A SIMULATION ANALYSIS OF BIVARIATE AVAILABILITY MODELS

Elise M. Caruso

Thesis submitted to the Faculty of Virginia Polytechnic Institute and State University in partial fulfillment of the requirements for the degree of

Master of Science
in
Industrial and Systems Engineering

Joel A. Nachlas, Chair
C. Richard Cassady
Bevlee A. Watford

July, 24, 2000
Blacksburg, Virginia

Keywords: Bivariate Failure Models, Availability, Simulation

A Simulation Analysis of Bivariate Failure Models

Elise M. Caruso

(ABSTRACT)

Equipment behavior is often discussed in terms of age and use. For example, an automobile is frequently referred to 3 years old with 30,000 miles. Bivariate failure modeling provides a framework for studying system behavior as a function of two variables. This is meaningful when studying the reliability/availability of systems and equipment.

This thesis extends work done in the area of bivariate failure modeling. Four bivariate failure models are selected for analysis. The study includes exploration of bivariate random number generation. The random data is utilized in estimating the bivariate renewal function and bivariate availability function. The two measures provide insight on system behavior characterized by multiple variables.

A method for generating bivariate failure and repair data is developed for each model. Of the four models, two represent correlated random variables; the other two, stochastic functionally dependent variables. Also, methods of estimating the bivariate renewals function and bivariate availability function are constructed. The bivariate failure and repair data from the four failure models is incorporated into the estimation processes to study various failure scenarios.

ACKNOWLEDGEMENTS

I would like to take this opportunity to thank the many people that contributed to the completion of thesis. In particular, I thank my advisor, Dr. Joel A. Nachlas, for his patience, understanding, support, and unique sense of humor. Without his encouragement in the classroom, I would not have spent the last year at Virginia Tech completing a Master's degree.

I would also like to thank my committee members, Dr. C. Richard Cassady and Dr. Bevlee A. Watford, for their comments, suggestions, and support. Dr. Cassady is owed a special thank you for his generosity and guidance during a very informative trip to Mississippi State University. Also, Dr. Watford deserves special acknowledgements for her help not only with this degree, but through my undergraduate years at Tech.

I thank the faculty and staff of the Industrial and Systems Engineering department for providing me the tools to succeed at Virginia Tech and in the future. This experience will be one to remember, not only for the trying times, but also for the good times. And the majority of those good times come from the close ISE family that grew over the last few years.

Most importantly, I would like to thank my parents, Mary and Enrico Caruso, for their support and encouragement, especially in the last few months. And I can't forget to mention the 5 people in my life, whose criticisms and comments are worth more than they will ever know, my brothers—Tony, Joe, Tom, Pete, and Rico. (And for you Rico, those names are in no particular order).

TABLE OF CONTENTS

CHAPTER 1 – INTRODUCTION 1

1.1 BACKGROUND 1
1.2 FAILURE MODELING 2
1.3 PROBLEM DESCRIPTION 4

CHAPTER 2 – LITERATURE REVIEW 6

2.1 BIVARIATE MODELING 6
2.1.1 Objectives 8
2.1.2 Definitions 9
2.2 BIVARIATE FAILURE MODEL CASES 12
2.2.1 Functionally Dependent Models 14
2.2.2 Correlated Models 14

CHAPTER 3 – PROBLEM STATEMENT 16

3.1 INTRODUCTION 16
3.2 GENERATING BIVARIATE FAILURE/REPAIR DATA 16
3.2.1 Bivariate Exponential Model 18
3.2.2 Bivariate Normal Model 19
3.2.3 Linear Stochastic Functional Relationship 20
3.2.4 Logistic Dependence Relationship 21
3.3 RENEWAL FUNCTION ESTIMATION 21
3.3.1 Bivariate Renewal Estimation 22
3.2.2 Univariate Projection 25
3.4 AVAILABILITY FUNCTION ESTIMATION 27
3.4.1 Bivariate Availability Estimation 27
3.5 EXPERIMENTAL METHODOLOGY 33

CHAPTER 4 – RESULTS AND CONCLUSIONS 35

4.1 BIVARIATE FAILURE DISTRIBUTIONS 35
4.2 RENEWAL ESTIMATION 39
4.2.1 Bivariate Exponential Model 40
4.2.2 Bivariate Normal Model 44
4.2.3 Linear Stochastic Functional Relationship 48
4.2.4 Logistic Stochastic Functional Relationship 53
4.2.5 Model Comparison 54
4.3 BIVARIATE AVAILABILITY ESTIMATION 55
4.3.1 Bivariate Exponential Model 55
4.3.2 Bivariate Normal Model 64
4.3.3 Linear Stochastic Functional Relationship 72
4.3.4 Logistic Stochastic Functional Relationship 79
4.3.5 Model Comparison 81
4.4 SUMMARY 82

CHAPTER 5 – FUTURE DIRECTIONS **83**

REFERENCES **84**

APPENDIX A – SIMULATION CODE **86**

A.1 LINEAR DEPENDENCE DATA GENERATION PROCEDURE 86
A.2 LOGISTIC DEPENDENCE DATA GENERATION PROCEDURE 87
A.3 BIVARIATE EXPONENTIAL DATA GENERATION PROCEDURE 89
A.4 BIVARIATE NORMAL DATA GENERATION PROCEDURE 90
A.5 BIVARIATE RENEWAL ESTIMATION 92
A.6 UNIVARIATE PROJECTION METHOD 94
A.7 BIVARIATE AVAILABILITY ESTIMATION (METHOD 1) 97
A.8 BIVARIATE AVAILABILITY ESTIMATION (METHOD 2) 101

APPENDIX B – ADDITIONAL CHARTS **104**

B.1 RENEWAL FUNCTION 104
B.1.2 Bivariate Exponential Model *105*
B.1.2 Bivariate Normal Model *110*
B.1.3 Linear Stochastic Functional Relationship *115*
B.2 AVAILABILITY FUNCTION 120
B.2.1 Bivariate Exponential Model *120*
B.2.2 Bivariate Normal Model *126*
B.2.3 Linear Stochastic Function *132*

LIST OF FIGURES

FIGURE 3-1 FLOWCHART FOR BIVARIATE RENEWAL ESTIMATION 24
FIGURE 3-2 FLOWCHART FOR UNIVARIATE PROJECTION METHOD 26
FIGURE 3-3 FLOWCHART FOR BIVARIATE AVAILABILITY ESTIMATION (METHOD 1) 28
FIGURE 3-4 FLOWCHART FOR BIVARIATE AVAILABILITY ESTIMATION (METHOD 2) 32
FIGURE 4-1 BIVARIATE EXPONENTIAL PROBABILITY DENSITY 35
FIGURE 4-2 BIVARIATE NORMAL PROBABILITY DENSITY FUNCTION (ρ = 0 .2) 36
FIGURE 4-3 BIVARIATE NORMAL PROBABILITY DENSITY FUNCTION (ρ = 0 .5) 37
FIGURE 4-4 BIVARIATE NORMAL PROBABILITY DENSITY FUNCTION (ρ = 0 .8) 37
FIGURE 4-5 LINEAR STOCHASTIC FUNCTION PROBABILITY DENSITY 38
FIGURE 4-6 LOGISTIC STOCHASTIC FUNCTION PROBABILITY DENSITY 39
FIGURE 4-7 RENEWAL FUNCTION (CASE 1, BVE) 40
FIGURE 4-8 UNIVARIATE PROJECTION (CASE1, BVE) 41
FIGURE 4-9 RENEWAL FUNCTION (CASE 2, BVE) 42
FIGURE 4-10 RENEWAL FUNCTION (CASE 3, BVE) 42
FIGURE 4-11 RENEWAL FUNCTION (CASE 4, BVE) 43
FIGURE 4-12 RENEWAL FUNCTION (CASE 5, BVE) 43
FIGURE 4-13 RENEWAL FUNCTION (CASE 6, BVE) 44
FIGURE 4-14 RENEWAL FUNCTION (CASE 1, BVN) 44
FIGURE 4-15 UNIVARIATE PROJECTION (CASE 1, BVN) 45
FIGURE 4-16 RENWAL FUNCTION (CASE 2, BVN) 46
FIGURE 4-17 RENEWAL FUNCTION (CASE 3, BVN) 46
FIGURE 4-18 RENEWAL FUNCTION (CASE 4, BVN) 47
FIGURE 4-19 RENEWAL FUNCTION (CASE 5, BVN) 47
FIGURE 4-20 RENEWAL FUNCTION (CASE 6, BVN) 48
FIGURE 4-21 RENEWAL FUNCTION (CASE 1, LISF) 49
FIGURE 4-22 UNIVARIATE PROJECTION (CASE 1, LISF) 50
FIGURE 4-23 RENEWAL FUNCTION (CASE 2, LISF) 51
FIGURE 4-24 RENEWAL FUNCTION (CASE 3, LISF) 51
FIGURE 4-25 RENEWAL FUNCTION (CASE 4, LISF) 52
FIGURE 4-26 RENEWAL FUNCTION (CASE 5, LISF) 52
FIGURE 4-27 RENEWAL FUNCTION (CASE 6, LISF) 53
FIGURE 4-28 RENEWAL FUNCTION (CASE 1, LOSF) 53
FIGURE 4-29 RENEWAL FUNCTION (CASE 2, LOSF) 54
FIGURE 4-30 AVAILABILITY FUNCTION (METHOD 1, CASE 1, BVE) 56
FIGURE 4-31 AVAILABILITY FUNCTION (METHOD 2, CASE 1, BVE) 56
FIGURE 4-32 AVAILABILITY FUNCTION (METHOD 1, CASE 2, BVE) 57
FIGURE 4-33 AVAILABILITY FUNCTION (METHOD 2, CASE 2, BVE) 58

FIGURE 4-34 AVAILABILITY FUNCTION (METHOD 1, CASE 2, BVE) 58
FIGURE 4-35 AVAILABILITY FUNCTION (METHOD 2, CASE 2, BVE) 59
FIGURE 4-36 AVAILABILITY FUNCTION (METHOD 1, CASE 4, BVE) 59
FIGURE 4-37 AVAILABILITY FUNCTION (METHOD 2, CASE 4, BVE) 60
FIGURE 4-38 AVAILABILITY FUNCTION (METHOD 1, CASE 5, BVE) 60
FIGURE 4-39 AVAILABILITY FUNCTION (METHOD 2, CASE 5, BVE) 61
FIGURE 4-40 AVAILABILITY FUNCTION (METHOD 1, CASE 6, BVE) 61
FIGURE 4-41 AVAILABILITY FUNCTION (METHOD 2, CASE 6, BVE) 62
FIGURE 4-42 AVAILABILITY FUNCTION (N = 500,000, CASE 2, BVE) 63
FIGURE 4-43 AVAILABILITY FUNCTION (DECREASED GRID, CASE 2, BVE) 63
FIGURE 4-44 AVAILABILITY FUNCTION (METHOD 1, CASE 1, BVN) 64
FIGURE 4-45 AVAILABILITY FUNCTION (METHOD 2, CASE 1, BVN) 65
FIGURE 4-46 AVAILABILITY FUNCTION (METHOD 1, CASE 2, BVN) 66
FIGURE 4-47 AVAILABILITY FUNCTION (METHOD 2, CASE 2, BVN) 66
FIGURE 4-48 AVAILABILITY FUNCTION (METHOD 1, CASE 3, BVN) 67
FIGURE 4-49 AVAILABILITY FUNCTION (METHOD 2, CASE 3, BVN) 67
FIGURE 4-50 AVAILABILITY FUNCTION (METHOD 1, CASE 4, BVN) 68
FIGURE 4-51 AVAILABILITY FUNCTION (METHOD 2, CASE 4, BVN) 68
FIGURE 4-52 AVAILABILITY FUNCTION (METHOD 1, CASE 5, BVN) 69
FIGURE 4-53 AVAILABILITY FUNCTION (METHOD 2, CASE 5, BVN) 69
FIGURE 4-54 AVAILABILITY FUNCTION (METHOD 1, CASE 6, BVN) 70
FIGURE 4-55 AVAILABILITY FUNCTION (METHOD 2, CASE 6, BVN) 70
FIGURE 4-56 AVAILABILITY FUNCTION (N = 500,000, CASE 2, BVN) 71
FIGURE 4-57 AVAILABILITY FUNCTION (DECREASED GRID, CASE 2, BVN) 71
FIGURE 4-58 AVAILABILITY FUNCTION (METHOD 1, CASE 1, LISF) 72
FIGURE 4-59 AVAILABILITY FUNCTION (METHOD 2, CASE 1, LISF) 73
FIGURE 4-60 AVAILABILITY FUNCTION (METHOD 1, CASE 2, LISF) 73
FIGURE 4-61 AVAILABILITY FUNCTION (METHOD 2, CASE 2, LISF) 74
FIGURE 4-62 AVAILABILITY FUNCTION (METHOD 1, CASE 3, LISF) 74
FIGURE 4-63 AVAILABILITY FUNCTION (METHOD 2, CASE 3, LISF) 75
FIGURE 4-64 AVAILABILITY FUNCTION (METHOD 1, CASE 5, LISF) 76
FIGURE 4-65 AVAILABILITY FUNCTION (METHOD 2, CASE 4, LISF) 76
FIGURE 4-66 AVAILABILITY FUNCTION (METHOD 1, CASE 5, LISF) 77
FIGURE 4-67 AVAILABILITY FUNCTION (METHOD 2, CASE 5, LISF) 77
FIGURE 4-68 AVAILABILITY FUNCTION (METHOD 1, CASE 6, LISF) 78
FIGURE 4-69 AVAILABILITY FUNCTION (METHOD 2, CASE 6, LISF) 78
FIGURE 4-70 AVAILABILITY FUNCTION (N = 500,000 CASE 2, LISF) 79
FIGURE 4-71 AVAILABILITY FUNCTION (DECREASED GRID, CASE 2, LISF) 79
FIGURE 4-72 AVAILABILITY FUNCTION (METHOD 1, CASE 1, LOSF) 80
FIGURE 4-73 AVAILABILITY FUNCTION (METHOD 2, CASE 1, LOSF) 80
FIGURE 4-74 AVAILABILITY FUNCTION (METHOD 1, CASE 2, LOSF) 81
FIGURE 4-75 AVAILABILITY FUNCTION (METHOD 2, CASE 2, LOSF) 81
FIGURE B-1 UNIVARIATE PROJECTION (CASES 1, 2, 3, BVE) 105
FIGURE B-2 RENEWAL FUNCTION (CASE 4A, BVE) 105
FIGURE B-3 RENEWAL FUNCTION (CASE 4B, BVE) 106
FIGURE B-4 UNIVARIATE PROJECTION (CASES 4, 4A, 4B, BVE) 106

