

An Optimal Approach to Geometric Trimming of B-Spline Surfaces

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(ABSTRACT)

Geometric trimming of a surface involves removal of unwanted portions of the surface and providing a new mathematical description for the trimmed patch. This entails creating a new geometry for the trimmed patch, which closely approximates the corresponding portion on the original patch. The procedure is shown to involve obtaining data points on the B-spline surface that lie within the region specified by the parameter values for the trimming curve and describing a new surface which interpolates this new set of data points. This research looks at optimizing the procedure described above by basing the choice of parameter values for the trimming curve, at points where curvature optima occur over the surface. A visualization tool kit has been developed using *OpenGL*, as a means to discern the difference between the two surfaces. In order to quantify and aid in minimizing the error (difference) in approximating the original surface with the trimmed patch, an error measurement tool developed in *MATLAB* has been employed.

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Always, Praise the Lord for giving us the wisdom.

Believe

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