

Variational Calculation of Optimum Dispersion Compensation For Non-linear Dispersive Fibers

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ABSTRACT

In fiber optic communication systems, the main linear phenomenon that causes optical pulse broadening is called *dispersion*, which limits the transmission data rate and distance. The principle nonlinear effect, called *self-phase modulation*, can also limit the system performance by causing spectral broadening. Hence, to achieve the optimal system performance, high data rate and low bandwidth occupancy, those effects must be overcome or compensated. In a nonlinear dispersive fiber, properties of a transmitting pulse: width, chirp, and spectra, are changed along the way and are complicated to predict. Although there is a well-known differential equation, called the *Nonlinear Schrodinger Equation*, which describes the complex envelope of the optical pulse subject to the nonlinear and dispersion effects, the equation cannot generally be solved in closed form. Although, the split-step Fourier method can be used to numerically determine pulse properties from this nonlinear equation, numerical results are time consuming to obtain and provide limited insight into functional relationships and how to design input pulses.

One technique, called the *Variational Method*, is an approximate but accurate way to solve the nonlinear Schrodinger equation in closed form. This method is exploited throughout this thesis to study the pulse properties in a nonlinear dispersive fiber, and to explore ways to compensate dispersion for both single link and concatenated link systems. In a single link system, dispersion compensation can be achieved by appropriately pre-chirping the input pulse. In this thesis, the variational method is then used to calculate the optimal values of pre-chirping, in which: (i) the initial pulse and spectral width are restored at the output, (ii) output pulse width is minimized, (iii) the

output pulse is transform limited, and (iv) the output time-bandwidth product is minimized.

For a concatenated link system, the variational calculation is used to (i) show the symmetry of pulse width around the chirp-free point in the plot of pulse width versus distance, (ii) find the optimal dispersion constant of the dispersion compensation fiber in the nonlinear dispersive regime, and (iii) suggest the dispersion maps for two and four link systems in which initial conditions (or parameters) are restored at the output end.

The accuracy of the variational approximation is confirmed by split-step Fourier simulation throughout this thesis. In addition, the comparisons show that the accuracy of the variational method improves as the nonlinear effects become small.

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Chapter 1

INTRODUCTION

High-speed data communications require a large bandwidth channel. Optical fiber is a medium providing huge channel bandwidth. The low attenuation region of a fiber corresponds to a bandwidth of about 30 THz, and experimentally more than 3 Tbps has been transmitted on a single fiber using a combination of time division and wavelength division multiplexing [1]. There have been a number of generations of fiber-optic communication systems, with each generation resulting in improvements of the system performance. Most of the prior generations considered the ways to increase repeater spacing by operating the system in the wavelength region near 1.3 or 1.55 μm , where the fiber loss is low. Then the optical amplification and wavelength-division multiplexing (WDM) systems were introduced in order to increase the repeater spacing and the transmission bit rate, respectively, in the next generation. As the serial transmission bit rate increases the effects of fiber dispersion (pulse spreading) becomes more severe and is receiving more attention [2]. Beside dispersion, fiber non-linearity also degrades the system performance by causing spectral broadening in a single channel system or Four-Wave Mixing in a multi-channel system. (Four wave mixing is what is called inter-modulation distortion in electrical systems. For example, given three signals at frequencies f_1 , f_2 , f_3 , nonlinearities will result in the generation of a new frequency component $f_4 = f_1 + f_2 - f_3$ which will also be in-band.)

Both dispersion and nonlinear effects limit transmission data rate and distance (or repeater spacing) in fiber-optic communication systems. Several techniques to counteract these problems have been developed. Pre-chirping is a pre-compensation technique [2], which can be used to overcome these effects. However, in nonlinear dispersive fiber, the properties of an optical pulse in terms of width, chirp, and spectra, are difficult to calculate [3, 4]. Consequently, it is difficult to determine the appropriate amount of pre-chirping. This motivates the application of the variational method for

calculating the suitable pre-chirping value in closed form for dispersion compensation in a nonlinear dispersive fiber. (The variational method is described in Chapter 2.)

It will be shown in Chapter 3 that, for a single link system, pre-chirping cannot always achieve perfect dispersion compensation, especially in a very long length system. An appropriate concatenated link system is needed in this case. Hence, it will be valuable to have the ideal dispersion map for a concatenated link system in order to achieve perfect dispersion compensation and restore the initial pulse conditions at the output. This motivates the application of the variational method to find the optimal dispersion maps for the new concatenated links systems.

The remainder of this chapter will discuss the basic ideas of fiber dispersion, non-linearity, and pre-chirping, which, hopefully, will be useful for the reader of this thesis. The outline of the thesis will be covered at the end of this chapter.

1.1 Fiber Dispersion

An optical pulse consists of a range of optical frequencies. Different frequencies (or spectral components) of the pulse travel at slightly different group velocities, a phenomenon referred to as *group-velocity dispersion (GVD)* [2]. In a single-mode fiber, dispersion is caused primarily by GVD¹. When an optical pulse is launched into an optical fiber, the pulse may spread outside its timing window due to dispersion, which causes pulse overlapping between adjacent timing windows and limits the transmission data rate. If we let $\Delta\omega$ be the spectral width of the pulse, the extent of pulse broadening for a fiber of length L is governed by [2] $\Delta T = L\beta_2\Delta\omega$, where β_2 is the second derivative of the propagation constant (β) with respect to frequency (ω) and is sometimes called the *GVD parameter* or *dispersion constant*. It measures the dispersion per unit length and per unit spectral width of the source, and consequently has the units of (time)²/distance.

¹ Any asymmetry in the circularity of the fiber will cause polarization-mode dispersion (PMD). The effects of PMD are not considered in this thesis.

The value of β_2 [ps²/km] depends on the design of the fiber and the operating wavelength. The dispersion constant goes through zero at a wavelength called the zero dispersion wavelength. (Dispersion is not zero at this wavelength, rather higher order terms (β_3) are then required to calculate dispersion. Throughout this thesis it is assumed that we are sufficiently removed from the zero dispersion wavelength that β_3 terms are negligible.) When β_2 is positive the dispersion is termed normal; when it is negative the dispersion is termed anomalous.

A parameter, called the *dispersion length* ($L_D = a_0^2 / |\beta_2|$), is often used to characterize dispersion in a fiber, where a_0 is the initial input pulse width (half-width at 1/e intensity point). Since (for a transform limited pulse) bandwidth is inversely proportional to a_0 , the dispersion length is essentially the length of fiber for which the dispersion is equal to the initial pulse width.

1.2 Fiber Non-Linearity

The response of any dielectric to light becomes nonlinear for intense electromagnetic fields [2]. Nonlinear effects in optical fibers are responsible for phenomena such as third-harmonic generation, four-wave mixing, and nonlinear refraction. However, most of the nonlinear effects in optical fibers, at high launching power, originate from the nonlinear refraction, a phenomenon that refers to the intensity dependence of the refractive index [5].

The type of nonlinear refraction that will be taken into account in this research is *self-phase modulation (SPM)*. SPM refers to the self-induced phase shift experienced by an optical field during its propagation in optical fibers. Optical power directly influences that nonlinear phase shift since the phase shift is influenced by the index of refraction and the index of refraction depends on the power. In practice, the time dependence of optical power makes the nonlinear phase shift vary with time, resulting in *frequency chirping*, that causes spectral broadening of the optical pulse. The nonlinear phase shift

for an input optical power P_0 , and a fiber of effective length L_{eff} is given by [2] $\phi_{NL} = \gamma P_0 L_{\text{eff}}$, where γ [$\text{W}^{-1} \text{ km}^{-1}$], called the *nonlinear coefficient*, determines how much nonlinear phase shift and hence frequency chirping would be introduced to the optical pulse by the nonlinear fiber. The effective length of the fiber is less than the actual fiber length to account for the power decrease caused by attenuation [2]. The *Nonlinear Length*, ($L_N = 1/\gamma P_0$) is the length for which the nonlinear phase shift is one radian. Thus, the dispersion length is the distance beyond which dispersion effects become important, and the nonlinear length the distance beyond which nonlinear effects become important. The square root of the ratio of these two lengths turns out to be a very important parameter. It is called the *Nonlinear Parameter* N , ($N^2 = L_D / L_N$). In general in this thesis we will do the analysis and numerical calculations in terms of normalized dimensionless parameters so that the results may then be applied to a wide range of physical situations.

1.3 Pre-Chirping

An optical pulse is said to be chirped if its carrier frequency changes with time in a deterministic fashion (If there are random variations in the carrier frequency, e.g. phase noise, this is generally not referred to as chirp) [2]. The effects of frequency chirping on the pulse and spectral broadening and compressing will be discussed after we have introduced the Nonlinear Schrodinger Equation in the next chapter. Pre-chirping is the process of introducing a chirp to an optical pulse before launching into a fiber-optic system. Pre-chirping can be done by passing an optical pulse into a phase modulator that introduces a time varying phase. If the phase is quadratic in time this corresponds to a linear chirp.

Optical pulse pre-chirping has emerged as a critical design tool in fiber optic systems, especially with the increased significance of the chirped return-to-zero format [3, 6, 7, 8]. Pre-chirping accomplishes various objectives including suppression of four wave mixing [3, 8], lowering pulse width fluctuations [9], spectral compression [10] and generating

transform limited output pulses [3]. In soliton systems a chirped pulse can experience pulse width and phase fluctuations but, with the correct pre-chirp, can be made to return to its exact initial conditions after a given propagation distance [11]. Likewise, equal input and output spectral widths can be arranged for an appropriately pre-chirped pulse [4].

Although pre-chirping can achieve dispersion compensation, as we will see in the next chapter, a chirped pulse has greater bandwidth (or wider spectral width) than that of un-chirped pulse, sometimes called a *transform-limited pulse*.

1.4 Outline of Thesis

In Chapter 2, Methodology, we will begin (2.1) with briefly introducing the *Nonlinear Schrodinger Equation (NSE)*, which is the differential equation for representing the propagation of an optical pulse in a nonlinear dispersive fiber. This section also gives the example of solving the NSE in the linear dispersive case. Then (2.2) we will briefly describe one technique called the *Split-Step Fourier Method (SSFM)*, which has been widely used to numerically solve the NSE. The SSFM will be used throughout the thesis to verify results obtained using the variational method. The variational method (2.3) will then be described which together with a power series expansion will then be used to obtain closed form approximate solutions for various parameters describing the pulse.

In Chapter 3, Single Link, we will use the variational method to predict the appropriate values of pre-chirping in various cases for a single link system. This will include the optimal pre-chirp, in which: (i) initial pulse and spectral width appear at the output, (ii) output pulse width is minimized, (iii) output pulse is transform limited, and (iv) output time-bandwidth product is minimized. In all cases, comparison will be made with the SSFM results.

Chapter 4 starts with demonstrating the symmetry of the pulse width around the chirp-free point, which is useful for constructing dispersion maps later on. Then the variational calculation is used to predict the optimal dispersion constant of the second section of fiber (dispersion compensation fiber), for which output pulse width is equal to the input pulse width for the case when non-linearity is significant. This chapter ends with the suggestion of two types of dispersion maps for concatenated link systems, in which the initial conditions: width, chirp, and spectra, are restored at the output.

Finally, the last chapter will be dedicated to the summary and conclusions of the thesis.

Chapter 2

METHODOLOGY

When non-linearities and dispersion are both present, the properties of an optical pulse propagating into an optical fiber are difficult to determine. Although one can study wave propagation from *Maxwell's Equations* and come up with a differential equation, namely the *Nonlinear Schrodinger Equation (NSE)* that expresses the behavior of pulse properties along the fiber, that equation is still too complicated to solve in a form that is useful. However, the NSE may be solved numerically by considering the non-linearity and dispersion to occur independently in small sections of fiber and then stepping along the fiber length. That technique, called the *Split-Step Fourier Method (SSFM)*, provides results in numerical form, which are accurate but slow and the method is of limited use for signal design. That motivates the idea of using *Variational Calculation (VC)* to solve the NSE in closed form. The results may then be used to determine how to compensate for the combined effects of dispersion and non-linearity, which is the objective of this thesis. Nevertheless, the accuracy of the VC needs to be confirmed by SSFM. This chapter will briefly discuss the nonlinear Schrodinger equation, the variational calculation, and the split-step Fourier method, the results of which will be used in the remainder of the thesis.

Note that we are not using the variational method as an alternative means of approximately solving the NSE. Instead, we will use the variational technique to obtain closed form analytic expressions among the fiber parameters when one or more parameters are selected to optimize certain propagation design features. The mathematical formulas thus obtained serve as predictors of optimal parameter values, which may then be methodically sharpened by split step calculations, but more importantly will yield the kind of insights into system behavior that can only come from simple algebraic expressions.

2.1 Nonlinear Schrodinger Equation (NSE)

2.1.1 Defining the Equation and Parameters

Like all electromagnetic phenomena, the propagation of optical fields in fiber is governed by Maxwell's equation [5]. However, the optical fiber has some particular characteristics: the absence of free charges and cylindrical symmetry, which are different from other media. In addition, in an optical fiber, the pulse envelope is assumed to change slowly in a distance and time corresponding to the optical wavelength and period, respectively (This is equivalent to taking the bandwidth to be small compared to the center frequency). Also pulse broadening results from the frequency dependence of the propagation constant, which can be expanded in a power series around the center frequency, i.e., [5]

$$\beta(\omega) = \beta_0 + (\omega - \omega_0)\beta_1 + \frac{1}{2}(\omega - \omega_0)^2\beta_2 + \frac{1}{6}(\omega - \omega_0)^3\beta_3 + \dots,$$

$$\text{where } \beta_n = \left(\frac{\partial^n \beta}{\partial \omega^n} \right)_{\omega=\omega_0}.$$

More specifically, $\beta_1 = 1/v_g$, where v_g is the group velocity, and β_2 , the dispersion constant, is related to fiber dispersion as discussed in section (1.1) of Chapter 1. The cubic and higher-order terms in the above expansion are related to the *dispersion slope* [2], which is generally negligible if the spectral width $\Delta\omega \ll \omega_0$, (the pulse width ≥ 0.1 ps) and operation is sufficiently removed from the zero dispersion wavelength ($|\beta_2| \gg |\beta_3(\omega_0 - \omega_{ZD})|$, where ω_{ZD} is the frequency corresponding to the zero dispersion wavelength) [5].

From Maxwell's equations, together with the previous propagation constant expansion and using the slowly varying approximation, the following equation is obtained for the envelope of the field [5]

$$i \frac{\partial A(z, T)}{\partial z} = -\frac{i}{2} \alpha A(z, T) + \frac{1}{2} \beta_2 \frac{\partial^2 A(z, T)}{\partial T^2} - \gamma |A(z, T)|^2 A(z, T), \quad (2.1)$$

where $A(z, T)$ is the slowly varying envelope of the field, z is the axial distance along the fiber, P_0 is the peak power of input pulse, T is the local time measured in a frame of reference moving with the pulse at the group velocity v_g ($T = t - z/v_g$), β_2 [ps²/km] is the dispersion constant representing the pulse broadening effect, γ [W⁻¹km⁻¹] is the nonlinear coefficient representing the fiber nonlinear effect, and α [km⁻¹] is the loss coefficient. Note that $U(z, T)$ is the normalized-amplitude field, in which $U(0,0) = 1$.

Proceeding, equation (2.1) can be written in normalized form by defining

$$U(z, T) = \frac{A(z, T)}{\sqrt{P_0} \exp(-\alpha z)}, \text{ so that it becomes}$$

$$i \frac{\partial U(z, T)}{\partial z} = \frac{1}{2} \beta_2 \frac{\partial^2 U(z, T)}{\partial T^2} - \gamma P_0 \exp(-\alpha z) |U(z, T)|^2 U(z, T) \quad (2.2)$$

This is called the *Nonlinear Schrodinger Equation (NSE)*.

Note that equation (2.2) includes only the nonlinear effect of Self-Phase Modulation (SPM), which is assumed to be the only nonlinear effect that needs to be considered. One might need to modify (2.2) for ultra-short optical pulses (width < 100 fs) by adding higher-order dispersion and non-linearity terms, which becomes important for ultra-short pulses [5]. We remark that no version of the variational method has been found that applies to the NSE with higher order dispersion terms; i.e., more complicated than (2.2).

In addition, we will assume exact loss compensation by using optical amplifiers and then replace the instantaneous power term, $P(z) = P_0 \exp(-\alpha z)$, by its average,

$$P_{avg} = \frac{1}{z_a} \int_0^{z_a} P_0 \exp(-\alpha z) dz, \text{ where } z_a \text{ is the amplifier spacing [12]. Then equation (2.2)}$$

can be rewritten as

$$i \frac{\partial U(z, T)}{\partial z} = \frac{1}{2} \beta_2 \frac{\partial^2 U(z, T)}{\partial T^2} - \gamma P_{\text{avg}} |U(z, T)|^2 U(z, T). \quad (2.3)$$

Note that, from now on, we will account for fiber loss by using average power but still use P_0 instead of P_{avg} for simple notation. Hence, for any optical pulse launched into a nonlinear dispersive fiber, its properties can be defined by solving (2.3). Let us consider the simple linear case as follows.

