

“Supervaluationism, Penumbral Connections, and the Nature of Higher-Order Vagueness”

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ABSTRACT

In this paper, I analyze Kit Fine’s account of the logic and semantics of vagueness. The overall aim of Fine’s project is to develop an account of the logic and semantics of vague language which accommodates distinctive characteristics of vagueness including penumbral connections and higher-order vagueness. I begin Chapter 1 with a discussion of what vagueness is and is not. Next, I trace the development of supervaluationism, and summarize Kit Fine’s supervaluationism and specification space approach to vagueness. I also discuss the more salient features of vagueness and I discuss them in relation to specification space models. I close with a look at the logic of vagueness and the logic of higher-order vagueness.

Chapter 2 deals with penumbrae and penumbral connections. I analyze Fine’s account of penumbral connections before arguing that his characterization of penumbral connections is too broad. Fine mistakenly identifies logically valid formulae and their instances as though they exhibited penumbral connections. After arguing that Fine’s misidentification of penumbral connections results in an analysis of penumbral connections which is built for too wide a notion of penumbral connections, I suggest a more refined characterization of penumbral connections.

I take up higher-order vagueness in Chapter 3. I begin with an overview of some characterizations of higher-order vagueness. Next, I revisit Fine’s accounts of the D operator and higher-order vagueness. Lastly, I argue that higher-order vagueness is not a distinct feature of the vagueness of natural language, but, rather, it is an artifact resulting from the analysis of the vagueness of natural language.

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## Introduction

In this paper, I examine some topics from Kit Fine's seminal paper "Vagueness, Truth and Logic". The overall aim of Fine's project is to develop an account of the logic and semantics of vague language which accommodates distinctive characteristics of vagueness including penumbral connections and higher-order vagueness. Vagueness, as I explain in the first chapter, is about borderline cases. For example, think of a man who is a borderline case of tall. As regards this man, you would not say that the sentence 'The man is tall' is true but you would not say that the sentence 'The man is tall' is false, either. Vagueness pervades ordinary English. We express vagueness through ordinary English sentences all the time. We cannot express vagueness in the language of classical logic.

I begin the first chapter with some background and with explanations of what vagueness is and is not. After that, I trace the development of supervaluationism, and summarize Kit Fine's supervaluationism and specification space approach to vagueness. Then I relate some of the most salient features of vagueness to specification space models. I close the chapter with a look at the logic of vagueness and the logic of higher-order vagueness.

Chapter 2 deals with penumbræ and penumbral connections. I commence Chapter 2 by discussing the notion 'penumbra'. Next, I analyze Fine's account of penumbral connections before arguing that his characterization of penumbral connections is too broad. I explain that Fine mistakenly identifies logically valid formulae and their instances as though they exhibited penumbral connections. After arguing that Fine's misidentification of penumbral connections results in an analysis of penumbral connections which is built for too wide a notion of penumbral connections, I suggest a more refined characterization of penumbral connections. Basically, I suggest that we should restrict the types of examples to which the characterization of penumbral connections applies exclusively to cases where the vagueness of statements matters.

In the third chapter, I take up another phenomenon Fine considers to be a distinct feature of vague language, namely, higher-order vagueness. I begin with an overview of some characterizations of higher-order vagueness. Next, I revisit Fine's accounts of the D operator

and higher-order vagueness. Lastly, I consider the source of vagueness in ordinary English in order to argue that higher orders of vagueness do not occur in ordinary English. After discussing the relation between the predicates 'is vague' and 'is a borderline case', I argue, in relation to Fine's account, that higher-order vagueness is not a distinct feature of the vagueness of natural language. Rather, it is an artifact resulting from the analysis of the vagueness of natural language. I close with a recommendation to theorists of vagueness and higher-order vagueness.

## Chapter 1

### Vagueness and the Development of Supervaluationism

#### §1 What is Vagueness?

##### 1.1 What Vagueness Is Not

Vagueness is not a matter of ambiguity or lack of information; rather, it has to do with borderline cases. An eye-witness to a robbery is uninformative, but not vague, when he reports that ‘the thief was no younger than nineteen years old but no older than forty-seven years old’. A second eye-witness reports that ‘the thief ran off toward the bank’ but failed to specify whether she intended ‘bank’ to refer to the financial institution due west from the crime scene or the river bank due east. This second piece of testimony is ambiguous.

A third eye-witness reports that ‘the thief was about six feet tall and was middle aged’, and this third piece of testimony is vague on two accounts. First, ‘about six feet tall’ is vague because it is unclear whether the man was less than, greater than, or exactly six feet tall. Second, ‘middle-aged’ is vague because it is unclear exactly which ages are included in the scope of ‘middle-aged’, so this description may equally refer to a man of forty-six years or a man of sixty-six.

Some philosophers claim that all words, hence all language, is more or less vague.<sup>1</sup> This remains an open question; however, it is safe to say that vagueness is not limited to predicates. For example, singular terms such as ‘Toronto’, ‘Mt. Everest’, and ‘the outback’ are vague. The geographic limits of Toronto are not precise, so the truth value of a statement like ‘Toronto has an odd number of trees’ depends on the way in which the truth-assigner interprets Toronto’s bounds.<sup>2</sup> Notice that the vagueness of ‘Toronto’ is inconsequential for the semantic evaluation of other statements, e.g., ‘Toronto is in Canada’. Hereafter, I shall be concerned mostly with vague predicates.

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<sup>1</sup> Bertrand Russell is an example. See Russell (1983: 149).

<sup>2</sup> The ‘Toronto’ example is from Mehlberg (1958: 257-258).

## 1.2 The Most Salient Features of Vagueness

### Borderline cases

Consideration of borderline cases is essential to any discussion of vagueness. In general, borderline cases are cases presumed to be *on the border* between the extensions and anti-extensions of vague predicates. ‘Presumed’ is important here. If the border is thick, so to speak, then there are cases that are *in between* the extension and the anti-extension rather than *within* one of them, and the thickness of the borderline containing the indeterminate borderline cases is itself indeterminate. Since thick-border theories admit this ‘in between’ area, they cannot define the range of borderline cases and so admit borderline borderline cases, borderline borderline borderline cases, etc. This phenomenon is known as ‘higher-order vagueness’.

Not all theorists think that borderlines are thick. Some theorists, such as Timothy Williamson, say that there is a sharp cut-off between the extension and anti-extension of every vague predicate.<sup>3</sup> This line of argument is much less ambitious than it sounds. Williamson does not tell us where or what the borderlines are. Instead, his account just tells us that we are ignorant of the sharp-cut-offs and cannot know them.

Specifically, Williamson writes, “Just one interpretation is correct, but speakers of the language could not know all the others to be incorrect.”<sup>4</sup> Here, vagueness is a phenomenon of epistemic ignorance. Vagueness occurs to us because we were ignorant of sharp cut-offs. Here, there are no borderline cases of borderline cases, i.e., there are no higher orders of vagueness, because there are no borderline cases.

### Penumbral Connections

Generally speaking, penumbral connections are logical relations which hold between vague sentences. They can also be thought of as logical relations between borderline case instances of

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<sup>3</sup> See Williamson (1994). I discuss Williamson’s view a bit more in this chapter and I analyze some of his objections to Fine (1975) in the second chapter.

<sup>4</sup> Williamson (1994: 164).



vague predicates.<sup>5</sup> Here's an example of just one type of penumbral connection. Consider that the predicate 'is expensive' admits borderline cases. Suppose you are considering buying a gift and the gift item is expensive. Now, suppose the gift item in question costs \$500. Surely, any alternative gift item which costs more than \$500 is expensive, too. There is a lot to say about penumbral connections. In the second chapter, I consider more closely just what penumbral connections are, and I discuss some other types of penumbral connections.

### Sorites Susceptibility

Another characteristic feature of vagueness is sorites susceptibility. Sorites sequences seem paradoxical because they raise the question of whether or not there are sharp cut-offs. Sorites sequences proceed from a clear case and move to another almost indistinguishable case; then, to another almost indistinguishable case, and to another, and another until a case is reached that is obviously a clear non-case. We are not comfortable calling the clear non case a clear case, but we cannot locate exactly where our reasoning lead us astray.

In terms of vagueness, we start with a single precisification which is taken to be true. Then we move to the truth of another precisification which is ever so slightly more controversial. As the reasoning goes, if the second sharpening is close to the first, then the second will be true if the first is. But repeating this reasoning from sharpening to sharpening to sharpening, etc. leaves us with a sharpening that is far from the initial sharpening and so cannot be true on any reasonable interpretation of the predicate.

Consider the sorites paradox:

P1: 5,000 grains makes a heap.

P2:  $\forall n$  (If  $n$  grains makes a heap, then  $n-1$  grains makes a heap)

Therefore, 0 grains makes a heap.

Suppose the first premise is true. Repeating the procedure in the second premise over and over eventually leaves us with the absurd conclusion that 0 grains make a heap. Where did we go

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<sup>5</sup> See Fine (1975: 270).

wrong? The truth of the first premise may not be controversial and the falsity of the conclusion is uncontroversial. The second premise is the matter of controversy.

If the P2 is true, then the argument is invalid, and the argument is valid but unsound if P2 is false. This is why sorites sequences raise the question of whether or not there are sharp cut-offs. All that is needed to make the second premise true and the argument invalid is the identification of a sharp cut-off between the extension and anti-extension of the predicate 'makes a heap'. But, as long as the sharp cut-off remains elusive, the validity of the paradoxical argument remains intact.

## §2 Philosophical Approaches to Vagueness

Since vagueness is about borderline cases, the nature of vagueness may be regarded as being about the nature of borderline cases. Epistemic theorists like Williamson think that the source of vagueness is ignorance: there are sharp boundaries between the positive and negative extensions of all predicates even if some such boundaries are unknown or unknowable.<sup>6</sup> Metaphysical theorists tend to be concerned with the vagueness of objects, i.e., ontic vagueness.<sup>7</sup> Linguistic theorists are concerned with the vagueness of meaning, including the vagueness of reference relations and truth.

Vagueness permeates philosophical discussions in epistemology, metaphysics, language and logic, and other philosophical areas, too. I do not seek to argue for the primacy of one or another of these dimensions. However, there is an important common thread among them. Each concerns borderline cases and seeks an account of borderlines, whether the account is positive or negative. A clear view of borderline cases in any one philosophical dimension may prove useful in the analysis of another.

I will work from the standpoint that vagueness is linguistic, mostly because that is the standpoint of the received view which I will be evaluating. Analyses which view vagueness as a

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<sup>6</sup> See Williamson (1994). Sorensen (2001) argues that we can know what (where) the sharp cut-offs are.

<sup>7</sup> Supporters of this approach include Tye (1990) and (2000) and van Inwagen (1990).

linguistic phenomenon differ in important respects. I will mention just a few competing views relating to linguistic vagueness. Of course, each view I mention admits variation.

Some theorists hold that vagueness is a matter of semantic indeterminacy.<sup>8</sup> This claim comes in two varieties. One is that a sentence like ‘Bobo is tall’ is neither true nor false where Bobo is borderline tall.<sup>9</sup> The other is that the sentence ‘Bobo is tall is either true or false, but it is neither determinately true nor determinately false.’<sup>10</sup> The question about determinate truth is an open one.<sup>11</sup>

Others theorists claim that vagueness is a kind of semantic indecision. On this view, vague sentences do not have determinate actual meanings; they have a range of determinate potential meanings. Each potential meaning is a semantic choice and our inability to choose exemplifies semantic indecision.<sup>12</sup> Still, others argue that vagueness is a matter of semantic under-determination. Here, the meanings of vague terms can be made more precise, and the operation by which they are made more precise should preserve truth.<sup>13</sup>

### §3 Supervaluationism

#### 3.1 The Inception and Development of Supervaluationism

Supervaluationism is a theory of linguistic vagueness whereby vagueness is viewed as semantic under-determination. At the center of the theory is the notion that the meaning of every statement containing a vague term admits a range of admissible interpretations. Since vague statements admit a range of admissible interpretations, a given vague statement will be true on some interpretations, false on others, and neither true nor false on others.

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<sup>8</sup> Williamson (1994) argues that semantic indeterminacy is a myth because there is only epistemic indeterminacy.

<sup>9</sup> McGee and McLaughlin (2004: 125-136) reply to Williamson’s discussion of this first variety of theorizing about semantic indeterminacy.

<sup>10</sup> McGee and McLaughlin (2004: 125) argue for this second version. See also McGee and McLaughlin (1995).

<sup>11</sup> Etchemendy (1999) suggests that in order to answer questions about determinate truth, we focus on the actual world, and consider different acceptable ways of interpreting the language.

<sup>12</sup> See Lewis (1986: 212).

<sup>13</sup> This is the view held by Fine. See Fine (1975: 267).

Statements containing vague terms are said to be true [false] *simpliciter* if they remain true [false] under every admissible interpretation of the vague term(s) they contain. The core of this idea goes back to Mehlberg (1956) and the idea was redeveloped in van Fraassen (1966, 1968, and 1969, Thomason (1970), Lewis (1970), and Kamp (1975). The term ‘supervaluationism’ was introduced by van Fraassen (1966), who defined truth *simpliciter* as truth on all assessments (interpretations) and falsity *simpliciter* as falsity on all assessments. Sentences assigned truth-value gaps are neither true nor false in the sense that they are neither true *simpliciter* nor false *simpliciter*.

Thomason (1970) defends a supervaluationistic future-tense semantics on which the future is open to various possible histories from the standpoint of the present, and we may consider there to be truths about the future insofar as these truths are common to each of the various possible histories. Lewis (1970) uses ‘delineation coordinates’, which are analogous to van Fraassen’s ‘assessments’, Thomason’s ‘histories’, and Mehlberg’s ‘interpretations’. Lewis (1970) and Kamp (1975) pioneered the application of supervaluationism to vagueness.

### 3.2 Fine’s Supervaluationism and Specification Space Approach

Kit Fine’s 1975 paper “Vagueness, Truth and Logic” is widely regarded as the *locus classicus* of supervaluationism as a theory of vagueness. Fine develops a thoroughly detailed account of the vagueness of sentences and presents a very interesting framework for the meanings of vague sentences. He calls this approach ‘the specification space approach’. Fine uses the specification space approach to motivate special notions of truth and validity, which he uses to analyze the logic and semantics of vague language.

On Fine’s view, vagueness is a matter of semantic under-determination. The meanings of vague sentences can be determined further. He writes:

A vague sentence can be made more precise; and this operation should preserve truth-value. But a vague sentence can be made to be either true or false, and therefore the original sentence can be neither.<sup>14</sup>

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<sup>14</sup> Fine (1975: 267).

So, Fine's supervaluation is based on the idea that every statement containing a vague term admits a range of admissible ways it can be made completely precise. These ways are called 'precisifications', 'specifications', 'sharpenings', etc.

