# Measurement System and Campaign for Characterizing of Theoretical Capacity and Cross-Correlation of Multiple-Input Multiple Output Indoor Wireless Channels

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# Abstract

The demands for greater capacity and lower transmitted power have historically motivated research in spatial diversity systems. Diversity techniques have been implemented in many current systems and have been shown to reduce the transmit power required to maintain acceptable system performance. Traditionally spatial diversity is based on the transmission and reception of a single stream of symbols through independent and spatially separated propagation channels. In more recent developments, space-time coding and array processing techniques use diversity concepts to resolve multiple independent streams of data and increase the potential data-rate. This new *space-time* research investigates the unprecedented ability to simultaneously transmit separate data streams from many closely-spaced antennas on a common carrier frequency. The effectiveness of these multi-element arrays in communication systems has been found to depend on antenna design and specific characteristics of the propagation channels. This thesis describes an effort to characterize an indoor office environment with respect to these applications.

Theoretical analyses have demonstrated a relationship between the theoretical capacity of multi-element array systems with the cross-correlation of spatially separated channels. Historical measurements have also shown that in the presence of Rayleigh fading, antenna spacing may be used to control the level of correlation between propagation channels and maximize the diversity gain, or potential system capacity of a space-time system. Both the design of the antenna arrays and characteristics of the propagation environment influence a system's potential capacity.

This thesis describes the construction of a measurement system and the use of this system to evaluate the capacity gains of multi-element arrays in a wireless communication system. The presented system is capable of measuring the channel gains between a number of transmitter and receiver antenna elements and calculating both the cross-correlation between channel gains and the theoretical system capacity. After a discussion of previous research, the measurement system and subsequent measurement results are described.

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# Chapter 1

# Introduction

### **1.1** Motivation

Mobile communication systems are continuously evolving to satisfy the growing consumer demand for higher data rates and a larger coverage area. Current estimates suggest there are over 114,000 cellular sites in the United States and that this number has grown by a factor of almost six since 1995 [1]. In December 2000, there were over 100 million cellular subscribers in the United States, with a 27% annual growth rate [2]. By 2002, third generation cellular networks will provide data rates of up to 2 Mbps, and offer applications such as internet browsing and video conferencing [3]. Current wireless networks support a large consumer population with significant demands on performance and coverage.

In contrast to modern networks, the original wireless systems provided limited performance and accommodated a select niche market. The first widely available portable systems were installed on ships for military and emergency operations, and by 1915 ship-to-ship and ship-to-shore communications were available on over two thousand navy and merchant vessels [4]. The system's transmitter consisted of an alternator, capable of producing radio frequencies up to 100kHz. This carrier wave signal was modulated with Morse code data and the system required between a 100 and 130 meter long antenna, hung from the ship's mast. Data rates were limited by the operator's skill, and the number of simultaneous, co-located communications was constrained by a few simple tuning components in the receiver [4]. Although wireless systems provided beneficial emergency and navigational communications on the sea, the quality of these systems was very limited compared to modern cellular networks.

Numerous innovations in the fields of electronics and computing enabled the development of these early wireless systems into the mobile devices available today. The binary keying of telegraphy was replaced with audio telephony, and then higher-keyed digital modulation. Digital modulation enabled engineers to incorporate error correction and data compression algorithms in the communication system [5]. Because a system's potential data rate depends on the received signal's power level, many modifications were implemented to improve the power efficiency of the transmitter amplifiers [6]. Also antennas were improved to point a larger fraction of the transmitted signal in the direction of the receiver [7]. The superheterodyne receiver was implemented to improve frequency selectivity and enable more users to simultaneously transmit over a limited frequency band. In addition, integrated semiconductor chips reduced device sizes and allowed greater complexity in the transmitter and receiver system architectures [8]. These and numerous unmentioned technological advances enabled the development of robust, high data-rate communication systems from the original wireless telegraphy systems of the early twentieth century.

Theoretical studies have predicted that antenna arrays hold even further potential for improved reliability and efficiency, by increasing the number of paths over which the signal propagates between the transmitter and receiver. Several receiver architectures have been developed to implement antenna arrays, such as diversity and space-time systems. Diversity systems contain more than one antenna at either the transmitter or receiver, and transmit data over many signal paths, or propagation channels. The signal passing through each channel fades by a different amount. With a larger number of channels, there is a greater probability that at least one signal will reach the receiver with sufficient power to decode the data. By increasing the probability of reception, diversity systems provide improved performance over single-antenna systems [7].

In more recent efforts, space-time architectures have been proposed which use the concept of diversity to improve the communication system's data rate. Instead of improving reliability by transmitting identical signals over many propagation channels, space-time systems transmit multiple signals simultaneously. The receiver uses the fading characteristics of each propagation channel to distinguish between the transmitted signals and recover each signal's independent data stream. With this approach, space-time systems, such as the space-time coding and the BLAST architectures attempt to improve data rates by simultaneously transmitting multiple signals over the same frequency band [9].

Both diversity and space-time systems assume that the propagation channels produced by spatially separated array-elements have unique fading characteristics [9]. Measurements can compare the fading characteristics between different array-elements and quantify the validity of this assumption. Measured data is helpful in designing a space-time system and necessary in predicting its data rate in a particular environment. Although numerous measurement campaigns have focused on diversity systems, with an antenna array at either the transmitter or receiver, few have implemented arrays at both ends of the propagation channel. This goal has been accomplished in this thesis work.

## **1.2** Overview and Purpose

This thesis describes the construction of a measurement system and the use of this system to evaluate the potential benefit of space-time architectures. The measurement system is composed of a transmitter and a receiver, and each contains a sixteen-element antenna array and sixteen independent signal chains. It simultaneously measures the propagation loss between each pair of array-elements at an RF frequency of 2.111GHz, and with a bandwidth of 32kHz. The measurement data is then statistically analyzed to predict the performance of multi-element array systems such as space-time architectures. A measurement campaign was conducted in an office building to verify the operation of the system and demonstrate its operation.

The second chapter presents an overview of diversity and space-time systems, and represents the result of a literature study. The chapter explains several of the recombination algorithms found in diversity techniques and significant space-time architectures, including space-time coding and the BLAST system. While using different algorithms to process the transmitted signal, both systems make similar assumptions about the propagation environment [9]. An explanation of these assumptions serves as an introduction to the propagation environment and the required characteristics of array-systems. The presented literature focuses on the system's operation, the potential for performance gains and their propagation requirements.

The third chapter introduces significant terminology used to evaluate propagation channels for array applications. The *cross-correlation* between the gains of two channels and the *multi-element array (MEA) capacity* can be used to predict the performance of diversity or space-time systems. The cross-correlation compares the fading characteristics of two propagation channels, and the MEA capacity estimates the theoretical data-rate which can be effectively transmitted between two antenna arrays [10]. The algorithms used to process measured data are presented and example calculations are provided. This chapter introduces terminology and calculations significant in the design of the constructed measurement system and subsequent measurement campaign.

Chapter 4 describes the design and construction of the measurement system. This system contains 16-element antenna arrays at both the transmitter and receiver, with an independent signal chain attached to each array element. Each transmitter array element produces a single frequency tone. The complex amplitude of each tone at each receiver element is recorded and processed to calculate the propagation loss between each of the 256 pairs of transmitter and receiver antenna elements. Chapter 4 describes the operation of both the baseband processing and the RF front end of the measurement system. Chapter 5 details the calibration procedure, which verifies the accuracy of the recorded data and the subsequent calculations.

A measurement campaign was conducted to measure correlation and theoretical capacity in the indoor office-building environment. Chapter 6 provides a detailed description of the measurements, conducted in Durham Hall on the Virginia Polytechnic Institute and State University campus. Specific measurement sites included small offices, a hallway, a large cubicle-partitioned room, and a lecture hall. The results, reported in Chapter 7, provide a sample of the capacity and correlation that may be available in these environments when using arrays with closely spaced elements.

This thesis describes research conducted to further understand the potential performance of multi-element array systems. A measurement system was constructed and measurements were conducted to observe the multi-element array capacity and the cross-correlation between propagation channels in an indoor office building. After an overview of previous investigations and relevant terminology, a detailed description of the measurement system and measurement campaign is provided, followed by a summary of significant results. This thesis work provides a measurement system that can be used to predict the performance of the emerging technology of space-time systems.

# Chapter 2

## Review of Previous Investigations

Many investigators in the telecommunications industry believe that antenna arrays hold enormous potential to improve the performance of wireless systems. In theory, a different data stream can be transmitted from each array element, producing a multiplicative gain in data rate over a system with only one transmitter antenna. In practice, this performance gain is limited by the system's ability to isolate each data stream from the received signal. Recent investigations in space-time architectures attempt to achieve this gain and separate the signals using array-processing and signal coding techniques, but performance is dependent on favorable characteristics in the propagation environment [9].

Space-time systems transmit a different signal from each element in the transmitter array. After propagating through their respective propagation channels, the signals are combined at each receiver array element. A propagation channel can be defined as the environmental conditions which affect a signal passing from one transmitter array element to one receiver array element. In this thesis, each pair of transmitter and receiver array elements defines a propagation channel. Space-time architectures assume that the fading characteristics of each channel are unique, and these systems use the differences in fading loss to separate the transmitted signals at the receiver [9].

The comparison of closely spaced propagation channels has been a topic of research for over three decades due to its importance in diversity techniques. Similar to space-time systems, diversity techniques transmit signals through multiple propagation channels and benefit from differences in their fading characteristics [7]. This chapter reviews diversity techniques, the results of this propagation research and the implementation of space-time systems. Significant algorithms used in diversity and space-time systems are included.

## **2.1** Diversity Techniques and Research

Much of the early work on antenna arrays and propagation through multiple channels was conducted to evaluate the performance gains provided by spatial diversity. The results from numerous theoretical analyses and measurement campaigns were published on this subject by the early 1970's. These studies analyzed the increase in reliability produced by augmenting either the transmitter or receiver with an additional antenna [7].

In a system with one transmitter and one receiver antenna, a communications outage occurs when the propagating signal fades below the minimum detectable power level of the receiver, preventing the receiver from effectively decoding the data. It was found that extra antennas serve to increase the number of paths over which the signal propagates from the transmitter to the receiver. Extra paths raise the probability that at least one received signal remains above the minimum detectable power level, improving the reliability of the system [7]. This section provides an overview of the environmental conditions, the benchmarks and common system algorithms pertaining to diversity systems.

### 2.1.1 The Rayleigh Environment

Mobile communication systems often operate in environments where the receiver is surrounded by a large number of scattering objects. In addition to the *line-of-sight* path, in which the signal travels directly from the transmitter to the receiver, multiple paths exist between the antennas that include reflections off scattering objects. A transmitted signal propagates along all existing paths between the transmitter and receiver antennas, and produces a number of *multipath components* at the receiver. Each of the multipath components arrives at the receiver antenna with a different amplitude and phase, depending on the path length and characteristics of the scattering objects. The received signal is a combination of the multipath components, and depending on the phase of each component, the combined signals can constructively interfere to produce a high received power, or destructively interfere, resulting in a smaller signal power. Movement in the propagation channel changes the amplitude and phase of each multipath component, causing rapid fluctuations in the received signal power [7].

