# 3 Development of a Fatigue Curve Database and Implementation

One of the updates required by DuPont to *Tmin* was the inclusion of four more alternating Stress-Number of Cycles (S-N) fatigue curves to its internal database. The four fatigue curves to be included are Nickel 200, Aluminum 3003-0, 1100, and 6061-T6. Upon research of these fatigue curves, it was found that many were created using different testing machines, which are described in detail in Section 3-2. Using these machines, the allowable stress of an engineering metal at a cycle number can be found. The allowable stress of a metal is based on the stress amplitude are discussed in Section 3-1. Figure 3-1 shows a typical comparison S-N curve created by using a stress-based fatigue test machine. This figure compares the strength difference between engineering metals 4340 Steel and 2024 Aluminum.

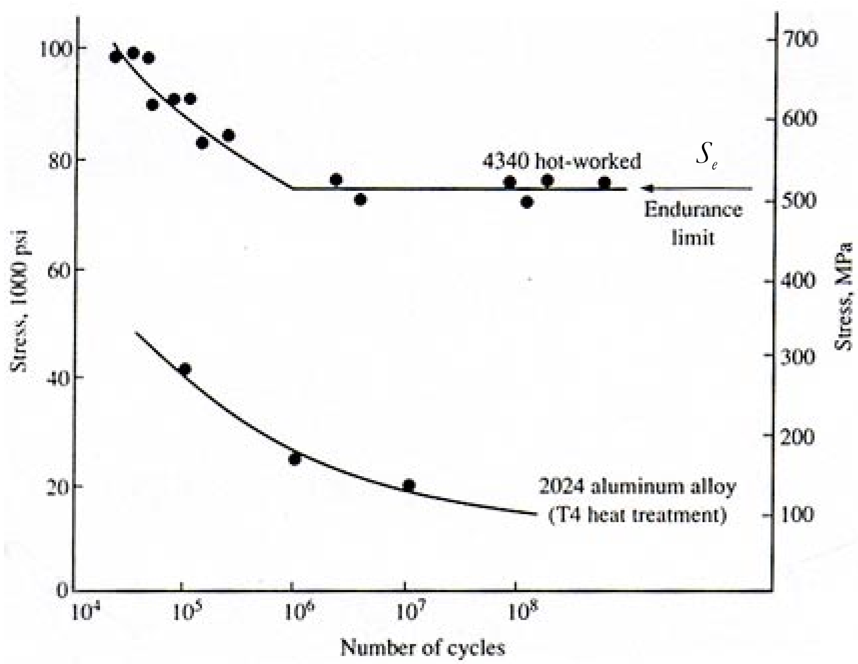



**Figure 3-1.** Stress Amplitude versus Number of Cycles to Failure Curves (after Flinn and Trojan [18])

There are several types of S-N curves, all of which arrive at the same goal, to obtain an important numeric value called the stress endurance limit, $S_e$, as seen in Figure 3-1. In order to arrive at the ASME S-N curve format discussed in Section 2-2, analysis of the methods of how these curves were created are shown in the next section.

The endurance limit is sometimes called the *fatigue strength,* which can also be called the allowed stress, $S_a$, at a given life. The endurance limit, seen in Figure 3.1, shows that stressing below $S_e$ of the AISI 4340 steel does not cause failure [18]. However, for Aluminum, the S-N curve continues to decline. As a result, the material must be tested for the number of cycles that it will be in service. The endurance limit is important because a design engineer can use this numerical value for the planning and design of a project. In Section 3-2 the procedures for the development of an S-N curve through the use of testing machines are detailed. Also in this section, the different stress states that are imposed on an engineering metal while using the testing machines are also explained. Section 3-3 details the creation of a strain-based fatigue curve and the cyclic hardening or cyclic softening of the engineering metal. In Section 3-4 the S-N curves that are to be input into the *Tmin* Microsoft Access database are detailed. And finally, the format for the inclusion of the S-N curves and how ASME requires the calculation of a minimum pipe-wall thickness are detailed in Section 3-5.

## 3.1 Stress-Based Fatigue Analysis and Stress States

The stresses that are placed on test specimens may be axial in character. This results in a tension-compression S-N curve. Two types of loading may occur to create a tension-compression S-N curve, axial or load based. When the loading is axial along a test-specimens length, the result is an axial S-N curve. Loading a test-specimen through displacement, which results in a bending compression S-N curve, creates another type of S-N curve. Both of these types of S-N curves are discussed in Section 3-2.

In order to identify these S-N curve types, the methods used in their creation will be discussed in Section 3.2 as well. Figure 3-2 shows various stress states against time. The stresses defined in this figure are mean stress, $\sigma_M$, stress amplitude, $\sigma_a$, minimum stress in the cycle, $\sigma_{Min}$, maximum stress in the cycle, $\sigma_{Max}$, stress range, $\Delta\sigma = \sigma_{Max} - \sigma_{Min}$, stress ratio, *R*, and amplitude ratio, *A* [19]; the stresses and ratios are defined in Section 3.2. By changing the stress states on the material in the test machine, various types of S-N curves are produced.

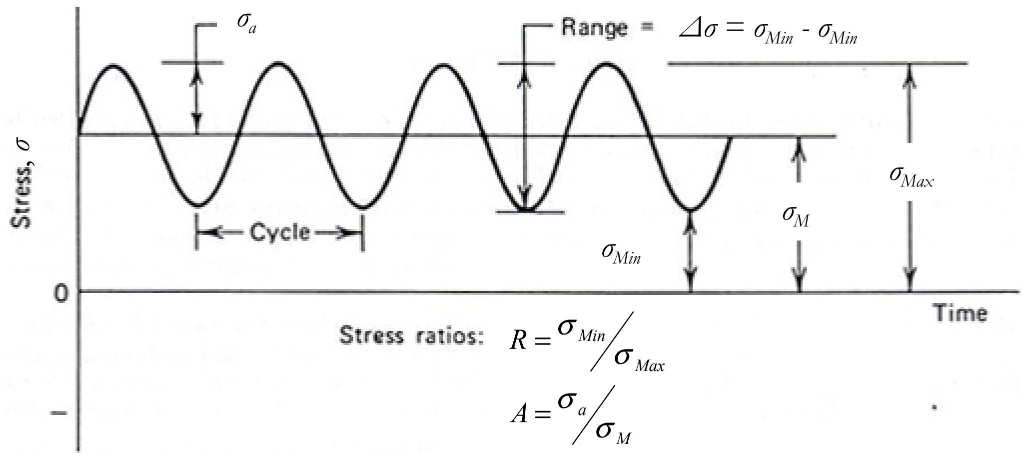



**Figure 3-2**. Schematic Illustrating Cyclic Loading Parameters (after Fuchs and Stephens [20])

As stated by Callister, there are three possible stresses per time. They are as follows: Completely Reversed (*R*=-*1*), Released Tension (*R*=*0*), and Non-zero Mean Stress [21]. The first is completely reversed stress with constant amplitude, where the alternating stress varies from a maximum stress, $\sigma_{Max}$, to a minimum compressive stress, $\sigma_{Min}$, of equal magnitude. The second, called released tension, occurs when the minimum stress is equal to zero. In the stress range seen in Figure 3.2, the stress ranges from zero up to some tensile maximum and then back to zero. For the third, the non-zero mean stress is similar to completely reversed except that the mean stress is either very tensile or compressive, but is not zero [21].