Fine adopts van Fraassen's definition of truth *simpliciter* as truth on all assessments and introduces 'supertruth' as being coextensive with 'truth on all ways of being made more precise', i.e., 'truth *simpliciter*'. Sentences are assigned truth-value gaps if they are neither supertrue nor superfalse but would be either true or false (on any given complete and admissible interpretation) if made more precise. Hence, Fine proposes a three-valued semantics. Sentences are assigned truth-value gaps if they are neither true *simpliciter*, i.e., not supertrue, nor false *simpliciter*, i.e., not superfalse.<sup>15</sup> Bivalence fails because it is not the case that every statement is either supertrue or superfalse.

Fine also develops a special notion of validity out of the theory's notion of supertruth. Whereas classical validity is necessary preservation of truth, Fine's proposed notion takes validity to be the necessary preservation of supertruth. The resulting logical system preserves the tautologies of classical logic despite being a non-truth functional, three-valued system for which bivalence fails.

Sentences containing vague predicates admit a range of precise potential meanings, i.e., a range of complete and admissible specifications. What is meant by 'complete' is that the specification is completely precise. What is meant by 'admissible' is that the precisification is consistent with ordinary use. I will define 'complete specification' more formally later in this chapter.

No matter how you interpret the predicate 'is tall', you'll assign 'true' to the sentence 'Yao Ming is tall' and 'false' to the sentence 'Danny DeVito is tall'. These are clear cases and are precise already because neither can be interpreted otherwise. However, what about a sentence like 'Bobo is tall', where Bobo is a borderline case of tall? Depending on the

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<sup>15</sup> The semantic valuations 'indefinite' and 'neither-true-nor-false' are used synonymously and are assigned to gappy sentences.

interpretation, this sentence may be either true or false, or, as Fine emphasizes, it may be neither-true-nor-false.

We can make explicit the potential meanings of the vague sentence ‘Bobo is tall’ by means of a specification space. A specification space is a set of partial models. Each model consists of a predicate and a domain of individuals. What is modeled are various ways to precisify the extension and anti-extension of the predicate in the given domain.

Let’s build a model for the predicate ‘is tall’ which consists of a domain of six individuals  $\{a, b, c, d, e, f\}$ . Two of the individuals,  $a$  and  $b$ , are clear cases of ‘is tall’, one of them,  $f$ , is a clear case of not-‘is tall’. Regardless of how we sharpen, interpret, precisify, specify [all synonymous] the meaning of the predicate ‘is tall’, objects  $a$  and  $b$  will be included in the predicate’s extension and object  $f$  will be included in its anti-extension.

We begin building our model by introducing the base point (root point). The clear cases of the extension and anti-extension of the predicate are indicated at the base point. In the model we are building, the extension of ‘is tall’ at the base-point contains individuals  $a$  and  $b$ , and the anti-extension of ‘is tall’ at the base-point includes only individual  $f$ . The clear cases of ‘is tall’ are fixed at the base-point and are preserved throughout the model. We consider the truth-values of the sentences ‘ $a$  is tall’, ‘ $b$  is tall’, and ‘ $f$  is not tall’, by interpreting the model.

So far, our specification space model for the predicate ‘is tall’ has the domain of individuals  $\{a, b, c, d, e, f\}$ , whereby the clear cases of ‘is tall’ have been identified as  $a$  and  $b$  and the clear non case has been identified as  $f$ . At the base-point, the extension of ‘is tall’ is represented as follows:  $\llbracket \_ \rrbracket_+ = \{a, b\}$ . Further, the anti-extension is represented as  $\llbracket \_ \rrbracket_- = \{f\}$ . The other three objects in the domain, i.e.,  $c$ ,  $d$ , and  $e$ , are the borderline cases, and we will account for them by gradually making the extension/anti-extension at the base point more precise.

We precisify by extending the base point to new specification points. A specification point is just a point within the specification space model. Each time we extend, we create a point

that accounts for more of the domain's individuals than the previous point accounted for. This is all very fast, so let me walk through it slowly.

Starting with the base point, specification points in the model *extend* to other specification points. To illustrate this, let's call the base-point 'MR' because it is the model's root. We represent MR within our model like this:

**MR (Model Root)**

$\mathbb{I}_+ = \{a, b\}$

$\mathbb{I}_- = \{f\}$

We can extend MR and create a new specification point in our model by (i) selecting an individual from the domain that is excluded from both the extension and anti-extension at the base-point and (ii) adding the selected individual to either the extension or anti-extension at the extended specification point.

Let's select  $e$  from the domain and add it to the anti-extension of MR. Call the new, more precise point 'MR1'. MR1 extends from the base-point MR and it appears in our model as follows:

**MR1**

$\mathbb{I}_+ = \{a, b\}$

$\mathbb{I}_- = \{e, f\}$

MR1 extends from MR because (i) its extension and anti-extension retain the same membership they had in MR and (ii) the membership of either MR's extension or MR's anti-extension is extended by one individual from the domain not accounted for in MR.

So, one specification point is said to *extend* to another when the extension and anti-extension at the *extending* point, i.e., the first point, are contained by the extension/anti-extension at the *extended* point, i.e., the second point. Here, an individual not accounted for by the extending point [MR] is introduced in the extended point [MR1]. So, MR extends to MR1, at which  $a$  and  $b$  are within the extension and  $e$  and  $f$  are in the anti-extension. The individual introduced in MR1 which was not accounted for in MR is  $e$ .

Consider again the domain  $\{a, b, c, d, e, f\}$ . The individuals in the subset  $\{c, d, \text{ and } e\}$  are the borderline cases at MR, and MR models this because it does not specify a place for  $c, d,$  or  $e$  in either the extension or anti-extension of ‘is tall’. When we extend MR to MR1, we preserve the extension and anti-extension from MR and introducing  $e$  to the anti-extension. We could create second specification point extending from MR, call it ‘MR2’, by preserving MR’s extension and anti-extension and adding  $d$  to the anti-extension at MR2. MR2 is represented in our model as follows:

**MR2**  
 $\llbracket \mathbb{I} \rrbracket_+ = \{a, b\}$   
 $\llbracket \mathbb{I} \rrbracket_- = \{d, f\}$

Since there are three borderline cases, i.e.,  $c, d,$  and  $e,$  and since each can be assigned either to the extension or to the anti-extension, our base-point MR immediately extends to six partial-points.

We can add to MR1 and MR2 the points MR3, MR4, MR5, and MR6, and the first level of specification points MR extends to will be finished. So far, the model will look like this:

<b>MR1</b> $\llbracket \mathbb{I} \rrbracket_+ = \{a, b\}$ $\llbracket \mathbb{I} \rrbracket_- = \{e, f\}$	<b>MR2</b> $\llbracket \mathbb{I} \rrbracket_+ = \{a, b\}$ $\llbracket \mathbb{I} \rrbracket_- = \{d, f\}$	<b>MR3</b> $\llbracket \mathbb{I} \rrbracket_+ = \{a, b\}$ $\llbracket \mathbb{I} \rrbracket_- = \{c, f\}$	<b>MR4</b> $\llbracket \mathbb{I} \rrbracket_+ = \{a, b, c\}$ $\llbracket \mathbb{I} \rrbracket_- = \{f\}$	<b>MR5</b> $\llbracket \mathbb{I} \rrbracket_+ = \{a, b, d\}$ $\llbracket \mathbb{I} \rrbracket_- = \{f\}$	<b>MR6</b> $\llbracket \mathbb{I} \rrbracket_+ = \{a, b, e\}$ $\llbracket \mathbb{I} \rrbracket_- = \{f\}$
<b>LEVEL 2</b>					
<b>MR</b> $\llbracket \mathbb{I} \rrbracket_+ = \{a, b\}$ $\llbracket \mathbb{I} \rrbracket_- = \{f\}$					
<b>LEVEL 1</b>					

Remember, one specification point is said to *extend* to another when the extension and anti-extension of the *extending* point, i.e., the first point, are respectively contained by the extension and anti-extension of the *extended* point, i.e., the second point, and when an individual not accounted for by the extending point is introduced in the extended point. With these conditions in mind, we can continue to build our specification space model.



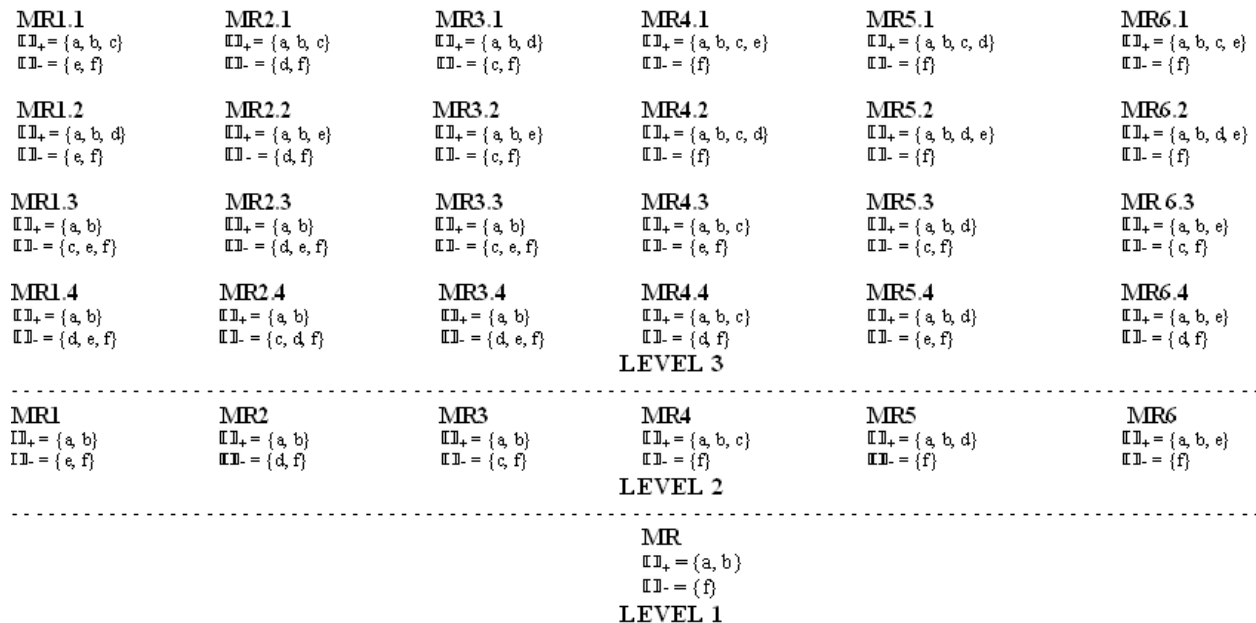
Each of the specification points on Level 2 only accounts for four individuals from the domain. This means that each Level 2 specification point immediately extends to four other specification points. For example, let's see how MR1 extends:

**MR1**  
 $\mathbb{I}_+ = \{a, b\}$   
 $\mathbb{I}_- = \{e, f\}$

MR1 does not account for  $c$  or  $d$ . Hence, we could extend MR1 to another specification point, call it 'MR1.1' by introducing  $c$  to the extension. Alternatively, we could extend MR1 to, say, MR1.2 by adding  $d$  to the extension. Also, we could extend MR1 a third way by introducing  $c$  to the anti-extension. Call this third one 'MR1.3'. Lastly, we could introduce  $d$  to the anti-extension. Call this last point 'MR1.4'. The four specification points extending from MR1 are represented as follows:

<b>MR1.1</b> $\mathbb{I}_+ = \{a, b, c\}$ $\mathbb{I}_- = \{e, f\}$	<b>MR1.2</b> $\mathbb{I}_+ = \{a, b, d\}$ $\mathbb{I}_- = \{e, f\}$	<b>MR1.3</b> $\mathbb{I}_+ = \{a, b\}$ $\mathbb{I}_- = \{c, e, f\}$	<b>MR1.4</b> $\mathbb{I}_+ = \{a, b\}$ $\mathbb{I}_- = \{d, e, f\}$
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In the same way as we extended MR1 to create four new specification points, we can extend MR2, MR3, MR4, MR5, and MR6 and create four new points extending from each.



Notice that points that extend from MR1 are directly above it, the points that extend from MR2 are directly above it, and so on, and so fourth. Each of the Level 3 specification points accounts for five individuals in the six individual domain  $\{a, b, c, d, e, f\}$ . This means that each extends to two further points and that each of the Level 4 specification points will account for the complete domain of individuals.

Since all of the specification points in Level 4 will be complete-points, our model will be complete once we extend the Level 3 points and explicate the Level 4 points. Following our procedure for extending specification points, we can complete our model and the result will be as follows:

