96	1.00	1.00	1.00	1.00	1.00	1.00
DMU 8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
DMU 9	0.90	0.90	0.90	0.90	0.90	1.00	1.00	1.00	1.00	1.00	1.00
DMU 10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
DMU 11	0.75	0.78	0.82	0.86	0.90	0.95	1.00	1.00	1.00	1.00	1.00
DMU 12	0.77	0.77	0.77	0.79	0.85	1.00	1.00	1.00	1.00	1.00	1.00
DMU 13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
DMU 14	0.62	0.64	0.67	0.71	0.75	0.81	0.87	0.94	1.00	1.00	1.00
DMU 15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
DMU 16	0.99	0.99	0.99	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.00
DMU 17	0.74	0.74	0.74	0.74	0.74	0.78	0.85	0.93	1.00	1.00	1.00
DMU 18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
DMU 19	0.73	0.73	0.73	0.73	0.73	0.73	0.73	0.77	1.00	1.00	1.00
DMU 20	0.84	0.84	0.84	0.84	0.84	0.84	0.84	0.84	1.00	1.00	1.00
DMU 21	0.78	0.78	0.78	0.78	0.78	0.78	0.78	0.78	1.00	1.00	1.00
DMU 22	0.81	0.81	0.81	0.81	0.81	0.81	0.81	0.82	1.00	1.00	1.00
DMU 23	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
DMU 24	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	1.00	1.00	1.00
DMU 25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
DMU 26	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.94	1.00	1.00	1.00
DMU 27	0.80	0.80	0.80	0.80	0.80	0.81	0.81	0.85	0.91	1.00	1.00
DMU 28	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
DMU 29	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
DMU 30	0.69	0.69	0.69	0.69	0.69	0.69	0.71	0.95	1.00	1.00	1.00
DMU 31	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	1.00	1.00	1.00
DMU 32	0.82	0.85	0.88	0.92	0.96	1.00	1.00	1.00	1.00	1.00	1.00

DMU 33	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
DMU 34	0.79	0.79	0.79	0.79	0.79	0.79	0.79	0.80	1.00	1.00	1.00
DMU 35	0.86	0.86	0.86	0.87	0.88	0.91	1.00	1.00	1.00	1.00	1.00
DMU 36	0.82	0.85	0.88	0.92	0.96	1.00	1.00	1.00	1.00	1.00	1.00
DMU 37	0.68	0.74	0.83	0.95	1.00	1.00	1.00	1.00	1.00	1.00	1.00
DMU 38	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
DMU 39	0.86	0.86	0.86	0.86	0.86	0.86	0.87	0.87	1.00	1.00	1.00
DMU 40	0.62	0.62	0.62	0.62	0.62	0.62	0.62	0.62	0.91	1.00	1.00
DMU 41	0.70	0.70	0.70	0.71	0.71	0.74	0.77	0.83	1.00	1.00	1.00
DMU 42	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
DMU 43	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	1.00	1.00	1.00
DMU 44	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
DMU 45	0.75	0.78	0.83	0.95	1.00	1.00	1.00	1.00	1.00	1.00	1.00
DMU 46	0.85	0.85	0.85	0.85	0.85	0.86	0.86	0.87	1.00	1.00	1.00
DMU 47	0.82	0.85	0.88	0.92	0.96	1.00	1.00	1.00	1.00	1.00	1.00
DMU 48	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
DMU 49	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.91	1.00	1.00
DMU 50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 4.7 Summaries of the Efficiency Scores of all DMUs when the Membership Value is Changed

Membership Value	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
DMU 5	0.69	0.75	0.75	0.75	0.84	1.00	1.00	1.00	1.00	1.00	1.00
DMU 14	0.62	0.64	0.67	0.71	0.75	0.81	0.87	0.94	1.00	1.00	1.00
DMU 16	0.99	0.99	0.99	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.00
DMU 27	0.80	0.80	0.80	0.80	0.80	0.81	0.81	0.85	0.91	1.00	1.00
DMU 35	0.86	0.86	0.86	0.87	0.88	0.91	1.00	1.00	1.00	1.00	1.00
DMU 36	0.82	0.85	0.88	0.92	0.96	1.00	1.00	1.00	1.00	1.00	1.00
DMU 37	0.68	0.74	0.83	0.95	1.00	1.00	1.00	1.00	1.00	1.00	1.00
DMU 49	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.91	1.00	1.00

Table 4.8 Summaries of the Efficiency Scores of specific DMUS when membership value is changed

To get a better visual understanding of the variation in technical efficiency, certain DMUs are plotted on a graph.

