5.0   Geometric trimming

5.1 General procedure

As described earlier, the number of control points affecting a single bicubic surface patch is 4 X 4, i.e., 16. In order to calculate a point on the surface, it is necessary to find which control points of the original surface affect the patch wherein the surface point is to be located. For the example shown in Figure 5.1, a point located in patch (*3,4*) is affected by the control points numbered *0, 1, 2* and *3* in the parametric *u*-direction and 1*, 2, 3* and *4* in the parametric v-direction. It should be noted that the first two and last two control points in each parametric direction are not considered, which follows from the definition of a B-spline surface. Hence, the mapping of points is started from patch (*3,3*) instead of (*1,1*).

Figure 5.1: Patch numbering scheme for a bi-cubic B-spline surface based on control point information.
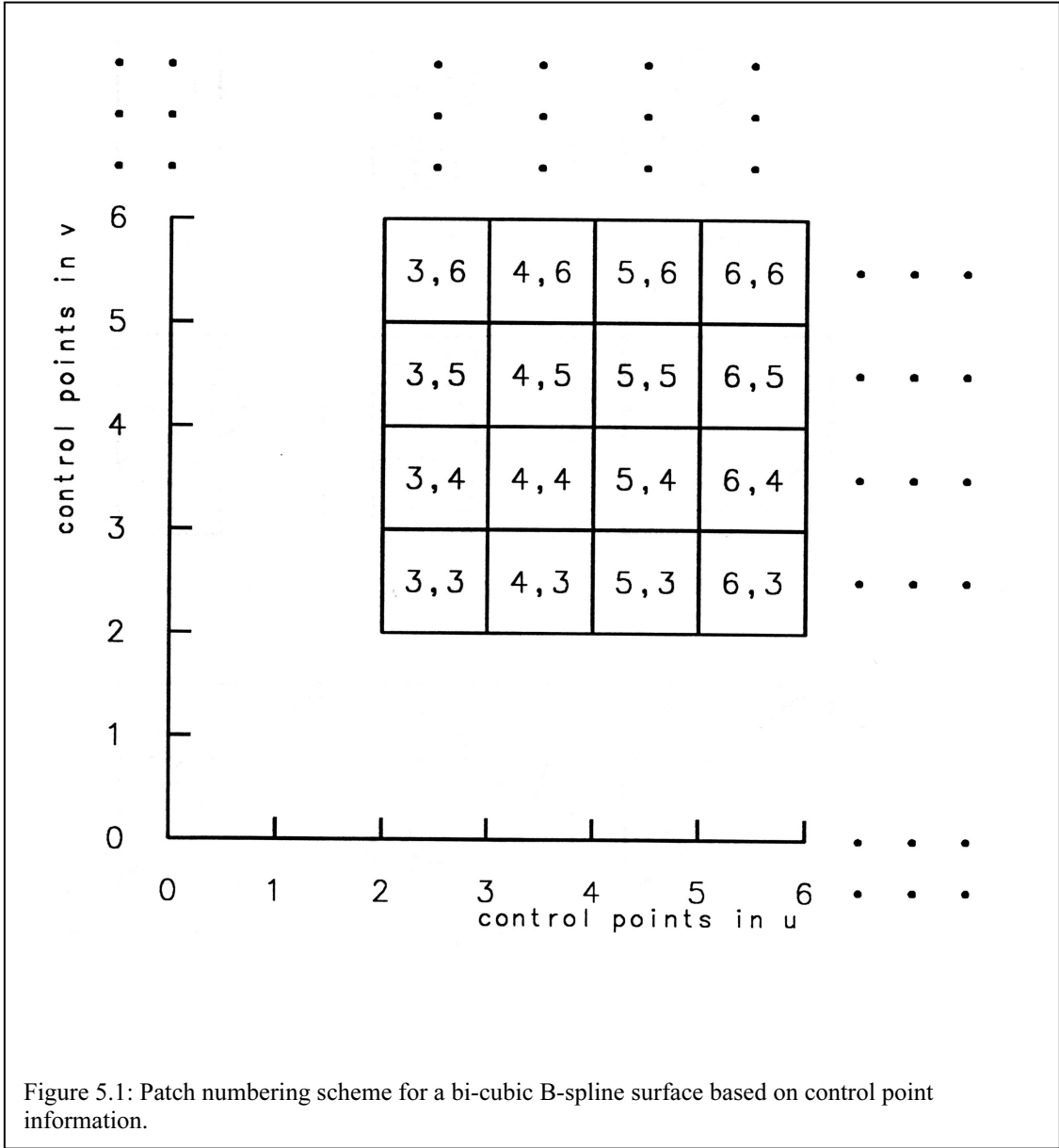
Consider a B-spline surface patch as shown in Figure 5.2. This surface patch was generated by Fleming's surface inversion algorithm [Flem92a] to interpolate a set of data points. This algorithm also defines the knot intervals and control points for this surface patch. Two of the characteristics of inversion algorithm are worth mentioning at this point, namely:

1. The number of knots that define a knot vector in each parametric direction is equal to the number of data points that has been considered for interpolation in that direction; and

2. The number of data points that can be considered along any parametric direction for any value of its orthogonal counterpart has to be constant.

