Modeling the United States Unemployment Rate with the Preisach Model of Hysteresis

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(ABSTRACT)

A system with hysteresis is one that exhibits path dependent but rate independent memory. Hysteresis can be observed physically through the magnetization of a ferromagnetic material. In order to mathematically describe systems with hysteresis, we use the Preisach model. A discussion of the Preisach model is given as well as a method for computing the hysteretic transformation of an input variable. The focus of this paper is hysteresis in economics, namely, unemployment. We consider essential time series techniques for analyzing time series data, i.e. unit root testing for stationarity. However, we point out problems in modeling hysteresis with these techniques and argue that unit root tests cannot capture the selective memory of a system with hysteresis. For that, hysteresis in economic time series data is modeled using the Preisach model. We test the explanatory power of the previous unemployment rate on the current unemployment rate using both a hysteretic and non-hysteretic model. We find that the non-hysteretic model is better at explaining current unemployment rates, which suggests hysteresis is not present in the United States unemployment rate.

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Chapter 1

Introduction

Hysteresis was first named by James Alfred Ewing [9] in 1885 when he coined the term to describe the lag-behind effect of magnetization in ferromagnetic materials. Since then, mathematical modeling of hysteresis has caught the attention of researchers in disciplines such as psychology [13, 15], marketing [14], biology [20], physics [11, 25], and engineering [19]. Hysteresis occurs when a system exhibits path dependent but rate independent memory. In systems that exhibit hysteresis, it is impossible to determine output without considering the history of the input, which is referred to as the memory effect. Multiple models have been formulated to describe systems that exhibit hysteresis. In this paper, we use the Preisach model.

The focus of this paper is hysteresis in economics; in particular, unemployment time series data. First, we will discuss two primary methods of modeling hysteresis: 1) the nonlinear Preisach model and 2) the linear unit root test for stationarity. The latter method has been used extensively in modeling hysteresis; however, we argue the unit root test fails to capture the nonlinearity and selective memory in times series data. Since the Preisach model is a “true” model of hysteresis, it performs better in this situation. Nonetheless, determining the stationarity of time series data is important and will be discussed. Finally, a brief discussion of two unemployment theories, the natural rate hypothesis and the hysteresis hypothesis, will be given as well as real world examples using unemployment time series data from the United States and the United Kingdom.

This paper is organized as follows. In chapter 2, we describe the physical phenomenon of hysteresis in a ferromagnetic body. Subsequently, the Preisach model is defined in chapter 3 as well as a description of important components such as a hysteron, the Preisach plane, and the wiping-out property. A computer program to compute the output of the Preisach model is given along with an example. Chapter 4 develops important concepts in time series analysis. In particular, we discuss the stationarity of the autoregressive model. Lastly, chapter 5 will focus on hysteresis in economics. Real-word unemployment time series data is used to investigate hysteresis in unemployment.

Chapter 2

Ferromagnetism: The Prototype of Hysteresis

Since magnetism in ferromagnetic bodies (i.e. iron, nickel, cobalt) is the prototype of the physical phenomenon called hysteresis, let us examine what happens to a ferromagnetic body in an applied magnetic field. Consider a magnetizing force H (input) and the induced magnetic flux density B (output). Let $+H_m$ and $-H_m$ be extreme values for H. As H increases from 0 to $+H_m$, the measured flux density of an initially non-magnetized ferromagnetic material (B = 0) will follow the nonlinear dashed line trajectory to point a (see Figure 2.1).

At point a, the material has reached a saturation point, and B will increase linearly. However, reducing the magnetic force H to zero will not cause the flux density B to return to the origin (the initial starting point), instead it will return to point b. This indicates there is magnetic flux in the material even when the magnetic force is zero, and the left over magnetic flux is called remanence. By increasing the magnetic force in the negative direction, the material will again become saturated in the opposite direction and reach point d. Note that when the magnetizing force is reversed, the hysteresis loop crosses the H-axis (i.e. where the flux is reduced to zero), which represents the coercivity of the material (point c). That is, how much magnetic force must be applied to reduce the magnetic flux to zero. Returning the magnetic force to zero will cause the material to return to point e instead of the origin. Finally, the curve takes a different path to point a than it did initially.

We call this type of path-dependent relation hysteresis. Thus, the hysteresis loop represents the history dependent nature of magnetization in a ferromagnetic material, i.e. the state of the system at a given input. For example, the value of B when $H = 0$ depends on the history or "path" by which $H = 0$ was reached. This behavior is independent of the rate at which the path is traversed (at least over a very large range of rates).

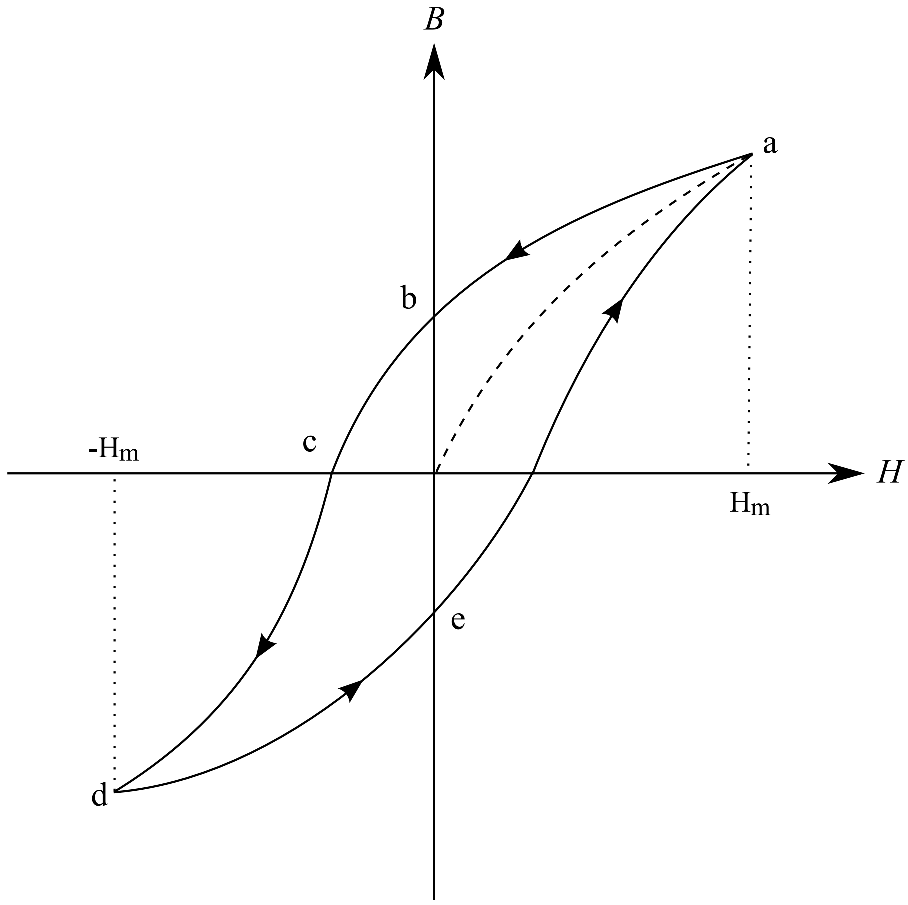

Figure 2.1: Magnetic hysteresis loop

Chapter 3

The Preisach Model

In an attempt to describe magnetization, F. Preisach [21] developed the Preisach model. The Preisach model was considered a physical model of hysteresis until two Russian mathematicians, Krasnoselskii and Pokrovskii, represented it in pure mathematical terms [16]. Since then, researchers have found that magnetics is not the only area where the model is applicable, which has led to the Preisach model being treated as an empirical mathematical model that provides a good fit to many systems with hysteresis. In this chapter, we will define the Preisach model and discuss properties associated with it. A method for applying the model using time series data is presented, and an example is given for illustration purposes.

3.1 Definition

The basic building block of the Preisach model is an operator $\hat{\gamma}_{\alpha,\beta}$ called a hysteron or relay. The hysteron $\hat{\gamma}_{\alpha,\beta}$ can take a value of either +1 or 0. Consider an input $u(t)$ and threshold values $\beta \geq \alpha$ such that if the input $u(t) \geq \beta$, then the hysteron takes on the value +1. Likewise, if the input $u(t) \leq \alpha$, then the hysteron takes on the value 0. We say that β is the "on" switch, and α is the "off" switch.

Each hysteron $\hat{\gamma}_{\alpha,\beta}$ can be represented on a switching points graph (see Figure 3.1). The trajectory a $\rightarrow$ b $\rightarrow$ c $\rightarrow$ d $\rightarrow$ e is followed as the input monotonically increases, and the trajectory e $\rightarrow$ d $\rightarrow$ f $\rightarrow$ b $\rightarrow$ a is followed as the input monotonically decreases. Notice, on the interval $[\alpha,\beta]$, the previously acquired position will determine whether or not $\hat{\gamma}_{\alpha,\beta}$ is "on" or "off." That is, if the input is approaching from below β, then $\hat{\gamma}_{\alpha,\beta}$ is in the "off" position, and if the input approaching from above α, then $\hat{\gamma}_{\alpha,\beta}$ is in the "on" position.

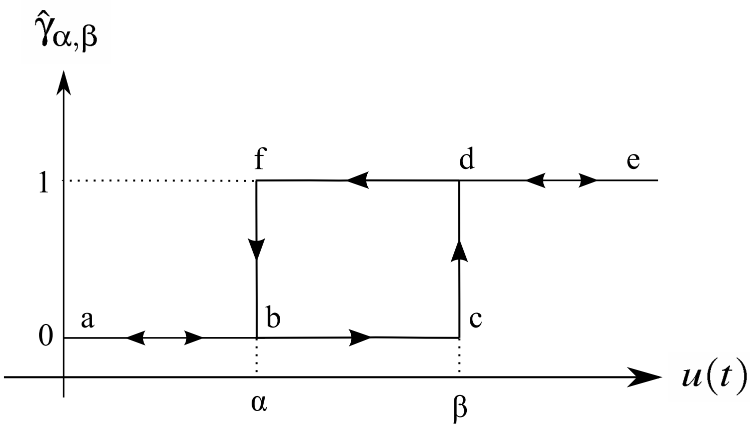

Figure 3.1: A single hysteron

Mathematically, the output of a hysteron $\hat{\gamma}_{\alpha,\beta}$ is expressed as

$$\hat{\gamma}_{\alpha,\beta} = \begin{cases} 1 & \text{if } u(t) \geq \beta \\ 0 & \text{if } u(t) \leq \alpha \\ k & \text{if } u(t) \in (\alpha,\beta) \end{cases}$$

where

$$k = \begin{cases} 1 & \text{if } u(t*) > \beta \\ 0 & \text{if } u(t*) < \alpha \end{cases}$$

for the final time t*, such that $u(t*) \notin (\alpha,\beta)$.

Now, consider an input $u(t)$, an output $v(t)$, a finite set of hysterons $\hat{\gamma}_{\alpha,\beta}$, and a weight function $\mu(\alpha,\beta)$, which is also referred to as the Preisach function. The output of the Preisach model represents a linear combination of the hysterons. Figure 3.2 shows a graphical representation of how a finite set of hysterons are used to approximate the continuous hysteresis loop.

Mathematically, the Preisach model with a finite collection of hysterons is written as

$$v(t) = \sum_{i=1}^{N} \mu(\alpha_i,\beta_i)\hat{\gamma}_{\alpha_i,\beta_i}(u(t)). \tag{3.1}$$

Figure 3.3 shows a graphical representation of the discrete Preisach model.