It has been demonstrated that the rapid signal fluctuations produced by a large number of multipath components are predictable when using a stochastic model. Both the in-phase and quadrature components of the combined signal may be represented as Gaussian random variables with a variance equal to the average received signal power. The signal voltage envelope, or the square-root of the received power, can be modeled with a Rayleigh density function. Equation (2-1) represents the probability that the signal envelope is equal to a certain value "r", where "b" is the mean received signal power. Equation (2-2) is an extension of the Rayleigh density function and represents the probability that the signal envelope is less than or equal to "R" [7]:

$$P(r) = \frac{r}{b} e^{-r^2/2b} \qquad \textbf{(2-1)}$$

$$P(r \le R) = 1 - e^{-R^2/2b} \qquad \textbf{(2-2)}$$

Figure 2-1 depicts the signal power of two independently faded signals in a Rayleigh fading environment. It can be noted that large simultaneous fades are rare when compared to the frequency of separate fades [11]. In this environment, Equations (2-1) (2-2) can be used to predict the performance of a communication system, or compare two systems [7].

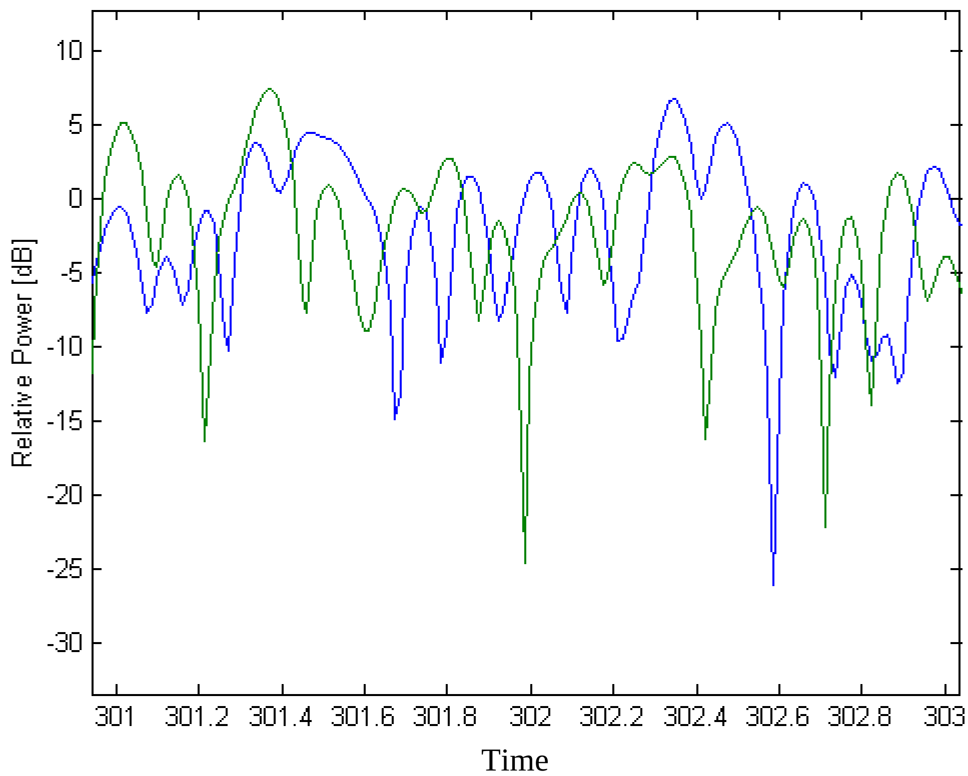



**Figure 2-1: The simulated envelopes of two uncorrelated Rayleigh fading signals**

### 2.1.2 Diversity Gain

The effectiveness of a diversity system can be measured by its reduction in the transmitter power required to maintain an acceptable rate of errors in the received data stream. Receivers are designed to provide this acceptable error rate provided that the received signal power is above a minimum threshold level. Additive noise and signal fading in the propagation channel produce decoding errors in the receiver when adequate power is not maintained. The communication system's error rate can be expressed as a function of the probability that the received signal power is below this threshold [5].

Diversity techniques improve this probability, without changing the transmit power, by transmitting the signal through many multiple propagation channels. The converse argument states a diversity system can transmit less power while maintaining a specified error probability. By improving the reliability of a communication system, diversity techniques effectively produce a *diversity gain* in the received signal [12]

Assuming a Rayleigh fading environment, diversity gain can be demonstrated using Equation (2-2). The variable “b” is the mean received signal power and can be measured or estimated for a particular communication system. “$R^2/2$” denotes the desired instantaneous signal power. With these variables, Equation (2-2) can be used to calculate the probability that the received signal power is below “$R^2/2$”, and provide an indication of when acceptable system performance is provided [7].

From Equation (2-2), it is known that in a Rayleigh environment the received signal power at a particular antenna has a 1 percent probability of dropping 20dB below the average power. This probability is reduced to 0.1 percent for 30dB and 0.01 percent for 40dB fades. If a receiver contains two antennas that receive independently faded signals, a simple stochastic analysis concludes that the probability of two simultaneously faded signals is equal to the product of the individual probabilities. The probability that two independent signals are faded more than 20dB below the average level is 0.01 percent, and more than 30dB is 0.0001 percent.

A receiver has been designed to produce an acceptable error probability when provided with a 0.01 percent probability that the instantaneous received power fades below the minimum detectable level. This receiver can withstand a 20dB fade using one antenna or a 40dB fade with two antennas before it fails to provide acceptable performance. With this example receiver, transmit power can be reduced by 20dB, without degrading the communication system’s performance, if the second antenna is utilized.

In the context of transmitter power, the performance of diversity systems is expressed in terms of diversity gain. Diversity techniques introduce redundancy into the communication system by transmitting a signal over many propagation channels, instead of a single channel. This redundancy raises the likelihood that the received signal contains a greater power than the receiver’s minimum detectable power level. Diversity techniques enable a designer to reduce the transmitter power without increasing the communication system's error rate. The reduction in transmitter power is referred to as diversity gain [12]

### 2.1.3 Types of Diversity Systems

Diversity systems are distinguished by the antenna array's placement, at either the transmitter or receiver, and by the system's processing algorithm. Both transmit and receive diversity systems are depicted in Figure 2-2.

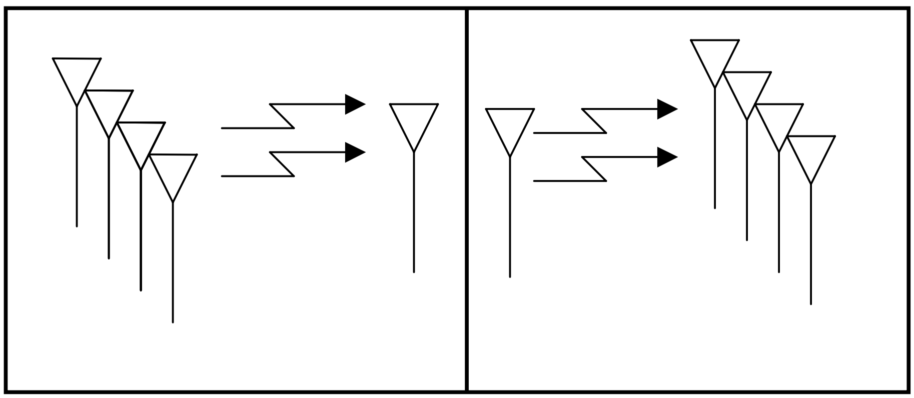


**Figure 2-2: Arrangement of antennas in diversity systems. Left: Transmit diversity, Right: Receive diversity.**

Receiver diversity schemes are often classified by the algorithm used to recombine the signal inputs from several receiver antennas. Three commonly used linear recombination techniques are *selection*, *maximal-ratio combining*, and *equal-gain combining*. These algorithms trade system complexity for performance, and each is advantageous in a specific environment [13].

In *selection diversity* the antenna with the largest received signal-to-noise ratio is chosen, and only that signal is passed to the decoder. This algorithm is used in receivers with stringent complexity requirements. For this reason, several variations have been developed to simplify the processing even further. One variant selects the receiver with the highest signal-plus-noise power. Instead of measuring both the signal and noise power levels separately, a simple power meter in the RF front end can be used to make the decision. Another method selects the antenna with the highest signal-to-noise ratio, but reduces the frequency of this decision. This scheme chooses an antenna and continuously evaluates its received power. The selection decision only occurs after the signal power drops below a predetermined threshold. Although selection combining may not provide the optimal combined signal-to-noise ratio, performance is raised above that of the single antenna receiver [13].

*Maximal-Ratio Combining* was specifically designed to maximize the signal-to-noise ratio of the combined signal. This technique assigns each receiver antenna a weighting factor, proportional to its corresponding signal-to-noise ratio, and then passes the sum of the weighted inputs to the receiver. *Equal-Gain Combining* also follows this format, but omits the weighting factors, or uses equal weights [11].

The performance of each algorithm often depends on the cross-correlation of fading between the propagation channels. Cross-correlation is a measure of the similarity between the channels' fading characteristics, and can be represented by the correlation coefficient. The details of this parameter will be provided in Chapter 3. For the following arguments, it is sufficient to state that highly correlated propagation channels exhibit similar fading characteristics and low correlation signifies contrasting fading characteristics [7].

Of the three recombining schemes, maximal-ratio combining theoretically obtains the optimum output signal-to-noise ratio with low cross-correlation. Maximal-ratio combining theoretically produces a signal with a signal-to-noise ratio (SNR) equal to the sum of the antennas' individual SNR's [11]. Equal-gain combining retains much of this performance using a simpler system design [13]. Both of these algorithms require that the input signals are co-phased and involve more complex signal processing than selection diversity. Producing co-phased signals may require phase shifters at the antenna outputs and phase detectors in the baseband subsystem. Another implementation involves a phase-lock loop. Both methods limit the response time of the system and require more complex hardware than selection diversity [13]. Each processing algorithm has benefits and drawbacks for different propagation conditions and system types.

Systems with transmit diversity contain two or more antennas at the transmitter and broadcast identical data from each antenna. One implementation switches between the two antennas, sending the signal from only one antenna at any given time. Unlike diversity at the receiver, transmit diversity requires performance information from the receiver to control the switching at the transmitter. The transmitter must acquire knowledge of the received signal-to-noise ratio, either through a feedback or a feedforward mechanism, usually implemented with a training sequence [14]. This complexity has often made receiver diversity schemes more desirable to the system designer.