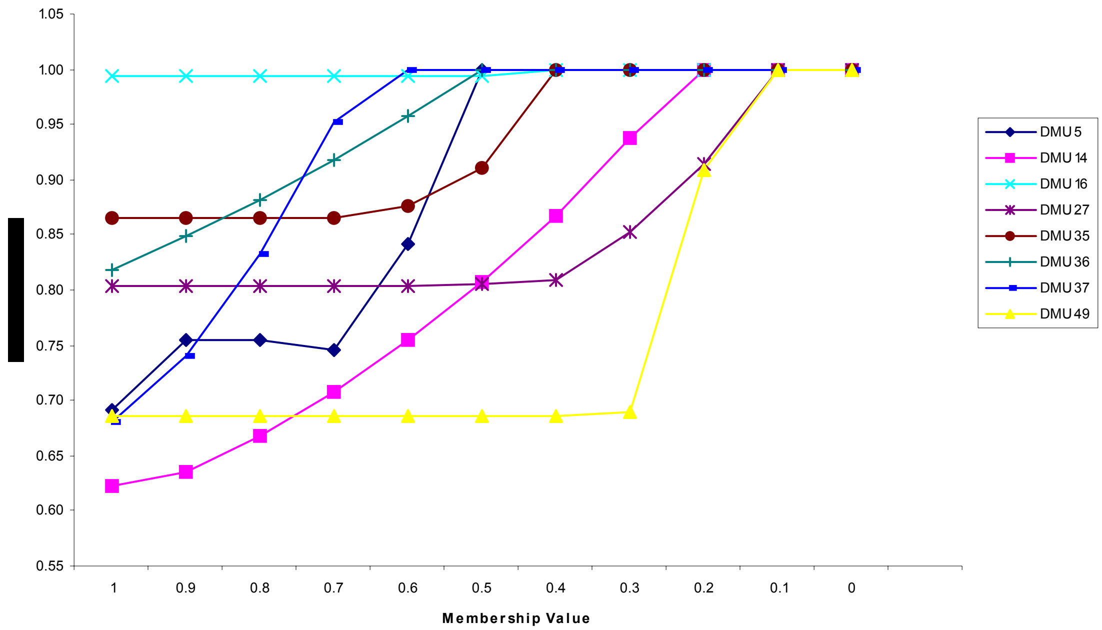
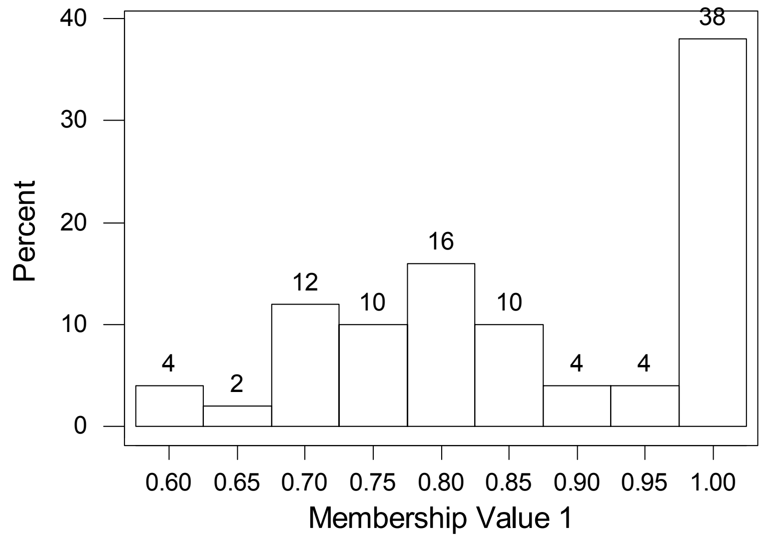
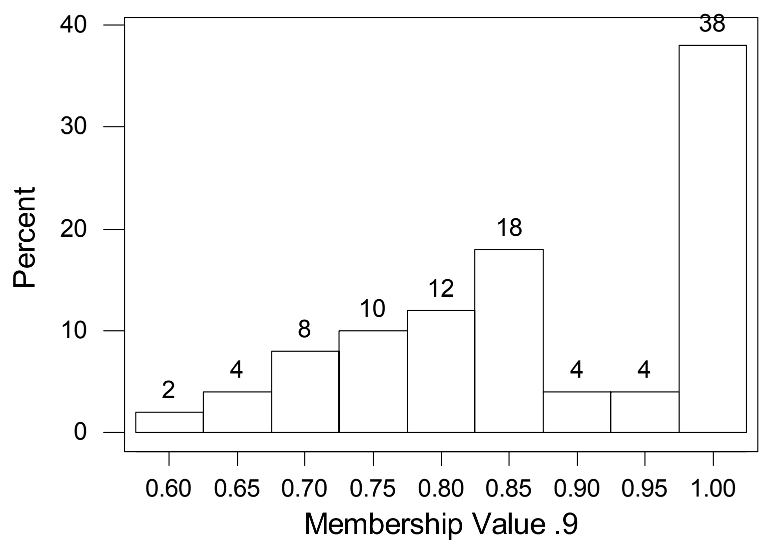
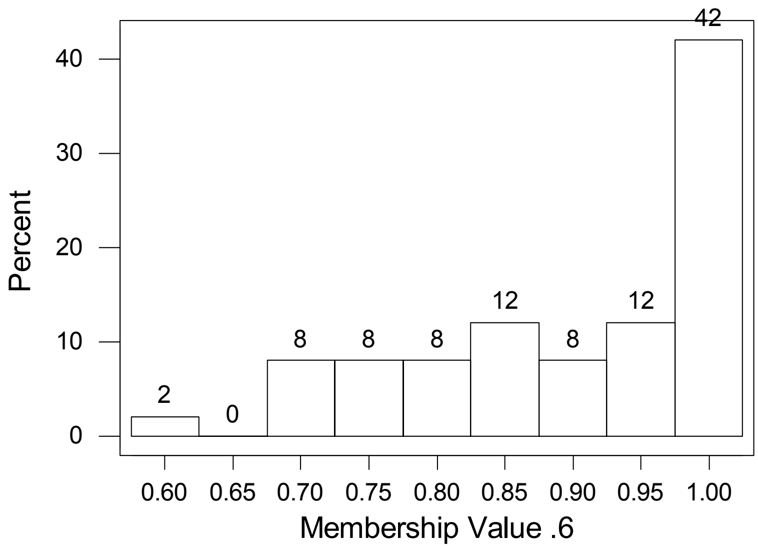