Figure 5.2: A bi-cubic B-spline surface obtained by interpolating a given set of data points and its parametric domain
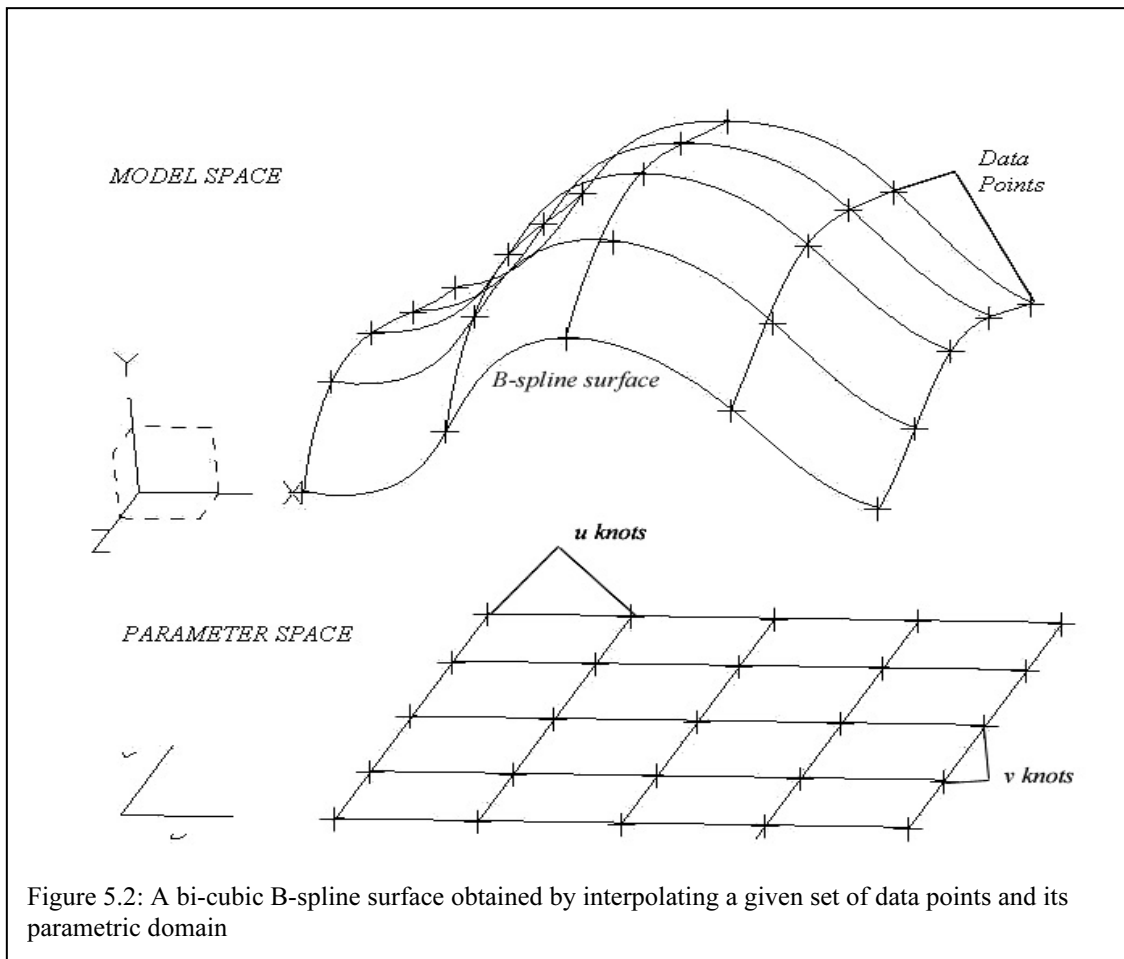
A trim curve is now required to assist in the trimming of the surface. A trim curve is the curve along which two surfaces intersect. A set of 21 parameter (u-v) values for the intersection (trim) points has been considered to describe the trim curve in the parametric domain of the surface. The choice of the number of points that can be used to describe the curve, however, is left to the designer.
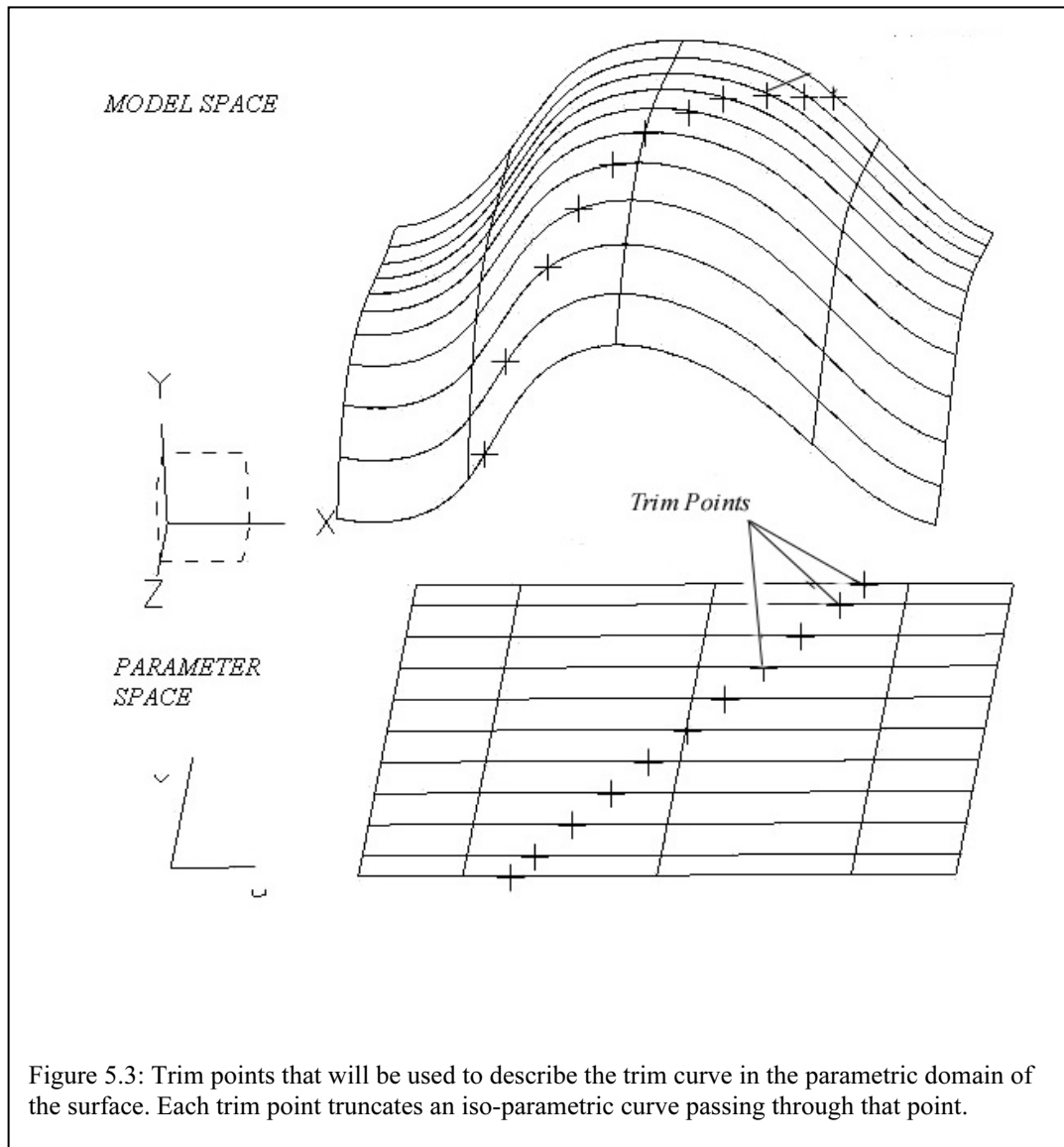

Figure 5.3: Trim points that will be used to describe the trim curve in the parametric domain of the surface. Each trim point truncates an iso-parametric curve passing through that point.

Having more number of closely spaced points would yield a curve that lies closer to the surface. Figure 5.3 shows the trim curve points in the parametric domain of the surface.

The next step is to find a new data point set for the trimmed patch with the trim curve, as its new boundary. The new data point set for the trimmed patch is found by subdividing the region of the parametric domain of the surface, that would remain after trimming and mapping the parametric points obtained from subdivision, onto model space. The process of subdividing the parametric domain of the surface is referred to as Parametric subdivision.