FIGURE B-5 RENEWAL FUNCTION (CASE 5A, BVE) 107
FIGURE B-6 RENEWAL FUNCTION (CASE 5B, BVE) 107
FIGURE B-7 UNIVARIATE PROJECTION (CASES 5, 5A, 5B, BVE) 108
FIGURE B-8 RENEWAL FUNCTION (CASE 6A, BVE) 108
FIGURE B-9 RENEWAL FUNCTION (CASE 6B, BVE) 109
FIGURE B-10 UNIVARIATE PROJECTION (CASES 6, 6A, 6B, BVE) 109
FIGURE B-11 UNIVARIATE PROJECTION (CASES 1, 2, 3, BVN) 110
FIGURE B-12 RENEWAL FUNCTION (CASE 4A, BVN) 110
FIGURE B-13 RENEWAL FUNCTION (CASE 4B, BVN) 111
FIGURE B-14 UNIVARIATE PROJECTION (CASES 4, 4A, 4B, BVN) 111
FIGURE B-15 RENEWAL FUNCTION (CASE 5A, BVN) 112
FIGURE B-16 RENEWAL FUNCTION (CASE 5B, BVN) 112
FIGURE B-17 UNIVARIATE PROJECTION (CASES 5, 5A, 5B, BVN) 113
FIGURE B-18 RENEWAL FUNCTION (CASE 6A, BVN) 113
FIGURE B-19 RENEWAL FUNCTION (CASE 6B, BVN) 114
FIGURE B-20 UNIVARIATE PROJECTION (CASES 6, 6A, 6B, BVN) 114
FIGURE B-21 UNIVARIATE PROJECTION (CASES 1, 2, 3, LISF) 115
FIGURE B-22 RENEWAL FUNCTION (CASE 4A, LISF) 115
FIGURE B-23 RENEWAL FUNCTION (CASE 4B, LISF) 116
FIGURE B-24 UNIVARIATE PROJECTION (CASES 4, 4A, 4B, LISF) 116
FIGURE B-25 RENEWAL FUNCTION (CASE 5A, LISF) 117
FIGURE B-26 RENEWAL FUNCTION (CASE 5B, LISF) 117
FIGURE B-27 UNIVARIATE PROJECTION (CASES 5, 5A, 5B, LISF) 118
FIGURE B-28 RENEWAL FUNCTION (CASE 6A, LISF) 118
FIGURE B-29 RENEWAL FUNCTION (CASE 6B, LISF) 119
FIGURE B-30 UNIVARIATE PROJECTION (CASES 6, 6A, 6B, LISF) 119
FIGURE B-31 AVAILABILITY FUNCTION (METHOD 1, CASE 4A, BVE) 120
FIGURE B-32 AVAILABILITY FUNCTION (METHOD 2, CASE 4A, BVE) 120
FIGURE B-33 AVAILABILITY FUNCTION (METHOD 1, CASE 4B, BVE) 121
FIGURE B-34 AVAILABILITY FUNCTION (METHOD 2, CASE 4B, BVE) 121
FIGURE B-35 AVAILABILITY FUNCTION (METHOD 1, CASE 5A, BVE) 122
FIGURE B-36 AVAILABILITY FUNCTION (METHOD 2, CASE 5A, BVE) 122
FIGURE B-37 AVAILABILITY FUNCTION (METHOD 1, CASE 5B, BVE) 123
FIGURE B-38 AVAILABILITY FUNCTION (METHOD 2, CASE 5B, BVE) 123
FIGURE B-39 AVAILABILITY FUNCTION (METHOD 1, CASE 6A, BVE) 124
FIGURE B-40 AVAILABILITY FUNCTION (METHOD 2, CASE 6A, BVE) 124
FIGURE B-41 AVAILABILITY FUNCTION (METHOD 1, CASE 6B, BVE) 125
FIGURE B-42 AVAILABILITY FUNCTION (METHOD 2, CASE 6B, BVE) 125
FIGURE B-43 AVAILABILITY FUNCTION (METHOD 1, CASE 4A, BVN) 126
FIGURE B-44 AVAILABILITY FUNCTION (METHOD 2, CASE 4A, BVN) 126
FIGURE B-45 AVAILABILITY FUNCTION (METHOD 1, CASE 4B, BVN) 127
FIGURE B-46 AVAILABILITY FUNCTION (METHOD 2, CASE 4B, BVN) 127
FIGURE B-47 AVAILABILITY FUNCTION (METHOD 1, CASE 5A, BVN) 128
FIGURE B-48 AVAILABILITY FUNCTION (METHOD 2, CASE 5A, BVN) 128
FIGURE B-49 AVAILABILITY FUNCTION (METHOD 1, CASE 5B, BVN) 129
FIGURE B-50 AVAILABILITY FUNCTION (METHOD 1, CASE 6A, BVN) 130

FIGURE B-51 AVAILABILITY FUNCTION (METHOD 2, CASE 6A, BVN)..............130
FIGURE B-52 AVAILABILITY FUNCTION (METHOD 1, CASE 6B, BVN)..............131
FIGURE B-53 AVAILABILITY FUNCTION (METHOD 2, CASE 6B, BVN)..............131
FIGURE B-54 AVAILABILITY FUNCTION (METHOD 1, CASE 4A, LISF)..............132
FIGURE B-55 AVAILABILITY FUNCTION (METHOD 2, CASE 4A, LISF)..............132
FIGURE B-56 AVAILABILITY FUNCTION (METHOD 1, CASE 4B, LISF)..............133
FIGURE B-57 AVAILABILITY FUNCTION (METHOD 2, CASE 4B, LISF)..............133
FIGURE B-58 AVAILABILITY FUNCTION (METHOD 1, CASE 5A, LISF)..............134
FIGURE B-59 AVAILABILITY FUNCTION (METHOD 2, CASE 5A, LISF)..............134
FIGURE B-60 AVAILABILITY FUNCTION (METHOD 1, CASE 5B, LISF)..............135
FIGURE B-61 AVAILABILITY FUNCTION (METHOD 2, CASE 5B, LISF)..............135
FIGURE B-62 AVAILABILITY FUNCTION (METHOD 1, CASE 6A, LISF)..............136
FIGURE B-63 AVAILABILITY FUNCTION (METHOD 2, CASE 6A, LISF)..............136
FIGURE B-64 AVAILABILITY FUNCTION (METHOD 1, CASE 6B, LISF)..............137
FIGURE B-65 AVAILABILITY FUNCTION (METHOD 2, CASE 6B, LISF)..............137

LIST OF TABLES

TABLE 3-1 RENEWAL MATRIX..23
TABLE 3-2 AVAILABILITY MATRIX (METHOD 1)..29
TABLE 3-3 AVAILABILITY MATRIX (METHOD 2)..30
TABLE 3-4 RELEVANT MATRIX (METHOD 2) 31
TABLE 3-5 MODEL PARAMETERS 34
TABLE 4-1 REPRESENTATIVE PARAMETER SETS..39
TABLE 4-2 DATA FOR UNIVARIATE PROJECTION VS. BIVARIATE ESTIMATE .41
TABLE 4-3 DATA FOR UNIVARIATE PROJECTION VS. BIVARIATE ESTIMATE .45
TABLE 4-4 DATA FOR UNIVARIATE PROJECTION VS. BIVARIATE ESTIMATE .50
TABLE 4-5 REPRESENTATIVE PARAMETER SETS..55
TABLE B-1 PARAMETER SETS..104

Chapter 1 – Introduction

1.1 Background

The study of reliability problems began as early as the late 1930s, largely in response to technological advances and problems encountered in some of the complex military systems used in World War II. Reliability theory has maintained a strong presence in the field of applied mathematics, each year increasing in popularity. An overall increase in system complexity has contributed to the growing interest in the field of reliability. Reliability, as defined by Nachlas [1998], is the probability that a device properly performs its intended function over time when operated within the environment for which it is designed. In the event that a system experiences no failures, we can say that system is absolutely reliable; however, that situation is unlikely to occur.

Each system experiences failures unique to that particular system. For example, the solenoid in an oven fails to function properly, thus causing the oven to heat improperly or not at all, or the oil pump in an automobile malfunctions causing the engine to halt. Failure distributions are used as a method of representing the life length of a system. "A failure distribution represents an attempt to describe mathematically the length of life of a material, structure, or a device (Barlow and Proschan [1965])." In reliability studies, various distributions, such as the exponential, the gamma, the normal, and the Weibull distributions are commonly used to represent the failure behavior of a system (component). The system behavior dictates the choice of the family of the probability distribution; the selection of an adequate failure distribution can sometimes be difficult. The goal is to select an appropriate distribution in an attempt to accurately represent the system behavior in order to determine various reliability measures for that system.

In a reliability context, failure models are used to characterize the lifetime behavior of a system. Over the last several years, much work has been done in the area of failure modeling; much of that work has focused on problems of a univariate nature. A univariate failure model focuses on failure as a function of a single index or variable, such as time to failure, cycles to failure, or usage to failure. It is apparent that univariate measures are adequate in many situations; however, situations exist where multiple measures seem to be more appropriate. Multivariate failure models combine two or more variables in order to describe the behavior of the system.

In both the univariate and multivariate cases, it may be possible to use failure behavior information to aid in maintenance planning. For example, in order to avoid unplanned work stoppages (due to failure), it may be beneficial to perform planned maintenance operations on a system with an increasing failure rate. Often failures are repairable through corrective maintenance operations; however, corrective maintenance actions typically demand more time and resources than a simple preventive maintenance activity. All types of maintenance actions contribute to the availability of a system; availability can be described as the probability a system will be operating at any given point in time. It is plausible that higher availability will result in increased production and/or system utilization—both desirable goals. Therefore, it is important to understand all aspects of systems behavior (i.e. failure processes, corrective maintenance, and preventive maintenance).

1.2 Failure Modeling

Aging is inherent in almost any operating system (component). Aging can be described as a gradual deterioration of the performance characteristics and/or gradual increase of the possibility of component failure (Gertsbakh [1989]). In terms of the system reliability, it is necessary to develop a mathematical description of the aging process. Reliability theory suggests several formal descriptions of aging, for example, increasing failure rate (IFR), increasing failure rate average (IFRA), and new better than used in expectation (NBUE). These classifications along with the lifetime distribution of the system allow for the determination of useful reliability measures.

The most commonly encountered failure models make use of univariate distributions, indexed by a single scale, to describe the failure of an item. Extensive work has been done in this area leading to many well-known concepts. All univariate distributions have a well-established relationship between the distribution and the aging process. For example, the exponential distribution displays a constant failure rate; the gamma and normal distributions, an increasing failure rate; the Weibull distribution, an increasing failure rate, a decreasing failure rate, and a special case of the exponential. The above-mentioned associations are commonly known in the univariate case; however, these concepts have not been extended to the multivariate case.

There are some situations where a univariate model does not adequately represent the system. A particularly useful example is the determination of failure points of an automobile. It is important to not only know that the car is expected to survive for 7 years, but it is also important to know that it will last for 75,000 miles. The significance lies in the fact that different users will reach usage values at varying points along the life of the unit. Clearly, multiple indices are useful; however, a single index model is typically used because it is mathematically more tractable. Some reliability models do account for the need to include multiple scales, but most are designed such that they may be reduced to a single scale.

A logical progression from the univariate case to the multivariate case is to consider reliability in terms of two variables or the bivariate case. Several researchers, including Mercer [1961], Birnbaum and Saunders [1969], Barlow and Proschan [1975], Lemoine and Wenocur [1985], and Yang [1999], have suggested that bivariate failure models appear to accurately represent system (equipment) behavior. In particular, Yang [1999], identifies five classes of bivariate reliability models. These models target situations that include maintenance operations and those that do not include maintenance. It is important to recall, maintenance activities can significantly impact the behvaior of a system. The models are developed sequentially and with an increasing level of detail. Equipment behavior is portrayed in terms of bivariate failure modeling, bivariate renewal modeling, bivariate corrective maintenance modeling, bivariate preventive maintenance modeling, and bivariate availability modeling. These objectives are not universal, but serve as the basis for the research presented here. Considerable advances were made in each of these areas. A general structure of bivariate probability models of system failure, which allows for numerical analysis of system behavior, was determined. Numerical integration techniques were used for the analysis. Additional analysis is necessary to more fully understand system behavior.

1.3 Problem Description

The purpose of the research presented here is to gain a better understanding of bivariate failure processes. Various bivariate failure models have been developed; however, the analysis of these models has been rather difficult. In particular, Yang [1999] was able to develop useful bivariate models, but was not able to obtain the corresponding performance measures—reliability and/or availability. The complexity of the Laplace transforms prohibited the determination of these measures. In order to provide further insight into the failure behavior of systems characterized by bivariate failure, four of Yang's models are examined.

Much of the work in the area of failure modeling has focused on univariate failure processes. For many of the univariate models a relatively good understanding of the relationship between the aging process and the lifetime distribution exist (e.g., the exponential distribution). It is not clear that similar relationships exist in the bivariate case and if the relationships exist the nature of each is unknown. Therefore, in order to provide useful applications of bivariate models, further study in this area is necessary.

Discrete-Event simulation is used to model system behavior. The first objective is to develop a method for computing bivariate random failure and repair data. Using the failure/repair data, simulation models are built to estimate the bivariate renewal function, when repair is instantaneous and the bivariate availability function, when repair is not instantaneous. For instantaneous repair, a sequence of failure events are generated and examined to find the expected number of renewals at a certain point in the time-use space. The failure data is used to gain a better understanding of the bivariate renewal process. For cases of non-instantaneous repair, a similar approach is used, but in addition to the generation of failure data, repair times are generated. The combination of failures and repairs allows for the estimation of system availability. The chief objective is to measure and analyze system availability. Also, the simulation results will be employed in an effort to strengthen and confirm analytic solutions.

Chapter 2 – Literature Review

2.1 Bivariate Modeling

Throughout the last several decades various researchers have addressed the idea of bivariate failure modeling and among these works many interpretations of bivariate failure modeling exist. Much of the early work in this area does not consider bivariate failure models in the sense studied here. The research presented in this thesis is based on the work of Yang [1999] and considers a classification scheme defined by the relationship between the two variables contributing to failure. In this case the two variables associated with the failure of a system are time and use. In one case the variables are treated as correlated random variables; the other, functionally dependent.

Researchers focusing on variables that are functionally dependent include Mercer [1961], Birnbaum and Saunders [1969], Barlow and Proschan [1975], and Lemoine and Wenocur [1985]. Within this body of work, the efforts of Mercer are the most closely related to the aims of the research here. Mercer studies failure as a function of both time and wear. He contends that a failure rate classified as a function of time alone ignores the underlying processes that can contribute to the failure of an item (in this study, the process of wear was considered). The resulting model includes a component related to the wear process in order to more accurately model failure. Mercer investigates alternative replacement strategies and provides insight on optimal replacement intervals.

There are several authors (Gumbel [1960], Marshall and Olkin [1967 a, b], Downton [1970], Baggs and Nagaraja [1996], and Signpurwalla and Wilson [1993]) who focus attention to the study of multivariate probability distributions in a reliability context. Marshall and Olkin concentrate on the discussion of life-length using a bivariate exponential (BVE) distribution to represent two components versus two distinct failure characteristics, such as time and usage. Other researchers (Block and Basu [1974] and Baggs and Nagaraja [1996]) extend the work of Marshall and Olkin to further investigate the intricacies of the BVE. In particular, Baggs and Nagaraja model a two-component system with dependent components whose lifetimes are characterized by the BVE. They consider dependent lifetimes in which failure is based on a single index—time. The work that is most closely related to that presented here was introduced by Singpurwalla and Wilson [1993].

Singpurwalla and Wilson discuss bivariate models in the context of warranty applications. They determine that the application of univariate failure models is sufficient in dealing with univariate warranties, but not appropriate for warranties such as automobile warranties that consider multiple failure criteria. In the case of an automobile, the warranty considerations may be based on model year and mileage. Singpurwalla and Wilson construct a generic bivariate model to reflect failure based on two scales. However, difficulty obtaining useful results led to a need for simulation studies. In later work [1998], simulation methods are used to determine optimal warranty periods based on the newly developed bivariate models.