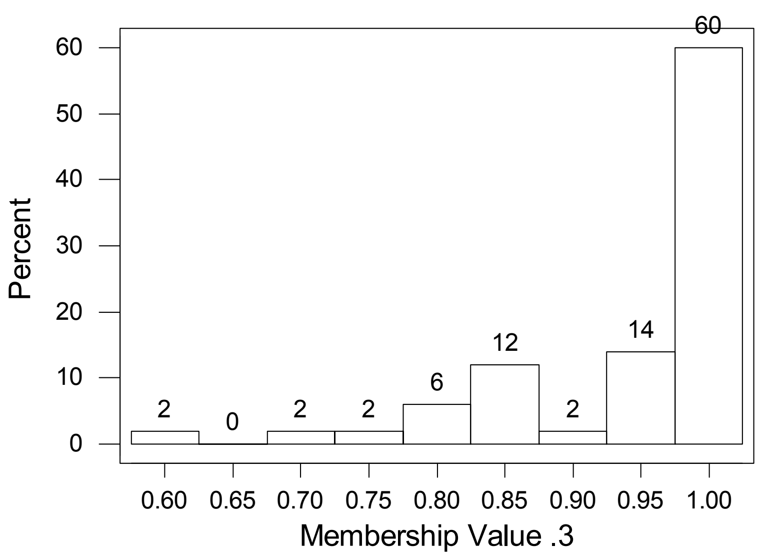
Figure 4.7 Efficiency Variation when the Membership Function is Varied

It can be seen in the graph that all the DMUs plotted in the graph change their efficiency as the membership value is varied between 1 and 0. Certain DMUs are more sensitive to the changes in the membership values compared to the other DMUs It can also seen that DMU 37 is the most sensitive to the change in the membership function than DMU 36. The least variation is that of DMU 16 which changes efficiency from .99 to 1. This whole process helps the decision maker make the right decisions with respect to the DMUs that are inefficient. The decision maker may change the membership function to a particular degree and then calculate the efficiency. In this way the CSR who is inefficient will not be penalized due to the uncertainty that is present in the data. It also gives him the opportunity to isolate these DMUs and deal with them separately. The variation in the percentage of DMUs as the membership function changes can be studied by using the figures below.


40
30
20
10
0
Percent
4
2
12
10
16
10
4
4
38
0.60 0.65 0.70 0.75 0.80 0.85 0.90 0.95 1.00
Membership Value 1


40
30
20
10
0
Percent
2
4
8
10
12
18
4
4
38
0.60 0.65 0.70 0.75 0.80 0.85 0.90 0.95 1.00
Membership Value .9


Percent
Membership Value .6
0
10
20
30
40
0.60 0.65 0.70 0.75 0.80 0.85 0.90 0.95 1.00
2 0 8 8 8 12 8 12 42


Percent
Membership Value .3
0
10
20
30
40
50
60
0.60 0.65 0.70 0.75 0.80 0.85 0.90 0.95 1.00
2 0 2 2 6 12 2 14 60