Parametric subdivision is an important step in this approach towards geometric trimming. The u-v parametric domain of the surface has to be subdivided along both the parametric directions in order to obtain the surface defining parametric points. The number of parametric points obtained from subdivision decides the number of data points that would be available for describing the trimmed patch upon mapping. Obviously, the more the number of surface points available, the higher the accuracy of the matching surfaces. But such an approach would sacrifice the computational speed. For this reason, the number of subdivisions for the v parametric direction was set equal to the number of points used to describe the trim curve and for the u parametric direction, it was set equal to the original number of points used to describe each isoparametric curve on the surface in that direction. Thereby, the original degree of computational complexity was maintained without significantly sacrificing the accuracy. Figure 5.4 illustrates this set up.

mapped data points
for the trimmed path

Truncated
curves

MODEL SPACE

Trim curve on the
surface in model
space

Mapping direction

PARAMETER
SPACE

Y

Z

Result of parametric subdivision of the region that would
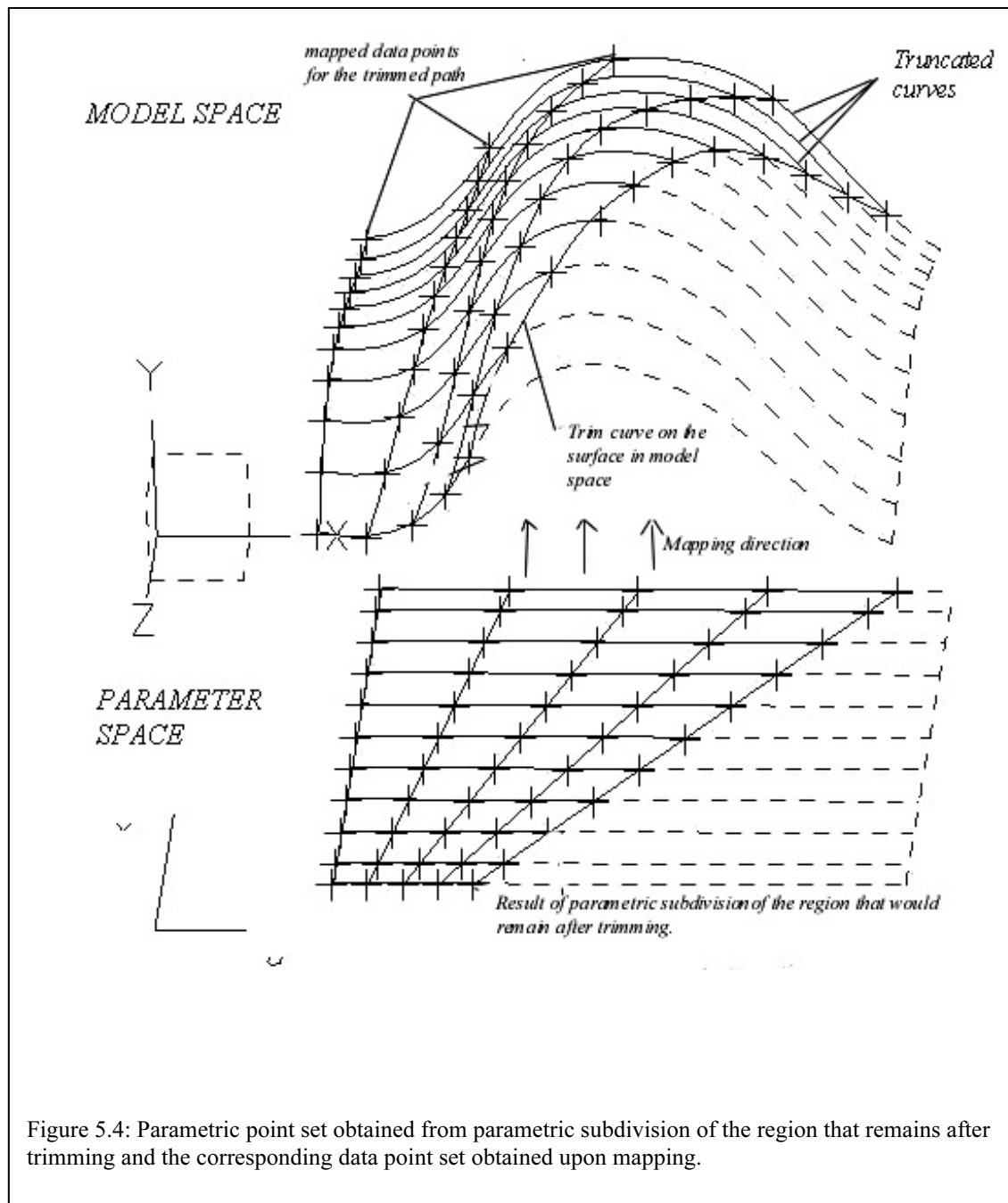remain after trimming.

Figure 5.4: Parametric point set obtained from parametric subdivision of the region that remains after trimming and the corresponding data point set obtained upon mapping.