Similar to the efforts of Singpurwalla and Wilson [1993], Yang [1999] develops generic bivariate failure models that focus on failure as a function of two variables. Yang proposes the use of bivariate models to characterize the behavior of equipment in the area of reliability. There is a progression from the generic failure model to models that include renewal and maintenance activities. As mentioned earlier, Yang identified five key efforts for the application of bivariate probability distributions to reliability/availability. Those efforts include:

i) Bivariate Failure Modeling
ii) Bivariate Renewal Modeling
iii) Bivariate Corrective Maintenance Modeling
iv) Bivariate Preventive Maintenance Modeling
v) Bivariate Availability Modeling

The next section includes a brief summary of Yang's modeling efforts, including useful definitions of each objective. The two subsequent sections provide explanations of the concepts used by Yang to apply bivariate probability distributions to reliability/availability.

2.1.1 Objectives

Yang [1999] identifies five objectives to progress through the construction of bivariate failure models to the development of models that allow for the measurement of system behavior. The work presented here is based on these efforts; it is important that the objectives are well understood.

i) Bivariate Failure Modeling

This step focuses on the construction of bivariate failure models, which consider a single-unit system with bivariate longevity. The goal of this step is to construct and evaluate bivariate failure models in an attempt to improve upon existing univariate failure models. The two model classes are defined and examples are included. Given the two model classes, the corresponding bivariate failure models are developed.

ii) Bivariate Renewal Modeling

Bivariate renewal models consider systems with independent and identically distributed (i.i.d) lifetimes, which are instantaneously repaired/replaced upon failure. It is assumed once a system is repaired/replaced the new system behaves identically to the repaired/replaced system. Under this assumption a bivariate renewal process may be used to describe system behavior. A bivariate renewal theory and a quasi-renewal theory are proposed and results are included.

iii) Bivariate Corrective Maintenance Modeling

Bivariate corrective maintenance models are an extension of the bivariate renewal models in that cases with non-instantaneous repair are considered. The lifetime distributions and the repair time distributions are i.i.d. Following a maintenance action, it is assumed the system is "*as good as new*." In these models, preventive maintenance time is not included, only corrective maintenance. Of the two model classes – functional relationships and correlated relationships – only the correlated models are examined. Yang is able to obtain the Laplace transform of the renewal function.

iv) Bivariate Preventive Maintenance Modeling

Bivariate preventive maintenance models build upon the results of the bivariate renewal models and the bivariate corrective maintenance models, by considering the effects of preventive maintenance. These models are similar to the corrective maintenance models, but include distinct preventive maintenance times. The purpose of the models is to capture the effects of preventive maintenance on a system. The models are investigated under an age-replacement policy and it is noted that other preventive maintenance policies may be considered.

v) Bivariate Availability Modeling

In this stage, bivariate availability models are provided; the models are derived from the developed bivariate corrective maintenance and preventive maintenance models. The corrective maintenance and preventive maintenance cases are considered separately; however, the availability measure for the preventive maintenance models is based on the results from the corrective maintenance models. The Laplace transforms for the bivariate availability models are presented, along with general results.

2.1.2 Definitions

The application of bivariate probability distributions to reliability is not a trivial task. The first step is to carefully define and interpret the bivariate probabilities. The following concepts are necessary to fully describe the bivariate failure models identified by Yang [1999]. Each model is defined by two variables—time to failure, T, and usage to failure, U. Time and usage are generic terms used to represent a variety of characteristics that contribute to system failure. System lifetimes are defined by the cumulative failure probability, $F_{T,U}(t,u)$, which is the probability failure occurs by time t and usage u, more formally:

$$F_{T,U}(t,u) = \Pr[T \leq t, U \leq u].$$

This probability represents the proportion of the population that have longevity vector values that do not surpass (t,u) in either vector component (Yang [1999]).

Each cumulative failure probability distribution has a corresponding reliability function, $\overline{F}_{T,U}(t,u)$, that represents the portion of the population whose failure age exceeds t, and failure usage exceeds u. The resulting reliability function is:

$$\overline{F}_{T,U}(t,u) = \Pr[T \geq t, U \geq u] = \int_t^\infty \int_u^\infty f_{T,U}(t,u)dudt,$$

where $f_{T,U}(t,u)$ is the joint probability density function of T and U. In addition to the joint probability density function, it is useful to determine the marginal distributions on t and u, $f_T(t)$ and $f_U(u)$, as well as the conditional distributions, $f_{T|U}(t \mid u)$ and $f_{U|T}(u \mid t)$.

2.1.3 Performance Measures

The effectiveness of a system can be measured in several ways. In this research, the two measures are the number of failures experienced by a certain point (or in a particular interval) and the availability of a system. Availability is the probability a system is in a functioning state at a particular point. It is important to note, there is an abundance of literature concerning each of these ideas in the univariate sense; however, there are no efforts to extend the results. Yang [1999] describes an approach for determining each of these measures for the bivariate case – the bivariate renewal models and the bivariate availability models, respectively. Yang provides several important terms to describe both of these processes.

The bivariate renewal function is an extension of the univariate renewal function. In either case, the basis is a counting process for which interarrival times are independent and identically distributed with a common distribution; this type of counting process is referred to as a *renewal process*. To define the renewal process in the univariate sense, let $\{X_n, n = 1,2,...\}$ be a sequence of nonnegative independent random variables with a common distribution F, where X_n is the time between the (n-1)st and the nth event. Also, let

$$S_0 = 0, \qquad S_n = \sum_{i=1}^{n} X_i, \quad n \geq 1,$$

where S_n is the time of the nth event. It is well known that

$$N(t) = \sup\{n : S_n \leq t\}$$

and the counting process $\{N(t), t \geq 0\}$ is termed a *renewal process*. The renewal function may is defined as the expected number of renewals by time t, or more formally:

$$M(t) = E[N(t)].$$

The bivariate renewal function is based on the same theory given in the univariate renewal function, only all definitions are extended to two include two variables. The following definitions are used to construct the bivariate renewal function. First, let $X_n = \{(T_n, U_n)\}, n = 1, 2...,$ be a sequence of independent and identically distributed non-negative bivariate random vectors, with common joint distribution function-- $F_{T,U} = \Pr\{T_n \leq t, U_n \leq u\}$ – where T_n and U_n represent the interarrival time and usage between the (n-1)st and nth, $n \geq 1$. The stochastic process $\{N_{T,U}(t,u), t > 0, u > 0\}$ is defined as a *counting process* that represents the total number of renewals by time t and usage u, or the *bivariate renewal counting process*. It follows that the *bivariate renewal function* may be stated as:

$$M_F(t,u) = E\left[N_{T,U}(t,u)\right].$$

For the bivariate availability measures, Yang [1999] defines four types of availability—bivariate point availability, bivariate limiting availability, bivariate average availability, and bivariate limiting average availability. This research will focus on *bivariate point availability*, $A(t,u)$, and is defined by:

$$A(t,u) = \Pr\{I(t,u) = 1\} = E[I(t,u)],$$

where $I(t,u)$ is the system status and is defined as:

$$I(t,u) = \begin{cases} 1, & \text{if the device is operating at time t and usage u} \\ 0, & \text{otherwise} \end{cases}.$$

In other words, bivariate point availability can be described as the probability that a system is functioning at any point – time *t* and usage *u*.

2.2 Bivariate Failure Model Cases

Yang [1999] addresses the problem of constructing bivariate failure, repair, and preventive maintenance models. He identifies two potentially important model classes and the difference between the model classes lies in the definition of the relationship between the two variables. One category is defined by variables that have a stochastic functional relationship. The other category serves to represent variables that are correlated versus stochastically dependent. The development of these types of models forms the cornerstone of this research.

There are many possible approaches to the development of bivariate failure models. In the case of those models with functionally dependent variables, the initial step in defining the model is to determine the representation of the stochastic element of the life variable. The function $U = g(t)$ represents the relation between the time and usage to failure, T and U respectively. It is assumed that the relationship between T and U can be determined by treating one or more of the parameters of $g(t)$ as random variables. Once the random feature is determined a transformation of variables is performed in order to obtain the marginal distribution on usage, which is then used to find the joint failure density (Yang [2000]). The following examples were suggested as possible functional relationships:

i) $g(t) = \alpha t + \beta$ (2.1)

ii) $g(t) = \alpha t^2 + \beta t + \gamma$ (2.2)

iii) $g(t) = \alpha t^n$ (2.3)

iv) $g(t) = \left(e^{\alpha t} - 1\right) / \left(e^{\alpha t} + \beta\right)$. (2.4)

The relationships shown in equations 2.1 and 2.4 will be explored here. For each of these models the parameter α is treated as a random variable, with probability distribution $\pi_\alpha(\cdot)$.

In addition to situations that warrant a functionally dependent set of variables, there are many scenarios where the two variables appear to be correlated. When the variables are correlated as opposed to functionally dependent it appears that the construction of bivariate models is less intensive; a bivariate distribution is selected and manipulated. It is important to note that the determination of the bivariate distribution must be done very carefully to ensure that it appropriately describes equipment behavior. Three candidate distributions that seem to accurately represent bivariate failure processes with correlated random variables are proposed. The distributions include a generalization of the bivariate exponential distribution, the bivariate normal distribution, and a model presented by Hunter [1974]. The density functions are as follows:

i) $f_{T,U}(t,u) = \lambda \eta e^{(\lambda t + \eta u)}\left(1 + \rho\left(1 - 2e^{-\lambda t} - 2e^{-\eta u} + 4e^{-(\lambda t + \eta u)}\right)\right)$ (2.5)

ii) $f_{T,U}(t,u) = \dfrac{1}{2\pi\sigma_t\sigma_u\sqrt{1-\rho^2}} \exp\left\{-\dfrac{1}{2(1-\rho^2)}\left[\dfrac{(t-\mu_t)^2}{\sigma_t^2} - 2\rho\dfrac{(t-\mu_t)(u-\mu_u)}{\sigma_t\sigma_u} + \dfrac{(u-\mu_u)^2}{\sigma_u^2}\right]\right\}$ (2.6)

iii) $f_{T,U}(t,u) = \dfrac{\lambda\eta}{1-\rho} I_0\left(\dfrac{2\sqrt{\rho}}{1-\rho}\sqrt{\lambda \eta t u}\right) \exp\left\{-\dfrac{\lambda t + \eta u}{1-\rho}\right\}$, (2.7)

where $I_n(\cdot)$ is the modified Bessel function of the first kind, of order *n*.

Yang [1999] explores four of the models (2 functionally dependent, 2 correlated) in great detail and uses those models as examples. For the example models, definitions are given and some basic results are provided. The focus here will be on the example models of Yang.

2.2.1 Functionally Dependent Models

The two functionally dependent models are a simple linear relationship,

$$g(t) = \alpha t + \beta,$$

and, the logistic model:

$$g(t) = \left(e^{\alpha t} - 1\right) / \left(e^{\alpha t} + \beta\right).$$

Eliashberg, Singpurwalla, and Wilson [1997] examined the logistic model in a multiple-index warranty application. Each of these models appears to be realistic option to represent equipment behavior in regard to multiple failure characteristics. Yang [1999] was able to compute probability values, reliability measures, and hazard function values. It is important to note there is no closed form expression of the cumulative distribution function (CDF) for either of these models.

2.2.2 Correlated Models

For the models representing correlated random variables, Yang [1999] chose the bivariate exponential distribution,

$$f_{T,U}(t,u) = \lambda\eta e^{(\lambda t + \eta u)}\left(1 + \rho\left(1 - 2e^{-\lambda t} - 2e^{-\eta u} + 4e^{-(\lambda t + \eta u)}\right)\right),$$

and the bivariate normal distribution,

$$f_{T,U}(t,u) = \frac{1}{2\pi\sigma_t\sigma_u\sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2\left(1-\rho^2\right)}\left[\frac{(t-\mu_t)^2}{\sigma_t^2} - 2\rho\frac{(t-\mu_t)(u-\mu_u)}{\sigma_t\sigma_u} + \frac{(u-\mu_u)^2}{\sigma_u^2}\right]\right\}.$$

The bivariate exponential (BVE) distribution is a logical choice since it has been studied in related applications (Marshall and Olkin [1967 a,b], Block and Basu [1974], and Baggs and Nagaraja [1996]). Also, the BVE exhibits the attractive features of having closed form expression for the CDF and having exponential marginal densities. These characteristics make the analysis more tractable. The second choice, the bivariate normal distribution, is relatively straightforward from an analytical standpoint. In addition, the marginal densities are normal, again making analysis less complicated Yang [1999] provides computational results for the probability values, reliability measures, and hazard function values.

Chapter 3 – Problem Statement

3.1 Introduction

Chapter 2 introduced the application of bivariate distributions to model failure with respect to two variables, time and usage. In this chapter, procedures are developed to generate random failure and repair data corresponding to the bivariate distributions. The processes are constructed for two classes of bivariate distributions – stochastic functional relationships and correlated random variables. The failure and repair data characterizes the behavior associated with each distribution and is then used to develop performance measure estimation methods. Procedures are created to estimate the bivariate renewal function and the bivariate availability function. The results from these tests are used to compare and contrast the 4 bivariate models.

3.2 Generating Bivariate Failure/Repair Data

The first step in developing the simulation models is the determination of a generation procedure for failure and repair data for each of the bivariate failure models selected. The failure/repair data is used to estimate two measures of system performance – reliability and availability. Each failure vector has two components (T, U) representing the time to failure and the use to failure, respectively. The same is true for the repair vectors; however, the components (R_t,R_u) represent the amount of time (or use) to complete a repair. Because the two components have dependence relationships (functional or correlated) it is necessary to employ a multivariate random number generation scheme.

Several algorithms that are candidates for the construction of multivariate random vectors exist. The most commonly used methods for multivariate random vector generation include the conditional method, transformation methods, and the acceptance-rejection method. Algorithms for each of these methods exist in the literature (Devroye [1986], Johnson [1987], and Law and Kelton [1991]) and are only valid when used in conjunction with a "good" uniform random number generator. Each has specific characteristics that warrant its use in various situations.

The conditional distribution method is a universal tool that can be used in many situations. It provides the ability to reduce the multivariate generation problem into a series of univariate generation problems (Devroye [1986]). This simplification allows for the use of numerous generation techniques that have been developed for the univariate case. In order to use this method, the distribution must be invertible and a series of conditional distributions must be obtainable. This is equivalent to knowing each marginal distribution since:

$$f_i(x_i \mid x_1,\ldots.x_{i-1}) = \frac{f_i^{\,*}(x_1,\ldots.x_i)}{f_{i-1}^{\,*}(x_1,\ldots.x_{i-1})},$$

where f^*_i is the marginal density of the first i components (i.e. the density of $(x_1,\ldots.x_i)$). The critical step in the conditional distribution method is the determination of the marginal distribution of the first vector element and subsequent conditional distributions for the remaining vector elements. For the cases analyzed here, the distributions $f_T(t)$ or $f_U(u)$ and $f_{t|u}(t \mid u)$ or $f_{u|t}(u \mid t)$ are obtainable when necessary. After the marginal distributions and corresponding conditional densities are found, the univariate generation technique, inversion, is implemented to generate T and U.