The literature also examines the Preisach model composed of continuously distributed infinite collections of hysterons described by

$$v(t) = \iint_{\beta \geq \alpha} \mu(\alpha,\beta)\hat{\gamma}_{\alpha,\beta}(u(t))\, d\alpha\, d\beta \tag{3.2}$$

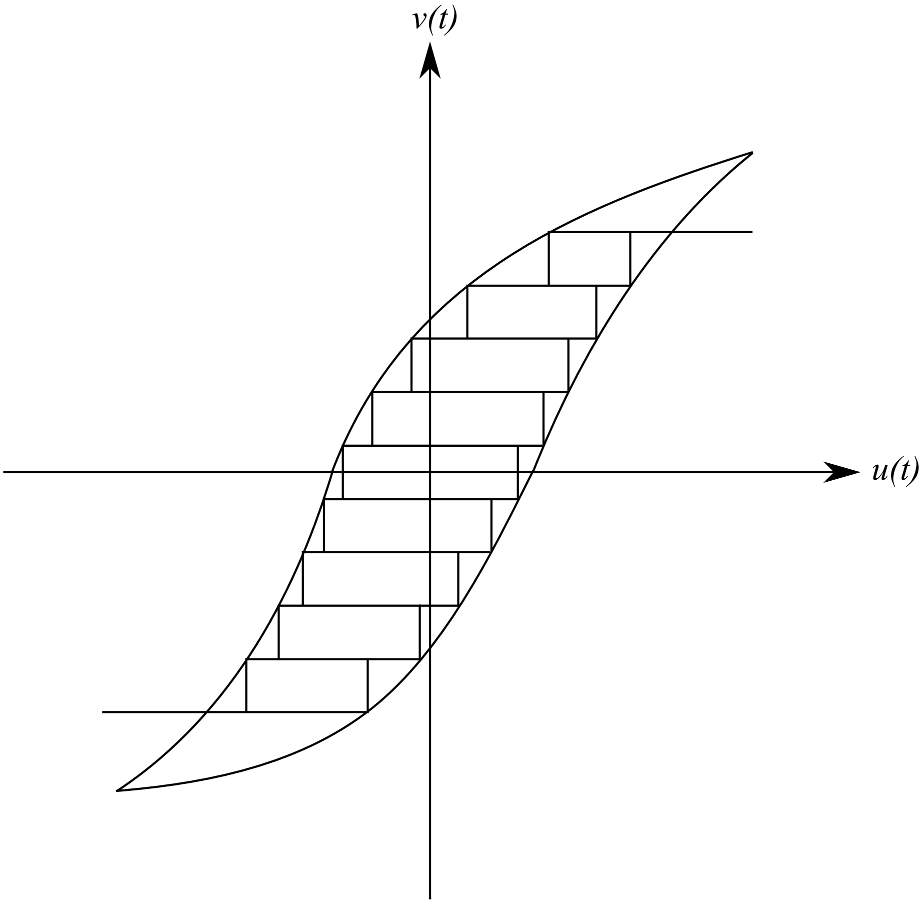

Figure 3.2: Approximation of a continuous hysteresis loop

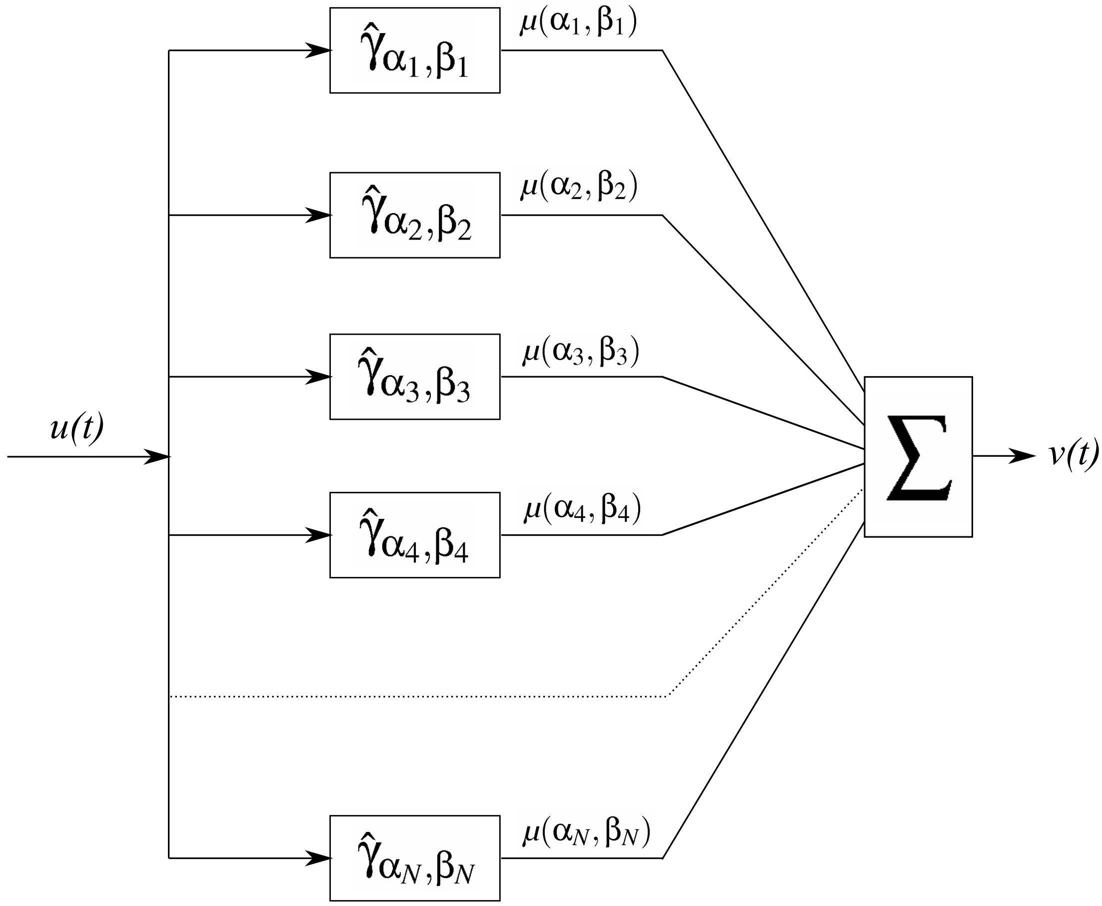

Figure 3.3: Graphical representation of the discrete Preisach model

3.2 The Preisach Plane

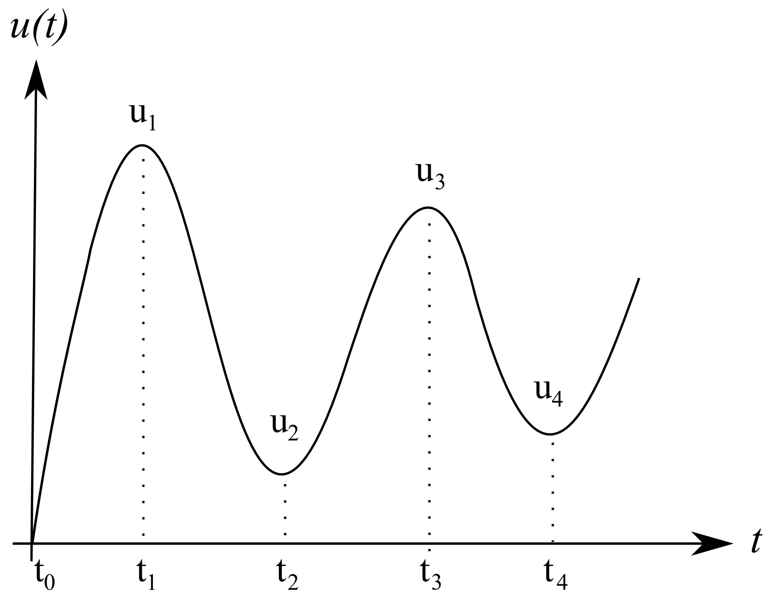

Figure 3.4: Input to Preisach model example

Having developed the idea of a hysteron, it is useful to have a method of keeping track of the "on" or "off" position of each hysteron. For this, the so-called Preisach plane is used. Since $\beta \geq \alpha$, the domain is a right triangle in the α-β plane. Each point in this plane represents an individual hysteron. This geometric interpretation is possible since a bijection exists between the hysterons $\hat{\gamma}_{\alpha,\beta}$ and the points (α,β) [17].

The simple hypothetical input shown in Figure 3.4 will be used to graph the Preisach plane and describe the evolving state of the hysterons represented by points in the plane (see Figure 3.5). Initially, assume every hysteron in the triangle is in the "off" position. Thus, for any (α,β) point in the triangle, the hysterons equal 0. Starting with t_0, the input is monotonically increased to a value u_1 at some time t_1. This creates a partition between the hysterons in the "on" position and the "off" position. We will call this partition $L(t)$ (see Figure 3.5a). Every point (α,β) below $L(t)$ turns "on" while the points above $L(t)$ remain "off".

Consider what happens when the input is monotonically decreased to a value u_2 at some time t_2. A vertical line is moved from right to left in the triangle until the value $\alpha = u_2$ is reached. The line $L(t)$ now has a new shape, and the boundaries for the "on" and "off" positions have changed. The points to the right of the vertical line have been turn "off" while the points to the left remain in the previously acquired position, i.e. the positions are preserved. (see Figure 3.5b).