MR1.1.1 $\mathbb{I}_+ = \{a, b, c, d\}$ $\mathbb{I}_- = \{e, f\}$	MR1.1.2 $\mathbb{I}_+ = \{a, b, c\}$ $\mathbb{I}_- = \{d, e, f\}$	MR2.1.1 $\mathbb{I}_+ = \{a, b, c, e\}$ $\mathbb{I}_- = \{d, f\}$	MR2.1.2 $\mathbb{I}_+ = \{a, b, c\}$ $\mathbb{I}_- = \{d, e, f\}$	MR3.1.1 $\mathbb{I}_+ = \{a, b, d, e\}$ $\mathbb{I}_- = \{c, f\}$	MR3.1.2 $\mathbb{I}_+ = \{a, b, d\}$ $\mathbb{I}_- = \{c, e, f\}$	MR4.1.1 $\mathbb{I}_+ = \{a, b, c, d, e\}$ $\mathbb{I}_- = \{f\}$	MR4.1.2 $\mathbb{I}_+ = \{a, b, c, e\}$ $\mathbb{I}_- = \{d, f\}$	MR5.1.1 $\mathbb{I}_+ = \{a, b, c, d, e\}$ $\mathbb{I}_- = \{f\}$	MR5.1.2 $\mathbb{I}_+ = \{a, b, c, d\}$ $\mathbb{I}_- = \{e, f\}$	MR6.1.1 $\mathbb{I}_+ = \{a, b, c, d, e\}$ $\mathbb{I}_- = \{f\}$	MR6.1.2 $\mathbb{I}_+ = \{a, b, c, e, d\}$ $\mathbb{I}_- = \{d, f\}$
MR1.2.1 $\mathbb{I}_+ = \{a, b, c, d\}$ $\mathbb{I}_- = \{e, f\}$	MR1.2.2 $\mathbb{I}_+ = \{a, b, d\}$ $\mathbb{I}_- = \{c, e, f\}$	MR2.2.1 $\mathbb{I}_+ = \{a, b, c, e\}$ $\mathbb{I}_- = \{d, f\}$	MR2.2.2 $\mathbb{I}_+ = \{a, b, e\}$ $\mathbb{I}_- = \{c, d, f\}$	MR3.2.1 $\mathbb{I}_+ = \{a, b, d, e\}$ $\mathbb{I}_- = \{c, f\}$	MR3.2.2 $\mathbb{I}_+ = \{a, b, e\}$ $\mathbb{I}_- = \{c, d, f\}$	MR4.2.1 $\mathbb{I}_+ = \{a, b, c, d, e\}$ $\mathbb{I}_- = \{f\}$	MR4.2.2 $\mathbb{I}_+ = \{a, b, c, d\}$ $\mathbb{I}_- = \{e, f\}$	MR5.2.1 $\mathbb{I}_+ = \{a, b, c, d, e\}$ $\mathbb{I}_- = \{f\}$	MR5.2.2 $\mathbb{I}_+ = \{a, b, d, e\}$ $\mathbb{I}_- = \{c, f\}$	MR6.2.1 $\mathbb{I}_+ = \{a, b, c, d, e\}$ $\mathbb{I}_- = \{f\}$	MR6.2.2 $\mathbb{I}_+ = \{a, b, e, d\}$ $\mathbb{I}_- = \{c, f\}$
MR1.3.1 $\mathbb{I}_+ = \{a, b, d\}$ $\mathbb{I}_- = \{c, e, f\}$	MR1.3.2 $\mathbb{I}_+ = \{a, b\}$ $\mathbb{I}_- = \{c, d, e, f\}$	MR2.3.1 $\mathbb{I}_+ = \{a, b, c\}$ $\mathbb{I}_- = \{d, e, f\}$	MR2.3.2 $\mathbb{I}_+ = \{a, b\}$ $\mathbb{I}_- = \{c, d, e, f\}$	MR3.3.1 $\mathbb{I}_+ = \{a, b, d\}$ $\mathbb{I}_- = \{c, d, e, f\}$	MR3.3.2 $\mathbb{I}_+ = \{a, b, e\}$ $\mathbb{I}_- = \{c, d, e, f\}$	MR4.3.1 $\mathbb{I}_+ = \{a, b, c, d\}$ $\mathbb{I}_- = \{e, f\}$	MR4.3.2 $\mathbb{I}_+ = \{a, b, c, e\}$ $\mathbb{I}_- = \{d, f\}$	MR5.3.1 $\mathbb{I}_+ = \{a, b, d, e\}$ $\mathbb{I}_- = \{c, f\}$	MR5.3.2 $\mathbb{I}_+ = \{a, b, d\}$ $\mathbb{I}_- = \{c, e, f\}$	MR6.3.1 $\mathbb{I}_+ = \{a, b, d, e\}$ $\mathbb{I}_- = \{c, f\}$	MR6.3.2 $\mathbb{I}_+ = \{a, b, e\}$ $\mathbb{I}_- = \{c, d, f\}$
MR1.4.1 $\mathbb{I}_+ = \{a, b, c\}$ $\mathbb{I}_- = \{d, e, f\}$	MR1.4.2 $\mathbb{I}_+ = \{a, b\}$ $\mathbb{I}_- = \{c, d, e, f\}$	MR2.4.1 $\mathbb{I}_+ = \{a, b, e\}$ $\mathbb{I}_- = \{c, d, f\}$	MR2.4.2 $\mathbb{I}_+ = \{a, b\}$ $\mathbb{I}_- = \{c, d, e, f\}$	MR3.4.1 $\mathbb{I}_+ = \{a, b, c\}$ $\mathbb{I}_- = \{d, e, f\}$	MR3.4.2 $\mathbb{I}_+ = \{a, b, e\}$ $\mathbb{I}_- = \{c, d, e, f\}$	MR4.4.1 $\mathbb{I}_+ = \{a, b, c, e\}$ $\mathbb{I}_- = \{d, f\}$	MR4.4.2 $\mathbb{I}_+ = \{a, b, c, d\}$ $\mathbb{I}_- = \{e, f\}$	MR5.4.1 $\mathbb{I}_+ = \{a, b, c, d\}$ $\mathbb{I}_- = \{e, f\}$	MR5.4.2 $\mathbb{I}_+ = \{a, b, d\}$ $\mathbb{I}_- = \{c, e, f\}$	MR6.4.1 $\mathbb{I}_+ = \{a, b, c, e\}$ $\mathbb{I}_- = \{d, f\}$	MR6.4.2 $\mathbb{I}_+ = \{a, b, e, d\}$ $\mathbb{I}_- = \{c, d, f\}$
<b>Level 4</b>											
-----											
MR1.1 $\mathbb{I}_+ = \{a, b, c\}$ $\mathbb{I}_- = \{e, f\}$	MR2.1 $\mathbb{I}_+ = \{a, b, c\}$ $\mathbb{I}_- = \{d, f\}$	MR3.1 $\mathbb{I}_+ = \{a, b, d\}$ $\mathbb{I}_- = \{c, f\}$	MR4.1 $\mathbb{I}_+ = \{a, b, c, e\}$ $\mathbb{I}_- = \{f\}$	MR5.1 $\mathbb{I}_+ = \{a, b, c, d\}$ $\mathbb{I}_- = \{f\}$	MR6.1 $\mathbb{I}_+ = \{a, b, c, e\}$ $\mathbb{I}_- = \{f\}$						
MR1.2 $\mathbb{I}_+ = \{a, b, d\}$ $\mathbb{I}_- = \{e, f\}$	MR2.2 $\mathbb{I}_+ = \{a, b, e\}$ $\mathbb{I}_- = \{d, f\}$	MR3.2 $\mathbb{I}_+ = \{a, b, e\}$ $\mathbb{I}_- = \{c, f\}$	MR4.2 $\mathbb{I}_+ = \{a, b, c, d\}$ $\mathbb{I}_- = \{f\}$	MR5.2 $\mathbb{I}_+ = \{a, b, d, e\}$ $\mathbb{I}_- = \{f\}$	MR6.2 $\mathbb{I}_+ = \{a, b, d, e\}$ $\mathbb{I}_- = \{f\}$						
MR1.3 $\mathbb{I}_+ = \{a, b\}$ $\mathbb{I}_- = \{c, e, f\}$	MR2.3 $\mathbb{I}_+ = \{a, b\}$ $\mathbb{I}_- = \{d, e, f\}$	MR3.3 $\mathbb{I}_+ = \{a, b\}$ $\mathbb{I}_- = \{c, e, f\}$	MR4.3 $\mathbb{I}_+ = \{a, b, c\}$ $\mathbb{I}_- = \{e, f\}$	MR5.3 $\mathbb{I}_+ = \{a, b, d\}$ $\mathbb{I}_- = \{c, f\}$	MR6.3 $\mathbb{I}_+ = \{a, b, e\}$ $\mathbb{I}_- = \{c, f\}$						
MR1.4 $\mathbb{I}_+ = \{a, b\}$ $\mathbb{I}_- = \{d, e, f\}$	MR2.4 $\mathbb{I}_+ = \{a, b\}$ $\mathbb{I}_- = \{c, d, f\}$	MR3.4 $\mathbb{I}_+ = \{a, b\}$ $\mathbb{I}_- = \{d, e, f\}$	MR4.4 $\mathbb{I}_+ = \{a, b, c\}$ $\mathbb{I}_- = \{d, f\}$	MR5.4 $\mathbb{I}_+ = \{a, b, d\}$ $\mathbb{I}_- = \{e, f\}$	MR6.4 $\mathbb{I}_+ = \{a, b, e\}$ $\mathbb{I}_- = \{d, f\}$						
<b>LEVEL 3</b>											
-----											
MR1 $\mathbb{I}_+ = \{a, b\}$ $\mathbb{I}_- = \{e, f\}$	MR2 $\mathbb{I}_+ = \{a, b\}$ $\mathbb{I}_- = \{d, f\}$	MR3 $\mathbb{I}_+ = \{a, b\}$ $\mathbb{I}_- = \{c, f\}$	MR4 $\mathbb{I}_+ = \{a, b, c\}$ $\mathbb{I}_- = \{f\}$	MR5 $\mathbb{I}_+ = \{a, b, d\}$ $\mathbb{I}_- = \{f\}$	MR6 $\mathbb{I}_+ = \{a, b, e\}$ $\mathbb{I}_- = \{f\}$						
<b>LEVEL 2</b>											
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MR $\mathbb{I}_+ = \{a, b\}$ $\mathbb{I}_- = \{f\}$											
<b>LEVEL 1</b>											

We can see which Level 4 points extend from which Level 3 points just by following the names of each point. For example, complete-points 4.3.1 and 4.3.2 extend partial point 4.3 .

Recall that earlier I promised to give a more formal definition of ‘complete specification’. In relation to the specification space, we may now understand ‘complete specification points’ to refer to specification points at which all the individuals in the domain are accounted for within either the extension or anti-extension. Complete specification points contrast with ‘partial specification points’, i.e., specification points at which at least one individual from the domain is not accounted for by either the extension or anti-extension. The base point is the most partial of the partial points in any model, and this is why, of all the points in the space, it is the most vague.

Within the space, Fine imposes three conditions on the ‘extends’ relation between specification points. The Completeness Condition requires that any specification point can be extended to a point at which there is a valuation for every proposition, i.e., to a complete point. The Fidelity Condition requires that all valuations at complete points are classical. This ensures that valuations are classical over complete specifications. Last, the Stability Condition requires that definite valuations on any given point are preserved in all extensions of that point. This guarantees that non borderline cases indicated at the base point stay non borderline all the way to the complete points.

We assign semantic values to sentences in the light of our interpretations of the complete specification points within the model. There are forty-eight complete specification points in the MR model. Of these, twenty-four have object  $c$  in the extension of ‘is tall’. If we interpret borderline case  $c$  as representing Bobo, then we can see that half of the complete specification points recommend that we assign ‘True’ to the sentence ‘Bobo is tall’. Then again, the other twenty-four complete points recommend the converse. The complete specification points in model MR represent all the ways the extension and anti-extension of ‘is tall’ can be made completely precise given the six individual domain  $\{a, b, c, d, e, f\}$ .

Fine suggests a notion of truth *simpliciter* whereby a sentence is true when it is true-at-all-complete-specification-points. He calls this special notion of truth ‘supertruth’. I will use the hyphenated term hereafter for the sake of clarity. Fine suggests that validity is the necessary preservation of truth-at-all-complete-specification-points. This notion of validity is called

‘supervalidity’. An argument is supervalid iff whenever the conjunction of premises is true-at-all-complete-specification-points, the conclusion is true-at-all-complete-specification-points.

### 3.3 Supervaluationism and the Features of Vagueness

Consider a blob that is borderline between red and pink.<sup>16</sup> The blob is a borderline case of ‘is pink’ and is a borderline case of ‘is red’. If we were to build a specification space model for these predicates, the blob would be within extension of ‘is red’ at some complete specification points and it would be within the extension of ‘is pink’ at other complete specification points. This is because some interpretations of these predicates include the blob and others do not. Whatever the model may look like, neither of these predicates will be vague at any of the complete specification points because the vagueness has been specified away.

Let’s now reconsider the most salient features of vagueness in the light of the specification space model presented in 3.2. The base point MR represents the clear cases of the ‘is tall’ predicate within the given domain. Level 1 is the vaguest because more members of the domain members are excluded by MR than by any other specification-points on any other level of the model, and Level 4 is the least vague. Each of the specification points on Level 4 is a complete point and complete points are the most precise among all the points in any model. The meaning of the predicate for the given domain is completely precise at each complete point, so there is no vagueness at individual *complete* specification points. Each complete point represents one potential sharp cut-off between the extension and anti-extension of the modeled predicate in the given domain.

Consider again the sorites paradox:

P1: 5,000 grains makes a heap.

P2:  $\forall n$  (If  $n$  grains makes a heap, then  $n-1$  grains makes a heap)

Therefore, 0 grains makes a heap.

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<sup>16</sup> This example is from Fine (1975: 269).

The second premise is not true true-at-every-complete-specification-point, so the conjunction of the premises cannot be true-at-all-complete-specification-points. Accordingly, sorites arguments cannot be invalid on Fine's conception of validity. Though valid, the argument cannot be sound for the same reason that it cannot be invalid, i.e., the second premise is not true-at-all-complete-specification-points.

What is needed to invalidate this kind of argument is a sharp cut-off between the extension and anti-extension of 'is a heap'. For Fine, the argument cannot be invalid because there is no uniform cut-off among *all* the complete specification points. Sorites arguments cannot be invalid in classical logic, either, because there, too, no sharp cut-off can be specified.

In the terms of supervaluationism, we can characterize sorites susceptibility as a logical procedure which moves from one sharpening of a predicate to a more controversial sharpening. Now, in the same spirit, we can notice that at least one group of penumbral connections proceeds from one sharpening of a vague predicate to any less controversial sharpening. If the first sharpening is acceptable, the second will be also. Consider two borderline tall men Joe and Jack. Jack is taller than Joe. Any interpretation of 'is tall' on which Joe is considered to be tall is an interpretation on which Jack is considered to be tall, also.

In the light of Fine's supervaluationism, then, we can characterize penumbral connections as logical relations which hold between sentences that are neither true-at-all-complete-specification-points nor false-at-all-complete-specification-points. Classical logic's failure to accommodate penumbral connections is one of the major motivations behind Fine's application of supervaluationism to vagueness.<sup>17</sup> Fine calls truths that come out of penumbral connections 'penumbral truths' and he states that "no natural truth-value approach respects penumbral truths."<sup>18</sup> I look closer at penumbral connections in the next chapter.

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<sup>17</sup> Fine (1975: 270).

<sup>18</sup> *Ibid.*

### 3.4 The Logic of Supervaluationism, Higher-Order Vagueness, and the D Operator

Supervalidity is the necessary preservation of truth-at-all-complete-specification-points. Fine establishes the following consequence relation between the logic of supervaluationism and classical logic: A set of sentences  $S$  superentails  $P$  if, for every complete specification point  $c$ , if  $S$  is true at  $c$ ,  $P$  is true at  $c$ . Now, the logic of truth-at-all-complete-specification-points is classical in the sense that it preserves all the tautologies of classical logic. In other words, for any set of premises  $\Gamma$  and proposition  $B$ ,  $\Gamma \models_{sv} B$  iff  $\Gamma \models_{cL} B$ .<sup>19</sup> So far, the system is sufficiently classical. Consider the following conjunction elimination argument:

P1: Joe is tall and Jack is tall.  
∴ Jack is tall.

Both men are borderline cases of tall and Jack is taller than Joe. The premise will not be true-at-all-complete-specification-points; however, any interpretation of ‘is tall’ for which P1 is true is an interpretation for which the conclusion will be true, also.

