Chapter 4 Heterodyne Interferometry Kinetics

4.1 Film thickness measurement

A plane wave light propagating in a direction $\tilde{s}$ can be expressed as electromagnetic waves[1] shown in equation 3.1.

$$\tilde{E} = \tilde{A} e^{i(\tilde{k} \cdot \tilde{r} + wt)}, \tilde{k} = \frac{w}{c} \approx{n}(w, \tilde{s})\tilde{s} \tag{4.1}$$

where $\tilde{E}$ is the electric field vector. $\tilde{A}$ is the complex vector (i.e. field amplitude), independent of coordinates and time, ω is the angular frequency and the angular frequency $\omega = 2\pi\nu$, where ν is light frequency and light frequency $\nu = c/\lambda$, c is the speed of light and λ is the wavelength of the light. $\tilde{k}$ is the wave vector, $\tilde{n}$ is the complex index of refraction and $\tilde{s}$ is a unit vector.

When electrical field $\tilde{E}$ is parallel to the principle direction of refraction index as shown in figure 4.1a, 4.1b and 4.1c. The refractive index can be reduced to scalar. Therefore, the reference beam and the sample beam as shown in figure 3.1 can be reduced to scalar function. The reference beam at the photodetector is phase modulated by Bragg cell and can be expressed as,

$$E_r = E_{r0} e^{i(wt + kx + f_r)} \tag{4.2}$$

Where E_r is the electric field of the reference beam. E_{r0} is the electric field amplitude of the reference beam. ω is the laser angular frequency and the laser angular frequency $\omega = 2\pi\nu$, where ν is laser frequency and laser frequency $\nu = c/\lambda$, c is the speed of laser and λ is the wavelength of the laser. k is the wave vector $k = 2\pi/\lambda$. ϕ_r is the phase of the reference beam and the phase of the reference beam $\phi_r = \omega_b t$.

The laser has multiple reflections at the film side shown in figure 4.1a, 4.1b, and 4.1c.

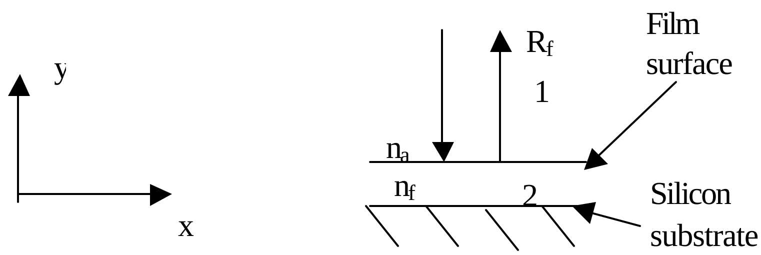
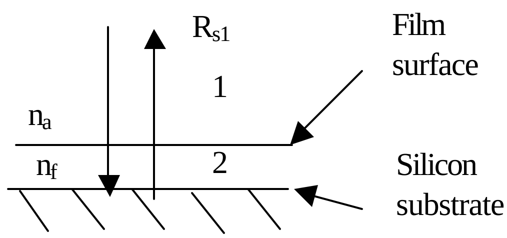


Figure 4.1a The first reflection at the film surface



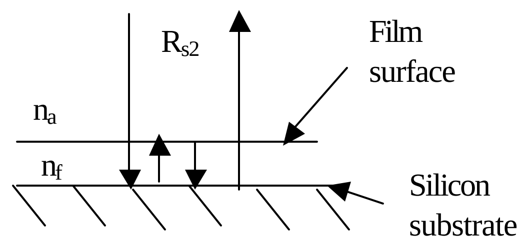


Figure 4.1b The first reflection at the silicon

Figure 4.1c The second reflection at the silicon

* The coordination is valid for all three figures 4.1a, 4.1b, and 4.1c. x axis is the principle direction which is parallel to the direction of applied electric field. y axis is the direction perpendicular to the film. z axis is the direction perpendicular to x and y axis and pointing out of the plane. Define R_f is the Fresnel reflectivity at the film. Rsi is the reflectivity of the silicon. R_{si2} are the reflectivity for the second reflection at the silicon.

