Appendix B

Equivalent Stiffness and Damping

In order to properly account for the AMD characteristics in the undamped critical speed analysis and forced response analysis, the equivalent stiffness and equivalent damping of the AMD used in this research were calculated.

First, the position stiffness (K_p) and the current stiffness (K_i) were calculated using a linearized model. Then, the model of the AMB plant developed by Josh Clements (unpublished work) was used to get the transfer function of the system and then the equivalent stiffness and damping were calculated.

Linearized Stiffness and Damping

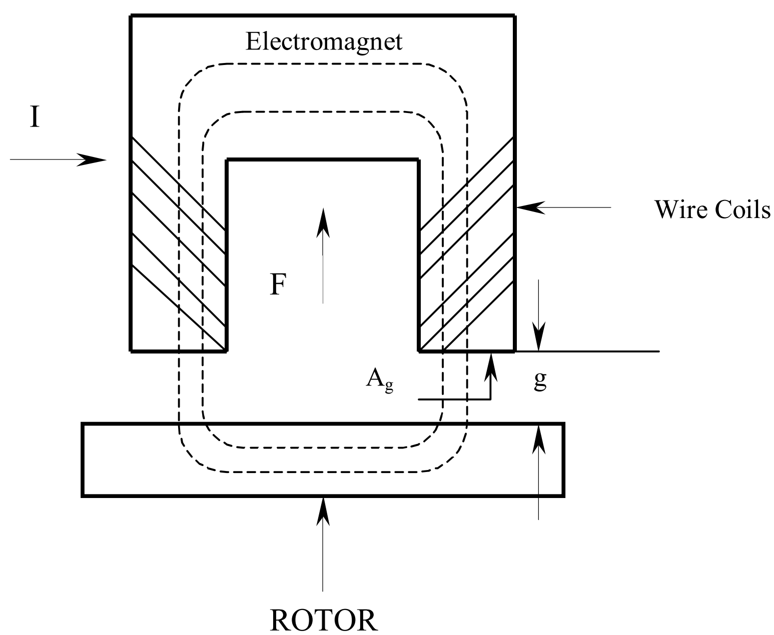


Figure B.1: Single horseshoe electromagnet

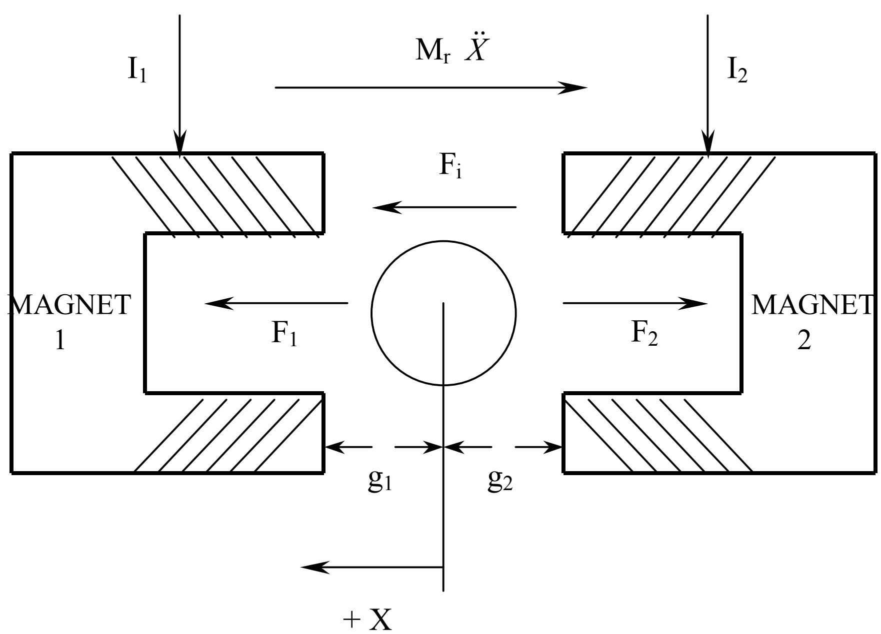
The attractive magnetic force applied to the rotor is given by the following equation

$$F = \frac{\varepsilon \mu_0 A_g N^2 I^2}{4g^2} \tag{B.1}$$

where ε is a geometric correction factor, μ_0 is the permeability of the air gap, A_g is a single pole face area, N is the total number of wire coils in a horseshoe, I is the current and g is the gap distance.