Once the parameter point set is obtained from subdivision, the knot interval to which each parameter point belongs and the associated control points are identified as described earlier. Equipped with this information, a mapping algorithm maps the parameter point set defining the trimmed patch, onto the surface in model space. This results in a data point set for the trimmed patch in model space. Figure 5.5 shows the data point set obtained upon mapping.
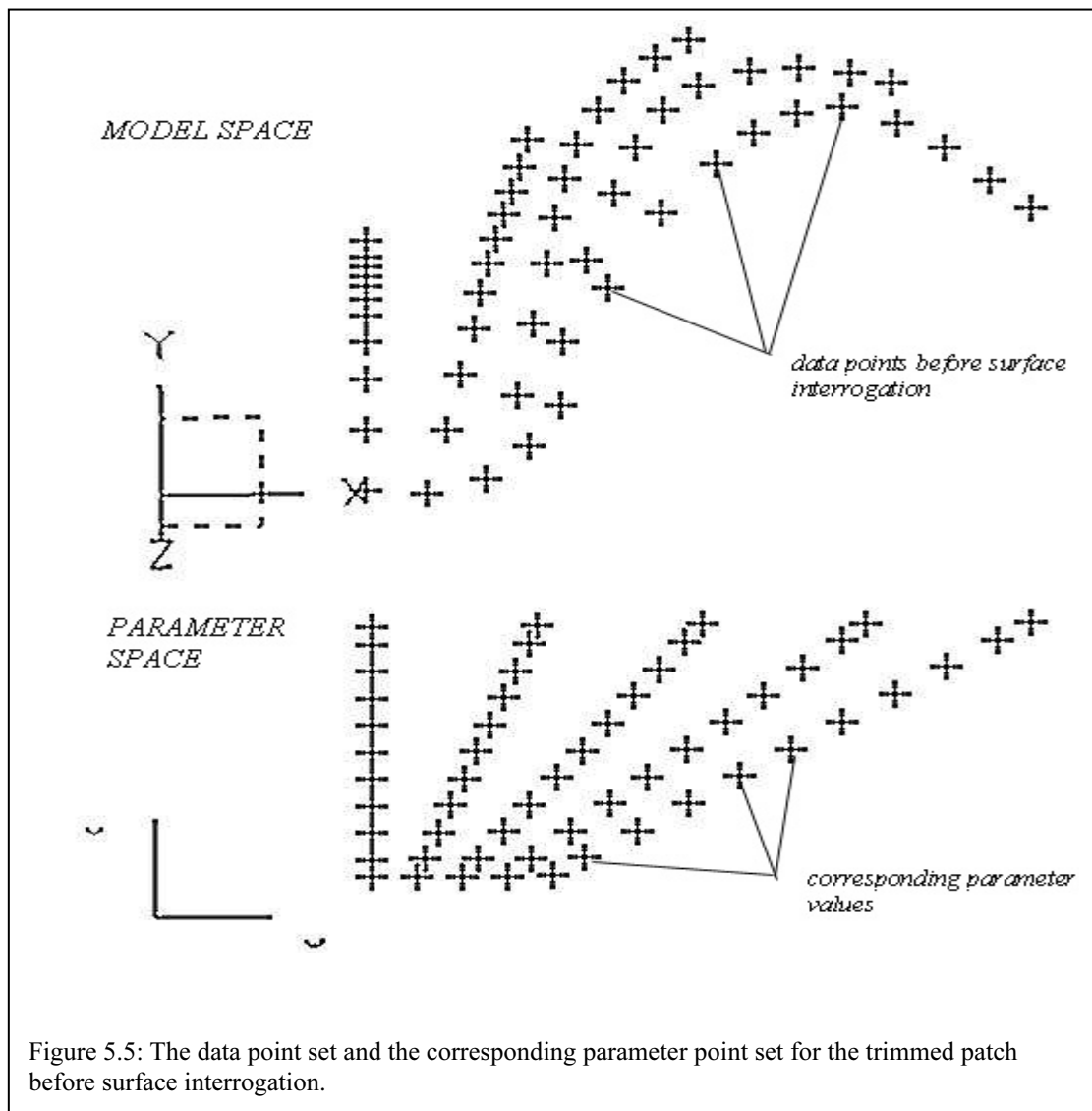

Figure 5.5: The data point set and the corresponding parameter point set for the trimmed patch before surface interrogation.
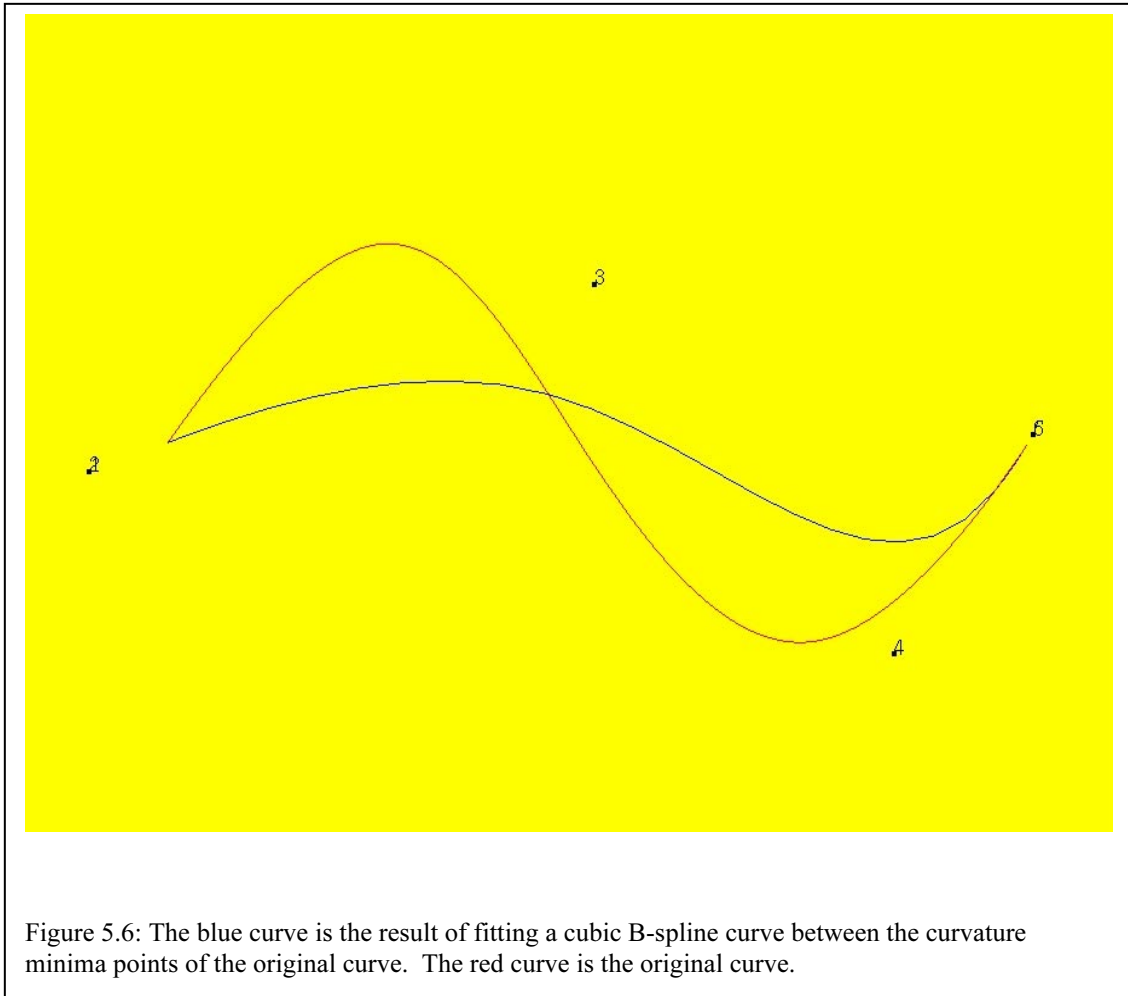
The data point set obtained at this stage can be used to describe the trimmed patch using the inversion algorithm. However, it has been felt that the number of points required can be optimized further. The argument here is that **if the surface is interrogated to find points at which there is a curvature optimum, these points would be enough to describe the trimmed surface patch and this patch should match the approximation that can be achieved by using the original data point set for the trimmed patch**.