The following section will show specific stress states that could occur to a test specimen. Because of the different stress states, the addition or removal of loads on test machines can achieve various types of S-N curves. As a result, the stress amplitude of the specimen could be lower or higher by the addition or removal of some stresses. To help understand what stresses are applied to a material, the following are defined: stress ratio, amplitude ratio, mean stress, stress range, alternating stress, and stress amplitude.

***Stress Ratio and Amplitude Ratio*** – Two ratios that are used in the evaluation of S-N curves are stress ratio, *R*, and amplitude ratio, *A*, seen as Equations (3.1) and (3.2).

$$R = \frac{\sigma_{Min}}{\sigma_{Max}} \tag{3.1}$$

$$A = \frac{\sigma_a}{\sigma_M} \tag{3.2}$$

When evaluating S-N curves that had been created many years ago, it is necessary to know one or both of these ratios. The use of these ratios can determine the state of stresses used on the specimen.

***Mean Stress –*** When developing an S-N curve for a specific application, the minimum and maximum stresses are changed. The mean stress is computed using Equation (3.3) [19].

$$\sigma_M = \frac{\sigma_{Max} + \sigma_{Min}}{2} \tag{3.3}$$

Fuchs states that there are two cases that can occur with mean stress: completely reversed cycling, non-zero-mean stress [20]. Once the minimum and maximum stress values are changed the influence of a mean stress on an S-N curve can be seen in Figure 3-3. When the mean stress is zero, seen in Figure 3-3 (*a*), completely reversed stressing occurs. Completely reversed stressing is defined as having a stress ratio $R$=*-1*. Next, with a non-zero-mean stress, shown in Figure 3-3 (*b*), the minimum and maximum stresses are of unequal amplitudes. As seen in this figure, if the stress amplitude is limited to specimen fatigue strength, then as the mean stress increases the alternating stress decreases.

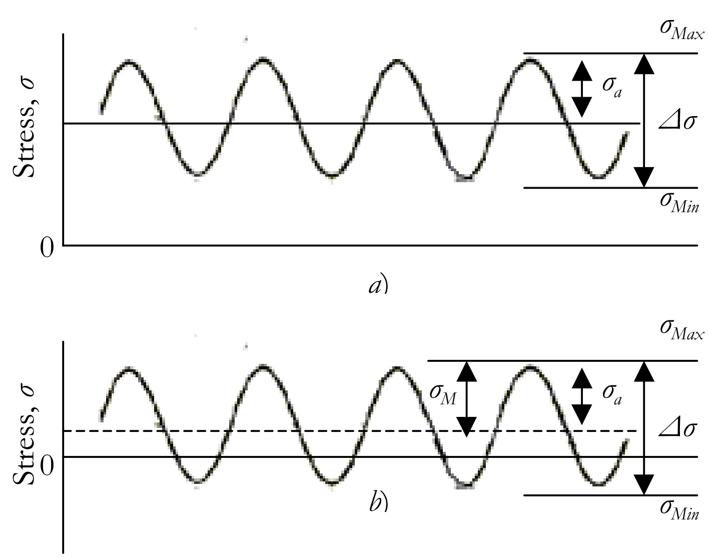



**Figure 3-3.** Mean Stress States Affect on Total Stress. (*a*) Zero Mean Stress, (*b*) Non-Zero Mean Stress (adapted from Dowling [22])

The three mean stress cases shown in Figure 3-4 illustrate that increased mean stress reduces the allowed alternating stress while increased compressive mean stress provides more room for an increasing alternating stress.

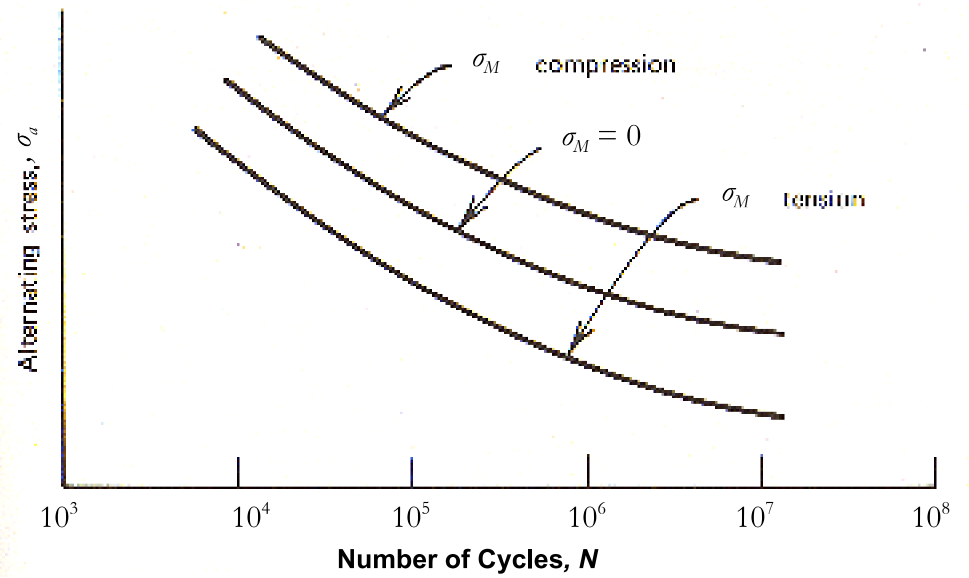



**Figure 3-4.** Effect of Mean Stress on Fatigue Life (after Fuchs and Stephens [20])

According to Fuchs and Stevens [20], in general when accounting for mean stresses, tensile stresses are detrimental while compressive stresses are beneficial [20]. Fuchs and Stevens also have studies that have found that compressive mean stresses can result in increases of up to 50 percent in alternating fatigue strength [20]. As stated in Section 3.3, reciprocating bending test machines are used to create mean stress fatigue curves.

As seen in Figure 3-4, the stress amplitude of a material is greatly reduced when subjected to a tension mean stress. In general, tensile mean stresses are harmful and compressive mean stresses are beneficial to a material [20]. This occurs when the minimum stress is zero and zero-to-maximum tension stressing occurs.

***Stress Range*** – The stress range is defined as the difference between the maximum and minimum stress, as seen in Equation (3.4).

$$\Delta\sigma = \sigma_{Max} - \sigma_{Min} \tag{3.4}$$

If the stress ratio, Equation (3.1) and the amplitude ratio, Equation (3.2), are provided with an S-N curve, the stress range can be found. However, because stress-states are different for most S-N curves, it would be difficult to reproduce a fatigue curve exactly because of possible mean stress effects.

***Alternating Stress*** – The term alternating stress is used by some authors and has the same meaning as stress amplitude (defined next), and are seen as Equations (3.5) and (3.6) [22]. Simple substitution can be used for the minimum and maximum stresses, using the mean stress and stress amplitude.

$$\sigma_{Max} = \sigma_M + \sigma_a \quad (3.5)$$

$$\sigma_{Min} = \sigma_M - \sigma_a \quad (3.6)$$

***Stress Amplitude*** – The minimum and maximum stresses that are placed on the test specimen by the test machine result in the stress amplitude. Stress amplitude is twice the stress range. Equation (3.7) uses the minimum and maximum stress values to calculate the allowable stresses that are used to create an S-N curve. In the following section, the creation of an S-N curve will be discussed.

$$\sigma_a = \frac{\sigma_{Max} - \sigma_{Min}}{2} \quad (3.7)$$

## 3.2 Creation of an Stress-Cycle Curve

In order to create an S-N curve, several machines were created. In this section, three machines that are still in use are described: Rotating Bending, Reciprocating Bending, and a Direct Force Fatigue Testing Machine.