2.1.2 Linear Dispersive Case

In the linear dispersive regime, equation (2.3) can be solved in closed form for the pulse width and spectra. Consider an input Gaussian pulse (with pre-chirping) transmitted into a fiber that has the form

$$U(0, T) = \exp\left[-\frac{T^2}{a_0^2} \left(\frac{1+iC}{2}\right)\right],$$

where C is called the initial or pre-chirp and a_0 is the initial pulse width (half-width at $1/e$ intensity point).

By taking the Fourier Transform, the initial spectrum is obtained as

$$\tilde{U}(0, \omega) = \left[\frac{2\pi a_0^2}{1+iC} \right]^{1/2} \exp\left[-\frac{\omega^2 a_0^2}{2(1+iC)}\right],$$

with the initial spectral width, $\Delta\omega_0 = (1+C^2)^{1/2} a_0^{-1}$. When the pulse has no chirp, it is called *transform limited*, and it has minimum spectral width, $\Delta\omega_0 = a_0^{-1}$. That means the spectral width increases with chirp.

The effect of pre-chirp on the output pulse width can be seen from the following calculations. When the last term of the right side of equation (2.3) is absent, we have the linear partial differential equation

$$i \frac{\partial U(z, T)}{\partial z} = \frac{1}{2} \beta_2 \frac{\partial^2 U(z, T)}{\partial T^2}, \quad (2.4)$$

and the solution of the propagating pulse at distance z is [2]

$$U(z, T) = \frac{a_0}{[a_0^2 - i\beta_2 z(1 + C)]^{1/2}} \exp\left[-\frac{T^2(1 + iC)}{2[a_0^2 - i\beta_2 z(1 + C)]^{1/2}}\right], \quad (2.5)$$

with the output pulse width

$$a_1 = a_0 \left[\left(1 + \frac{C\beta_2 z}{a_0^2}\right)^2 + \left(\frac{\beta_2 z}{a_0^2}\right)^2 \right]^{1/2}. \quad (2.6)$$

Note from equation (2.5) that one may consider the complex argument within the exponential term as the real and imaginary terms: $\exp[\text{Re} + i \text{Im}]$, where Re and Im represent real and imaginary terms, respectively. Then the ratio of imaginary term and the square of time, Im/T^2 , will be later called $b(z)$ in the variational calculation. In general, when $\gamma \neq 0$, $b(z)$ cannot be found; $b(z)T^2$ (or Im) is called the quadratically varying phase.

We can see from equation (2.6) that dispersion causes the pulse to broaden (β_2 term makes $a_1 > a_0$). In addition, if the term $C\beta_2 > 0$, then the optical pulse is more spread out than when there is no pre-chirp or $C = 0$. In contrast, if the term $C\beta_2 < 0$, the optical pulse initially compresses. The idea of using appropriate pre-chirp to compensate dispersion will be discussed later. Similarly, equal input and output spectral widths can be arranged with an appropriately pre-chirped pulse.

Again, by taking Fourier Transform, we obtain the optical spectrum as

$$\tilde{U}(z, \omega) = \tilde{U}(0, \omega) \exp\left[\frac{i}{2} \beta_2 \omega^2 z\right] = \left[\frac{2\pi a_0^2}{1+iC}\right]^{1/2} \exp\left[-\frac{\omega^2}{2} \left(\frac{a_0^2}{(1+iC)} - i\beta_2 z\right)\right].$$

Note that dispersion changes only the phase of each spectral component of an optical pulse (it does not affect the spectral width). In the other words, in the linear dispersive regime, the spectral width of an optical pulse remains constant.

For the nonlinear dispersive fiber case, equation (2.3) is too complicated to be directly solved in closed form. The following SFFM will show how to solve (2.3) numerically.

2.2 Split-Step Fourier Method (SFFM)

The SFFM is the widely accepted and reliable numerical approach to study pulse propagation in nonlinear dispersive fibers [5]. In general, dispersion and nonlinear effects act together along the length of fiber. The SFFM assumes that in a short propagation distance, dispersion and non-linearity can be assumed to act independently [5]. The idea is to consider a long length of fiber to be the connection of many small pieces of fiber as in Figure 2.1, where h is the step size, and D or N represents the section that has only the dispersion or nonlinear effect, respectively.

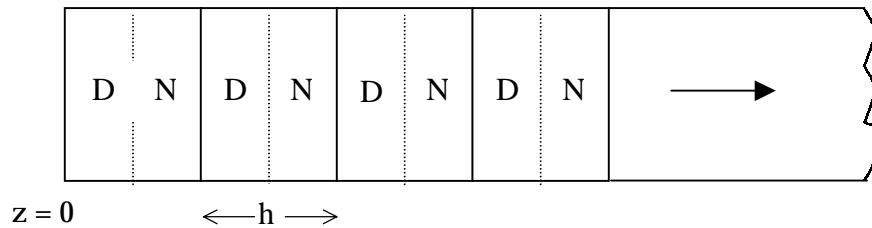


Figure 2.1: Schematic illustration of the SFFM used for numerical simulation

More specifically, from distance 0 to $h/2$, the dispersion effect acts alone, therefore (2.3) can be solved to get the pulse conditions at distance $h/2$, which will be the initial conditions for the next section governed by the nonlinear effect. Continuing, from distance $h/2$ to h , setting the dispersion term to zero makes the equation (2.3) solvable to get the pulse conditions at distance h , which will be the input conditions for the next section governed by the dispersion effect. These processes, done by computer simulation, will repeatedly go on until the end of fiber. Note that the smaller the step size, the slower the simulation time, but the greater the accuracy. A key to making the SSFM work is the fact that the NSE can be solved in essentially closed form [5] in the two separate cases of pure linear and pure nonlinear (dispersionless). The computational speed of the method comes from using the fast Fourier transform.

Although SFFM is an accurate way to study pulse propagation, again the results are in numerical form, which may not be convenient in some aspects. However, it will be used to confirm the accuracy of variational calculation throughout the thesis.

2.3 Variational Calculation (VC)

2.3.1 Defining Equations and Parameters

Pulse propagation in nonlinear dispersive fibers can be analyzed using the variational calculation generated by the Lagrangian for NSE (2.3), which is [12, 13]

$$L(z, T) = \frac{i}{2} \left[U(z, T) \frac{\partial U^*(z, T)}{\partial z} - U^*(z, T) \frac{\partial U(z, T)}{\partial z} \right] - \frac{\beta_2}{2} \left| \frac{\partial U(z, T)}{\partial T} \right|^2 - \frac{\mathcal{P}_0}{2} |U(z, T)|^4.$$

Anderson [10] shows that the solution $U(z, T)$ of (2.3) provides an extremum of the functional $J = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} L(z, T) \partial z \partial T$. In the ‘Ritz’ method one extremizes J within a class of functions meant to approximate $U(z, T)$. Anderson [10] replaces $U(z, T)$ by a Gaussian defined by three parameters, peak amplitude, pulse width, and chirp, and then selects

values of the parameters which give a local maximum and minimum of J . More specifically,

$$U(z, T) = A(z) \exp\left[T^2 \left(-\frac{1}{2a^2(z)} + ib(z)\right)\right], \quad (2.7)$$

where $A(z)$ is the pulse center height, with $A(0) = 1$, $a(z)$ is the pulse half-width at (1/e-intensity point), and $b(z)$ is the frequency chirp parameter. Note that, for relatively small non-linearity, pulses will remain approximately Gaussian, and all parameters are allowed to vary along the length of fiber.

Proceeding, by substituting (2.7) into the functional J and minimizing the latter, the following equations for $A(z)$, $a(z)$, and $b(z)$ are obtained [12, 13]

$$\frac{\partial a}{\partial z} = \mp 2\beta_2 a(z)b(z) \quad (2.8)$$

$$\frac{\partial b}{\partial z} = \pm 2\beta_2 b^2(z) \mp \frac{\beta_2}{2a^4(z)} - \frac{\gamma P_0 a_0}{2\sqrt{2}a^3(z)} \quad (2.9)$$

$$a(z)|A(z)|^2 = E_0 = \text{constant} \quad (\text{Energy Conservation}) \quad (2.10)$$

where $a_0 = a(0)$; in addition to (2.10), which connects the modulus of $A(z)$ to $a(z)$, the complex quantity $A(z)$ in (2.7) is expressible in terms of $a(z)$ [13]. In (2.8-9), “+” is used for $\beta_2 > 0$, “-” for $\beta_2 < 0$. It is well known [5] that the variational method provides a good approximation to (2.3) when the non-linearity level is not too high; in the (ideal) linear case, (2.7)-(2.10) are exact.

Differentiating (2.8) and substituting in (2.9), we get

$$\frac{\partial^2 a}{\partial z^2} = \frac{\beta_2^2}{a^3(z)} \pm \frac{\beta_2 \gamma E_0}{\sqrt{2}a^2(z)}, \quad (2.11)$$

where $E_0 = P_0 a_0$. Note that Chapter 4 (4.1) will show the symmetry of the equation (2.11), which was also pointed out in [14]. At this time, we will consider only the normal dispersion case, $\beta_2 > 0$.

2.3.2 Power Series for the Pulse Width

Equation (2.11) does not permit a simple solution; it can be integrated but the result gives $a(z)$ only implicitly. To get a handle on $a(z)$, we then expand $a^2(z)$ in a power series of z where the coefficients are found by equations (2.8)-(2.11). The power series of the square of the pulse width can be written as

$$a^2(z) = c_0^{(1)} + c_1^{(1)}z + c_2^{(1)}z^2 + c_3^{(1)}z^3 + c_4^{(1)}z^4 + \dots \quad 0 \leq z \leq L_1, \quad (2.12)$$

and for the case of multiple sections of fibers, the power series of the adjacent section is

$$a^2(z) = c_0^{(2)} + c_1^{(2)}(z - L_1) + c_2^{(2)}(z - L_1)^2 + c_3^{(2)}(z - L_1)^3 + c_4^{(2)}(z - L_1)^4 + \dots \quad L_1 \leq z \leq L_2, \quad (2.13)$$

where $c_i^{(j)}$ represents the i th coefficients of the j th section of fiber, L_1 and L_2 are the length of the 1st and 2nd section of fiber, respectively. Moreover, from the variational calculation, it can be shown that the pulse width grows linearly with distance z along the fiber (see Appendix A). Since the squared width is therefore asymptotically quadratic, (2.12) and (2.13) can be rewritten as

$$a^2(z) = c_0^{(1)} + c_1^{(1)}z + c_2^{(1)}z^2 \dots \quad 0 \leq z \leq L_1 \quad (2.14)$$

$$a^2(z) = c_0^{(2)} + c_1^{(2)}(z - L_1) + c_2^{(2)}(z - L_1)^2 \dots \quad L_1 \leq z \leq L_2 \quad (2.15)$$

A quadratic series method was used in [15] to model general pulse behavior in fibers. The authors assumed unchirped Gaussian pulses and derived governing equations for the width based on explicitly computable integral moments. Their approach is independent of [3]. Liao [11] used a variational technique and a quadratic power series

to find the unique pre-chirp which induces a pulse propagating in a single link under anomalous dispersion to return to its exact initial width and chirp; [11] does not consider normal dispersion. The quadratic series in [11] is an expansion in a perturbation parameter whose terms a_1 , a_2 , etc, are corrections to the pulse width. The equivalent of our equations (2.8)-(2.10) in [11] leads to coupled equations for a_2 and the chirp, from which the latter may be inferred. The approach of this thesis is thus independent of [11].

The process of finding the coefficients is given in Appendix B. Equations (2.14) and (2.15) (with explicit expressions for the coefficients) will be used in the following chapters to predict the optimal ways of dispersion compensation; optimal pre-chirp values for a single link are considered in Chapter 3, and dispersion constants and dispersion maps for multiple links are considered in Chapter 4.

Chapter 3

SINGLE LINK

In modern optical communication systems, information in the form of optical pulses is transmitted from a light source, which is either a laser or light emitting diode (LED). After the optical pulses are launched into an optical fiber, they spread out by the effect of dispersion. Moreover, if the optical power is high enough, nonlinear effects can also cause spectral broadening, which may increase the dispersion and cause pulse distortion. Our goal here is to produce a received pulse identical to the transmitted pulse by means of making both output time and spectral width equal to the corresponding input quantities. (It is assumed that the principal effect of dispersion and non-linearity is to change temporal and spectral width but not greatly alter the shape of the pulse). In the presence of the combined effects of dispersion and non-linearity, it is not generally possible to obtain an analytic expression for the output pulse in terms of the input pulse. However, if the pulse shape is assumed to be invariant, then the variational method allows an analytic expression to be obtained for the parameters characterizing the pulse.

One method of counteracting dispersion is connecting another piece of optical fiber to compensate dispersion, called *dispersion compensation fiber*. However, in the presence of nonlinear effects, the optimal conditions or parameters for the compensating fiber might be different from those with only the effect of dispersion. This will be discussed in Chapter 4.

Another way to compensate dispersion is to *pre-chirp* the optical pulse before launching into an optical fiber. In this Chapter, we will calculate the values of pre-chirping for which (i) the initial pulse and spectral width are restored at the output, (ii) output pulse width is minimized, (iii) the output pulse is transform limited, and (iv) the output time-bandwidth product is minimized. It turns out that there is also a maximal length at which the output width can be made equal to the input width via pre-chirp. This has not been noted in the literature. This chapter will calculate this maximal length as well.

3.1 Optimal Pre-Chirping for Equal Input and Output Width

Considering only one link, we can make the output width equal to the input width with a suitable value of pre-chirp (C_{opt}), which will be found as follows.

The analysis begins with the square of optical width in a quadratic equation (B.3) from the variational approximation (see Appendix B for derivation)

$$a^2(z) = a_0^2 + 2\beta_2^{(1)}Cz + \left(\frac{(\beta_2^{(1)})^2}{a_0^2} + \frac{\beta_2^{(1)}\gamma E_0}{\sqrt{2}a_0} + \frac{(\beta_2^{(1)})^2 C^2}{a_0^2} \right) z^2 \quad 0 \leq z \leq L_1, \quad (3.1)$$

where, a_0 and $a(z)$ are the initial and instant pulse widths (half-width at 1/e intensity point) along the distance z , respectively, C represents the value of pre-chirp, γ is the material non-linearity and always with the initial pulse energy (E_0), $\beta_2^{(1)}$ is the dispersion constant. Note that the superscript, (1), is for the first section, which is the only one considered in this chapter. Since, in Chapter 4, there are multiple sections of fiber, we then use the superscripts, (i), of the term $\beta_2^{(i)}$ represents the dispersion constants of the i-th section.

Now some simplifications are obtained by defining some normalized parameters.

$$\text{Let } \Delta_1 = \beta_2^{(1)}L_1, \quad F_1 = \frac{\gamma E_0 L_1}{\sqrt{2}a_0}, \quad u = \frac{\Delta_1}{a_0^2}, \quad N^2 = \frac{a_0^2 \gamma P_0}{|\beta_2^{(1)}|} = \frac{\sqrt{2}F_1}{u}.$$

Note that the subscripts '1' of the parameters; Δ , L , and F represent the values of the first section, whereas, for the multiple sections in Chapter 4, the subscripts (i) is used for the i-th sections of fibers.

Hence, the new simplified equation of pulse width (square) at the output end of fiber (length L_1) is

$$a_1^2 = a^2(L_1) = a_0^2 + 2\Delta_1 C + \left(\frac{\Delta_1^2}{a_0^2} + \Delta_1 F_1 + \frac{\Delta_1^2 C^2}{a_0^2} \right) = a_0^2 + \Delta_1(2C + F_1) + \frac{\Delta_1^2(1 + C^2)}{a_0^2}. \quad (3.2)$$

Equivalently,

$$a_1^2 / a_0^2 = 1 + u(2C + F_1) + u^2(1 + C^2). \quad (3.3)$$

From equation (3.2), by setting $a_1^2 = a_0^2$ at $C = C_{opt}$, we obtain

$$\Delta_1(2C_{opt} + F_1) + \frac{\Delta_1^2(1 + C_{opt}^2)}{a_0^2} = 0,$$

or
$$C_{opt}^2 + 2\frac{a_0^2}{\Delta_1}C_{opt} + \frac{a_0^2 F_1}{\Delta_1} + 1 = 0,$$

or
$$C_{opt}^2 + \frac{2}{u}C_{opt} + \frac{N^2}{\sqrt{2}} + 1 = 0,$$

which is a quadratic equation having solutions

$$C_{opt} = -\frac{1}{u} \pm \frac{1}{u} \sqrt{1 - \left(1 + \frac{N^2}{\sqrt{2}}\right) u^2}.$$

From a practical standpoint, a small value of C_{opt} is preferred to minimize spectral width; hence, by taking the “+” sign for C_{opt} , the optimum pre-chirp will be

$$C_{opt} = -\frac{1}{u} + \frac{1}{u} \sqrt{1 - \left(1 + \frac{N^2}{\sqrt{2}}\right) u^2}. \quad (3.4)$$

However, a real-valued C_{opt} will exist if and only if the following relation holds.

$$1 - \left(1 + \frac{N^2}{\sqrt{2}}\right) u^2 \geq 0,$$

that is
$$\left(1 + \frac{N^2}{\sqrt{2}}\right) u^2 \leq 1,$$

or equivalently $u \leq \sqrt{\frac{1}{\left(1 + \frac{N^2}{\sqrt{2}}\right)}}$.

