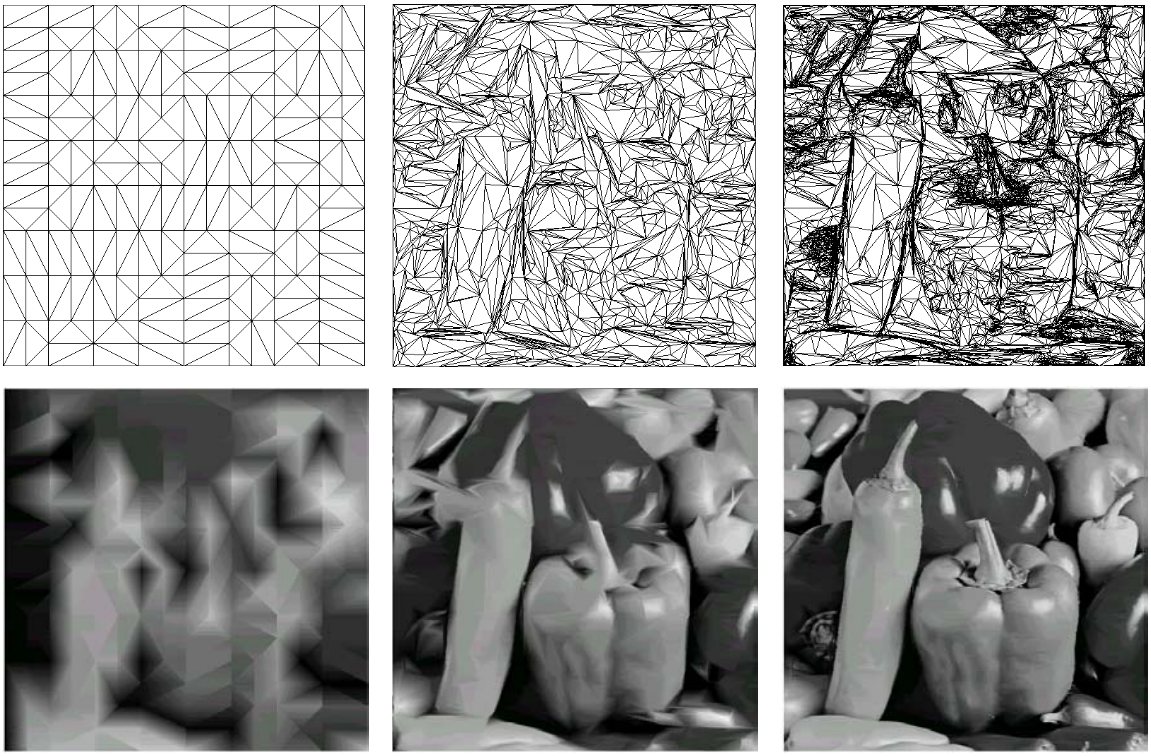
CHAPTER 1
INTRODUCTION

1.1 Motivation

Linear approximation has been used in many applications to represent signal data. This reduces not only the storage space for data, but also the amount of data that must be accessed for later processing. In his dissertation, Dr. Sang-Mook Lee [Lee 02] has proposed an algorithm to approximate height field datasets using a form of linear approximation known as triangulation. Using his procedure, a rectangular grid of distance values is approximated by a set of planar triangles. The algorithm employs wavelet coefficients to construct the initial triangulation for the approximation as well as to refine the triangular mesh. One of the main advantages of using this linear planar representation is the considerable reduction in data.

This thesis will extend the work of Dr. Lee to approximate gray-scale images. In spite of the similarity between height fields and visual images, one of the main characteristic differences is that there tends to be more abrupt changes in image data than in height field data. Sudden changes in height occur only rarely in such fields as geography, but can commonly occur in gray-scale images. Therefore there is a need to modify and adapt the algorithm to handle image data. Figure 1.1 shows an example of triangular mesh and its image approximations for an image.