***Rotating Bending Testing Machine*** **–** The type of S-N curve created by this machine is identified as a Rotating-Bending (R-B), stress-controlled fatigue data curve. This machine is shown in Figure 3-2. The rotating bending test machine is used to create a S-N curve by turning the motor at a constant Revolution-Per-Minute (RPM), or frequency. To create a failure on the engineering specimen, seen in Figure 3-5, a constant-stationary load is applied on the specimen, which creates a constant bending moment [19]. A stationary moment applied to a rotating specimen causes the stress at any point on the outer surface of the specimen to go from zero to a maximum tension stress back to zero and finally to a compressive stress. Thus, the stress state is one that is completely reversed in nature.

Next, the numbers of turns of the motor are counted until failure of the specimen occurs. Using the failure number and the applied alternating stress one obtains a point on the S-N curve. Stressing multiple times at the current stress level and many times at other a stress levels provides the data for a complete S-N curve.

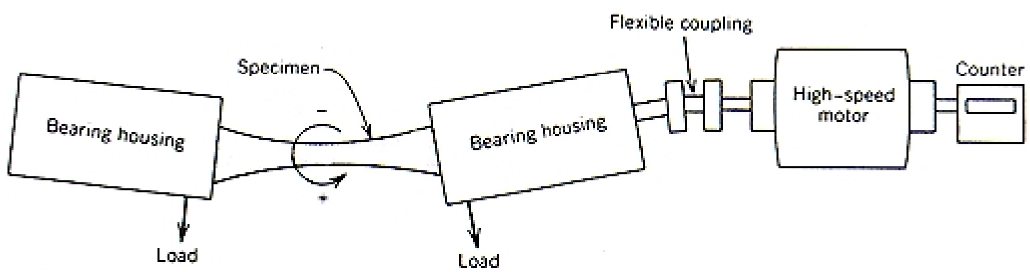



**Figure 3-5.** Rotating Bending Testing Machine (after Callister [21])

A typical test specimen that is used for a rotating bending machine is shown in Figure 3-6. Normally these specimens are ground and polished to a specific finish, which depends on the type of S-N curve that is desired.

**Figure 3-6.** Rotating-Bending Machine Test Specimen (after Fuchs and Stephens [20])

The rotating bending test machine ensures that the bending moment is constant along the entire specimen length. To keep a constant bending moment on the test specimen a constant load is applied on both bearing housings. The tests completed are performed using a constant load applied to the specimen. As a result, each point on the surface of the test specimen is subjected to fully reversed cycling. Collins [19] states that any point on the specimen is subjected to a completely reversed stress-time pattern by following the stress history of a point on the specimen. Doing this it can be seen that as the point rotates from maximum compression at the top position down through center position with zero stress, then down to maximum tension at the bottom.

***Reciprocating Bending Test Machine –*** The type of S-N curve produced is identified as a Tension-Compression (T-C), strain-controlled fatigue data curve. This machine is capable of producing either completely reversed stresses or non-zero mean cyclic stresses by positioning the specimen clamping vise with respect to the mean displacement position of the crank drive as shown in Figure 3-7 [19]. This machine relies on controlled geometric deflections from the rotating eccentric crank to keep the alternating strain level zero.

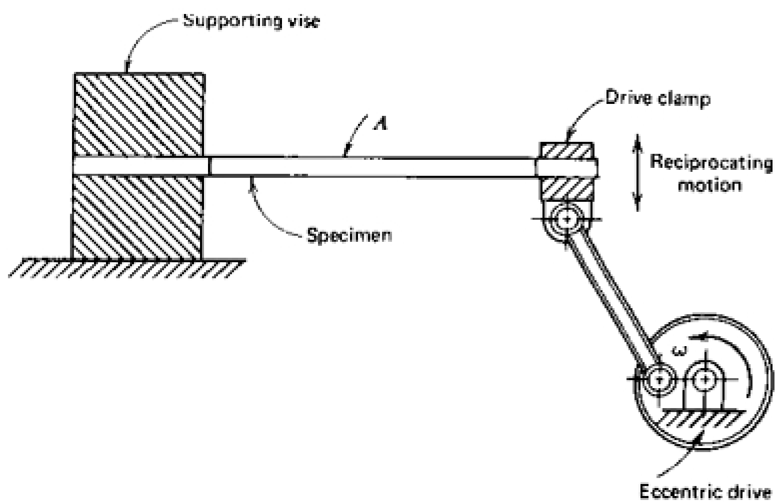



**Figure 3-7.** Test Machine for Reciprocating Bending (after Collins [19])

Dowling [22] states that geometric changes in the apparatus alter the length of the connecting rod from the eccentric drive to give different mean deflections, thus different mean stresses. A top view of the test specimen used in the reciprocating bending test machine is shown in Figure 3-8.

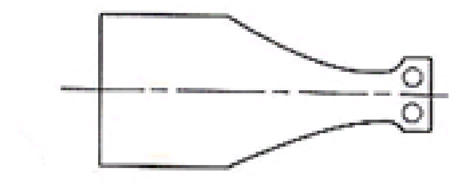


**Figure 3-8.** Reciprocating Bending Machine Test Specimen (after Fuchs and Stephens [20])

***Direct Force Testing Machine*** **–** The type of S-N curve produced is by this machine is identified as a Tension-Compression (T-C), constant-strain fatigue curve. This type of machine is similar to the reciprocating bending machines. However, this machine applies direct tensile or compressive stress on the specimen by pushing and pulling in the direction of the specimen axis [19]. The direct force fatigue machine is seen in Figure 3-9.

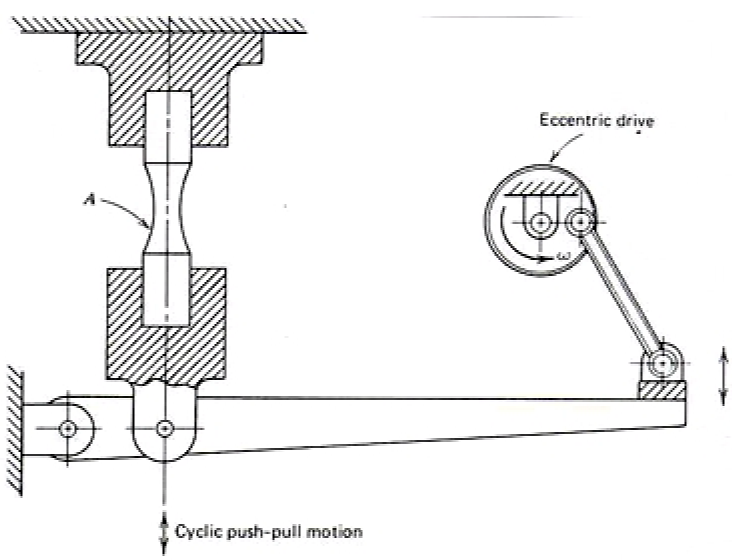



**Figure 3-9.** Machine for Direct-Force Testing (after Collins [19])

The specimens used in this machine are similar in geometry to the Rotating-Bending test specimen, as shown in Figure 3-10.

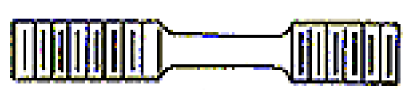


**Figure 3-10.** Direct-Force Machine Test Specimen (after Fuchs and Stephens [20])

As a result of the different test specimen shapes and sizes, these specimens can be used to obtain the desired T-C or R-B stress (strain)-number of cycles curve. The following section details the relationship between stress and strain that are used in obtaining various S-N curves using stress-based analysis.