For the generation of R_{ti} and R_{ui}, it is important to note, the procedures are identical to those for the generation of T and U. The difference appears in the parameter selection. The parameters for the repair distribution are selected such that the repair rate is 10 percent of the failure distribution. It is important to note, the values generated from the above procedures are individual failure (repair) vectors. A series of 10 failure (repair) vectors is generated for each machine and used to determine the lifetime of the machine. Each failure vector represents the amount of time and usage that has passed since the previous failure. So, failure i is determined by adding (T_i, U_i) to the longevity vector for failure i-l. For example, given the following failure vectors:

$(T_1, U_1) = (1750, 2345)$

$(T_2, U_2) = (3420, 2190)$,

the first failure would occur at (1750, 2345) and the second failure would occur at (5170, 4535). The same method is used to generate machine lifetimes when repair is included. However, to find the i^{th} failure point, the values of the i-1st failure and repair vectors are added to the i^{th} failure vector. For instance, using the same failure vectors mentioned above and including the following repair vector (R_{t1}, R_{u1}) = (355, 195), the second failure would occur at (5525, 4730). It is important to note that in all model instances, the repair distribution is taken from the same family as the failure distribution. The code used to generate failure/repair data can be found in Appendix A. The subsequent sections describe the various generation techniques in greater detail.

3.2.1 Bivariate Exponential Model

To address the issue of generating failure/repair behavior for the bivariate exponential model,

$$f_{T,U}(t,u) = \lambda\eta e^{(\lambda t+\eta u)}\left(1+\rho\left(1-2e^{-\lambda t}-2e^{-\eta u}+4e^{-(\lambda t+\eta u)}\right)\right) \qquad (3.1)$$

the conditional distribution method is utilized. It is well known the marginal distributions of the bivariate exponential distribution are exponential. Using this result, the conditional distribution on either variable is found using the definition of conditional density function, formally stated as:

$$f_{Y|X}(y \mid x) = \frac{f_{X,Y}(x,y)}{f_X(x)}. \qquad (3.2)$$

For the purposes of this study, time was considered as the first vector component. Therefore, following the conditional distribution procedure, T is generated from the marginal distribution, $f_T(t) = \lambda e^{-\lambda t}$, using the inversion method as follows:

$$T = -\frac{1}{\lambda}\ln(1-Z_1), \qquad (3.3)$$

where Z_1 is a uniform random number on the interval (0,1). The next step is to construct the conditional distribution, $f_{u|t}(u \mid t)$. The conditional distribution is found to be:

$$
\begin{aligned}
f_{U|T}(u\mid t) &= \frac{f_{U,T}(u,t)}{f_T(t)} \\
&= \frac{\lambda\eta e^{-(\lambda t+\eta u)}\left(1+\rho\left(1-2e^{-\lambda t}-2e^{-\eta u}+4e^{-(\lambda t+\eta u)}\right)\right)}{\lambda e^{-\lambda t}} \qquad (3.4)\\
&= \eta e^{-\eta u}\left(1+\rho\left(1-2e^{-\lambda t}-2e^{-\eta u}+4e^{-(\lambda t+\eta u)}\right)\right)
\end{aligned}
$$

Using the conditional distribution, $f_{u|t}(u\mid t)$, and the value of *T*, *U* may be generated using the inverse transformation technique as follows:

$$
U = -\frac{1}{\eta}\ln\left[\frac{.5}{c_2}\left(-c_1-\sqrt{c_1^2+4c_2 y-4c_2}\right)\right], \qquad (3.5)
$$

where

$$
c_1 = -1-\rho+2\rho e^{-\lambda t} \qquad (3.6)
$$

$$
c_2 = \rho-2\rho e^{-\lambda t} \qquad (3.7)
$$

3.2.2 Bivariate Normal Model

For the bivariate normal distribution,

$$
f_{T,U}(t,u) = \frac{1}{2\pi\sigma_t\sigma_u\sqrt{1-\rho^2}}\exp\left\{-\frac{1}{2(1-\rho^2)}\left[\frac{(t-\mu_t)^2}{\sigma_t^2}-2\rho\frac{(t-\mu_t)(u-\mu_u)}{\sigma_t\sigma_u}+\frac{(u-\mu_u)^2}{\sigma_u^2}\right]\right\}, \qquad (3.8)
$$

the conditional distribution method is a valid generation technique. However, Scheuer and Stoller [1962] develop a simpler method. The method is applicable when generating a n-dimensional multivariate normal distribution with mean vector $\mu = (\mu_1, \mu_2, \ldots, \mu_n)^T$ and covariance matrix Σ, where the $(i,j)^{th}$ entry is σ_{ij}. Also, Σ must be symmetric and positive definite; this gives us the ability to factor it uniquely as $\Sigma = CC^T$, where C is a lower triangular *n* x *n* matrix (Law and Kelton [1991]). The resulting algorithm is formally stated as:

1. Generate $Z_1, Z_2, \ldots, Z_n$ as IID N(0,1) random variates
2. For $i = 1,2,\ldots n$, let $X_i = \mu_i + \sum_{j=1}^{i} c_{ij} Z_j$ and return $X = (X_1, X_2, \ldots X_N)^T$.

Barr and Slezak [1972] assert that this method is the most appropriate of a number of methods of generating the multivariate normal. Based on this method, Banks, Carson, and Nelson [1996] provide a specific procedure for the bivariate case. The algorithm is as follows:

1. Generate Z_1 and Z_2 as IID N(0,1) random variates
2. Set $X_1 = \mu_1 + \sigma_1 Z_1$
3. Set $X_2 = \mu_2 + \sigma_2 \left(\rho Z_1 + \sqrt{1-\rho^2} Z_2\right)$

For the case presented here, the following substitutions are made: $X_1 = T$, $X_2 = U$, $\mu_1 = \mu_t$, $\mu_2 = \mu_u$, $\sigma_1 = \sigma_t$, and $\sigma_2 = \sigma_u$.

3.2.3 Linear Stochastic Functional Relationship

The first of the functionally dependent relationships studied is a linear relationship between *T* and *U*, denoted by:

$$g(t) = \alpha t + \beta, \tag{3.9}$$

where $g(t) = u$, β is a constant, and α is a random variable with distribution $\pi_\alpha(\cdot)$. In all cases considered α is assumed to have the exponential distribution of the form:

$$\pi_\alpha(\cdot) = ce^{-c\alpha}. \tag{3.10}$$

It is important to recognize the difficulty in generating the linear stochastic functional relationship. An appropriate method using the joint distribution function was not obtained. Therefore, an initial attempt of generation was made by assuming a distribution for *T* and using the linear function to generate *U*. The vector component, *t*, is assumed to behave according the exponential distribution, with parameter λ. Using these two assumptions the following procedure is implemented to generate the failure vector components (*T*,*U*):

1. Set β = constant
2. Generate Z_1 and Z_2 as IID U(0,1) random variates.
3. Set $T = -\frac{1}{\lambda}\ln(1-Z_1)$
4. Set $\alpha = -\frac{1}{c}\ln(1-Z_2)$

5. Set $U = \alpha t + \beta$.

3.2.4 Logistic Dependence Relationship

The last model under consideration is the logistic model studied by Eliashberg, Singpurwalla, and Wilson [1997]. Recall, the model is of the form:

$$\mathrm{g(t)} = \left(\mathrm{e}^{\alpha t} - 1\right) / \left(\mathrm{e}^{\alpha t} + \beta\right), \tag{3.11}$$

where *g(t) = u,* β is a constant, and α is a random variable with distribution $\pi_\alpha(\cdot)$. Again, a useful generation technique using the joint distribution function for this relationship was not used. So, the two assumptions from the previous section (α and t are exponential random variables) are made in this case as well. The resulting random vector generation procedure is:

1. Set β = constant
2. Generate Z_1 and Z_2 as IID U(0,1) random variates.
3. Set $T = -\frac{1}{\lambda}\ln(1 - Z_1)$
4. Set $\alpha = -\frac{1}{c}\ln(1 - Z_2)$
5. Set $U = \left(\mathrm{e}^{\alpha t} - 1\right) / \left(\mathrm{e}^{\alpha t} + \beta\right)$.

3.3 Renewal Function Estimation

A bivariate renewal model is provided by Yang [1999]; however, direct analysis of the model is rather difficult due to the complexity of the Laplace transform. The goal here is to develop a simulation procedure to portray the renewal behavior of a system that undergoes instantaneous repair/replacement. A bivariate renewal estimation procedure is developed to gather insight on the behavior the four failure models. The characteristics are then used to compare the behavior across the model types.

The estimation procedure considers the failure behavior of a fixed number of machines for a fixed number of failures, over incremental observations in the time-usage plane. The sample population consists of 1,000 machines with 10 failures per machine. The first step in the estimation procedure is to determine an appropriate grid resolution (for the observation points). In order to capture the cumulative behavior, the grid size is selected based on the failure rate for either time or usage. Three observation grids are constructed corresponding to the three failure rate levels (low, medium, and high); each grid consists of two hundred observation points. To illustrate the process, consider a case where the mean time to failure and usage are 3,000 and 18,500, respectively. Using these values, statistics are collected every 2000 time (usage) units until 200 observations are complete (i.e. lattice point (400000, 400000)). The observation intervals for the two alternative grids are 300 and 800 units. The remainder of the renewal estimation procedure is based on the observation grid.

A second method is developed to estimate the projection of the bivariate renewal process on either the time or usage axis. The same random data generation procedure is used; however, comparisons are made according to only one variable. The observation interval corresponds to the interval used in the bivariate renewal estimation procedure. This method is provides insight on the renewal behavior when considering only one of the two variables.

3.3.1 Bivariate Renewal Estimation

The renewal estimation process begins by generating failure data for the sample population. Then each of the 10 failures is compared, individually, against all observation points, $O(j,k)$, for all $j = 1, 2, \ldots, 200$ and $k = 1, 2, \ldots, 200$, where j represents observation times, and k represents observation usages. The comparison statistics are used to generate a matrix of cumulative renewals across all machines, or the renewal matrix. The comparison of failure point to observation point determines the location(s) in the matrix that are updated to reflect the failure. An update to the matrix is performed only if the following condition is true: the vector components of failure i are less than or equal to $O(j,k)$. In other words, the failure time component is less than or equal to $O(j)$

and the failure usage component is less than or equal to *O(k)*. The updated entries correspond to the total number of renewals experienced until that point.

The following example illustrates two iterations of the bivariate renewal estimation procedure. Given the failure points:

i	Failure (i)	Cumulative Renewals
1	(3282, 2736)	1
2	(6303, 5591)	2

the resulting renewal matrix is as follows:

Time Usage	300	600	….	3000	3300	3600	….	6300	6600	6900
300										
600										
. .	. .	. .		. .						
2700										
3000					1	1	….	1	1	1
3300					1	1	….	1	1	1
. .					. .	. .	…. ….	. .	. .	. .
5400					1	1	1	1	1	1
5700									2	2
6000									2	2

Table 3-1 RENEWAL MATRIX

The above example represents the matrix updates of 1 machine for 2 failures. The procedure is repeated for the remaining failure vectors (i = 3, 4, …, 10) and for the remaining machines (N = 10,000). The renewal updates from all N machines are accumulated in one renewal matrix. Then an average over N machines is taken to find the average cumulative number of renewals for a given parameter set. Figure 3-1 provides a graphical representation of the method described here and the Matlab code used for this estimation procedure can be found in Appendix A.5.

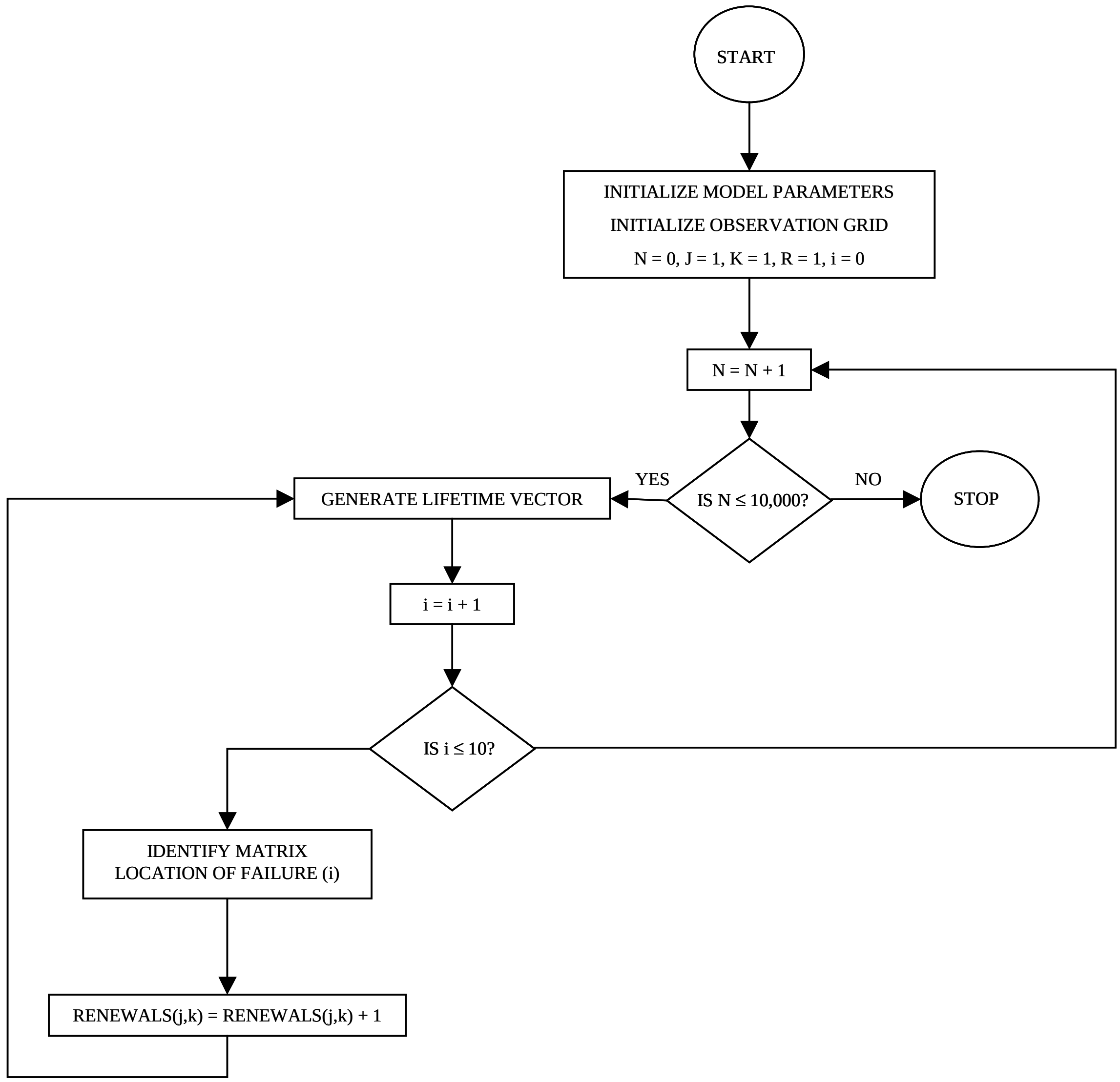

Figure 3-1 FLOWCHART FOR BIVARIATE RENEWAL ESTIMATION

3.2.2 Univariate Projection

The final renewal estimation procedure estimates the projection of the bivariate renewal function, $M_F(t,u)$, onto one axis (time or use). Hunter [1974] addresses renewal theory in two dimensions and finds a simplified method of addressing the bivariate renewal process. First, he defines $N_x^{(1)}$ and $N_y^{(2)}$ as the univariate renewal counting process for the X-renewals and the Y-renewals; $N_{x,y}$ is defined as the two-dimensional renewal counting process. In the cases presented here, X and Y are equivalent to T and U, respectively. Hunter determines a relationship between the univariate counting process and the bivariate counting process: $N_{x,y} = \min\{N_x^{(1)}, N_y^{(2)}\}$. In this research a procedure is developed to analyze bivariate failure data in one dimension and the results are compared to those of Hunter. The projection of the bivariate process onto one axis (time or use) is informative for counting the number of renewals in one dimension.