Suppose instead that Jack is a clear case of tall and Joe is a borderline case. The sentence ‘Jack is tall’ is true-at-all-complete-specification-points, but the sentence ‘Joe is tall’ is not. Accordingly, their conjunction, i.e., P1, will not be true-at-all-complete-specification-points. So, the argument is supervalid in this case, too. Suppose again that both Jack and Joe are borderline cases of tall and consider the following disjunction introduction argument:

P1: Joe is tall.  
∴ Joe is tall or Jack is tall

Here, if P1 is true-at-all-complete-specification-points, then the conclusion is also true-at-all-complete-specification-points. If P1 is not true-at-all-complete-specification-points, the argument will still be valid.

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<sup>19</sup> Keefe (2000: 175-176) discusses this two way consequence relation at length.

Fine's logic preserves classical logic and accounts for vague predicates at the level of first-order vagueness. But his account of vagueness leads to higher-order vagueness. Fine thinks that this is as it is supposed to be because he thinks that higher-order vagueness is a distinctive feature of vagueness. If higher-order vagueness is an essential feature, the logic of vagueness should cover it, and Fine introduces the Definitely operator (D) for this very reason.

Unfortunately, the presence of the D operator significantly modifies the logic of supervaluationism and compromises its ability to preserve classical logic. With the admission of higher-order vagueness and the introduction of D, the logic of truth-at-all-complete-specification-points goes south. Specifically, the two way consequence relation between supervaluational consequence and classical consequence no longer holds. I consider more closely higher-order vagueness and the D operator in the third chapter.

Fine identifies penumbral connections and higher-order vagueness as being two distinctive features of vagueness.<sup>20</sup> In the next two chapters, I examine and analyze penumbral connections and higher-order vagueness, respectively, and I analyze Fine's accounts of these phenomena. In Chapter 2, I argue that Fine applies the category 'penumbral connection' too widely. In Chapter 3, I argue that higher-order vagueness is not a phenomenon of the vagueness of natural language, but is, rather, an artifact of analysis.

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<sup>20</sup> Fine (1975: 287)

## Chapter 2

### Penumbral Connections vs. Pseudo Penumbral Connections

#### §1 Penumbrae and Penumbral Connections

In Chapter 1, I introduced penumbral connections and situated them in relation to vagueness and Fine's supervaluationism. The time is ripe for analysis of penumbral connections and Fine's account of them. Most generally, I agree that penumbral connections are a distinct feature of vague predicates. The aim of the present chapter is two-fold.

First, I seek to reconsider Fine's characterization of penumbral connections in order to examine the different types of cases he thinks exemplify this characterization. In the course of doing so, I argue that Fine categorizes some kinds of statements as exemplifying penumbral connections which do not have this characteristic. Specifically, he considers instances of logically valid formulae as being expressions that exhibit penumbral connections.

According to Fine, a formula is valid iff it takes a certain designated semantic valuation, or supervaluation, for every specification. I don't think that Fine's account of logically valid formulae is controversial. However, I argue that although instances of logically valid formulae may be satisfiable in any domain for every interpretation, that this is the case has nothing to do with any relations which hold between indefinite predicates. In short, penumbral connections are not merely instances of valid formulae.

Second, I discuss why it is significant that Fine mistakes instances of logically valid formulae for expressions that can potentially exhibit penumbral connections. The worry is that Fine's analysis does not cut to the heart of what penumbral connections are because he misidentifies the *analysandum*. Because of the misidentification, his analysis targets too wide a range of phenomena, and, hence, is too weak an account. Since Fine's supervaluationism treats phenomena which do not exemplify penumbral connections as though they did, it is not a



sufficient account of penumbral connections. I argue for a refinement of Fine’s characterization of penumbral connections. My refined characterization is more exact.

### 1.1 Russell on Penumbrae

As far as I can track, the notion ‘penumbra’ is introduced to the vagueness literature by Bertrand Russell in his 1923 paper “Vagueness”. Of penumbra, Russell writes:

[A]ll words are attributable without doubt over a certain area, but become questionable within a penumbra ... Someone might seek to obtain precision in the use of words by saying that no word is to be applied in the penumbra, but unfortunately the penumbra is itself not accurately definable, and all the vagueness which apply to the primary use of words apply also when we try to fix a limit to their indubitable applicability.”<sup>21</sup>

Here, Russell observes that there are some words for which we cannot determine the limits of their applications. What is more, the boundaries of the penumbra are not rigidly definable. The truth-value assignment of a vague statement  $A$  is questionable for the non-clear cases of  $A$  and non-clear non-cases of  $A$ . The borders between the clear and non-clear cases, on the one hand, and the non-clear non-cases and the clear non-cases, on the other, are imprecise. I shall return to this insight in the next chapter.

Also, Russell contrasts vagueness with precision and explains that the two are contraries. Vagueness occurs where there is a lack of precision, and precision is to be found where there is a lack of vagueness. For any vague statement  $A$ , the penumbral area is located between the non-clear cases of  $A$  and the non-clear non-cases of  $A$ .

### 1.2 Three Ways to Handle Vagueness Precisely

Classical logic demands precision but ordinary English does not. In the light of the demand for precision, there are at least three ways to handle vagueness. The first is to deny that there are borderline cases in the first place.<sup>22</sup> Handling vagueness in this way amounts to the view that ‘vagueness’ fails to refer. Moreover, this first method calls for the rejection of any account

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<sup>21</sup> Russell (1983: 149).

<sup>22</sup> This strategy is taken up by epistemicists, a.k.a., epistemic theorists. See Williamson (1994) and Sorensen (2001).

which works from the standpoint that ‘vagueness’ does refer. Since consideration of this view bankrupts the value of positive accounts of borderline cases, I shall cast it aside in order to discuss other views in relation to which I may analyze Fine’s account.

A second option is to acknowledge the legitimacy of vagueness as a target of analysis yet deny that vague statements can be assigned truth-values. This method is doomed given that it is unproblematic to assign truth-values to the clear cases and the clear non-cases, whatever their ranges may happen to be. Truth assignments may be problematic for borderline cases, and our inability to discriminate a penumbra’s boundaries may be troublesome, but it does not follow that assignments cannot be given to the cases outside the penumbra.

A third option embraces the fact that assignments may be given to extra-penumbral cases unproblematically and suggests that the borderline cases be assigned values in relation to specific interpretations. Fine’s approach is an example of this third method. Clear cases are those which come out true for every interpretation, clear non-cases are those which come out false for every interpretation, and the borderline, i.e., penumbral, cases are those which are neither true for every interpretation nor false for every interpretation.

### 1.3 Fine’s Characterization of Penumbral Connections

Fine extends the notion ‘penumbra’ by discussing penumbral connections. He “refer[s] to the possibility that logical relations hold among indefinite predicates as *penumbral connection*[s].”<sup>23</sup> He calls ‘truths on a penumbra’, i.e., ‘penumbral truths’, the truths that arise from penumbral connections.

Fine claims that no natural (two-valued) truth-value approach respects penumbral truths. This claim is tantamount to the claim that no two-valued approach can accommodate penumbral connections. For example, according to Fine, two-valued approaches cannot distinguish between

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<sup>23</sup> Fine (1975: 270).

‘red’ and ‘pink’, for example, as being independent and exclusive upon their common penumbra.<sup>24</sup>

Classical logic’s inability to accommodate and express penumbral connections is perhaps the principal motivation behind Fine’s three-valued semantics of truth-at-all-complete-specification-points, falsity-at-all-complete-specification-points, or neither-truth-nor-falsity-at-all-complete-specification-points. In the light of Fine’s three-valued semantics, then, we may restate his characterization of penumbral connections as logical relations which hold between sentences that are neither-true-nor-false-at-all-complete-specification-points.

### 1.4 Three Types of Penumbral Connections

In Chapter 1, I explained that penumbral connections proceed from one sharpening of a vague predicate to any less controversial sharpening. If the first sharpening is acceptable, the second will be also. I shall label this group of penumbral connection ‘sequenced examples’. Let’s draw upon the specification space outlined in Chapter 1 in order explicate the notion ‘less controversial’ since it is a crucial part of sequenced examples of penumbral connections.

Consider again the Level 4 of the MR model, i.e., the level of complete specification points:

MR1.1.1 [ ] = {a, b, c, d} [ ] = {e, f}	MR1.1.2 [ ] = {a, b, c} [ ] = {d, e, f}	MR2.1.1 [ ] = {a, b, c, e} [ ] = {d, f}	MR2.1.2 [ ] = {a, b, c} [ ] = {d, e, f}	MR3.1.1 [ ] = {a, b, d, e} [ ] = {c, f}	MR3.1.2 [ ] = {a, b, d} [ ] = {c, e, f}	MR4.1.1 [ ] = {a, b, c, d, e} [ ] = {f}	MR4.1.2 [ ] = {a, b, c, e} [ ] = {d, f}	MR5.1.1 [ ] = {a, b, c, d, e} [ ] = {f}	MR5.1.2 [ ] = {a, b, c, d} [ ] = {e, f}	MR6.1.1 [ ] = {a, b, c, d, e} [ ] = {f}	MR6.1.2 [ ] = {a, b, c, e} [ ] = {d, f}
MR1.2.1 [ ] = {a, b, c, d} [ ] = {e, f}	MR1.2.2 [ ] = {a, b, d} [ ] = {c, e, f}	MR2.2.1 [ ] = {a, b, c, e} [ ] = {d, f}	MR2.2.2 [ ] = {a, b, e} [ ] = {c, d, f}	MR3.2.1 [ ] = {a, b, d, e} [ ] = {c, f}	MR3.2.2 [ ] = {a, b, e} [ ] = {c, d, f}	MR4.2.1 [ ] = {a, b, c, d, e} [ ] = {f}	MR4.2.2 [ ] = {a, b, c, d} [ ] = {e, f}	MR5.2.1 [ ] = {a, b, c, d, e} [ ] = {f}	MR5.2.2 [ ] = {a, b, d, e} [ ] = {c, f}	MR6.2.1 [ ] = {a, b, c, d, e} [ ] = {f}	MR6.2.2 [ ] = {a, b, e, d} [ ] = {c, f}
MR1.3.1 [ ] = {a, b, d} [ ] = {c, e, f}	MR1.3.2 [ ] = {a, b} [ ] = {c, d, e, f}	MR2.3.1 [ ] = {a, b, c} [ ] = {d, e, f}	MR2.3.2 [ ] = {a, b} [ ] = {c, d, e, f}	MR3.3.1 [ ] = {a, b, d} [ ] = {c, e, f}	MR3.3.2 [ ] = {a, b} [ ] = {c, d, e, f}	MR4.3.1 [ ] = {a, b, c, d} [ ] = {e, f}	MR4.3.2 [ ] = {a, b, c} [ ] = {d, e, f}	MR5.3.1 [ ] = {a, b, d, e} [ ] = {c, f}	MR5.3.2 [ ] = {a, b, d} [ ] = {c, e, f}	MR6.3.1 [ ] = {a, b, d, e} [ ] = {c, f}	MR6.3.2 [ ] = {a, b, e} [ ] = {c, d, f}
MR1.4.1 [ ] = {a, b, c} [ ] = {d, e, f}	MR1.4.2 [ ] = {a, b} [ ] = {c, d, e, f}	MR2.4.1 [ ] = {a, b, e} [ ] = {c, d, f}	MR2.4.2 [ ] = {a, b} [ ] = {c, d, e, f}	MR3.4.1 [ ] = {a, b, c} [ ] = {d, e, f}	MR3.4.2 [ ] = {a, b} [ ] = {c, d, e, f}	MR4.4.1 [ ] = {a, b, c, e} [ ] = {d, f}	MR4.4.2 [ ] = {a, b, c} [ ] = {d, e, f}	MR5.4.1 [ ] = {a, b, c, d} [ ] = {e, f}	MR5.4.2 [ ] = {a, b, d} [ ] = {c, e, f}	MR6.4.1 [ ] = {a, b, c, e} [ ] = {d, f}	MR6.4.2 [ ] = {a, b, e} [ ] = {c, d, f}

Level 4

Suppose the members of our domain, i.e., {a, b, c, d, e, f}, are arranged in sequence according to height, where the tallest is first in the sequence and the shortest is last. The sequence, then, would be < a, b, c, d, e, f >. Here, *a* and *b* are clear cases of the vague predicate ‘is tall’ and *f* is a clear non case of ‘is tall’. Hence, regarding the borderline cases *c*, *d*, and *e*, the four types of admissible and complete specification points are exhibited by the following complete points:

<sup>24</sup> *Ibid.*

MR1.1.1    MR1.1.2    MR4.1.1    MR2.3.2  
 $\llbracket \cdot \rrbracket_+ \{a, b, c, d\}$     $\llbracket \cdot \rrbracket_+ \{a, b, c\}$     $\llbracket \cdot \rrbracket_+ \{a, b, c, d, e\}$     $\llbracket \cdot \rrbracket_+ \{a, b\}$   
 $\llbracket \cdot \rrbracket_- \{e, f\}$      $\llbracket \cdot \rrbracket_- \{d, e, f\}$     $\llbracket \cdot \rrbracket_- \{f\}$      $\llbracket \cdot \rrbracket_- \{c, d, e, f\}$

Complete points similar to MR2.3.2 are ones which precisify ‘is tall’ such that only *a* and *b*, i.e., the clear cases, count as being tall. Here, presumably, *a* is taller than *b* even if both *a* and *b* are clear cases of ‘is tall’. Complete points similar to MR1.1.2 are ones which precisify ‘is tall’ to the effect that, in addition to the clear cases, *c* is tall. Here, of course, it is less controversial to say that *b* is tall than it is to say that *c* is tall. In the same way, at complete points similar to MR1.1.1, *e* is a less controversial case of non tall than *d* is.

MR’s complete points precisify ‘is tall’ to the effect that *c* is interpreted as being a case of tall and *d* interpreted as being a case of is tall, too. Given our ordered sequence, and in relation to complete points similar to MR1.1.1, the interpretation that *c* is a case of tall is less controversial than the interpretation that *d* is tall. Lastly, complete points similar to MR4.1.1 are ones at which all the borderline cases are precisified as being cases of ‘is tall’. Here, *c* is a less controversial case than *d*, which is a less controversial case than *e*.

Recall that I illustrated the idea that at least one type of penumbral connection proceeds from one sharpening of a vague predicate to any less controversial sharpening with the example of Joe and Jack, both of whom are borderline cases of tall. Joe and Jack are each borderline cases of tall but Jack is taller than Joe; so, Jack is a less controversial borderline case than Joe is. Hence, any interpretation of ‘is tall’ for which the sentence ‘Joe is tall’ is true is also an interpretation on which the sentence ‘Jack is tall’ is true.