Then the maximum value of u is $u_{\max} = \sqrt{\frac{1}{\left(1 + \frac{N^2}{\sqrt{2}}\right)}}$. (3.5)

But from the definition of the normalized length u , $u_{\max} = \frac{\beta_2^{(1)} L_{\max}}{a_0^2}$.

Therefore, the maximal length of fiber for existence of an optimal pre-chirp to make the output equal to the input width is

$$L_{\max} = \frac{a_0^2 u_{\max}}{\beta_2^{(1)}} = \frac{a_0^2}{\beta_2^{(1)}} \sqrt{\frac{1}{\left(1 + \frac{N^2}{\sqrt{2}}\right)}}$$

or $L_{\max} = L_D \sqrt{\frac{1}{\left(1 + \frac{N^2}{\sqrt{2}}\right)}}$. (3.6)

This means one can make a_1 equal to a_0 by pre-chirping the optical pulse if and only if the length of fiber is less than L_{\max} . Otherwise, a_1 will always be greater than a_0 .

For the special case when there is small non-linearity $\left(\frac{N^2}{\sqrt{2}} \ll 1\right)$, the limitation will be

$L_{\max} \cong L_D$, which means in the case of small non-linearity, the maximum length of fiber at which the output pulse width can be forced equal to the input width by introducing pre-chirp is L_D (the *Dispersion Length*).

Simulation Results

The following illustrative plot shows the result from the above calculations of optimum pre-chirp by using the variational method, where the initial width $a_0 = 50$ ps, dispersion constant $\beta_2^{(1)} = 2$ ps²/km, length of fiber $L_1 = 100$ Km, material non-linearity $\gamma = 2.432$ W⁻¹-Km⁻¹, pulse energy $E_0 = 0.1$ ps-Watt.

In Figure (3.1), the dashed line shows the evolution of un-chirped pulse width, which broadens from 50 ps to 50.85 ps within 100 Km length of fiber. On the other hand, the compensation of pulse dispersion by optimal pre-chirping is shown in the solid line. The figure is meant only for illustrative purposes; the small amount of pulse broadening is of course due to the low dispersion.

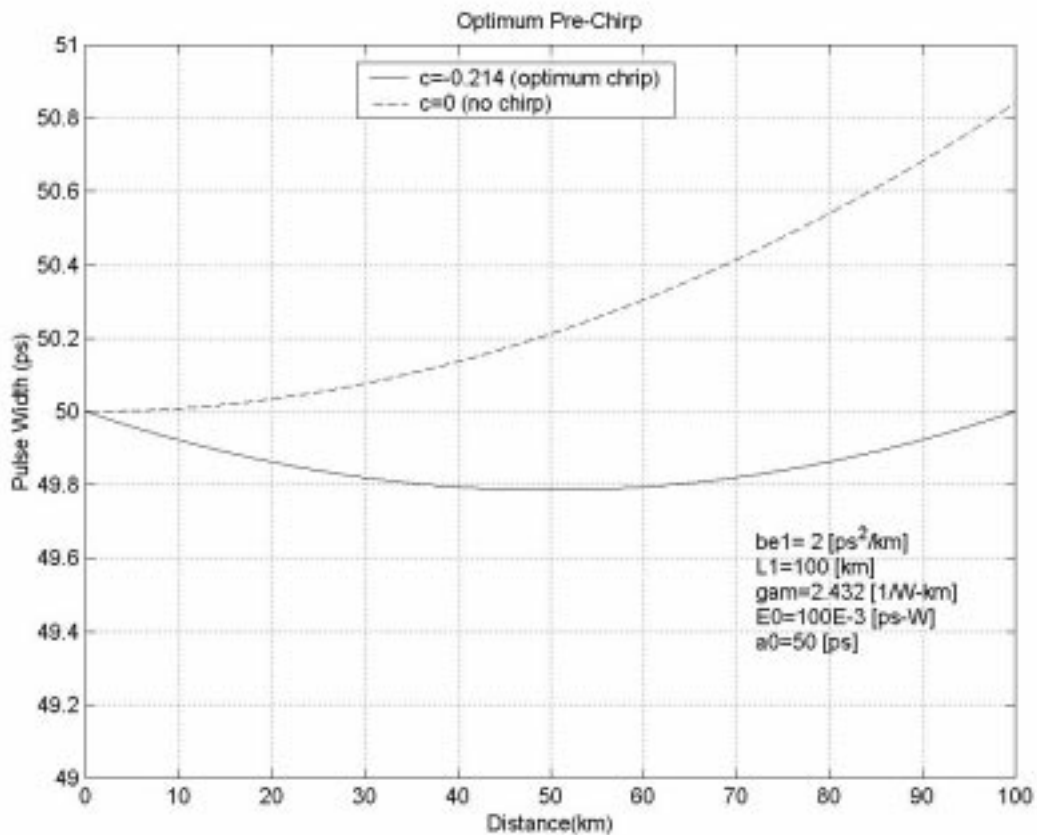


Figure 3.1: Plot of Optimum pre-chirp

The following table shows the comparison with Split-Step Fourier (SSF) simulation results for C_{opt} that forces $a_0 = a_1$.

Table 3.1 is for the case where $a_0 = 20$ ps, $\beta_2^{(1)} = 2$ ps²/km, $\gamma = 2.432$ W⁻¹Km⁻¹, $E_0 = 0.1$ ps-W, and various lengths (L_1). For these parameter values it follows that

$$N^2 = \frac{a_0^2 \gamma P_0}{|\beta_2^{(1)}|} = \frac{20^2 \times 2.432 \times 0.1 / 20}{|2|} = 2.432$$

$$L_{max} = \frac{a_0^2}{\beta_2^{(1)}} \sqrt{\frac{1}{\left(1 + \frac{N^2}{\sqrt{2}}\right)}} = \frac{20^2}{2} \sqrt{\frac{1}{1 + \frac{2.432}{\sqrt{2}}}} = 121.3 \text{ Km}$$

Length of fiber, L_1 (Km)	C_{opt} from Variational Calc.	C_{opt} from SSF. Simulation
123	No real value of C_{opt}	-1.218
122	No real value of C_{opt}	-1.193
121	-1.542	-1.168
120	-1.426	-1.145
115	-1.187	-1.045
110	-1.053	-0.960
105	-0.952	-0.887
100	-0.869	-0.822
90	-0.733	-0.714
80	-0.621	-0.606

Table 3.1. Comparison of C_{opt} between VC & SSF to make $a_0 = a_1$

It is seen from the table that the values of C_{opt} predicted from the variational method are quite closed to those from the split-step Fourier simulation, particularly for a small length of fiber (< 115 Km from Table 3.1). In addition, from SSF simulation results, the

maximum length for this purpose is 132 Km, which is a little bit greater than 121.3 Km from the variational approximation.

In addition, to confirm the consistency between two methods, we make further comparisons. In Figure 3.2, we plot the values of C_{opt} from variational calculations compared to those of split-step simulations for various N and u .

It can be concluded from Figure 3.2 that the results of C_{opt} from variational calculation (lines) technique are accurate and consistent with Split-Step Fourier method (markers) when the nonlinear effect is not too high ($N < 1$), and especially when the parameter u is small (short length of fiber).

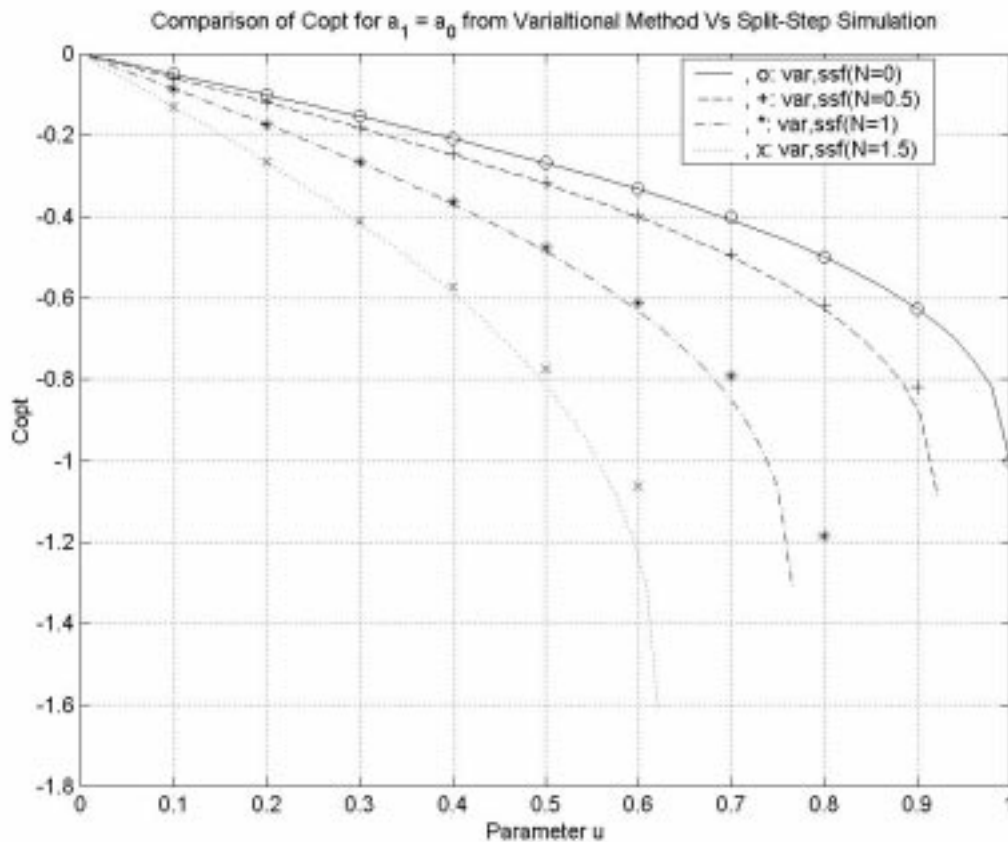


Figure 3.2: The Comparison of C_{opt} for making a_1 equal to a_0 between VC & SSF.

3.2 Equal pulse width and equal spectral width

In the variational analysis, equal input and output pulse (time) widths is equivalent to equal input and output spectral (frequency) widths, which can be shown by the following calculations.

As shown in Appendix C (Eq. C.10), the RMS spectral width square from the variational method is given by

$$\sigma_{\omega}^2(z) = \sigma_{\omega,0}^2 + \frac{\sqrt{2}N^2}{2a_0^2} \left(1 - \frac{a_0}{a(z)} \right). \quad (3.7)$$

Note that when the output pulse width $a(z)$ is equal to the input width a_0 at the end of fiber, of length L_1 , the last term of equation (3.7) is equal to zero, and then the square of the output RMS spectral width will be $\sigma_{\omega}^2(L_1) = \sigma_{\omega,0}^2$, which is equal to the square of the initial spectral width at the input of fiber.

In the other words, one can make the spectral width of an output equal to the input by making the pulse widths equal (at least within the context of the variational approximation). Therefore, the optimal pre-chirp, C_{opt} , which causes pulse widths equal will also make spectral widths equal.

Simulation Result

Figure 3.3 shows 2 plots; pulse width ratio (a_1/a_0) and spectral width ratio ($\sigma_{\omega,1}/\sigma_{\omega,0}$) versus pre-chirp (C) of variational approximations (solid line) and split-step simulations (circle dots), in which $u = 0.5$ and $N = 0.5$. From the upper figure, at $C = 0.32$, pulse width equality occurs ($a_1/a_0 = 1$). For that same value of pre-chirp, in the lower figure, the input and output spectral widths are also equal ($\sigma_{\omega,1}/\sigma_{\omega,0} = 1$), and those equalities can be confirmed by both methods.

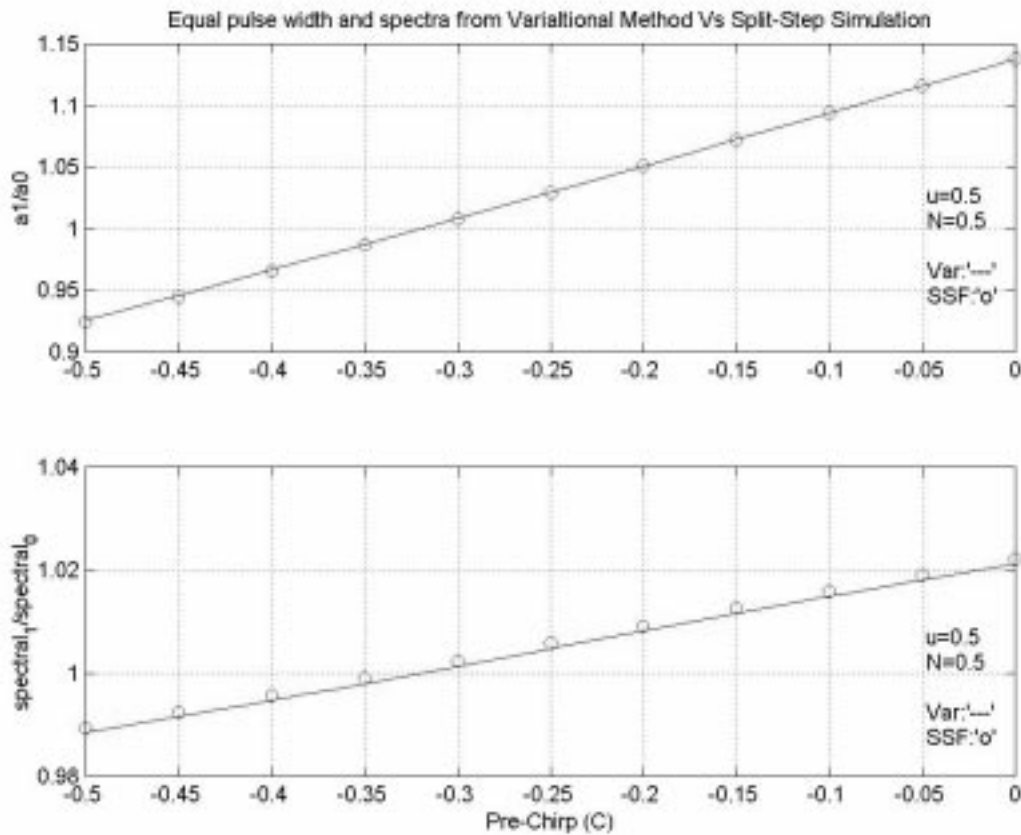


Figure 3.3: The Comparison of Equality of pulse width and spectra between VC & SSF.

3.3 Pre-Chirping for Minimum Output Width

Although the previous results show that there exists a limited length of fiber to achieve equal width by optimum pre-chirp ($L_1 \leq L_{\max}$), for any length of fiber we can still find the new optimum pre-chirp that makes the output width minimum.

By differentiating equation (3.2) with respect to C and setting it equal to zero, we obtain

$$\frac{\partial a_1^2}{\partial C} = 2\Delta_1 + \frac{2\Delta_1^2 C}{a_0^2} = 0,$$

that is $C_{opt} = -\frac{a_0^2}{\Delta_1} = -\frac{1}{u}$. (3.8)

This value of pre-chirp will lead to the smallest output width, and this minimum output width can be found by substituting C_{opt} in equation (3.2). Doing so we obtain

$$a_{1,\min}^2 = a_0^2 + \Delta_1 \left(\frac{-2a_0^2}{\Delta_1} + F_1 \right) + \frac{\Delta_1^2 \left(1 + \frac{a_0^4}{\Delta_1^2} \right)}{a_0^2} = a_0^2 + (-2a_0^2 + \Delta_1 F_1) + \frac{\Delta_1^2}{a_0^2} + a_0^2,$$

or $a_{1,\min}^2 = \Delta_1 F_1 + \frac{\Delta_1^2}{a_0^2} = \Delta_1 u \left(1 + \frac{N^2}{\sqrt{2}} \right)$, (3.9)

and also $\frac{a_{1,\min}^2}{a_0^2} = \frac{\Delta_1}{a_0^2} u \left(1 + \frac{N^2}{\sqrt{2}} \right) = \frac{u^2}{u_{\max}^2}$. (> 1 if $u > u_{\max}$)

That is, in the case where $L_1 > L_{\max}$ or $u > u_{\max}$, the minimum output width ($a_{1,\min}$) is always greater than input width (a_0).

