APPENDIX A:

ALGEBRA

A.1) CLOSED ECONOMY MODEL

ε: Supply elasticity
e: Absolute value of demand elasticity

Supply function: $Q_s = a + b(P+K)$
Demand function: $Q_d = c - dP$
$a<0$, $b>0$, $c>0$ and $d>0$

At equilibrium: $Q_s = Q_d$
$a + b(P+K) = c - dP$
$bP + dP = c - a - Kb$
$P(b + d) = c - a - Kb$
$P = (c - a - Kb)/(b + d)$

Situation without research: $K=0 \rightarrow P = (c - a)/(b + d)$
Situation with research: $K\neq0 \rightarrow P' = (c - a - Kb)/(b + d)$

$P - P' = Kb/(b + d)$
$P - P' = (Kb*P/Q)/[(b + d)*P/Q]$
$P - P' = K\varepsilon/(\varepsilon + e) = E$

$Ee = (P-P')[(Q'-Q)/(P-P')]*(P/Q) = (Q'-Q)*(P/Q) \rightarrow Q'-Q = Ee\ Q/P \rightarrow (Q'-Q)/Q = Ee/P$

$P'-f = (P-f)-(P-P') = K-E$

1) PARALLEL SHIFT OF THE SUPPLY CURVE

K: Vertical shift of the supply function ($)

$\Delta CS = (P-P')\ Q + ½\ (P-P')(Q'-Q) = (P-P')\ Q\ [1 + ½(Q'-Q)/Q] = EQ\ (1 + ½\ Ee/P)$
$\Delta PS = (P'-f)Q + ½\ (P'-f)(Q'-Q) = (P'-f)Q\ (1 + ½\ (Q'-Q)/Q) = (K-E)Q\ (1 + ½\ Ee/P)$
$\Delta TS = \Delta CS + \Delta PS = EQ\ (1 + ½\ Ee/P) + (K-E)Q\ (1 + ½\ Ee/P) = KQ\ (1 + ½\ Ee/P)$

2) PIVOTAL SHIFT OF THE SUPPLY CURVE

k is the proportionate vertical shift (%)$\rightarrow$ $K = kP$ ($)

$\Delta CS = EQ\ (1 + ½\ Ee/P)$
$\Delta PS = \Delta TS - \Delta CS$
$\Delta TS = ½\ KQ + ½\ K(Q'-Q) = ½\ KQ\ [1 + (Q'-Q)/Q] = ½\ KQ\ (1 + Ee/P)$

3) PARALLEL SHIFT OF THE DEMAND CURVE

L: Vertical shift in demand curve ($)
P_0: Initial equilibrium price (before demand shifts)

Q_0: Initial equilibrium quantity (before demand shifts)
P: Final equilibrium price (after demand shifts)
Q: Final equilibrium quantity (after demand shifts)
ε: Supply elasticity
e: Absolute value of demand elasticity

Supply function: $Q_s = a + bP$
Demand function: $Q_{do} = c_o - dP_o$ (before shift)
Demand function: $Q_d = c - dP$ (after shift)
$a<0$, $b>0$, $c>0$, $c_o>0$ and $d>0$ (the slopes of the demand curves are the same because a parallel shift is assumed)

At equilibrium:
$Q_s = Q_{do}$ and $Q_s = Q_d$
$a + bP_o = c_o - dP_o$ and $a + bP = c - dP$
$P_o = (c_o - a)/(b+d)$ and $P = (c-a)/(b+d)$

$P-P_o = (c-c_o)/(b+d)$
$P-P_o = L/b+d$
$P-P_o = (LP_o/Q_o)/[(b+d)(P_o/Q_o)] = (LP_o/Q_o)/(\varepsilon+e)$
$P = P_o + (LP_o/Q_o)/(\varepsilon+e)$
$P = P_o\,(1 + (L/Q_o)/(\varepsilon+e))$

$Q-Q_o = (c - dP) - (c_o - dP_o) = c-c_o - d(P-P_o) = L - d(LP_o/Q_o)/(\varepsilon+e) = L - Le/(\varepsilon+e)$
$Q = Q_o + L - Le/(\varepsilon+e)$
$Q = Q_o\,(1 + L/Q_o - (Le/Q_o)/(\varepsilon+e))$

4) INTRODUCTION OF A PRODUCER BASE PRICE

P_b: Producer base price
P: Consumer price before supply shift
P': Consumer price after supply shift
Q: Quantity supplied and consumed before supply shift
Q': Quantity supplied and consumed after supply shift
ε: Supply elasticity
e: Absolute value of demand elasticity
$\varepsilon = [(Q'-Q)/K]*[P_b/Q] \rightarrow (Q'-Q)/Q = \varepsilon K/P_b$