Let’s abstract from this example and explicate the schema of which it is an instance. For every *x* and for every *y*, where *x* and *y* are non-identical borderline cases of a vague predicate *Fx*, and where *x* is a less controversial case of *F* than *y* is, if *Fy*, then *Fx*. This schema and its instances are true for every interpretation in any domain. But this is just one type of penumbral connection, namely, sequenced examples. I now turn some other types of penumbral connections which I did not discuss in Chapter 1.

Recall the blob example.<sup>25</sup> A certain colored blob is on the border of pink and red. As regards this blob, the statements ‘The blob is pink’ and ‘The blob is red’ are neither-true-nor-false-at-all-complete-specification-points. The blob is in the penumbra of the predicate ‘is pink’ and it is in the penumbra of ‘is red’. The penumbrae of the two predicates overlap.

On some interpretations, the blob is pink. On others, it is red. There are no interpretations for which the blob is both pink and red. Since the conjunction ‘The blob is both pink and red’ is false on every interpretation, it is a *penumbral falsity* that the blob is both pink and red. On any particular interpretation, the blob is either pink or red.

Notice here that penumbral falsity would not be exhibited by the conjunction of two predicates which lack a mutual exclusion upon their common penumbra. For example, the predicates ‘is tall’ and ‘is red’ do not share a common penumbra. So for some individual *a*, which is a borderline case of ‘is tall’ and ‘is red’, the conjunction ‘Both *a* is tall and *a* is red’ is neither-true-nor-false-at-all-complete-specification-points.

Now, consider the disjunction ‘The blob is pink or red’. This is true on every interpretation, i.e., it is a *penumbral truth* that the blob is pink or red. Given that the predicates ‘is pink’ and ‘is red’ share a common penumbra, cases on the border of pink and red may be interpreted as being cases of one or the other color. Here, none of the borderline cases can be interpreted as being both pink and red but each of them will be interpreted as being either pink or red.

Notice further that penumbral truth would not be exhibited by the conjunction of two predicates which lacked a mutual exclusion upon their common penumbra. The predicates ‘is tall’ and ‘is red’ do not share a common penumbra; so, for some individual, *a*, which is a borderline case of ‘is tall’ and ‘is red’, the disjunction ‘Either *a* is tall or *a* is red’ is neither-true-nor-false-at-all-complete-specification-points.

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<sup>25</sup> Again, I use Fine’s (1975: 269) example.

The three examples illustrated above, namely, sequenced examples, examples of penumbral truth, and examples of penumbral falsity, are genuine cases of penumbral connections. Since there are legitimate examples of penumbral connections, we now can say with justification that penumbral connections are legitimate phenomena. Accordingly, Fine's motivations behind his project of seeking an analysis of vagueness which accommodates penumbral connections are well warranted. So far, so good.

Given the blob examples of penumbral truths and penumbral falsities, we can abstract a bit in order to uncover the logical schema of which these examples are instances. The conjunction 'The blob is both pink and red' is false for every interpretation because no blob in the shared penumbra between pink and red can be both pink and red on any interpretation. That is, for each interpretation of 'is pink' and 'is red', there is a mutual exclusion between the extensions of the two predicates on each interpretation.

We may say that if there is a mutual exclusion between the extensions of any two predicates  $Fx$  and  $Gx$ , the formula  $\exists x (Fx \wedge Gx)$  is true for no precisification. Particular instantiations of the formula  $\exists x (Fx \wedge Gx)$ , given the constraints above stated, will be false for any specific precisification and so will be examples of penumbral falsity. Note that instances of this formula have no model because they exhibit some relation between the vague predicates involved.

The disjunction 'The blob is pink or red' is true for every precisification because any given blob in the shared penumbra between pink and red will be either pink or red on any interpretation. Implicit here is the requirement that the predicates 'is pink' and 'is red' share a common penumbra but this common penumbra is not common to any other predicate, e.g., 'is yellow'. I find this implicit requirement to be acceptable.

Here, we may say that if two predicates  $Fx$  and  $Gx$  share a common penumbra which is not common to any other predicate  $Hx$ , where  $Hx \neq Fx$ ,  $Hx \neq Gx$ , and  $Fx \neq Gx$ , the formula  $\forall x (Fx \vee Gx)$  is true for every precisification. Particular instantiations of the formula

$\forall x (Fx \vee Gx)$ , given the constraints above stated, will be true for any specific precisification and so will exhibit penumbral truth. Each instance of this formula has a model, but, more importantly, each has a model because each exhibits some relation between the vague predicates involved.

Given their respective above stated constraints, quantified formulae such as  $\exists x (Fx \wedge Gx)$  and  $\forall x (Fx \vee Gx)$  express generalized penumbral connections. These quantified formulae have truth conditions for a given precisification. The first is false for every interpretation in any domain, while the second is true for every interpretation in any domain. Consequently, instances of these formulae will be false-at-all-complete-specification-points and true-at-all-complete-specification-points, respectively. While instances of these formulae exhibit penumbral connections between vague predicates with respect to particular individuals in the domain, the generalized formulae exhibit, more generally, penumbral connections between predicates.

So far I've discussed three legitimate types of penumbral connections. The first, namely, sequenced examples, occur when we move from one complete sharpening to a less controversial complete sharpening. The second typifies penumbral falsity in that on any given interpretation no object in some relevant domain can be predicated by one of two predicates that share a common penumbra. The third typifies penumbral truths in that on every interpretation each object in some relevant domain will be predicated by one of two mutually predicates that share a common penumbra.

To say that each variety of penumbral connection discussed so far is a legitimate type of penumbral connection is to say that each exhibits a relation which holds *between vague predicates*. I turn now to another group cases which Fine considers to exhibit penumbral connections. I shall argue that such cases do not exhibit penumbral connections although they happen to be cases for which uniform truth-assignments are given across interpretations.

### 1.5 Pseudo Penumbral Connections

Consider the blob again, but, this time, consider it as being only a borderline case of red. In any domain and for any given interpretation, the conjunction ‘The blob is red and the blob is not red’ will be false. Despite the fact that ‘The blob is red’ conjoined with its negation yields a falsity for every interpretation, that this is so, I argue, is not a result of any penumbral connection between the vague predicates involved.

The conjunction ‘The blob is red and the blob is not red’ is false on any interpretation in virtue of the fact that its negation is an instance of a logically valid formula, namely, the principle of non-contradiction. The principle of non-contradiction and its instances are true for every interpretation in any domain regardless of the natures and contents of the predicates involved. Instances of the formula  $\exists x (Fx \wedge \neg Fx)$  are false for every interpretation whether or not the components involve indefinite predicates.

The principle of non-contradiction may be represented by the formula  $\neg \exists x (Fx \wedge \neg Fx)$ . This formula acts as a schema, and this schema is true for every interpretation in any domain. But, I argue, ‘The blob is red and the blob is not red’ *does not exhibit a penumbral connection between vague predicates*. Of course, ‘The blob is red and the blob is not red’ will be assigned false uniformly for every interpretation, however, this is a consequence of the fact that their conjunction is an instance of a logically invalid formula and so should not be misidentified as a exhibiting some penumbral connection.

Moreover, the formula  $\neg \exists x (Fx \wedge \neg Fx)$  does not express a generalized logical relation *between vague predicates*. Not only does  $\neg \exists x (Fx \wedge \neg Fx)$  fail to express general logical relation between vague predicates, instances of this formula fail to exhibit penumbral connections between vague predicates with respect to particular individuals in the domain. For this reason, I say that this formula and its instances exhibit not penumbral connections but pseudo penumbral connections.

The analysis of logically valid formulae and their instances should be separate from the analysis of the penumbral connections among vague predicates. Otherwise, we will erroneously develop an account of penumbral connections which is not fit to uncover the essence of meaningful relations which hold between vague predicates. Any application of the label



‘penumbral connections’ to what are really pseudo penumbral connections requires a failure to separate logically valid formulae and their instances from the analysis of the relations that hold between vague predicates. Let me illustrate this point with a second example of pseudo penumbral connections.

In any domain and for any given interpretation, the disjunction ‘The blob is red or the blob is not red’ will be true. Despite the fact that ‘The blob is red’ disjoined with its negation yields a truth for every interpretation, this is not a result of any logical relation between the *vague predicates* involved. Such a disjunction is true for any interpretation in virtue of the fact that the disjunction ‘The blob is red or the blob is not red’ is an instance of a logically valid formula, namely, excluded middle. Excluded middle and its instances are true in any domain for every interpretation regardless of the natures and contents of the sentences involved. Sentences such as these are true for every interpretation whether or not the components are vague predicates. So these, too, do not really exhibit penumbral connections.

Excluded middle may be represented by the formula  $\forall x (Fx \vee \neg Fx)$ . This formula acts as a schema, and this schema and its instances are true for every interpretation in any domain. That the disjunction is true for any interpretation is a consequence of the fact that their disjunction is an instance of a logically valid formula, which is true on all precisifications. Again, the analysis of logically valid formulae and their instances should be separate from the analysis of the logical relations that hold between vague predicates. Hence, ‘The blob is red or the blob is not red’ *does not exhibit a logical relation between vague predicates*.

Cases of penumbral connections are exclusively cases of meaningful relations which hold between vague predicates. We ought not to lose sight of this. Just because vague predicates occur in a complex sentence for which truth (falsity) is assigned for every interpretation, this does not guarantee that such a complex sentence exemplifies some penumbral connection. Penumbral connections occur only when such sentences are true (false) for any interpretation *because of* some relation between the component sentences. Sure, instantiations of logically valid statement forms will have uniform assignments across all interpretations; but, as I’ve emphasized, this is the case regardless of whether or not vague predicates are involved.

It is a sufficient condition for the logical validity of a given formula that in any domain for any interpretation it be assigned ‘True’. Nevertheless, this is only one of two individually necessary and jointly sufficient conditions for penumbral connections. The other individually necessary condition is that the uniform assignment be a consequence of some relation *between vague predicates*. Fine does not recognize this. In fact, he considers instantiations of non-contradiction and excluded middle involving vague statements to be examples of penumbral connections. This is a notable mistake.

I have not yet challenged Fine’s characterization of penumbral connections, *viz.*, logical relations which hold between sentences that are neither-true-nor-false-at-all-complete-specification-points. So far, my disagreement with Fine on the subject of penumbral connections strictly has regarded the kinds of statements that count as exhibiting penumbral connections. Let me now draw a distinction in order to suggest a refined, more exact, characterization of penumbral connections.

## §2 Rethinking Penumbral Connections

Fine characterizes penumbral connections as logical relations which hold between sentences that are neither-true-nor-false-at-all-complete-specification-points. I wish to distinguish between *logical relations* and *analytic relations*. Of course, it is not incorrect to say that analytic relations are a variety of logical relations, but it is important to observe that not all logical relations are analytic ones. I shall argue for a refinement of Fine’s characterization of penumbral connections whereby we substitute ‘analytic relations’ for ‘logical relations’. The proposed characterization, then, is as follows: Penumbral connections are analytic relations which hold between the meanings of vague predicates.

Consider again the three legitimate types of penumbral connections (PC) and the two examples of pseudo penumbral connections (PPC).

PC1: Sequence Examples

PC2: Penumbral Truths

PC3: Penumbral Falsities

PPC1: Principle of Non-Contradiction and its Instances  
PPC2: Law of Excluded Middle and its Instances

Formulae exemplifying any of the above five categories are formulae to which uniform supervvaluations are assigned across interpretations in any domain. So, what distinguishes the first three as PCs and the last two as PPCs? The first three categories are groups of formulae to which uniform valuations are assigned across interpretations in any domain as consequence of some *analytic relation* between predicates. The latter two categories are groups of formulae to which uniform valuations are assigned across interpretations in any domain as a consequence of some *logical relation* between predicates.

The Principle of Non-Contradiction, the Law of Excluded Middle, and their instances are logically valid independently of the properties of the predicates contained in their components. That they receive uniform valuations across interpretations in any domain is a consequence of the structure of the formulae, the logical meanings and arrangement of the connectives, and the logical relations between the connectives as given in the structures of the formulae.

The reason why the formula  $\exists x (Fx \wedge \neg Fx)$  is false for every interpretation in any domain is to be found in the meaning of the quantifier, the logical meanings and arrangement of the connectives, and the fact that one and the same predicate function appears unnegated on one side of the conjunction but negated on the other. For cases of the quantified formula or its instances, the vagueness or precision of the meaning of the predicate is inconsequential.

In the same way, the reason why the formula  $\forall x (Fx \vee \neg Fx)$  is true for every interpretation in any domain is to be found in the meaning of the quantifier, the logical meanings and arrangement of the connectives, and the fact that one and the same predicate function appears non-negated on one side of the disjunction but negated on the other. Here, too, for cases of the quantified formula or its instances, the vagueness or precision of the meaning of the predicate is inconsequential.

But, cases of sequence examples, penumbral truths, and penumbral falsities receive uniform assignments across interpretations in any domain as a consequence of something subtler

than the meanings and arrangement of the connectives and other symbols. They receive uniform assignments across interpretations as a consequence of some analytic relation between the predicates involved. Consider sequence examples. When the individuals in the domain are sequenced, e.g., by height in the case of the predicate ‘is tall’, there is an analytic relation at hand. In virtue of the meaning of ‘is tall’ in the sequenced domain, there is an analytic relation between a given precisification and all less controversial precisifications.

An analytic relation is also exhibited in the blob cases. There, the relation holds between the predicate ‘is red’ and the predicate ‘is pink’. We can observe that the mutual exclusion upon a common penumbra indicates an analytic relation between two predicates. For example, the fact that that the borderline pink, borderline red, blob must be one or the other (red or pink) on any interpretation illustrates an analytic relation responsible for the penumbral truth exhibited by Excluded Middle and its instances.

Also, we can observe that the mutual exclusion upon a common penumbra indicates, perhaps the same, analytic relation in cases of Non Contradiction and penumbral falsity. It is false for every interpretation in any domain that the borderline pink, borderline red, blob is both pink and red. Penumbral truths (falsities) are true (false) for every interpretation in any domain because of some analytic relation between predicates, not because of some logical relation between operators, etc.

### §3 Why the Refined Characterization Matters

Fine’s supervaluationism is intended to offer an analysis of the logic and semantics of the vague predicates of ordinary language. Logical relations, as I have distinguished them, only hold between the meanings and arrangement of operators, etc. The logical properties of operators and other arranged symbols are not features of ordinary English predicates. Hence, an account of penumbral connections which fails to discriminate between penumbral connections and pseudo penumbral connections is one which fails to accurately identify penumbral connections in the salient way.

Fine's account of penumbral connections is not subtle enough to account for the relations at the heart of penumbral connections. Fine's over-determination of what counts as penumbral connections weakens his account's ability to express penumbral connections. If we're going to get an account of penumbral connections right, it must be geared toward classifying and systematizing the various analytic relations which hold among vague predicates. In the present work, I will not embark on a systematic botanizing project. I only wish to make clear that what supervenience explains when it explains penumbral connections is something which will not help us to uncover the nature of penumbral connections.