### 3.3 Strain-Based Fatigue Analysis

According to Dowling [22], this strain-based analysis was developed in the early 1950's in order to analyze engineering metals that had fairly short fatigue lives. The approach used to obtain a strain-life curve is the use of a completely reversed cycling machine with constant minimum or maximum strain limits. Adapting a figure from Shigley and Mitchell [10], Figure 3-11 shows a hysteresis loop where various strain states are present verses the stress range, $\Delta\sigma$. The strains seen in this figure are plastic strain range, $\Delta\epsilon_p$, elastic strain range, $\Delta\epsilon_e$, and the total strain range, $\Delta\epsilon = \Delta\epsilon_p + \Delta\epsilon_e$. At point *A* in this figure is the fatigue ductility coefficient, $\varepsilon_F^{'}$, which is true strain corresponding to a fracture in one reversal. Point *B* gives the fatigue strength exponent which is the slope of the elastic strain line, and is proportional to the true stress amplitude [10].

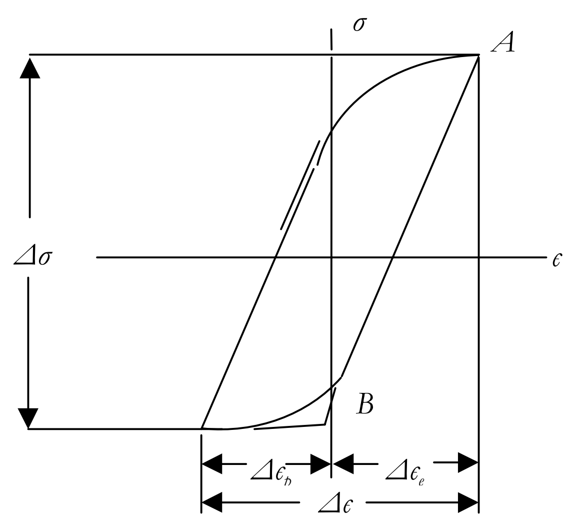



**Fig 3-11.** True Stress, True Strain Hysteresis Loop (adapted from Shigley and Mitchell [10])

Using the previous figure, in the left side of the $\sigma$-axis is the plastic region of the material, and the right side is the elastic region. Each of these data points obtained through failure, occur at corresponding an alternating strain, $\varepsilon_a$, and give a final S-N fatigue curve. Alternating strain is defined as the summation of elastic and plastic amplitude strains seen as Equation (3.8). The elastic strain is related to stress amplitude by Equation (3.9) [22].

$$\varepsilon_a = \varepsilon_{ea} + \varepsilon_{pa} \tag{3.8}$$

$$\varepsilon_{ea} = \frac{\sigma_a}{E} \tag{3.9}$$

Plastic strain is the measure of the width of a stress-strain hysteresis loop, seen in Figure 3-12. Hysteresis loops stabilize after repeated cycling so that the stress amplitude remains relatively constant over a large portion of fatigue life. Based on stable hysteresis loops at different constant strain amplitudes, a curve passed through the tops of the loops defines a cyclic stress-strain curve [23]. A typical hysteresis loop produced through cyclic-strain will either will increase or decrease in strain amplitude depending on the engineering metal being tested. In this figure, it is shown that at low cycles the specimen will either cyclically harden or soften. From the hysteresis loops seen in these figures, the total length of the hysteresis curve is identified as the total longitudinal strain, $\varepsilon_{el}^{l}$, which will be used in more detail in Section 3-4.

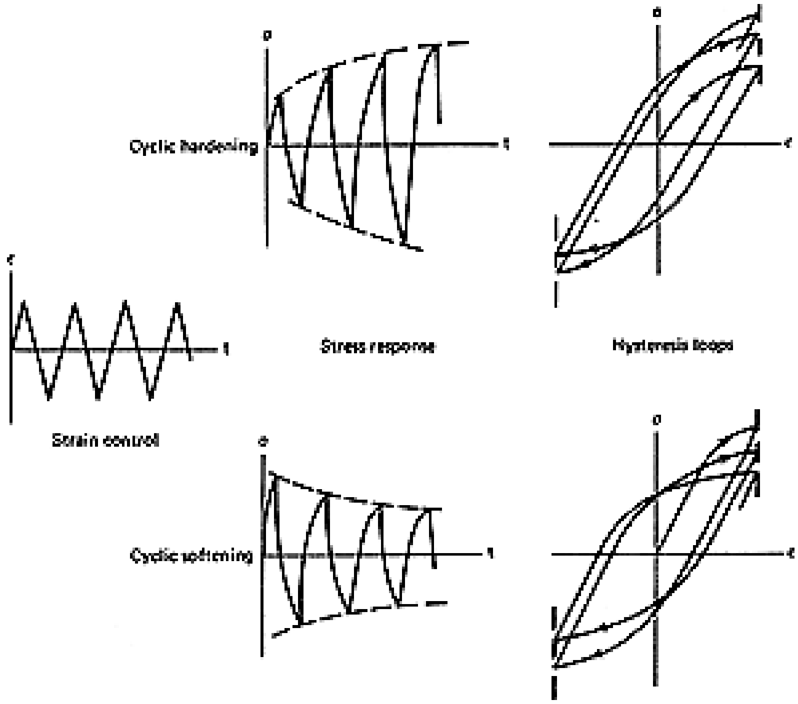



**Figure 3-12.** Illustration of Cyclic Strain Hardening or Strain Softening (after Collins [19])

One material that benefits through cyclic strain hardening is aluminum. Choosing aluminum for an application where strain loading is frequent will ensure that the material will not fail prematurely [18]. In the other case, where a material cyclically softens, such as copper, the material will be used only in situations where there is very little, if any, cyclic strain observed [18].

As was shown in the previous sections, there are many types of S-N fatigue curves that can be produced. Therefore, an understanding of how these S-N fatigue curves were produced was necessary. In the following section, the alternating stress S-N curves to be input into *Tmin* are detailed. In this section, the fatigue curves that are to be implemented into *Tmin* are shown to be of either a rotating-bending or tension-compression, stress-based or strain-based format according to the ASME [12]. To properly use the fatigue data, a correction factor to convert to or from a rotating-bending or tension-compression S-N curve will also be discussed in Section 3.4. Only upon the understanding of how these S-N curves were created, could the conversion to a conservative ASME fatigue curve be created.

## 3.4 Fatigue Data Obtained for *Tmin*

The additional S-N fatigue curves to be input into *Tmin* had to be converted to the conservative ASME fatigue curve format. Following the procedure required by ASME in Section 2-2, the following fatigue curves obtained were then adjusted according to the conservative ASME format [16]. In this section, the process for the evaluation and conversion of the four fatigue curves to ASME format (Aluminum 1100, 3003-0, 6061-T6, and Nickel 200) are detailed. Once all the fatigue curves were converted they were then incorporated into the *Tmin* Microsoft Access® database.

***Nickel 200:***

The fatigue data for this material was obtained from an Inconel data sheet [23]. However, in order to use this data, it had to be converted. D. J. McAdam Jr. [24] recorded fatigue testing of Nickel 200 in 1927. McAdam tested the Nickel 200 circular rod in air, fresh water, and salt water to give three different S-N curves. He obtained the fatigue data through a rotating-bending testing machine that was stress-controlled. Since this data was not in the tension-compression ASME format, it had to be converted.