In this method, the same basic failure generation scheme is employed. To maintain consistency between the two methods, the process is repeated for N = 1,000 machines. The renewal matrix is reduced to a renewal vector with the same observation coverage as the bivariate method. The observation increment is chosen according to the mean value for the given bivariate model; the larger of the two mean values is used in order to capture the full extent of the behavior. Figure 3-2 describes the general outline of the procedure and the simulation code used to generate the univariate renewal data can be found in Appendix A.6

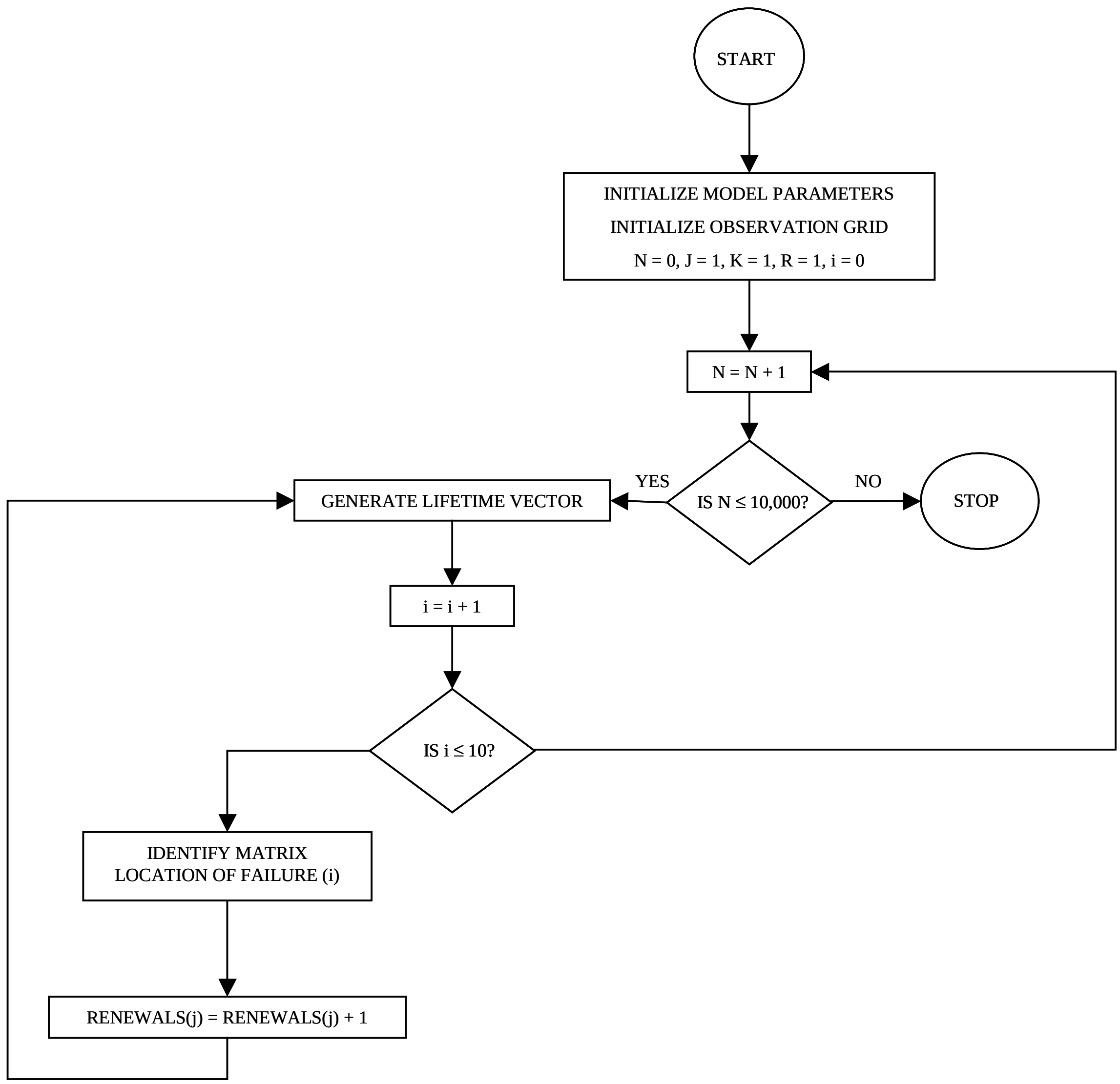

Figure 3-2 FLOWCHART FOR UNIVARIATE PROJECTION METHOD

3.4 Availability Function Estimation

As is the case with the bivariate renewal model provided by Yang [1999], direct analysis of the bivariate availability model is challenging. Estimation of the availability function follows the same approach used in the estimation of the renewal function. Simulation models are constructed to capture the lifetime behavior of a system that undergoes corrective maintenance. Two methods of bivariate availability estimation are developed. The basic procedures used in the availability estimation models are the same as those used in the renewal estimation models, except a larger number of machines (N = 100,000) are considered. The two bivariate models consider a fixed number of machines, each with a predetermined number of failures.

3.4.1 Bivariate Availability Estimation

The two estimation procedures begin by selecting a distribution and the corresponding parameters. Given this information, an observation grid is initialized from the mean of the distribution on time or usage. The observation grid is a matrix representing various lattice points in the time-usage plane where the system behavior is monitored. After the parameters and the grid resolution are fixed, 10 failure and repair vectors are generated for each machine (N = 100,000). The failure vector (T_i, U_i) represents the time and usage failure i occurs; the repair vector (R_{ti}, R_{ui}), the time and usage repair i is completed. Each component (time, usage) of the failure and repair vectors is compared to O(j) or O(k) ($O(j)$ refers to time, $O(k)$ refers to usage). The information gained here is used to determine the availability matrix, which is similar to the renewal matrix. The availability matrix entries represent the number of machines functioning at a particular observation point.

The comparison process identifies the matrix location corresponding to a failure or repair completion. In method 1, all entries of the availability matrix are initialized to N, meaning all machines are functioning. Given the matrix location of any failure and repair sequence, all matrix location between the two points are decremented by 1 to reflect the non-functioning machine. An overview of the process is shown in Figure 3-3 and the Simulation code appears in Appendix A.6.

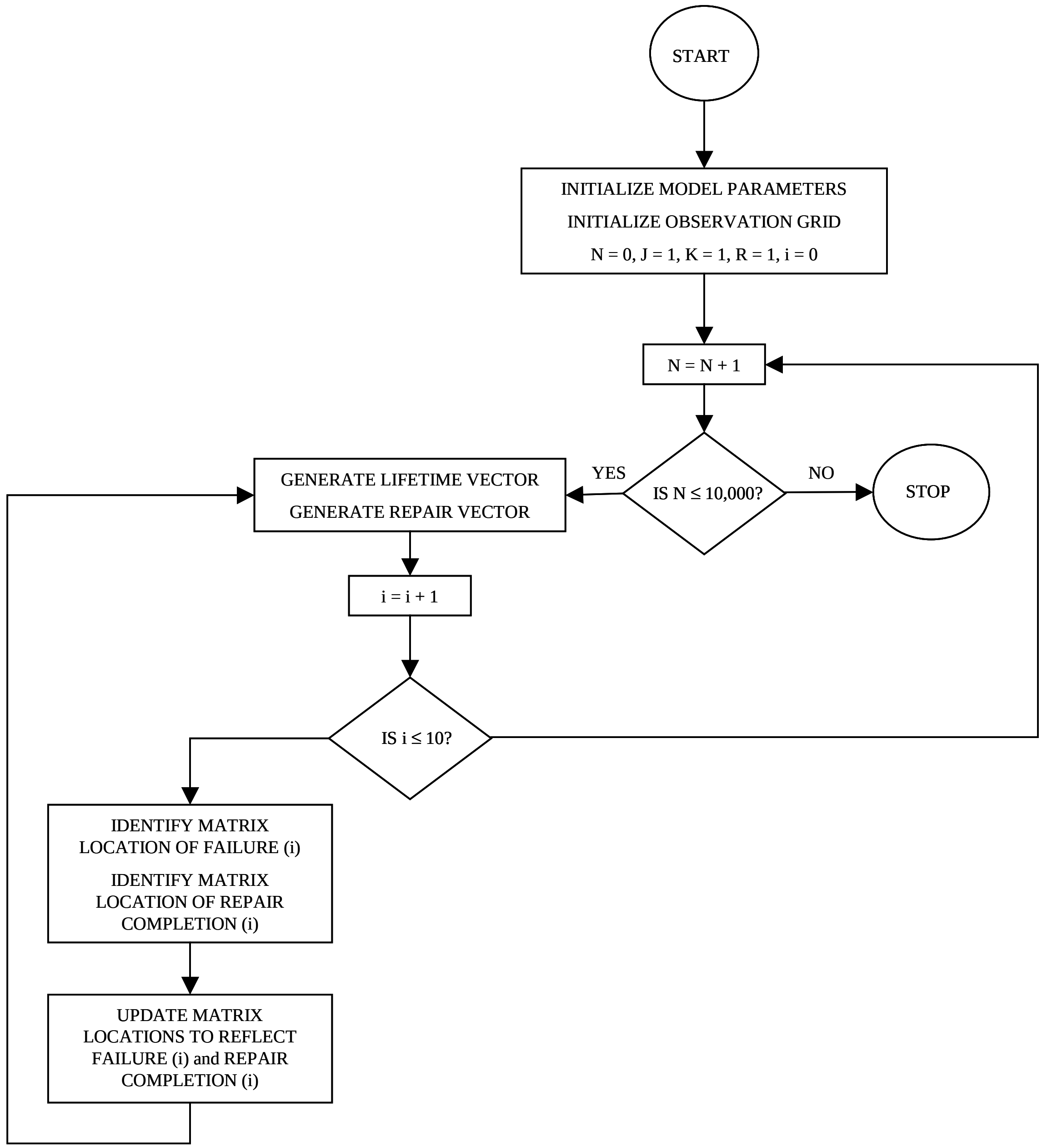

Figure 3-3 FLOWCHART FOR BIVARIATE AVAILABILITY ESTIMATION (METHOD 1)

The following example illustrates the process for 1 machine and the first two failure/repair sequences. Suppose the following two sequences occur:

i	Failure (i)	Repair Completion
1	(2038, 3205)	(2274, 3606)
2	(4791, 7088)	(5104, 7358)

Using method 1, the resulting availability matrix is:

Time / Usage	300	600	….	1800	2100	2400	….	4500	4800	5100	5400
300	100000	100000	….	1000000	100000	100000	….	100000	100000	100000	100000
600	100000	100000	….	100000	100000	100000	….	100000	100000	100000	100000
.	.	.	.	.	.	.	….	.	.	.	.
.	.	.	.	.	.	.	….	.	.	.	.
3000	100000	10000	….	100000	100000	100000	….	100000	100000	100000	100000
3300	100000	10000	….	100000	99999	99999	….	100000	100000	100000	100000
3600	100000	10000	….	100000	99999	100000	….	100000	100000	100000	100000
.	.	.	….	.	.	.	….	.	.	.	.
.	.	.	….	.	.	.	….	.	.	.	.
6900	100000	100000	….	100000	100000	100000	….	100000	100000	100000	100000
7200	100000	100000	….	100000	100000	100000	….	100000	99999	99999	99999
7500	100000	100000	….	100000	100000	100000	….	100000	99999	99999	100000
7800	100000	100000	….	100000	100000	100000	….	100000	100000	100000	100000

Table 3-2 AVAILABILITY MATRIX (METHOD 1)

For some matrix locations, the availability may be overestimated due to the assumption all machines are functioning unless a failure occurs; this occurs because all machines do not necessarily pass through all observation points. For example, O(7,10) – (2400, 3000) reflects an availability of 1, dividing the number of functioning machines by the total number of machines (N = 100,000). However, failure 1 occurs at (2038, 3205) so the point (2400, 3000) is not accessible by this particular machine and at this point 10,000 machines are not functioning. A second method is developed to improve the estimation procedure. The second method follows the same initial procedure, but takes a different approach to obtaining the availability statistics.

For method 2, two matrices are maintained – the availability matrix and the relevant matrix. Similar to the second renewal estimation method, the relevant matrix tracks the locations that are accessible by each machine. The availability matrix is similar to that of availability estimation method 1; however, the assumption that all machines are functioning is relaxed. In this case, the availability matrix accumulates the total number of working machines that access a particular observation location. The two matrices are then used to estimate the bivariate availability function by averaging over the total number of machines that have visited each observation point. The following example, using the same failure and repair information from above:

i	Failure (i)	Repair Completion
1	(2038, 3205)	(2274, 3606)
2	(4791, 7088)	(5104, 7358)

shows the difference in the availability matrix (Table 3-3) from method 1 and method 2. Also, the relevant matrix is presented in Table 3-4.

Time \ Usage	300	600	….	1800	2100	2400	2700	….	4500	4800	5100	5400
300	1		….	1				….				
600	1		….	1				….				
. .	. .	. .	. .	. .	. .	. .		…. ….	. .	. .	. .	. .
3000	1	1	….	1				….				
3300			….					….				
3600			….					….				
3900						1	1	….	1			
. .	. .	. .	…. ….	. .	. .	. .	. .	…. ….	. .	. .	. .	. .
6900			….			1	1	….	1			
7200			….					….				
7500			….					….				1
7800			….					….				1

Table 3-3 AVAILABILITY MATRIX (METHOD 2)

It is important to notice that, only some of the entries are filled, as opposed to all of the entries in the availability matrix of method 1. In particular, for method 2 the entry in O(7,10) – (2400, 3000) – reflects a zero because the machine is not functioning at that particular point. Recall, the same entry O(7,10) is equal to 10,000 in method 1, implying that all machines are functioning. Given the above availability matrix, the relevant matrix (Table 3-4) is used to provide the number of machines working at a given observation point.

Time / Usage	300	600	….	1800	2100	2400	2700	….	4500	4800	5100	5400
300	1		….	1				….				
600	1		….	1				….				
. .	. .	. .	. .	. .	. .	. .		…. ….	. .	. .	. .	. .
3000	1	1	….	1				….				
3300			….		1			….				
3600			….		1			….				
3900						1	1	….	1			
. .	. .	. .	…. ….	. .	. .	. .	. .	…. ….	. .	. .	. .	. .
6900			….			1	1	….	1			
7200			….					….		1	1	
7500			….					….				1
7800			….					….				1

Table 3-4 RELEVANT MATRIX (METHOD 2)

The difference between the two methods can be made more clear by the observations highlighted in Table 3-4. Although these points are not accessible by the machine, method 1 considers them as accessible and the family of machines as fully functioning. The degree of difference between the estimates is not clear. Figure 3-4 provides a graphical representation of the second method of bivariate availability estimation. The data generation code is available in Appendix A.9.