These diversity schemes were designed to reduce the probability and duration of fades and consequently reduce the required transmitter power. The actual diversity gain provided by each system depends, in part, on the correlation of fading between antennas. Transmit diversity may be efficient if low correlation exists between transmitter antennas. With receive diversity, maximal-ratio combining performs well in low correlation environments, but performance degrades with high values of correlation. Due to its simple implementation, selection diversity may present the most efficient use of system resources in some propagation environments. Predicting or measuring the correlation between propagation channels can improve the effectiveness of a diversity system [7].

### 2.1.4 Investigations in Diversity

Over the past three decades, many investigators have published results on cross-correlation and diversity systems. Many of these articles have either measured the correlation of fading or measured the effectiveness of a particular diversity scheme in a particular environment. A number of researchers compared the two parameters and drawn conclusions on the effects of cross-correlation on diversity gain. Each measurement campaign focused on a specific environment or antenna arrangement, and most results are specific to that scenario. This section presents an overview of many of the significant investigations.

Some of the most fundamental research linking cross-correlation and the effectiveness of diversity was published by William C.Y. Lee. These publications represent a consolidated study involving almost a decade of theoretical analyses and measurement campaigns. Lee was successfully able to demonstrate the benefits of diversity between an elevated base station and a mobile vehicle, as found in typical cellular networks [15]. Later publications specified that antenna spacing and local area scatterers affect cross-channel correlation, and he described their relationship with diversity system performance [16] [17]. In these articles, Lee describes both theoretical analyses and supporting measurements to substantiate his conclusions. He also presents a thorough derivation of diversity gain for specific diversity techniques in terms of the correlation between the two antennas in [18].

In these publications, Lee described the relationship between antenna spacing, local scatterers and the correlation of fading between antennas. One of his earliest conclusions states that the correlation of two received signals decreases when the two receiver antennas are separated by farther distances. A reader may find this statement substantiated after accepting that two antennas should receive identical signals if they are ideally collocated. Lee builds on this statement by observing that cross-correlation also decreases with the presence of local area scatterers. He supports this finding by explaining that nearby scatterers increase the beamwidth of the incoming signal and decorrelate the phase offsets between the individual multipath components. Simulations were presented to this effect in [17].

Lee demonstrated that cross-channel correlation directly affects the performance of diversity systems, and although optimal diversity gain is produced with uncorrelated signals, much of the provided advantage is retained with correlation coefficients up to 0.7. Measurements demonstrated that without the presence of local scatterers, an antenna separation of $70\lambda$-$80\lambda$ is required with broadside reception and a separation of $15\lambda$-$20\lambda$ is required with in-line reception to maintain this correlation value on an elevated base station. Antenna separations less than these values produced an unacceptable combined signal-to-noise ratio in an equal-gain diversity system. Broadside reception occurs when the line between the two antennas is perpendicular to the direction of propagation, and in-line reception occurs when this line is parallel. The wavelength of the propagating carrier signal is denoted by $\lambda$ in the previous statements. Also, a base station was specified as elevated above the majority of nearby obstacles, thus free from the effects of scattering objects. Lee's research provides a good overview of the currently accepted relationship between channel fading, correlation and diversity performance [15].

Measurements in a variety of environments have since supported Lee's findings. In a typical mobile environment, a vehicle is surrounded by scatterers and thus mobile antennas only need to be spaced a fraction of a wavelength apart to have uncorrelated fading [11]. Indoor measurements, where scattering objects are prevalent, have demonstrated that antenna spacings of less than one wavelength may produce highly decorrelated fading. Specifically, a minimum antenna spacing of only $0.4\lambda$ was necessary to produce correlation coefficients of less than 0.2 in [19]. Also, theoretical analysis and measurement campaigns have demonstrated that diversity gain is reduced only negligibly with a correlation coefficient of up to 0.3 [17] [20]. These findings were further supported by [21], in which ray-tracing simulations tested the three diversity recombination techniques described above. It was concluded that only a $0.4\lambda$ antenna spacing is required to produce significant diversity gains, corresponding to around a 0.5 correlation coefficient. A measurement campaign in which two antennas were mounted on a vehicle demonstrated that a spacing of only one wavelength is required to produce a 0.8 correlation coefficient [22]. It should be noted that in this system, the vehicle itself acted as a local scatterer. These measurements demonstrated that local scatterers near the antenna array reduce the correlation coefficient from the values observed in Lee's base station measurements, even with small antenna separations.

In the literature discussed above, diversity systems have been demonstrated to improve reliability and reduce the necessary transmitted power to maintain acceptable system performance. The use of diversity is particularly effective when cross-correlation between the two propagation paths is low, as determined by both environmental characteristics and antenna design. Many additional resources are available which further explain these topics. A theoretical analysis, presented in [23], studies the effects of antenna coupling and channel correlation. The relationship between the correlation coefficient and the potential advantage of diversity techniques may be found in [15]. In a recent thesis, Durgin developed a theory that predicts the correlation between spatially separated antennas in a Rayleigh environment if the direction of arrival of the multipath signals is known [24]. Also, a thorough explanation of diversity recombination schemes is provided in [7], along with an analytical comparison assuming pure Rayleigh fading. The outlined references provide a good representation of background literature relating environmental conditions to cross-correlation between channels and the performance of diversity systems.

## **2.2** Space-Time Codes

Transmit diversity can improve the performance of mobile communications, specifically in systems with large base-stations and significant demands on the downlink data-rate. The downlink represents the data transmitted from the base-station to the mobile station. In a system that requires high-speed transmission from a base-station with few size restrictions, the use of transmit diversity at the base station may provide the performance gains necessary to improve data-rate on the downlink. Research is currently underway to achieve transmit diversity by combining diversity techniques with channel coding, in the space-time coding architecture.

Space-time codes use coding techniques to effectively transmit multiple streams of encoded data simultaneously. Channel coding introduces extra bits into a communication system's transmitted data to produce redundancy. This redundancy reduces the system's data rate but can be used to correct errors in the received data and improve reliability [5]. Space-Time codes attempt to achieve transmit diversity, by extending the code processing to include both the dimensions of time and space, or spatially separated antennas. In space-time coding systems, different symbols are transmitted from each antenna or array element. The decoder uses the redundancy produced by the space-time code to separate the symbols at the receiver and recover the original stream of data [9].

Space-time codes attempt to maximize both diversity and coding gains with a single algorithm while limiting the system's processing complexity. Currently developing systems are classified by their processing algorithms as space-time block codes or space-time trellis codes [9]. An overview of these systems is presented in the following section.

### 2.2.1 Space-Time Block Codes

Similar to traditional single-antenna block codes, a space-time block code can be designated by its transmission matrix. This matrix describes the arrangement of symbols transmitted from each antenna, at each time interval. The block diagram in Figure 2-3 depicts the use of space-time block codes in a communication system. The figure includes an example transmission matrix from [25] in the upper right corner.

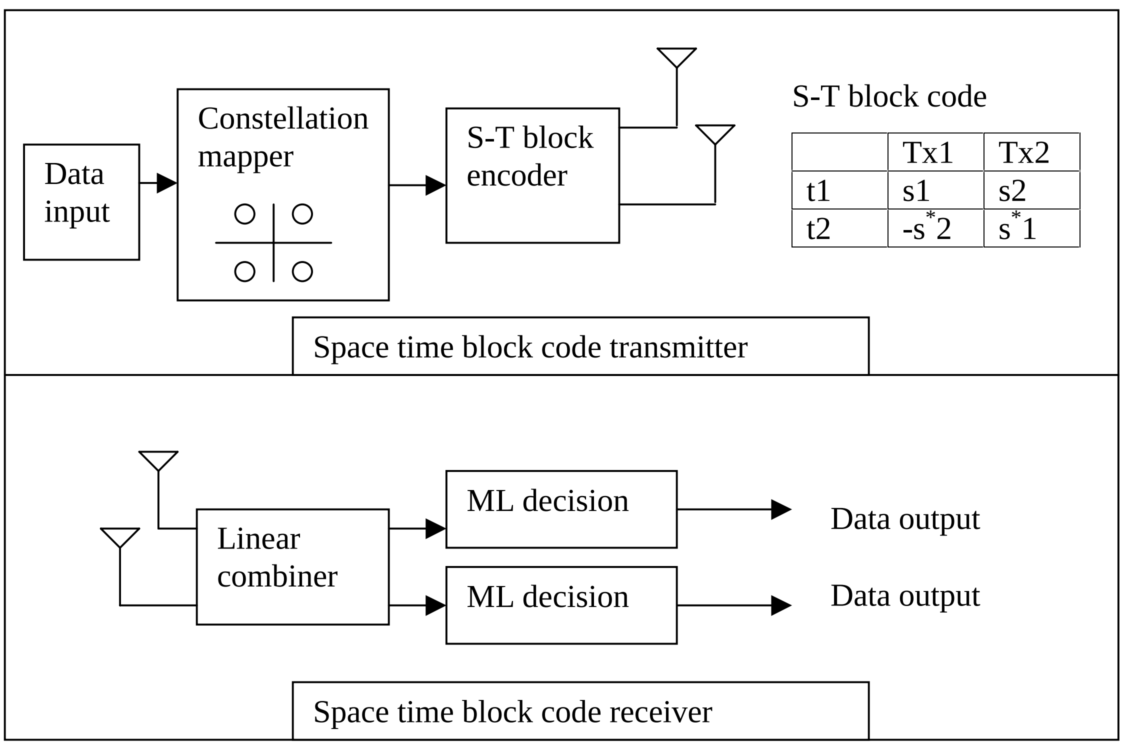



**Figure 2-3: Block diagram of a space-time block code transmitter and receiver**

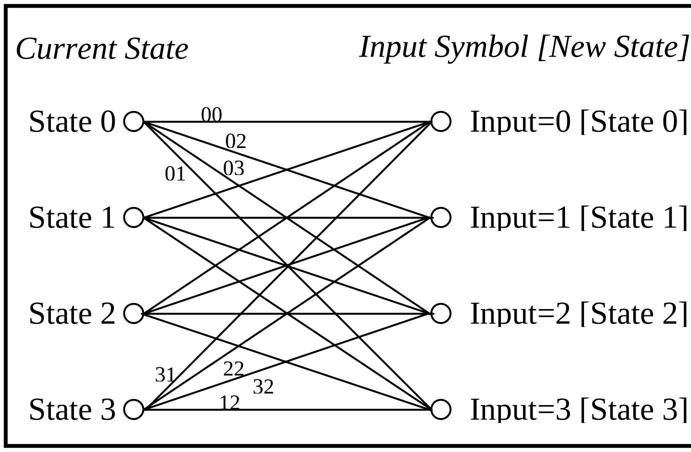
In the transmitter, data is mapped into symbols and $n_T$ symbols are input into the space-time encoder, where $n_T$ is equal to the number of transmitter antennas. From the input symbols, the space-time block encoder produces a matrix "block", which represents the symbols output from each antenna over the duration of the block. For this example code, the input symbols "s1" and "s2" are transmitted in the time frame "t1", from antennas one and two, respectively. In the second time frame, the symbols "–s*2" and "s*1" are transmitted, where "s*1" denotes the complex conjugate of "s1"[25].