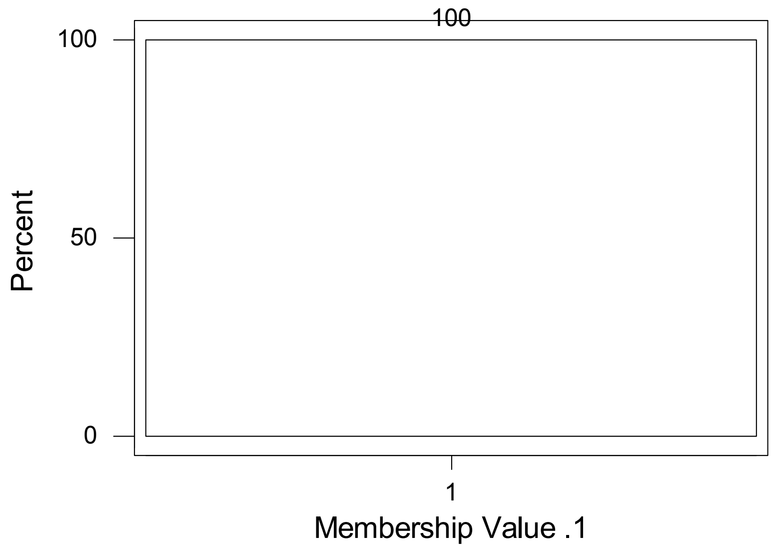


Figure 4.8 (a)-(e) Snap Shot of Technical Efficiency Score Variation

4.3.5 Identifying the Peer groups

When we measure the efficiency of the CSRs, we can also identify the peer members of the inefficient CSR. This will aid the decision maker in grouping the CSR for training sessions.

DMU	**Peer Members**													efficiency
DMU1	0	0	0	0	0	a1	a10	a23	a31	a50	z15	z25	z44	1
DMU2	0	0	0	0	0	0	0	0	a10	a2	a29	z28	z44	0.71
DMU3	0	0	0	0	a10	a23	a29	a3	a5	z18	z25	z3	z48	1
DMU4	0	0	0	0	a1	a10	a14	a23	a31	a4	a5	z28	z44	0.92
DMU5	0	0	0	0	0	0	0	0	0	a23	a5	a8	z44	0.69
DMU6	0	0	0	0	a1	a14	a23	a3	a31	a5	a6	z28	z6	0.65
DMU7	0	0	0	0	0	0	0	a1	a10	a5	z1	z13	z5	0.82

DMU8	0	0	0	0	0	0	0	0	a23	a5	a8	z44	z8	1
DMU9	0	0	0	0	a1	a10	a20	a23	a5	a9	z25	z3	z8	0.9
DMU10	0	0	0	0	0	0	0	0	0	a10	a23	a7	z44	1
DMU11	0	0	0	0	a1	a10	a11	a5	a7	z12	z25	z42	z44	0.75
DMU12	0	0	0	0	0	a1	a11	a12	a14	a29	a5	a50	z28	0.77
DMU13	0	0	0	a11	a12	a23	a31	a36	a5	a50	z13	z18	z44	1
DMU14	a1	a10	a14	a23	a25	a3	a31	a38	a5	a7	z14	z28	z42	0.62
DMU15	0	0	0	0	0	0	a1	a10	a14	a43	a5	a50	z15	1
DMU16	0	0	0	0	a10	a16	a31	a36	a43	a5	z18	z28	z44	0.99
DMU17	0	0	0	a1	a17	a23	a26	a29	a5	a8	z18	z3	z8	0.74
DMU18	0	0	0	0	a1	a10	a14	a17	a18	a29	a8	z18	z28	1
DMU19	0	0	0	0	0	a12	a19	a23	a28	a31	a43	a5	z28	0.73
DMU20	0	0	0	0	a1	a14	a19	a23	a31	a5	z18	z28	z44	0.84
DMU21	0	0	0	a1	a14	a23	a29	a31	a5	a8	z18	z28	z44	0.78
DMU22	0	0	0	0	a10	a14	a22	a23	a28	a5	a50	z18	z28	0.81
DMU23	0	0	0	0	0	0	0	a10	a23	a29	a3	a38	z13	1
DMU24	0	0	0	0	0	0	a29	a31	a38	a5	z18	z28	z44	0.85
DMU25	0	0	0	0	0	0	0	a14	a31	a38	a43	a5	z25	1
DMU26	0	0	a1	a20	a25	a26	a29	a43	a5	a8	z18	z28	z44	0.94
DMU27	0	0	a10	a14	a20	a25	a27	a29	a31	a43	a5	z28	z44	0.8
DMU28	0	0	0	0	0	0	0	a10	a14	a31	a38	a43	z28	1
DMU29	0	0	0	0	0	0	a10	a23	a29	a38	a5	z12	z44	1
DMU30	0	0	0	0	0	0	a1	a10	a30	a40	a5	z28	z44	0.69
DMU31	0	0	0	0	0	a10	a14	a26	a29	a31	a8	z28	z44	0.99
DMU32	0	0	0	0	0	a23	a29	a31	a5	a50	a8	z28	z44	0.82
DMU33	0	0	0	0	0	a1	a14	a23	a3	a33	a50	z25	z44	1
DMU34	0	0	0	0	0	0	a10	a23	a29	a34	z18	z28	z44	0.79
DMU35	a1	a10	a14	a29	a31	a35	a36	a38	a5	a8	z18	z28	z44	0.86
DMU36	a1	a23	a31	a35	a36	a43	a5	a50	a8	z25	z42	z43	z5	0.82
DMU37	0	0	0	a1	a10	a23	a37	a43	a5	z14	z25	z28	z8	0.68
DMU38	0	0	0	0	0	0	0	a23	a27	a29	a38	a5	z44	1
DMU39	0	0	0	0	a10	a27	a29	a31	a39	a5	z18	z44	z8	0.86
DMU40	0	0	0	0	0	a1	a23	a27	a31	a40	a5	z18	z28	0.62
DMU41	0	0	a1	a10	a27	a3	a31	a38	a41	a5	z28	z44	z8	0.7
DMU42	0	0	0	0	a10	a14	a25	a27	a29	a3	a41	a50	z42	1
DMU43	0	0	0	0	0	a14	a23	a27	a29	a43	a5	z28	z44	0.97
DMU44	0	0	0	0	0	a1	a10	a23	a27	a5	a50	z18	z44	1
DMU45	0	a1	a10	a23	a27	a3	a31	a38	a45	a5	z12	z42	z44	0.75