n_a is the refractive index of air, n_a=1.0. n_f is the refractive index of the film, n_f=1.453. The reflectivity of the silicon R_{s1} =0.72 for 633nm. The reflectivity of R_f can be calculated as,

$$R_f = \frac{(n_f - n_a)^2}{(n_f + n_a)^2} = 0.034 \tag{4.3}$$

The reflectivity of R_{s2} can be calculated as,

$$R_{s2} = R_{s1} \cdot R_f \cdot R_{s1} = 0.017 \tag{4.4}$$

From above calculation, we can see that the multiple reflections are small. Consider the absorption of the film. Define I_0 is the intensity of incident sample beam. I_{Rf} is the intensity of light reflected at the film surface. I_{Rs1} is the intensity of light reflected first time from the silicon. I_{Rs2} is the intensity of light reflected second time on silicon. α is the absorption coefficient of polymer. Z is the thickness of the film.

$$
\begin{aligned}
I_{Rf} &= I_0 R_f = 0.034 I_0 \\
I_{Rs1} &= I_0 R_{s1} e^{-\alpha 2Z} = 0.72 I_0 e^{-\alpha 2Z} \\
I_{Rs2} &= I_0 R_{s2} e^{-2\alpha Z} e^{-2\alpha Z} = 0.017 I_0 e^{-4\alpha Z}
\end{aligned}
\qquad (4.5)
$$

The absorption coefficient of polymer and the thickness of the film is very small, $\alpha Z \approx 0$, therefore, we can neglect the film absorption. Since the intensity of the reflected sample beam is different from the sample incident beam. We include a factor γ to modify the intensity of the sample reflected beam which interferes with the reference beam. The reflected sample beam is expressed as,

$$
E_s = \gamma E_{s0} e^{i(\omega t + kx + \phi_s + \Delta\phi)} \qquad (4.6)
$$

Where E_s is the electric vector of the sample beam. ϕ_s is the phase of the reflected sample beam. E_{s0} is the electric field amplitude of the incident sample beam. Define E_{s0}' as the amplitude of the reflected beam and set $E_{s0}' = \gamma E_{s0}$ to simplify the derivation. $\Delta\phi$ is the phase change of reflected beam due to the optical path of the sample which equals to the refractive index of the film times film thickness and can be expressed as,

$$
\Delta\phi = 2nkZ_0(1 - \cos\omega_g t)/2 = nkZ_0 - n\kappa Z_0 \cos(\omega_g t) \qquad (4.7)
$$

Where Z_0 is the initial film thickness, ω_g is the frequency of galvano mirror and $k = 2\pi/\lambda$ Total power at the detector is given by,

$$
\begin{aligned}
P &= (E_r + E_s)(E_r + E_s)^* \\
P &= E_r^2 + E_s^2 + E_r E_s^* + E_r^* E_s
\end{aligned}
\qquad (4.8)
$$

By substituting equation 4.2 and 4.6 into equation 4.8, and rearrange, we obtain,

$$P = E_{r0}^{2} + E_{s0}^{'2} + 2E_{r0}E_{s0}^{'}\cos(w_b t - f_s - \Delta f) \tag{4.9}$$

Substituting equation 4.7 into equation 4.9, we obtain,

$$P = E_{r0}^{2} + E_{s0}^{'2} + 2E_{r0}E_{s0}^{'}\cos(w_b t - f_s - nkZ_0 + nKZ_0\cos(w_g t)) \tag{4.10}$$

Next we set

$$f_s - nkZ_0 = f_1 \tag{4.11}$$

Simplify equation 4.10 to obtain,

$$P = E_{r0}^{2} + E_{s0}^{'2} + 2E_{r0}E_{s0}^{'}\cos(w_b t - f_1 + nKZ_0\cos(w_g t)) \tag{4.12}$$