The block diagram can be scaled to include an arbitrary number of antennas at either the transmitter or receiver. The example code does not explicitly specify the number of antennas at the receiver, but in practice this number depends on the desired system performance and the parameters of the code. Most space-time block codes have been designed assuming the channel gains are known, or estimated, at the receiver. The linear combiner uses the estimates of the channel gains and the known transmission matrix to produce estimates of the symbols "s1"and "s2" from the received signals. A maximum likelihood detector then compares each estimate to the transmitted signal constellation and outputs the received data [25].

Space-time block codes are attractive because they offer a significant transmit diversity gain and can be decoded with simple linear algorithms. While most block codes do not achieve the substantial coding gain of space-time trellis codes, they require less complexity at the receiver [9]. This characteristic would be beneficial in achieving diversity gains on the downlink of cellular systems, where the size and power constraints limit the processing capacity of mobile units.

### 2.2.2 Space-Time Trellis Codes

Space-time trellis codes are able to provide both diversity gain and coding gain, but require more processing that space-time block codes. Trellis codes are finite-length convolutional type codes, in which the output symbols depends on both the encoder's current state and the input data. These codes can be described by their trellis diagram and output table, as depicted for an example code in Figure 2-4 [26].




| | | | |
|---|---|---|---|
| 00 | 02 | 03 | 01 |
| 11 | 13 | 21 | 33 |
| 10 | 23 | 20 | 30 |
| 31 | 22 | 32 | 12 |

**Figure 2-4: Space-time trellis code (left) and an output table (right) for an $n_T$=2, 4-ary system with four states.**

The trellis depicts the transition from the encoder's current state to its next state. This example uses 4-ary modulation, corresponding to symbols 0,1,2 and 3. The current state of the encoder, depicted as nodes on the left side of the trellis, is determined by previous input symbols. In this case, the current state is equal to the last symbol input into the encoder. If the encoder state depends on more than one symbol, the trellis would contain additional states. For each input symbol, the encoder shifts to a new state and outputs a symbol at each transmitter antenna. The input symbol is denoted at the right side of the trellis and the encoder's new state is written next to it in brackets. The output symbols are associated with each branch between the current and new states, and are listed in matrix form in the output table. The output table has "m" columns and an equal number of rows as states in the trellis [26]. A few of the branches in Figure 2-4 have been labeled with their corresponding output symbols.

The output is represented by $n_T$ m-ary digits, corresponding to the m-ary symbols transmitted from the $n_T$ transmitter antennas. For each input symbol, one symbol is simultaneously transmitted from each antenna. After propagating to the receiver, these symbols can be decoded using the reverse process with the Viterbi algorithm. As in space-time block coding, the channel gains must be known at the receiver [26].

Both space-time block coding and space-time trellis coding depend on favorable characteristics in the propagation environment. Simulations in [27] and [28], which focus on cross-correlated Rayleigh fading channels, suggest that performance degradation is insignificant with a correlation coefficient under 0.4. According to these articles, the reasons for degradation are similar to those for diversity systems and described by Lee. In addition, because of specific issues in the trellis-coding algorithm, correlation between receiver antennas may affect the system more than identical correlation values between transmitter antennas. These publications indicate that knowledge of an environment's characteristics, either from measurements or theoretical analyses, may be beneficial to the design of effective space-time codes.

## **2.3** BLAST

The Bell-labs LAyered Space Time (BLAST) architecture is a Multiple-Input Multiple-Output (MIMO) scheme which uses linear signal processing techniques, instead of the error correction codes described above, to reconstruct transmitted symbols from the receiver inputs. This system has the potential to increase system capacity by a factor of the number of transmitter antennas. Requirements include an equal or greater number of antennas at the receiver than at the transmitter, and an accurate knowledge of the channel gains at the receiver. As shown in [10] describes the following processing algorithm used in the BLAST architecture.

### 2.3.1 Transmitter Signal Processing

Signal processing at the transmitter is relatively straightforward, since no processing is required between antenna channels. First, the data is input into the system and demultiplexed into $n_T$ data streams, where $n_T$ corresponds to the number of transmitter antennas. Next, this data is encoded to introduce redundancy, similar to the currently used channel coding. Each data stream is independently coded over time, requiring only the currently available one-dimensional codecs. The purpose of the coding will be discussed further in the receiver processing section. Each of the $n_T$ data streams then passes through the $n_T$ RF front ends and is transmitted from its respective antenna [10].

Some complexity is introduced in assigning transmitter antennas to data streams. The transmitter contains no knowledge about the channel, and emits a constant power level from each antenna. With these restrictions, the data streams must be rotated between antennas to ensure that each data stream receives an equal received signal-to-noise ratio after fading. Each data stream is transmitted from one antenna for a certain length of time. In the next time frame, the stream of data continues from a different antenna, and the rotation is repeated until the data stream has been assigned to every antenna. This cycle continues for the duration of transmission. The cycling prevents any one data stream from experiencing a permanent deep fade, and acts as an interleaver over the spatial dimension [10].

It should also be noted that transmission occurs as a series of long bursts. A long burst is defined to be shorter than the channel's correlation time, and long enough to contain the large ensemble of symbols necessary for statistical averaging. If the duration of a burst contains only a few data symbols, the system can not be described with stochastic random variables, such as those used in theoretic capacity calculations. A large but finite number of symbols must be transmitted in each burst to ensure the accuracy of these calculations [10].

### 2.3.2 Receiver Signal Processing

Receiver complexity compensates for the lack of processing between data streams in the transmitter. The signal produced at each receiver antenna is a linear combination of the signals from all transmitter antennas. The order in which each transmitted symbol is isolated from the combination is central to the BLAST architecture.

The receiver isolates each transmitted data stream by repeatedly *canceling* and *nulling* interfering data streams from the combined signal. Canceling and nulling are signal processing algorithms commonly found in array-based architectures which remove interfering signals from the desired signal, similar to the case of interfering transmitters in the BLAST system. The matrix depicted in Figure (2-5) relates the transmitted and received signals for a particular time frame. The channel gain, $h_{ij}$, corresponds to transmitter "i" and receiver "j" at a particular time frame [10].

$$\begin{aligned} r_1 &= t_1 h_{11} + t_2 h_{12} + t_3 h_{13} + t_4 h_{14} \\ r_2 &= t_1 h_{21} + t_2 h_{22} + t_3 h_{23} + t_4 h_{24} \\ r_3 &= t_1 h_{31} + t_2 h_{32} + t_3 h_{33} + t_4 h_{34} \\ r_4 &= t_1 h_{41} + t_2 h_{42} + t_3 h_{43} + t_4 h_{44} \end{aligned}$$

**Figure 2-5: Received signal matrix in terms of transmitted signals and channel gains (nT=4 nR=4)**

Canceling mathematically removes a known signal from the received signal. If the channel gains are known at the receiver and the receiver has already decoded the signals from transmitters $t_1$ and $t_2$, half of the terms in Figure (2-5) can be subtracted from the received signals. The signals from the remaining transmitters can be then be isolated using the nulling technique [10] [30].

Nulling involves the generation of orthonormal basis functions from the known $h_{ij}$ values and projecting the received signals onto these bases. Because the bases are orthogonal, the undesired terms of the received signal may then be subtracted out by removing their matrix elements. This process does not require knowledge about the nulled data, but assumes that the channel gains can be made into orthogonal basis functions. It should be noted that in the previous $n_T$,$n_R$=4 example, this assumption is supported by the fact that an equal number of equations and undetected transmitted signals exist in each nulling calculation. Linear algebra reveals that a matrix system with an equal number of variables as rows produces a single unique solution, or a unique estimate of the desired signal [10] [30].

Before implementing the canceling and nulling operations, the receiver must first store and rearrange the signals received in $n_T$ time-frames to produce the diagonal processing sequence depicted in Figure (2-6). Each block in the figure represents the signals from all of the receiver antennas. The data streams, denoted by letters "A" through "D", are rotated between transmitter antennas one through four as detailed in the last section. In this figure, the transmitters are numbered in order of increasing signal-to-noise ratio, and the arrow indicates the order in which symbols are processed at the receiver [10].

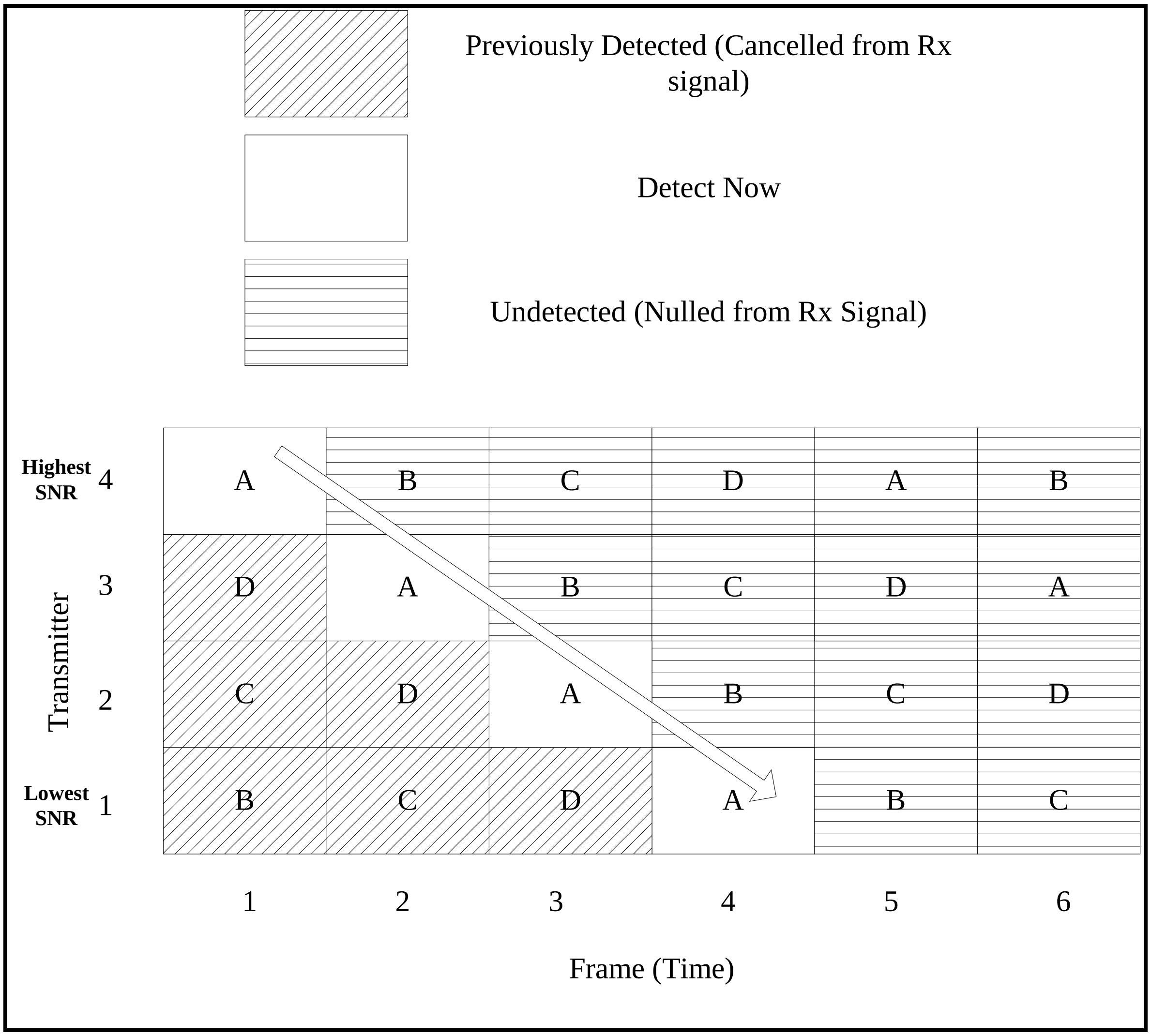



**Figure 2-6: The relationship between data streams and transmitter antennas. The transmitter numbering constitutes the spatial dimension of the space-time system.**