DMU46	0	0	0	0	a27	a3	a31	a43	a46	a5	a50	z44	z8	0.85
DMU47	0	a27	a29	a3	a31	a43	a46	a47	a5	a50	z14	z28	z42	0.82
DMU48	0	0	0	a14	a27	a31	a36	a43	a5	z3	z44	z48	z8	1
DMU49	0	0	a1	a14	a2	a25	a27	a31	a36	a49	a5	z28	z44	0.69
DMU50	0	0	0	0	0	0	0	0	a23	a36	a50	z25	z44	1

Figure 4.9 Peer Members of the CSRs when the level of satisfaction is 1

In this figure if there is a1 besides a specific DMU, then it indicates that that DMU has the DMU 1 as a peer member representing service quality node. If there is z1, then it indicates that DMU 1 is a peer member in the overall production node. The level of satisfaction can also be changed and the changes in the peer group can be studied.

4.3.5 Statistical Validation of the Fuzzy DEA Network model

To check that the efficiency scores obtained from the Fuzzy DEA Network model is significantly different from the DEA Network model, the paired two-sample t-test is applied.

	Fuzzy Model	*Precise Model*
Mean	1	0.864336
Variance	0	0.016458
Observations	50	50
Hypothesized Mean Difference	0	
Df	49	
t Stat	7.47747756	
P(T<=t) one-tail	6.0617E-10	
t Critical one-tail	1.67655116	

Table 4.9 Results of Paired two-sample t-tests comparing efficiency scores of the DEA Network model and the Fuzzy DEA Network model with membership function 0.

	Fuzzy Model	*Precise Model*
Mean	0.94249799	0.864336013
Variance	0.00825446	0.01645845
Observations	50	50
Hypothesized Mean Difference	0	
Df	49	
t Stat	5.06056583	
P(T<=t) one-tail	3.1414E-06	
t Critical one-tail	1.67655116	

Table 4.10 Results of Paired two-sample t-tests comparing efficiency scores of Fuzzy DEA Network model and the DEA Network model with membership function 0.3.

	Fuzzy Model	*Precise Model*
Mean	0.899620234	0.864336013
Variance	0.012770133	0.01645845
Observations	50	50
Hypothesized Mean Difference	0	
Df	49	
t Stat	3.445042143	
P(T<=t) one-tail	0.000589872	
t Critical one-tail	1.676551165	

Table 4.11 Results of Paired two-sample t-tests comparing the efficiency scores of DEA Network model and the Fuzzy DEA Network model with membership function 0.6.

	Fuzzy Model	Precise Model
Mean	0.871069535	0.864336013
Variance	0.014988673	0.01645845
Observations	50	50
Hypothesized Mean Difference	0	
Df	49	
t Stat	3.136260203	
P(T<=t) one-tail	0.001446105	
t Critical one-tail	1.676551165	

Table 4.12 Results of Paired two-sample t-tests comparing efficiency scores of the DEA Network model and the Fuzzy DEA Network model with membership function 0.9.

In all the above cases we reject the null hypothesis and accept the alternate hypothesis that there is a significant variation between the mean efficiency scores of the fuzzy and the precise model.