In equation 4.12, (E_{r0}^2 + $(E_{s0}^{'})^2$) is constant power, invariant of time, (i.e. DC signal). The time dependent component (P′) is given by:

$$P^{'} = 2E_{r0}E_{s0}^{'}\cos(w_b t - f_1 + nKZ_0\cos(w_g t)) \tag{4.13}$$

Due to the reflectivity change and scattering at the edge of film, the amplitude modulation factor is considered. Wafer may absorb moisture on the surface. All these factors will introduce a phase lag of amplitude with the galvano mirror rocking frequency. To correct this effect, a background factor ϕ_g is introduced. S is the reflectivity of the sample surface and S_0 is the initial reflectivity of the sample. S and S_0 can be measured by the reflectivity measurement. Expand S/S_0 at S_0 and taking two terms for approximation.

$$s/s_0 = 1 + a\cos(w_g t + f_g) \tag{4.14}$$

where a = S/S_0.

By substituting equation 4.13 into 4.14, we obtain,

$$P^{'} \approx 2E_{r0}E_{s0}^{'}(1 + a\cos(w_g t + f_g))(\cos(w_b t - f_1) - nKZ_0\cos(w_g t)\sin(w_b t - f_1)) \quad (4.15)$$

By expansion and simplification of equation 4.15, we obtain,

$$\begin{aligned} P^{'} \approx\ & 2E_{r0}E_{s0}^{'}(\cos(w_b t - f_1) + a\cos(w_g t + f_g)\cos(\sin(w_b t - f_1)) - \\ & nkZ_0/2\sin((w_b + w_g)t - f_1) - nkZ_0/2\sin((w_b - w_g)t - f_1) \end{aligned} \quad (4.16)$$

Further expand equation 4.16 and obtain,

$$\begin{aligned} P^{'} \approx\ & 2E_{r0}E_{s0}^{'}(\cos(w_b t - f_1) - (nkZ_0 + a\sin f_g)/2\sin((w_b + w_g)t - f_1)) + \\ & a/2\cos f_g\cos((w_b + w_g)t - f_1) + (a\sin f_g - nkZ_0)/2\sin((w_b - w_g)t - f_1) \\ & + a/2\cos((w_b - w_g)t - f_1)\cos f_g) \end{aligned} \quad (4.17)$$

The spectrum analyzer only measures the time-dependent components in the total power at the photodetector. Therefore, spectrum analyzer measures main signal (i.e. the carrier, the power portion at frequency ω_b), the first order left sideband (the power portion at $(\omega_b - \omega_g)t$), the first order right sideband, (the power portion at $(\omega_b + \omega_g)t$).

Let ratio of first order left sideband over carrier $= R_1$

Let ratio of first order right sideband over carrier $= R_2$

Using equation 4.16 and definition of R_1, R_2, we obtain,

$$R_1 = 1/2\sqrt{((nkZ_0)^2 + a^2 - 2nkZ_0 a\sin f_g)} \quad (4.18)$$

$$R_2 = 1/2\sqrt{((nkZ_0)^2 + a^2 + 2nkZ_0 a\sin f_g)} \quad (4.19)$$

Manipulating the equation 4.18 and 4.19 and the values of Z_0 and ϕ_g are obtained.

$$Z_0 = 1/(kn)\sqrt{2(R^2{}_1 + R_2{}^2) - a^2/2} \quad (4.20)$$

$$f_g = \arcsin((R_2^2 - R_1^2)/nkZ_0 a) \quad (4.21)$$

Equation 4.19 indicates that if either the film thickness is known, the refractive index can be calculated and vice versa.