The symbols of one data stream are processed from every transmitter antenna before the next data stream is begun. The frame with the highest signal power is processed first, and the frame with the lowest signal power is processed last. This processing arrangement appears as a diagonal across the spatial and temporal dimensions, when the spatial dimension is represented by the locations of the transmitter antennas. It can be shown that this processing sequence produces a minimum symbol error rate when using the following symbol estimation technique [10].

As shown in Figure (2-6), the transmitters have been numbered in order of increasing signal-to-noise ratio. It can be shown that the following processing calculations result in a reduced probability of symbol error when the transmitter with the highest signal-to-noise ratio is detected first [10].

For the $n_T=4$, $n_R=4$ example shown in Figure (2-6), data stream "A" is decoded first because its signal has the highest signal-to-noise ratio in the first time frame. In each time frame there exists a number of previously detected transmitters and a complementary number of undetected transmitters. Because all other transmitters have been previously detected in the first frame, they can be *cancelled* from the received signal. After cancellation of transmitters one through three, the remainder of the input is the desired symbol, produced by transmitter four. For the second frame, transmitter three is to be isolated, containing the next largest signal-to-noise ratio. The two previously detected signals, "C" and "D", are cancelled, leaving transmitters three and four superimposed in the received data. The desired transmitter is isolated by *nulling* out the component produced by transmitter four. At the third frame, one transmitter can be cancelled and two must be nulled to isolate transmitter two. Finally at the fourth frame, all of the interfering transmitters must be nulled [10].

After $n_T$ frames of data stream "A" are isolated, the receiver will repeat the process for data stream "B" and then the subsequent data streams. It should be noted that the canceling and nulling operations require that the receiver has knowledge of the channel gains. To accommodate this, a training sequence must be transmitted between data bursts. The transmitter broadcasts a predefined data sequence, allowing the receiver to calculate these channel gains [10].

### 2.3.3 Implementation Options and Performance

Numerous implementation options are available for the BLAST architecture which may be used to enhance performance or system capacity in specific environments. These options usually reduce the system's data rate, but improve reliability or processing gain. For propagation environments that suffer from shadowing, feedback may be used to control the transmitted power from individual antennas. This technique can increase the average signal-to-noise ratio and more efficiently utilize power consumption [10].

Small or mobile devices which are limited in processing power may require a simplified processing algorithm. Vertical BLAST omits the rotation of data streams between transmitter antennas for this purpose. Although this scheme achieves a reduced capacity gain, as compared to the previously described Diagonal BLAST, it may be applicable in products with limited processing ability. Measured results conducted in a controlled indoor environment have boasted spectral efficiencies of up to 40bits/sec/Hz with an $n_T$=8, $n_R$=12 narrowband system [31]. As will be shown in Chapter 3, these greater spectral efficiencies are achieved by exploiting multiple propagation channels with uncorrelated fading.

Temporal channel coding may be performed to reduce the required signal-to-noise ratio for a given error probability. Because each data stream is encoded independently, this extra redundancy is useful in combating signal fades, but may be omitted to satisfy system requirements [10]. Simulation results have demonstrated that with an $n_T$=2, $n_R$=2 system, uncoded BLAST is capable of 4.0bits/sec/Hz with an SNR of 18.0dB, while coded BLAST requires an SNR of 10.1dB but provides only 2.0bits/sec/Hz [32]. It should be noted that channel coding protects against fades across both the spatial and temporal domains, but requires only a one-dimensional codec. Investigators have suggested that this presents a complexity advantage over architectures using two-dimensional coding algorithms, as may be required for some space-time systems [10].

# Chapter 3

## Analytical Parameters

This chapter describes specific parameters used to predict the performance of narrowband *multiple-input multiple-output* (MIMO) communication systems. MIMO systems contain antenna arrays at both the transmitter and the receiver and generate concurrent signals in the same frequency band. A propagation channel exists between each pair of transmitter and receiver array-elements, which can be characterized by a channel gain. Parameters that compare the gains of two channels or describe multiple channels as a group can be beneficial in predicting the performance of MIMO communication systems.

Specific parameters introduced in this chapter are *cross-correlation* and *multi-element array (MEA) capacity*. Both parameters are used to analyze the data recorded by the measurement system, constructed in this thesis project. Mathematical expressions are presented for each parameter in terms of the channel gains, and the data analysis process is discussed. The terminology developed in this section will be repeated throughout this thesis to specify the accuracy of the measurement system and the characteristics of the propagation environment.

## **3.1** Cross-Correlation

Cross-correlation is a statistical quantity often used to compare two propagation channels. For the purposes of spatial diversity, propagation channels are defined by the locations of their transmitter and receiver antenna elements. Identical signals passing through two propagation channels may acquire different values of amplitude or phase, depending on the gain of each channel. Cross-correlation can describe the similarity between the two channel gains and the similarity between the amplitude and phase characteristics of the two received signals. When the channel gains are time-varying, cross-correlation can also be used to calculate the probability of concurrent fades. This section discusses the cross-correlation of fading between two propagation channels and its application to spatial diversity [7].

### 3.1.1 Explanation and Application

Cross-correlation is a statistical parameter used to describe the similarity between two random variables or processes. A random variable is defined as a single observed event. Two variables with a high cross-correlation are very likely to have equivalent values, and a low cross-correlation indicates that very little information can be learned about one variable from knowledge of the other [33]. If the two variables X and Y are uncorrelated, and it is known that X has a large positive value, Y may have a large positive value, a small value or a large negative value. An observation of X provides no indication of Y if the two variables are uncorrelated.

In stochastic analyses, the cross-correlation function is equal to the expected value of the product of the two random variables. Equation (3-1) expresses the expected value function as "E( )" and calculates the cross-correlation function of two random variables "$r_1$" and "$r_2$" in terms of their probability density functions. The expected value represents the average of the variable over a large number of observations. In this thesis, the symbols "$^*$" and "$^+$" denote the complex-conjugate and the complex-conjugate transpose operations, respectively. It should be noted that this general form of the cross-correlation function makes no assumptions about the properties of the random variables [34].

$$R_{r_1,r_2} = E\left[r_1 r_2^*\right] \qquad \textbf{(3-1)}$$

In addition to comparing a pair of random variables, the cross-correlation function can also be used to compare a pair of random processes. A *random process* is a random variable which changes and can be expressed as a function of an *independent variable*. Another definition states that a random process is an infinite number of random variables, corresponding to an infinite number of instances of the independent variable. Whether the random process is viewed as a changing function or a stream of instances, it can be sampled at different values of the independent variable [33]. Time and distance are two independent variables often recorded in measurement campaigns.

Cross-correlation represents the degree of accuracy with which an observer can guess the value of one process, from a measured sample in the second process [34]. Two highly correlated processes tend to follow each other's movements, and share the same maximums and minimums. On the other hand, uncorrelated processes share no common frequency, and the minimum and maximum values occur at different points in time. In this case, an observation of one process can not be used as an indication of the value of the second process. Equation (3-1) is expanded to specify the random processes, $r_1$ and $r_2$, as functions of their independent variables, "n" and "m", respectively in Equation (3-2). It should be noted that both of the processes may be expressed in terms of time indeces, designated by "n" and "m", but these independent variables do not necessarily represent the same instant [33].

$$R_{r_1,r_2}(n,m) = E\left[r_1(n)\, r_2^*(m)\right] \qquad \textbf{(3-2)}$$

This equation may be expressed in a more practical form after specifying the structure of the two processes. A common method of conducting measurements involves observing a process at known points in time, and then compiling the sampled data into a list, or vector. Each observation represents a snapshot of the random process at a discrete time-instant. After compiling a large, but finite, list of values for each of the two processes, the expected value function can be expressed as the summation of the observations, normalized by the size of the list. Equation (3-3) represents the expected value calculation for a list of observations of the random process "X". In this expression, the independent variable "n" denotes the observation number, and the summation includes a total of "N" observations. Equation (3-3) can be substituted into Equation (3-2) to produce Equation (3-4) [33].

$$E(X) = \frac{1}{N}\sum_{n=1}^{N} X(n) \qquad \textbf{(3-3)}$$

$$R_{r_1 r_2}(n, n+lag) = \frac{1}{N}\sum_{n=1}^{N} r_1(n) r_2^*(n+lag) \qquad \textbf{(3-4)}$$

The cross-correlation function can be expressed in terms of a lag value if the two processes are jointly stationary. The joint wide-sense stationarity condition states that the expected values are constants and the cross-correlation of the processes depend only on the difference between the independent variables of the two processes. If the processes $r_1$ and $r_2$ in Equation (3-2) are wide-sense stationary, the cross-correlation depends on (n-m) instead of the individual values of "n" and "m". Equation (3-5) specifies the *zero lag* element of the cross-correlation function, signifying that both processes have been observed simultaneously, or at the same value of the independent variable [34].

$$R_{r_1,r_2}(0) = R_{r_1,r_2}(n,n) = \frac{1}{N}\sum_{n=1}^{N} r_1(n) r_2^*(n) \qquad \textbf{(3-5)}$$

Non-zero lag values would be important if the correlation between observations taken at different points in time were of interest. For instance if two receivers were located at different distances from the transmitter, the signal would require longer to reach the farther receiver, and a constant delay would exist between the first received signal and the second. In this case, a significant correlation should exist between non-zero lag elements of the cross-correlation function. It is assumed that only the zero lag element of correlation is of interest in this investigation [33].