4.4 CONCLUSION

It was found that the efficiency score generated by the Network model was always lower or equal to the efficiency score generated by the Non Network model. Then using the Fuzzy DEA network model the variation of the efficiency of the CSRs were studied. It is also found that the mean efficiency scores of the fuzzy model varied significantly when compared with the precise model. The Fuzzy DEA Network model provides an opportunity by which the uncertainty associated with the input variables can be studied. This will help the decision maker make better decisions.

CHAPTER 5 CONCLUSION

In this chapter, the conclusion of this research effort is presented. The first section of this research summarizes the entire research. The next section describes the areas in which future research can be done.

5.1 SUMMARY

5.1.1 Modeling Service Quality using DEA/Network Model

One of the primary objectives of this research was to come up with a performance evaluation model that accounts for service quality using the Data Envelopment Analysis. The Network model was used to capture service quality, and DEA was used to evaluate the performance of CSRs.

The input and the output variables denoting service quality variables was decided using attributes from the SERVQUAL model designed by Parasuraman, Zeithaml, and Berry (1985). The main construct in the model is reliability, responsiveness, assurance, empathy and tangibles. Kayanma and Black (2000) used the framework of SERVQUAL to capture the service quality of Online Travel Agencies.

The DEA /Network model had been previously used to model instances, where there were sub processes involved. Färe (1991) proposed the network model and modeled the intermediate output. Färe and Grosskopf (1996) extended the concept and introduced a model in which the inputs need not be allocated efficiently between the nodes. Färe and Whittaker (1995) also implemented the network model to model the production process of a diary farm. In one instance, Lõthgren and Tambour (1999) applied the network model to incorporate customer satisfaction as a sub process in the main process.

In this research, the DEA/ Network model was used to model service quality as a sub process to the main production process. The Network Model introduced in this research is unique to the literature.

Some of the advantages of the Network Model are listed as follows:

One of the advantages in using the Network model is that multiple processes can be modeled in it.

The Network Model also takes care of the issue of dimensionality. When the number of input and the output variables are more, then the number of DMUs that were deemed efficient was more. The number of DMUs that were efficient is fewer in the Network Model compared to the Non Network Model.

5.1.2 Capturing uncertainty in Service Quality variables using Fuzzy Set theory

The second objective was to capture the uncertainty associated perceived among the input and the output quality variables using fuzzy set theory. Sengupta (1992) had explored the use of fuzzy theory in the context of DEA. Later Triantis and Girod (1998) developed a model for measuring technical efficiency in a Fuzzy Environment.

In this research, the uncertainty associated with the service quality variables is captured using Fuzzy sets. Linguistic variables like “Pleasant”, “ More Pleasant” is modeled into the model using fuzzy sets. In the present research, Triangular membership functions are employed to capture these quality variables. Because of the uncertainty associated with some of the input and the output variables, the model has fuzzy constraints.

In the model that Sengupta proposed, the objective function and the constraints were fuzzy. Zimmerman (1996) had suggested an Unsymmetrical Fuzzy LP, which was used in this DEA/Network Model

5.1.2 Evaluating the efficiency of CSRs in a call center

The third objective of the research was to implement the model to the data that was collected at US Airways. The call center in an airline is the first point of contact and is the point where the sale is made. So maintaining a very high standard of service becomes imperative for an airline company. The model was developed, by considering service quality a sub process of the overall process. The model focused on reducing the amount of input used and so the input reducing DEA model was used.

The input variables and the output variables in the whole process are identified. The constructs in the SERVQUAL model are used as a guiding tool to identify the service quality variables. The data that was collected in the first quarter of 2002 was then run using the model.

The number of CSRs in the study was 50. The Fuzzy /DEA/Network model, and BCC models were implemented and the results were tabulated and presented graphically in various charts.

5.1.3 Provide Insights on the data set

The fourth objective of the research was to suggest improvements on the allocation of input resources and to focus on the unique insight that the model that the model provided. Based on the results obtained from running the models, the inefficient CSRs were identified and improvements were suggested. The peer members for the CSRs were also identified. The advantage of identifying the peer members is that the inefficient CSRs can be grouped with their more efficient CSRs. Grouping the CSRs in like groups will also help the company in training them more effectively.

The CSRs that were most affected by the fuzzy variables were also identified. This will aid the decision maker in making the right decisions when evaluating the performance of the CSR. A term "Level of satisfaction" was introduced in the model. This term in addition to the efficiency score provides a realistic view on the efficiency score of the CSR. The change in the efficiency score when the "Level of satisfaction" is varied can be viewed graphically in the graphs to help the decision maker make better decisions.

In addition, the fuzzy model helps in identifying the borderline DMUs. These are the DMUs that move from the inefficient set to the efficient set when the constraints are relaxed.