4.2 Measurement of electric field effect

Under electrical field (E-field), the polymer film will orient in the E-field direction. The poling of polymer thin film will change the refractive index and the thickness of the polymer film along the E-field. Since polymers were poled uniformly along the field direction (i.e. principle direction x), therefore we can reduce light propagation as scalar equations 4.2 and 4.6 for electric properties measurements too. Since the film absorption and multiple reflections effect is very small, we can neglect the reflectivity change across the film. The phase change due to the refractive index and thickness change can be expressed similar to equation 4.7,

$$\Delta f = 2k(nZ - n_0 Z_0) \quad (4.22)$$

Where Z_0, n_0 are the initial polymer film thickness and refractive index. Z and n are the instantaneous film thickness and the refractive index along the applied field direction. Z and n are functions of time. The optical length of laser traveling in the film equals the film refractive index times film thickness and is expresses as,

$$nZ = 1/2(n_0 Z_0 + n_1 Z_1) - 1/2(n_1 Z_1 - n_0 Z_0)\cos(w_E t) \quad (4.23)$$

Where n_1 and Z_1 are the maximum refractive index and film thickness under certain frequency and amplitude of electric field. Z_0, n_0 are the initial polymer film thickness and refractive index. ω_E is the electric field frequency. By substituting equation 4.22 into 4.9, the total power is obtained,

$$P = E_{r0}{}^2 + E'_{s0}{}^2 + 2E_{r0}E_{s0}{}'\cos(w_b t - f_s - 2k(nZ - Z_0 n_0)) \quad (4.24)$$

where E_{r0}, E'_{s0} are the magnitudes of the reference beam and the reflected sample beam; ω_E is electric field frequency.

As we discussed before, $(E_{r0}^2 + E'^2_{s0})$ is DC component, invariant of time. The time dependent component P′ is given by:

$$P' = 2E_{r0}E'_{s0}\cos(\omega_b t - \phi_s - 2k(nZ - n_0 Z_0)) \quad (4.25)$$

Further expanding equation 4.24 after substituting equation 4.23 into 4.25,

$$P' = 2E_{r0}E'_{s0}\cos(\omega_b t - \phi_s + k(n_0 Z_0 - n_1 Z_1) - k(n_1 Z_1 - n_0 Z_0)\cos\omega_E t) \quad (4.26)$$

Set

$$\phi_s - k(Z_0 n_0 - n_1 Z_1) = \phi' \quad (4.27)$$

Equation 4.26 is simplified as,

$$P' = 2E_{r0}E'_{s0}\cos(\omega_b t - \phi' + k(Z_1 n_1 - n_0 Z_0)\cos\omega_E t) \quad (4.28)$$

From equation 4.28,

$$P' = 2E_{r0}E'_{s0}(\cos(\omega_b t - \phi') * \cos(k(Z_1 n_1 - Z_0 n_0)\cos(\omega_E t)) - \sin(\omega_b t - \phi')\sin(k(Z_1 n_1 - Z_0 n_0)\cos(\omega_E t))) \quad (4.29)$$

Using expansion series, the following terms are expressed as,

$$\sin(k(n_1 Z_1 - Z_0 n_0))\cos(\omega_E t) \approx k(n_1 Z_1 - Z_0 n_0)\cos(\omega_E t) \quad (4.30)$$

$$\cos(k(n_1 Z_1 - Z_0 n_0)\cos(\omega_E t)) \approx 1 - k^2(n_1 Z_1 - Z_0 n_0)^2\cos^2(\omega_E t)/2 \quad (4.31)$$
$$= 1 - k^2(n_1 Z_1 - Z_0 n_0)^2(1 + \cos(2\omega_E t))/4$$

By substituting Equation 4.30 and 4.31 into equation 4.29 and simplifying, we obtained,

$$P' = 2E_{s0}E'_{s0}((1 - k^2(n_1Z_1 - n_0Z_0)^2/4)\cos(w_b t - f')$$
$$- k(n_1Z_1 - Z_0n_0)/2\sin(w_b t - f' + w_E t)$$
$$- k(n_1Z_1 - Z_0n_0)/2\sin(w_b t - f' - w_E t) \quad (4.32)$$
$$- k^2(n_1Z_1 - Z_0n_0)^2/8\cos(w_b t - f' - 2w_E t)$$
$$- k^2(n_1Z_1 - Z_0n_0)^2/8\cos(w_b t - f' + 2w_E t))$$

The spectrum analyzer measures only the time-dependent components of power as discussed earlier with film thickness measurements. This time-dependent power will include the carrier (i.e. main signal) at $\omega_b t$; the first order left sideband at $(\omega_b t-\omega_E t)$; the first order right sideband at $(\omega_b t+\omega_E t)$; the second order left sideband at $(\omega_b t-2\omega_E t)$; the second order right sidebands at $(\omega_b t+2\omega_E t)$.