It is important to Fine's account that it be able to evaluate complex sentences of which vague sentences are components. An analysis of penumbral truths (falsities) which have uniform assignments across all interpretations is very important because it allows Fine to assign the values 'true-at-all-complete-specification-points' and 'false-at-all-complete-specification-points' to complex statements even if their component statements are neither-true-nor-false-at-all-complete-specification-points.

It may be that Fine was eager to uncover as many types of sentences as possible which have vague predicates as components but come out either true-at-all-complete-specification-points or false-at-all-complete-specification-points. Here, this eagerness may have led him to mistake instances of logically valid formulae containing vague predicates for statements exhibiting penumbral connections. Fine's mischaracterization results in an analysis of penumbral connections not built explicitly for penumbral connections. For this reason, the misidentification does not facilitate the overall aim of his project, which is to develop an account of vagueness and the distinctive characteristics of vague language. When we misidentify the phenomena of penumbral connections, we cease to be analyzing vagueness or its phenomena.

## Chapter 3

### The Nature of Higher-Order Vagueness

Fine aims to extend classical logic in order to accommodate vagueness. When we extend classical logic in the way Fine suggests, we encounter the phenomenon of higher-order vagueness. Fine regards the possibility of higher-order vagueness as being a distinct feature of vagueness, and so he seeks to account for it in his extension of classical logic.

In this chapter, I argue that the possibility of higher-order vagueness is not a distinct feature of vagueness, that is, it is at least not a distinct feature of the vagueness of natural language. Higher-order vagueness, I claim, only occurs when vagueness is analyzed. It is, hence, an artifact of analysis. I proceed as follows. First, I consider some characterizations of higher-order vagueness. Second, I revisit Fine's accounts of the D operator and higher-order vagueness. Third, I discuss the relation between the predicates 'is vague' and 'is a borderline case', among other things, and I argue that higher-order vagueness is an artifact of analysis.

#### §1 Characterizations of Higher-Order Vagueness

##### 1.1 Russell

Recall from Chapter 2 Russell's observation that penumbral boundaries are not rigidly definable. This observation relates to the phenomenon now known as 'higher-order vagueness'. Earlier I cashed out Russell's observation by saying that the borders between clear and non-clear cases, on the one hand, and clear and non-clear non-cases, on the other, are imprecise. But what does it mean to say that the boundaries of the penumbra are not rigidly definable? Russell does not answer this question, but I aim to shed light on this in what follows.

## 1.2 Some Characterizations of Higher-Order Vagueness

The issue of how best to characterize vagueness has been the burning topic in the vagueness literature.<sup>26</sup> Also a burning topic, but only recently, is the issue of how best to characterize higher-order vagueness.

It seems to have become common practice for theorists to develop characterizations of higher-order vagueness by extending characterizations of vagueness. *Prima facie*, this way of developing a characterization of higher-order vagueness may seem plausible, but I see no reason why this method must be followed. Whatever the case may be, let's move forward and run through a few of the more popular ways to characterize higher-order vagueness, each of which grows out of some characterization of vagueness.

A very crude and general characterization of higher-order vagueness refers to the analysis of the vagueness of an object language by means of a vague meta-language. The problem of vagueness does not go away just by analyzing a vague object language by means of a meta-language. We may call 'first-order vagueness' the vagueness of some vague object-language, and we may call 'second-order vagueness' the vagueness of the meta-language which is used to analyze some vague object language. On this first characterization, third-order vagueness occurs when a vague meta-meta-language is used in the analysis of some second-order vague meta-language, and so on and so forth.

We can observe a second characterization by calling on a recent blog post on vagueness in which Brian Weatherson considers for the sake of argument the following line of thought:

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<sup>26</sup> See Schiffer (1998), Eklund (2007), and Wright (forthcoming (a)).

One of the driving intuitions that motivates the rejection of bivalence, in the context of vagueness, is the intuition that there are no sharp cut off points... The intuition extends further [to higher-order vagueness]. Surely there is no sharp cut off point between being young, and being borderline young (and between being borderline and being not young.) There are borderline borderline cases. And similarly there shouldn't be sharp cut off points between youngness, and borderline borderline youngness etc... Thus there should be all kinds of orders of vagueness - at each level we escape sharp cut off points by positing higher levels of vagueness.<sup>27</sup>

Here, vagueness is characterized as a lack of sharp cut-off points and a presence of borderline cases. Extending this, higher-order vagueness can be characterized as borderline cases of borderline cases.

Another way to characterize higher-order vagueness is to say that 'vague' is vague.<sup>28</sup> Fine, himself, states that "the vague may itself be vague, or vaguely vague, and so on."<sup>29</sup> This amounts to saying that the predicates 'is vague' and 'is nonvague' have borderline cases in their extensions and anti-extensions. A similar characterization of higher-order vagueness has it that the predicates 'is a borderline case' and 'is a clear case' are vague. Both of these latter two characterizations are similar to Russell's observation that the bounds of penumbrae are indeterminate and not rigidly definable. I shall return to these latter characterizations of vagueness and borderline cases in §3.

Higher-order vagueness stems from the lack of sharp cut-offs regarding which complete specifications counts as complete and admissible specifications. Fine calls 'admissible' precise interpretations that are compatible with the ordinary uses of language. Clearly, though, there are often many precise interpretations compatible with our ordinary usage. Fine acknowledges that "the set of admissible specifications is itself intrinsically vague."<sup>30</sup>

Generally speaking, higher-order vagueness is occurs when vagueness is compounded in some way or other. For example, when a sentence is vague, there is a blurred boundary between its positive semantic value and its negative semantic value. It is not enough to say that the

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<sup>27</sup> See <http://possiblyphilosophy.wordpress.com/2008/09/01/higher-order-vagueness-and-sharp-boundaries/> .

<sup>28</sup> Those who pursue this characterization include Sorensen (1985), Hyde (1994) and (2003), and Varzi (2003). Those who are hostile to it include Wright (forthcoming (b)). For a response to Varzi (2003) see Hull (2005).

<sup>29</sup> See Fine (1975: 287).

<sup>30</sup> Fine (1975: 297). See also p. 281.



boundary is a fuzzy, as doing so merely requires us to locate the boundaries between the clear cases and the fuzzy ones and the clear non cases and the fuzzy ones.

Here, the problem of vagueness is compounded in the sense that, instead of there being one cut-off point to decipher, there are two. In other words, there is no sharp border between the borderline cases and the clear cases, nor is there a sharp border between the borderline non cases and the clear non cases. Considerations such as these are exactly the sort that motivated Fine to introduce the D operator. Let's now turn to Fine's account the D operator and higher-order vagueness.

## §2 Fine's Account of the D Operator and Higher-Order Vagueness

To say that an atomic sentence is true-at-all-complete-specification-points or false-at-all-complete-specification-points is tantamount to saying that it is a clear case or a clear non case, respectively. Where  $A$  is a vague sentence, Fine's formal machinery allows the supervaluationist to superevaluate some complex sentences, e.g., ' $A \vee \neg A$ ' and ' $A \rightarrow A$ ', as being true-at-all-complete-specification-points. It also allows the supervaluationist to superevaluate some complex sentences, e.g., ' $\neg(A \vee \neg A)$ ', ' $\neg(A \rightarrow A)$ ', ' $A \rightarrow \neg A$ ', as being false-at-all-complete-specification-points. But Fine's supersemantics also calls for the attribution of the superevaluation 'neither-true-nor-false-at-all-complete-specification-points' to a subset of both atomic and complex sentences.

Let's step back a moment to refresh ourselves on why we might worry about higher-order vagueness. It is unusual for a predicate, for example, to admit borderline cases but do so where there are sharp boundaries between the borderline cases and the clear and non clear cases.<sup>31</sup> However, it's not unusual for a predicate to admit a range of borderline cases the borders of which admit borderline cases. On the specification space approach, as expounded in the first chapter, when a sentence is neither-true-nor-false-at-all-specification points, we interpret this and

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<sup>31</sup> Sainsbury (1991: 173) argues that any predicate the borderline cases of which are rigidly defined should not be considered vague. Fine (1976: 266) disagrees.

assign it ‘neither true nor false’ or ‘indefinite’. It is only with the introduction of the D operator that we can express that it is true that a sentence is neither true nor false, or indefinite.

Definite truth is truth on all models. Clear cases, i.e., cases which are true-at-all-complete-specification-points, and clear non cases, i.e., cases which are false-at-all-complete-specification-points, exemplify definite truth and definite falsity, respectively. Hence, a statement which is not definitely true is one that is not true-at-all-complete-specification points, and a statement which is not definitely false is one that is not false-at-all-complete-specification-points. Let’s proceed to a discussion of the D operator.

## 2.1 The D Operator

Fine introduces the Definitely operator (**D**) in order to express higher-order vagueness. The D operator is inter-definable with the Indefinitely operator (**I**). To say that a statement ‘*A*’ is true is the same as to say that it is definitely the case that *A*.<sup>32</sup> Where the statement *A* is neither-true-at-all-complete-specification-points, the D and **I** operators are inter-definable as follows:

$$\begin{aligned} \mathbf{I}A &=_{\text{def}} \neg \mathbf{D}A \wedge \neg \mathbf{D}\neg A \\ \mathbf{D}A &=_{\text{def}} \neg \mathbf{I}A \wedge \neg \mathbf{I}\neg A \end{aligned}$$

Let me explain how D is intended to express higher-order vagueness.

Consider a statement *B*. **DB** expresses that it is definitely the case that *B* is true-at-all-complete-specification-points and **D** $\neg$ *B* expresses that it is definitely the case that *B* is false-at-all-complete-specification-points. Suppose *A* is a borderline case. **D** allows us to express that it is true that *A* is a borderline case. The formula  $\neg \mathbf{D}A \wedge \neg \mathbf{D}\neg A$  tells us that both it is not true that *A* is true-at-all-complete-specification-points and it is not true that *A* is false-at-all-complete-specification-points. In other words, it tells us that it is true that *A* is neither-true-nor-false-at-all-complete-specification-points. But, just as  $\neg \mathbf{D}A \wedge \neg \mathbf{D}\neg A$  expresses that *A* is a borderline case,

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<sup>32</sup> Fine (1975: 295).

$\neg DD\neg A \wedge \neg DDA$  expresses that  $A$  is a borderline case of being a borderline case. As it turns out, a formula with  $n$  number of iterations of the D operator is an expression about something at the  $n$ th-order.

For example, the formula  $D\neg DDD\neg A$  has four iterations of the D operator and so expresses something about the fourth-order. What it expresses is that it is definitely the case that it is not definitely the case that it is definitely the case that that it is definitely the case that it is not the case that  $A$ . The main operator is the leftmost D, so the statement as a whole is true for all interpretations of the formula  $D\neg DDD\neg A$ .

## 2.2 The Not-So Classical Logic of D

There is a mutual consequence relation between classical logic and Fine's D-free supervaluationist (SV) logic, and it is as follows: a wff is classically true iff it is true-at-all-complete-specification-points. Similarly, a wff is classically false iff it is false-at-all-complete-specification-points. More specifically and formally, we can say the following:

If  $\Gamma \models_{\text{CL}} B$ , then  $\Gamma \models_{\text{SV}} B$

If  $B$  is a classical consequence of the set of wffs  $\Gamma$ , then  $B$  is a supervaluational consequence of the set of wffs  $\Gamma$ .

If  $\Gamma \models_{\text{SV}} B$ , then  $\Gamma \models_{\text{CL}} B$

If  $B$  is a supervaluational consequence of the set of wffs  $\Gamma$ , then  $B$  is a classical consequence of the set of wffs  $\Gamma$ .

This consequent relation does not hold between classical logic and Fine's SV logic with D (SV+D). Logical consequence is defined for the D-enhanced system as follows:  $B$  is a consequence of  $A$  if and only if the set  $\{\neg A, B, DB, DDB, \dots\}$  is not satisfiable.<sup>33</sup> But the D enhanced supervaluationist logic violates several classical rules that the D-free system does not. The trouble with this is that the logic of supervaluationism goes south with the introduction of the D

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<sup>33</sup> See Fine (1975: 290).

operator because the mutual consequence relation between supervalutary consequence and classical consequence no longer holds once D is included in the logic of supervalutism.

The classical rules that SV+ D violates include Contraposition,  $\rightarrow$ Intro.,  $\vee$ Elim., and  $\neg$ Intro. Let me discuss these in order.<sup>34</sup> First, given that  $A$ , along with any premises and/or justified derivations from the premises, if  $B$  can be classically inferred, then  $\neg A$  can be classically inferred from  $\neg B$ , along with any premises and/or justified derivations. Now, in SV+ D, although we can infer  $DA$  from  $A$ , along with any premises and/or justified derivations, we cannot infer  $\neg A$  from  $\neg DA$ , along with any premises and/or justified derivations. Hence, Contraposition fails for SV+ D.

For example, suppose that it is true that John is tall. That is, suppose that John is a clear case of tall. Since it is true that John is tall, it follows that he is definitely tall. Now suppose that it is not the case that John is definitely tall. If it is not the case that John is definitely tall, then it does not follow that John is not tall. Here, Contraposition does not work.

Second, in classical logic, if we can validly infer  $B$  from  $A$ , along with any premises and/or justified derivations, then we can validly infer  $A \rightarrow B$  from any or no premises and/or justified derivations. This does not hold in SV+ D because, in SV+ D, the inference from the combination of any or no premises and/or justified derivations and  $A$  to  $DA$ , and hence infer  $A \rightarrow DA$  from any or no premises and/or justified derivations does not preserve truth-at-all-specification-points.

Third, the classically valid method of proof by cases, i.e.,  $\vee$ Elim., also fails for SV+ D. In classical logic, if we can infer from  $A$ , along with any premises and/or justified derivations, to  $C$ , and if we can infer from  $B$ , along with any premises and/or justified derivations, to  $C$ , then we can validly infer from the disjunction ' $A \vee B$ ', along with any premises and/or justified derivations, to  $C$ . In SV+ D, although we can infer from  $DA$  from  $A$  to, and although we can

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<sup>34</sup> For another discussion of the failure of these four classically valid inference rules, see Williamson (1994: 151-152).

infer  $\neg DA$  from  $\neg A$ , but we cannot infer  $DA \vee \neg DA$  from  $A \vee \neg A$ , along with any premises and/or justified derivations. Doing so would not preserve truth-at-all-specification-points.