A simple example involving two six-sided dice can be used to demonstrate cross-correlation. Each die is rolled 1000 times and the results are recorded in two lists, as depicted in Figure (3-1). The outcome of the rolls are random processes, and can be expressed as a function of the roll index. The roll index varies from one to 1000. Each number in a list indicates a sample, or observation, of one of the random processes and takes the form of an integer between one and six. Even though the outcome of a roll does not depend on an independent variable such as time or distance, the example can be used as a simple demonstration of cross-correlation between two random processes.

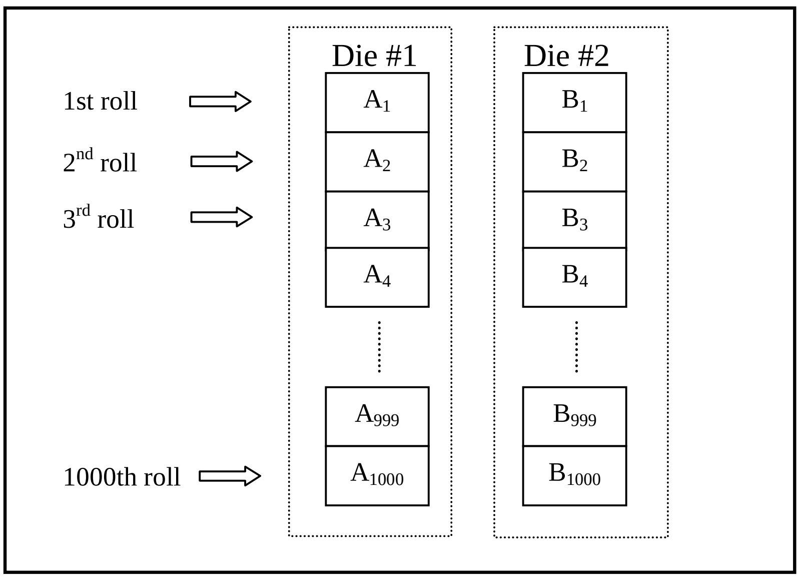



**Figure 3-1: Two lists containing the observed data from 1000 rolls of two dice**

The cross-correlation of the two random processes can be calculated by inserting the observed data into Equation (3-5). Each value in the first list is multiplied by its corresponding value in the second list, and the sum of the results is divided by 1000 to produce the zero lag of the cross-correlation function. As a note, this calculation is simplified because both lists are composed of real numbers, and the complex conjugate of a real number is equal to itself.

In this example, two uncorrelated processes were demonstrated with two fair dice, and Equation (3-5) should produce a result close to the theoretical value of 12.25. In theory, uncorrelated processes are defined to have a cross-correlation value equal to the product of the expected values of the two individual processes. This definition is represented by Equation (3-6) where X and Y are uncorrelated processes [34]. A fair die has an equal probability of landing on each of the six possible outcomes. With dice numbered from one to six, the mean is equal to 3.5, and the product of the means of two dice is 12.25.

$$R(X,Y) = E(X,Y) = E(X)E(Y) \tag{3-6}$$

In a similar experiment, correlated processes can be demonstrated with two weighted dice. For example, two dice which are weighted to continuously land on the same number would represent two fully-correlated random processes. By definition, the probability density functions of two fully-correlated processes are identical, as represented by $r_1$=$r_2$ in Equation (3-5). The value of the cross-correlation function depends on the weighted number itself. A repeated six produces a cross-correlation of 36, and a repeated one produces a cross-correlation of one.

The result of the cross-correlation function is dependent on both the mean and variance values of the random processes, and experiments observing different mechanisms or utilizing different measurement systems may obtain varying cross-correlation values [33]. A test that uses two fair dice numbered from three to eight instead of one to six will produce a cross-correlation of 30.25 instead of the previously calculated 12.25, even though both pairs of dice emulate uncorrelated random processes. By changing the numbering on the dice, it can be demonstrated that the cross-correlation function does not clearly reflect the similarity between the two dice experiments. The cross-correlation function represents the similarity between two random processes, but does not produce consistent results for all experiments.

### 3.1.2 Correlation in Diversity Applications

The cross-correlation of fading between two spatially distinct channels can be used to predict the performance of diversity and space-time systems. Diversity systems combine multiple received signals with varying instantaneous signal power levels to improve the reliability of communication systems. Uncorrelated signals contain a lower probability of simultaneous fades than signals propagating through highly correlated channels, maximizing the effectiveness of diversity techniques [7].

An analysis of the probability of simultaneous fades can provide insight into how cross-correlation affects diversity systems. The probability $F_A(c)$ is defined as the chance that, in a particular measurement, the gain of channel A is reduced and the power the received signal falls below a minimum value "c". Equation (3-7) relates the conditional probability that both channels A and B are faded below a specific value, "c", if it is known that channel A is faded by this value. The right side of the expression defines the conditional probability as the ratio of the joint probability, that both channels are faded, over the probability that channel A is faded [5].

$$f\left(B < c \mid A < c\right) = \frac{F_{AB}\left(c, c\right)}{F_A(c)} \quad \textbf{(3-7)}$$

In the fully-correlated case, the joint probability in the numerator is equal to the individual probability in the denominator, because the two propagation channels always fade simultaneously [7]. The conditional probability is equal to unity, signifying that if channel A is faded, then it is known with absolute certainty that channel B is also faded. Because both channels fade simultaneously, transmitting a signal across multiple propagation channels will not reduce the probability of at least one unfaded signal at the receiver, limiting the reliability improvement of diversity techniques.

If two channels contain uncorrelated fading, no information can be predicted about the path loss of one propagation channel from measured data of a second channel [34]. To simplify the analysis, it can be assumed that the channel gains are mutually independent, making the joint probability, or the probability of simultaneous fades, equivalent to the product of the two individual probabilities. Because probability values must always be less than unity, the product of two probabilities is smaller than the individual values [33]. The conditional probability of Equation (3-7) is smaller than P(A), implying that simultaneous fades are less probability than individual fades. In the uncorrelated case, diversity techniques are effective because the redundant antennas take advantage of the reduced probability of simultaneous fades to improve the system's reliability [7].

In summary, if the channels are uncorrelated the probability of two simultaneous fades is equal to the product of the probabilities of individual fades, and if the channels are fully-correlated the channels the probability is equal to that of an individual fade. In this way, diversity systems take advantage of channel independence to reduce the frequency and duration of detrimental fades.

### 3.1.3 Correlation Coefficient

The *correlation coefficient* is a normalized value of cross-correlation, which more intuitively represents the similarity between two random processes. It has a range from zero to positive one, where a positive one indicates that the two observations are always equal, and a correlation coefficient of zero signifies the uncorrelated case. For the remainder of this thesis, the following expression for the correlation coefficient will be used to calculate the cross-correlation of two random processes. The term "$R_{r1,r2}(0)$" denotes the zeroth lag element of the cross-correlation function of the random processes "$r_1$" and "$r_2$". The mean and standard deviation of a random process X are denoted by "$\mu_X$" and "$\sigma_X$" and are defined by Equations (3-3) and (3-8) respectively [34]. Equation (3-9) expresses the correlation coefficient of the random processes "$r_1$" and $r_2$" in terms of cross-correlation, mean and variance [33].

$$\sigma_X = \sqrt{E\left(X^2\right) - E^2\left(X\right)} \qquad \textbf{(3-8)}$$

$$\rho = \left| \frac{R_{r_1,r_2}(0) - \mu_{r_1}\mu_{r_2}}{\sigma_{r_1}\sigma_{r_2}} \right| \qquad \textbf{(3-9)}$$

The correlation coefficient can be calculated from recorded data using Equations (3-3), (3-5) and (3-8) and (3-9). Similar to cross-correlation, the correlation coefficient represents the likelihood that observations of two random processes will produce equal values. This parameter is advantageous because the results do not depend on the mean or variance of the observed random processes. In the dice example of Section 3.1.1, it can be shown that the correlation coefficient is not dependent on the numbering of the dice. The correlation results presented in this thesis are expressed as correlation coefficients.

### 3.1.4 Assumptions in the Calculation

The cross-correlation has been introduced as a stochastic function, which represents the similarity between the two random processes, and the correlation coefficient has been defined as a normalized version of the cross-correlation that provides a more intuitive assessment. As explained in Chapter 2, channel fading can be modeled with a Rayleigh fading distribution, producing rapid fluctuations in the received signal amplitude when the receiver is moved over short distances. The following discussion further develops the fading model and defines the measurement data input into the correlation calculation [7].

This thesis investigation uses the assumption of a flat-fading channel to simplify the mathematical fading model. Flat fading occurs when the transmitted signal bandwidth is significantly smaller than the channel's coherence bandwidth and, inside this bandwidth, the channel gains are not frequency dependent [35]. The narrowband assumption is supported by indoor measurements conducted at 2.4GHz in which the coherence bandwidth was found to be greater than 650kHz. The measurement campaign defined coherence bandwidth as the frequency range in which the correlation between amplitudes of two frequency components was observed above 0.9 [35]. The signals transmitted in this investigation lie within a 32kHz bandwidth, validating a narrowband assumption.

In a flat-fading channel, a narrowband signal can be represented by a sinusoid and fully characterized by its amplitude and phase, or complex amplitude. In addition, channel gain can be expressed as a multiplicative fading coefficient, which is factored with the transmitted signal to produce the received signal. As discussed in Chapter 2, this channel gain can be expressed as the quadrature sum of two Gaussian probability distributions, producing a fading envelope with a Rayleigh distribution [7]. The following notation represents the channel gain between a pair of transmitter and receiver antenna elements. The gain $h_{ij}$ represents the propagation gain between transmitter antenna "i" and receiver antenna "j", the transmitted signal's amplitude from antenna "i" is represented as "$t_i$" and the received signal's amplitude at antenna "j" is "$r_j$". Equation (3-10) expresses the signal amplitudes at two receivers in terms of a transmitted signal's amplitude and channel gains.

$$\begin{aligned} r_1 &= h_{11}t_1 \\ r_2 &= h_{21}t_1 \end{aligned} \qquad \textbf{(3-10)}$$

It can be shown using Equation (3-10) that in specific measurements the received signal amplitudes can accurately represent the channel gains in the correlation coefficient calculation. The correlation coefficient has been defined in Equation (3-9) as a normalized form of the cross-correlation. Because the cross-correlation is normalized by the product of the means of the two measured processes, a constant amplitude shift does not affect the result of the calculation. Equation (3-10) demonstrates that the received signal amplitudes are equal to the fading gains multiplied by a common factor equal to the transmitted signal amplitude. The result of the calculation will not be affected if this common factor is removed. Assuming that all transmitted signal amplitudes are equal, the correlation coefficient of the channel gains can be calculated from the received signal amplitudes.