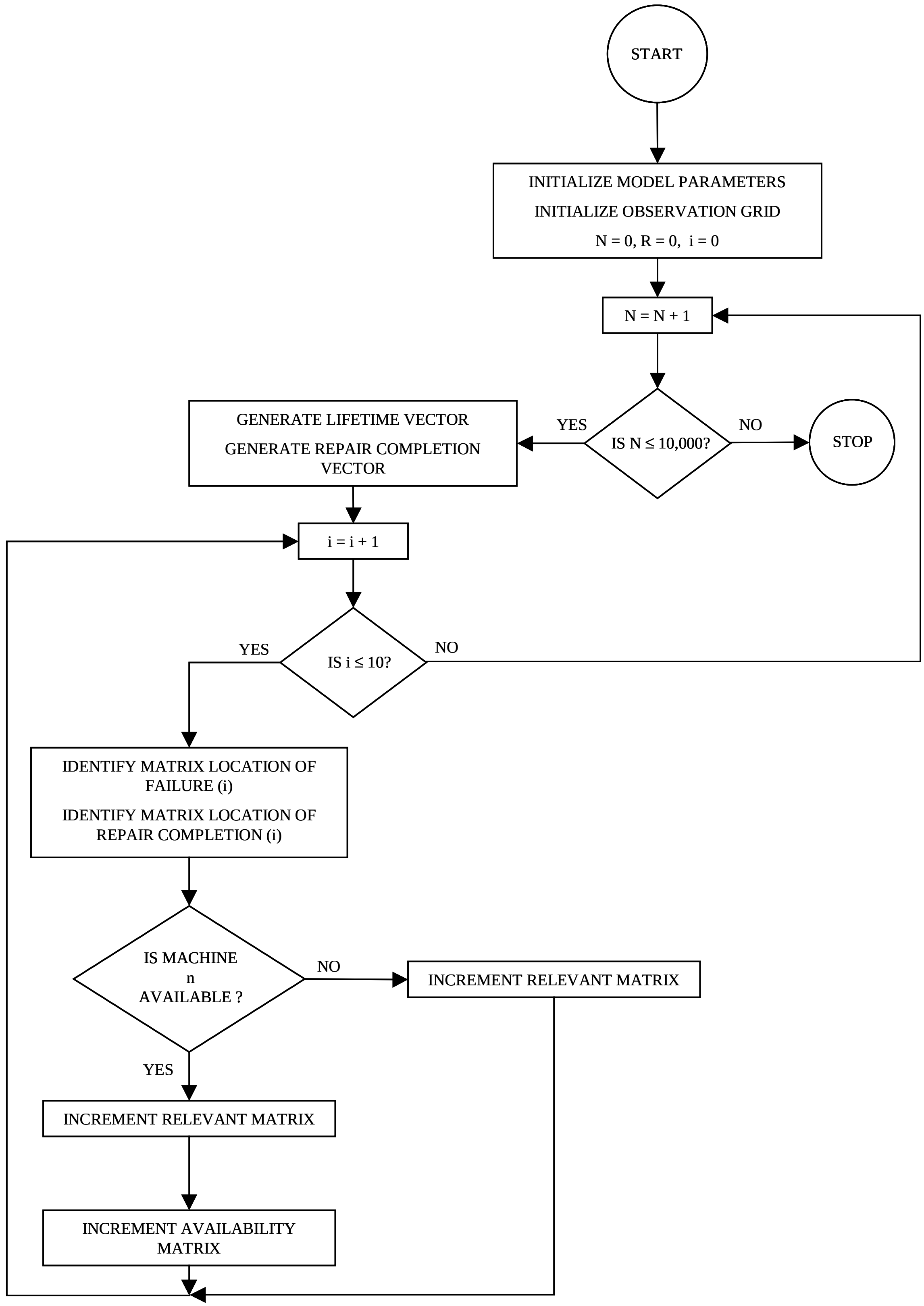

Figure 3-4 FLOWCHART FOR BIVARIATE AVAILABILITY ESTIMATION (METHOD 2)

3.5 Experimental Methodology

Simulation experiments are constructed such that a wide variety of potential systems are presented. For each of the correlated models and the linear stochastic functional relationship, the parameter values are varied on three levels in order to represent low, medium, and high failure/repair rates. All possible parameter combinations are considered, giving 27 cases. The parameter selection allows for comparison across models, by ensuring the mean values for time and use are similar. In the case of the logistic stochastic functional relationship, identification of appropriate parameters is not clear. Only two parameter sets are selected and comparison to other models is attempted. In all cases, the parameters for the repair distribution are chosen based on those for the lifetime distribution. The cases considered for each model are identified in Table 3-5.

	Bivariate Exponential Model (BVE) (λ, η, ρ)	Bivariate Normal Model (BVN) (μ_T, μ_U, σ_T, σ_U, ρ)	Linear Stochastic Function (LISF) (λ, c, β)	Logistic Stochastic Function (LOSF) (λ, c, β)
1	(0.00033, 0.00031, 0.2)	(3000, 3250, 600, 650, 0.2)	(0.00033, 1, 0)	(0.00033, 416, 1511)
2	(0.00033, 0.00031, 0.5)	(3000, 3250, 600, 650, 0.5)	(0.00033, 1, 250)	(0.00014, 1087, 1511)
3	(0.00033, 0.00031, 0.8)	(3000, 3250, 650, 600, 0.8)	(0.00033, 1, 500)	----
4	(0.00033, 0.00014, 0.2)	(3000, 7400, 600, 1480, 0.2)	(0.00033, 0.4, 0)	----
5	(0.00033, 0.00014, 0.5)	(3000, 7400, 600, 1480, 0.5)	(0.00033, 0.4, 250)	----
6	(0.00033, 0.00014, 0.8)	(3000, 7400, 600, 1480, 0.8)	(0.00033, 0.4, 500)	----
7	(0.00033, 0.000054, 0.2)	(3000, 18500, 600, 3700, 0.2)	(0.00033, 0.167, 0)	
8	(0.00033, 0.000054, 0.5)	(3000, 18500, 600, 3700, 0.5)	(0.00033, 0.167, 250)	
9	(0.00033, 0.000054, 0.8)	(3000, 18500, 600, 3700, 0.8)	(0.00033, 0.167, 500)	
10	(0.00013, 0.00031, 0.2)	(8000, 3250, 1600, 650, 0.2)	(0.00013, 2.5, 0)	
11	(0.00013, 0.00031, 0.5)	(8000, 3250, 1600, 650, 0.5)	(0.00013, 2.5, 250)	
12	(0.00013, 0.00031, 0.8)	(8000, 3250, 1600, 650, 0.8)	(0.00013, 2.5, 500)	
13	(0.00013, 0.00014, 0.2)	(8000, 7400, 1600, 1480, 0.2)	(0.00013, 1, 0)	
14	(0.00013, 0.00014, 0.5)	(8000, 7400, 1600, 1480, 0.5)	(0.00013, 1, 250)	
15	(0.00013, 0.00014, 0.8)	(8000, 7400, 1600, 1480, 0.8)	(0.00013, 1, 500)	
16	(0.00013, 0.000054, 0.2)	(8000, 18500, 1600, 3700, 0.2)	(0.00013, 0.5, 0)	
17	(0.00013, 0.000054, 0.5)	(8000, 18500, 1600, 3700, 0.5)	(0.00013, 0.5, 250)	
18	(0.00013, 0.000054, 0.8)	(8000, 18500, 1600, 3700, 0.8)	(0.00013, 0.5, 500)	
19	(0.00005, 0.00031, 0.2)	(20000, 3250, 4000, 600, 0.2)	(0.00005, 6.67, 0)	
20	(0.00005, 0.00031, 0.5)	(20000, 3250, 4000, 600, 0.5)	(0.00005, 6.67, 250)	
21	(0.00005, 0.00031, 0.8)	(20000, 3250, 4000, 600, 0.8)	(0.00005, 6.67, 500)	
22	(0.00005, 0.00014, 0.2)	(20000, 7400, 4000, 1480, 0.2)	(0.00005, 2.85, 0)	
23	(0.00005, 0.00014, 0.5)	(20000, 7400, 4000, 1480, 0.5)	(0.00005, 2.85, 250)	
24	(0.00005, 0.00014, 0.8)	(20000, 7400, 4000, 1480, 0.8)	(0.00005, 2.85, 500)	
25	(0.00005, 0.000054, 0.2)	(20000, 18500, 4000, 3700, 0.2)	(0.00005, 1, 0)	
26	(0.00005, 0.000054, 0.5)	(20000, 18500, 4000, 3700, 0.5)	(0.00005, 1, 250)	
27	(0.00005, 0.000054, 0.8)	(20000, 18500, 4000, 3700, 0.8)	(0.00005, 1, 500)	

Table 3-5 MODEL PARAMETERS

The parameters for the repair distributions are selected so that the repair rate is approximately 10 percent of the failure rate. Also, the repair distribution for each model is from the same family.

Chapter 4 – Results and Conclusions

4.1 Bivariate Failure Distributions

Examples of the bivariate probability densities for each of the candidate models are shown in Figure 4-1 through Figure 4-6. The density plots provide an indication of the behavior expected of each distribution. For instance, with the bivariate exponential distribution (Figure 4-1), the observations are well dispersed throughout the plane, but are concentrated near the origin and at values less than the mean on time and usage. From this one can conclude there is a significantly high probability of early failure, but also late failures are not uncommon. For the bivariate exponential distribution, variations in the correlation coefficient do not greatly impact the shape of the distribution.

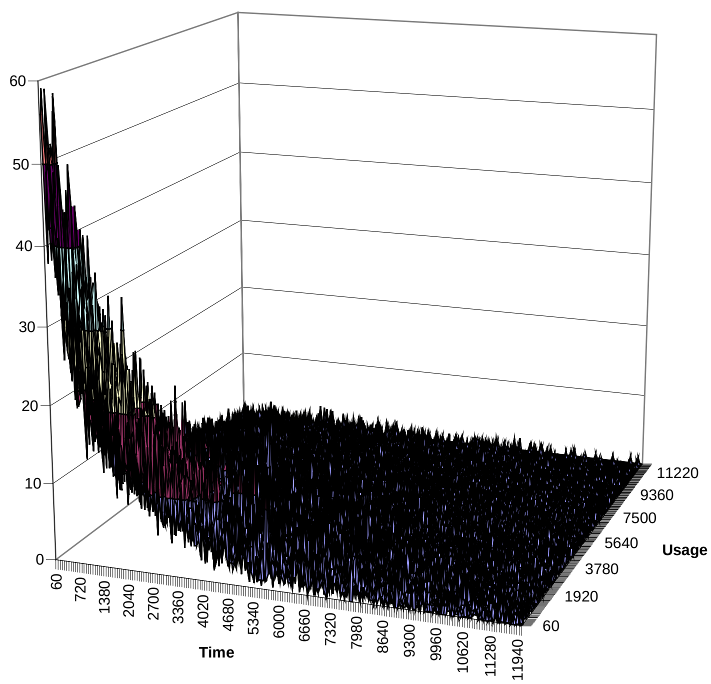

Figure 4-1 BIVARIATE EXPONENTIAL PROBABILITY DENSITY

However, with the bivariate normal distribution a noticeable change in the shape of the distribution occurs after varying the correlation coefficient. Notice in Figure 4-2, for a low correlation value the function peaks around the point (μ_t, μ_u) and the base of the function is symmetric about this point.

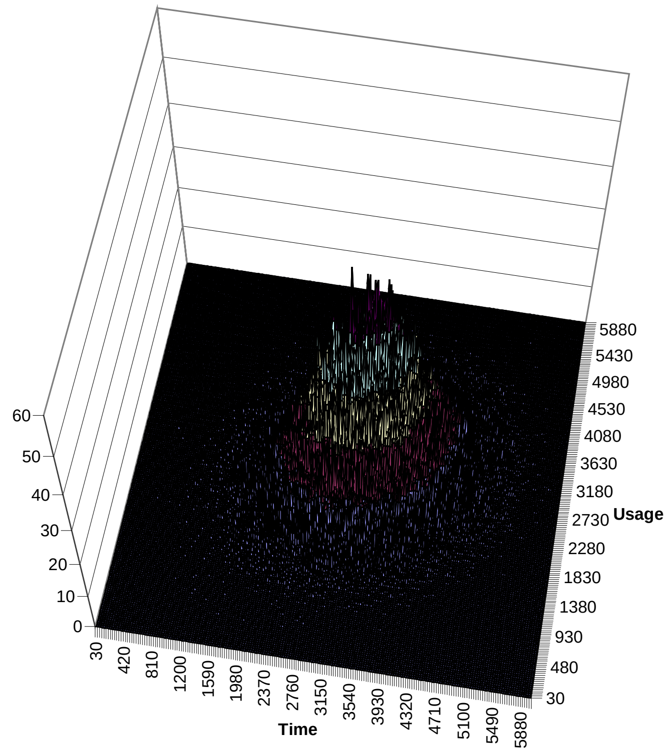

Figure 4-2 BIVARIATE NORMAL PROBABILITY DENSITY FUNCTION (ρ = 0 .2)

In contrast, in

Figure 4-3 and

Figure 4-4, the function peak shifts toward increased usage values. Also, the base of the function becomes narrower.

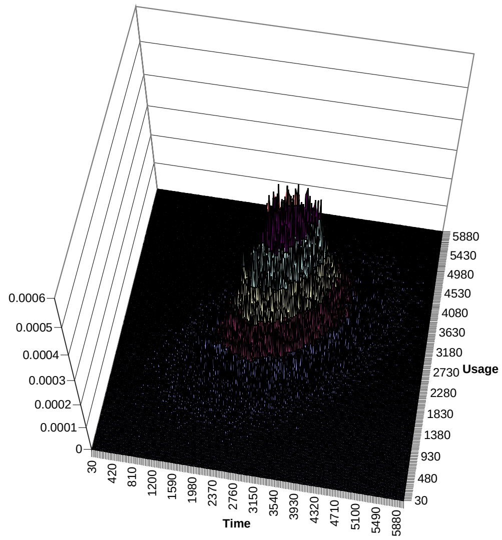

Figure 4-3 BIVARIATE NORMAL PROBABILITY DENSITY FUNCTION (ρ = 0 .5)

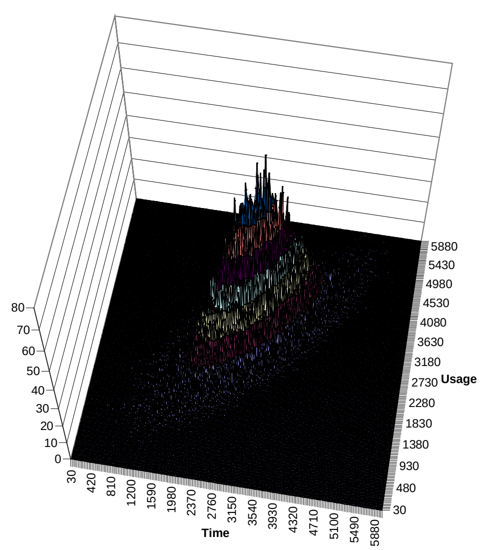

Figure 4-4 BIVARIATE NORMAL PROBABILITY DENSITY FUNCTION (ρ = 0 .8)

It is important to recognize the lack of dispersion in the plane for the bivariate normal density. There is little dispersion as a result of the choice of a small standard deviation for time and usage. The standard deviations, σ_t and σ_u, were chosen as 1/5 of μ_t and μ_u, in order to avoid negative values in the generation procedure. Larger values for the standard deviations would have provided more representation throughout the plane.