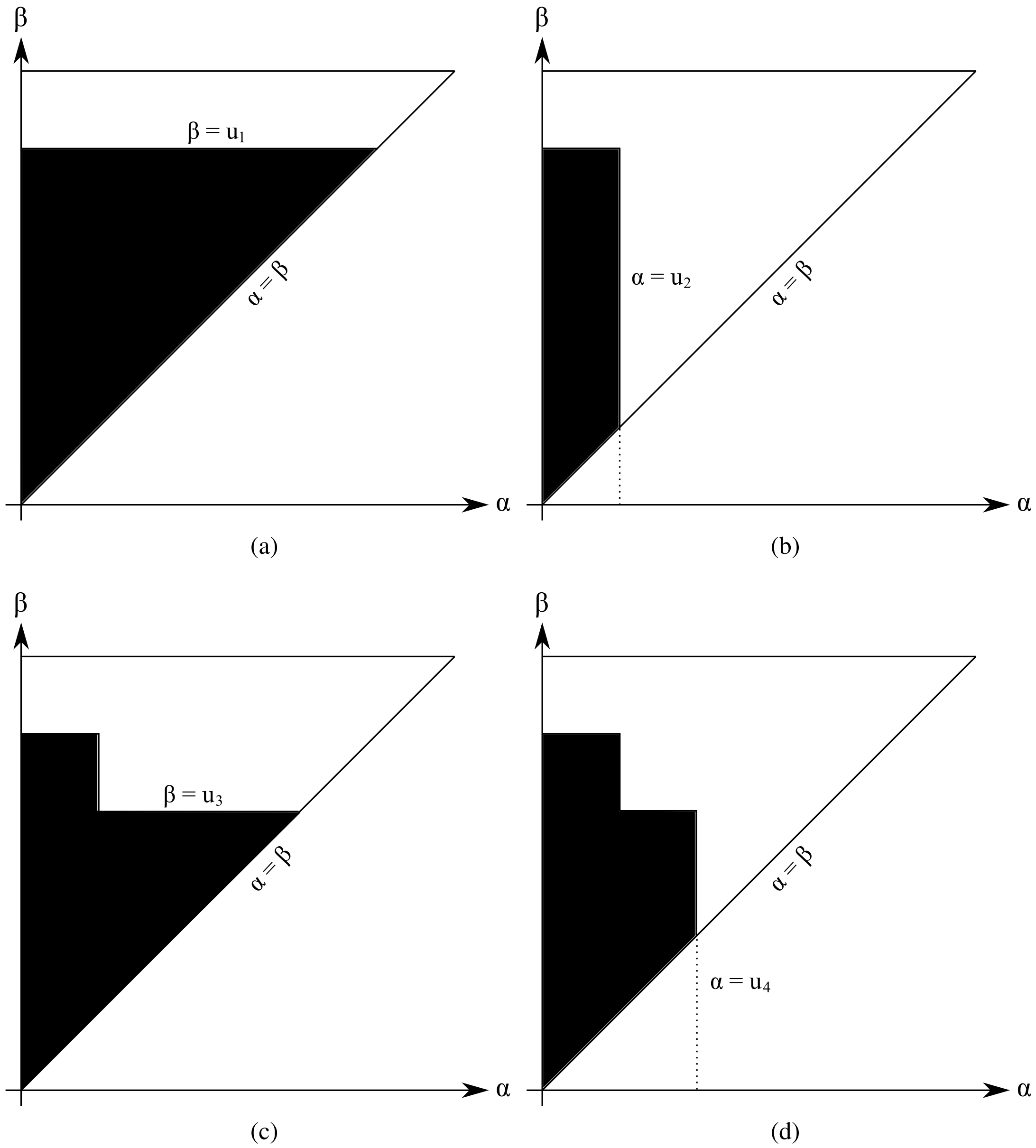

Figure 3.5: Preisach plane

The input is then monotonically increased to a value u_3 at some time t_3. Note the input $u_3 < u_1$. The process is the same as before. The horizontal line is increased until $\beta = u_3$ is reached. Every point below the line that was in the "off" position is now turned "on" and every point that was already "on" is left "on." (see Figure 3.5c). Finally, the input is monotonically decreased to a value of u_4 at some time t_4 where $u_4 > u_2$. Like before, the vertical like moves right to left until the value $\alpha = u_4$ is reached. The points to the right of the vertical line are turned "off" and every point to the left is preserved. (see Figure 3.5d).

Hysteresis loops formed by a uniform $\mu(\alpha, \beta)$ are graphed in the input-output plane. Figure 3.6 shows the input-output graph for the previous input data. The output is the shaded area in the Preisach plane.

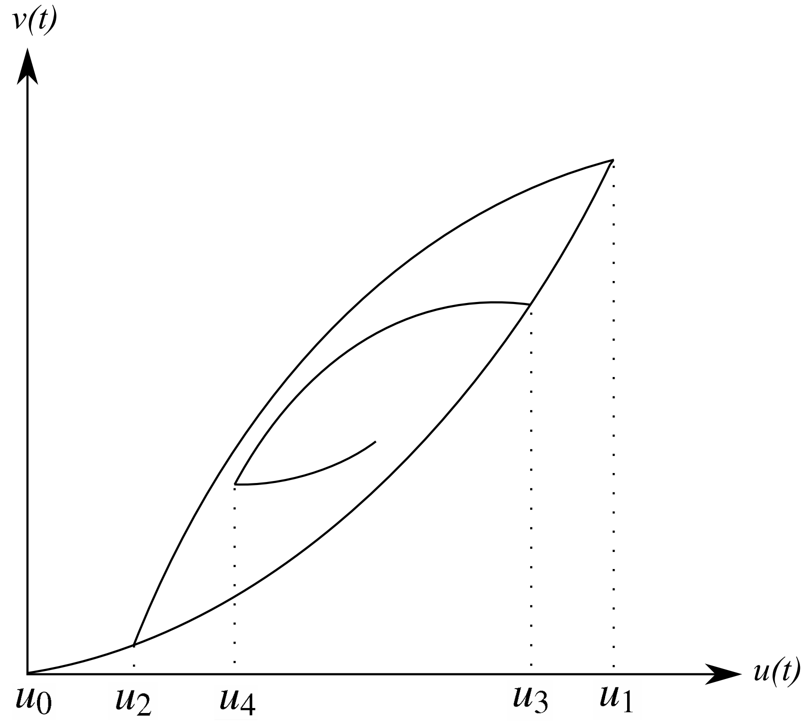

Figure 3.6: Graphical representation of the discrete Preisach model

3.3 The Wiping-Out Property

The Preisach model has several properties which are discussed in Mayergoyz [17]. One useful property for us to consider is the Wiping-Out Property proved by Mayergoyz [17, p.13], which will be necessary to understand nondominated extrema and selective memory later on.

Theorem 1. *(Wiping-Out Property) Each local input maximum wipes out the vertices of L(t) whose* β*-coordinates are below this maximum, and each local minimum wipes out the vertices whose* α*-coordinates are above this minimum.*

Consider the situation depicted in Figure 3.7a. The wiping out property states that by sweeping to points above vertices that are already in memory, it causes those points to be wiped out. Figure 3.7b illustrates this property. When sweeping from a → b, the previous vertices on $L(t)$ are wiped out, and a new $L(t)$ is formed. However, we do not wipe out vertices above a, and point c is still the nondominated maximum.

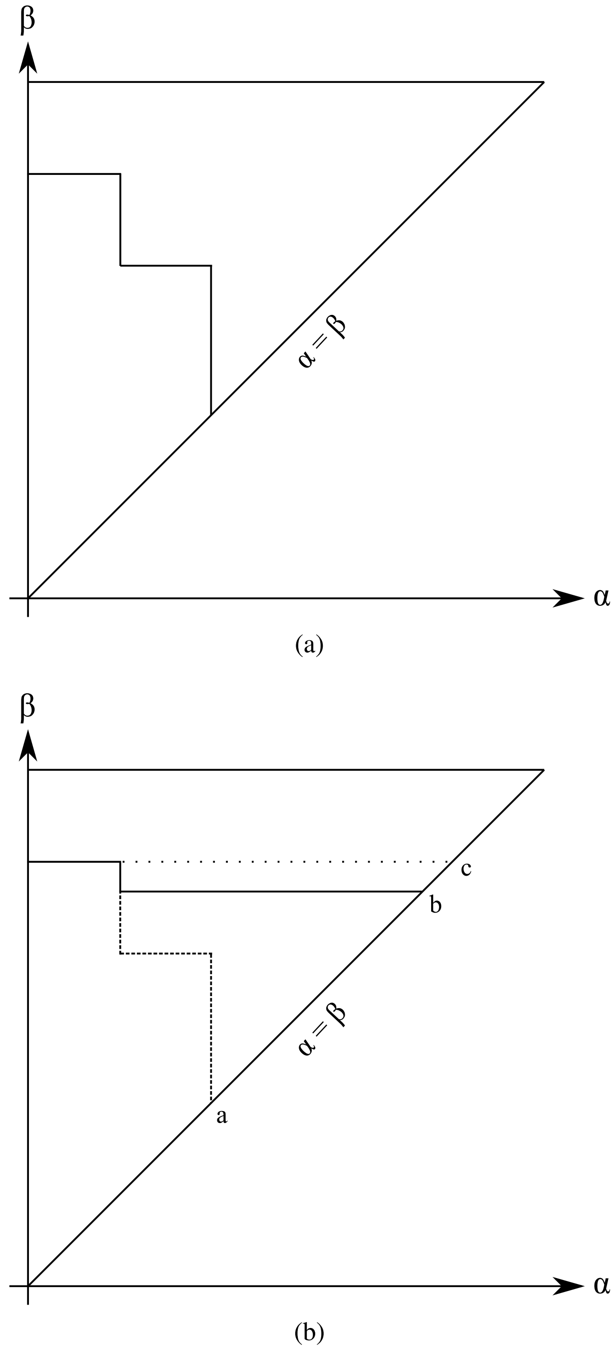

Figure 3.7: Wiping-Out Property

3.4 Computing the Hysteretic Transformation

Darby, Cross & Piscitelli [8] developed a computer program to compute the hysteretic transformation (see Appendix A); that is, the output $v(t)$ in equation 3.1. In developing the program, the authors assume a uniform distribution, i.e. $\mu(\alpha,\beta) = 1$. They perform sensitivity analysis with economic time series data and determine the hysteresis operators are insensitive to the specification of the weight function. Therefore, we also assume $\mu(\alpha,\beta) = 1$.

The hysteretic transformation is calculated by 1) determining the dimensions of the right triangle, 2) determining the nondominated extrema, and 3) calculating the area of the triangles.

Step 1: Determining the dimensions of the right triangle:

$$\alpha_0 = \min\{u(t), t = 1, 2, ..., N\}$$

$$\beta_0 = \max\{u(t), t = 1, 2, ..., N\}$$

Step 2: Determining the nondominated extrema:

Let M(i, t) be a matrix of nondominated maxima and m(i, t) be a matrix of nondominated minima where i = 1, 2, ... and t = 1, 2, ..., T_{max}. Assume $t_0 = 1$. A maximum or minimum is nondominated if it remains in memory, i.e. not wiped out.

The first nondominated maximum is

$$M(1,t) = \max_{j\in[1,\,t]} u(j) = u(t_1^+).$$

The first nondominated minimum is

$$m(1,t) = \min_{j\in[t_1^+,\,t]} u(j) = u(t_1^-).$$

The second nondominated maximum is

$$M(2,t) = \max_{j\in[t_1^-,\,t]} u(j) = u(t_2^+).$$

The second nondominated minimum is

$$m(2,t) = \min_{j\in[t_2^+,\,t]} u(j) = u(t_2^-).$$

In general,

$$M(k,t) = \max_{j \in [t_{k-1}^-,\, t]} u(j) = u(t_k^+),$$

and

$$m(k,t) = \min_{j \in [t_k^+,\, t]} u(j) = u(t_k^-).$$

Step 3: Calculating the area of the triangles:

Define the area to be T(t), and let $T_p(t)$ be the previous value of T(t). First, consider the situation in Figure 3.8 where the shaded part has already been calculated, i.e. T_p.

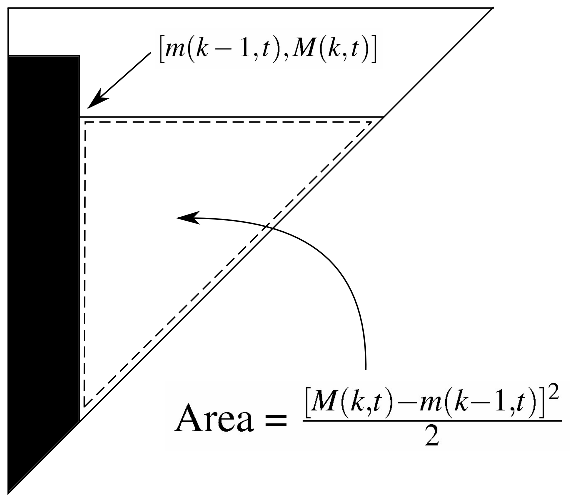

Figure 3.8: Computing the area: Part 1

The area of the triangle with vertex $[m(k-1,t), M(k,t)]$ is computed as follows

$$\int_{m(k-1,t)}^{M(k,t)} \int_{\alpha}^{M(k,t)} 1 \; d\beta d\alpha = \frac{[M(k,t) - m(k-1,t)]^2}{2}. \tag{3.3}$$

Thus, to obtain the area T(t) we compute the following

$$T(t) = T_p(t) + \frac{[M(k,t) - m(k-1,t)]^2}{2}. \tag{3.4}$$

The area of the shaded portion of the Preisach triangle in Figure 3.9 has been calculated.