Figure B.2 shows two identical opposing magnets constituting a single axis within a magnetic bearing actuator.

The equation of motion that describes the system is given by



$$M_r \ddot{X} + F_2 - F_1 = F_i \tag{B.2}$$

where M_r is the mass of the rotor, F_1 and F_2 are the forces from each opposing magnet and F_i is an external forced applied to the mass of the rotor. The stiffness and damping are derived from the net forced (F_{NET}) applied to the mass by the opposing magnets. The net force is giving by the equation

$$F_{NET} = F_2 - F_1 \tag{B.3}$$

Substituting Equation B.1 into Equation B.3 results

$$F_{NET} = F_2 - F_1 = \varepsilon \frac{\mu_0 A_g N^2}{4} \left(\frac{I_2^2}{g_2^2} - \frac{I_1^2}{g_1^2} \right) \tag{B.4}$$

where I_1 and I_2 are the currents in magnets 1 and 2 and g_1 and g_2 are the gap distance between the mass of the rotor and the magnets 1 and 2 respectively.

The terms g_1 and g_2 can be replaced by

$$g_1 = g_0 - X \tag{B.5}$$

and

$$g_2 = g_0 + X \tag{B.6}$$

where g_0 is the nominal gap assuming that the rotor is centered in the axis and X represents a perturbation in the position of the rotor measured from the center. The difference in sign is because when the rotor moves towards any of the magnets, moves away from the other one.

In a similar fashion, the currents I_1 and I_2 can be expressed as

$$I_1 = I_B - I_P \tag{B.7}$$

and

$$I_2 = I_B + I_P \tag{B.8}$$

where I_B is a bias current and I_P is a perturbation current.

Substituting Equations B.6, B.7, B.8 and B.8 into Equation B.5 results in the following

$$F_{NET} = \frac{\varepsilon\mu_0 A_g N^2}{4}\left[\frac{(I_B + I_P)^2}{(g_0 + X)^2} - \frac{(I_B - I_P)^2}{(g_0 - X)^2}\right] \tag{B.9}$$

The linearized model of the net force $F_{NET,}$ assumes that the perturbation current I_P, and the perturbation position X, are small compared to the bias current $I_{B,}$ and the nominal gap g_0, respectively. This allows the exclusion of the higher order terms of the perturbation current I_P, and the perturbation position X, resulting in the following equation.

$$F_{NET} = \left(\frac{\varepsilon\mu_0 A_g N^2 I_B}{g_0^2}\right) I_P - \left(\frac{\varepsilon\mu_0 A_g N^2 I_B^2}{g_0^3}\right) X \tag{B.10}$$

The position stiffness in now calculated taking the partial derivative of Equation B.10 with respect to the perturbation position X and evaluating the resulting expression at the bias current I_B and the nominal gap g_0. In a similar fashion, the current stiffness is calculated taking the partial derivative of Equation B.10 with respect to the perturbation current I_P and evaluating the resulting expression at the bias current I_B and the nominal gap g_0. The position stiffness is given by

$$K_P = \left.\frac{\partial F_{NET}}{\partial X}\right|_{g_0, I_B} = -\frac{\varepsilon\mu_0 A_g N^2 I_B^2}{g_0^3} \tag{B.11}$$

and the current stiffness is given by

$$K_I = \left.\frac{\partial F_{NET}}{\partial I_P}\right|_{g_0, I_B} = -\frac{\varepsilon\mu_0 A_g N^2 I_B}{g_0^2} \tag{B.12}$$

now, substituting the position stiffness and the current stiffness into equation B.2 yields

$$M_r \ddot{X} + K_P X + K_I I_P = F_i \qquad \text{B.13}$$

Equivalent Stiffness and Damping

The transfer function of the controller expresses the relationship between the output, perturbation current and the input, position