Figure 4-5 depicts the realization of the linear stochastic functional relationship. Similar to the BVE, there is a high concentration of observations near the origin and good representation throughout the plane. Although, it appears the linear relationship tends to have a higher proportion of early values than the bivariate exponential. Another interesting point is the sparcity of values near the mean values on time and usage, as seen in the BVE.

The last distribution, the logistic stochastic functional relationship Figure 4-6 also resembles the BVE. There is a considerable increase in the number of observations near the origin, implying a high percentage of early failures. An important distinction between the behavior of this model and the others is the scale on the usage variable. The values of U fall in the interval (0, 1]. Additional instances of the stochastic functional probability densities can be found in Appendix B.

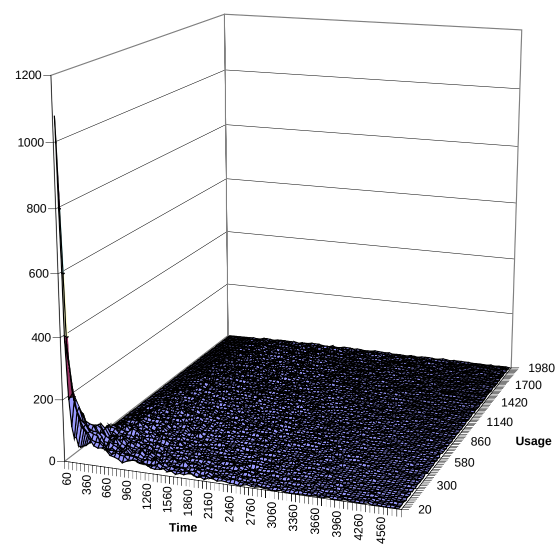

Figure 4-5 LINEAR STOCHASTIC FUNCTION PROBABILITY DENSITY

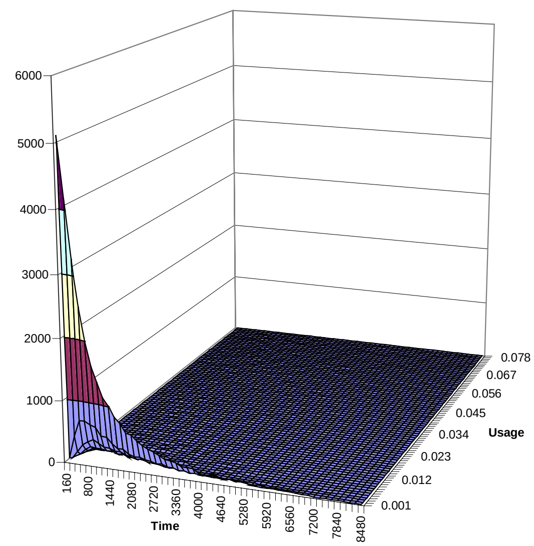

Figure 4-6 LOGISTIC STOCHASTIC FUNCTION PROBABILITY DENSITY

4.2 Renewal Estimation

After initial analysis of the results for renewal estimation, the collection of parameter sets can be reduced to the cases enumerated in Table 4-1:

	Bivariate Exponential Model (BVE) (λ, η, ρ)	Bivariate Normal Model (BVN) (μ_T, μ_U, σ_T, σ_U, ρ)	Linear Stochastic Function (LISF) (λ, c, β)	Logistic Stochastic Function (LOSF) (λ, c, β)
1	(0.00033, 0.00031, 0.2)	(3000, 3250, 600, 650, 0.2)	(0.00033, 1, 0)	(0.00033, 416, 1511)
2	(0.00033, 0.00031, 0.5)	(3000, 3250, 600, 650, 0.5)	(0.00033, 1, 250)	(0.00014, 1087, 1511)
3	(0.00033, 0.00031, 0.8)	(3000, 3250, 650, 600, 0.8)	(0.00033, 1, 500)	----
4	(0.00033, 0.00014, 0.5)	(3000, 7400, 600, 1480, 0.5)	(0.00033, 0.4, 250)	----
5	(0.00033, 0.000054, 0.5)	(3000, 18500, 600, 3700, 0.5)	(0.00033, 0.167, 250)	----
6	(0.00005, 0.00031, 0.5)	(20000, 3250, 4000, 600, 0.5)	(0.00005, 6.67, 250)	----

Table 4-1 REPRESENTATIVE PARAMETER SETS

The reduction is made because qualitatively the behavior is the same across parameter sets. Plots for the cases corresponding to variations in ρ or β, for cases 4-6, are provided in Appendix B.1.

4.2.1 Bivariate Exponential Model

In general, the renewal function for the bivariate exponential distribution demonstrates reasonable behavior. The renewal function exhibits gradually increasing behavior throughout the plane. However, there is a distinct area of greater increase, particularly for low values of time and usage; this is understandable considering the bivariate exponential density. By the nature of the bivariate exponential distribution, there is a significant probability of early failures as well as a reasonably high likelihood of late failure, which accounts for the shape of the renewal function. Figure 4-7 shows the renewal function behavior for case 1, where $\lambda = 0.0003$, $\eta = 0.00031$, and $\rho = 0.2$.

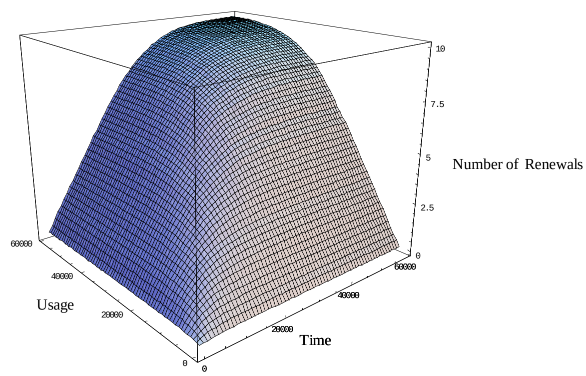

Figure 4-7 RENEWAL FUNCTION (CASE 1, BVE)

Figure 4-8 and Table 4-2 represents the results for the univariate projection method for case 1. The results verify the behavior exhibited in the bivariate renewal estimation procedure. Qualitatively the behavior is the same across all parameter sets and all model instances. The results for the representative parameter sets can be found in Appendix B.1. A distinction between Hunter's univariate renewal counting process and that used here is evident: in all instances, $N_{t,u} \neq \min\left\{N_t^{(1)}, N_y^{(2)}\right\}$. Hunter's univariate process accumulates renewals until the particular time or use value in question; however, the method used here counts the number of renewals across all values of time or use while holding the second variable constant.

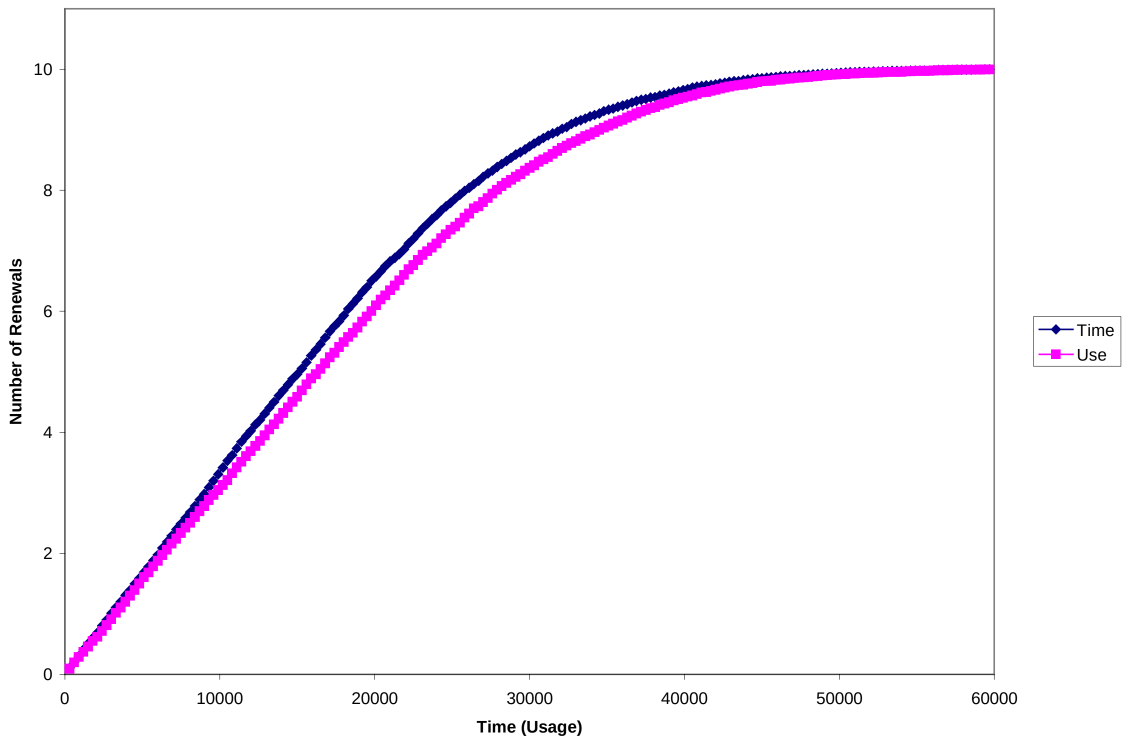

Figure 4-8 UNIVARIATE PROJECTION (CASE1, BVE)

Parameter Values	t	u	N_t	N_u	min $\{N_t, N_u\}$	$N_{t,u}$
(0.00033, 0.00031, 0.2)	300	300	0.103	0.091	0.091	0.016
	4200	4200	1.387	1.296	1.296	0.778
	36000	36000	9.403	9.162	9.162	8.755

Table 4-2 DATA FOR UNIVARIATE PROJECTION vs. BIVARIATE ESTIMATE

Figure 4-9 and Figure 4-10 show the effects of varying the degree of correlation between the variables. The variations in correlation do not appear to cause significant differences in renewal function behavior. Upon increasing the correlation coefficient, ρ, the function behavior is altered in the use component, but not time. In particular, the number of renewals increases more quickly in the usage variable for $\rho = 0.5$ and $\rho = 0.8$, than for $\rho = 0.2$. In all cases the difference is nominal.

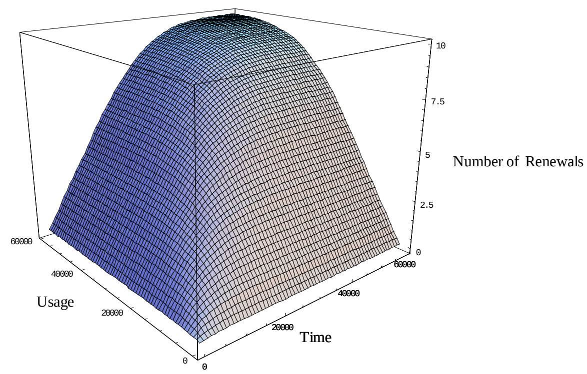

Figure 4-9 RENEWAL FUNCTION (CASE 2, BVE)

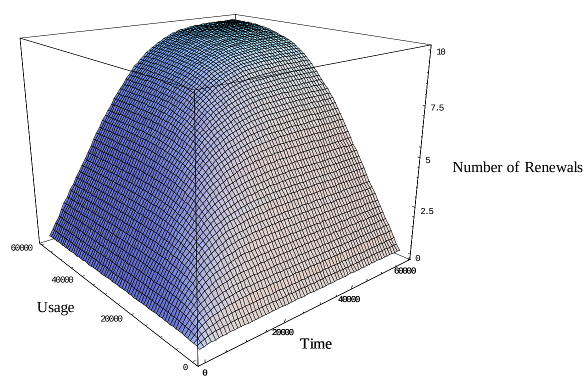

Figure 4-10 RENEWAL FUNCTION (CASE 3, BVE)

In the cases that vary the failure rates on different levels, the results are comparable and the differences achieved are not surprising. For example, when the mean for t is low and the mean for u is high it is expected that the number of renewals will increase more quickly with respect to time. As seen in Figure 4-11 the area of steepest increase is deflected toward the usage axis (i.e. low time, high usage). As the distance between the failure rates increases, the shift toward the axis corresponding to the larger value is more apparent (Figure 4-12 and Figure 4-13). In other words, the number of renewals increases more slowly in the variable with the larger mean value.

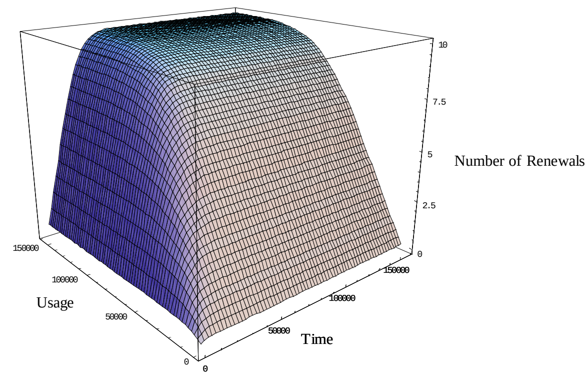

Figure 4-11 RENEWAL FUNCTION (CASE 4, BVE)

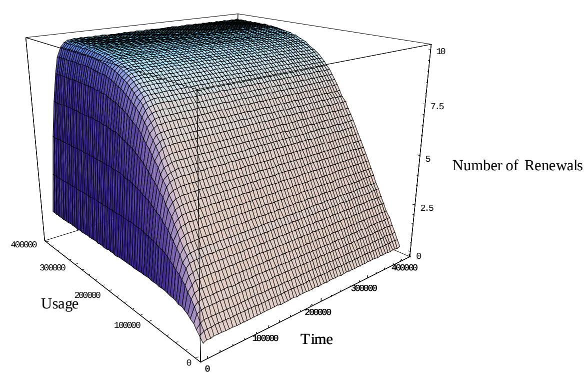

Figure 4-12 RENEWAL FUNCTION (CASE 5, BVE)

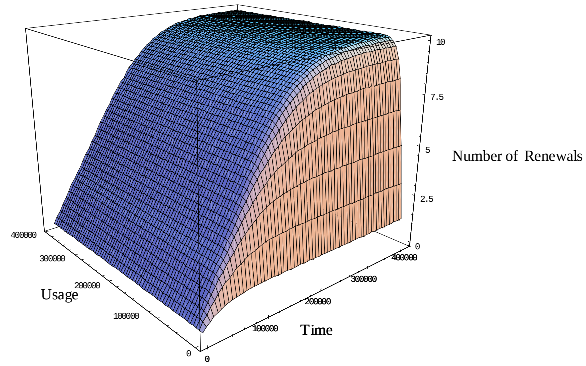

Figure 4-13 RENEWAL FUNCTION (CASE 6, BVE)

4.2.2 Bivariate Normal Model

In all instances of the bivariate normal distribution, at points early in the plane the renewal function is not smooth. Early there are jumps in the number of renewals as time and usage increase. However, as the renewal function approaches the observation point ($5\mu_t$, $5\mu_u$) the ridges become smooth. The smoothness of the function implies a more gradual increase in the number of renewals as opposed to the early discrete jumps. Figure 4-14 shows the renewal function for case 1 (μ_t = 3000, μ_u = 3250, ρ = 0.2); this chart is representative of all parameter sets with comparable mean values for *t* and *u* and low correlation.