As shown from Equation 4.14 in the previous chapter, the curve obtained by points on the surface along a particular parametric direction, say u-direction, for a fixed parameter value of v can be treated as individual B-spline curves and this fact has been used to aid in the interrogation of the surface.
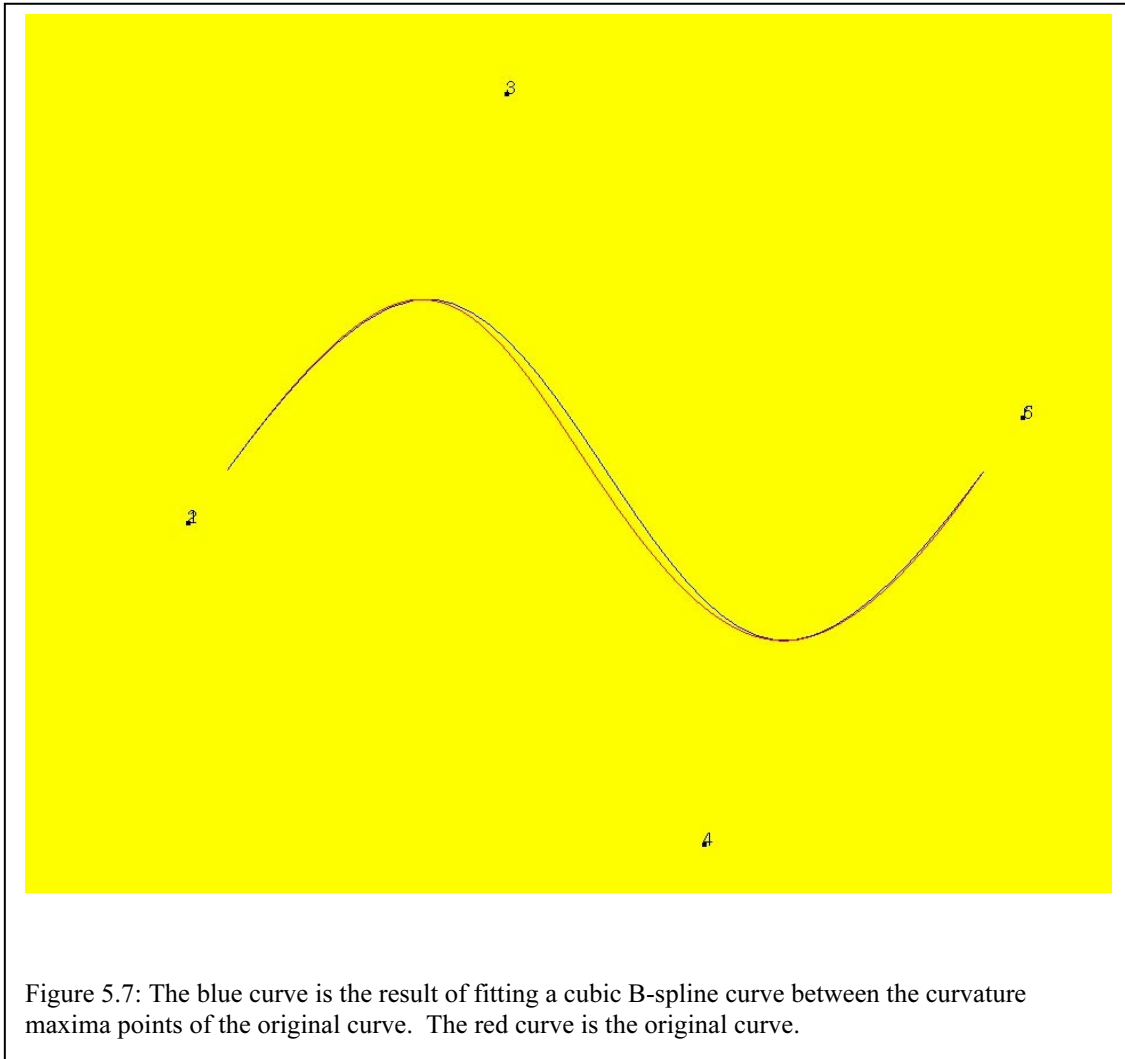
Thus, from the mapped set of data points for the trimmed patch, iso-parametric curves are constructed in both the u and the v directions as shown in Figure 5.4. The idea is to have the maximum number of curves that can be possible from the available information, to lie on the surface, so that curvatures changes can be more comprehensively recorded over the whole surface. Curvature at various points along each curve is calculated.

Curvature optima on a curve can be characterized in two ways. There is a curvature optimum occurring at a point on the curve when there is curvature minima or curvature maxima at that point. Initially, it was felt that identifying the points where an inflection (curvature minima) occurs and using those points to describe the trimmed patch would give a good approximation. Figure 5.6 shows that this information was not enough to
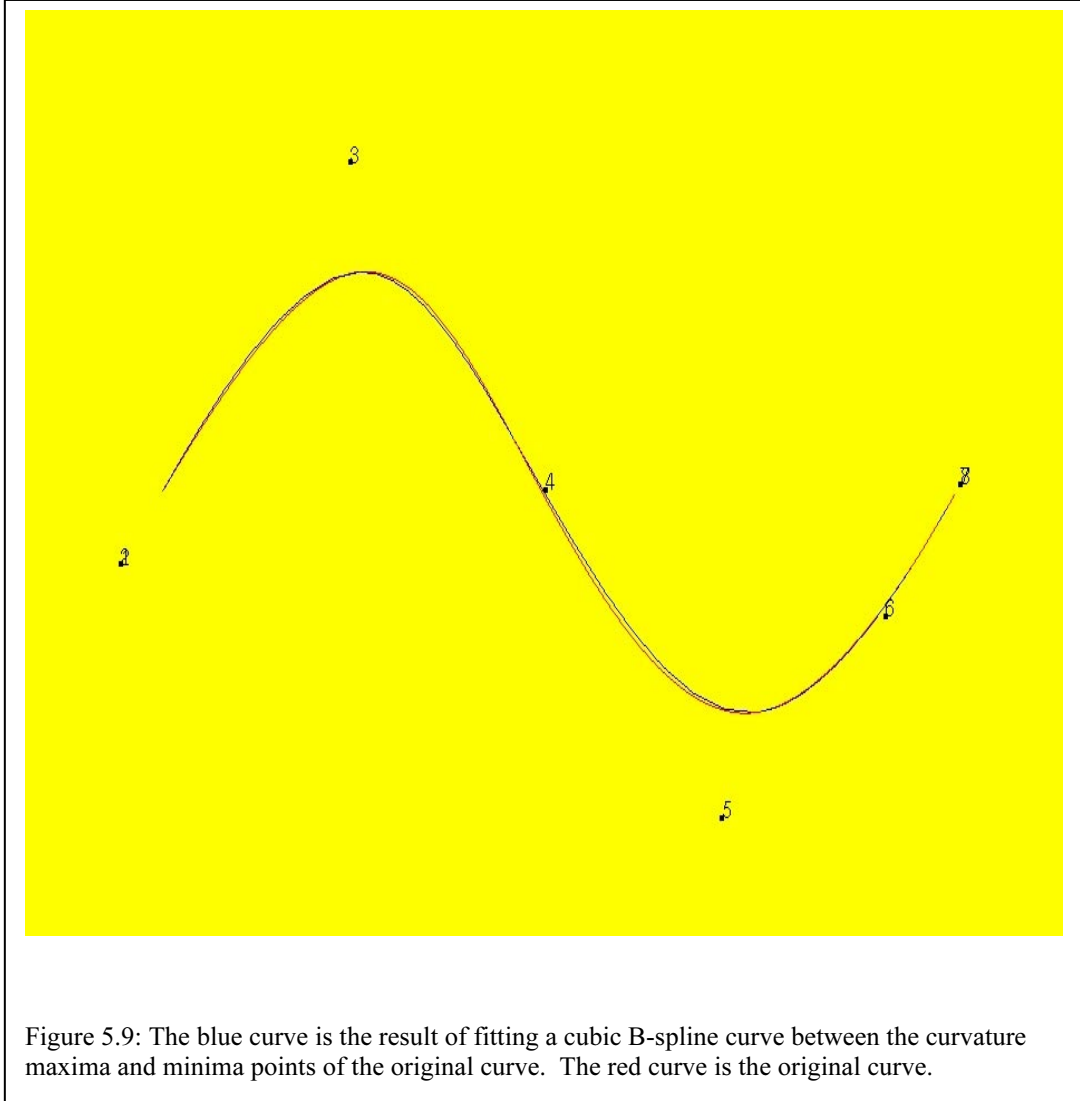
characterize the curve/surface. As seen from the figure, there may be a case where the curve that needs to be approximated is of a very high degree, that approximating it with a cubic curve, fit between the inflection points of the original curve does not provide with good results.