5.1.4 Validation of the Models Developed

In the first scenario, the DEA/ Network Model is compared with the BCC model. It was statistically validated using the paired two-sample t-tests that there was a difference among the results. In the next case, the fuzzy model is compared with the crisp model.

Using the paired two-sample t-tests, it was found that the two models varied statistically.

5.2 RECOMMENDATION FOR FUTURE RESEARCH

The current research can be investigated and extended to a great degree in the area of Fuzzy DEA.

Research could be done on the membership function that was applied in the fuzzy model. Research needs to be done on the impact of the form of the membership function on the efficiency score. The linear membership function may not be the most suitable membership function in all cases. The membership function that was chosen depends on the application and the decision maker. The triangular membership function was used in this research work. Research needs to be done on the various kinds of membership functions that could be employed. As the membership functions are presently defined, the input and the output variables vary linearly. A study needs to be conducted when the membership functions are non linear. The model could be run on various models and scenarios. There might be a more appropriate membership function that can be used in the other scenarios like the S shaped membership, trapezoidal membership function etc.

Research could also be done to include multiple linguistic variables in the model. In the present research, when modeling the service quality variables three options were modeled like “pleasant”, “unpleasant” and “most unpleasant”. Models can be created that has more than three options in the membership function.

The model that was developed, in this research work has been applied to only one application i.e., the data that was collected at the call center of a major Airline. This model needs to be applied in varied situations. By applying it to various other scenarios, it would enable us to better validate the Fuzzy DEA/Network model.

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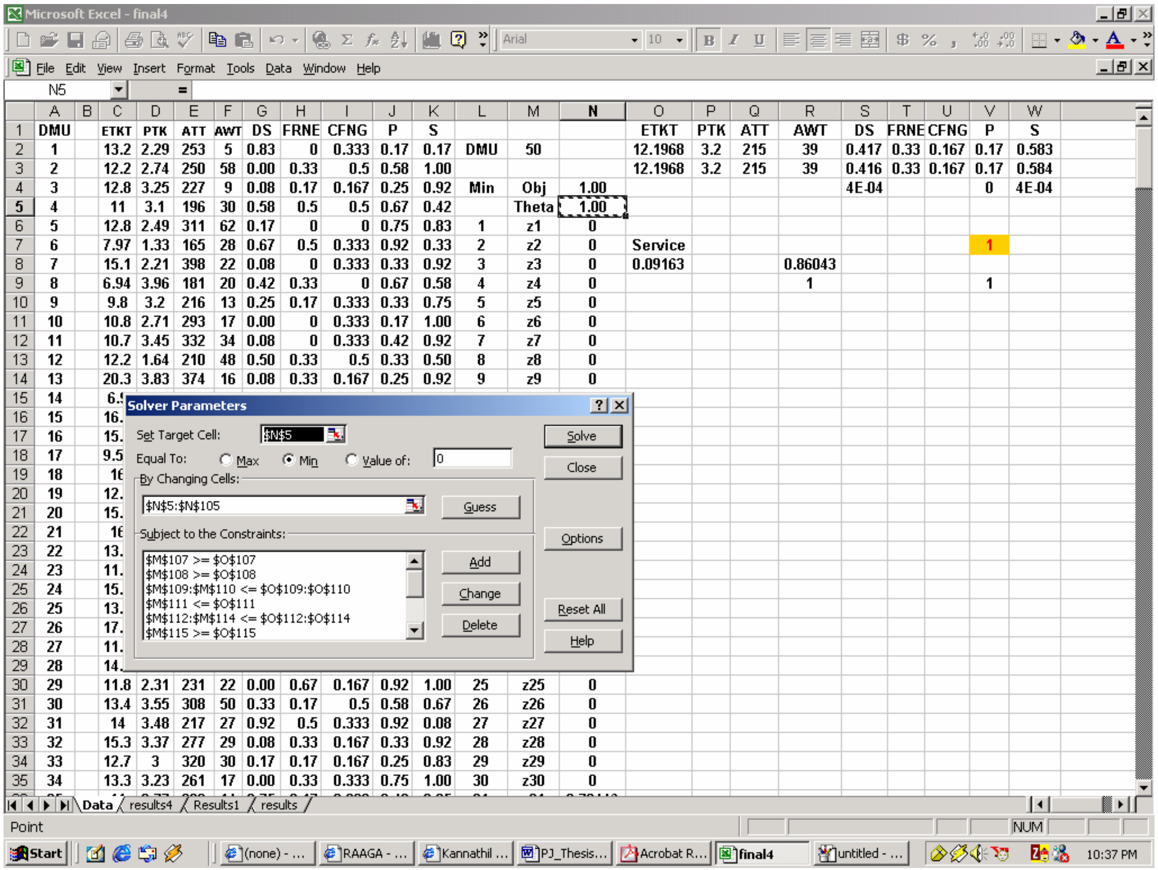
Appendix 1