A theory to convert the R-B data to a T-C format was created by Von Phillip [25]. This theory states that converting a material with a circular cross-section profile can be converted from R-B data to a T-C approximate data using in Equation (3-10). Fatigue curves are required by ASME are to be in T-C format. Therefore, a conversion from rotating-bending to a tension-compression format was performed using Equation (3-10). This equation changes the S-N data from rotating-bending, to a tension compression format. However, this conversion factor does not affect whether the data was strain or stress controlled. ASME, luckily, allows the use of stress-controlled data when strain-controlled data is not possible, as detailed in Appendix 5 of Section VIII of the Unfired Boiler and Pressure Vessel standards and codes [12].

$$\frac{S_{al\,TC}}{S_{al\,RB}} = 0.59 \tag{3-10}$$

Using this conversion factor, the rotating-bending S-N curve obtained was then converted to the correct ASME format by simple multiplication. However, since the data was stress-controlled before conversion to a T-C stress-controlled format, the data must also be mean-stress corrected using Equation 2.22 in Chapter 2. First, however, according to ASME Article III-2000 of Section III of the Boiler and Pressure Vessel Code, a best-fit curve to the experimental data is obtained by applying the method of least squares to the logarithms of the stress values [16]. Therefore, once the curve fit was applied using the least squares to the logarithms of the stress data, a curve was formulated. First, however, the stress data was converted to a log-log linear equation. Using the log-log linear regression allowed for a linear regression curve fit to be performed on the stress data; this is done by making the assumption of a straight line for the log-log fatigue data. Equation (3.11) shows the format starting the log-log linear regression. The logarithms of the stress and number of cycles were first obtained, and then a curve fit was which is done by taking the logarithm to the base 10 of the alternating stress and the number of cycles, *N*.

$$Log_{10}(\sigma_a), Log_{10}(N) \tag{3.11}$$

The data points resulting from these logarithm values are then passed through a mathematical solver for an equation that fits the line. Multiple order polynomial equations, from 1st to 3rd order were used to fit the log-log data. Once this was done, the differences in the curve fits were compared using the statistical $R^2$ value. The highest $R^2$ was one of the deciding factors in choosing the curve fit equation. Also, the statistical error of the data, which is seen in Equation (3.12), was checked. This calculation is called the *Signal-to-Noise* ratio (*S/N*), where the lower the numerical value, the better the curve fit.

$$S/N = \frac{\mathrm{R}^2}{Max(Log_{10}(\sigma_a))} \tag{3.12}$$

One final check was used as well, the numerical difference between the fatigue data and the curve-fit data, called the residuals, were compared. By comparing multiple polynomial curve fit cases, the case that was chosen was dependent on the scatter, or randomness of the residuals in conjunction with the highest *S/N* ratio. The curve fit and the logarithms of the stress and number of cycles are seen in Figure 3-13. Although the data appears to indicate an endurance limit, the numerical values of stress continue to fall, even below $10^{10}$ cycles. No endurance limit is therefore seen.

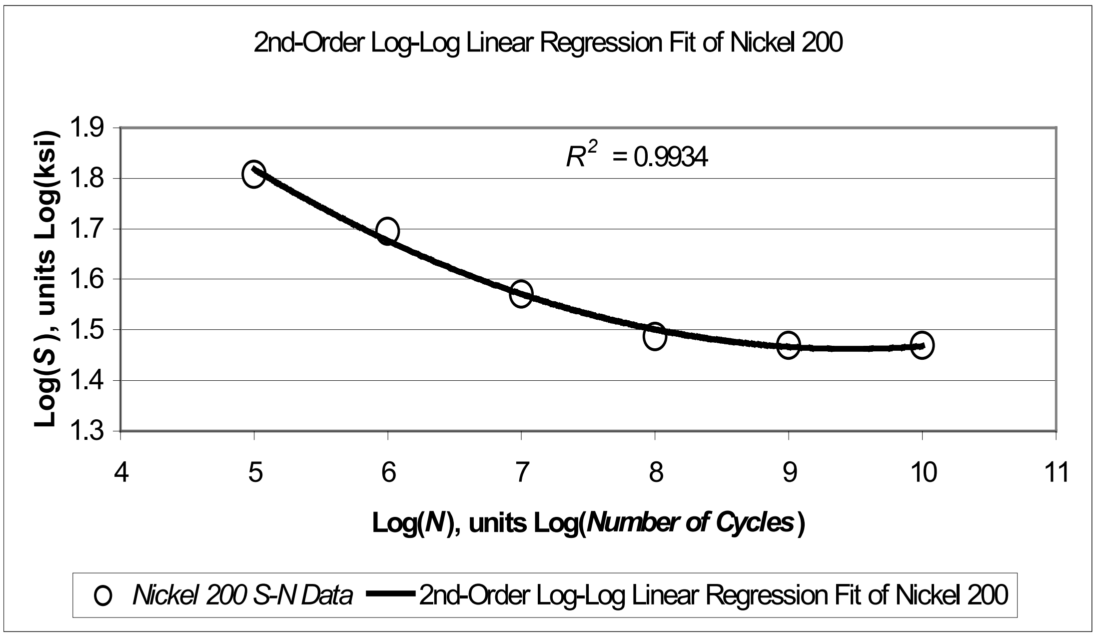



**Figure 3-13.** Second-Order Log-Log Linear regression Curve Fit of Nickel 200 Fatigue Data with $R^2$ Value

A *Matlab* plot is seen using the same curve fit data, but with the residuals seen in the lower figure and the resulting stress data can be found in Appendix B. Checking the residual values of the data to see that they are random ensures that a possible good fit for the data has been found. From consideration of the residuals, the $R^2$, and the *S/N* ratio, the second-order polynomial equation resulting from the curve fit was considered the best fit to the data and was used for representation of the fatigue data.

The fatigue curves that are supplied by ASME in Appendix 5 of Section VIII of the Boiler and Pressure Vessel code all start at 10 cycles and end at $10^6$ or greater number of cycles [12]. DuPont has entered in the data from these fatigue curves within the Microsoft

Access Database of *Tmin*. However, in the computer code of *Tmin*, they limit the number of cycles used for final evaluation from 360 to 7000 cycles [2]. For the Nickel 200 data supplied by Inconel and tested by McAdams, the fatigue data did not start until $10^4$ cycles [24]. However, upon direction of DuPont, the resulting curve produced could be extended to 10 cycles by extending the allowable stress curve fit, which was divided by a factor of 2 according to ASME guidelines, to 10 cycles. The justification from DuPont for extending the allowable stress curve fit was that *Tmin* is a screening tool and that using this lower stress value would be conservative. As a result of this direction, the log-log linear equation was again used with a second-order polynomial, but with the number of cycles starting at 10 cycles and ending at $10^6$ cycles, instead of $10^4$ to $10^8$ cycles as supplied by McAdams and Inconel [23, 24]. Upon application of this new direction, an S-N curve was produced that used the conservative ASME format, and the number of cycles extended from 10 to $10^6$ and is seen in Figure 3-14. However, as seen in the figure, the mean stress is not adjusted from use of the mean-stress adjustment equation, because there is no point at which the yield strength and the allowable stress cross. This is because the fatigue data supplied never is less than the yield strength value supplied by Shigley and Mishke [9].