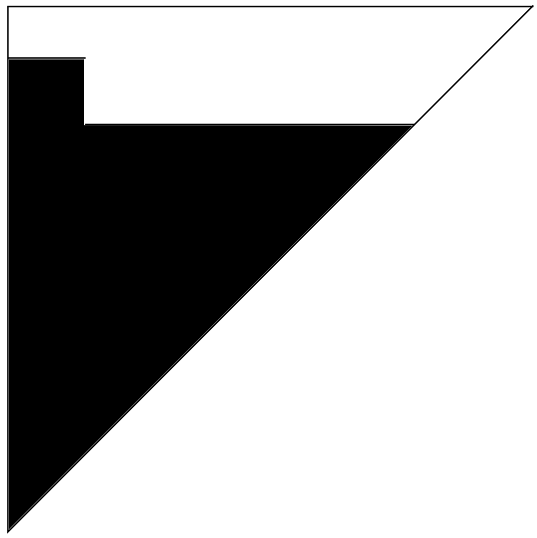

Figure 3.9: The computed area

Next, the area of the triangle with vertex $[m(k,t), M(k,t)]$ is subtracted from the value computed in the previous equation by computing

$$T(t) = T_p(t) - \frac{[M(k,t) - m(k,t)]^2}{2}. \tag{3.5}$$

Figure 3.10 shows the area computed by the above two processes. The same procedure is continued until the area has been computed.

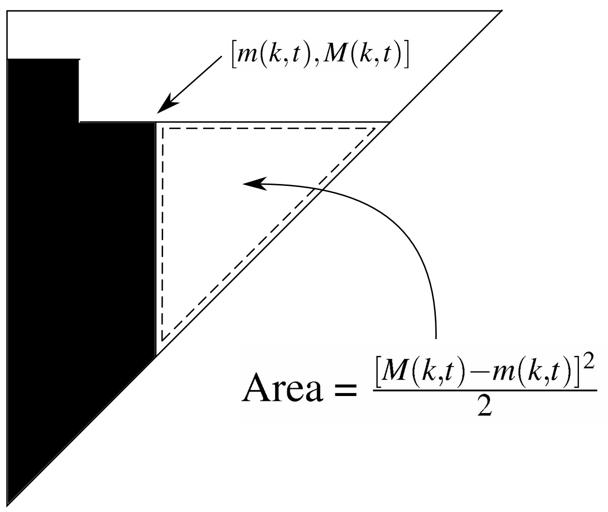

Figure 3.10: Computing the area: Part 2

3.5 Example

Consider a periodic function:

$$u(t) = \frac{sin(t)}{\sqrt{t+1}} + 1 \tag{3.6}$$

where $t \in [0, 12]$.

Figure 3.11 shows the hysteresis loops in the input-output graph.

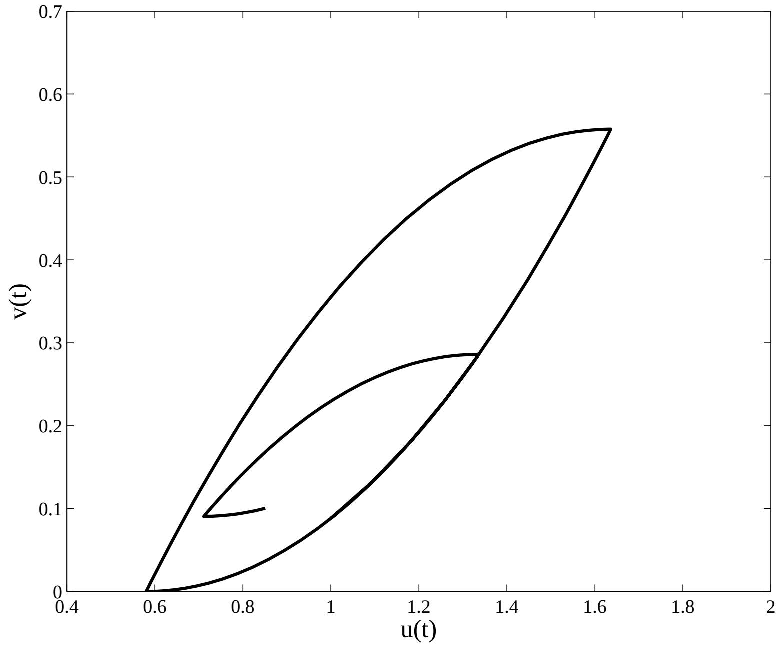

Figure 3.11: Input-output graph for example

Figure 3.12a shows the graph of the function, and Figure 3.12b shows the output produced by the hysteresis variable program. Notice the lag behind property in the output.

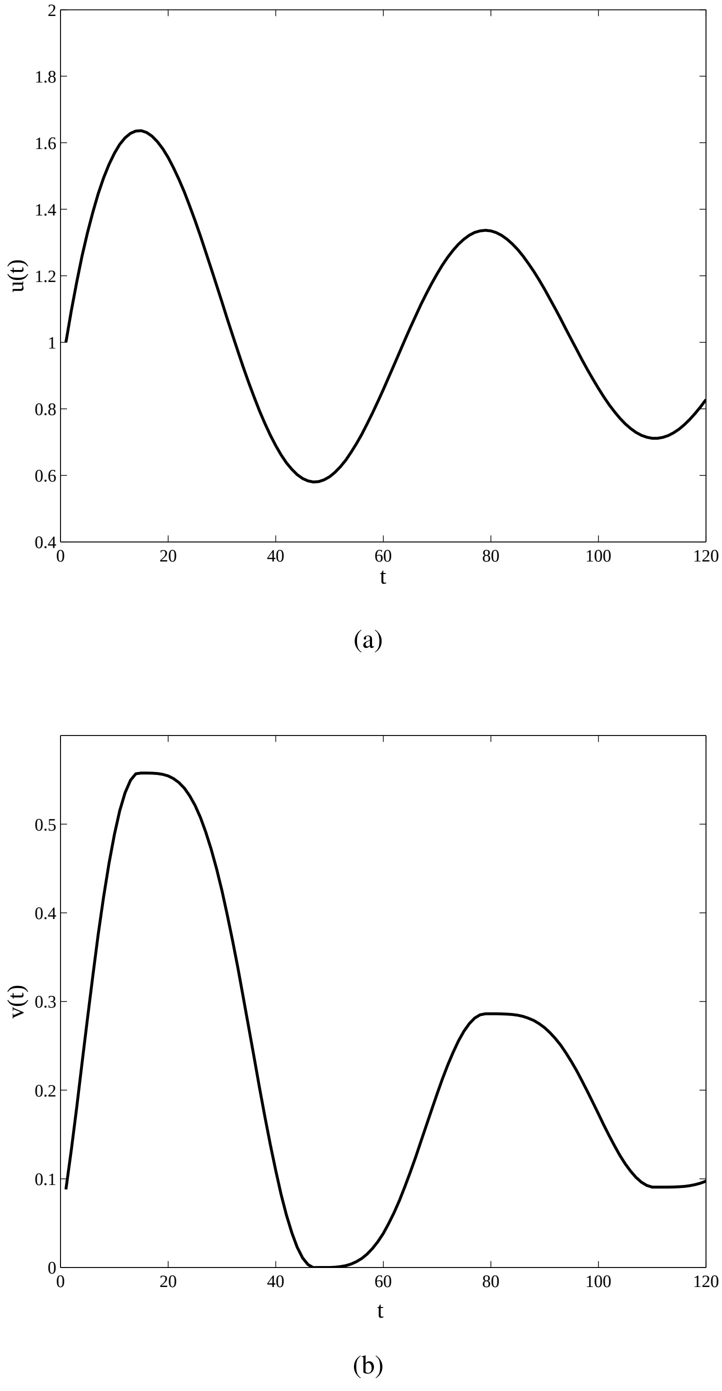

Figure 3.12: Graphs of the input and output for the example

Chapter 4

Econometric Methods

In this chapter, we will describe the autoregressive model. An autoregressive model of order p, denoted AR(p) for the time series x_t, is defined as

$$x_t = \sum_{i=1}^{p} \varphi_i x_{t-i} + \varepsilon_t \tag{4.1}$$

where φ_i are parameters and ε_t is an error term. The error term ε_t has the following assumptions:

1) zero mean,
2) variance σ^2,
3) independent and identically distributed (i.i.d.),
4) uncorrelated for different time values.

The independent variables in this linear regression are prior values of the time series, x_{t-i} for i = 1,2,...,p, and the dependent variable is the value at the present time x_t.

This linear regression equation takes the history of the time series x_t into account, i.e. x_{t-i}. This concept is like the Preisach model introduced in the last chapter, i.e. to understand the value of x_t, one has to take the previous value x_{t-1} into account. However, the lag behind property of the hysteresis transformation should not be confused with a time lag in the autoregressive model. In other words, the "lag" in the hysteretic transformation from the Preisach model is not necessarily a time lag. Many authors use autoregressive modeling to model hysteresis. The major difference between the two approaches is the autoregressive model is linear while the Preisach model is nonlinear. A discussion of the problems that arise in modeling hysteresis with the autoregressive model is given in chapter 5.

The following section will discuss a method for estimating the parameters φ_i , which is called Ordinary Least Squares. This will be followed by a review of basic probability theory leading up to the definition of a stationary time series. Finally, we will define a nonstationary unit root process and describe how to test time series data for a unit root.

4.1 Ordinary Least Squares

Ordinary Least Squares is a standard method for estimating the parameters of a linear regression model. Consider the following AR(1) process (an intercept γ, called a drift, is included for completeness)

$$x_t = \gamma + \varphi x_{t-1} + \varepsilon_t. \tag{4.2}$$

Using time series data $\{(x_{t-1}, x_t); t = 1, 2, ..., n\}$, we can estimate the parameters γ and φ using a technique called Ordinary Least Squares (OLS). The basic idea behind OLS is to fit a line through the sample points (x_{t-1}, x_t) that minimizes the sum of squared residuals (ε_t). In other words, we want to minimize $\sum_{t=1}^{n} (\hat{\varepsilon}_t)^2 = \sum_{t=1}^{n} (x_t - \hat{\gamma} - \hat{\varphi} x_{t-1})^2$. Taking the partial derivatives, we obtain:

$$\begin{aligned} 1) \quad \frac{\partial \hat{\varepsilon}_t^2}{\partial \gamma} &= -2\sum_{t=1}^{n} (x_t - \hat{\gamma} - \hat{\varphi} x_{t-1}), \\ 2) \quad \frac{\partial \hat{\varepsilon}_t^2}{\partial \varphi} &= -2\sum_{t=1}^{n} (x_t - \hat{\gamma} - \hat{\varphi} x_{t-1}) x_{t-1}. \end{aligned}$$

Minimization is obtained by setting the derivatives equal to zero. It follows that

$$\sum_{t=1}^{n} (x_t - \hat{\gamma} - \hat{\varphi} x_{t-1}) = 0 \implies \sum_{t=1}^{n} x_t - \hat{\gamma} n - \hat{\varphi} \sum_{t=1}^{n} x_{t-1} = 0$$

$$\implies \frac{\sum_{t=1}^{n} x_t}{n} - \hat{\varphi} \frac{\sum_{t=1}^{n} x_{t-1}}{n} = \hat{\gamma}.$$

Therefore,

$$\hat{\gamma} = \bar{x}_t - \hat{\varphi} \bar{x}_{t-1} \tag{4.3}$$

Similarly,

$$\sum_{t=1}^{n} (x_t - \hat{\gamma} - \hat{\varphi} x_{t-1}) x_{t-1} = 0 \implies \sum_{t=1}^{n} (x_t - (\bar{x}_t - \hat{\varphi} \bar{x}_{t-1}) - \hat{\varphi} x_{t-1}) x_{t-1} = 0$$

$$\implies \sum_{t=1}^{n} x_{t-1}(x_t - \bar{x}_t) - \hat{\varphi} \sum_{t=1}^{n} x_{t-1}(x_{t-1} - \bar{x}_{t-1}) = 0$$

$$\implies \sum_{t=1}^{n} (x_{t-1} - \bar{x}_{t-1})(x_t - \bar{x}_t) = \hat{\varphi} \sum_{t=1}^{n} (x_{t-1} - \bar{x}_{t-1})^2 = 0.$$

Therefore,

$$\hat{\varphi} = \frac{\sum_{t=1}^{n}(x_{t-1}-\bar{x}_{t-1})(x_t-\bar{x}_t)}{\sum_{t=1}^{n}(x_{t-1}-\bar{x}_{t-1})^2} \tag{4.4}$$

Here $\hat{\varphi}$ and $\hat{\gamma}$ are called the OLS estimators. A standard way to test for statistical significance of the OLS estimators is using the t-test, which assumes x_t is normally distributed. The following hypothesis test is performed

$$\begin{aligned} H_0 &: \varphi = 0, \\ H_a &: \varphi \neq 0, \end{aligned} \tag{4.5}$$

using the test statistic

$$t = \frac{\hat{\varphi}}{se(\hat{\varphi})} \tag{4.6}$$

where the se($\hat{\varphi}$) denotes the standard error of $\hat{\varphi}$.