Supply function: $Q_s = a + b(P+K)$
Demand function: $Q_d = c - dP$
$a<0$, $b>0$, $c>0$ and $d>0$
Situation without research: $K=0 \rightarrow Q_s = a + bP_b$
Situation with research: $K\neq 0 \rightarrow Q_s = a + b(P_b+K)$

$Q_d = c - dP \rightarrow dP = c - Q_d \rightarrow P = c/d - Q_d/d$

$P = c/d - 1/d\ (a + bP_b)$
$P' = c/d - 1/d\ (a + bP_b + bK)$
$P-P' = bK/d = \varepsilon K/e = E$

$Ee = (P-P')[(Q'-Q)/(P-P')]*(P/Q) = (Q'-Q)*(P/Q) \rightarrow Q'-Q = Ee\ Q/P \rightarrow (Q'-Q)/Q = Ee/P$

PARALLEL SHIFT OF THE SUPPLY CURVE

K: Vertical shift of the supply function ($)

$\Delta CS = (P-P')Q + ½\ (P-P')(Q'-Q) = (P-P')Q\ (1 + ½\ (Q'-Q)/Q) = EQ\ (1 + ½\ Ee/P)$
$\Delta PS = KQ + ½\ K(Q'-Q) = KQ\ (1 + ½\ (Q'-Q)/Q) = KQ\ (1 + ½\ \varepsilon K/P_b)$
$\Delta GC = (P-P')Q + (P_b-P')(Q'-Q) = EQ + (P_b-P')EeQ/P = EQ\ (1 + (P_b-P')e/P) = EQ\ (1 + ((P_b-P) + (P-P'))e/P) = EQ\ (1 + ((P_b-P) + E)e/P)$
$\Delta NSW = \Delta CS + \Delta PS - \Delta GC$

PIVOTAL SHIFT OF THE SUPPLY CURVE

k is the proportionate vertical shift (%)$\rightarrow$ K = kP

$\Delta CS = (P-P')Q + ½\ (P-P')(Q'-Q) = (P-P')Q\ (1 + ½\ (Q'-Q)/Q) = EQ\ (1 + ½\ Ee/P)$
$\Delta PS = ½\ KQ + ½\ K(Q'-Q) = ½\ KQ\ (1 + (Q'-Q)/Q) = ½\ KQ\ (1 + \varepsilon K/P_b)$
$\Delta GC = (P-P')Q + (P_b-P')(Q'-Q) = EQ + (P_b-P')EeQ/P = EQ\ (1 + (P_b-P')e/P) = EQ\ (1 + ((P_b-P) + (P-P'))e/P) = EQ\ (1 + ((P_b-P) + E)e/P)$
$\Delta NSW = \Delta CS + \Delta PS - \Delta GC$

PARALLEL SHIFT OF THE DEMAND CURVE

L: Vertical shift in demand curve ($)
P_0: Initial consumer price (before demand shifts)
Q_0: Initial quantity consumed (before demand shifts)
P: Final consumer price (after demand shifts)
Q: Final quantity consumed (after demand shifts)
ε: Supply elasticity
e: Absolute value of demand elasticity

Demand function: $Q_{do} = c_o - dP_o$ (before shift)
Demand function: $Q_d = c - dP$ (after shift)
$c>0$, $c_o>0$ and $d>0$ (the slopes of the demand curves are the same because a parallel shift is assumed)
At quantity supplied Q_s at price P_b:
$Q_{do} = Q_d$
$c_o - dP_o = c - dP$
$dP - dP_o = c - c_o$
$P-P_o = (c-c_o)/d$
$P-P_o = L/d$

$P = P_o + (L/d)$
$P = P_o (1 + L/dP_o)$
$P = P_o (1 + (L/dP_o)(Q_o/Q_o))$
$P = P_o (1 + (L/eQ_o)$

$Q_o = Q$

A.2) FARM HOUSEHOLD CONSUMPTION MODEL

P: Equilibrium price before supply shift
P': Equilibrium price after supply shift
Q_p: Quantity consumed on farm
ε: Supply elasticity
e: Absolute value of demand elasticity
$e = 0 \rightarrow P\text{-}P' = E = K\varepsilon/(\varepsilon + e) = K$

1) PARALLEL SHIFT OF THE SUPPLY CURVE

K: Vertical shift of the supply function ($)

