Chapter 3

Freestanding Tube

3.1 Introduction

In this chapter, a model to predict the shape of a freestanding tube as a function of the internal pressure in the tube is formulated. All derivations in this chapter were performed by Dr. Plaut. Mathematica was used to solve for any unknowns and produce data in order to plot the final shape of the tube. The x and y coordinates produced by Mathematica were then transferred to Excel and graphs were generated.

3.2 Assumptions

Some basic assumptions were made in order to derive the equations that would model the behavior of the freestanding tube. The foundation was considered to be undeformable and the friction between it and the tube was neglected. The tube was assumed to be inextensible with no bending stiffness present. The weight of the tube material was neglected and the tube was assumed to be infinitely long. Any fluid within the tube was assumed to be incompressible.

3.3 Derivation

Consider the diagram shown in Figure 3-1, where a single tube rests on a rigid foundation. The diagram depicts the cross section of a long slender tube. The horizontal position is X and the vertical position is Y, with the origin placed as shown. The variable S represents the arc length of the tube and the circumference of the tube is L. The angle θ represents the angle of the tangent to the tube measured from the horizontal. The length of contact between the tube and the foundation is denoted B, and the internal pressure head of the tube is H_{int}. The tube is filled with water with specific weight γ_{int}. Not labeled in the figure, T represents the tension within the tube material.

Other variables defined:

$P = P_{bot} - \gamma_{int} Y$,

$P_{bot} = \gamma_{int} H_{int}$, where

P = pressure head anywhere in the tube

P_{bot} = pressure at the bottom of the tube

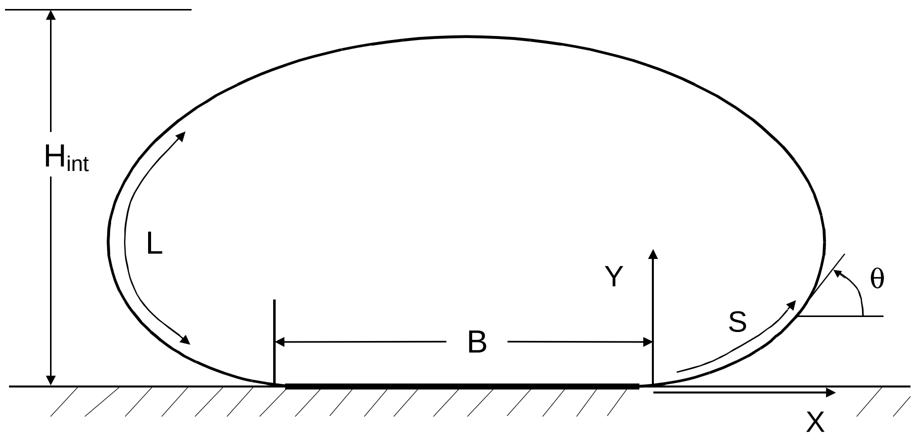

Figure 3-1: Freestanding Tube

From an element of the tube, the following can be derived:

$$\frac{dX}{dS} = \cos\theta, \qquad \frac{dY}{dS} = \sin\theta, \qquad \frac{d\theta}{dS} = \frac{\gamma_{int}(H_{int} - Y)}{T}. \qquad (3.1, 3.2, 3.3)$$

For simplicity, the following nondimensional variables are used:

$$x = \frac{X}{L}, \qquad y = \frac{Y}{L}, \qquad s = \frac{S}{L},$$

$$h_{int} = \frac{H_{int}}{L}, \qquad b = \frac{B}{L}, \qquad t = \frac{T}{\gamma_{int} L^2},$$

$$p = \frac{P}{\gamma_{int} L}, \qquad p_{bot} = h_{int} = \frac{P_{bot}}{\gamma_{int} L} = \frac{H_{int}}{L}. \qquad (3.4)$$

Then equations 3.1-3.3 become

$$\frac{dx}{ds} = \cos\theta, \qquad \frac{dy}{ds} = \sin\theta, \qquad \frac{d\theta}{ds} = \frac{h_{int} - y}{t}. \qquad (3.5, 3.6, 3.7)$$