4.2 Probability Theory

In order to describe the properties of the autoregressive model, we need some basic definitions.

Definition 1. *A **probability space** (Ω, A, P) is defined by the sample space Ω, event space $A \subset \Omega$, and the probability distribution $P : A \to \mathbb{R}$. The probability distribution satisfies*

1) $P(a) > 0, \quad \forall a \in A$,
2) $P(\Omega) = 1$,
3) $P(a \bigcup b) = P(a) + P(b)$ if $a \bigcap b = \emptyset \quad, \forall a, b \in A$.

Definition 2. *A **random variable** X is a function mapping a sample space $\Omega \to \mathbb{R}$.*

Definition 3. *A **cumulative distribution function** of a random variable X is defined by*

$$F(x) = P(\omega \in \Omega : X(\omega) \leq x) \tag{4.7}$$

where F is continuous when approached from the right, monotonically increasing, and $\lim_{x\to-\infty} F(x) = 0$, $\lim_{x\to+\infty} F(x) = 1$.

Definition 4. *In time series, a **stochastic process** is a time indexed sequence of random variables. Consider a probability space (Ω, A, P) and indexed set T. A time series $X(t, \omega)$ is defined on $T \times \Omega$ where $X(t, \omega)$ is a random variable for each fixed t.*

Definition 5. *A* ***joint distribution function*** *of a finite set of random variables* $\{X_{t_1}, X_{t_2}, ..., X_{t_n}\}$ *from the collection* $\{X(t, \omega) : t \in T\}$ *is defined by*

$$P\{\omega : X(t_1, \omega) \leq x_{t_1}, ..., X(t_n, \omega) \leq x_{t_n}\}. \tag{4.8}$$

Definition 6. *(Population Mean) The* ***expected value*** *of a discrete random variable X is defined as*

$$E(X) = \mu = \sum [xP(X = x)].$$

Lemma 1. *(Linearity) The following* ***linearity*** *properties hold for expected value*

1) *E(aX) = aE(X),*
2) *E(X + b) = E(X) + b,*
3) *E(X+Y) = E(X) + E(Y),*

where X, Y are discrete random variables and a, b are constants.

Fuller [10, pg. 31] proves the following lemma.

Lemma 2. *Let* $\{X_j\}$ *be a sequence of random variables defined on the probability space* (Ω, A, P)*. Assume*

$$\sum_{j=-\infty}^{\infty} E\{|X_j|\} < \infty.$$

Then $\sum_{j=-\infty}^{\infty} X_j$ *is defined as an almost sure limit and*

$$E\left\{\sum_{j=-\infty}^{\infty} X_j\right\} = \sum_{j=-\infty}^{\infty} E\{X_j\}. \tag{4.9}$$

Definition 7. *(Population Variance) The* ***variance*** *of a discrete random variable X is defined as*

$$Var(X) = \sigma^2 = E[(X - E(X))^2].$$

Definition 8. *(Covariance) The* ***covariance*** *between two discrete random variables X and Y is defined as*

$$Cov(X,Y) = E[(X - E(X))(Y - E(Y))].$$

Note: Cov(X,X) = Var(X).

4.3 Stationarity

Definition 9. *(Stationary Stochastic Process) A stochastic process* $\{x_{t_i} : i = 1, 2, ...\}$ *is* ***stationary*** *if for all* $1 \leq t_1 < t_2 < ... < t_n$*, the joint distribution of* $(x_{t_1}, x_{t_2}, ..., x_{t_n})$ *is equal to the joint distribution of* $(x_{t_1+h}, x_{t_2+h}, ..., x_{t_n+h})$ *for all* $h \geq 1$.

This definition is often hard to apply for probabilistic time series because the cumulative distribution is not known. For that, we define a weaker form of stationarity called weakly stationary or covariance stationary. The use of the term stationary from this point on will be used to describe a weakly stationary process.

Definition 10. *A stochastic process* $\{x_t : t = 1, 2, ...\}$ *is* ***covariance stationary*** *or weakly stationary if*

1) $E\{x_t\}$ *is constant,*
2) *Var*(x_t) *is constant,*
3) *Cov*(x_t, x_{t+h}) *does not depend on time for all t.*

Fuller [10, pg. 59] proves the following theorem regarding stationarity.

Theorem 2. *Let* $\{x_t\}$ *be a time series defined on the integers with* $E\{x_t^2\} < K$ *for all t. Suppose* $\{x_t\}$ *satisfies*

$$x_t - \sum_{j=1}^{p} \varphi_j x_{t-j} = \varepsilon_t, \qquad t = 0, 1, 2, ...,.$$

Here $\{\varepsilon_t\}$, $t = 0, 1, 2, ...,$ *is a sequence of uncorrelated (0,* σ^2*) random variables. Let* $\lambda_1, \lambda_2, ..., \lambda_p$ *be the roots of*

$$\lambda^p - \sum_{j=1}^{p} \varphi_j \lambda^{p-j} = 0. \tag{4.10}$$

and suppose $|\lambda_i| < 1$, *i = 1, 2, ..., p. Then* x_t *is covariance stationary.*

Definition 11. *A* ***nonstationary process*** *is a stochastic process that is not stationary.*

For illustration of this, consider the following AR(1) model

$$x_t = \varphi x_{t-1} + \varepsilon_t, \tag{4.11}$$

which is a first order, constant-coefficient, linear finite difference equation.

Note that the solution to the homogeneous difference equation (i.e. $\varepsilon_t = 0$) is

$$x_t = \varphi^t x_0$$

where x_0 is the initial value.

Using repeated substitution, the solution to the inhomogeneous difference equation is

$$
\begin{aligned}
x_1 &= \varphi x_0 + \varepsilon_1 \\
x_2 &= \varphi x_1 + \varepsilon_2 \\
&= \varphi(\varphi x_0 + \varepsilon_1) + \varepsilon_2 \\
&= \varphi^2 x_0 + \varphi\varepsilon_1 + \varepsilon_2 \\
x_3 &= \varphi x_2 + \varepsilon_3 \\
&= \varphi(\varphi^2 x_0 + \varphi\varepsilon_1 + \varepsilon_2) + \varepsilon_3 \\
&= \varphi^3 x_0 + (\varphi^2\varepsilon_1 + \varphi\varepsilon_2 + \varepsilon_3) \\
&\cdot \\
&\cdot \\
x_t &= \varphi^t x_0 + \sum_{j=0}^{t-1} \varphi^j \varepsilon_{t-j}.
\end{aligned}
$$

Consider the following cases:

Case 1: $|\varphi| < 1$.

Since $\lim_{t\to\infty} \varphi^t = 0$, we have

$$\lim_{t\to\infty} x_t = \sum_{i=1}^{\infty} \varphi^i \varepsilon_{t-i}.$$

Proposition 1. *x_t is stationary.*

Proof.

(i) $E(x_t) = E\left(\sum_{i=0}^{\infty} \varphi^i \varepsilon_{t-i}\right) = E(\varepsilon_t) + \varphi E(\varepsilon_{t-1}) + \varphi^2 E(\varepsilon_{t-2}) + \ldots = 0.$

(ii) $\text{Var}(x_t) = \text{Var}\left(\sum_{i=0}^{\infty} \varphi^i \varepsilon_{t-i}\right) = \text{Var}(\varepsilon_t) + \varphi\ \text{Var}(\varepsilon_{t-1}) + \varphi^2\ \text{Var}(\varepsilon_{t-2}) + \ldots$

$$= \sigma^2(1 + \varphi + \varphi^2 + \ldots) = \frac{\sigma^2}{1-\varphi}.$$

using the geometric series $\sum_{i=0}^{\infty} r^i = \frac{1}{1-r}$, and the i.i.d. assumption for $\{\varepsilon_t\}$.

(iii) $\text{Cov}(x_t, x_{t+h}) = E(x_t x_{t+h}) = E\left\{\left(\sum_{i=0}^{\infty} \varphi^i \varepsilon_{t-i}\right)\left(\sum_{j=0}^{\infty} \varphi^j \varepsilon_{t+h-j}\right)\right\}$

$$= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \varphi^i \varphi^j E(\varepsilon_{t-i}\varepsilon_{t+h-j}) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \varphi^i \varphi^j \gamma(h-j+i)$$

$$= \sigma^2 \sum_{i=0}^{\infty} \varphi^i \varphi^{i+h} = \sigma^2 \varphi^h \sum_{i=0}^{\infty} (\varphi^2)^i = \frac{\sigma^2 \varphi^h}{1-\varphi^2}.$$

where $\text{Cov}(\varepsilon_{t-i}, \varepsilon_{t+h-j}) = E(\varepsilon_{t-i}\varepsilon_{t+h-j}) = \gamma(h-j+i) = \begin{cases} \sigma^2 & h-j+i=0 \\ 0 & \text{otherwise} \end{cases}$

using the property Cov(X,X) = Var(X) = σ^2 and $E(\varepsilon_t \varepsilon_{t*}) = 0$ for $t \neq t*$.

Furthermore, if $\gamma(h-j+i) = \sigma^2$, then h - j + i =0 $\implies$ j = i + h.
Therefore, the conditions for stationary have been satisfied.

□

Case 2: $|\varphi| = 1$.

$$x_t = x_0 + \sum_{j=0}^{t-1} \varepsilon_{t-j}$$

This is nonstationary since the variance depends on time t. Using the i.i.d. assumption for $\{\varepsilon_t\}$,

$$\begin{aligned} \text{Var}(x_t) &= \text{Var}(x_0) + \text{Var}(\varepsilon_1) + \text{Var}(\varepsilon_2) + \ldots + \text{Var}(\varepsilon_t) \\ &= 0 + \sigma^2 + \sigma^2 + \ldots + \sigma^2 \\ &= t\sigma^2 \end{aligned}$$

Note: when $|\varphi| > 1$, $\lim_{t\to\infty} \varphi^t = \infty$ so x_t diverges, which is termed an explosive process in the econometric literature. This process is very rare in time series data and is not considered.

4.4 Unit Roots

From the previous discussion, we know the following AR(1) process is a nonstationary

$$x_t = x_{t-1} + \varepsilon_t \tag{4.12}$$

This process is called a random walk.

