Appendix A

Mathematica Code for SMA Ring Heating Model

The following is the Mathematica code used to evaluate the heating of SMA rings in bolted joints as described by equations 4.2 and 4.3.

```
Tsurround = 300 ; (*K*)
Current = 300;  (*amps*)
tfin = 60; (*seconds*)

(*Ring Properties*)
(*Ring must be heated to 438 K (165 C) but
  kept less than 573 k (300 C).*)
R = .0018; (*ohms*)
mass = 6.778 / 1000; (*kg*)
l = .3815 * .0254 ; (*m*)
id = .959 * .0254; (*m*)
od = 1.054 * .0254; (*m*)
k =  .558 * 90.7 + .442 * 21.9; (*W/m K*)
cp = .558 * 444 + .442 * 522; (*J/m K*)
k = 18;
cp = 837;
ringarea = π * ((od / 2) ^ 2 - (id / 2) ^ 2)

(*alumina washer properties*)
kwasher = 36 ; (*W/m K*)
ρwasher = 3970; (*kg/m^3*)
cpwasher = 765 ; (*J/kg K*)
lwasher = 3 / 16 * .0254; (*m*)

(*Cu lead properties*)
klead = 401 ; (*W/m K*)
ρlead = 8933 ; (*kg/m^3*)
cplead = 385 ; (*J/kg K*)
llead = 1 / 8 * .0254; (*m*)
```

```
vol = l * π ((od / 2) ^ 2 - (id / 2) ^ 2);
qdot = Current ^ 2 * R / vol  (*W/m^3*)
power = Current ^ 2 * R
ρ = mass / vol
ρ2 = .558 * 8900 + .442 * 4500  (*kg/m^3*)
```

one dimensional heat equation with constant k and insulated ends

```
var1 = NDSolve[{∂t T[x, t] * ρ * cp / k == ∂x,x T[x, t] + qdot / k, ((∂x T[x, t]) /. x → 0) == 0,
   ((∂x T[x, t]) /. x → l) == 0, T[x, 0] == Tsurround}, T, {x, 0, l}, {t, 0, tfin}]

Tsolved = First[var1]
Plot3D[Evaluate[T[x, t] /. Tsolved], {x, 0, l}, {t, 0, tfin}, PlotPoints → 30,
 AxesLabel → {"x", "t", "T"}]
Evaluate[T[x, t] /. Tsolved] /. {x → 0, t → tfin}

(*Ring must be heated to 438 K (165 C) but
  kept less than 573 k (300 C).*)
```

one dimensional heat equation with constant k and heat sink ends

```
var1 = NDSolve[{∂t T[x, t] * ρ * cp / k == ∂x,x T[x, t] + qdot / k,
   ((T[x, t]) /. x → 0) == Tsurround, ((T[x, t]) /. x → l) == Tsurround,
   T[x, 0] == Tsurround}, T, {x, 0, l}, {t, 0, tfin}]

Tsolved = First[var1]
Plot3D[Evaluate[T[x, t] /. Tsolved], {x, 0, l}, {t, 0, tfin}, PlotPoints → 30,
 AxesLabel → {"x (m)", "time (s)", "Temp. (K)"}]
Plot[Evaluate[T[x, t] /. Tsolved] /. x → l / 2, {t, 0, tfin}, PlotPoints → 30,
 AxesLabel → {"x (m)", "time (s)", "Temp. (K)"}]
Evaluate[T[x, t] /. Tsolved] /. {x → l / 2, t → tfin}

(*Ring must be heated to 438 K (165 C) but
  kept less than 573 k (300 C).*)
```

one dimensional heat equation with constant k and heat sink ends (check (eq3.42 in Incropera)

```
Tcheck = -qdot / (2 * k) * x ^ 2 + qdot / (2 * k) * (l / 2) ^ 2 + Tsurround
Plot[Tcheck, {x, -l / 2, l / 2}]
Tcheck /. x → 0
```

one dimensional heat equation with nonconstant k (includes washers) and heat sink ends

```
ktot = kwasher + UnitStep[(x)] * (k - kwasher) - UnitStep[(x - (l))] * (k - kwasher);
ρtot = ρwasher + UnitStep[(x)] * (ρ - ρwasher) - UnitStep[(x - (l))] * (ρ - ρwasher);
cptot = cpwasher + UnitStep[(x)] * (cp - cpwasher) -
   UnitStep[(x - (l))] * (cp - cpwasher);
qdottot = 0 + UnitStep[(x)] * (qdot - 0) - UnitStep[(x - (l))] * (qdot - 0);
ltot = l + 2 * lwasher;
Plot[cptot, {x, -lwasher, lwasher + l + l / 8}]

var1 = NDSolve[{∂t T[x, t] * ρtot * cptot == ∂x (ktot * ∂x T[x, t]) + qdottot,
   ((T[x, t]) /. x → -lwasher) == Tsurround, ((T[x, t]) /. x → l + lwasher) == Tsurround,
   T[x, 0] == Tsurround}, T, {x, -lwasher, l + lwasher}, {t, 0, tfin}]
Tsolved = First[var1]
Plot3D[Evaluate[T[x, t] /. Tsolved], {x, 0, l}, {t, 0, tfin}, PlotPoints → 30,
 AxesLabel → {"x (m)", "time (s)", "Temp. (K)"}]
Plot[Evaluate[T[x, t] /. Tsolved] /. x → l / 2, {t, 0, tfin}, PlotPoints → 30,
 AxesLabel → {"x (m)", "time (s)", "Temp. (K)"}]
Evaluate[T[x, t] /. Tsolved] /. {x → l / 2, t → tfin}
Evaluate[T[x, t] /. Tsolved] /. {x → l / 4, t → tfin}
Evaluate[T[x, t] /. Tsolved] /. {x → 0, t → 11}

(*Ring must be heated to 438 K (165 C) but
  kept less than 573 K (300 C).*)
```

Additional analyses were performed to evaluate various other configurations of the joint however, these varied only in the formulation of the nonconstant material properties.