

We now consider the discrete ship dimension problem that arises from the fact that containers are built in discrete sizes. Hence the number of TEUs given by (3.3) and hence the objective function are discontinuous functions of the principal dimensions. We solve this problem using a linear response surface fit, using the method of least squares as explained in section 3.5, that makes the number of TEUs and hence the objective function continuous. This is shown in Figure 3.4.

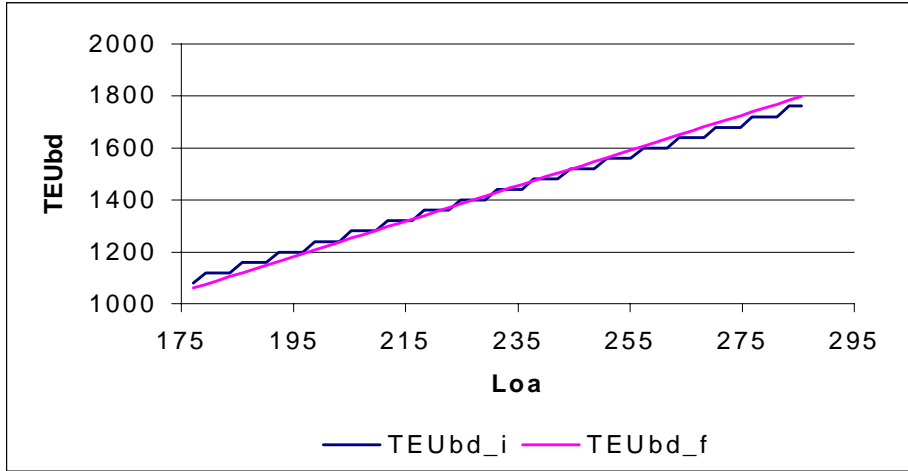
The resulting expression for the continuous number of TEUs is as follows.

$$\text{TEUbd}_f = ((0.0196 \times \text{Loa} \times \text{B} \times \text{D}) - 148.6129) \times (0.8479 \times \text{Cb} - 0.0918) \quad (3.4)$$

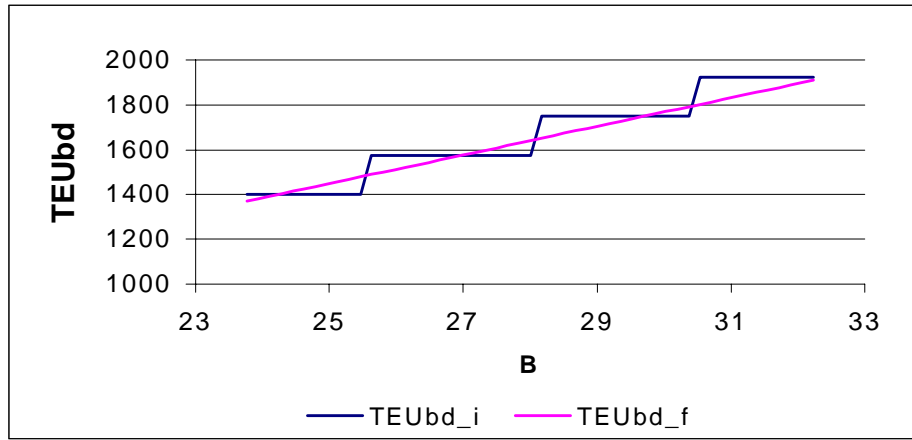
### 3.3 TEUs above deck

We discretize the space available for container stowage in the lengthwise and beamwise directions. The geometry of the surface area is accounted for by a “Stowage factor” for TEUs above deck. The product of the number of TEUs that can be stowed in these two directions, the number of tiers above deck and the stowage factor gives us the total number of TEUs that can be stowed above deck.