Figure 5.6: The blue curve is the result of fitting a cubic B-spline curve between the curvature minima points of the original curve. The red curve is the original curve.

An alternative to this was to consider the curvature maxima as a means of characterizing the curve. This provided with slightly better results but still was not enough to give a good approximation. Figure 5.7 illustrates this argument.

Figure 5.7: The blue curve is the result of fitting a cubic B-spline curve between the curvature maxima points of the original curve. The red curve is the original curve.

Considering both the curvature maxima and minima to characterize the curve provided the best approximation. Hence both the curvature maxima and minima were adopted as the basis for characterizing a curve/surface. Figure 5.8 shows the results obtained on adopting the new basis for curve/surface characterization.

Figure 5.9: The blue curve is the result of fitting a cubic B-spline curve between the curvature maxima and minima points of the original curve. The red curve is the original curve.

For all practical purposes, it is seen that a curvature minima occurs at a point on a curve for the following condition:

$$\kappa(u_i) - \kappa(u_{i-1}) < 0 \;\land\; \kappa(u_{i+1}) - \kappa(u_i) \geq 0$$

And the curvature maxima occurs at a point on a curve for the following condition:

$$\kappa(u_i) - \kappa(u_{i-1}) > 0 \ \wedge \ \kappa(u_{i+1}) - \kappa(u_i) \leq 0$$

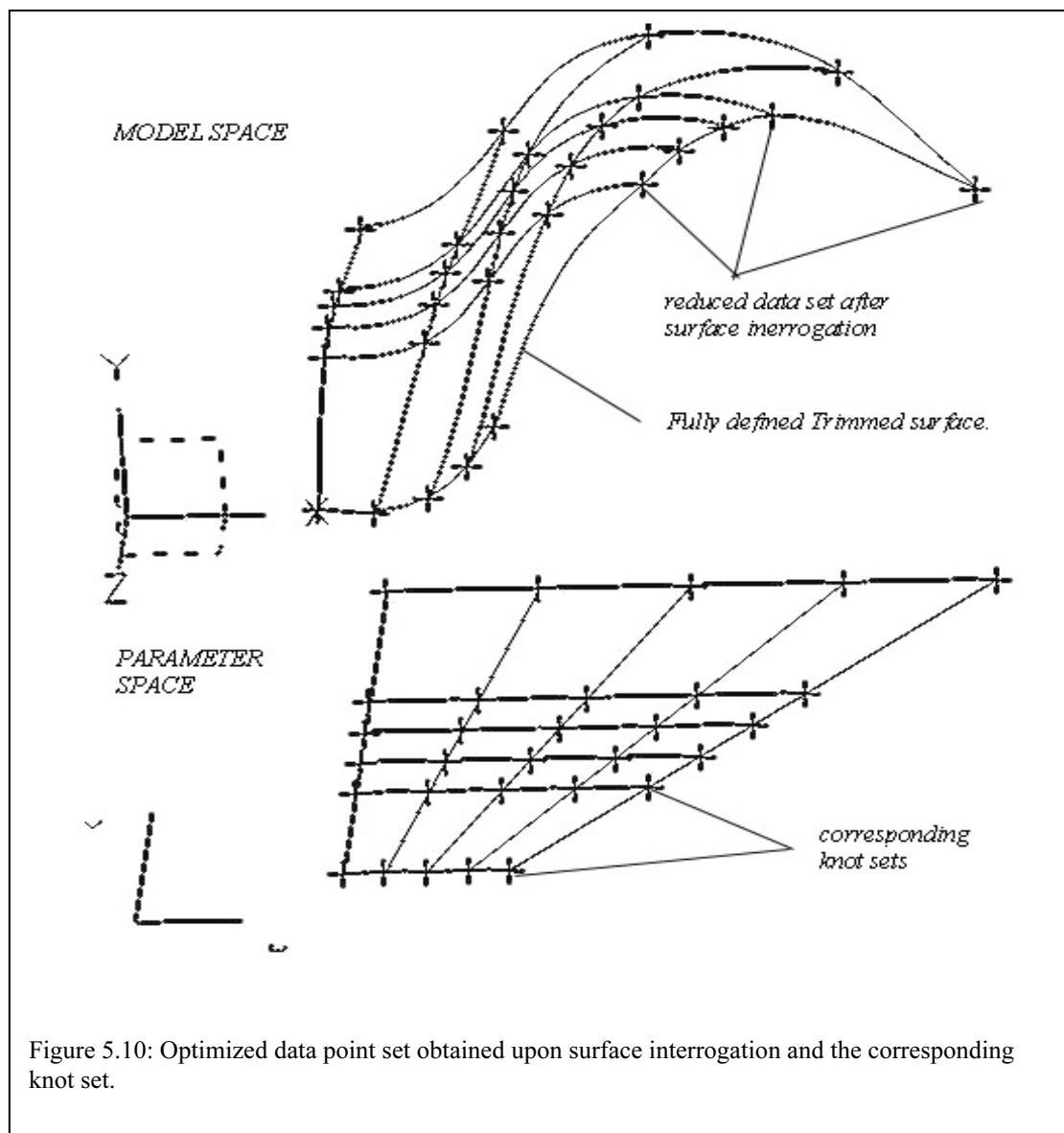The points of curvature maxima or minima for any given curve can thus be identified.