"Differencing" equation (4.12), we obtain

$$x_t - x_{t-1} = \Delta x_t = \varepsilon_t,$$

which is stationary since $E(\varepsilon_t) = 0$, Var(ε_t) $= E(\varepsilon_t^2) = \sigma^2$, and $E(\varepsilon_t \varepsilon_{t_*}) = 0$ for $t \neq t_*$.

Definition 12. *The **order of integration**, written I(d), is defined as the number of differences, d, required to make a stationary process. A stationary process is integrated of order zero, I(0).*

Thus, the first difference of the random walk process is stationary, which means it is integrated of order 1, or I(1). We call this a unit root process, which is nonstationary primarily because there is no fixed (equilibrium) level of the variable [23].

As an example, consider the following equation AR(2) process

$$x_t - 1.5x_{t-1} + .5x_{t-2} = \varepsilon_t$$

where the characteristic equation $\lambda^2 - 1.5\lambda + .5 = 0$ has roots $\lambda = .5, 1$.

Since $\lambda = 1$, this process contains a unit root. Taking the first difference we obtain

$$\Delta x_t - .5\Delta x_{t-1} = \varepsilon_t,$$

which is stationary ($\lambda = .5 < 1$).

4.5 Testing for Unit Roots

At this point, the logical "test" for a unit root would be to estimate the parameters of the autoregressive process. However, when using OLS estimation, we assume the time series data is stationary. This is because with non-stationary data, the Var(x_t) $\to \infty$ as $t \to \infty$ so the standard tests (i.e. t-test, F-test) are not valid since x_t is not normally distributed. Therefore, if we have no knowledge of the stationarity of x_t, the inferences made about statistical significance of the parameters using the t-distributions may not be correct.

Given these problems with parameter estimation, multiple tests have been developed in order to test for unit roots. There is ongoing research and debate in the econometrics literature as to which unit root test is "best" or the most useful. For the purposes of this paper, we choose the Augmented Dickey-Fuller (ADF) test since it is used by Darby, Cross & Piscitelli [8].

The ADF test provides a strategy for determining whether the time series data has a unit root or not. Three cases are considered: zero mean, single mean (also referred to as a drift) and linear trend. By subtracting x_{t-1}, we can rewrite equation 4.11 in a "nicer" form for hypothesis testing

$$\Delta x_t = \tau x_{t-1} + \varepsilon_t,$$

where $\tau = \varphi - 1$.

Referring to the hypothesis test in equation 4.5, failing to reject the null hypothesis (H_0) indicates a unit root and that x_t is non-stationary. That is, $\tau = 0$ indicates the data has to be differenced to make it stationary. Since x_t is I(1) under H_0, the standard t-distribution does not apply for the OLS estimator. For this, the so-called Dickey-Fuller distribution is used where the test statistic is calculated the same way as equation 4.6. The critical values for the test were derived by Dickey and Fuller using Monte Carlo experiments.

The test performed above is called the Dickey-Fuller test. However, an "augmented" version was developed to help eliminate serial correlation, the correlation of a variable with itself over time, by adding additional lags.

In order to develop the Augmented Dickey-Fuller test, consider the following random walk AR(2) process

$$x_t = \varphi_1 x_{t-1} + \varphi_2 x_{t-2} + \varepsilon_t.$$

This can be rewritten as

$$x_t = (\varphi_1 + \varphi_2)x_{t-1} - \varphi_2(x_{t-1} - x_{t-2}) + \varepsilon_t.$$

Now, subtracting x_{t-1} from both sides, we get

$$\Delta x_t = \tau x_{t-1} + \delta_1 \Delta x_{t-1} + \varepsilon_t$$

where $\Delta x_{t-k} = x_{t-k} - x_{t-(k+1)}, \tau = \varphi_1 + \varphi_2 - 1,$ and $\delta_1 = -\varphi_2$.

In general, for the AR(p) random walk model, we have

$$\Delta x_t = \tau x_{t-1} + \sum_{i=1}^{p-1} \delta_i \Delta x_{t-i} + \varepsilon_t \tag{4.13}$$

where $\delta_i = -\sum_{k=i+1}^{p} \varphi_i$ and $\tau = (\sum_{i=1}^{p} \varphi_i) - 1$.

If the time series data has a deterministic linear trend (i.e. a linear trend line, βt, can be fitted to the data), then we use the following equation to test for a unit root:

$$\Delta x_t = \beta t + \tau x_{t-1} + \sum_{i=1}^{p-1} \delta_i \Delta x_{t-i} + \varepsilon_t \tag{4.14}$$

Furthermore, a time series with a drift γ can be written as

$$\Delta x_t = \gamma + \tau x_{t-1} + \sum_{i=1}^{p-1} \delta_i \Delta x_{t-i} + \varepsilon_t \tag{4.15}$$

where δ_i and τ have the same meaning as before.

The hypothesis test for the Augmented Dickey-Fuller test is the same as described above for the Dickey-Fuller test (i.e. $\tau = 0$ indicates a unit root). Note: from equation 4.10, a unit root implies that $\sum_{i=1}^{p} \varphi_i = 1$, which causes $\tau = 0$.

Chapter 5

Hysteresis in Economics

After introducing the Preisach model and unit root processes, the focus now is applying these methods to model hysteresis in economic time series data. We describe the natural rate hypothesis and hysteresis hypothesis in unemployment and explain why distinguishing between the two is important for policymakers. Next, we define the important role of economic shocks in hysteresis and consider the problems that arise when modeling hysteresis with economic shocks present. A discussion of how the Preisach model can fix these problems is given. Finally, unemployment time series data is used to model hysteresis in the unemployment rates of the United States and the United Kingdom.

5.1 Natural Rate Hypothesis

The natural rate of unemployment states that unemployment has a natural equilibrium point, which is termed the 'natural rate.' The United States natural rate is estimated to be approximately 5 percent [12]. The natural rate of unemployment is caused by 'supply-side' factors, and the hypothesis does not consider demand policies in the reduction of unemployment rates [5]. This leads policy makers to believe that higher unemployment will not arise from anti-inflation policies. However, the hypothesis has experienced controversy, especially in European countries [4].

It is important to understand two economic principles: 1) classical dichotomy and 2) the neutrality of money. In economics, classical dichotomy is the division of real and nominal variables. Real variables are adjusted for inflation; whereas, nominal variables do not account for inflation. In other words, nominal variables are measured at the current price level. Real variables include unemployment rates and GDP; however, nominal variables include price and wage levels. This is important in the study of unemployment because the natural rate hypothesis assumes a classical dichotomy [4].

Secondly, the natural rate hypothesis applies the neutrality of money principle [4]. In classical economic thought, the neutrality of money principle follows the classical dichotomy idea and states that real variables are not affected by changes in the money supply; therefore, only nominal variables will be affected. New Keynesian economic thought believes that short-term money supply changes have an impact on real variables; however, the neutrality of money principle is a good indicator of the economy's behavior over the long-term. Therefore, changes in the money supply cannot affect the equilibrium rate of unemployment; however, it can temporarily affect the actual rate of unemployment. There is a great deal of economic literature on this idea of persistence, and Cross [5] argues the term 'hysteresis' has been misused by some economists to explain persistence. This is important to unemployment rates because central banks, such as the Federal Reserve in the United States, use this information to monitor and make decisions on the money supply.

5.2 Hysteresis Hypothesis

5.2.1 Introduction

In contrast to the natural rate approach, the hysteresis hypothesis takes into account the history of the actual unemployment rate when determining the equilibrium point [2]. The distinguishing feature of the hysteresis approach is the path-dependency of unemployment rates. This implies that temporary economic shocks have the potential to affect unemployment rates permanently. There is less debate about the hysteresis hypothesis in European countries, where Europe was hit with multiple shocks raising unemployment to record levels in the 1980s. Authors have attempted to explain the apparent change in equilibrium unemployment using various hysteresis theories [22]. However, many authors have bastardized the term 'hysteresis' [5].

5.2.2 Shocks

Economic shocks are an important component of the hysteresis hypothesis. An economic shock is an unpredictable event that generates a significant change in the economy, either positively or negatively. An expansionary shock is positive and leads to an increase in output; whereas, a contractionary shock leads to a decrease in output (see e.g. [5]). Therefore, in the case of unemployment, rates will be lower with expansionary shocks and higher with contractionary shocks. This is illustrated for the natural rate hypothesis in Figure 5.1 where the trajectory $a \to b \to a \to c \to a$ is followed. In contrast, Figure 5.2 shows the hysteresis hypothesis where the trajectory is $a \to b \to \bar{a} \to c \to \underline{a}$. Remanence, $|a - \bar{a}|$, is displayed in the hysteresis hypothesis because when the temporary shock is removed, there is a new unemployment rate $\bar{a}$.

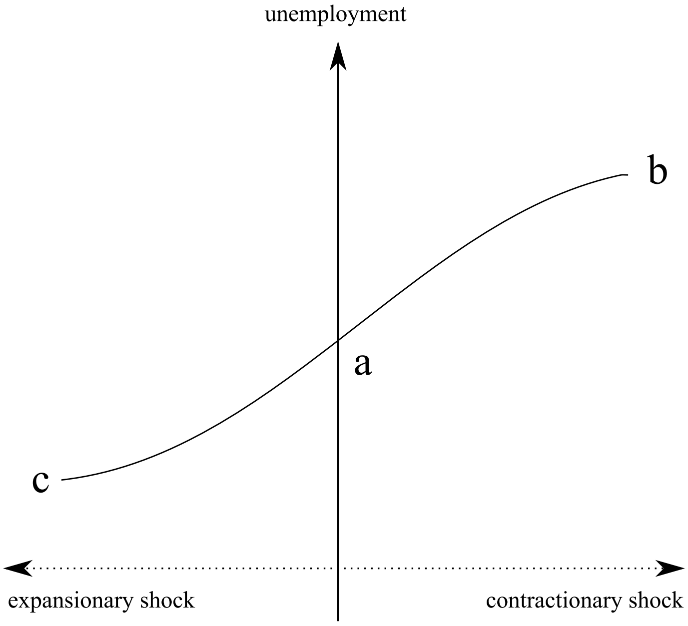

Figure 5.1: Shocks: Natural Rate

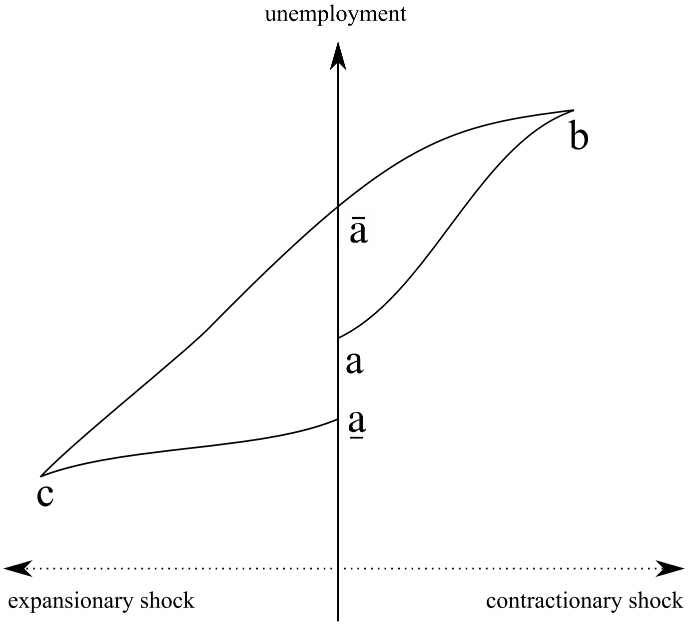

Figure 5.2: Shocks: Hysteresis

5.2.3 The Problems in Modeling Hysteresis

As mentioned before, the term 'hysteresis' has been bastardized in the literature. Many models assume linearity, which is in direct contradiction to nonlinear property of hysteresis [5]. Furthermore, modeling the selective memory that is present in hysteretic behavior has been difficult [18]. One approach is testing for a unit root and determining the stationarity of the time series data. If the data is nonstationary, many authors describe this as hysteresis. However, the typical unit root tests fail to capture selective memory. For instance, consider a shock followed by another shock that is in the opposite direction but same in magnitude. Using a unit root test, this sequence of events will vanish and everything will return to its initial position [1]. In other words, the equilibrium has not changed [18]. However, hysteretic systems will display remanence after this kind of sequence.