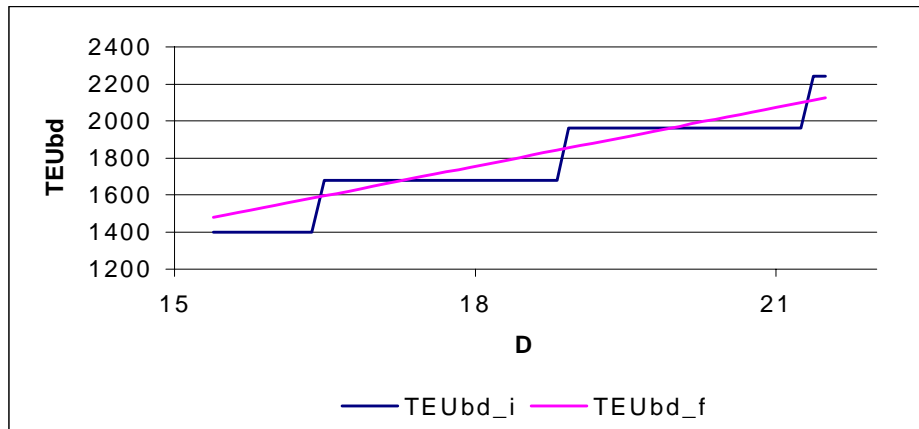
$$\text{TEUbd}_i = \left[ \frac{\text{Loa}}{1.05 \times 6.1} \right] \times \left[ \frac{\text{B}}{2.44} \right] \times \text{Ntd} \times \text{STWd} \quad (3.5)$$



**TEUs below deck as a function of Loa**



**TEUs below deck as a function of Beam**



**TEUs below deck as a function of Depth**

**Figure 3.4: The Fit for TEUs below deck checked at point:  
Loa = 224.84, B = 24, D=14.5**

where

- TEUd<sub>i</sub> = Number of TEUs above deck.
- Ntd = Number of tiers above deck
- STWd = Stowage factor for TEUs above deck obtained from a single ship as we have only one source of data for the number of tiers and the total number of TEUs above deck which are required to calculate STWd : 0.7534

We could allocate a fixed number for the number of tiers on deck. However, data from Panamax and post Panamax ships [12] indicate otherwise. A Panamax containership, measuring 292 meters overall by 32.2 meters wide, can have a stated capacity of 4000 TEU carrying 5 high on deck for the aft hatches. The forward hatches taper down to 4 high for visibility purposes. The post Panamax containerships, that are almost all that are built now, have capacities of 5200 TEU, carrying containers 5 high on deck and 10 deep below deck with an overall length of 292 meters and a width of 40 meters. Using the same length and 43 meters wide, 10 deep below deck and 5-6 high on deck, the rated capacity is 6600 TEU. Further, the fact that the containers need not be evenly distributed over the surface area on deck implies that the number of tiers on deck need not be an integer.

To account for these factors we choose to express the number of tiers as a function of the beam based on interpolation. The following expression has been formulated based on the above data.

**Table 3.2: Number of tiers on deck as a function of beam**

Beam	Ntd
$\leq 32.2$	4.0
$32.2 < B \leq 40.0$	$4.0 + \frac{(B - 32.2)}{(40.0 - 32.2)}$
$40.0 < B \leq 43.0$	$5.0 + \frac{(B - 40.0)}{(43.0 - 40.0)}$
$43.0 < B$	6.0

Again the number of TEUs as expressed in (3.5) is a discontinuous function of the Length and the Beam of the ship. The continuous expression obtained through a linear response surface fit is as follows:

$$TEUd_f = (Loa \times B \times 0.050117 \times Ntd) - 82.6702 \quad (3.6)$$

### 3.4 Center of Gravity of Containers

#### Containers below deck

To account for the geometry of the hull form, the moment of the containers below deck has to be expressed as a function of the block coefficient. Let us define a parameter, “MArmfactor\_TEUbd” as the number with which the height of the number of tiers below deck must be multiplied to get its center of gravity.