Fourth, we introduce negations in classical logic by the method of *reductio ad absurdum*, i.e.,  $\neg$ -Intro. For example, if we can validly infer  $B$  from  $A$ , along with any premises and/or justified derivations, and we can validly infer  $\neg B$  from  $A$ , along with any premises and/or justified derivations, then we can validly infer  $\neg A$  from any or no premises and/or justified derivations. But, in SV+D, even if we can validly infer  $DA$  from  $A \wedge \neg DA$ , plus any premises and/or justified derivations, and even if we can validly infer  $\neg DA$  from  $A \wedge \neg DA$ , plus any premises and/or justified derivations, we cannot validly infer  $\neg(A \wedge \neg DA)$  from the premises and/or justified derivations. Here, too, doing so would not preserve truth-at-all-specification-points.<sup>35</sup>

In short, Fine's system needs enhancement by the D operator in order to express higher-order vagueness. However, one of the major aims of his project is to render a logic of vagueness that is sufficiently classical. The logic of SV+D is not sufficiently classical. Unfortunately, the supervaluationist is faced with a trade off. There is a mutual consequent relation between SV and classical logic, but SV does not account for higher-order vagueness. The supervaluationist must either forget about an analysis of higher-order vagueness, or, instead, forget about having a sufficiently classical logic of vagueness. Forgetting either would undermine the motivations behind Fine's supervaluationism.

### §3 The Nature of Higher-Order Vagueness

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<sup>35</sup> It worth noting that Keefe (2000: 179-180) argues that we should not expect these classical inference rules to hold for SV+D by appealing to the idea that D is essentially non classical. Keefe suggests alternative forms of each of these four rules. I will not discuss these alternative versions, but they are as follows:

Contrap\*: If  $A \models_{\text{CL}} B$ , then  $\neg B \models_{\text{D}} \neg DA$   
 $\rightarrow$ Intro\*: If  $A \wedge B \models_{\text{CL}} C$ , then  $B \models_{\text{D}} DA \rightarrow C$   
 $\vee$ Elim\*: If  $A \models_{\text{CL}} C$  and  $B \models_{\text{CL}} C$ , then  $(DA \vee DB) \models_{\text{D}} C$   
 $\neg$ -Intro\*: If  $A \wedge B \models_{\text{CL}} C \wedge \neg C$ , then  $B \models_{\text{D}} \neg DA$

I now seek to expose the nature of higher-order vagueness. I begin by tracing the source of higher-order vagueness as it occurs in Fine’s account of vagueness. Next, I consider the source of vagueness in ordinary English in order to argue that higher orders of vagueness do not occur in ordinary English. I close with a recommendation to theorists of vagueness and higher-order vagueness.

### 3.1 Is Higher-Order Vagueness Born Out of Supervaluationism?

Consider again Level 4 of the specification space I presented in the first chapter:

MR1.1.1 $\llbracket \llbracket =\{a, b, c, d\} \rrbracket \rrbracket =\{a, b, c, d\}$ $\llbracket \llbracket =\{e, f\} \rrbracket \rrbracket =\{e, f\}$	MR1.1.2 $\llbracket \llbracket =\{a, b, c\} \rrbracket \rrbracket =\{a, b, c\}$ $\llbracket \llbracket =\{d, e, f\} \rrbracket \rrbracket =\{d, e, f\}$	MR2.1.1 $\llbracket \llbracket =\{a, b, c, e\} \rrbracket \rrbracket =\{a, b, c, e\}$ $\llbracket \llbracket =\{d, f\} \rrbracket \rrbracket =\{d, f\}$	MR2.1.2 $\llbracket \llbracket =\{a, b, c\} \rrbracket \rrbracket =\{a, b, c\}$ $\llbracket \llbracket =\{d, e, f\} \rrbracket \rrbracket =\{d, e, f\}$	MR3.1.1 $\llbracket \llbracket =\{a, b, d, e\} \rrbracket \rrbracket =\{a, b, d, e\}$ $\llbracket \llbracket =\{c, f\} \rrbracket \rrbracket =\{c, f\}$	MR3.1.2 $\llbracket \llbracket =\{a, b, d\} \rrbracket \rrbracket =\{a, b, d\}$ $\llbracket \llbracket =\{c, e, f\} \rrbracket \rrbracket =\{c, e, f\}$	MR4.1.1 $\llbracket \llbracket =\{a, b, c, d, e\} \rrbracket \rrbracket =\{a, b, c, d, e\}$ $\llbracket \llbracket =\{f\} \rrbracket \rrbracket =\{f\}$	MR4.1.2 $\llbracket \llbracket =\{a, b, c, e\} \rrbracket \rrbracket =\{a, b, c, e\}$ $\llbracket \llbracket =\{d, f\} \rrbracket \rrbracket =\{d, f\}$	MR5.1.1 $\llbracket \llbracket =\{a, b, c, d, e\} \rrbracket \rrbracket =\{a, b, c, d, e\}$ $\llbracket \llbracket =\{f\} \rrbracket \rrbracket =\{f\}$	MR5.1.2 $\llbracket \llbracket =\{a, b, c, d\} \rrbracket \rrbracket =\{a, b, c, d\}$ $\llbracket \llbracket =\{e, f\} \rrbracket \rrbracket =\{e, f\}$	MR6.1.1 $\llbracket \llbracket =\{a, b, c, d, e\} \rrbracket \rrbracket =\{a, b, c, d, e\}$ $\llbracket \llbracket =\{f\} \rrbracket \rrbracket =\{f\}$	MR6.1.2 $\llbracket \llbracket =\{a, b, c, e\} \rrbracket \rrbracket =\{a, b, c, e\}$ $\llbracket \llbracket =\{d, f\} \rrbracket \rrbracket =\{d, f\}$
MR1.2.1 $\llbracket \llbracket =\{a, b, c, d\} \rrbracket \rrbracket =\{a, b, c, d\}$ $\llbracket \llbracket =\{e, f\} \rrbracket \rrbracket =\{e, f\}$	MR1.2.2 $\llbracket \llbracket =\{a, b, d\} \rrbracket \rrbracket =\{a, b, d\}$ $\llbracket \llbracket =\{c, e, f\} \rrbracket \rrbracket =\{c, e, f\}$	MR2.2.1 $\llbracket \llbracket =\{a, b, c, e\} \rrbracket \rrbracket =\{a, b, c, e\}$ $\llbracket \llbracket =\{d, f\} \rrbracket \rrbracket =\{d, f\}$	MR2.2.2 $\llbracket \llbracket =\{a, b, e\} \rrbracket \rrbracket =\{a, b, e\}$ $\llbracket \llbracket =\{c, d, f\} \rrbracket \rrbracket =\{c, d, f\}$	MR3.2.1 $\llbracket \llbracket =\{a, b, d, e\} \rrbracket \rrbracket =\{a, b, d, e\}$ $\llbracket \llbracket =\{c, f\} \rrbracket \rrbracket =\{c, f\}$	MR3.2.2 $\llbracket \llbracket =\{a, b, e\} \rrbracket \rrbracket =\{a, b, e\}$ $\llbracket \llbracket =\{c, d, f\} \rrbracket \rrbracket =\{c, d, f\}$	MR4.2.1 $\llbracket \llbracket =\{a, b, c, d, e\} \rrbracket \rrbracket =\{a, b, c, d, e\}$ $\llbracket \llbracket =\{f\} \rrbracket \rrbracket =\{f\}$	MR4.2.2 $\llbracket \llbracket =\{a, b, c, d\} \rrbracket \rrbracket =\{a, b, c, d\}$ $\llbracket \llbracket =\{e, f\} \rrbracket \rrbracket =\{e, f\}$	MR5.2.1 $\llbracket \llbracket =\{a, b, c, d, e\} \rrbracket \rrbracket =\{a, b, c, d, e\}$ $\llbracket \llbracket =\{f\} \rrbracket \rrbracket =\{f\}$	MR5.2.2 $\llbracket \llbracket =\{a, b, d, e\} \rrbracket \rrbracket =\{a, b, d, e\}$ $\llbracket \llbracket =\{c, f\} \rrbracket \rrbracket =\{c, f\}$	MR6.2.1 $\llbracket \llbracket =\{a, b, c, d, e\} \rrbracket \rrbracket =\{a, b, c, d, e\}$ $\llbracket \llbracket =\{f\} \rrbracket \rrbracket =\{f\}$	MR6.2.2 $\llbracket \llbracket =\{a, b, e, d\} \rrbracket \rrbracket =\{a, b, e, d\}$ $\llbracket \llbracket =\{c, f\} \rrbracket \rrbracket =\{c, f\}$
MR1.3.1 $\llbracket \llbracket =\{a, b, d\} \rrbracket \rrbracket =\{a, b, d\}$ $\llbracket \llbracket =\{c, e, f\} \rrbracket \rrbracket =\{c, e, f\}$	MR1.3.2 $\llbracket \llbracket =\{a, b\} \rrbracket \rrbracket =\{a, b\}$ $\llbracket \llbracket =\{c, d, e, f\} \rrbracket \rrbracket =\{c, d, e, f\}$	MR2.3.1 $\llbracket \llbracket =\{a, b, c\} \rrbracket \rrbracket =\{a, b, c\}$ $\llbracket \llbracket =\{d, e, f\} \rrbracket \rrbracket =\{d, e, f\}$	MR2.3.2 $\llbracket \llbracket =\{a, b\} \rrbracket \rrbracket =\{a, b\}$ $\llbracket \llbracket =\{c, d, e, f\} \rrbracket \rrbracket =\{c, d, e, f\}$	MR3.3.1 $\llbracket \llbracket =\{a, b, d\} \rrbracket \rrbracket =\{a, b, d\}$ $\llbracket \llbracket =\{c, e, f\} \rrbracket \rrbracket =\{c, e, f\}$	MR3.3.2 $\llbracket \llbracket =\{a, b\} \rrbracket \rrbracket =\{a, b\}$ $\llbracket \llbracket =\{c, d, e, f\} \rrbracket \rrbracket =\{c, d, e, f\}$	MR4.3.1 $\llbracket \llbracket =\{a, b, c, d\} \rrbracket \rrbracket =\{a, b, c, d\}$ $\llbracket \llbracket =\{e, f\} \rrbracket \rrbracket =\{e, f\}$	MR4.3.2 $\llbracket \llbracket =\{a, b, c\} \rrbracket \rrbracket =\{a, b, c\}$ $\llbracket \llbracket =\{d, e, f\} \rrbracket \rrbracket =\{d, e, f\}$	MR5.3.1 $\llbracket \llbracket =\{a, b, d, e\} \rrbracket \rrbracket =\{a, b, d, e\}$ $\llbracket \llbracket =\{c, f\} \rrbracket \rrbracket =\{c, f\}$	MR5.3.2 $\llbracket \llbracket =\{a, b, d\} \rrbracket \rrbracket =\{a, b, d\}$ $\llbracket \llbracket =\{c, e, f\} \rrbracket \rrbracket =\{c, e, f\}$	MR6.3.1 $\llbracket \llbracket =\{a, b, d, e\} \rrbracket \rrbracket =\{a, b, d, e\}$ $\llbracket \llbracket =\{c, f\} \rrbracket \rrbracket =\{c, f\}$	MR6.3.2 $\llbracket \llbracket =\{a, b, e\} \rrbracket \rrbracket =\{a, b, e\}$ $\llbracket \llbracket =\{c, d, f\} \rrbracket \rrbracket =\{c, d, f\}$
MR1.4.1 $\llbracket \llbracket =\{a, b, c\} \rrbracket \rrbracket =\{a, b, c\}$ $\llbracket \llbracket =\{d, e, f\} \rrbracket \rrbracket =\{d, e, f\}$	MR1.4.2 $\llbracket \llbracket =\{a, b\} \rrbracket \rrbracket =\{a, b\}$ $\llbracket \llbracket =\{c, d, e, f\} \rrbracket \rrbracket =\{c, d, e, f\}$	MR2.4.1 $\llbracket \llbracket =\{a, b, e\} \rrbracket \rrbracket =\{a, b, e\}$ $\llbracket \llbracket =\{c, d, f\} \rrbracket \rrbracket =\{c, d, f\}$	MR2.4.2 $\llbracket \llbracket =\{a, b\} \rrbracket \rrbracket =\{a, b\}$ $\llbracket \llbracket =\{c, d, e, f\} \rrbracket \rrbracket =\{c, d, e, f\}$	MR3.4.1 $\llbracket \llbracket =\{a, b, c\} \rrbracket \rrbracket =\{a, b, c\}$ $\llbracket \llbracket =\{d, e, f\} \rrbracket \rrbracket =\{d, e, f\}$	MR3.4.2 $\llbracket \llbracket =\{a, b\} \rrbracket \rrbracket =\{a, b\}$ $\llbracket \llbracket =\{c, d, e, f\} \rrbracket \rrbracket =\{c, d, e, f\}$	MR4.4.1 $\llbracket \llbracket =\{a, b, c, e\} \rrbracket \rrbracket =\{a, b, c, e\}$ $\llbracket \llbracket =\{d, f\} \rrbracket \rrbracket =\{d, f\}$	MR4.4.2 $\llbracket \llbracket =\{a, b, c\} \rrbracket \rrbracket =\{a, b, c\}$ $\llbracket \llbracket =\{d, e, f\} \rrbracket \rrbracket =\{d, e, f\}$	MR5.4.1 $\llbracket \llbracket =\{a, b, c, d\} \rrbracket \rrbracket =\{a, b, c, d\}$ $\llbracket \llbracket =\{e, f\} \rrbracket \rrbracket =\{e, f\}$	MR5.4.2 $\llbracket \llbracket =\{a, b, d\} \rrbracket \rrbracket =\{a, b, d\}$ $\llbracket \llbracket =\{c, e, f\} \rrbracket \rrbracket =\{c, e, f\}$	MR6.4.1 $\llbracket \llbracket =\{a, b, c, e\} \rrbracket \rrbracket =\{a, b, c, e\}$ $\llbracket \llbracket =\{d, f\} \rrbracket \rrbracket =\{d, f\}$	MR6.4.2 $\llbracket \llbracket =\{a, b, e\} \rrbracket \rrbracket =\{a, b, e\}$ $\llbracket \llbracket =\{c, d, f\} \rrbracket \rrbracket =\{c, d, f\}$

Level 4

Each individual complete specification point exhibits a sharp boundary between the extension and anti-extension of a given predicate (in the case of MR, the predicate is ‘is tall’). Notice that there is no vagueness, nor is there higher-order vagueness, at any of the complete specification points. But, Fine’s suggested semantics for vagueness is not about individual specification points. Instead, the semantics of supervaluationism requires that we generalize over all complete points within a specification space.<sup>36</sup>

Recall that Fine defines truth *simpliciter* as truth-at-all-complete-specification-points and falsity *simpliciter* as falsity-at-all-complete-specification-points. Defining truth and falsity

<sup>36</sup> Here, there is a strong analogy between the way we are to generalize over a specification space and the generalizations we make when evaluating a completed classical truth table. Each row of a classical truth table marks a complete interpretation. When the column under the main operator of some classical wff contains all true values, we abstract from individual interpretations and call the wff a tautology. When the final column contains all false values, we abstract and call the wff a contradiction, and when it contains a mixture of true and false values, we abstract and call it a contingency. Similarly, when a statement is true-at-all-complete-specification-points, Fine interprets the statement as being true *simpliciter*. When a statement is false-at-all-complete-specification-points, Fine interprets it as being false *simpliciter*. But, when a statement is neither-true-nor-false-at-all-complete-specification-points, Fine does not interpret it as being anything *simpliciter*.