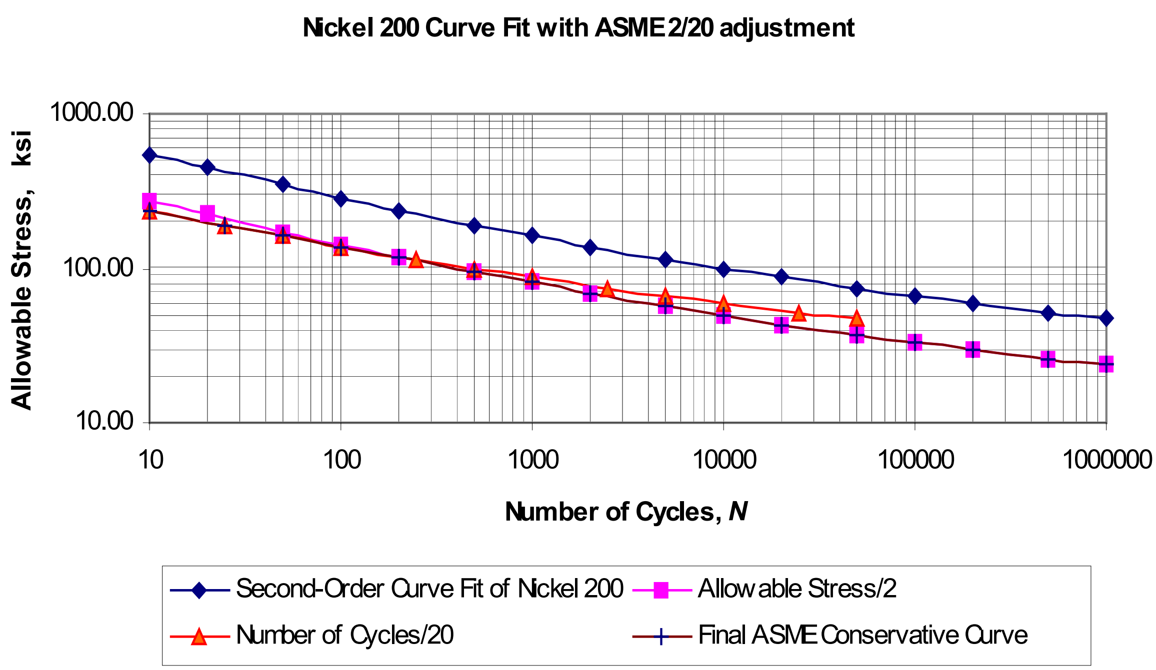



**Figure 3-14.** Fatigue Curve of Nickel 200, with Allowable Stress, $S_a$ Divided by a Factor of 2, and Number of Cycles, $N$, Divided by a Factor of 20.

***Aluminum 6061-T6:***

The fatigue data was obtained from a document written by G. T. Yahr [26], the document can be found in Appendix B. In this document Yahr supplies the entire fatigue curve equation, seen as Equation (3-13) that he derived from multiple sources for this aluminum alloy. From the multiple sources of data, which start at 10 cycles and end at $10^8$ cycles, he was able to use the ASME "best-fit" Equation (2.20). One of the stipulations of using this best-fit equation was the necessity to provide the value *A*, the percent area reduction of the specimen.

The value *A* that Yahr used in this equation was then solved for and found to be 12%. This value obtained was compared to another percent area reduction recorded by Shigley and Mishke [9]. They state that the percent area reduction for this material is 15%, which results in a 20% difference between the two. Since this percent difference is not very high, the best-fit equation was found to be valid for use. Since aluminum does not have a true stress endurance limit, which must be provided as *B* in the best-fit equation, it is actually impossible to provide this value. The usual method is to provide the fatigue strength at the highest know number of cycles. Yahr used a value of *14,000* psi, which was obtained from the S-N data at $10^8$ cycles.

$$S_{al} = \frac{2{,}100{,}00}{\sqrt{N}} + 14{,}000 Psi \tag{3-13}$$

Once the best-fit Equation (3-13) was confirmed to fit aluminum data by reproduction of the curve, and comparing it to the S-N curve provided in the document, the curve was ready for conversion to the conservative ASME format. The best-fit Equation (3-13) provided by Yahr was then corrected for mean-stress. Finally, the new mean-stress adjusted curve was then adjusted using the equation provided, and was converted to the conservative ASME format. The final curves produced by the entire process can be seen in Figure 3-15.

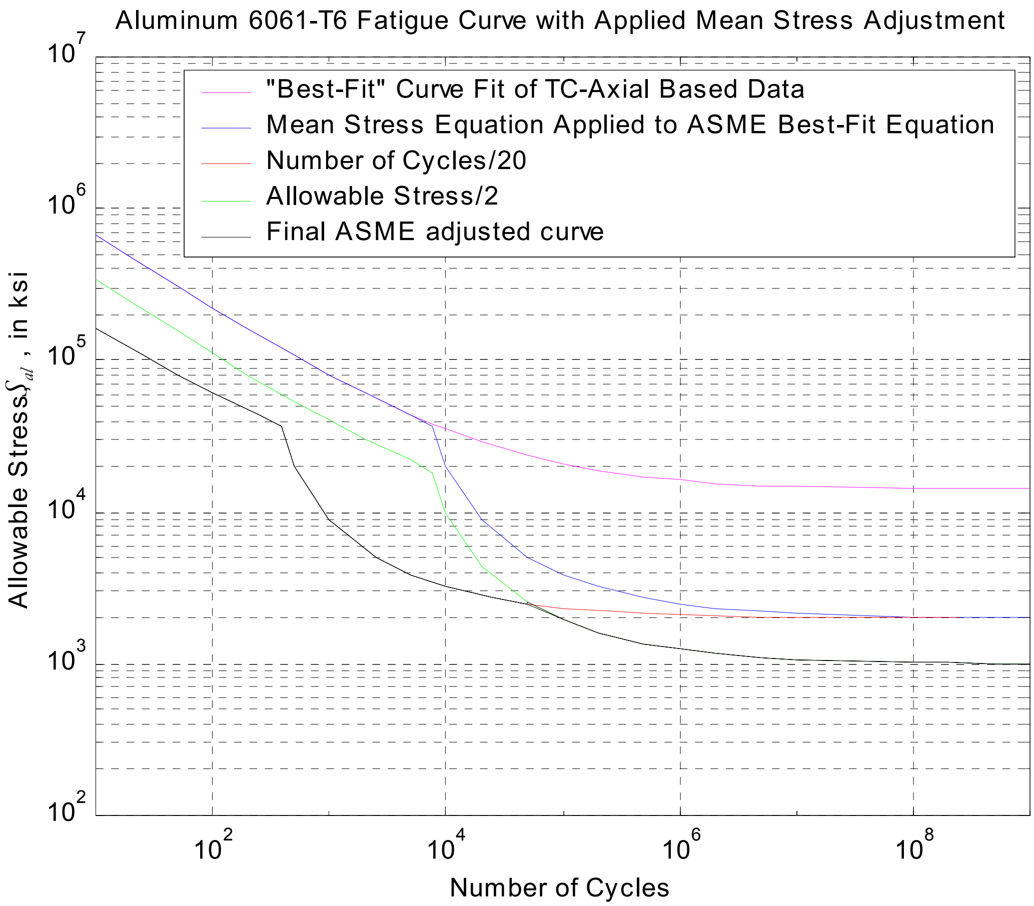



**Figure 3-15.** Fatigue Curve of Aluminum 6061-T6 with Mean Stress Adjustment, and $S_{al}$ Divided by a Factor of 2, and Number of Cycles Divided by a Factor of 20

The fatigue data supplied for this figure can be found in Appendix B and has been entered into the Microsoft Access database in *Tmin*.