Figure 5.10: Optimized data point set obtained upon surface interrogation and the corresponding knot set.

These points are mapped back to the parametric domain of that curve to obtain the parameter value at which the curvature optima occur. This process is repeated for all the curves being considered for interrogation.

It is found that, for any two adjacent curves on any given free-form surface, the curvature maxima or minima may not occur at a point, which has the same parameter value. Also, the number of such points may not be the same on both curves. This can be attributed to the non-uniform geometry of the surfaces being considered. This would then amount to an uneven subdivision of the parametric domain of the trimmed patch. But Fleming's surface inversion algorithm cannot handle uneven subdivision. This situation is contrary to the second characteristic of the inversion algorithm. The dilemma is addressed by mapping the parameter values obtained from interrogation of each curve in any given direction, onto the parametric boundary in that direction. A uniform subdivision of the parametric domain of the trimmed patch can now be performed based on the resulting number of mapped points in each parametric direction and their respective locations in the domain. The parameter point set thus, obtained from subdivision is then mapped on to the surface in model space resulting in a new optimized data point set for the trimmed patch as shown in Figure 5.10.

Once the optimized data point set for the trimmed patch is obtained, the end constraints are specified and will correspond to the first derivatives of the original surface at the trimming curve. With these components, the points and derivative constraints, a new

mathematical surface can be computed by using the continuity constraint based inversion procedure (see section 4.7).

It was incidentally found that because of the nature of the continuity based inversion algorithm, if a continuity constraint is specified at a certain location, the same continuity value is set for either the entire row in v or column in the u parametric direction depending on the case. Thus, specifying continuity constraints at the trim points on the trimming curve in both parametric directions was found not to be feasible. This is because, setting a v-direction C^1 continuity constraint on a point on the trimming curve makes the entire v row on the surface C^1 continuous, thus, disrupting the original C^2 continuity in the u-direction. Hence, only u-direction continuities were specified at the trim points. Since for the cases considered, individual iso-parametric curves were disrupted only in the u-parametric direction when trimmed, setting a continuity constraint only in the u-direction provided with reasonable accuracy.

In order to calculate the first derivatives used as end continuity constraints when defining a new set of trimmed isoparametric curves, the following equations were used.

$$\frac{d}{du}s(u) = \frac{d}{du}\sum_{i=0}^{n}d_iN_i^k(u) = \sum_{i=0}^{n}d_i\frac{d}{du}(N_i^k(u) \tag{5.1}$$

$$\frac{d}{du}s(u,v) = \sum_{i=0}^{n}\sum_{j=0}^{m}d_{ij}(\frac{d}{du}N_i^k(u))N_j^l(v) \tag{5.2}$$

$$\frac{d}{dv}s(u,v) = \sum_{i=0}^{n}\sum_{j=0}^{m} d_{ij} N_i^k(u)(\frac{d}{dv} N_j^l(v)) \qquad (5.3)$$

Equation (5.1) shows a mathematical representation for the first derivative of a B-spline curve while equation (5.2) and (5.3) expands this relationship for surfaces. Both, basis functions and the derivatives of the basis functions are numerically computed using a non-recursive procedure based on an algorithm by deBoor [deBo78] and modified by Gloudemans [Glou89].

Cases with open trim curves have been tested in this research. Open trimming curves are defined as curves that intersect at exactly two different boundaries of the parametric domain as shown in Figure 5.11.

In summary, the following steps are followed in order to trim a surface using an open trimming curve.

▪ Points describing the original surface are read from a data file "*datasurf_xx*". The files used in this research are compiled in Appendix B.

▪ A bicubic B-spline surface that interpolates the surface data is created and defined as the original surface (Figure 5.12). The surfaces considered are easy to visualize and don't have a predictable behavior like conics do.

▪ A trimming curve is approximated from a set of points in the file "*datatrimxx*" (Appendix B)

- Parametric points obtained from subdivision of the retained portion of the parametric domain of the surface are mapped onto the surface resulting in a data point set for the trimmed patch.

Figure 5.11: Different trimming criterion considered in this research
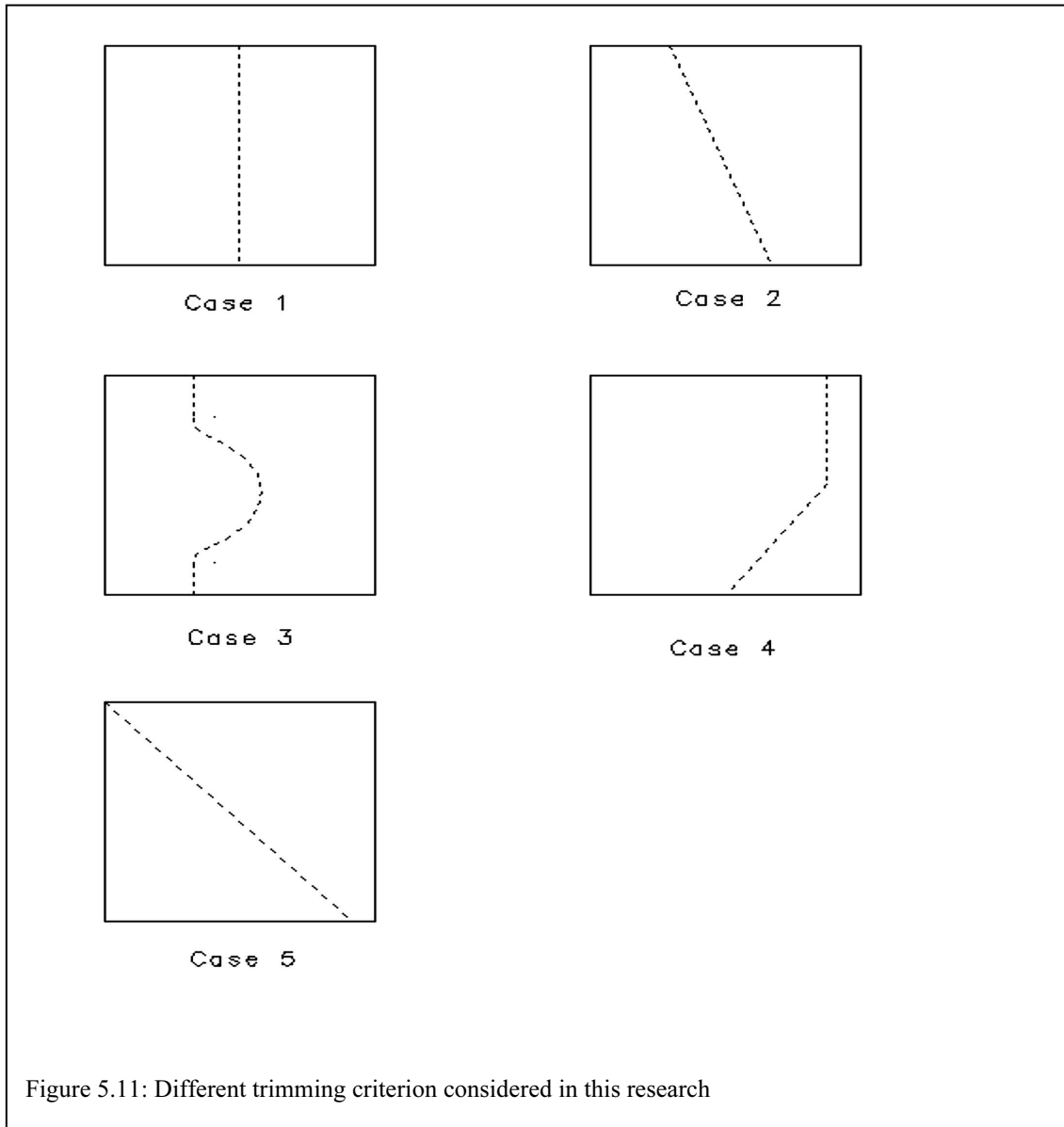
- The trimmed patch is further interrogated to identify curvature changes using the data point information from the previous step. Points at which curvature changes occur are garnered to obtain the new optimized data point set for the trimmed patch.