Simulation Results for $C_{opt} = -1/u$

The following plots show the results of optimal pre-chirp for making the output pulse widths minimal. Again, the results are for various nonlinear parameters $N = 0, 0.5, 1,$ and 1.5 . For these cases, the greatest u_{max} is equal to 1. To show the effect of operating with $u > u_{max}$, we set u equal to 2, which leads to $C_{opt} = -0.5$.

Figure 3.4 shows the ratio of output width and input widths (a_1/a_0) versus pre-chirp (C). From the variational calculations (lines), the smallest ratio of pulse width occurs when pre-chirp equal to $C_{opt} = -0.5$, which is consistent with that from split-step method (markers) for small N (less than 1). On the other hand, for large N (more than 1), the magnitude of the optimal pre-chirp from split-step method tends to be a little bit larger than 0.5.

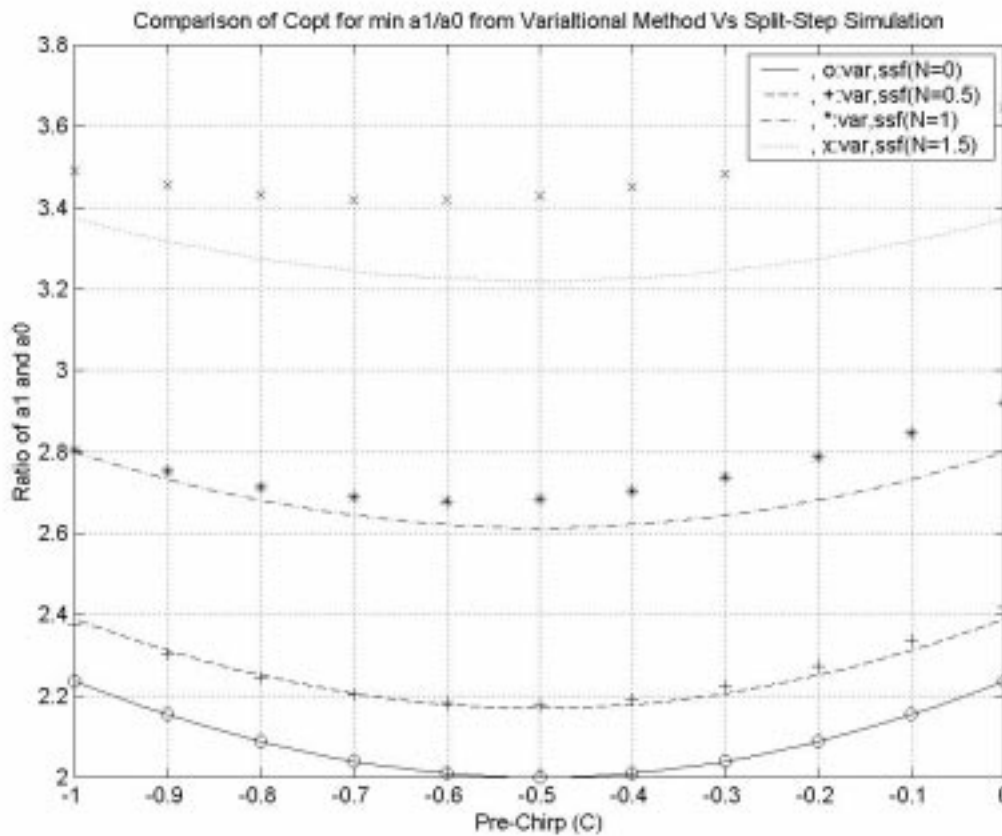


Figure 3.4: Comparison of VC & SSF for minimizing a_1

3.4 Transform Limited Pulse at Output

As previously mentioned, an optical pulse is called transform limited when the pulse has no chirp, $b(z) = 0$. Such a pulse has minimal spectral width, which is desirable in a communication system. In this section the variational method will be used to find the pre-chirp for making the output pulse transform limited.

Note in the variational equation (2.8) that the pulse chirp is related to the derivative of pulse width with respect to propagation distance as

$$a_z(z) = -2\beta_2 a(z)b(z),$$

where the subscript z denotes derivative with respect to z .

Therefore, the optical pulse will be transform limited ($b(z) = 0$) when $a_z(z) = 0$.

Beginning by differentiating the pulse width equation (3.1) with respect to z , we obtain

$$2a(z)a_z(z) = 2\beta_2^{(1)}C + 2\left(\frac{(\beta_2^{(1)})^2}{a_0^2} + \frac{\beta_2^{(1)}\gamma E_0}{\sqrt{2}a_0} + \frac{(\beta_2^{(1)})^2 C^2}{a_0^2}\right)z, \quad 0 \leq z \leq L_1.$$

Substituting $z = L_1$ in this expression gives

$$2a_1 a_z(L_1) = 2\beta_2^{(1)}C + 2\left(\frac{(\beta_2^{(1)})^2}{a_0^2} + \frac{\beta_2^{(1)}\gamma E_0}{\sqrt{2}a_0} + \frac{(\beta_2^{(1)})^2 C^2}{a_0^2}\right)L_1. \quad (3.10)$$

By setting the right hand side of equation (3.10) equal to zero, we obtain

$$\begin{aligned} \beta_2^{(1)}C + \left(\frac{(\beta_2^{(1)})^2}{a_0^2} + \frac{\beta_2^{(1)}\gamma E_0}{\sqrt{2}a_0} + \frac{(\beta_2^{(1)})^2 C^2}{a_0^2}\right)L_1 &= 0, \\ C + \left(\frac{\beta_2^{(1)}}{a_0^2} + \frac{\gamma E_0}{\sqrt{2}a_0} + \frac{\beta_2^{(1)} C^2}{a_0^2}\right)L_1 &= 0, \end{aligned}$$

or $C + [u(1 + C^2) + F_1] = 0,$

$$C^2 + \frac{1}{u}C + \frac{F_1}{u} + 1 = 0,$$

that is $C^2 + \frac{1}{u}C + \frac{N^2}{\sqrt{2}} + 1 = 0.$

Therefore, the value of C for making output pulse transform limited is

$$C = -\frac{1}{2u} \pm \frac{1}{u} \sqrt{\frac{1}{4} - \left(1 + \frac{N^2}{\sqrt{2}}\right) u^2}.$$

Again, a small value of chirp is preferred to minimize the spectral width; hence, the desired pre-chirp will be

$$C_{TL} = -\frac{1}{2u} + \frac{1}{u} \sqrt{\frac{1}{4} - \left(1 + \frac{N^2}{\sqrt{2}}\right) u^2}. \quad (3.11)$$

However, C_{TL} will exist if and only if the following relation holds,

$$\frac{1}{4} - \left(1 + \frac{N^2}{\sqrt{2}}\right) u^2 \geq 0,$$

that is $u \leq \frac{1}{2} \sqrt{\frac{1}{\left(1 + \frac{N^2}{\sqrt{2}}\right)}}.$

Then the maximum value of u is

$$u_{\max} = \frac{1}{2} \sqrt{\frac{1}{\left(1 + \frac{N^2}{\sqrt{2}}\right)}},$$

with the corresponding maximum length of fiber given by

$$L_{\max} = \frac{L_D}{2} \sqrt{\frac{1}{\left(1 + \frac{N^2}{\sqrt{2}}\right)}} \cong \frac{L_D}{2} \quad (\text{for small } N). \quad (3.12)$$

Notice that the maximum length for which one can make the output pulse transform-limited by pre-chirping is one half of the maximum length for making equal input and output widths. This argument can also be verified by using the symmetry of the variational equation ($a_{zz}(z)$), which will be illustrated in the next chapter.

Moreover, the value of pre-chirping required to make the output transform-limited is larger than that of making equal input and output widths. That means the spectral width of the optical pulse at the front end of fiber is also larger. This difference will be used to trade-off for the optimal way of conserving time-bandwidth product; this is discussed further in section 3.5.

Simulation Results

The following plots show the results of C_{TL} for making the output pulse transform limited. Again, the results are for various nonlinear parameters $N = 0, 0.5, 1,$ and $1.5,$ for which the greatest u_{max} , is equal to 0.5 . Consequently, the parameter u is varied from zero to 0.5 ($u < u_{max}$).

The results from both the VC and SSF indicate that when N increases, the maximum value of u for which a transform limited output may be achieved reduces. When N is large ($N > 0.5$), the VC somewhat overestimates this maximum distance. For example, at N equal to 1 , the variational method shows u_{max} equal to 0.38 ; on the other hand, SSF predicts that the maximum u for which transform limited can be achieved is equal to only 0.3 .

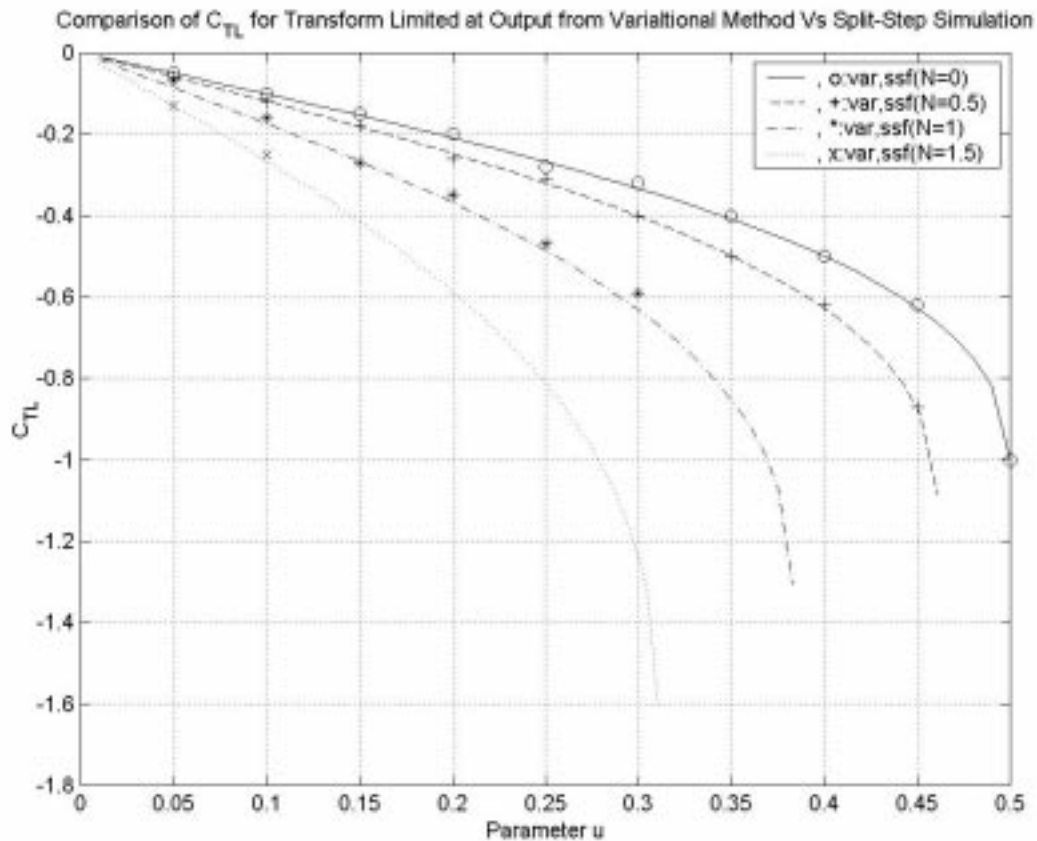


Figure 3.5: The Comparison of VC & SSF for making output Transform Limited

3.5 Minimizing Output Time-Bandwidth Product (TBP)

Time-Bandwidth Product (TBP) is the product of the RMS pulse and spectral widths of an optical pulse propagating in a fiber. Physically, TBP measures the bandwidth efficiency of any communication system. Normally, a higher transmission rate system requires sending shorter pulses resulting in a higher bandwidth. The minimal time-bandwidth product is obtained when we are transform limited. However, it may not be possible to be transform limited at the output of the fiber. Thus, we wish to consider signal design to obtain an optimal time-bandwidth product. In this section we show how to use the variational method to determine the time-bandwidth product, and find the appropriate pre-chirp required to minimize it.

From the previous section, we know that the half-width at 1/e- intensity point of an optical pulse is represented by $a(z)$, which is related to the RMS pulse width by the relationship

$$\sigma_t^2(z) = \frac{a^2(z)}{2}.$$

As shown in Appendix C (Eq. C.8), the RMS spectral width square from the variational method

$$\sigma_\omega^2(z) = \frac{1}{2} \left(\frac{1}{a^2} + \frac{a_z^2}{(\beta_2^{(1)})^2} \right), \quad (3.13)$$

then,
$$\sigma_t^2(z)\sigma_\omega^2(z) = \frac{a^2(z)}{2} \times \frac{1}{2} \left(\frac{1}{a^2(z)} + \frac{a_z^2}{(\beta_2^{(1)})^2} \right) = \frac{1}{4} \left(1 + \frac{a^2(z)a_z^2}{(\beta_2^{(1)})^2} \right).$$

Finally, we will have time-bandwidth product as

$$\text{TBP: } \sigma_t(z)\sigma_\omega(z) = \frac{1}{2} \sqrt{1 + \frac{a^2(z)a_z^2}{(\beta_2^{(1)})^2}}. \quad (3.14)$$

Note from equation (3.14) that TBP will have its minimal value equal to 0.5, which occurs when the pulse is transform limited, in which case $a^2(z)a_z^2(z)$ or $a(z)a_z(z) = 0$. Therefore, we can find the appropriate pre-chirping for which TBP is minimal at the output of fiber as follows.

Note that to make the condition $a(z)a_z(z) = 0$ at the end of fiber length L_1 by pre-chirping is the same procedure as making the output pulse transform-limited ($a_z(z) = 0$), which has already been discussed. Therefore, the pre-chirp required to make the time-bandwidth product minimal at the output is the same as the pre-chirp required to make the output pulse transform-limited, which is

$$C_{TBP} = C_{TL} = -\frac{1}{2u} + \frac{1}{u} \sqrt{\frac{1}{4} - \left(1 + \frac{N^2}{\sqrt{2}}\right) u^2}. \quad (3.15)$$

Thus, there exists the same maximum length for doing this, which is,

$$L_{\max} = \frac{L_D}{2} \sqrt{\frac{1}{\left(1 + \frac{N^2}{\sqrt{2}}\right)}}.$$

Simulation Results

The following plot shows the result of Time Bandwidth Products (TBP) at the output of a fiber versus pre-chirping values from variational calculation (lines) and split-step simulation (markers). Notice that when non-linearity is large ($N > 1$), the minimum time bandwidth product is greater than 0.5; however, the minimum points from both methods are still the same. Consequently, the variational calculation and split-step simulation are consistent in predicting the pre-chirp for minimizing the time bandwidth product of the output pulse. Note in the plot that the parameter u is equal to 0.2, which is less than u_{\max} for all N values.

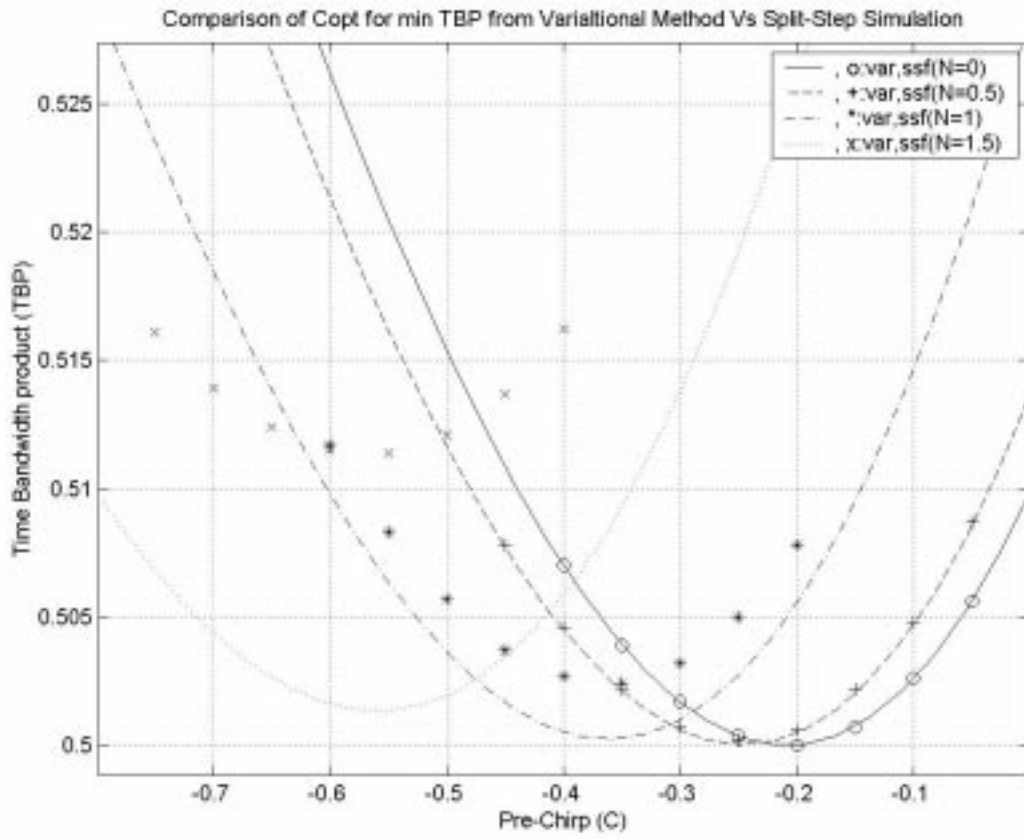


Figure 3.6: The Comparison of VC & SSF for minimizing output TBP.