For illustration of this, consider the situation in Figure 5.3 where $x_1 = x_3 = x_5 = 0$ and $x_2 = -x_4$. Recall the AR(1) process: $x_t = \varphi x_{t-1} + \varepsilon_t$. It is easy to compute that $\varphi = 0$ using the OLS estimators in equation 4.4 . In other words, history, i.e. x_{t-1}, is not important because the shocks cancel each other out, which wipes away any memory. Consider the same situation described in Figure 5.3 using the Preisach model of hysteresis. Figure 5.4 shows the effect of these shocks on the Preisach plane. It is obvious that these shocks do not vanish and restore the system to the status quo. Thus, the Preisach model is able to capture the memory of these shocks; whereas, the unit root test would fail to do so. This is very troublesome when modeling hysteresis since remanence is a fundamental property.

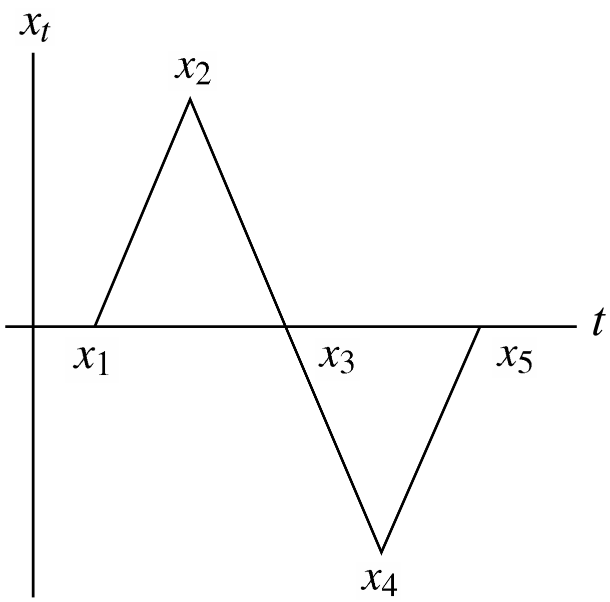

Figure 5.3: Hypothetical shocks

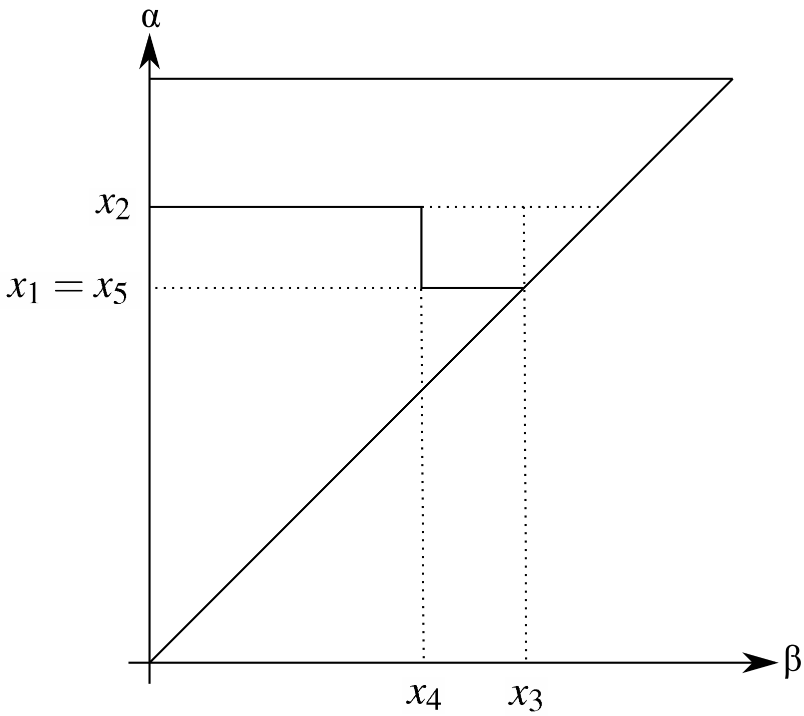

Figure 5.4: Preisach plane for shocks

5.2.4 The Preisach Model in Economics

When modeling hysteresis, it is important to consider the properties of systems with hysteresis. That is, 1) elements in a system respond to disturbances in non-linear fashion and 2) elements are heterogeneous [6]. Both of these properties are accounted for in the Preisach model. The idea of switching values provides a method to model decision making of market entry and exit. Since many decisions in economics are binary, they lend themselves to be modeled by hysteresis operators. For example, buying or selling a stock, producing a commodity or not, etc. Piscitelli, Cross, Grinfeld, & Lamba [18] argue the hysteretic transformation can be used to determine its explanatory power on a variable. Finally, the hysteretic transformation provides a way to capture the selective memory (i.e. nondominated extrema) in a variable.

5.3 Modeling US Unemployment

Annual unemployment rate time series data for the United States was obtained from the Bureau of Labor Statistics for 1949-08, which is graphed in Figure 5.5.

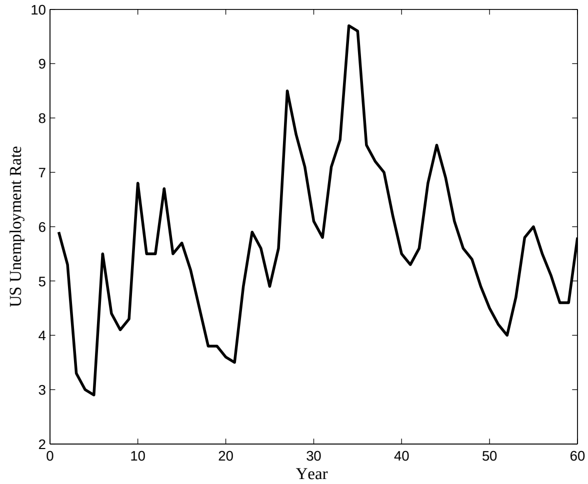

Figure 5.5: US unemployment rate (1949-08)

Using SAS, the Augmented Dickey-Fuller test is performed on the US unemployment data. Generally, lag lengths are determined by the frequency of the data [24]. Since the data is annual, it is appropriate to choose a lag of 1. Furthermore, we consider the single mean case because the unemployment rate does not have a zero mean. Using these specifications, we reject the null hypothesis of a unit root (τ = -3.12, p = .0304), and conclude the unemployment rate is stationary.

In contrast, it will be useful to consider a country where hysteresis has been shown to exist. For that, annual unemployment rate time series data for the United Kingdom is considered, which was obtained from the Office of National Statistics. Figure 5.6 shows a graph of the unemployment rate for 1971-07.

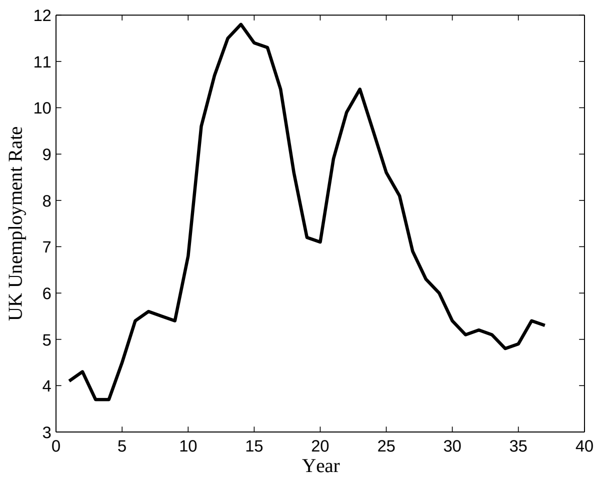

Figure 5.6: UK unemployment rate (1971-07)

Performing the Augmented Dickey-Fuller test using the same specifications as before, we are unable to reject the null hypothesis of a unit root (τ = -2.36, p = .1604). Therefore, a unit root is present in the UK unemployment rate.

Now, it will be useful to consider the "true" hysteresis model, i.e. where selective memory can be properly captured. For this, we use the Preisach model. Consider the behavior of an individual firm within an economy. Let the threshold values (α,β) represent the inactivity level of the firm. If the unemployment rate rises above β, we say the firm will become inactive and will cease employment. That is, a higher unemployment rate indicates a failing economy so businesses start to cease operations. Likewise, if the unemployment rate drops below α, this indicates a strong economy and more businesses begin to hire. We assume these switching values (α, β) are different for each firm, which satisfies the property of heterogeneity. Nonlinearity is accounted for in the hysteresis operators. See Figure 5.7 for illustration.

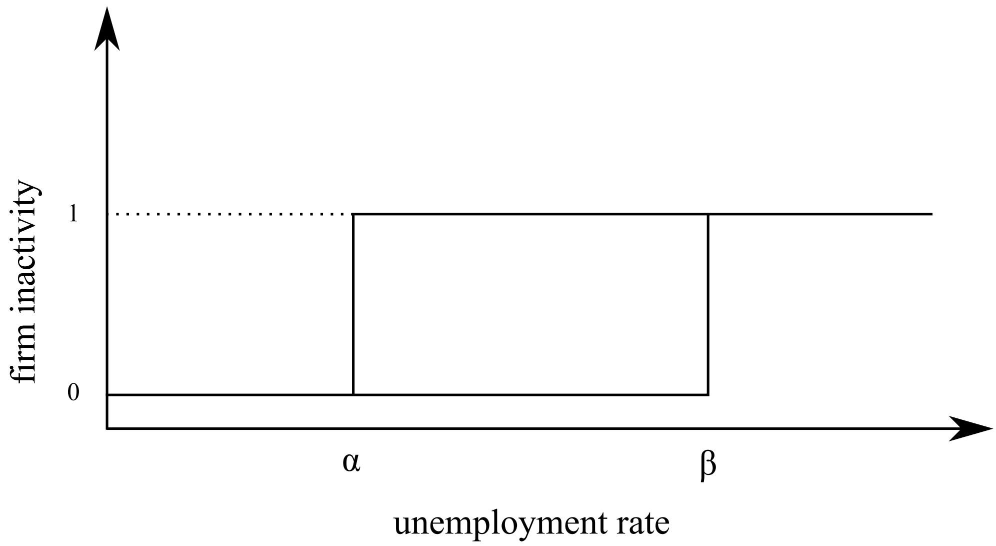

Figure 5.7: Behavior of an individual firm

Many authors use prices of various economic output variables (i.e. oil prices) or interest rates as the input. Higher prices/interest rates indicate a stronger economy, and lower prices/interest rates indicate a weaker economy. Therefore, firms will become active in a strong economy and inactive in a weak economy. Our definition is the exact opposite. We consider the "on" position to describe a firm that is inactive, which is certainly counterintuitive. The reason for this is that higher unemployment rates indicate a weak economy and lower unemployment rates indicate a strong economy. Therefore, the direction is switched.