### 3.1.5 Organization of the Measured Data

The random processes input into the correlation coefficient calculation are structured as two vectors, or lists of observations of the received signal amplitude. The two vectors represent the signal amplitudes measured at two antennas on the receiver array. Each vector element consists of the complex amplitude value measured at a specific point in time and with the transmitter and receiver arrays at particular locations. Chapter 6 further describes the measurement process. This section discusses the construction of the two vectors from recorded data.

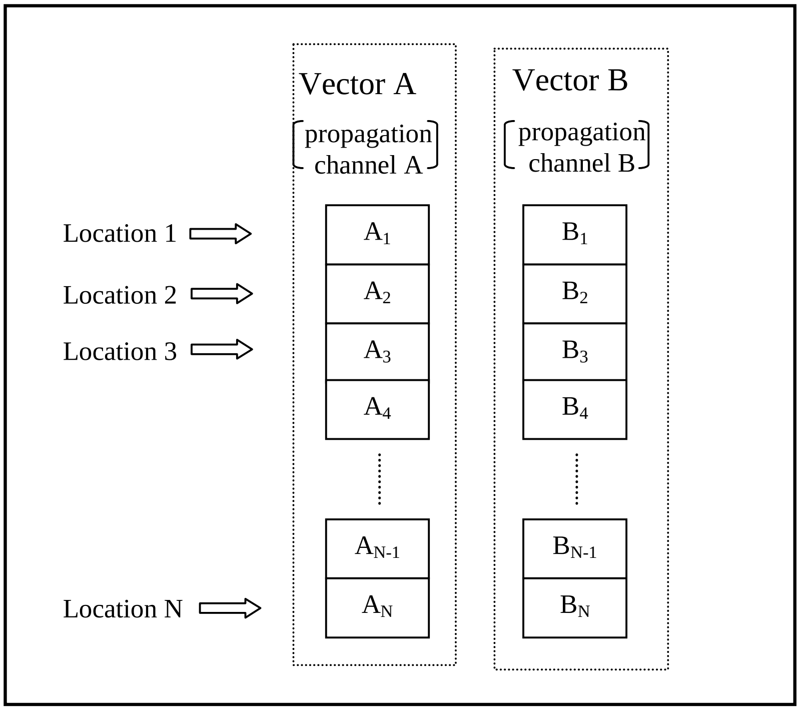



**Figure 3-2: Two vectors are created from measured data and input into the correlation calculation.**

The following two assumptions were implied in calculating the correlation coefficient and guide the construction of the data vectors. In order to obtain an accurate correlation value, the data must contain **many independent samples** of an **unchanging propagation channel**. The first assumption, that each vector contains a large number of independent observations, was maintained by moving the receiver array slightly between each measurement.

The transmitter and receiver were placed in stationary positions, and a measurement was recorded. This data constituted a single complex-amplitude data point for each propagation channel, as labeled by “Location 1” in Figure (3-2). The receiver array was moved a small distance to “Location 2”, and another measurement was recorded. This process was repeated nine times, producing a single data point for each propagation channel at nine receiver locations. The receiver array was moved approximately six inches between each measurement in the pattern of an eighteen-inch square grid. In Figure (3-2), the elements in Vector A correspond to the channel gains between a single pair of transmitter and receiver array-elements at nine receiver locations in the grid.

The second assumption states that the two propagation channels input into the correlation calculation are jointly stationary. As explained Section 3.1.1, this indicates that the correlation coefficient is constant over the entire set of recorded data. To validate this assumption, measurements were conducted at night, when there is a minimum of pedestrian traffic. Also the movement of the receiver array was restricted to an eighteen-inch square area, a significantly smaller distance than the transmitter-receiver separation. The absence of movement, the stationary transmitter, and the relatively stationary receiver array should have prevented substantial variations in the propagation channel's characteristics. The movement of the receiver array in the grid pattern, the transmitter-receiver separation and significant environmental characteristics are further described in the measurement plan in Chapter 6.

The assumptions of independent samples and joint-stationarity could not be analytically verified due to the limited number of measurements. Because the array was moved manually between each measurement, only nine measurements were recorded in each grid area. Investigations by Lee suggest that a quantitative analysis of these assumptions requires a significantly larger number of samples. In his investigations on diversity techniques, Lee recorded data from an array attached to a slow-moving van. By continuously recording a stream of data while driving a constant speed and direction, Lee was able to estimate the distance between each measurement. To verify that a finite-length block of sampled data represents a large number of independent samples, he calculated the autocorrelation of the data for a number of delay lag values. It should be recalled that the lag value describes the number of samples, or delay, between the compared measurements. By observing the *minimum* delay required to produce a null in the autocorrelation function, he concluded that samples with this delay constitute independent measurements. Lee then calculated the cross-correlation with different lag values, and found the *maximum* lag over which the cross-correlation is constant. By choosing a vector size between the minimum and maximum lag values, he was able to verify that his calculations contained a large number of independent samples while maintaining the joint-stationarity requirement [15].

Although Lee's method is useful when the receiver is moving at a constant speed throughout the measurement, neither the transmitter nor receiver arrays were mobile in this measurement campaign.  A numerical validation of the two assumptions with this measurement technique could be the focus for further research.  Lee's methods could be implemented and sufficient data could be collected by attaching the receiver array to a automated track or automobile.

A significant point should be noted about the construction of vectors depicted in Figure (3-2) from the measured data.  The zeroth lag element of the cross-correlation assumes that the samples are arranged with the same order in every vector.  If the two vectors are arranged in the same order, then each location's measurements of the first propagation channel will be compared to the corresponding measurements of the second channel, when the two vectors are input into Equation (3-9).  Even though the ordering of locations does not hold the same intuitive significance as the ordering of sampled measurements over time, a discrepancy between two vectors can produce errors in the cross-correlation calculation.

Previous investigations suggest a further simplification of the vector construction.  It has been demonstrated that the performance of diversity systems is dependent on the correlation of signals' voltage envelopes and is insensitive to relative phase variations. For this reason, recorded phase values of the channel gain are omitted from the cross-correlation analysis.  This simplification has been employed in many previous measurement campaigns [23] [15].

### 3.1.6 Calculation of the Correlation Coefficient

In this investigation, the following procedure will be used to calculate the correlation coefficient from the measured channel gains.  Two vectors are constructed with a pair of channel gains from each measurement in a nine point grid. Each vector contains nine elements, one from each measurement location. Equation (3-9) is then used to produce an estimate of the correlation coefficient from the two vectors. The resulting parameter has a range from zero to positive one and intuitively represents the cross-correlation between two fading channels. This process can be repeated for any pair of channels measured in the same nine-point grid.

A comparison of correlation coefficients can be used to analyze the affects of antenna spacing, environmental conditions, or to compare the cross-correlation between receiver antennas with transmitter antennas. A large number of correlation coefficients can be calculated for the channels produced by two sixteen element arrays and multiple measurement locations. The following sections discuss the motivation for these measurements.

### **3.2** H-Matrix Representation of Channel Fading

The channel representation provided in Equation (3-10), consisting of a pair of channel gains, is now expanded to include an arbitrary number of channels as determined by the number of antennas in the communication system. The channel gains between each transmitter and receiver pair may be combined into an H-matrix of size $n_T$ by $n_R$, with the element $h_{ij}$ representing the channel gain between transmitter "i" and receiver "j". For example, a system with four transmitter and four receiver array elements can be represented by the channel gains depicted in Figure (2-5) or expressed in matrix form. The gains matrix, or *H-matrix,* is represented as "H" in Equation (3-11), where "$\bar{r}$" is the vector of received signals and "$\bar{t}$" is the vector of transmitted signals.

$$\bar{r} = H\bar{t} \qquad \textbf{(3-11)}$$

#### 3.2.1 Statistical Representation

It should be reiterated from Chapter 2 that each Rayleigh fading propagation channel can be modeled by the quadrature sum of two Gaussian random variables. Each quadrature component is represented by a probability density function, and together they represent the magnitude and phase of the channel gain. A stochastic representation of the H-matrix can be written in the form of an $n_T$ by $n_R$ matrix of channel gains, where each gain is expressed in terms of its probability density function.

Cross-correlation between channels can be explicitly defined in the H-matrix if statistical models of the propagation channels are known. Each probability density function would be expressed in terms of the mean and variance of every channel gain and the correlation coefficient between each pair of channels. An analysis of this stochastic representation grows complex with a large number of propagation channels, and is beyond the scope of this thesis.

The stochastic analysis is significant because it provides a simple explanation of the H-matrix representing fully-correlated propagation channels. Fully-correlated propagation channels are modeled by channel gains with identical probability density functions and can be considered equivalent in theoretical analyses. If all propagation channels are correlated, with a correlation coefficient of one, all elements in the H-matrix have equal gains and can be represented by identical density functions. This fact will be used in the following sections to simplify the H-matrix and demonstrate the relationship between correlation and channel capacity [36].

### 3.2.2 Representation of Measured Data

In a measurement campaign, the sampled channel gain data is arranged into H-matrix form for each measurement. This sampled H-matrix represents a single observation of the propagation channels between the measurement system's transmitter and receiver antenna elements. After a large number of independent observations are collected, an accurate probability density function of each channel can be produced to define the stochastic model described in the last section. The distinction between the stochastic model and a single observation should be noted. In theoretical discussions about cross-correlation and system capacity, the H-matrix refers to a stochastic model of the propagation channels. In specifying measurement data and data processing techniques, the H-matrix represents observations of the random process.

### **3.3** Multi-Element Array Capacity

A system with multiple transmit antennas has the potential to transmit data with a dramatically faster rate than a similar system with a single transmitter antenna. The following section outlines an analysis demonstrating that this potential capacity gain may be as high as a factor of $n_T$, where $n_T$ is the number of transmit antennas. From this upper limit, capacity may be degraded with the introduction of non-ideal channel characteristics or antenna coupling. Cross-correlation between propagation channels constitutes a major concern in maintaining this capacity gain. This section presents an overview of the theoretical capacity gain provided by multi-element arrays, including examples and expressions found in data analysis.

#### 3.3.1 Background

The term *channel capacity* was originally used by Claude Shannon in the late 1940's as an abstract concept to calculate the amount of binary information which could be transmitted over a specific propagation channel. Because no assumptions are made about the communication system in this derivation, this capacity describes the rate attainable only by an ideal system, and provides a theoretic upper bound on the rate of data communications. Channel capacity has since been found useful in comparing the potential throughput provided by different propagation channels and communication techniques [5].

Shannon's channel capacity was derived using the model of a binary symmetric channel, with a discrete memoryless response. The theoretical communication system transmits binary data stream over the channel and records a distorted data stream at the receiver. The model ignores transceiver operations such as frequency conversion and signal amplification and calculates the probability of error directly from the signal-to-noise ratio of the received signal. The Shannon channel capacity in the presence of additive white Gaussian noise, is represented by Equation (3-12) the following expression, with the units of bits/sec/Hz [5] [37].

$$C = \log_2(1 + P_{sig} / P_{noise}) \qquad \textbf{(3-12)}$$

The capacity calculated in this thesis refers to a derivative of Shannon's original capacity expression, extending it to include multiple antennas at both the transmitter and receiver. The channels between transmitter-receiver antenna pairs are defined to contain Rayleigh distributed fading envelopes, such as found in highly scattering environments. The Rayleigh fading distributions may be independent or correlated, and the calculation takes this correlation into account by describing the channels with an H-matrix as described in the previous section [10].