An important observation from these calculations is that although we can make the output pulse transform-limited (least bandwidth consumption) by pre-chirping an input pulse, if the required value of that pre-chirp is large, then the spectral width at the beginning of propagation is also large. Then cross-talk interference might occur if there are multiple channels. Alternatively, the optimal way to conserve bandwidth might be achieved by making the output spectral width equal to the input. Making the output pulse width equal to the input leads to that equilibrium. This method uses a smaller pre-chirp than that of making the output pulse transform-limited.

Chapter 4

SYMMETRY & MULTIPLE LINKS

In the previous chapter, pre-chirping was used to overcome pulse dispersion in an optical fiber. However, there is a limit to the fiber length to which this method applies. Another way to compensate dispersion for long distance ($L_1 > L_{\max}$) communication systems is to use concatenated links, which may comprise one or more additional sections. In the ideal linear case, perfect compensation occurs when the sum of the dispersion-distance products of the multiple links is zero. However, in the presence of non-linearity a new relationship is found to govern many operating systems, and that will be discussed in this chapter

This chapter will begin with the proof of the symmetry of pulse width evolution while propagating in fiber, and its use in predicting dispersion phenomena and dispersion maps within the variational approximation.

4.1 Symmetry

Firstly, let z_0 be the distance, measured from the front end of a fiber, where the slope of the plot of pulse width, $a(z)$, versus distance, z , is equal to zero ($a_z(z_0) = 0$).

Let $r(z) = a(z_0 + z)$ and $l(z) = a(z_0 - z)$. Then $r_z(z) = a_z(z_0 + z)$, $l_z(z) = -a_z(z_0 - z)$, by differentiation. Also $r_{zz}(z) = a_{zz}(z_0 + z)$, $l_{zz}(z) = -a_{zz}(z_0 - z)(-1) = a_{zz}(z_0 - z)$.

Then

$$r_{zz}(z) = a_{zz}(z_0 + z) = \frac{(\beta_2^{(1)})^2}{a^3(z_0 + z)} + \frac{\beta_2^{(1)} \gamma E_0}{\sqrt{2} a^2(z_0 + z)} = \frac{(\beta_2^{(1)})^2}{r^3(z)} + \frac{\beta_2^{(1)} \gamma E_0}{\sqrt{2} r^2(z)},$$

and similarly

$$l_{zz}(z) = \frac{(\beta_2^{(1)})^2}{l^3(z)} + \frac{\beta_2^{(1)}\gamma E_0}{\sqrt{2}l^2(z)}.$$

Since $r(z)$ and $l(z)$ satisfy the same differential equation,

$$y_{zz}(z) = \frac{(\beta_2^{(1)})^2}{y^3(z)} + \frac{\beta_2^{(1)}\gamma E_0}{\sqrt{2}y^2(z)},$$

and the same initial conditions $y(0) = a(z_0)$, $y'(0) = 0$, then they are identical; i.e., $l(z) = r(z)$. This is the same as $a(z_0 + z) = a(z_0 - z)$, so $a(z)$ is symmetric about chirp free points z_0 where $a_z(z_0) = 0$.

Simulation Results

The following plots show the symmetry of the pulse width from variational calculations (lines) compared with split-step simulations (markers). In Figure 4.1, the minimum point of pulse width is at a normalized distance $z/L_d = 0.5$, for which the slope (a_z) is equal to zero. It may be noticed that pulse widths are symmetric around this point as predicted by the variational method. Although, for large non-linearity ($N > 1$), pulse widths simulated from the split-step method are not exactly the same as those of the variational method, the symmetry of the curve is still clearly noticed.

By recognizing the symmetry of pulse width, one might easily interpret the previous result in Chapter 3 for the maximum distance for which a transform limited output pulse can be obtained. It is one half of the maximum distance in which the equal input and output width can be achieved by pre-chirping.

In addition, another advantage of symmetry is to readily design appropriate dispersion maps in concatenated link systems, for which the chirp, pulse width, and spectrum can be brought back to their initial values at the output. That design will be discussed later in this chapter.

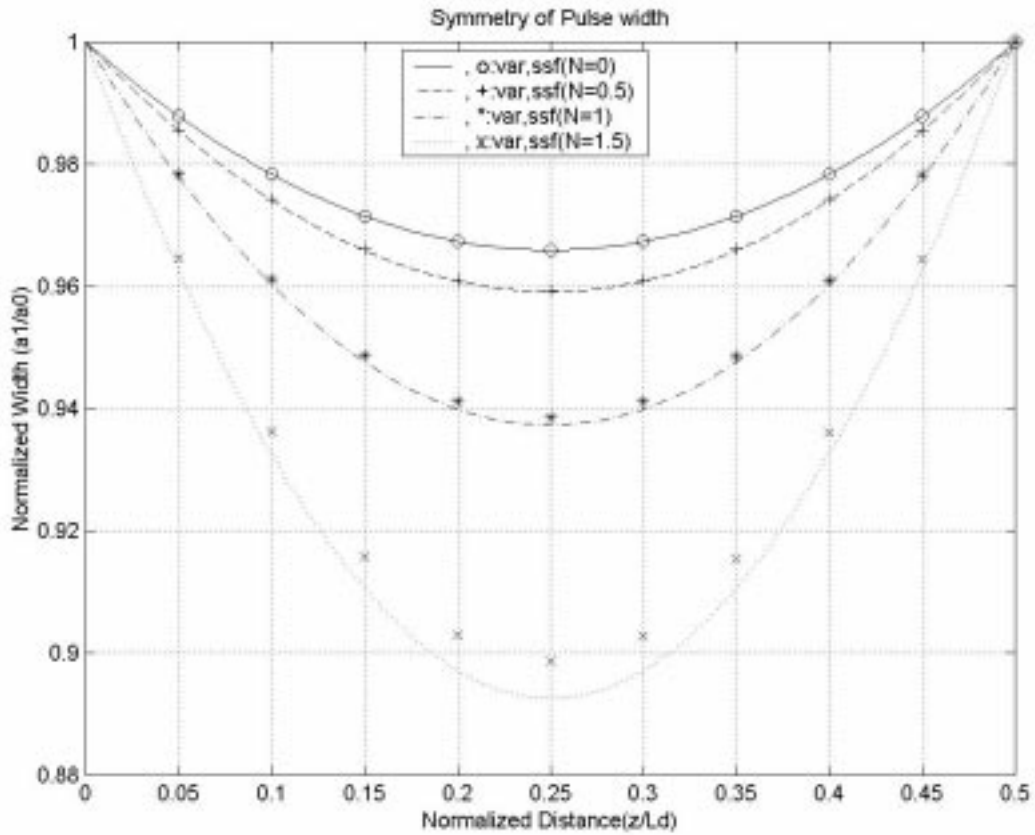


Figure 4.1: The symmetry of pulse width

4.2 Optimal Dispersion Constant for Dispersion Compensation Fiber

This section covers the calculation of optimal dispersion constant for dispersion compensating fiber. We consider a system with two sections of fiber. The first section is the main fiber that has nonlinear and (normal) dispersion effects. The second (shorter) section is the compensation fiber with anomalous dispersion ($\beta_2^{(2)} < 0$). The objective is to find the value of $\beta_2^{(2)}$ in closed form for which pulse broadening is compensated by the compensation fiber in the presence of the nonlinear effect. First, we will define the pulse width equation for the second section of fiber, which is derived in Appendix C, and show that non-linearity in the second fiber, is negligible which simplifies the equation. Then that equation is used to find $\beta_2^{(2)}$.

4.2.1 Pulse Width Equation with Simplification

Beginning with the pulse width equation (derived in Appendix B, see Eqs. B.7 and B.8)

$$a^2(z) = a_1^2 + c_1(z - L_1) + \left(\frac{(\beta_2^{(2)})^2}{a_1^2} + \frac{\beta_2^{(2)}\gamma E_0}{\sqrt{2}a_1} + \frac{c_1^2}{4a_1^2} \right) (z - L_1)^2 \quad \dots L_1 \leq z \leq L_1 + L_2, \quad (4.1)$$

$$\text{where} \quad c_1 = 2\beta_2^{(2)}C + 2 \left(\frac{\beta_2^{(1)}\beta_2^{(2)}}{a_0^2} + \frac{\beta_2^{(2)}\gamma E_0}{\sqrt{2}a_0} + \frac{\beta_2^{(1)}\beta_2^{(2)}C^2}{a_0^2} \right) L_1. \quad (4.2)$$

Define the normalized parameters

$$\Delta_1 = \beta_2^{(1)}L_1, \quad F_1 = \frac{\gamma E_0 L_1}{\sqrt{2}a_0}, \quad u = \frac{\Delta_1}{a_0^2}, \quad N^2 = \frac{a_0^2 \gamma P_0}{|\beta_2^{(1)}|} = \frac{\sqrt{2}F_1}{u}, \quad \Delta_2 = \beta_2^{(2)}L_2, \quad F_2 = \frac{\gamma E_0 L_2}{\sqrt{2}a_0}.$$

We then have the new simplified equation of the pulse width at distance $L_1 + L_2$, where L_2 is the length of the second section of fiber, as

$$a^2(L_1 + L_2) = a_2^2 = a_1^2 + \Gamma + \left(\frac{\Delta_2^2}{a_1^2} + \Delta_2 F_2 + \frac{\Gamma^2}{4a_1^2} \right), \quad (4.3)$$

$$\text{where } \Gamma = 2\Delta_2 C + 2 \left(\frac{\Delta_1 \Delta_2}{a_0^2} + \Delta_2 F_1 + \frac{\Delta_1 \Delta_2 C^2}{a_0^2} \right) = 2\Delta_2 (C + F_1) + \frac{2\Delta_1 \Delta_2}{a_0^2} (1 + C^2). \quad (4.4)$$

It is reasonable to suppose that nonlinear effects are small in the second link because the peak power decreases in the first link and because the second link is usually short by comparison. Indeed, the nonlinear effect, which is strongest in the beginning of the first link, is expected to be small by the end. Therefore, we are justified in treating the second link as linear. Hence, the nonlinear term $\Delta_2 F_2$ in equation (4.3) can be eliminated, and we have

$$a_2^2 = a_1^2 + \Gamma + \left(\frac{\Delta_2^2}{a_1^2} + \frac{\Gamma^2}{4a_1^2} \right). \quad (4.5)$$

We will confirm this linear behavior by split-step simulations.

Simulation Results

The following plots show the output pulse width a_2 versus L_2 with various values of pulse energies. In this plot, we use equation (4.5), which neglects the non-linear term, for the variational approximation (solid lines), and compare the results with split-step (circle points) simulation with the parameters; $C = 0$, $a_0 = 20\text{ps}$, $\beta_2^{(1)} = 2\text{ps}^2/\text{km}$, $L_1 = 100\text{km}$, $\beta_2^{(2)} = -20\text{ps}^2/\text{km}$, and $\gamma = 2.432 \text{ W}^{-1}\text{km}^{-1}$. Note that $E_0 = 10, 30, 70, \text{ and } 100\text{ps-mW}$ correspond to $N = 0.49, 0.99, 1.30, \text{ and } 1.56$, respectively. The lengths of the second fiber are between 0 and 20km.

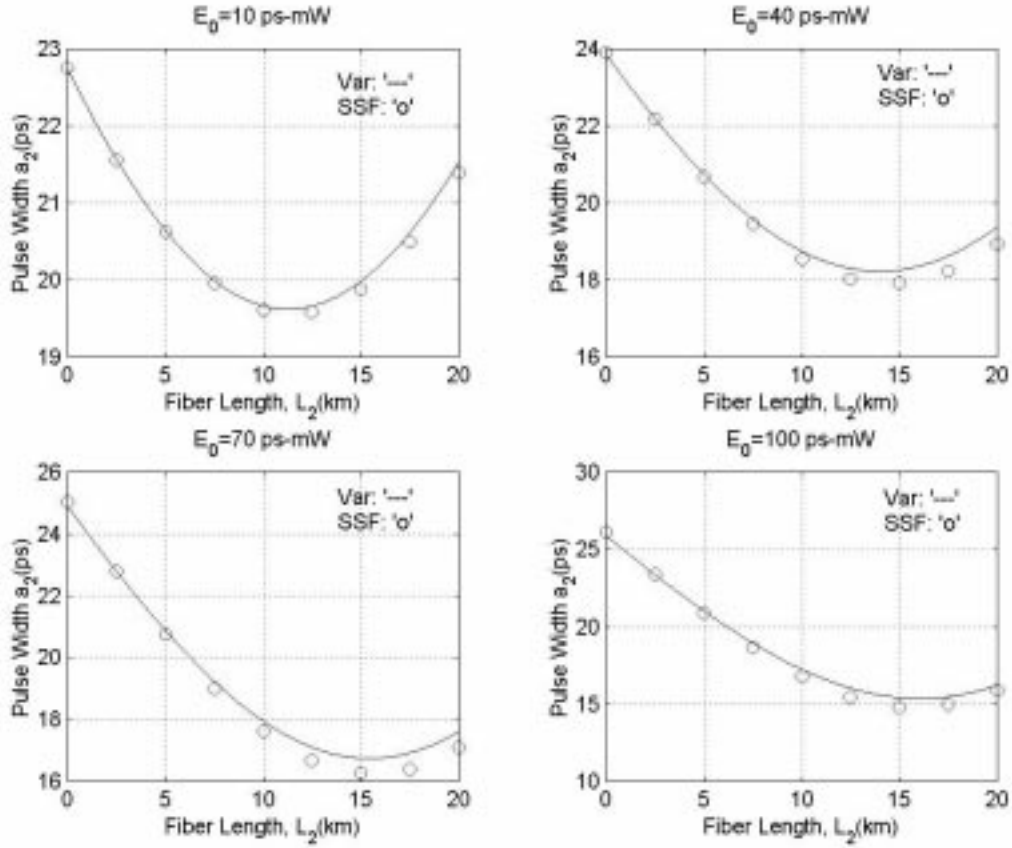


Figure 4.2: Showing that non-linearity in the second fiber is negligible

4.2.2 Optimal Dispersion Constant

In the ideal linear case, perfect dispersion compensation occurs when the sum of the dispersion constant-distance products of the two links is zero ($\Delta_2 = -\Delta_1$). However, in the presence of non-linearity, the relationship is changed, which will now be discussed.