Let R_1 and R_2 be the ratio of first order left and right sidebands versus carrier. R_3 and R_4 are the ratio of second order left and right sidebands versus carrier.

The ratio of first order and second order sidebands versus the carrier can be calculated from equation 4.32 as,

$$R_1 = R_2 = -k(n_1Z_1 - Z_0n_0)/2(1 - k^2(n_1Z_1 - n_0Z_0)^2/4) \quad (4.33)$$

$$R_3 = R_4 = -k^2(n_1Z_1 - Z_0n_0)^2/8(1 - k^2(n_1Z_1 - n_0Z_0)^2/4) \quad (4.34)$$

R_3 and R_4 can be further simplified as,

$$R_3 = R_4 = -1/(8/k^2(n_1Z_1 - n_0Z_0)^2) - 2) \quad (4.35)$$

Equation 4.34 shows when $(n_1Z_1-n_0Z_0)$ increase, R_3 and R_4 also increase. Furthermore, the absolute change of (nZ) as defined earlier (the refractive index along the applied electric field direction times the instantaneous film thickness) can be calculated.

The changes in refractive index times thickness due to the first order sidebands is,

$$(n_1Z_1 - n_0Z_0) = (1 - \sqrt{(1 + 4R_{1_1}^2)})/R_1k = l * (1 - \sqrt{(1 + 4R_{1_1}^2)})/2 * R_1 * p \quad (4.36)$$

The changes in refractive index times film thickness in terms of second order sidebands is given by,

$$(n_1 Z_1 - n_0 Z_0) = -1/k * \sqrt{8R_3 / (1+2R_3)} = -\frac{l}{2*p}\sqrt{8R_3 / (1+2R_3)} \qquad (4.37)$$

From equation 4.36 and 4.37, if the initial film thickness and refractive index are known, the percent change of (nZ) of polymer film under electric field can be calculated.

$$\%change = (n_1 Z_1 - n_0 Z_0)/(n_0 Z_0) = n_1 Z_1 / n_0 Z_0 - 1 \qquad (4.38)$$

This differential interferometer design also allows us directly measure the film thickness changes by removing the two polarizers as shown in figure 3.1. This special function of our interferometer is further discussed in chapter 5. Without polarizers, the laser beam is not polarized along the E-field, thus the interferometry cannot measure the change of polymer orientation along the E-field. The change of (nZ) is all contributed by film thickness change instead of both refractive index and thickness change. In this case, it is reasonable to assume $n_1=n_0=n$. By simplifying equations 4.36 and 4.37 with $n_1=n_0$, the thickness change is calculated as,

$$R_1 = R_2 = -kn_0(Z_1 - Z_0)/2(1 - k^2 n_0^2 (Z_1 - Z_0)^2/4) \qquad (4.39)$$

$$R_3 = R_4 = -1/(8/k^2 n_0^2 (Z_1 - Z_0)^2 - 2) \qquad (4.40)$$

The above two equations can also be written as,

$$(Z_1 - Z_0) = (1 - \sqrt{(1+4R_{1_1}^2)})/n_0 R_1 k = l * (1 - \sqrt{(1+4R_{1_1}^2)})/2 * R_1 * p * n_0 \qquad (4.41)$$

$$(Z_1 - Z_0) = -1/n_0 k * \sqrt{8R_3 / (1+2R_3)} = -\frac{l}{2*p*n_0}\sqrt{8R_3 / (1+2R_3)} \qquad (4.42)$$

Thus from equation 4.41 and 4.42, if R_1, R_2, R_3, R_4 are known, the change in thickness due to E-field (i.e. electrorestrictive effect) can be measured.

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[1] E. Hecht, Optics, 2nd edition, Addision-Wesley publishing company, (1987).