$\Delta CS_p = EQ_p (1 + \frac{1}{2} Ee/P) = EQ_p = KQ_p$

2) PIVOTAL SHIFT OF THE SUPPLY CURVE

k is the proportionate vertical shift (%)$\rightarrow$ K = kP ($)

$\Delta CS_p = \frac{1}{2}EQ_p (1 + Ee/P) = \frac{1}{2} EQ_p = \frac{1}{2} KQ_p$

3) PARALLEL SHIFT OF THE DEMAND CURVE

L: Horizontal shift in demand curve ($)
P_0: Initial equilibrium price (before demand shifts)
Q_0: Initial equilibrium quantity (before demand shifts)
P: Final equilibrium price (after demand shifts)
Q: Final equilibrium quantity (after demand shifts)
ε: Supply elasticity
e: Absolute value of demand elasticity

Supply function: $Q_s = a + bP$
Demand function: $Q_{do} = c_o$ (before shift)
Demand function: $Q_d = c$ (after shift)
$a<0$, $b>0$, $c>0$, $c_o>0$ and $d>0$ (the slopes of the demand curves are the same because a parallel shift is assumed)

At equilibrium:

$Q_s = Q_{do}$ and $Q_s = Q_d$
$a + bP_o = c_o$ and $a + bP = c$
$P_o = (c_o – a)/b$ and $P = (c – a)/b$

$P-P_o = (c-c_o)/b$
$P-P_o = L/b$
$P-P_o = (LP_o/Q_o)/b(P_o/Q_o) = (LP_o/Q_o)/\varepsilon$
$P = P_o + (LP_o/Q_o)/\varepsilon$
$P = P_o\,(1 + (L/Q_o)/\varepsilon)$

$Q-Q_o = c – c_o = L$
$Q = Q_o + L$
$Q = Q_o\,(1 + L/Q_o)$

4) INTRODUCTION OF A PRODUCER BASE PRICE

P_b: Producer base price
P: Consumer price before supply shift
P': Consumer price after supply shift
Q_p: Quantity consumed on farm
e: Absolute value of demand elasticity
$e = 0 \rightarrow E = P-P' = K\varepsilon/(\varepsilon + e) = K$

$\Delta CS_p = 0$ because $\Delta P_b = 0$ (parallel and pivotal shift of the supply curve)

A.3) SMALL OPEN ECONOMY MODEL

P_w: World price
Q_0: Quantity consumed
Q: Quantity supplied before supply shift
Q': Quantity supplied after supply shift
ε: Supply elasticity
$\varepsilon = [(Q'-Q)/K]*[P_w/Q] \rightarrow (Q'-Q)/Q = \varepsilon K/P_w$

1) PARALLEL SHIFT OF THE SUPPLY CURVE

K: Vertical shift of the supply function ($)

$\Delta CS = (P-P')\,Q + ½\,(P-P')(Q'-Q) = 0$ because $\Delta P = \Delta Q = 0$
$\Delta TS = \Delta PS = KQ + ½\,K(Q'-Q) = KQ\,(1 + ½\,(Q'-Q)/Q) = KQ\,(1 + ½\,\varepsilon K/P_w)$

2) PIVOTAL SHIFT OF THE SUPPLY CURVE

k is the proportionate vertical shift (%)$\rightarrow$ $K = kP$ ($)
$\Delta CS = (P-P')\,Q + ½\,(P-P')(Q'-Q) = 0$ because $\Delta P = \Delta Q = 0$

$\Delta TS = \Delta PS = ½ KQ + ½ K(Q'-Q) = ½ KQ [1 + (Q'-Q)/Q] = ½ KQ (1 + \varepsilon K/P_w)$

A.4) OTHER FORMULAS

1) SUPPLY SHIFT K

$K = [\Delta Y/\varepsilon Y - \Delta C/C(1+\Delta Y/Y) – f \Delta F/F] p A_t (1-\delta)^t P$

K: Vertical shift of the supply curve ($)

$\Delta Y/Y$: Expected relative increase in experimental yield per hectare
$\Delta Y/\varepsilon Y$: Conversion of experimental yield increase (horizontal shift) into a gross proportionate reduction in marginal cost per ton of output (vertical shift)

$\Delta C/C$: Proportionate variable input cost change per hectare
$\Delta C/C(1+\Delta Y/Y)$: Proportionate variable input cost change per ton of output

$\Delta F/F$: Proportionate fixed input cost change per ton of output
f: Fraction of pre-research cost per ton of output that accounts for allocatable fixed factors

p: Probability of success

A_t: Adoption rate

$(1-\delta)$: Depreciation factor

P: Pre-research equilibrium price

t: year t