Multiplying the pulse width equation (3.2) of the first fiber by a_0^2 ,

$$a_0^2 a_1^2 = a_0^4 + \Delta_1 (2C + F_1) a_0^2 + \Delta_1^2 (1 + C^2). \quad (4.6)$$

Multiplying the pulse width equation (4.5) of the second fiber by a_1^2 ,

$$a_1^2 a_2^2 = a_1^4 + \Gamma a_1^2 + \left(\Delta_2^2 + \frac{\Gamma^2}{4} \right). \quad (4.7)$$

Writing down all terms in (4.7),

$$\begin{aligned} a_1^4 &= \left[a_0^2 + \Delta_1(2C + F_1) + \frac{\Delta_1^2(1 + C^2)}{a_0^2} \right]^2 \\ &= a_0^4 + \Delta_1^2(2C + F_1)^2 + \frac{\Delta_1^4(1 + C^2)^2}{a_0^4} + 2a_0^2\Delta_1(2C + F_1) + 2(2C + F_1)\frac{\Delta_1^3(1 + C^2)}{a_0^2} + 2\Delta_1^2(1 + C^2), \end{aligned}$$

$$\begin{aligned} \Gamma a_1^2 &= \left[2\Delta_2(C + F_1) + \frac{2\Delta_1\Delta_2}{a_0^2}(1 + C^2) \right] \times \left[a_0^2 + \Delta_1(2C + F_1) + \frac{\Delta_1^2(1 + C^2)}{a_0^2} \right] \\ &= 2\Delta_2(C + F_1)a_0^2 + 2\Delta_1\Delta_2(2C + F_1)(C + F_1) + \frac{2\Delta_1^2\Delta_2(C + F_1)(1 + C^2)}{a_0^2} + 2\Delta_1\Delta_2(1 + C^2) \\ &\quad + \frac{2\Delta_1^2\Delta_2}{a_0^2}(2C + F_1)(1 + C^2) + \frac{2\Delta_1^3\Delta_2}{a_0^4}(1 + C^2)^2 \\ &= 2\Delta_2(C + F_1)a_0^2 + 2\Delta_1\Delta_2 \left[(2C + F_1)(C + F_1) + (1 + C^2) \right] + \frac{2\Delta_1^2\Delta_2(3C + 2F_1)(1 + C^2)}{a_0^2} \\ &\quad + \frac{2\Delta_1^3\Delta_2}{a_0^4}(1 + C^2)^2, \end{aligned}$$

$$\begin{aligned} \Gamma^2 &= \left[2\Delta_2(C + F_1) + \frac{2\Delta_1\Delta_2}{a_0^2}(1 + C^2) \right]^2 \\ &= 4\Delta_2^2(C + F_1)^2 + \frac{4\Delta_1^2\Delta_2^2}{a_0^4}(1 + C^2)^2 + \frac{8\Delta_1\Delta_2^2}{a_0^2}(1 + C^2)(C + F_1). \end{aligned}$$

Therefore,

$$\begin{aligned}
a_1^2 a_2^2 &= a_1^4 + \Gamma a_1^2 + \left(\Delta_2^2 + \frac{\Gamma^2}{4} \right) \\
&= a_0^4 + \Delta_1^2 (2C + F_1)^2 + \frac{\Delta_1^4 (1 + C^2)^2}{a_0^4} + 2\Delta_1 (2C + F_1) a_0^2 + 2(2C + F_1) \frac{\Delta_1^3 (1 + C^2)}{a_0^2} \\
&\quad + 2\Delta_1^2 (1 + C^2) + 2\Delta_2 (C + F_1) a_0^2 + 2\Delta_1 \Delta_2 \left[(2C + F_1)(C + F_1) + 2(1 + C^2) \right] \\
&\quad + \frac{2\Delta_1^2 \Delta_2 (3C + 2F_1)(1 + C^2)}{a_0^2} + \frac{2\Delta_1^3 \Delta_2 (1 + C^2)^2}{a_0^4} + \Delta_2^2 + \Delta_2^2 (C + F_1)^2 + \frac{\Delta_1^2 \Delta_2^2 (1 + C^2)^2}{a_0^4} \\
&\quad + \frac{2\Delta_1 \Delta_2^2 (1 + C^2)(C + F_1)}{a_0^2} \\
&= a_0^4 + 2[\Delta_1 (2C + F_1) + \Delta_2 (C + F_1)] a_0^2 + \Delta_1^2 [(2C + F_1)^2 + 2(1 + C^2)] \\
&\quad + 2\Delta_1 \Delta_2 [(2C + F_1)(C + F_1) + 2(1 + C^2)] + \Delta_2^2 [1 + (C + F_1)^2] \\
&\quad + \frac{2(1 + C^2)}{a_0^2} [\Delta_1^3 (2C + F_1) + \Delta_1^2 \Delta_2 (3C + 2F_1) + \Delta_1 \Delta_2^2 (C + F_1)] \\
&\quad + \frac{\Delta_1^2 (1 + C^2)^2}{a_0^4} [(\Delta_1^2 + \Delta_2^2) + 2\Delta_1 \Delta_2] \tag{4.8}
\end{aligned}$$

For perfect dispersion compensation ($a_2 = a_0$), equation (4.6) is equal to (4.8). Note that the terms a_0^4 cancel. Carrying out operations leads to a complicated expression involving all the fiber parameters in the two links. We can solve for, or optimize, one parameter in terms of the others by using mathematical software.

The complicated expression for $a_2 = a_0$ simplifies in an important special case. For large a_0 let us retain only the dominant terms of $a_0^2 a_2^2 = a_1^2 a_2^2$. In the limit of large a_0 the dominant terms are

$$\Delta_1 (2C + F_1) a_0^2 = 2[\Delta_1 (2C + F_1) + \Delta_2 (C + F_1)] a_0^2,$$

which gives
$$\Delta_2 = -\frac{\Delta_1 (2C + F_1)}{2(C + F_1)} = -\frac{\Delta_1}{2} \left(1 + \frac{C}{(C + F_1)} \right). \tag{4.9}$$

Notice that the values of Δ_2 in (4.9) are between $-\Delta_1$ and $-\Delta_1/2$ depending on pre-chirp (C) and non-linearity (F₁). When there is no pre-chirp (C = 0), we have

$$\Delta_2 = -\frac{\Delta_1}{2}, \text{ whereas in the linear case (F}_1 = 0), \Delta_2 = -\Delta_1.$$

In figure 4.3, we plot the pulse width versus propagation distance (z) with parameters; C = 0 and 0.1, $a_0 = 50\text{ps}$, $\beta_2^{(1)} = 2\text{ps}^2/\text{km}$, $L_1 = 100\text{km}$, $L_2 = 10\text{km}$, $\gamma = 2.432 \text{ W}^{-1}\text{km}^{-1}$, and $E_0 = 100\text{ps-mW}$. Note that for C = 0 and 0.1, the variational approximation predicts that the corresponding $\beta_2^{(2)} = -10$ or $-5.9 \text{ ps}^2/\text{km}$, respectively, will make $a_2 = a_0$. From the figure, the pulse width broadens in the first section ($0 < z < 100$), and then compresses to its original value at the output of the second fiber ($z = 110\text{km}$) with the values of $\beta_2^{(2)}$ as predicted from the variational method.

The following Split-Step simulations confirm the results of the variational calculations. In figure 4.4, we plot the compensation length L_2 , to make the output width equal to the input width, for input widths a_0 ranging from 10 to 100ps. Different curves in the figure are for the variational and split-step methods for fixed total pulse energies 100, 500, and 1000ps-mW, respectively. We set C = 0, $\beta_2^{(1)} = 2\text{ps}^2/\text{km}$, $L_1 = 100\text{km}$, $\beta_2^{(2)} = -20\text{ps}^2/\text{km}$, and $\gamma = 2.432 \text{ W}^{-1}\text{km}^{-1}$. Note that we expect $L_2 = -\frac{\Delta_1}{2\beta_2^{(2)}} = \frac{2 \times 100}{2 \times 20} = 5\text{km}$ for high input width in all cases (energies).

We can see from Figure 4.4 that energy does not effect the compensation length at high input width, which is expected since the energy term F₁ is gone when C = 0 in (4.9). However, in the small input width range, the compensation lengths are larger than 5km, which is also expected since nonlinear effects are so small that the fiber is nearly linear. Note that if this fiber is operating in the linear regime, the compensation length will be 10km.

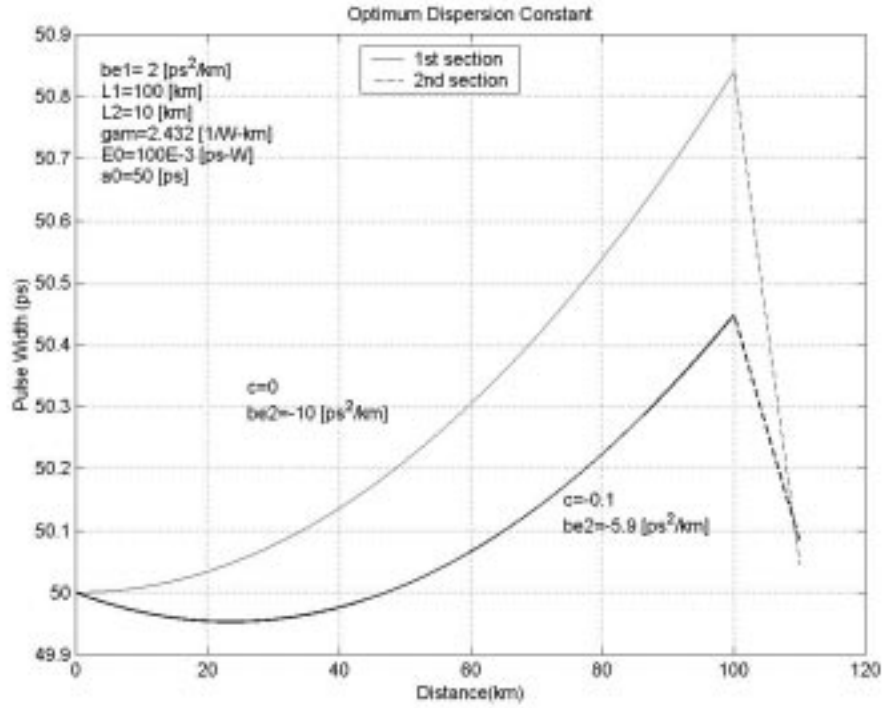


Figure 4.3: Optimal Dispersion Constant of the 2nd fiber.

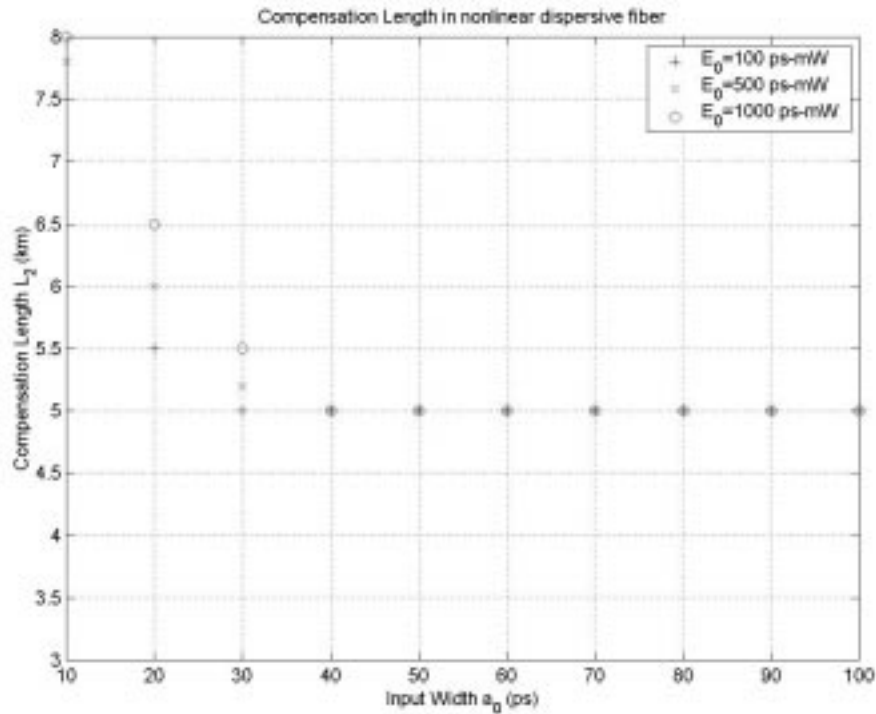


Figure 4.4: Compensation length (L_2) in nonlinear dispersive fiber by SSF.

Although using a second section of fiber with an appropriate dispersion constant (4.9) and length can compensate dispersion from the first section and bring back the initial width at the output, the slope of plot at the output of the second fiber (which represents chirp), is not equal to that of the input pulse. Since the chirps are not the same, the spectra are also not the same. Thus, although the pulse width is restored, the pulse has not been restored to its initial conditions. This motivates consideration of system design in which the initial conditions of an input pulse will be restored at the output. Means for accomplishing this are discussed in the following section.

4.3 Dispersion Map

The objective here is to suggest the dispersion map for the system consisting of concatenated links of fibers, in which the pulse at the end is identical to the input pulse. Two types of dispersion maps are discussed below.

4.3.1 Two-Link System

A two-link system can be exploited when the length of the first section is less than the maximum length in which a_1 can be equal to a_0 by pre-chirping ($L_1 \leq L_{\max}$). The idea is to pre-chirp the input pulse in order to make a_1 equal to a_0 . Then we use symmetry and the previous derivation of the second link to predict the parameters of the second link in which $a_2 = a_1$. Note that when $a_0 = a_1$, the spectral widths will then be the same at the input and the end of the first section; see section (3.2) in Chapter 3. The condition $a_2 = a_1$ will also lead to the equality of spectral widths of the second fiber. Therefore, the previous two equalities of pulse width will then cause the equality of all spectral widths; $\sigma_{\omega,0} = \sigma_{\omega,1} = \sigma_{\omega,2}$. In Figure 4.5 we demonstrate the two-link compensation scheme taken through three iterations. The inset figure shows the two-link configuration. In the first link we have $a_0 = a_1$ at the output and the sign of the chirp

(a_z) is reversed at L_1 . Link two, from L_1 to $L_1 + L_2$, gives $a_2 = a_1$ and another reversed chirp sign. By symmetry about the chirp free point in link two, the input pulse at $L_1 + L_2$ has identical width, chirp, and spectrum to the original input pulse at $z = 0$. Hence the two link setup provides a periodic dispersion map.

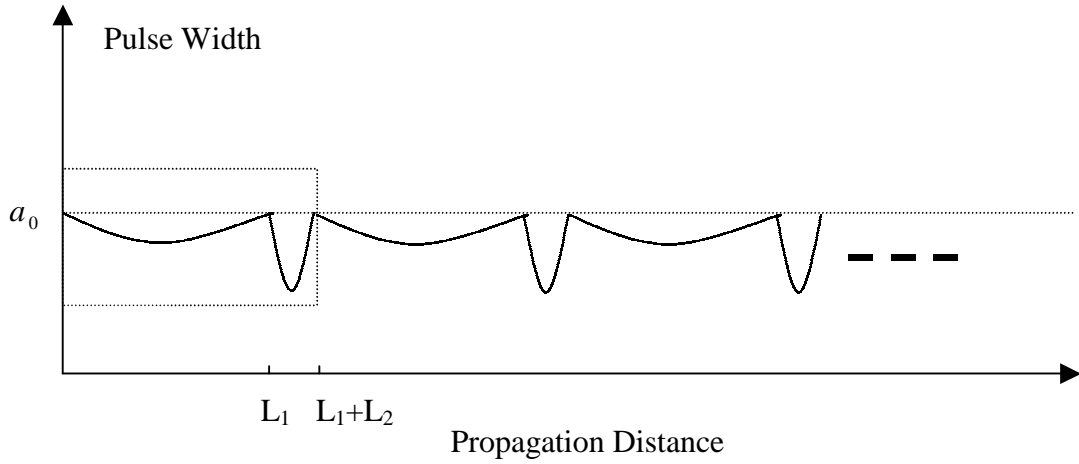


Figure 4.5: Two Link Consecutive System

Calculations of Δ_2 for $a_2 = a_1$

Beginning with equation (4.5), we have

$$a_2^2 = a_1^2 + \Gamma + \left(\frac{\Delta_2^2}{a_1^2} + \frac{\Gamma^2}{4a_1^2} \right). \tag{4.5}$$

Let $a_2 = a_1$, we then have

$$\Gamma a_1^2 = - \left(\Delta_2^2 + \frac{\Gamma^2}{4} \right),$$

or
$$\left[2\Delta_2(C + F_1) + \frac{2\Delta_1\Delta_2}{a_0^2}(1 + C^2) \right] \times \left[a_0^2 + \Delta_1(2C + F_1) + \frac{\Delta_1^2(1 + C^2)}{a_0^2} \right]$$

$$= -\Delta_2^2 - \left[\Delta_2(C + F_1) + \frac{\Delta_1\Delta_2}{a_0^2}(1 + C^2) \right]^2.$$

Dividing Δ_2 into both sides,

$$\left[2(C + F_1) + \frac{2\Delta_1}{a_0^2}(1 + C^2) \right] \times \left[a_0^2 + \Delta_1(2C + F_1) + \frac{\Delta_1^2(1 + C^2)}{a_0^2} \right]$$

$$= -\Delta_2 \left(1 + \left[(C + F_1) + \frac{\Delta_1}{a_0^2}(1 + C^2) \right]^2 \right).$$

Then we will have

$$\Delta_2 = -2 \frac{\left[(C + F_1) + \frac{\Delta_1}{a_0^2}(1 + C^2) \right] \times \left[a_0^2 + \Delta_1(2C + F_1) + \frac{\Delta_1^2(1 + C^2)}{a_0^2} \right]}{\left(1 + \left[(C + F_1) + \frac{\Delta_1}{a_0^2}(1 + C^2) \right]^2 \right)},$$

that is
$$\Delta_2 = -2a_0^2 \frac{[(C + F_1) + u(1 + C^2)] \times [1 + u(2C + F_1) + u^2(1 + C^2)]}{1 + [(C + F_1) + u(1 + C^2)]^2}. \quad (4.10)$$

Equation (4.10) is the general form of Δ_2 for making $a_2 = a_1$. However, in this case, we select pre-chirp for which $a_1 = a_0$, therefore we have

$$\left[1 + u(2C + F_1) + u^2(1 + C^2) \right] = \frac{a_1^2}{a_0^2} = 1. \quad (4.11)$$

And by symmetry,

$$a_z(L_1^-) = -a_z(0) = -(-2\beta_2^{(1)} a_0 b_0) = 2\beta_2^{(1)} a_0 \left(-\frac{C}{a_0^2} \right) = -2\beta_2^{(1)} \frac{C}{a_0}.$$

From $c_1 = 2a_1a_z(L_1^+) = \frac{\beta_2^{(2)}}{\beta_2^{(1)}} \times 2a_1a_z(L_1^-)$, we have

$$\Gamma = c_1L_2 = \frac{\Delta_2}{\beta_2^{(1)}} \times 2a_1a_z(L_1^-) = \frac{\Delta_2}{\beta_2^{(1)}} \times 2a_0 \left(-2\beta_2^{(1)} \frac{C}{a_0} \right) = -4\Delta_2C.$$

Therefore, we get

$$\left[(C + F_1) + u(1 + C^2) \right] = \frac{\Gamma}{\Delta_2} = -4C. \quad (4.12)$$

After substituting equation (4.12) and (4.11) into (4.10), we obtain

$$\Delta_2 = -2a_0^2 \frac{(-4C) \times 1}{(1 + (-4C)^2)} = \frac{8C}{(1 + 16C^2)} a_0^2. \quad (4.13)$$

The above relation is the appropriate value of Δ_2 for making $a_2 = a_1$ in the case where $a_1 = a_0$ is achieved by optimal pre-chirping. Notice that because the value of pre-chirp is negative, the value of Δ_2 is also negative which means the second fiber must have anomalous dispersion ($\beta_2^{(2)} < 0$). In addition, one might express equation (4.13) in the form of $\beta_2^{(2)}$ as

$$\beta_2^{(2)} = \frac{8C}{(1 + 16C^2)L_2} a_0^2. \quad (4.14)$$

Also, note that in this case $C = C_{opt} = -\frac{1}{u} + \frac{1}{u} \sqrt{1 - \left(1 + \frac{N^2}{\sqrt{2}}\right) u^2}$.