The theoretical capacity of this multi-element array system can be calculated with Equation (3-13). The " $^+$ " denotes the complex-conjugate transpose operation, "$n_T$" symbolizes the number of transmitters, "I" represents the identity matrix and "$\rho_{sys}$" is the average signal-to-noise ratio at the receiver. In this equation, the H-matrix represents the relative channel voltage gains and is normalized to have a total energy of $n_T$ squared. It should also be noted that the matrix multiplication requires the system to have an equal number of transmitter and receiver antennas. A derivation of this expression may be found in [10].

$$C = \log_2\left[\det\left(I + \frac{\rho_{SYS}}{n_T} HH^+\right)\right] \qquad \textbf{(3-13)}$$

When comparing Equations (3-12) and (3-13), it can be noted that the received signal-to-noise ratio in the single-antenna system is replaced by $\rho_{SYS}$, $n_T$ and the H-matrix in the MEA capacity equation. The variable $\rho_{SYS}$ represents the SNR of one transmitted signal, averaged over all receiver antennas. In Equation (3-13) it is assumed that each transmitted signal has an equal SNR. The product of the normalized H-matrix with its complex conjugate represents the gain of each channel expressed in terms of power, instead of voltage. When combined with $\rho_{SYS}$ and $n_T$ in Equation (3-13), this product represents a matrix of signal-to-noise ratios for each transmitted signal at each receiver element.  Equation (3-13) is an extension of the original Shannon capacity applied to MEA systems [10].

It can be shown that the minimum MEA capacity is equivalent to the capacity of a single-antenna system, but the total signal-to-noise ratio of the two communications systems must be equal to maintain an objective comparison.  With an SNR of 20dB, Equation (3-12) results in a system capacity of 6.7bits/sec/Hz for a single-antenna system.  If the SNR is increased by a factor of four to 26dB, the system capacity rises to 8.6bits/sec/Hz, and a factor of sixteen produces a 32dB SNR and a capacity of 10.6bits/sec/Hz.  Increasing the transmitted power by a factor of sixteen significantly improves the potential capacity of a system.

The performance of a single-antenna system is identical to one that divides the same amount of energy between $n_T$ transmitters, assuming both systems experience identical propagation conditions [5]. The measurement system constructed in this thesis work contains sixteen transmitter antennas.  Each transmitted signal was designed to maintain an average SNR of 20dB at the receiver, producing a $\rho_{SYS}$ of 100.  If each transmitted signal has a 20dB SNR, then the total SNR of sixteen signals is 32dB.  With this signal-to-noise ratio, a single-antenna system produces a system capacity of 10.6bits/sec/Hz, as calculated with Equation (3-12).  In an identical environment, where all propagation channels are fully-correlated, every element of the H-matrix equals one, Equation (3-13) produces an equal capacity value.

The minimum and maximum capacity values of an MEA system with an average SNR of 20dB and $n_T$ equals sixteen are 10.6bits/sec/Hz and 106.5 bits/sec/Hz, depending on the value of the H-matrix.  The results recorded in the measurement campaign and presented in Chapter 7 are within this range.  With a large total signal-to-noise ratio, it would not be uncommon for a sixteen-transmitter system to obtain unusually high capacity rates, as compared to common single-antenna systems.

### 3.3.2 Explanation and Application

The theoretical H-matrix elements are represented by Gaussian random variables, and two channels that are fully-correlated are expressed with identical matrix elements. If more than one pair of channels are fully-correlated, the matrix can be reduced to a more simple form by performing the linear row operations commonly used in matrix algebra. Row reduction minimizes the number of non-zero matrix elements by canceling linearly dependent rows with each other.

A matrix represented by its row-reduced form has mutually orthogonal rows, with the left-most element in each non-zero row equal to unity. In its reduced form, the number of non-zero, or independent, rows is called the *rank* of the matrix. If this process is performed on an H-matrix, the rank represents the number of transmitter antennas with uncorrelated propagation channels to the receiver. Of the available transmitter antennas, this is the number of transmitted signals that the receiver can effectively isolate [38].

With an H-matrix consisting of partially correlated probability distribution functions, this reduction process becomes complex, and the matrix's rank is difficult to estimate. Similarly these calculations can become problematic if the propagation channel is represented by a number of H-matrices, instead of a statistical function. Rank is usually calculated from a single matrix with integer element values. Although equivalent parameters exist which estimate the rank for more complex H-matrix representations, their numerical analyses are beyond the scope of this thesis. A thorough analysis of the relationship between rank and system capacity may be found in [39]. For the purposes of this thesis, a sufficient relationship between cross-correlation, the rank of the H-matrix and the MEA capacity can be demonstrated with the following examples.

An environment which produces the minimum MEA capacity can be represented by the all-ones matrix and is characterized by a rank of one. To produce this H-matrix, all of the transmitters and receivers must be combined into a single propagation channel. A single transmission cable would result in fully-correlated received signals and has been named a *keyhole environment* in a recent publication [40]. All elements in the H-matrix are represented by identical values. Because a matrix with identical elements produces a minimum in the determinant function, the MEA capacity calculation described by Equation (3-13) is also minimized [30]. The receiver cannot use the variations between the channel gains to distinguish between different transmitted signals, and only one transmitter antenna can be used effectively with space-time techniques. With an all-ones H-matrix, capacity deteriorates to the level produced with a single transmitter antenna, and Equation (3-12) is equal to Equation (3-13), after an adjusting $\rho_{SYS}$ to maintain an equal transmitter power.

The opposite scenario, in which the MEA capacity is maximized, can be represented by an identity matrix. This H-matrix consists of diagonal elements of one, and zero elements elsewhere, and is characterized by a rank of $n_T$. A simple analysis can show both the determinant function and Equation (3-13) are maximized by the identity matrix [30]. An identity matrix may be produced by connecting each transmitter to a single receiver through transmission cables. Each receiver observes the signal from only one transmitter, without interference from adjacent transmitter signals. In this propagation environment, the transmitted signals are uncorrelated at the receiver and the data from every transmitter can be effectively decoded with a space-time system. An environment whose H-matrix can be reduced to the identity matrix can achieve $n_T$ times the capacity of a single-antenna system.

### 3.3.3 MEA Capacity Gain

The latter example results in the minimum multi-element array capacity, equivalent to the value calculated with Shannon's original channel capacity expression. This minimum MEA system capacity is identical to that of a single-transmitter system, when the two systems transmit the same total power. Using the single-transmitter capacity as a reference, *capacity gain* can be expressed as the extra capacity attainable by a multi-transmitter system above this reference. It should be noted that the theoretical channel capacity can only be realized by an ideal communication system, capable of resolving the signals from different transmitters; this is the motivation of the currently developing space-time systems.

### 3.3.4 Calculating MEA Capacity

Equation (3-13) is used to calculate the MEA capacity for each H-matrix recorded in the measurement campaign. A signal-to-noise ratio of 20dB was assumed as a nominal value for a realistic communication system, and translates into a $\rho_{SYS}$ of 100. The value of $n_T$ was set to sixteen to match the dimensions of the measured H-matrix. These values are used to predict the potential capacity of a communication system with sixteen transmitter and sixteen receiver antennas and an average SNR of 20dB.

The distinction between measured and system signal-to-noise ratios must be noted. The *measured signal-to-noise ratio*, $\rho_{MEAS}$, is the value measured by the device which records the H-matrix, such as the measurement system constructed in this investigation. The *system signal-to-noise ratio,* $\rho_{SYS}$, is that of the predicted communication system. Both values are calculated as the SNR averaged over all receiver antennas. Because $\rho_{SYS}$ in Equation (3-13) is predicted and not measured, the calculated capacity is determined by the correlation between propagation channels and is independent of the measurement system's transmit power level.

The H-matrix input into Equation (3-13) is normalized to accurately represent the signal-to-noise ratio of the predicted communication system. The normalized H-matrix has a total power equal to the number of matrix-elements, and an average per-element power of unity. When multiplied by the average signal-to-noise ratio of $\rho_{SYS}$ and divided by $n_T$, the magnitude-squared value of each element in the H-matrix represents the signal-to-noise ratio at one receiver antenna due to a single propagation channel. In addition, the total signal-to-noise ratio at a receiver antenna, due to all transmitter antennas can be calculated with Equation (3-14). Note that in this equation, $\rho_{SYS}$ is the average of all $\rho_i$ because by definition the system SNR is the average over all receiver antennas. Normalization of the H-matrix produces a calculated MEA capacity which is independent of the measurement system's SNR, and dependent only on the correlation between propagation channels.

$$\rho_i = \rho_{SYS} \sum_{j=1}^{n_T} \frac{1}{n_T} \left| h_{i,j} \right|^2 \qquad \textbf{(3-14)}$$

In addition to this normalization, it can be shown that the product of an H-matrix and its complex-conjugate transpose is insensitive to a constant phase shift of all matrix elements. This insensitivity and the normalization demonstrate that MEA capacity depends only on the relative values between the H-matrix elements, and disregards a constant factor of either magnitude or phase over the entire matrix. This property is used calculating the H-matrix, as described in Chapter 4 [10].

Simulations have demonstrated that this capacity calculation provides an acceptable accuracy when the measured SNR is at least 10dB greater than the system SNR. In a sixteen-transmitter system, the margin of error at the minimum and maximum capacity levels of 10.6 and 106.53bits/sec/Hz are 2 and 6bits/sec/Hz respectively. Calculations performed in this investigation assume a system signal-to-noise ratio of 20dB, and require a measured signal-to-noise ratio greater than 30dB.

The channel capacity formula is not specific to any particular wireless communication standard, but estimates the maximum achievable capacity in the presence of correlated Rayleigh fading channels. The predicted capacity is dependent on both the level of correlation between channels and the communication system's received signal-to-noise ratio. The previously discussed BLAST architecture has been designed to achieve this capacity, and constitutes the original motivation for this research.

### **3.4** Summary

This chapter has provided an overview of cross-correlation and the potential capacity provided by multi-element arrays. An expression for the correlation coefficient has been presented which quantifies the similarity between two channel fading envelopes. This parameter is calculated from two vectors of measured channel gains, each containing one vector-element per local area measurement. A MEA capacity expression was also provided which determines the theoretical upper limit of the data rate for a measured environment. The channel gains input into both calculations are recorded by the measurement system in the form of H-matrices. Both calculations provide single observations of statistical channel parameters, and a large number of measurements are required to obtain accurate results. The concepts explained in this chapter are repeated throughout this thesis and employed in data analysis.