CHAPTER 7

CONCLUSION AND FUTURE DIRECTIONS

7.1 Summary and Conclusion

In this thesis, the new fast algorithm proposed by Dr. Sang-Mook Lee is adapted and modified for approximating gray-scale images. The main goal of the algorithm is to use triangular meshes with a small number of triangles to represent two-dimensional images within a given error bound. First the algorithm extracts information at different resolution levels by performing wavelet transforms at several scales. The wavelet coefficients from the coarsest level are analyzed for constructing the initial triangulation. Predefined basic and dual templates are needed at the coarsest scale.

After initial triangulation, with the help of wavelet coefficients at different levels, refinement and mesh enhancement are performed iteratively to achieve image approximation that tries to minimize both the number of triangles and the error. In this thesis, the L^2 norm is used while Gouraud shading is used to reconstruct an image from the triangular mesh. In the refinement process, the algorithm selects the candidates by comparing total wavelet energy and absolute wavelet coefficients with some threshold values.

The previous step sometimes results in redundancy of vertices or triangles that can be removed. Therefore, mesh enhancement is necessary to delete these vertices and triangles from the triangular mesh. This process also regularizes the mesh by deleting some thin sliver triangles. The other common mesh enhancement operator is removing the vertices that are shared by triangles whose normal surface vectors are approximately parallel.

This thesis also includes studies in the choice of triangles for edge swapping. Although the data-dependent scheme tends to produce a variety of types of triangles, it can result in many thin sliver triangles, which cannot be further refined in the next iteration. On the other hand, Delaunay triangulation does not produce optimal results. However one of its advantages is its efficiency in execution time. The study shows that the combination of the Delaunay approach and a data-dependent scheme produces the best results from the test images. The algorithm tends to ignore the choice of triangles that are too thin and may prevent the future refinement processes.

Since the memory required by the algorithm depends largely on the image size and the number of triangles, *n*, and vertices, *r*, while the number of triangles is close to twice the number of vertices, the required memory can be approximately expressed in term of the number of triangles and image size as $120n+5N^2$. Normally less than five percent of the total pixels are sufficient to represent the image with a PSNR higher than 30 dB. This will cost the algorithm about $11N^2$, which is equivalent to the memory space of 11 images. However, in the worst-case situation, where all vertices are needed to represent the entire image pixel, the memory cost will rise to $125N^2$.

The algorithm's execution time is affected by two main factors, the size of the image and the number of vertices. If the image is too large, the dominant time spent will go to edge swapping, and its time complexity can be expressed as $O(HW)$. If processing time is dominated by the refinement and mesh enhancement step, the time complexity is $O(h^2r)$. The vertex degree, *h*, usually starts at 5 at the initial triangulation and increases during the refinement step. Even though this value cannot exceed 8 and can be accounted as constant, it is still very difficult to assume that the algorithm is linear in time with the number of vertices. This is because the number of triangles can increase by up to a factor of 3 after each refinement step.

The algorithm can also be used to control the level of detail in the reconstructed image. The demand for progressive transmission and level-of-detail control in multimedia applications has made triangulation an area of interest during the last few decades. Transmission of data can begin with the low-resolution initial triangulation, and this is progressively improved by transmitting detailed information. Level-of-detail control can be used to adjust the amount of transmission per image according to the supply of network bandwidth. With the help of geometry and coordinate compression, the algorithm can be used for image compression as well.

The algorithm can also be used to refine triangulation for region-of-interest (ROI) applications, and for segmentation. Although the algorithm has been developed for two-dimensional gray-scale images, it can be adapted to multi-dimensional signal data, including color images.

7.2 Future Work

One extension to improve the result is to use the polyphase wavelet transform [Strang and Nguyen 96]. Since wavelet filters can have null points in the frequency response, using polyphase filters will ensure that all frequency components will be detected. However these filters should guarantee that their absolute response does not cross the tolerable margin. Furthermore, the study of the combination of horizontal, vertical and diagonal wavelet coefficients to detect corners will be useful for locating significant positions for the vertices.

Another possible extension to this work is to provide efficient progressive improvement. Although the data structure used for this triangular mesh is a hierarchical ring connection, there is no connection between resolution levels that could help the algorithm to perform immediate progressive transmission. To solve this problem, a hierarchical data structure could be introduced

Finally, it should be possible to implement the algorithm on fast hardware. Since the templates can be stored within a small amount of memory, initial triangulation is relatively simple. However, refinement operators, although simple and straightforward, might need some code optimization to ensure that the DSP or embedded system can reliably yield a satisfactory running time.