Microsoft Excel - final4
Solver Parameters
Set Target Cell: $N$5
Equal To: Max Min Value of: 0
By Changing Cells:
$N$5:$N$105
Subject to the Constraints:
$M$107 >= $O$107
$M$108 >= $O$108
$M$109:$M$110 <= $O$109:$O$110
$M$111 <= $O$111
$M$112:$M$114 <= $O$112:$O$114
$M$115 >= $O$115
Solve
Close
Guess
Options
Add
Change
Delete
Reset All
Help

Macro 1

```
Sub themain()
'
' themain Macro
' Macro recorded 7/14/2002 by
'
Sheets("Results").Select
    Range("A1").Select
    For n = 1 To 50
    Sheets("Data").Select
    Range("M2").Select
    Selection.Value = n
    SolverSolve (True)
    Range("n4:n105").Select
    Selection.Copy
    Sheets("Results").Select
    Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:= _
        False, Transpose:=False
    Range("A1").Offset(0, n).Select
    Next n
'
End Sub
```

Macro 2

```
Dim Number, Digits, MyString

    Sub Macro1()

    For n = 1 To 50


    Call second(n)

    Next n


    End Sub
    Sub second(n)

    Sheets("Data").Select
```

```
    Range("M2").Select
    Selection.Value = n
    Sheets("Data").Select
    SolverSolve (True)



    'Get and store the value for 0
    Sheets("Data").Select

    Range("V7").Select
    Selection.Value = 0
    SolverSolve (True)
    ObjectiveValueAtZero = Sheets("Data").Range("N4").Value
       Sheets("Results4").Select
       Range("A4").Offset(o, n - 1).Select
       Selection.Value = "" & ObjectiveValueAtZero


'Get and store the value for 1
    Sheets("Data").Select
    Range("V7").Select
    Selection.Value = 1
    SolverSolve (True)
    ObjectiveValueAtOne = Sheets("Data").Range("N4").Value
   Sheets("Results4").Select
     Range("A5").Offset(o, n - 1).Select
     Selection.Value = "" & ObjectiveValueAtOne

    If ObjectiveValueAtOne = 1 Then Exit Sub


    If ObjectiveValueAtZero < 1 Then Exit Sub


'If both the above case are not true then follow the half section method
    LowerValue = 0
    UpperValue = 1
    NumberOfIterations = 0
    Sheets("Results1").Select
    Range("A20").Select
    Selection.Value = "LowerValue"
    Range("A21").Select
    Selection.Value = "UpperValue"
    Range("A22").Select
    Selection.Value = "Obj Value"
```

```
'Set Accuracy value here
    Accuracy = 0.001
    Do While (UpperValue - LowerValue) > Accuracy

        NumberOfIterations = NumberOfIterations + 1
        Sheets("Data").Select
        Range("V7").Select
        InBetweenValue = ((UpperValue + LowerValue) / 2)
        Selection.Value = InBetweenValue
        SolverSolve (True)
        ObjectiveValueInBetween = Sheets("Data").Range("N4").Value
        If ObjectiveValueInBetween < 1 Then
            LowerValue = LowerValue
            UpperValue = InBetweenValue
        Else
            LowerValue = InBetweenValue
            UpperValue = UpperValue
        End If
        Sheets("Results1").Select
        Range("A20").Offset(0, NumberOfIterations).Select
        Selection.Value = LowerValue
        Range("A21").Offset(0, NumberOfIterations).Select
        Selection.Value = UpperValue
        Range("A22").Offset(0, NumberOfIterations).Select
        Selection.Value = ObjectiveValueInBetween
    Loop

        Sheets("Results4").Select
        Range("A1").Offset(o, n - 1).Select
        Selection.Value = "" & LowerValue
        Range("A2").Offset(o, n - 1).Select
        Selection.Value = "" & UpperValue

        Sheets("Results1").Select
        Range("A25").Select




    End Sub
```

VITA

Abhishek Poykayil Jayananda Panicker was born in Coimbatore, India in 1978. He graduated with his Mechanical Engineering from Sri Ramakrishna Engineering College in1999. He enrolled for his Masters Degree in Industrial and Systems Engineering at Virginia Tech in fall 1999.

In the summer of 2000, he enrolled in the internship program a major Airline company to gain corporate experience. He worked as a Database Specialist and a Lead performance Analyst in the seven months that he worked there. Back in school, the whole of 2001 he spent his time working as research assistant, working on the Supply Chain of Lockheed Martin. In the spring of 2002, he went back to US Airways and did his third rotation. In summer of 2002, he obtained a full time position as a Senior Analyst and is presently working at US Airways.