***Aluminum 1100:***

A technical note developed by Robert W. Smith et al. [17] recorded this fatigue curve. They recorded a series of strain range tests for various materials, but only Aluminum 1100 was of interest to my sponsor. The data recorded was in a tension-compression, strain-based format. In addition, the data Smith recorded was in longitudinal strain format, $\varepsilon_{el}^{l}$ [17]. Recorded in this document was the conversion to a stress-number of cycles format is given in Equation (3-14) using the stress amplitude. Using this equation, the data was converted to the conservative ASME format.

$$S_{al} = \frac{E\varepsilon_{el}^{l}}{2} \qquad (3\text{-}14)$$

Using the data supplied in the document the total longitudinal strain-range was then converted to allowable stress. Then the converted data was subject to the curve fitting procedure described earlier and was finally converted to an ASME approved S-N curve. The fatigue data, the log-log regressive curve fit, residuals, and final ASME format adjusted curve can be seen in Appendix B.

***Aluminum 3003-0:***

The final fatigue curve that was to be incorporated into *Tmin* was Aluminum 3003-0, and was obtained by from Alcoa and adapted by the American Society of Metals (ASM). Alcoa supplied the rotating-bending, fully reversed fatigue curve [27]. The data obtained was then converted to T-C format using the Von-Phillips theory. Once the conversion was completed, a log-log linear regression to the data curve was performed. Using the curve-fit polynomial, the data was then mean-stress adjusted, and the 2/20 formats were applied. The curve-fit as well as the original data, residuals, and the final implemented fatigue curve can be found in Appendix B. The fully adjusted ASME stress-number of cycles curve was then implemented into the *Tmin* Microsoft Access database.

In the next section, the implementation of the fatigue data curves and the calculations required by ASME for solution of a safe pipe-wall thickness will be explained.

## 3.5 Calculation of Piping Fatigue Information and Implementation

The American Society of Mechanical Engineers developed another code that is used for the evaluation of pressurized piping. This standard and code, Process Piping, ASME B31.3-1999 Edition is used for the evaluation of piping stresses and pipe-wall thicknesses [28]. Within *Tmin,* DuPont has developed an order of operations that is used for the calculation of horizontal piping span, pipe-wall thickness, which follows the ASME B31.3

code. This calculation process can be seen in Figure 3-16. This figure shows an additional procedure for 2-D vertical piping span developed for this project. These procedures will be used for the solution of a minimum pipe-wall thickness.

The first block in the figure obtains an allowable static stress, $S_a$. When the end user chooses a material and an operating temperature for *Tmin*, an allowable stress at this temperature is chosen. The allowable static stress is obtained through Table A-1, which contains the allowable stresses of materials at a specified temperature, in the B31.3 standards and code [28]. DuPont renames this allowable static stress as $S_b$ for ease of later calculations within the computer program. According to Part 2, paragraph 304.1 (a) in the B31.3 code, the required thickness of straight sections of pipe shall be determined in accordance with the following Equation (3-15) [28]. The minimum thickness $t_{Hoop}$ for the pipe selected shall not be less than the nominal thickness, $t_m$, minus the required thickness, $t_m$ ($t_m$ – $c_{Hoop}$). In Equation (2.11) $c$ is the added material allowance to account for progressive deterioration or thinning of the pipe-wall in service, due to the effects of corrosion, erosion, and wear [29].

$$t_m = t_{Hoop} + c \tag{3-15}$$

Using the allowable static stress value obtained from Table A-1, in the B31.3 document, the pipe-wall thickness will be calculated. The next block is the calculation of the pipe-wall thickness, $t_{Hoop}$, which is derived from solving the hoop stress equation for the pipe-wall thickness. The two other factors seen in block 2 of the design procedure are $E_w$, and $Y$ [28]. The factor $E_w$ is a weld joint factor chosen by the user upon selection of the piping material, and $Y$, is a temperature dependant coefficient whose values can be found in Table 304.1.1 of the B31.3 ASME code and standards [28].

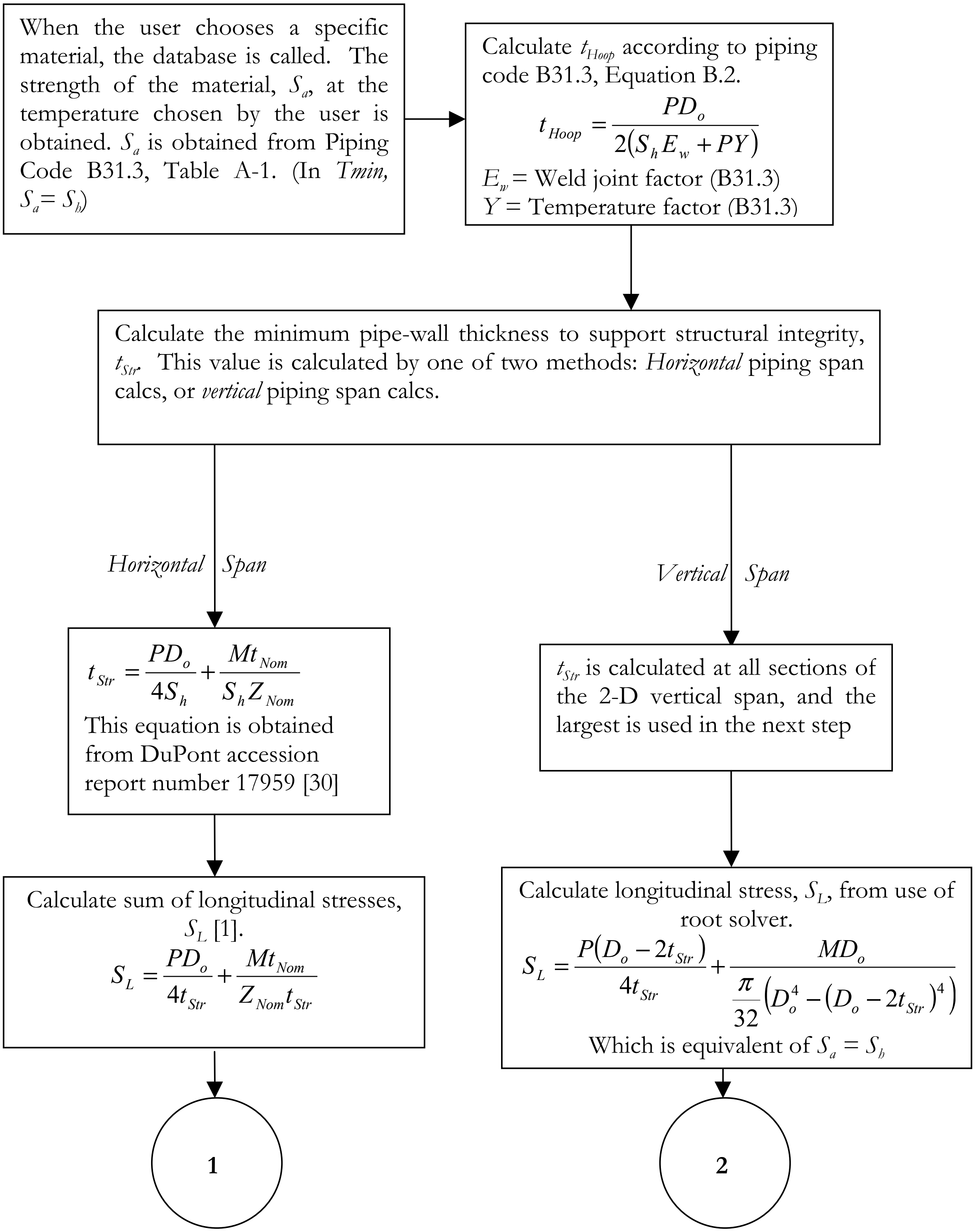



**Figure 3-16.** Order of Operations for Pipe-Wall Thickness within *Tmin*. Bullets 1 and 2 will be continued in Figure 3-17.