In a general form the controller transfer function can be written as

$$G(i\omega) = a_G(\omega) + ib_G(\omega) \qquad \text{B.14}$$

where a_G and b_G represent the real and imaginary part of the transfer function respectively and ω represents the frequency. This transfer function multiplied by the position X yields the control current. Substituting the control current as I_P into Equation B.13 and assuming a harmonic forcing function results

$$-M_r X\omega^2 + \left[K_P + K_I\left(a_G + ib_G\right)\right]X = F_i \qquad \text{B.15}$$

now, equating the force produced by an equivalent stiffness and damping with the net force produced by the position stiffness, the current stiffness and the controller transfer function results in the following

$$\left(K_{eq} + C_{eq} i\omega\right)X = \left[K_P + K_I\left(a_G + ib_G\right)\right]X \qquad \text{B.16}$$

and now, equating real and imaginary terms in both sides of the equation results in the equivalent stiffness as

$$K_{eq} = K_P + K_I a_G \qquad \text{B.17}$$

and the equivalent damping as

$$C_{eq} = \frac{K_I b_G}{\omega} \tag{B.18}$$

Controller Transfer Function

A model for the controller transfer function developed by Josh Clements (unpublished work) was reviewed and it was considered appropriate for the AMD system used in the present research. Clements considered in the controller transfer function the effect of the sensor sensitivity, the PID filter, the Low Pass filter and the amplifier.

Equivalent stiffness and damping values

The parameters used to tune the AMDs, shown in table 2.3, were introduced in a MATLAB file developed by Josh Clements (unpublished work) resulting in the plots of the equivalent stiffness and damping as a function of the frequency shown in Figure B.3.

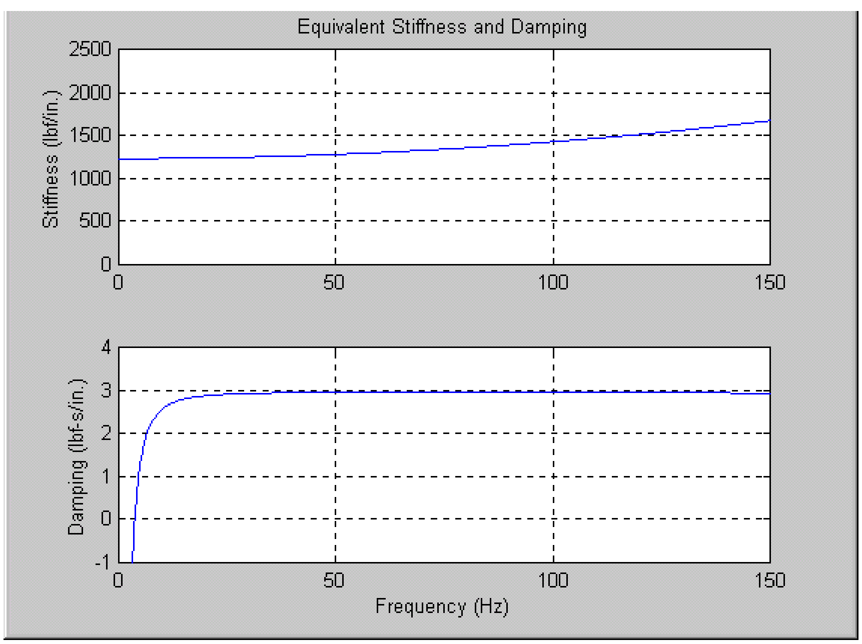


Figure B.3: Equivalent Stiffness and Damping of the AMD