One of the advantages of using wavelet coefficients in his algorithm is to assist in multiresolution analysis (MRA). This feature enhances the algorithm to be able to support progressive data transmission, which is highly favorable in network applications. Furthermore the algorithm can take advantage of level-of-detail (LOD) control. These properties are very desirable to network media, where low-bandwidth capacity can occur.



(a) (b) (c)

Figure 1.1 - Approximation using triangulation at different resolution levels. (a) Initial triangulation. (b) An approximation at an intermediate level (c) Finest-level approximation. Only 2% of the original input data is needed to provide a good approximation of the Peppers image.

Furthermore, using a triangular mesh to represent image data can assist in segmentation. Since each triangular patch contains redundancy in information, it can be used to indicate the characteristics of a region. This advantage is very useful in computer vision and pattern recognition applications to describe the information present in the image. Figure 1.2 shows examples of segmentation based on individual pixels of the original data and of individual triangles of the triangular mesh.

Another advantage of using triangular meshes is a large reduction in data for processing. For example, only 2% of the storage used for the original image is needed to represent the Peppers image with good accuracy. This reduction in data is very useful in many applications. For example, instead of transforming (translation, scaling or rotation) the entire data, only the representative vertex data in the triangular mesh is required in the calculation. This can significantly speed processing time. This triangulation can also be efficiently used to detect objects in the image. Figure 1.3 shows the result of eye detection on the Elaine image. This algorithm will be discussed in Section 6.8.

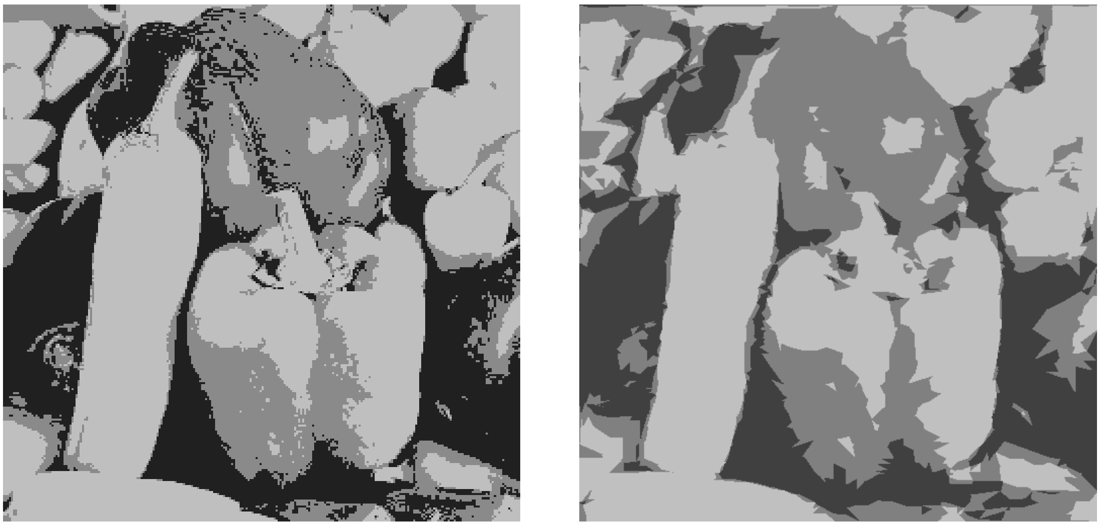


Figure 1.2 – Segmentation of the Peppers image. (a) Segmentation based individual pixel values. (b) Segmentation using individual triangle.

Figure 1.3 – Example of eye detection. Triangular mesh representation can simplify this processing.

1.2 Approach

The goal of this study is to modify and adapt the height field approximation algorithm developed by Dr. Sang-Mook Lee [Lee 02] to two-dimensional gray-scale images. Besides obtaining good approximation output, the algorithm should also prove to be practical in terms of processing time and memory requirements.