The next block in Figure 3-16 is to calculate the minimum pipe-wall thickness to support structural integrity, which is called $t_{Str}$. For the horizontal span and conversations with DuPont, they have formulated an equation that calculates the minimum pipe-wall thickness from summation of longitudinal stresses [30]. While in the 2-D vertical piping span calculations, detailed in Chapter 4, the minimum pipe-wall thickness is calculated using a root-solver in most cases because of the complexity of the equations used. From the ASME B31.3 standards and codes, the sum of the longitudinal stresses Equation (3.12) has been modified by DuPont [29, 30]. The modified equation, seen in the horizontal calculation block, includes the pipe-wall thickness needed for structural integrity, $t_{Str}$, for longitudinal stresses. Note that the nominal pipe wall thickness is used in the solution to facilitate a direct solution for the $t_{str}$. This, of course, is not strictly correct but the approach has been shown to be conservative.

$$S_h = \frac{PD_o}{4t} + \frac{Mc}{Z} \tag{3-16}$$

Upon comparison of the DuPont derived equation for longitudinal stresses to the 2-D vertical piping span longitudinal stresses using true pipe wall thicknesses instead of nominal thicknesses, a percent error between two pipe-wall thicknesses calculated was found. When compared, all the variables used in both equations were identical, and a resultant error of +8.4% in wall thickness was found. An increase in the minimum wall thickness is considered conservative. Thus, the DuPont derived equation was found to be more conservative than those used in the 2-D vertical piping span calculations that are passed through the root-solver.

The next value to be calculated is the sum of longitudinal stresses, $S_L$, as seen in the two bottom boxes in Figure 3-16. For the horizontal piping span, the stress-life equation is seen to include the original nominal pipe-wall thickness to obtain the nominal section modulus and the effective pipe-wall thickness. When calculated, the stress life is always less than or equal to the allowable static stress found from the material choice at operating temperature.

As seen in the next Figure 3-17, which is the continuation of Figure 3-16 shows in the top box that the sum of longitudinal stresses is used in the evaluation of the thermal expansion stress range $S_A$ [28]. The thermal expansion stress range uses the static strength, $S_C$, at a temperature of 70° F, and the static strength, $S_h$, at the operating temperature. Within this equation the values for the factor *f* is the ASME stress-range reduction factor, which are found in Table 302.3.5 in the ASME B31.3 codes and standards [28]. It is observed that this is not a true thermal stress calculation since it depends on a mechanically induced stress and a pair of static strengths. This raises concerns for its validity. This will be further addressed in the recommendation section of this thesis. Moreover, it is suspected that this relationship may have been developed for steel piping and may not be valid for ferrous metals.

Now that the thermal expansion stress range has been calculated, the next box in Figure 3-17 will be detailed. When a material chosen by the user does not contain a fatigue curve in the database, no additional calculations will occur. However, when the material chosen does have a fatigue curve in the database, the next calculations will occur. The default number of cycles in the *Tmin* program is 360 cycles and is compared to the number of cycles input by the user. If the number of cycles input by the user, $N_{User}$, is greater than 360, then the allowable stress, $S_{al}$, will be obtained from the fatigue curve in the database [2].

In the next box, the cyclic pipe-wall thickness, $t_{Cyc}$ will be calculated. Using the number of cycles, the allowable stress is obtained at that cycle number., and used in a conversion factor created by DuPont in their accession report [30]. Using the equation to calculate the thickness at the number of cycles, a code safety factor, $K_f$, created by DuPont, was input into the *Tmin* source code [2, 30]. While the other thickness is solved by finding the allowable stress at the default number of cycles, and then the conversion factor created by DuPont is again used to solve for the pipe-wall thickness due to fatigue, $t_{fat}$, is used to avoid fatigue failure [30]. Once all pipe-wall thickness calculations have been completed, the largest thickness value is found through all of the previous calculations and is seen in the logic box at the bottom of Figure 3-17. The largest pipe-wall thickness value is then given a safety factor by Dupont of $t_{Min}$ multiplied by 1.3 and is now called the screening pipe-wall thickness, $t_{Scr}$.

1

2

Calculate $S_A$ from ASME B31.3 1996a 302.3.5.d Equation 1a & 1b.

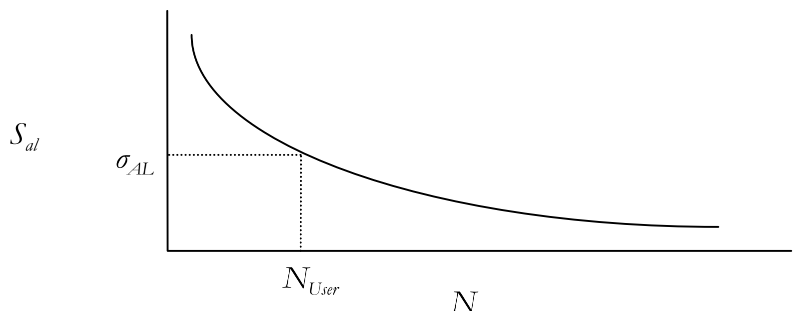
$$S_A = f\left(1.25\left(S_c + S_h\right) - S_L\right)$$

Where: $S_A$ = Thermal Expansion Stress Range

$S_c$ = Basic Allowable Stress at Minimum Metal Temperature. ASME B31.4.

$f$ = Stress-Range Reduction Factor. ASME B31.3 1996a 302.3.5.d Equation 1c.

Get the number of cycles, $N$, input by the user if different from default of 360 cycles. From the number of cycles, go to the fatigue curve at that cycle number, and return an allowable stress, $S_{al}$, value.




Calculate $t_{Cyc}$ from the number of cycles, $N_{User}$, input from the user. Use the $S_A$ from previous calculation in addition to the $S_{al}$ obtained from the fatigue calculations.

$$t_{Cyc} = K_f \frac{S_A}{S_{al}}\left(\frac{E_c}{E_H}\right)t_{Nom}$$

$K_f$ = Fatigue Strength Reduction Factor

$E_c$ = Elastic Modulus at 70º F

$E_H$ = Elastic Modulus at Temperature

Calculate $t_{fat}$ from the number of cycles, $N$, from default of 360 cycles. Use the $S_A$ from previous calculation in addition to the $S_{al}$ obtained from the fatigue calculations.

$$t_{Fat} = K_f \frac{S_A}{S_{al}}\left(\frac{E_c}{E_H}\right)t_{Nom}$$

$K_f$ = Fatigue Strength Reduction Factor

$E_c$ = Elastic Modulus at 70º F

$E_H$ = Elastic Modulus at Temperature

$t_{Cyc}$ is only used for final thickness comparison. $t_{Fatl}$ is seen again at the final output page of the Tmin program.

To find the largest *Tmin*, run the *Tmin* calcs through a bunch of IF-Then statements to find the largest TMin

$t_{min} = 1.3 * t_{str}$

If $t_{Hoop} > t_{min}$ *Then* $t_{min} = t_{Hoop}$

If $t_{mech} > t_{min}$ *Then* $t_{min} = t_{mech}$

If Fatigue Calculations are done and $t_{Cyc} > t_{Min}$ *Then* $t_{Min} = t_{Cyc}$

**Figure 3-17.** Continuation of Figure 3-16 of Order of Operations for Pipe-Wall Thickness within *Tmin*

In this chapter the methodologies of obtaining fatigue data and the conversion to ASME format have been presented, as well as the solution for determining the minimum pipe-wall thickness of a piping span in the presence of fatigue. All of the S-N curves converted to ASME format, as well as the data points, can be found in Appendix B. Chapter 4 details the analysis of a 2-D vertical piping span.