Finally, the dispersion map of this system will be as shown in Figure 4.6.

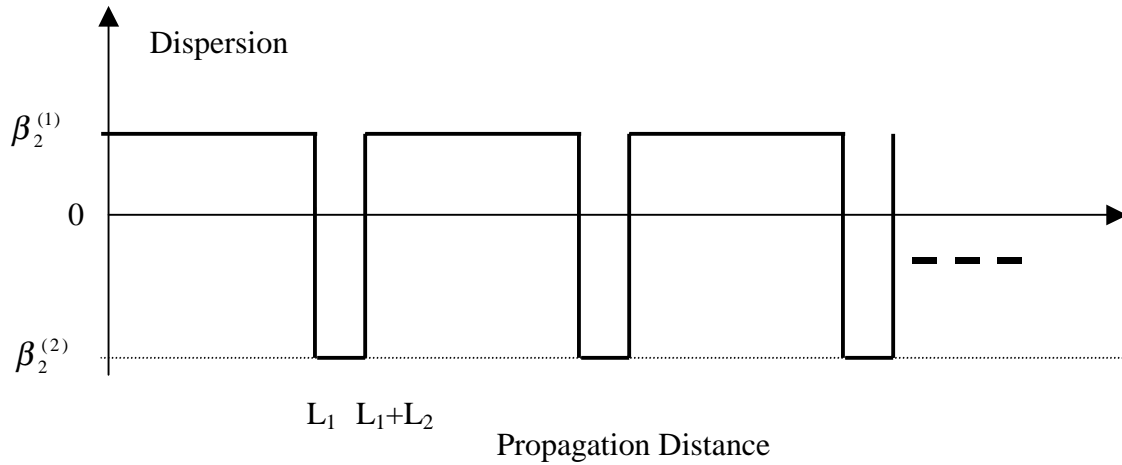


Figure 4.6: Dispersion Map of Two-Link Consecutive System

4.3.2 Four-Link System

Again, the previous two-link system can be achieved only when L_1 is less than L_{\max} . However, there is an alternative way of constructing a dispersion map for a system requiring a long length of the first (main) fiber, $L_1 > L_{\max}$. A four-link system is illustrated in Figure 4.7.

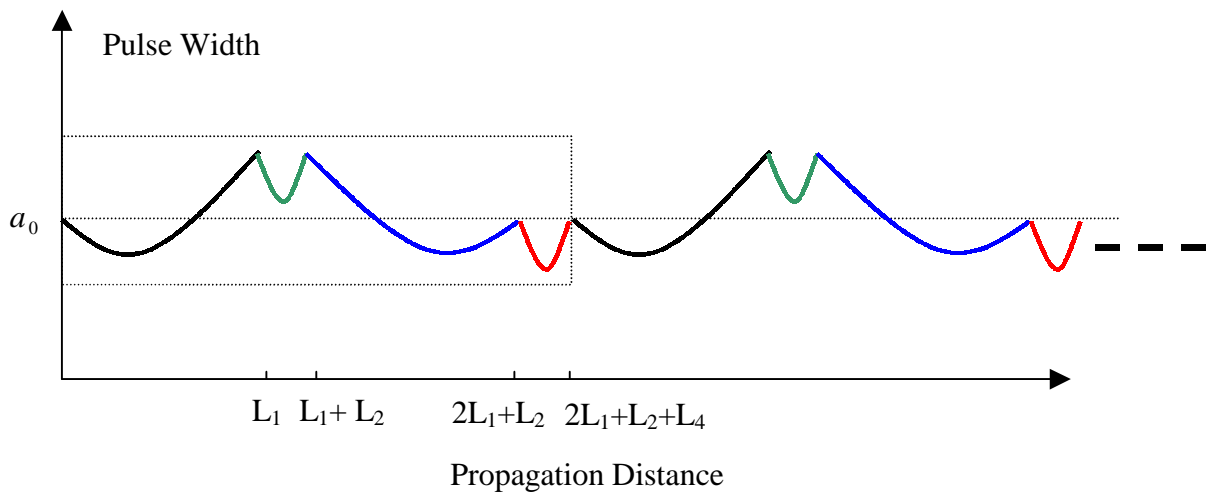


Figure 4.7: Four-Link Consecutive System

In Figure 4.7, whatever the output a_1 at $z = L_1$, we can arrange $a_2 = a_1$, where a_2 is the output width of the second link, as previously shown in equation (4.10). In the variational method, the solution of $a(z)$ is always symmetric about the chirp-free points, where $a_z(z) = 0$ (middle point of the second section). Therefore, the pulse width plot of the first section will be symmetric to the third section, given the same dispersion constants and lengths ($\beta_2^{(1)} = \beta_2^{(3)}$, $L_1 = L_3$) of those two fibers. Since the output width $a_0 = a_3$, the issue in the fourth link reduces to making the output pulse width equal to its input width, as in the second link, and also returning the initial chirp. Since the result of this map returns both width and chirp to the exact initial conditions, it can be repeated indefinitely. Thus, what we need now is to show the existence of the optimal pre-chirp for minimizing the required $\beta_2^{(2)}$, and to determine $\beta_2^{(4)}$ for which $a_4 = a_3 = a_0$.

Optimal Pre-Chirp for four consecutive link system

Recalling equation (4.10), the value of Δ_2 ($\beta_2^{(2)} L_2$) for making $a_1 = a_2$ is

$$\Delta_2 = -2a_0^2 \frac{[(C + F_1) + u(1 + C^2)] \times [1 + u(2C + F_1) + u^2(1 + C^2)]}{1 + [(C + F_1) + u(1 + C^2)]^2}, \quad (4.10)$$

which can be made minimal (in modulus) by selecting the appropriate value of pre-chirp C . If we differentiate equation (4.10) with respect to C and set the derivative equal to zero, then we can solve for the value of C to minimize $|\Delta_2|$. However, the result is so complicated that one might prefer to use symbolic software to solve this problem instead, or to simply plot $|\Delta_2|$ vs C .

In figure 4.8 we plot the values of $\beta_2^{(2)}$, against various values of C , which yield $a_2 = a_1$ for the following parameter values: $u = 0.5$, $a_0 = 20\text{ps}$, $L_1 = 100\text{km}$, $\beta_2^{(1)} = 2\text{ps}^2/\text{km}$, $L_2 = 20\text{km}$, $F_1 = 0.86$. The solid line curve is the optimal $\beta_2^{(2)}$ predicted by the variational

method. To generate the data for the figure we started with the optimum $C = C_{\text{opt}}$ predicted by the variational method and calculated the corresponding $\beta_2^{(2)}$ by split-step (circle dots) for various values of C near C_{opt} , which yield $a_2 = a_1$.

Note from the plot that although the values of $\beta_2^{(2)}$ from both methods are different (differences of the order of 1 to 2ps²/km), the minimum from both methods occurs at the same value of C equal to -3.05.

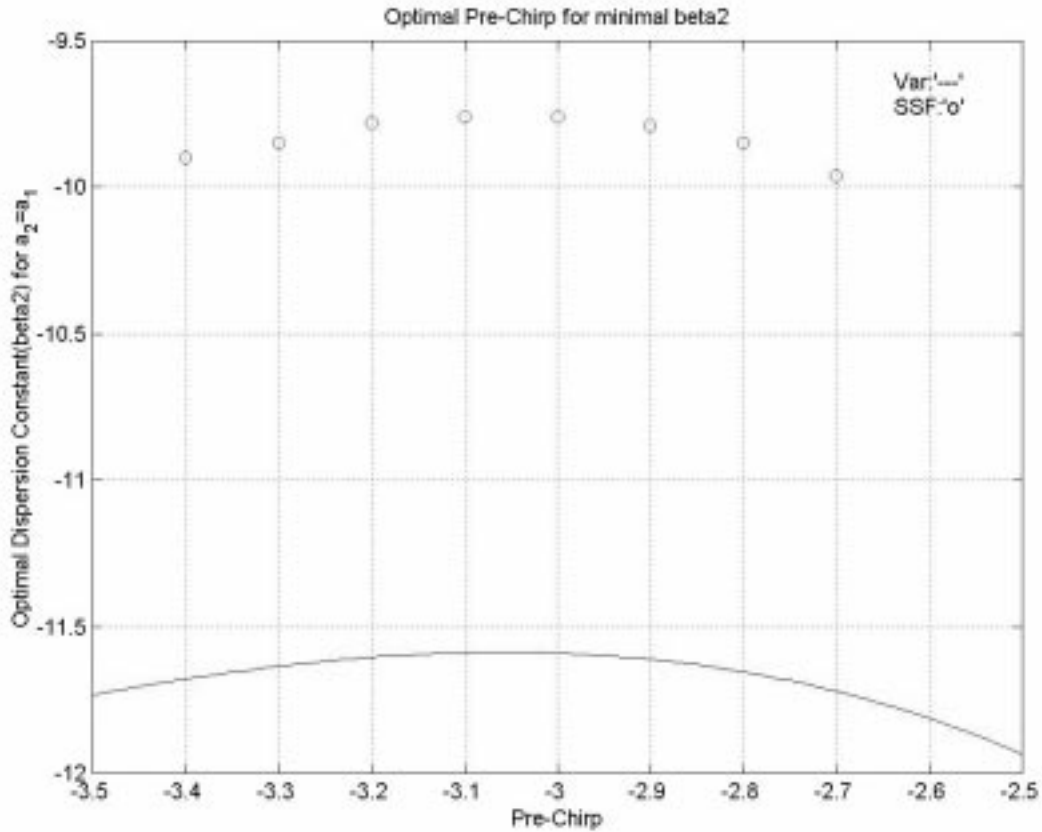


Figure 4.8: Optimal Pre-Chirp for minimal $|\beta_2^{(2)}|$ from VC and SSF

We have discussed how to select the initial chirp C so that $a_1 = a_2$ and $\beta_2^{(2)}$ has the smallest possible modulus. To complete the dispersion map, there remains only to find

the required dispersion $\beta_2^{(4)}$ in link #4, which will guarantee $a_4 = a_3 = a_0$. Tracing through the segments and using symmetry, it follows that the slope at the input in #4 is

$$a_z([2L_1 + L_2]^+) = \frac{\beta_2^{(4)}}{\beta_2^{(3)}} a_z([2L_1 + L_2]^-) = -\frac{\beta_2^{(4)}}{\beta_2^{(1)}} a_z(0),$$

that is

$$= -\frac{\beta_2^{(4)}}{\beta_2^{(1)}} (-2\beta_2^{(1)} a_0 b_0) = -\frac{\beta_2^{(4)}}{\beta_2^{(1)}} \left(-2\beta_2^{(1)} a_0 \left[-\frac{C}{2a_0^2} \right] \right) = -\frac{C\beta_2^{(4)}}{a_0}.$$

Then we can write the pulse width in the fourth section, by substituting a_1 by a_0 , c_1 by $2a_0 a_z([2L_1 + L_2]^+) = -2C\beta_2^{(4)}$, L_1 by $2L_1 + L_2$, and by neglecting the nonlinear term in equation (4.1). This results in

$$a^2(z) = a_0^2 - 2C\beta_2^{(4)}(z - (2L_1 + L_2)) + \left(\frac{(\beta_2^{(4)})^2}{a_0^2} + \frac{C^2(\beta_2^{(4)})^2}{a_0^2} \right) (z - (2L_1 + L_2))^2, \quad 2L_1 + L_2 < z < 2L_1 + L_2 + L_4. \quad (4.15)$$

Setting equation (4.15) equal to a_0 at $z = 2L_1 + L_2 + L_4$, where L_4 is the length of segment #4, gives

$$-2C\beta_2^{(4)}L_4 + \left(\frac{(\beta_2^{(4)})^2}{a_0^2} + \frac{C^2(\beta_2^{(4)})^2}{a_0^2} \right) L_4^2 = 0.$$

That is
$$\beta_2^{(4)}L_4 = \frac{2Ca_0^2}{(1+C^2)}. \quad (4.16)$$

We can either take $\beta_2^{(4)}$ as fixed and solve for L_4 or vice-versa.

Thus (4.10) and (4.16) are algorithms for selecting C , $\beta_2^{(2)}$, and $\beta_2^{(4)}$ so as to form a four-cycle dispersion map that returns an output state identical to the input state. To test the

algorithms, we ran split step simulations for various parameter values and obtained results very close to those predicted by the variational method.

For example, if $L_1 = L_3 = 200\text{km}$, $L_2 = L_4 = 20\text{km}$, $\beta_2^{(1)} = 2\text{ps}^2/\text{km}$, $a_0 = 20\text{ps}^2$, $N = 0.7$, one calculates from the variational equations (4.10) and (4.16) that $C = -1.41$, $\beta_2^{(2)} = -23.6\text{ps}^2/\text{km}$, $\beta_2^{(4)} = -18.65\text{ps}^2/\text{km}$.

Split-Step calculations then yield the actual optimal C to be $C = -1.46$, $\beta_2^{(2)} = -23.6\text{ps}^2/\text{km}$, $\beta_2^{(4)} = -18.75\text{ps}^2/\text{km}$ and the sequence of successive pulse widths as

$$a_1 = 23.96\text{ps}^2, a_2 = 23.94\text{ps}^2, a_3 = 20.15\text{ps}^2, a_4 = 20.00\text{ps}^2.$$

In this example, $u = 1$ and $u_{\max} = 0.86$ for the first segment.

Chapter 5

SUMMARY AND CONCLUSION

5.1 Summary

We have demonstrated throughout this thesis that the variational method can be exploited to obtain pulse properties (pulse width, spectral width, and chirp) whose evolution otherwise requires numerical solution of the nonlinear Schrodinger equation. With the power series estimation, one can predict the propagation patterns of an optical pulse launched into an optical fiber, by means of pulse width, frequency changing (Chirp), and spectra.

In Appendix B, we have illustrated the uses of the variational method to find the equation (B.3) of an optical pulse propagating in a single link fiber optic communication system. From that equation, we can find the optimal value of pre-chirp (3.4) to compensate the dispersion effect and restore the initial width and spectra at the output for any system parameters; initial pulse width, dispersion constant, fiber length, and nonlinear parameter. The variational method also showed the limitation of fiber length in accomplishing this goal. In the case where fiber length is longer than the maximum length in (3.6), there also exists an optimal pre-chirp (3.8), which make the output width minimal (but still larger than the initial one).

With equation (B.3) we can also find the pre-chirp value (3.11) for making the output pulse transform limited, which leads to the minimal time bandwidth product (3.15) as well. Again, there is also a maximum length (3.12) corresponding to this case. Note that although we can make the pulse transform limited at the output, doing that requires pre-chirping at the input, which may cause channel cross-talk at the front end of a fiber in WDM system. In addition, the variational method can also predict the spectral width (C.10) of a pulse while propagating in a fiber as shown in Appendix C.

In Chapter 4, we have used the variational method to verify the symmetry of a pulse width around the chirp-free point in the plot of pulse width versus distance. For concatenated links, we have found the equations (B.7) and (B.8) for the pulse propagating in the concatenated link by using the variational method. Then we showed the optimal dispersion constant of the compensation fiber (4.9) for which the output pulse width is equal to the input one. Note that the result is different from that of the ideal linear case.

By using the knowledge of symmetry and equations (B.7) and (B.8), we have suggested two types of dispersion maps for an optimal system, in which initial conditions (pulse width, chirp, and spectra) are restored at the output. Note that in both types of dispersion maps, pre-chirping has to be limited, to achieve the optimal dispersion maps, by means of small dispersion constant or shorter length of fiber.

We have also showed the consistency of the results from the variational method by comparison with the split-step Fourier simulation. Note again that the variational approximation is accurate for small non-linearity of the fiber. Moreover, in this thesis we have approximately taken account of the loss of the fiber by using the average optical power instead of the instantaneous power (2.3). Hence, in future work, one might want to include the loss in a variational calculation by beginning with equation (2.2) and finding the new variational equations.