Assuming θ increases from $\theta = \theta_o$ to $\theta(s)$, it can be shown (Plaut and Klusman 1999) that

$$u = \sqrt{h_{int}^2 - 2t(\cos\theta_o - \cos\theta)} \tag{3.8}$$

where

$$u = t\frac{d\theta}{ds} > 0\,, \tag{3.9}$$

and then, with $\theta_o = 0$,

$$y = h_{int} - u\,, \qquad x = t\int_0^{\theta}\frac{\cos\theta}{u}d\theta\,, \qquad s = t\int_0^{\theta}\frac{1}{u}d\theta\,. \tag{3.10, 3.11, 3.12}$$

At $\theta = 2\pi$: $y = 0$, $x = -b$, and $s = 1 - b$. Therefore,

$$-b = t\int_0^{2\pi}\frac{\cos\theta}{u}d\theta\,, \qquad 1 - b = t\int_0^{2\pi}\frac{1}{u}d\theta\,. \tag{3.13, 3.14}$$

Eliminating b gives:

$$t\int_0^{2\pi}\frac{(1-\cos\theta)}{\sqrt{h_{int}^2 - 2t(1-\cos\theta)}}d\theta = 1\,. \tag{3.15}$$

3.4 Analysis

Because of the complexity of the given equations, a powerful computing software package, Mathematica (Wolfram 1996), was utilized to solve for any unknowns. Once data were produced, Excel was used to generate plots.

In order to generate graphs depicting the shape of the freestanding tube, x and y coordinates were needed. Both x and y coordinate equations contain the unknown tension t. To solve for the unknown, the "FindRoot" command in Mathematica was carried out. Placing equation 3.15 as the conditional equation and specifying an internal hydrostatic pressure h_{int}, the tension in the tube material was calculated.

There may be multiple solutions for the freestanding tube case, depending on the internal hydrostatic pressure. Therefore, initial guesses for the variable t were required in the "FindRoot" command. If the initial guess was not near the actual value, the program experienced difficulty converging to a solution. All solutions were verified by checking equation 3.15.

Once equation 3.15 was satisfied, corresponding x and y values were calculated using Mathematica and then transferred to Excel. Graphs were then generated to plot and observe the behavior of the tube at varying h_{int} values. Table 3-1 presents a list of several different solutions for a freestanding tube case. It is apparent that as the internal pressure of the tube (h_{int}) increases, the total height of the tube, as well as the tension, increases and the contact length (b) between the tube and the foundation decreases. Three of the solutions are plotted in Figure 3-2 to compare the change in shape of the tube as h_{int} increases.

Freestanding tube			
h_{int}	**height**	**b**	**t**
0.2	0.1761	0.3057	0.0099
0.3	0.2205	0.2340	0.0209
0.4	0.2457	0.1854	0.0340
0.5	0.2611	0.1523	0.0482

Table 3-1: Solutions for Freestanding Tube

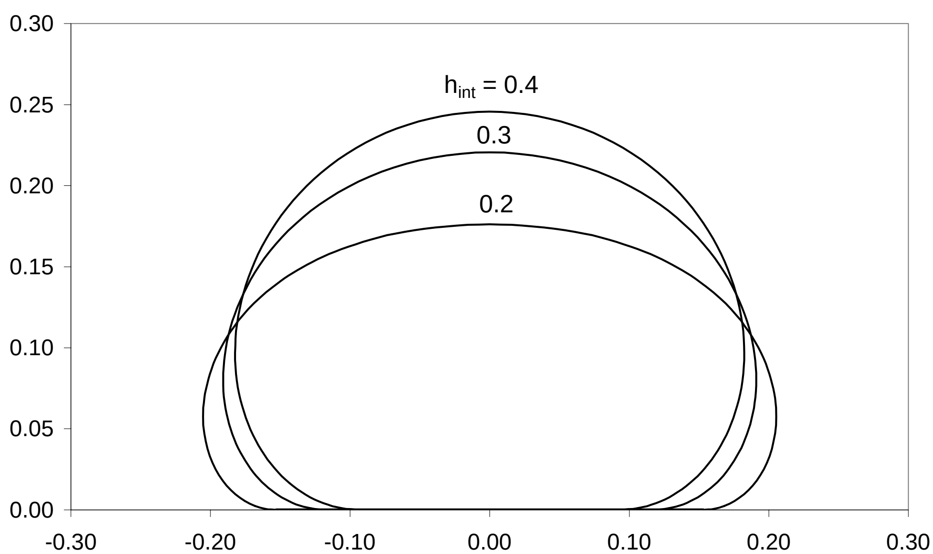

Figure 3-2: Shapes of Freestanding Tube with Varying h_{int} Values