The problem can be formulated as minimizing the energy function

$$\varepsilon = \alpha E_{error}\left(\Gamma\right) + \beta E_{memory}\left(\Gamma\right) + \gamma E_{speed} \tag{1.1}$$

where Γ is the set of triangular mesh elements and α , β and γ are constants, which assign weights to the total squared error of the triangulation, the memory for the triangulation and the speed to construct the triangulation, respectively.

Most of the studies in data approximation are considered to be either refinement [Lu, Le and Yun 00] [Gross, Staadt and Gatti 96] or decimation [Schroeder, Zarge and Lorensen 92] approaches. In his dissertation, Dr. Lee has proposed a refinement approach that utilizes wavelet coefficients as the main features that guide triangle refinement. His high-level algorithm can be summarized as follows:

1. Wavelet transform on image input data
2. Construction of initial triangular mesh using the wavelet coefficients at the coarsest level
3. Repeat:
    a. Region selection
    b. Triangulation refinement
    c. Mesh simplification

In this thesis, the algorithm will be adapted and modified to handle the characteristics of natural-scene gray-scale image data, which tends to have larger changes in value when compared to height field data. The adapted algorithm will be discussed in Section 5.1.2.

1.3 Significance of Research

This research has focused mainly on the features that support the algorithm as well as the factors that will have influence on the reconstructed image. The study can be divided into areas as follows:

- This thesis adapts Dr. Lee's height field approximation algorithm to process image data. Beginning with the construction of initial triangulation, the algorithm is followed by iterations of image enhancement and mesh simplification operators. The new algorithm will be discussed in Section 6.2.2.

- The comparison of two triangulation schemes, Delaunay and data-dependent triangulation, will be presented. While the Delaunay approach can greatly increase computation speed, it does not produce good results in terms of the number of triangles and total squared error. On the other hand, although the data-dependent scheme is able to find an optimal result, its production of thin triangles does not guarantee optimal results for successive levels, and especially for the final level. The study will also cover the benefits obtained from combining both schemes.

- Two underlying data structures, the hierarchical ring and the half-edge data structures were developed and implemented. All the linked lists are doubly connected so that neighborhood accesses can be achieved in constant time. The algorithm's time complexity and memory requirements will be analyzed in detail for feasibility in real world applications.

- The results show that a considerable reduction in data for processing is possible, while providing good approximations of the image data. This reduction in data is very useful in many applications such as data compression and processing. The algorithm is also found to be practical in time and memory space and, therefore, can be implemented efficiently on hardware systems.

1.4 Overview of Material

The material in this thesis is organized as follows. Chapter 2 gives a brief review of the mathematical background of discrete Fourier and wavelet transforms. The wavelet transform will be further discussed for its concept of multiresolution and for the feasibility of using wavelet coefficients to construct the triangular mesh. The chapter will end with a comparison of the different types of wavelet signals.

Chapter 3 will present the background knowledge required for constructing triangular meshes. The chapter starts with the definition of images and triangulation before presenting

Delaunay and data-dependent schemes. Gouraud shading for image reconstruction and an error metric will also be discussed. Finally the chapter ends with the basics of graph theory, with an application focusing on triangulation.

Chapter 4 exhibits the use of wavelet coefficients for initial triangulation as developed by Dr. Lee. The chapter focuses on designing the basic, dual and variant templates, which are used to solve vertex incompatibility problems. Chapter 5 first compares the SML algorithm (algorithm developed by Dr. Sang-Mook Lee) and the new algorithm. It is followed by the description of each image enhancement and mesh simplification operators that are used to improve the image quality while, at the same time, trying to minimize the number of elements in the mesh.

In Chapter 6, time complexity and memory requirements of the algorithm are evaluated. Results will be presented and analyzed using comparisons in many aspects such as multiresolution analysis (MRA) and level-of-detail (LOD). Applications using triangular meshes to represent gray-scale image will also be presented in this chapter. Lastly, Chapter 7 will conclude the work and discuss directions for future work.