- The optimized data point set is fit with a bi-cubic B-spline surface to obtain the trimmed patch. Figure 5.13 shows the trimmed patch.
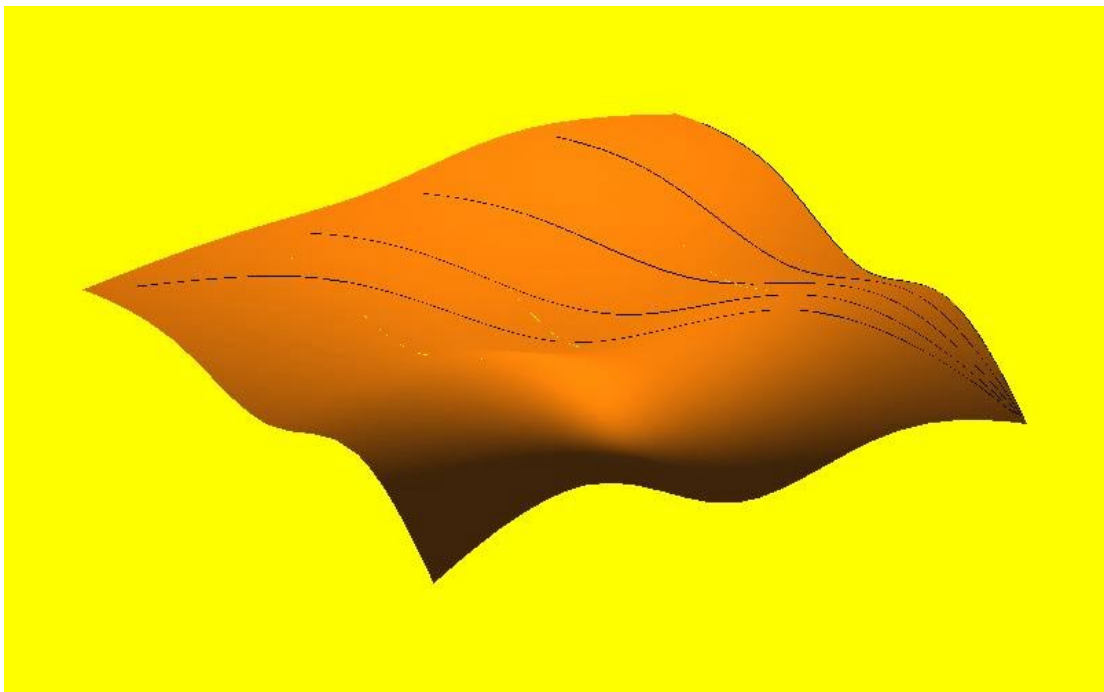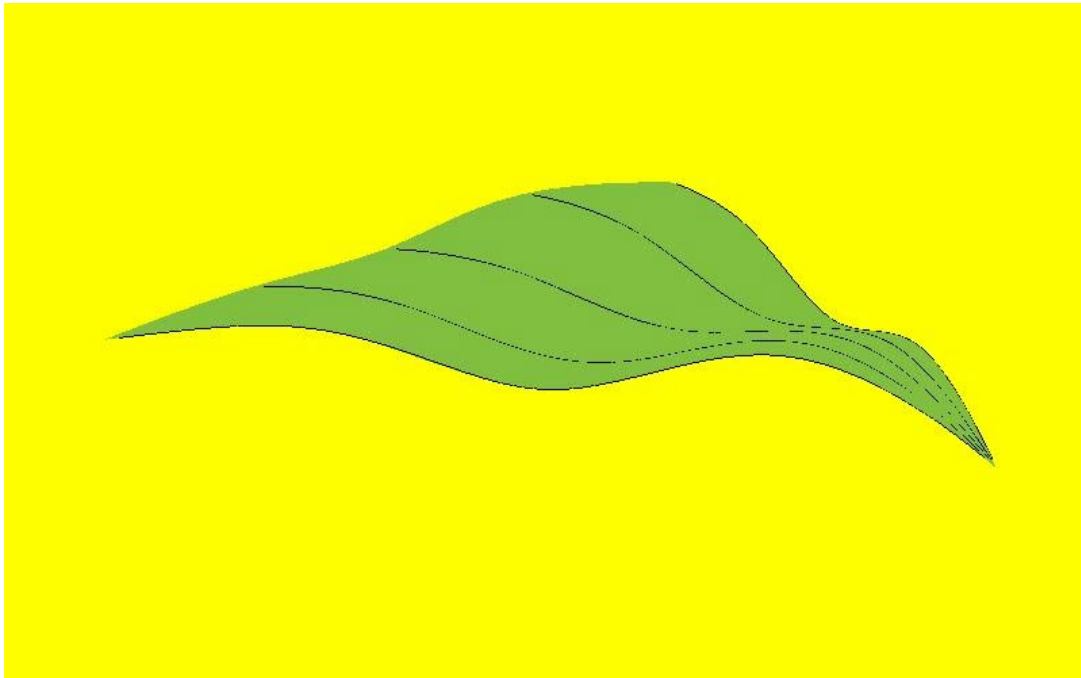
Figure 5.12: Rendered image of the original surface patch.

Figure 5.13: Rendered image of the trimmed patch.