Using the Matlab code in Appendix A, we compute the hysteresis variable for both the US and UK unemployment rate. The unemployment rate and its hysteretic transformation is then graphed. Figures 5.8 and 5.9 show these graphs for the US and UK, respectively.

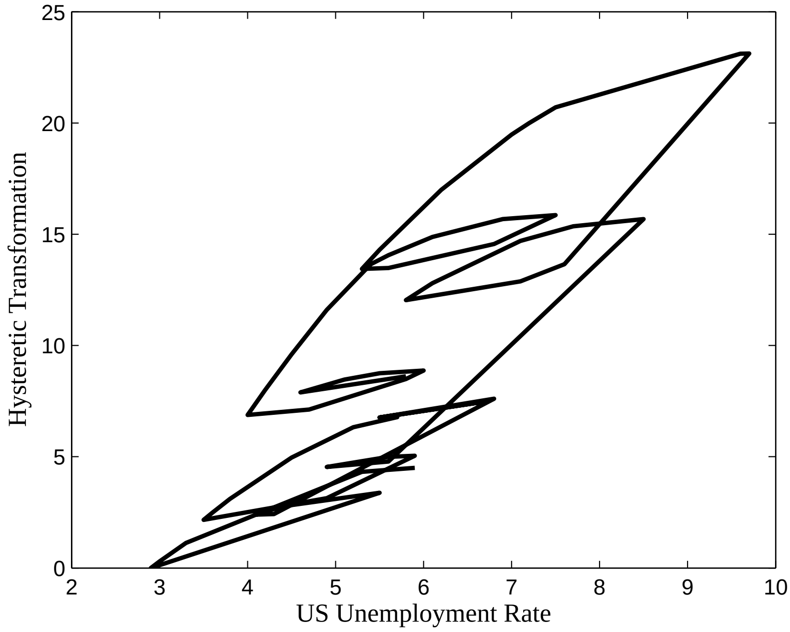

Figure 5.8: US hysteresis loops

Figure 5.9: UK hysteresis loops

By visual inspection, the UK unemployment rate behaves more like hysteresis than the US unemployment rate. However, it will be useful to asses this relationship using time series analysis. This will only be performed for the US unemployment rate. We have already shown the UK unemployment rate is nonstationary and in fact it's hysteretic transformation is nonstationary as well (τ = -2.26, p = .1915). For two nonstationary time series, a technique called cointegration is needed, which is outside the scope of this paper.

For the US unemployment data, two models will be considered, a hysteretic model and a non-hysteretic model. Let $unemp_t$ be the US unemployment rate at year t.

The non-hysteretic model is

$$unemp_t = \gamma + \varphi unemp_{t-1} + \varepsilon_t \tag{5.1}$$

where $unemp_{t-1}$ is the US unemployment rate of the previous year.

The hysteretic model is

$$unemp_t = \gamma + \varphi H(unemp_{t-1}) + \varepsilon_t \tag{5.2}$$

where $H(unemp_{t-1})$ is the hysteretic transformation of the US unemployment rate of the previous year.

Using OLS regression, the following regressions are obtained (see Figures 5.10 & 5.11)

$$\widehat{unemp}_t = 1.331 + 0.763\widehat{unemp_{t-1}}$$

with R^2 = .57 ($F_{1,57} = 79.39, p < .0001$).

$$\widehat{unemp}_t = 4.149 + 0.157H(\widehat{unemp_{t-1}})$$

with R^2 = .39 ($F_{1,57} = 37.85, p < .0001$).

The past value of the unemployment rate explains 57% of the change in the current unemployment rate; however, the hysteretic transformation of the past value explains only 39% of the current unemployment rate. Obviously, the non-hysteretic model is better at explaining current unemployment.

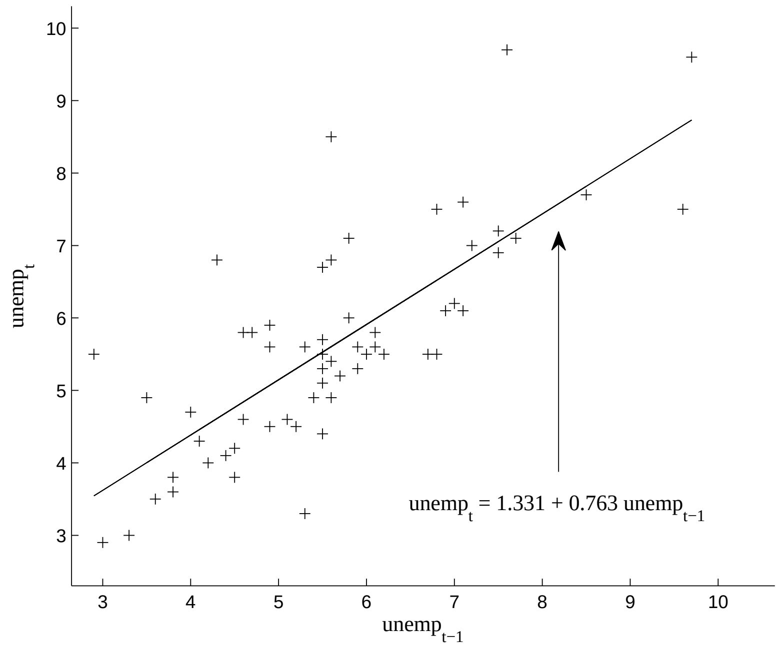

Figure 5.10: Scatterplot of $unemp_t$ vs. $unemp_{t-1}$

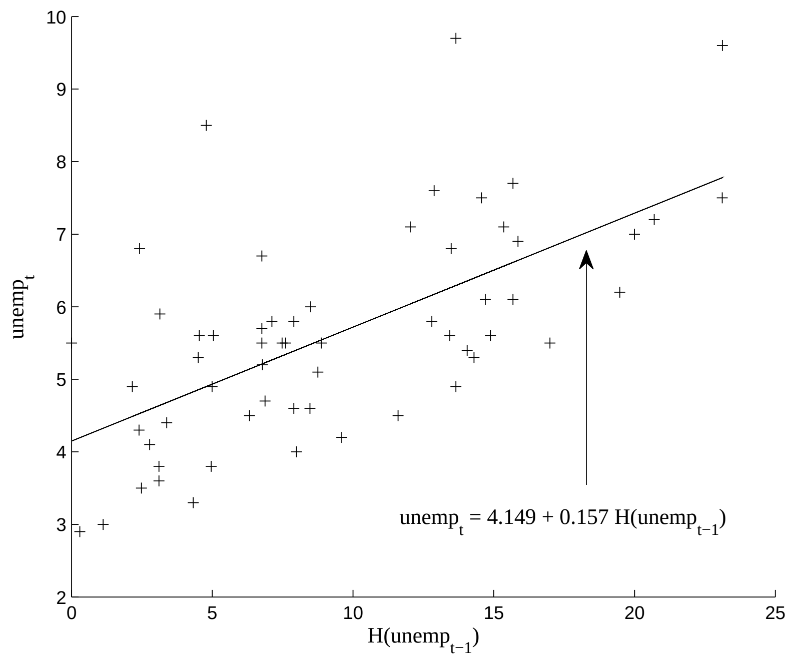

Figure 5.11: Scatterplot of $unemp_t$ vs. $H(unemp_{t-1})$

Chapter 6

Conclusion

Hysteresis has been and continues to be a widely studied phenomenon outside of the physical sciences. Many economists use linear unit root tests to examine hysteresis in economic time series data, but the unit root tests are unable to capture the selective memory of systems with hysteresis. Determining the stationarity of time series data is important; however, nonstationary data does not necessarily indicate hysteresis. Instead, it means the variable of interest displays persistence in reacting to shocks, i.e. temporary change in the equilibrium. If the persistence is permanent, then we call that hysteresis.

In this paper, we chose to investigate the United States unemployment using the Preisach model. We modeled the time series of unemployment rates where a firm was either hiring or firing based upon unemployment rates. Next, the hysterons were aggregated and the hysteretic transformation was computed. A hysteretic model to determine the explanatory power that the hysteretic transformation of the previous unemployment rate had on the current unemployment rate was compared to a non-hysteretic model, i.e. the explanatory power of the previous unemployment rate on the current unemployment rate. We found the non-hysteretic model explained current unemployment rates better than the hysteretic model. Thus, given these findings and the fact that the US unemployment rate is stationary, hysteresis does not appear to be an important factor in the US unemployment rate.

Using the Preisach model, economists are better able to model hysteresis; however, there are problems that have to be considered. Primarily, specification of the weight function $\mu(\alpha, \beta)$ is difficult. We followed Piscitelli, Cross, Grinfeld, & Lamba [18] and used a uniform distribution. The authors determined that the hysteretic output is insensitive to the specification of the weight function. However, it would be interesting to see if that held true for other times series data from other countries and different time periods. Another problem is that switching points of the hysterons may be time dependent. That is, firms may adjust threshold values over time given new information or changes in strategy. Future research needs to consider a time-dependent Preisach model, which is introduced by Cross, Krasnoselskii, and Pokrovskii [7].

Knowing the hysteresis hypothesis has been accepted by many economists in the United Kingdom, we chose to analyze the unemployment rate in the UK as well. Unfortunately, we were limited in statistically analyzing the UK data. This is because both the unemployment rate and the hysteretic transformation were nonstationary. Therefore, a technique called cointegration is needed. Two nonstationary processes are cointegrated if there exists a stationary linear combination, which is referred to as a conintegrating vector. If a cointegrating vector can be found, then even though the variables may wander in different directions, they will come back to a mean distance apart over time. This technique was not in the scope of this paper, and could not be used on the United States data since unemployment was stationary. However, future research could investigate statistical significance of the UK unemployment data and its hysteretic transformation.

We were also limited in the availability of recorded time series data. Thus, it is hard to make inferences over long periods of time. Moreover, we used annual data but future research could consider quarterly data since the hysteretic transformation might be altered because there may be nondominated economic shocks which occur within a year's time. A lag length of 4 would probably be the most appropriate for the ADF test. The hysteretic and non-hysteretic model would consider the unemployment rate from the previous quarter. Finally, it would be interesting to see if hysteretic behavior occurs during particular time periods. This may indicate there has been a phase reset due to some kind of economic recovery.

Many authors choose to use other economic output variables in their model; we chose the unemployment rate. Future research could determine the relationship between unemployment and the hysteretic transformation of real economic output variables such as oil prices, interest rates, or GDP growth. Economic theory would be important in determining the appropriate model. Following Darby, Cross, & Piscatelli [8], it would be interesting to see if a nominal variable could be used to explain changes in the United States unemployment rate, which contradicts the natural rate hypothesis.

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Appendix A

Matlab Code for Calculating the Hysteretic Transformation

```
v = data;
n=length(v);
u=v;
a0=(min(u));
b0=(max(u));
for t=1:length(u)
   k=1;
   [M(k,t), tp(k)]=max(u(1:t));
   T(t) = ((M(k,t)-a0)^2)/2;
   tt=tp(k);
   while tt<t
      z=u(tp(k):t);
      [m(k,t), Iz]=min(z);
      tm(k)=tp(k)-1+Iz;
      T(t)=T(t)-((M(k,t)-m(k,t))^2)/2;
      if tm[k] < t
         w=u(tm(k):t);
         [M(k+1, t), Iw]=max(w);
         tp(k+1)=tm(k)-1+Iw;
         T(t)=T(t)+((M(k+1,t)-m(k,t))^2)/2;
         tt=tp(k+1);
         k=k+1;
      else
            tt=tm(k);
            k=k+1;
      end
   end
end
```