5.2 List of Contributions and Conclusions of Thesis

- For a single link system, this thesis has shown the uses of variational method to predict the values of pre-chirp in determinable form for the following needs
 - Initial pulse and spectral widths are restored at the output
 - Output pulse width is minimized
 - Output pulse is transform limited and has minimum time-bandwidth product

This also includes the calculations of the limited length of fiber in achieving those needs.

- For the multiple link systems, variational method has been exploited to design two types of the optimal dispersion maps in order to counteract the nonlinear and dispersive effects and restore the identical input pulse conditions at the output.
 - Two link system: For a short main link
 - Four link system: For any length of main link

These include the calculations of optimal dispersion constant for the compensating fiber in the presence of nonlinear effect.

These results are valuable for improving system performance by means of increasing transmitting data rate and then the number of channels or users in the fiber-optic system.

Hopefully, the use of the variational method, which is a fast and accurate way of predicting the optical pulse phenomena in nonlinear dispersive fibers, will be useful for either improving existing system performance or designing new systems.

Appendix A

VALIDATON OF QUADRATIC SERIES

In the variational equation (Eq. 2.11) for the normal dispersion case, which is

$$\frac{\partial^2 a}{\partial z^2} = \frac{\beta_2^2}{a^3(z)} + \frac{\beta_2 \gamma E_0}{\sqrt{2} a^2(z)}, \quad (\text{A.1})$$

when $\beta_2^{(1)} > 0$, the argument on the right side is always greater than zero. Therefore, $a_{zz}(z) \geq 0$, which means $a_z(z)$ is increasing and eventually positive with z . So $a(z)$ is unbounded. As propagation distance becomes large ($z \rightarrow \infty$), the pulse width becomes large ($a(z) \rightarrow \infty$). From (A.1), $a_{zz}(z)$ will become small. Then $a_z(z)$ will eventually approach a constant. That means $a(z)$ asymptotically grows linearly with z , and so the quadratic series for $a^2(z)$ are valid. In the other words,

$$a(z) = \tilde{c}_0^{(1)} + \tilde{c}_1^{(1)} z \quad \Rightarrow \quad a^2(z) = c_0^{(1)} + c_1^{(1)} z + c_2^{(1)} z^2.$$

This quadratic series for the pulse width is used throughout Chapters 3 and 4, with the coefficients in the expansion as obtained in Appendix B.

Appendix B

POWER SERIES EXPANSION

B.1. One Section of fiber

Recalling equation (2.14)

$$a^2(z) = c_0^{(1)} + c_1^{(1)}z + c_2^{(1)}z^2 \quad \dots \quad 0 \leq z \leq L_1. \quad (\text{B.1})$$

First, we clearly have

$$c_0^{(1)} = a^2(0) = a_0^2$$

Differentiating equation (B.1)

$$2a(z)a_z(z) = c_1^{(1)} + 2c_2^{(1)}z \quad (\text{B.2})$$

$$c_1^{(1)} = 2a(0)a_z(0) = 2a_0[-2\beta_2^{(1)}a(0)b(0)] = -4a_0^2\beta_2^{(1)}\left(\frac{-C}{2a_0^2}\right) = 2\beta_2^{(1)}C$$

Note that $a_z(z)$ represents $\frac{\partial a(z)}{\partial z}$ and $a_z(0)$ can be found by substituting $z = 0$ in equation (2.8).

Differentiating equation (B.2)

$$2a(z)a_{zz}(z) + 2a_z^2(z) = 2c_2^{(1)}$$

$$c_2^{(1)} = \frac{1}{2}[2a(0)a_{zz}(0) + 2a_z^2(0)] = a_0a_{zz}(0) + 4a_0^2(\beta_2^{(1)})^2\left(\frac{C^2}{4a_0^4}\right)$$

By substituting $z = 0$ in equation (2.11) to get $a_{zz}(0)$, then we have

$$c_2^{(1)} = \left(\frac{(\beta_2^{(1)})^2}{a_0^2} + \frac{\beta_2^{(1)}\gamma E_0}{\sqrt{2}a_0} + \frac{(\beta_2^{(1)})^2 C^2}{a_0^2} \right)$$

Finally, the solution for the square of the pulse width in nonlinear dispersive fibers by using the variational method is

$$a^2(z) = a_0^2 + 2\beta_2^{(1)}Cz + \left(\frac{(\beta_2^{(1)})^2}{a_0^2} + \frac{\beta_2^{(1)}\gamma E_0}{\sqrt{2}a_0} + \frac{(\beta_2^{(1)})^2 C^2}{a_0^2} \right) z^2 \dots \quad 0 \leq z \leq L_1. \quad (\text{B.3})$$

This series for the square of the pulse width is used to find the optimal values of pre-chirps for various cases in Chapter 3.

B.2. Multiple Sections of Fibers

Because in Chapter 4 multiple sections systems are discussed, the expansion of the power series for the second section ($L_1 \leq z \leq L_1 + L_2$) is needed. It is obtained as follows.

Recalling equation (2.15),

$$a^2(z) = c_0^{(2)} + c_1^{(2)}(z - L_1) + c_2^{(2)}(z - L_1)^2 \dots \quad L_1 \leq z \leq L_1 + L_2 \quad (\text{B.4})$$

The above coefficients can be found by recalling the variational equations (2.8) to (2.11) and realizing that $a_z(z)$ has a discontinuity at $z = L_1$, the interface between the two links. The reason why is that physical parameters such as $a(z)$ and $b(z)$ are continuous because pulse width and chirp (or pulse spectrum) can not change abruptly, or experience a variation in zero time. However, it then follows from equation (2.8) that $a_z(z)$ is discontinuous and that the discontinuity is given by

$$\left. \begin{aligned} a_z(L_1^-) &= -2\beta_2^{(1)}a(L_1)b(L_1) \\ a_z(L_1^+) &= -2\beta_2^{(2)}a(L_1)b(L_1) \end{aligned} \right\} \Rightarrow \frac{a_z(L_1^+)}{a_z(L_1^-)} = \frac{\beta_2^{(2)}}{\beta_2^{(1)}}. \quad (\text{B.5})$$

If $\beta_2^{(1)} \neq \beta_2^{(2)}$ then $a_z(L_1^+) \neq a_z(L_1^-)$. Note that by $a_z(L_1^{+-})$, we mean the right and left limits of the derivative

$$a_z(L_1^+) = \lim_{\substack{\varepsilon \rightarrow 0 \\ \varepsilon > 0}} \frac{\partial a(L_1 + \varepsilon)}{\partial z}, \quad a_z(L_1^-) = \lim_{\substack{\varepsilon \rightarrow 0 \\ \varepsilon > 0}} \frac{\partial a(L_1 - \varepsilon)}{\partial z};$$

By setting $z = L_1$ in equation (B.4), we obtain

$$c_0^{(2)} = a^2(L_1) = a_1^2.$$

Differentiating equation (B.4),

$$2a(z)a_z(z) = c_1^{(2)} + 2c_2^{(2)}(z - L_1), \tag{B.6}$$

so $c_1^{(2)} = 2a(L_1)a_z(L_1^+),$

which can be found from equation (B.3) and (B.5).

In fact, for $z < L_1$, differentiating (B.3),

$$2a(z)a_z(z) = 2\beta_2^{(1)}C + 2\left(\frac{(\beta_2^{(1)})^2}{a_0^2} + \frac{\beta_2^{(1)}\gamma E_0}{\sqrt{2}a_0} + \frac{(\beta_2^{(1)})^2 C^2}{a_0^2}\right)z$$

and $c_1^{(2)} = 2a(L_1)a_z(L_1^+) = \left(\frac{\beta_2^{(2)}}{\beta_2^{(1)}}\right) \times 2a(L_1)a_z(L_1^-).$

Therefore,

$$c_1^{(2)} = 2\beta_2^{(2)}C + 2\left(\frac{\beta_2^{(1)}\beta_2^{(2)}}{a_0^2} + \frac{\beta_2^{(2)}\gamma E_0}{\sqrt{2}a_0} + \frac{\beta_2^{(1)}\beta_2^{(2)}C^2}{a_0^2}\right)L_1 \equiv c_1.$$

Note that we are defining a new parameter c_1 , which will be used throughout this derivation.

Differentiating equation (B.6),

$$2a(z)a_{zz}(z) + 2a_z^2(z) = 2c_2^{(2)}.$$

From equation (2.11),

$$a_{zz}(z) = \frac{(\beta_2^{(2)})^2}{a^3(z)} + \frac{\beta_2^{(2)}\gamma E_0}{\sqrt{2}a^2(z)},$$

or $a_{zz}(L_1) = \frac{(\beta_2^{(2)})^2}{a_1^3} + \frac{\beta_2^{(2)}\gamma E_0}{\sqrt{2}a_1^2},$

and $a_z(L_1^+) = \frac{c_1}{2a_1},$

then, $c_2^{(2)} = a(L_1)a_{zz}(L_1) + a_z^2(L_1) = \frac{(\beta_2^{(2)})^2}{a_1^2} + \frac{\beta_2^{(2)}\gamma E_0}{\sqrt{2}a_1} + \frac{c_1^2}{4a_1^2}.$

Therefore, in the second link, the width $a(z)$ is given by

$$a^2(z) = a_1^2 + c_1(z - L_1) + \left(\frac{(\beta_2^{(2)})^2}{a_1^2} + \frac{\beta_2^{(2)}\gamma E_0}{\sqrt{2}a_1} + \frac{c_1^2}{4a_1^2} \right) (z - L_1)^2 \quad \dots L_1 \leq z \leq L_1 + L_2, \quad (\text{B.7})$$

where $c_1 = 2\beta_2^{(2)}C + 2\left(\frac{\beta_2^{(1)}\beta_2^{(2)}}{a_0^2} + \frac{\beta_2^{(2)}\gamma E_0}{\sqrt{2}a_0} + \frac{\beta_2^{(1)}\beta_2^{(2)}C^2}{a_0^2} \right) L_1.$ (B.8)

This series for the pulse width is used in Chapter 4 to find: (i) the optimal dispersion constant $\beta_2^{(2)}$ of the compensation fiber in the high non-linearity regime, and (ii) dispersion maps.

Appendix C

SPECTRAL WIDTH DERIVATION

The variational method can be used to derive the spectral width of an optical pulse in a nonlinear dispersive fiber as follows.

Step 1: Define $a_z^2(z)$

Recalling the variational equation (2.11) for the normal dispersion case,

$$a_{zz}(z) = \frac{(\beta_2^{(1)})^2}{a^3(z)} + \frac{\beta_2^{(1)}\gamma E_0}{\sqrt{2}a^2(z)}, \quad (\text{C.1})$$

Multiply $a_{zz}(z)$ of equation (C.1) by $a_z(z)$ and integrate,

$$a_{zz}(z)a_z(z) = \frac{(\beta_2^{(1)})^2 a_z(z)}{a^3(z)} + \frac{\beta_2^{(1)}\gamma E_0 a_z(z)}{\sqrt{2}a^2(z)},$$

$$\text{then, } \frac{1}{2} a_z^2(z) = -\frac{1}{2} \frac{(\beta_2^{(1)})^2}{a^2(z)} - \frac{\beta_2^{(1)}\gamma E_0}{\sqrt{2}a(z)} + K, \quad K = \text{constant,}$$

$$\text{that is } a_z^2(z) = -\frac{(\beta_2^{(1)})^2}{a^2(z)} - \frac{\sqrt{2}\beta_2^{(1)}\gamma E_0}{a(z)} + K.$$

$$\text{From (2.8), } a_z^2(0) = 4(\beta_2^{(1)})^2 a_0^2 b_0^2 = \frac{(\beta_2^{(1)})^2 C^2}{a_0^2},$$

$$\text{therefore, } K = \frac{(\beta_2^{(1)})^2 C^2}{a_0^2} - \left(-\frac{(\beta_2^{(1)})^2}{a_0^2} - \frac{\sqrt{2}\beta_2^{(1)}\gamma E_0}{a_0} \right) = \frac{(\beta_2^{(1)})^2}{a_0^2} (1 + C^2) + \sqrt{2}\beta_2^{(1)}\gamma P_0.$$

$$\text{Then, } a_z^2(z) = \frac{(\beta_2^{(1)})^2 C^2}{a_0^2} + (\beta_2^{(1)})^2 \left(\frac{1}{a_0^2} - \frac{1}{a^2(z)} \right) + \sqrt{2}\beta_2^{(1)}\gamma P_0 \left(1 - \frac{a_0}{a(z)} \right). \quad (\text{C.2})$$

Step 2: Define $\sigma_{\omega}^2(z)$

Let $\tilde{U}(z, \omega)$ be the Fourier transform of $U(z, T)$, which is a Gaussian pulse of the form,

$$U(z, T) = A(z) \exp\left[T^2 \left(-\frac{1}{2a^2(z)} + jb(z)\right)\right]. \quad (\text{C.3})$$

From Fourier transform tables, $\mathfrak{F}\left[e^{-\frac{T^2}{2x}}\right] = \sqrt{2\pi x} e^{-\frac{x\omega^2}{2}}$. (C.4)

By writing the new form of equation (C.2) as $e^{-\frac{T^2}{2x}}$, we will have

$$\frac{1}{2x} = \frac{1}{2a^2} - ib \Rightarrow x = \frac{a^2}{1 - 2ia^2b} = \frac{a^2 + 2ia^4b}{1 + 4a^4b^2},$$

that is $x = \frac{a^2}{1 + 4a^4b^2} + i \frac{2a^4b}{1 + 4a^4b^2}$. (C.5)

Taking the Fourier Transform of (C.3) by using (C.4) with the corresponding parameter x in (C.5), we obtain the spectra of an optical pulse propagating in a fiber as

$$\tilde{U}(z, \omega) = \sqrt{2\pi x} A(z) e^{\left[\frac{\omega^2}{2} \left(\frac{a^2}{1 + 4a^4b^2} + i \frac{2a^4b}{1 + 4a^4b^2} \right) \right]},$$

or $= \sqrt{2\pi x} A(z) e^{-\frac{1}{2} \left(\frac{a^2\omega^2}{1 + 4a^4b^2} \right)} e^{-i \frac{a^4b\omega^2}{1 + 4a^4b^2}}$, (C.6)

which has half-spectral width square at 1/e intensity point as

$$\Delta\omega^2(z) = \frac{1 + 4a^4b^2}{a^2} = \frac{1}{a^2} + 4a^2b^2.$$

From the variational equation (2.8), we can see that $a^2 b^2 = a_z^2 / 4(\beta_2^{(1)})^2$, therefore,

$$\Delta\omega^2(z) = \frac{1}{a^2} + \frac{a_z^2}{(\beta_2^{(1)})^2}. \quad (\text{C.7})$$

The square of the RMS spectral width, which is one half of $\Delta\omega^2(z)$, given by

$$\sigma_\omega^2(z) = \frac{\Delta\omega^2(z)}{2} = \frac{1}{2} \left(\frac{1}{a^2} + \frac{a_z^2}{(\beta_2^{(1)})^2} \right). \quad (\text{C.8})$$

Step 3: Final Form of $\sigma_\omega^2(z)$

By substituting (C.2) into equation (C.8), we obtain

$$\sigma_\omega^2(z) = \frac{1}{2} \left(\frac{1}{a^2(z)} + \frac{1}{(\beta_2^{(1)})^2} \left[\frac{(\beta_2^{(1)})^2 C^2}{a_0^2} + (\beta_2^{(1)})^2 \left(\frac{1}{a_0^2} - \frac{1}{a^2(z)} \right) + \sqrt{2} \beta_2^{(1)} \mathcal{P}_0 \left(1 - \frac{a_0}{a(z)} \right) \right] \right),$$

that is
$$\sigma_\omega^2(z) = \frac{1}{2} \left[\frac{C^2}{a_0^2} + \frac{1}{a_0^2} + \frac{\sqrt{2} \mathcal{P}_0}{\beta_2^{(1)}} \left(1 - \frac{a_0}{a(z)} \right) \right].$$

Equivalently,

$$\sigma_\omega^2(z) = \frac{1}{2} \left[\frac{(1+C^2)}{a_0^2} + \frac{\sqrt{2} N^2}{a_0^2} \left(1 - \frac{a_0}{a(z)} \right) \right], \quad (\text{C.9})$$

or
$$\sigma_\omega^2(z) = \sigma_{\omega,0}^2 + \frac{\sqrt{2} N^2}{2a_0^2} \left(1 - \frac{a_0}{a(z)} \right) \quad (\text{C.10})$$

where $\sigma_{\omega,0}^2$ is the initial RMS spectral width (square) of an input pulse.

Note that (C.6-9) are valid within a single fiber link but can be modified to cover concatenated links. For example, in (C.6) the term a_z is discontinuous but $a_z / \beta_2^{(1)}$ is continuous.

This form of RMS spectral width is used in Chapter 3.

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