From this definition it follows that:

**Table 3.3: Moment arm factor for TEUs below deck as a function of Cb**

Cb	MArmfactor_TEUbd
1.0 ( Rectangular block )	0.5
0.5 ( Triangular block )	0.6667

Without loss of generality, it can be stated that:

$$\text{Marmfactor\_TEUbd} = 0.5 + \frac{(1 - Cb)}{3} \quad (3.7)$$

The following equations express the moment arm, weight and the moment of the TEUs below deck respectively.

$$\text{MArm\_TEUbd} = \text{DBH} + (\text{Ntb} \times 2.44 \times \text{MArmfactor\_TEUbd}) \quad (3.8)$$

where

Ntb = Number of tiers below deck

$$\text{Wt\_TEUbd} = \text{TEUbd} \times \text{Wpc} \quad (3.9)$$

where

Wpc = weight per TEU. This is defined by the user. In this work it is defined as 12 metric tons.

$$\text{Moment\_TEUbd} = \text{Marm\_TEUbd} \times \text{Wt\_TEUbd} \quad (3.10)$$

#### Containers above deck.

The following equations express the moment arm, weight and the moment of the TEUs above deck respectively.

$$\text{MArm\_TEUd} = D + \text{HCH} + \left( \frac{\text{Ntd} \times 2.44}{2} \right) \quad (3.11)$$

where

HCH = Hatch coaming height defined by the user. In this work, it is defined as 1.83 meters.

$$Wt\_TEUd = TEUd \times Wpc \quad (3.12)$$

$$Moment\_TEUd = MArm\_TEUd \times Wt\_TEUd \quad (3.13)$$

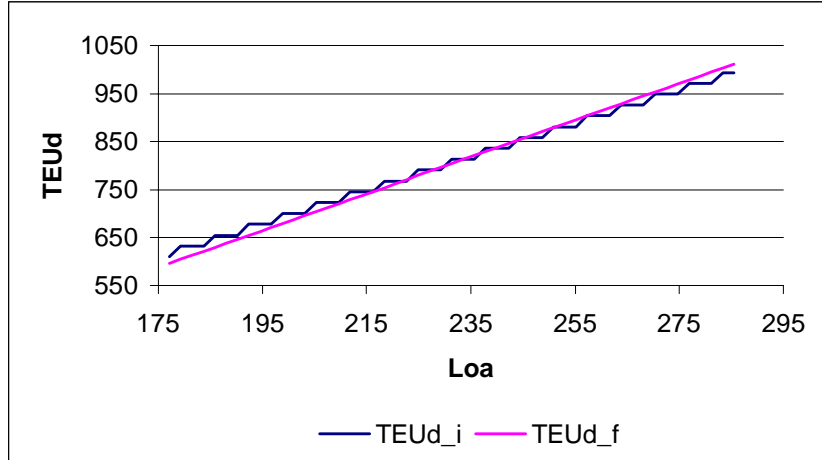


Figure 3.5.a: TEUs above deck as a function of Loa

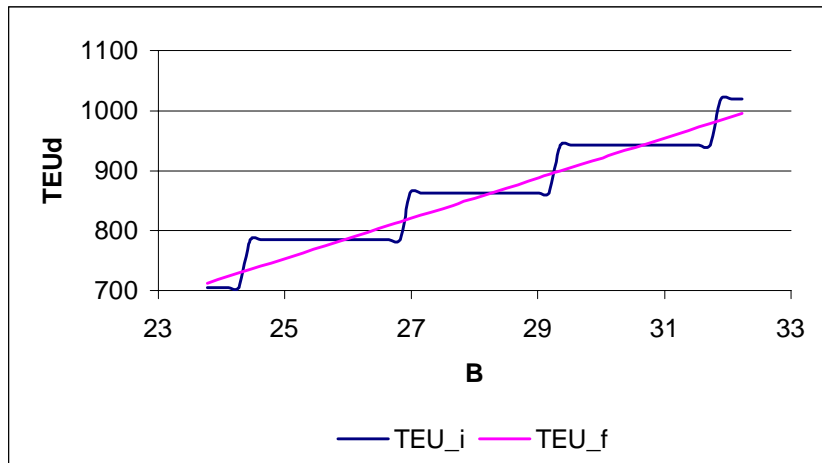


Figure 3.5.b: TEUs above deck as a function of Beam

Figure 3.5: The fit for TEUs on deck checked at point: Loa = 222.5, B = 25.5

### 3.5 Multiple Regression Analysis

In this section the general multiple regression model is presented [14]. In this explanation, the matrix notation is followed. Let us define the following matrix and vectors:

$$X = \begin{bmatrix} X_{01} & X_{11} & \dots & X_{p1} \\ X_{02} & X_{12} & \dots & X_{p2} \\ \dots & \dots & \dots & \dots \\ X_{0n} & X_{1n} & \dots & X_{pn} \end{bmatrix}, \quad Y = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{bmatrix}, \quad u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}$$

and

$$\beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}$$

The linear model that represents the data is

$$Y = X \beta + u$$

where

$$x_{0i} = 1 \text{ for all } i.$$

The assumptions made about  $u$  for least square estimation are

$$E(u) = 0, \text{ Var}(u) = E(uu') = \sigma^2 I_n;$$

that is, the  $u_i$ 's are independent, have zero mean and constant variance. This implies

$$E(y) = X\beta$$

The least square estimator  $b$  of  $\beta$  is obtained by minimizing the sum of squared deviations of the observations from their expected values. Hence, the least squares estimators are obtained by minimizing  $S(\beta)$ , where

$$S(\beta) = u'u = (Y - X\beta)'(Y - X\beta).$$

Minimization of  $S(\beta)$  leads to the system of equations

$$(X'X)b = X'Y.$$

This system of equations is called the normal equations. Assuming that  $(X'X)$  has an inverse,  $b$  can be written explicitly as

$$B = (X'X)^{-1} XY.$$

The vector of predicted values  $\hat{y}$  corresponding to the observed value  $y$  are  
 $\hat{y} = X b$ .

The vector of residuals  $e$  is given by  
 $e = y - \hat{y} = y - X b$ .

### 3.6 Evaluating the Goodness of a Fit

Once the response surface model has been determined, it is desirable to evaluate the goodness of fit of the model to the observed data. The indexes most commonly used for this purpose are:

- 1.) The sample correlation coefficient computed for the model and the observed data defined [14] as:

$$R = \frac{\sum (y_i - \bar{y})(\hat{y}_i - \bar{\hat{y}})}{\left[ \sum (y_i - \bar{y})^2 \sum (\hat{y}_i - \bar{\hat{y}})^2 \right]^{1/2}} \quad (3.14)$$

where

- $y_i$  = actual value of the response
- $\bar{y}$  = mean of  $y_i$
- $\hat{y}_i$  = predicted value of the response
- $\bar{\hat{y}}$  = mean of  $\hat{y}_i$

The numerical value of  $R$  lies between 1 and  $-1$ . The goodness of the fit may be viewed as a measure of the strength of the linear relationship between  $y$  (actual value of dependent variable) and  $x$  (independent variable).

- 2.) The root mean square error estimate is the square root of the sum of squares of the differences between the actual ( $y_i$ ) and the predicted ( $\hat{y}_i$ ) values of the response at the data points divided by the difference between total number of data points ( $N$ ) and the number of terms ( $n$ ) in the response surface model.

$$RMSE = \sqrt{\frac{\sum_{i=1}^N (y_i - \hat{y}_i)^2}{N - n}} \quad (3.15)$$

The expression above is valid when the points used in the equation are the same points used to fit the data. If otherwise ( $N-n$ ) is replaced by  $N$

In our case (3.14) and (3.15) give:

**Table 3.4: Sample correlation coefficient and RMSE for the fits**

	<b>R</b>	<b>RMSE</b>
<b>TEUbd</b>	0.9883	126.92
<b>TEUd</b>	0.9921	29.89

Notice that the sample correlation coefficient in both cases is close to unity indicating the strength of the linear relationship of the dependent variables on the independent variables.

Therefore the number of TEUs below deck and above deck are linear functions of the variables as indicated in equations (3.4) and (3.6) respectively.