*simpliciter* in this way leaves us with the borderline cases, which are neither true *simpliciter* nor false *simpliciter*. We do not gain a clearer understanding of vagueness in virtue of some abstraction from a specification space.

But, if we notice that specification space abstractions do not help illuminate the nature of vagueness, it may help us gain a clearer understanding about the nature of higher-order vagueness. Are there borderline cases of truth *simpliciter*? Falsity *simpliciter*? For both questions, the answer is, “No”. A given formula is either true-at-all-complete-specification-points or it is not. Alternatively, a given formula either is or is not false-at-all-complete-specification-points.

Fine’s specification space semantics is based on a special concept of truth, and this concept of truth carries significant commitments. Let me explain. The concept of truth-at-all-specification-points is the root of Fine’s motive for considering higher-order vagueness as being a distinctive feature of vagueness. Moreover, the phenomenon of higher-order vagueness emerges out of Fine’s special notion of truth-at-all-complete-specification-points.

If we were not to consider something as being neither-true-nor-false-at-all-complete-specification-points, we would not consider that something as being a borderline case of being neither-true-nor-false-at-all-specification-points. To reject higher-order vagueness as a distinct feature of vagueness is to reject the concept of truth-at-all-specification-points. For these reasons, I claim that higher-order vagueness is an artifact of Fine’s analysis.<sup>37</sup> More details to come.

### 3.2 Higher-Order Vagueness and Ordinary English

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<sup>37</sup> It occurs to me that the claim that higher-order vagueness is an artifact of Fine’s system may be symptomatic of a more general point. It may be that higher-order vagueness only arises when vagueness is analyzed. Here, higher-order vagueness is an artifact of analysis in the general sense that it is exclusively a property of the analysis of some first-order language rather than being a property of the language itself. I shall not pursue this hypothesis presently. Instead, I will confine the remainder of my discussion to the potentially symptomatic claim that higher-order vagueness is an artifact of Fine’s system.

In ordinary English, when we refer to borderline cases, we say that some individual is a borderline case of *something*, viz., some category of things. So, the predicate ‘is a borderline case’ is not a unary predicate of the form ‘ $x$  is a borderline case’. Rather, the predicate ‘is a borderline case’ is a binary predicate of the form ‘ $x$  is a borderline case of  $y$ ’. Here,  $y$  is some category of things. Presumably, this category of things must admit borderline cases, so it may be represented as some vague predicate  $\Psi x$ . Accordingly, when we say that some individual is a borderline case of some predicate  $\Psi x$ , the logical form of what we say is ‘ $a$  is a borderline case of  $\Psi x$ ’. This is the logical form of first-order vague statements. We will take a critical look at this form in a moment.

Where  $Tx$  stands for ‘ $x$  is tall’, the structure of the sentence ‘ $a$  is a borderline case of the category of tall things’ is ‘ $a$  is a borderline case of  $Tx$ ’. Let’s consider how we might reveal the structure of a second-order vague statement, i.e., one that expresses that some individual is a borderline case of a borderline case of tall. Since the predicate ‘is a borderline case’ is not unary, we cannot just say ‘It is a borderline case that  $a$  is a borderline case of  $Tx$ ’. But this observation is not worth pursuing as it is not in accordance with our ordinary use of language.

Another option for expressing a potentially second-order vague statement is to say ‘ $a$  is a borderline case of a borderline case of  $Tx$ ’. This seems preferable to the ‘It is a borderline case that  $a$  is a borderline case of  $Tx$ ’ construction. But, how can we reveal the logical structure of ‘ $a$  is a borderline case of a borderline case of  $Tx$ ’ if we understand the predicate ‘is a borderline case’ to be of the form ‘ $a$  is a borderline case of  $\Psi x$ ’? We cannot. Is this because we are wrong about the ‘ $a$  is a borderline case of  $\Psi x$ ’ construction? I don’t think so. The predicate ‘is a borderline case of a borderline case’ cannot be the result of some iteration of the predicate ‘is a borderline case’.

What I’m suggesting is that ‘is a borderline case’ and ‘is a borderline case of a borderline case’ are, nontrivially, distinct predicates. Neither is reducible to the other. The logical form of the second seems to be ‘ $a$  is a borderline case of a borderline case of  $\Psi x$ ’. Fine claims that “higher-order vagueness is a species of first-order vagueness.”<sup>38</sup> I disagree. Unfortunately, Fine does not provide an argument for the claim that higher-order vagueness is a species of first-order

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<sup>38</sup> Fine (1975: 288).



vagueness. It would be very instructive to know the sense in which he means to use ‘species’. Unfortunately, here, too, nothing is provided.

Here is my argument for why higher-order vagueness is a *not* a species of first-order vagueness. If higher-order vagueness is a species of first-order vagueness, then we can formulate the logical form of the expression ‘ $a$  is a borderline case of a borderline case of  $\Psi x$ ’ in terms of the logical form of the expression ‘ $a$  is a borderline case’. But, we cannot formulate the logical form of the expression ‘ $a$  is a borderline case of a borderline case of  $\Psi x$ ’ in terms of the logical form of the expression ‘ $a$  is a borderline case’. This is because ‘ $a$  is a borderline case of a borderline case of  $\Psi x$ ’ is not producible from ‘ $a$  is a borderline case of  $\Psi x$ ’ or any number of iterations of ‘ $a$  is a borderline case of  $\Psi x$ ’. Here, ‘ $a$  is a borderline case of a borderline case of  $\Psi x$ ’ is an expression relating to second-order vagueness and ‘ $a$  is a borderline case of  $\Psi x$ ’ is an expression relating to first-order vagueness. Neither of these is expressible in terms of the other. I will not speculate about Fine’s intended sense of ‘species’.

Fine also claims that “most, if not all, vague predicates in natural language are higher-order vague.”<sup>39</sup> Here, too, I disagree, and here, too, Fine provides no argument. Again, I will provide an argument for my disagreement. The predicate ‘is a borderline case’ is not a unary predicate. But the predicate ‘is vague’ is a unary predicate, and it takes predicates as its arguments. Vagueness may have a lot to do with borderline cases, but, ‘is vague’ is not tantamount to ‘is a borderline case’.

For some object  $a$ , it only makes sense to say that  $a$  is a borderline case of some predicate  $\Psi x$  if it is true that  $\Psi x$  is vague. But, it seems that the predicate ‘is vague’ is not a vague predicate. Suppose we considered the domain of all predicates. The set of all vague predicates is a proper subset of the set of predicates. Are there borderline cases of what counts as a member of the subset, i.e., the set of vague predicates? I do not think so. Here’s why.

Suppose we constructed a specification subspace for every predicate. We would then have to consider a domain for each predicate. Each individual in the domain of each nonvague

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<sup>39</sup> Fine (1975: 288).

predicate, when taken as an argument for its respective predicate, will come out either true-at-all-complete-specification-points or false-at-all-complete-specification-points. We will consider any predicate the subspace of which does not yield this result for every individual in its domain as being a vague predicate. There will not be any borderline cases of nonvague predicates, nor will there be any borderline cases of what counts as a vague predicate. Hence, there are no borderline cases of what counts as a vague predicate. Therefore, the predicates ‘is vague’ and ‘is nonvague’ are not vague predicates. From this it follows that it is false to say that most, if not all, vague predicates in natural language are higher-order vague.

Russell is correct that penumbral boundaries are not rigidly definable. But this just means that the boundaries of what counts as a borderline case are blurry. To say that penumbral boundaries are not rigidly definable is not to posit that there are orders of vagueness beyond first-order vagueness. But saying that penumbral boundaries are not rigidly definable does posit that there are orders of borderline cases beyond first-order borderline cases. Hence, even though it is false to say that most, if not all, vague predicates are higher-order vague, it is ok to say that most, if not all, borderline cases admit higher-order borderline cases. Let’s link this back to ordinary English.

Since penumbral boundaries are not rigidly definable, the boundaries of what counts as a borderline case are blurry. Hence, if a predicate admits borderline cases, it admits borderline borderline cases. Also, no matter how many times we iterate ‘borderline case’, the result is still going to be a borderline case. Even the most elaborately nested iterations of ‘borderline case’ do not eliminate vagueness since there is no limit to how diminutive the difference between an  $n$ th-order borderline case and an  $n+1$ th-order borderline case can be.

For example, a 100<sup>th</sup> order borderline case of ‘is tall’ might be only a millimeter shorter than a 101<sup>st</sup> order borderline case. But a 10,000<sup>th</sup>-order borderline case of ‘is tall’ might be only 1/1000<sup>th</sup> of a millimeter shorter than a 10,001<sup>st</sup>-order borderline case. But, orders of borderline cases are distinguished by theorists concerned with the analysis of vagueness and borderline cases. Speakers of ordinary English do not distinguish orders of borderline cases even if the penumbrae of the vague predicates they regularly utilize are not rigidly definable.

In ordinary English, vagueness is just vagueness. Similarly, borderline cases, whatever order the theorist claim them to be, are just considered to be borderline cases. In ordinary English, we do not discriminate a borderline tall man from a borderline borderline tall man from a borderline borderline borderline borderline, etc., tall man. We call each of these men a borderline case of tall and that's it. We do not iterate the notion 'borderline case'.

I argue that to distinguish between orders of vagueness is to draw a distinction within some kind of extra-ordinary language of analysis. Such distinctions are drawn only in well-defined languages of analysis, e.g., the language of Fine's supervaluationism. Higher-order vagueness, then, in the sense of second-order, third-order...  $n$ th-order vagueness, only occurs in analysis. Higher-order vagueness is not a characteristic feature of the vagueness of ordinary English; rather, it is an artifact of some analysis of the vagueness of ordinary English.

It is only when vagueness is analyzed that talk of third-order, twelfth-order, twentieth-order vagueness is even intelligible. In ordinary English, vagueness is not so carefully shaped. Vagueness pervades ordinary English. We often approximate in ordinary English, and we often use a vague term with some intended interpretation. Ordinary English would be quite cumbersome and would not be recognizable as the ordinary language we know if it were completely precise. To require that there be intelligible talk of third-order, twelfth-order, twentieth-order vagueness in ordinary English is to over-intellectualize ordinary English.

Ordinary English may be guilty of generalizing higher-orders of borderline cases by just calling them borderline cases. For example, even if something is a borderline case of a borderline case of a borderline case, we just ordinarily call it 'a borderline case'. Even if we do generalize in this way, it is no consequence of such generalizing that higher-order vagueness is a phenomenon of ordinary English.

Vagueness, as a phenomenon of ordinary English, does not have the property of having the potential for higher-order vagueness. To attribute such a property to the vagueness of ordinary English is to mischaracterize the phenomena of vagueness that occur in ordinary

English. Even though analyses of the vagueness of ordinary English are apt for the identification of higher orders of vagueness and borderline cases, the target of analysis should be restricted to vagueness as it occurs in ordinary English. In closing, I recommend that theorists of vagueness and higher-order vagueness observe the following warning: Analyses which target higher-order vagueness as if it were a property of the vagueness of ordinary English misidentify their *analysandum*. If we wish to analyze the vagueness of natural language, we must first get right just what the vagueness of natural language amounts to.

### Conclusion

The overall aim of Fine's project is to extend classical logic in order to develop an account of the logic and semantics of the vague predicates of ordinary language which accommodates distinctive characteristics of vagueness including penumbral connections and higher-order vagueness. In the second chapter, I argued that Fine's account of penumbral connections fails to discriminate between penumbral connections and pseudo penumbral connections, and, consequently, Fine's account fails to accurately identify penumbral connections in the salient way.

After reconsidering Fine's characterization of penumbral connections, I argued that Fine categorizes some kinds of formulae as exemplifying penumbral connections which do not have this characteristic. Such formulae and their instances receive uniform value assignments across interpretations, however, as I argued, penumbral connections are not merely instances of valid formulae. Fine's characterization of penumbral connections requires that there be logical relations between vague predicates. Logical relations, as I distinguished them, only hold between the logical meanings and arrangement of operators, etc. The logical properties of operators and other arranged symbols are not features of ordinary English predicates.

Here, I said that formulae that receive uniform value assignments across interpretations independently of considerations relating to analytic relations between vague predicates do not exhibit penumbral connections but pseudo penumbral connections. I also discussed why it is

significant that Fine mistakes instances of logically valid formulae as being sentences which can potentially exhibit penumbral connections and argued that Fine's analysis does not cut to the heart of what penumbral connections are because he misidentifies the *analysandum*. Because of the misidentification, his analysis targets too wide a range of phenomena, and, hence, is too weak of an account. Since Fine's supervaluationism treats phenomena which do not exemplify penumbral connections as though they did, it is not a sufficient account of penumbral connections. When we misidentify the phenomena of penumbral connections, we cease to be analyzing vagueness and its phenomena.

I took up higher-order vagueness in the third chapter and argued that the possibility of higher-order vagueness is not a distinct feature of the vagueness of natural language. After giving an overview of some characterizations of higher-order vagueness, I revisited Fine's accounts of the D operator and higher-order vagueness. In relation to Fine's account, I argued that higher-order vagueness is not a distinct feature of the vagueness of natural language, but, rather, it is an artifact resulting from the analysis of the vagueness of natural language.

In an effort to expose the nature of higher-order vagueness, I traced the source of higher-order vagueness back to the special notion of truth proposed by Fine's account, and I considered the source of vagueness in ordinary English in order to argue that higher orders of vagueness do not occur in ordinary English. I discussed the relation between the predicates 'is vague' and 'is a borderline case', among other things, in order to argue that higher-order vagueness is an artifact of analysis.

To attribute higher-order vagueness to the vagueness of ordinary English is to mischaracterize the phenomena of vagueness that occur in ordinary English. Even though analyses of the vagueness of ordinary English are apt for the identification of higher orders of vagueness and borderline cases, the target of analysis should be restricted to vagueness as it occurs in ordinary English. I closed the chapter by recommending that theorists of vagueness and higher-order vagueness observe the following warning: Analyses which target higher-order vagueness as if it were a property of the vagueness of ordinary English misidentify their *analysandum*.

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