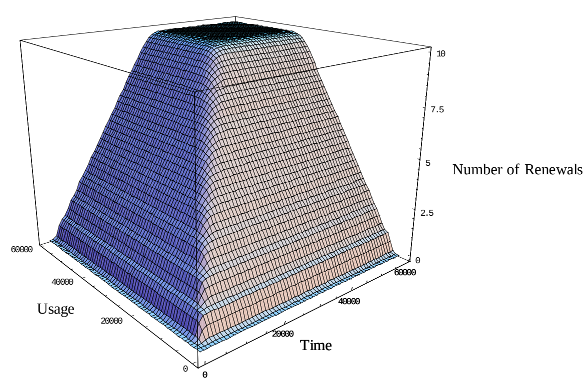

Figure 4-14 RENEWAL FUNCTION (CASE 1, BVN)

Figure 4-15 and Table 4-3, represent the analysis of the univariate projection method and the bivariate estimation procedure. The results confirm the behavior seen in the bivariate estimation procedure. The bivariate normal case reiterates the difference between Hunter's method and that used here. The results for the additional parameter sets may be found in Appendix B.1.

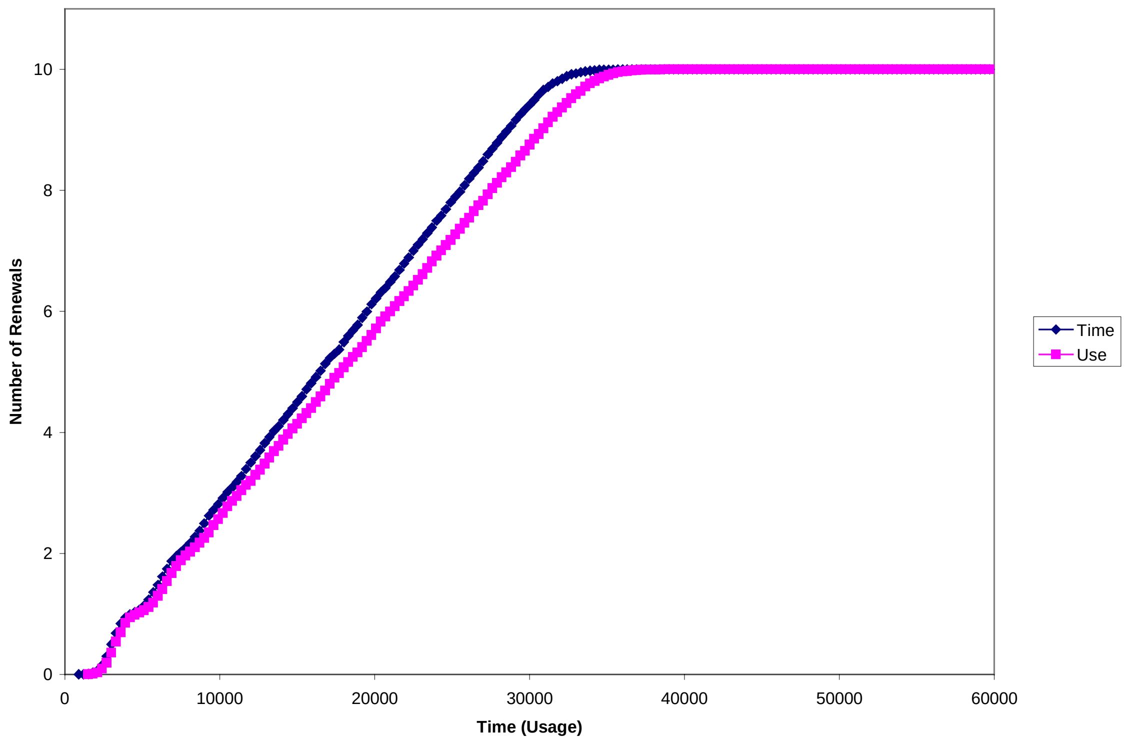

Figure 4-15 UNIVARIATE PROJECTION (CASE 1, BVN)

Parameter Values	t	u	N_t	N_u	min $\{N_t, N_u\}$	$N_{t,u}$
(3000, 3250, 600, 650, 0.2)	300	300	0	0	0	0
	4200	4200	0.992	0.941	0.941	0.914
	36000	36000	10	9.975	9.975	9.955

Table 4-3 DATA FOR UNIVARIATE PROJECTION vs. BIVARIATE ESTIMATE

As a point of comparison, Figure 4-16 and Figure 4-17 depict the effects of increases in the value of ρ. The overall shape of the function is unchanged; however, the location of the function is altered. The base of the renewal function shifts away from the time axis as the correlation increases from 02. to 0.5 and 0.2 to 0.8. This indicates a slower rate of increase in the usage variable as ρ increases. This behavior occurs because an increase in ρ results in increased values for *U*. Less of a distinction between the renewal function for ρ = 0.5 and ρ = 0.8 exists.

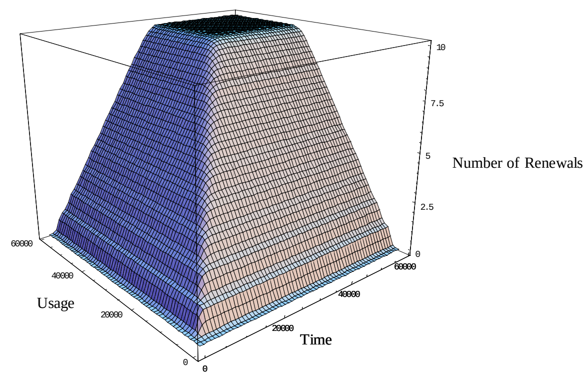

Figure 4-16 RENWAL FUNCTION (CASE 2, BVN)

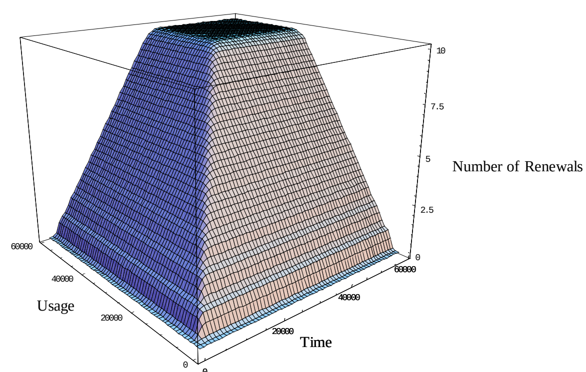

Figure 4-17 RENEWAL FUNCTION (CASE 3, BVN)

A final observation can be made concerning the behavior of the renewal function as the mean failure rates are examined at varying levels. The average cumulative number of renewals increases more quickly in relation to the variable with a smaller mean. This is not surprising since a smaller mean implies to earlier failures. It is important to note the relative shape of the function is the same. However, the function plateau is elongated and the peak occurs closer to the axis associated with the larger mean. As the deviation between the means increases, the magnitude of the shifts increases (Figure 4-18 - Figure 4-20). Figure 4-18 and Figure 4-19 represent a low failure rate for time paired with a high usage failure rate, whereas Figure 4-20 shows the opposite relationship—high time failure rate and low usage failure rate.

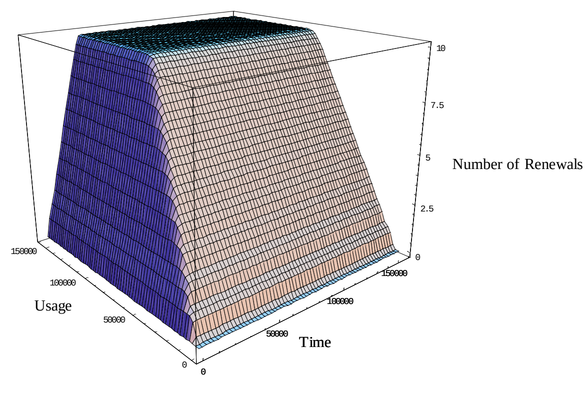

Figure 4-18 RENEWAL FUNCTION (CASE 4, BVN)

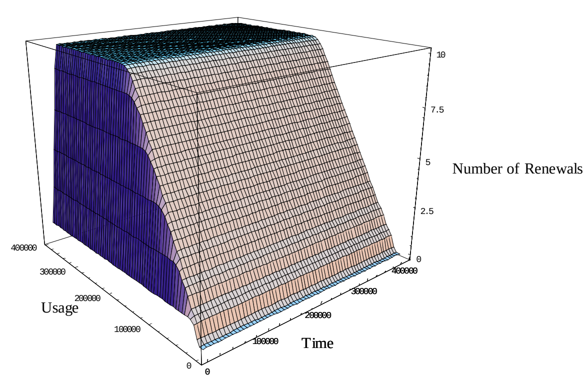

Figure 4-19 RENEWAL FUNCTION (CASE 5, BVN)

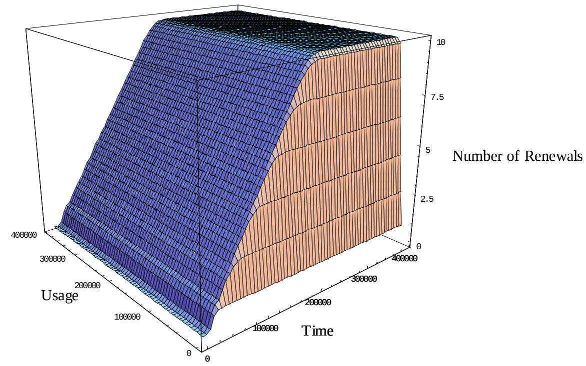

Figure 4-20 RENEWAL FUNCTION (CASE 6, BVN)

4.2.3 Linear Stochastic Functional Relationship

The renewal function for the linear relationship exhibits many of the properties seen with the bivariate exponential distribution. The function is smooth in all areas of the plane, implying gradual increases throughout the plane. However, the number of renewals appears to increase faster early in the plane. Except for extremely early values of time and use, the rate of increase along the axes is constant. This is expected since failures with a high time (use) and low use (time) occur much less frequently than those with low time and low use. Figure 4-21 depicts the general renewal behavior exhibited by the linear stochastic function when $\lambda = 0.0003$, $\alpha = 1$, and $\beta = 0$.

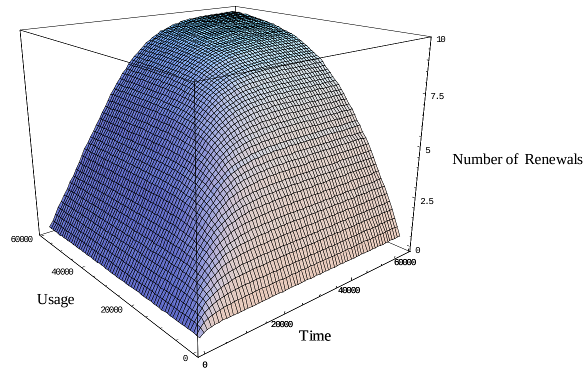

Figure 4-21 RENEWAL FUNCTION (CASE 1, LISF)

Figure 4-22 and Table 4-4, represent the analysis of the univariate projection method and the bivariate estimation procedure. The behavior exhibited in Figure 4-22 is comparable to that seen in the bivariate estimate (Figure 4-21). It is important to note that, for earlier values of time and use, the number of renewals increases more quickly in the usage variable; however, as time and use progress the opposite is true. The results for the additional parameter sets may be found in Appendix B.1.

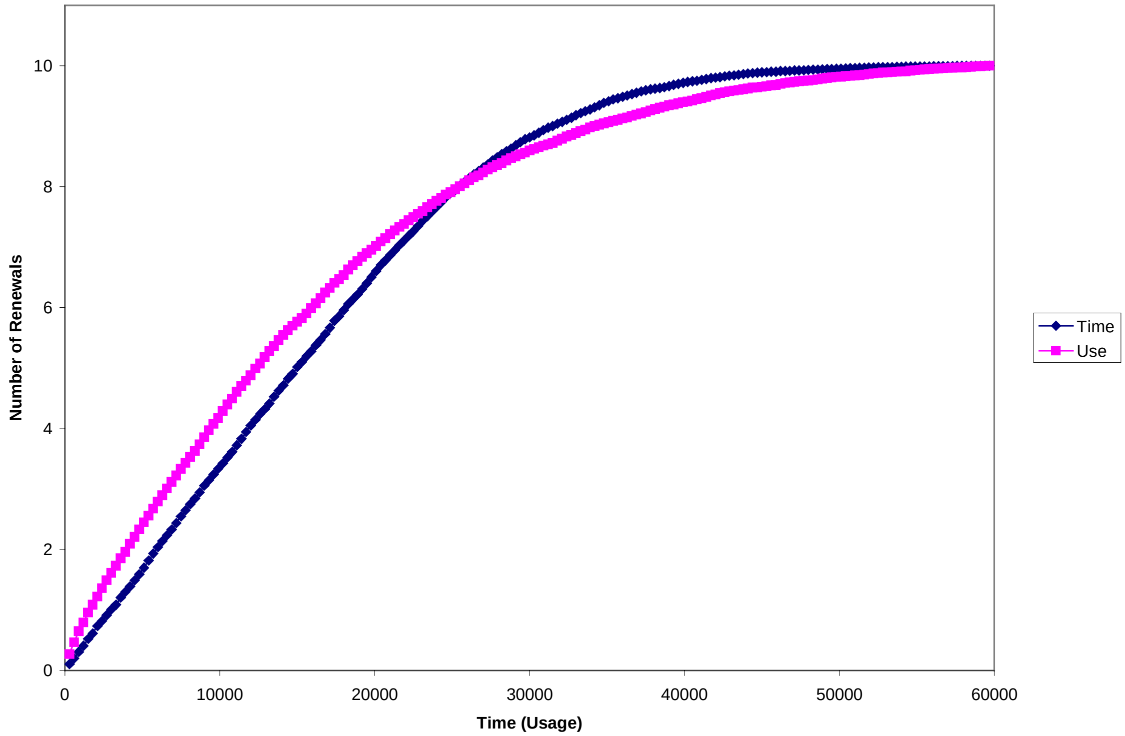

Figure 4-22 UNIVARIATE PROJECTION (CASE 1, LISF)

Parameter Values	t	u	N_t	N_u	min $\{N_t, N_u\}$	$N_{t,u}$
(0.00033, 1, 0)	300	300	0.104	0.268	0.104	0.091
	4200	4200	1.387	2.092	1.387	1.178
	36000	36000	9.488	9.129	9.129	8.755

Table 4-4 DATA FOR UNIVARIATE PROJECTION vs. BIVARIATE ESTIMATE

Figure 4-23 and Figure 4-24 represent the changes in the renewal behavior as a result of variations in the constant, β. The global behavior mimics case 1, although differences exist in relation to the use variable. In general, the number of renewals increases more quickly for smaller values of β. This is understandable considering the constant, β, increases the amount of use between failures. It is interesting, however, for β = 0 and β 250, renewals increase faster in usage than in time, but for β = 500 the number of renewals tends to increase faster in time than in use.

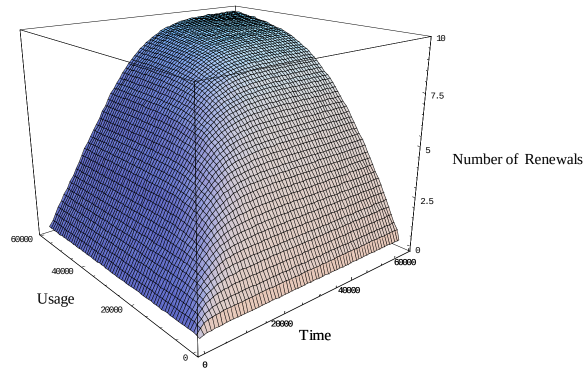

Figure 4-23 RENEWAL FUNCTION (CASE 2, LISF)

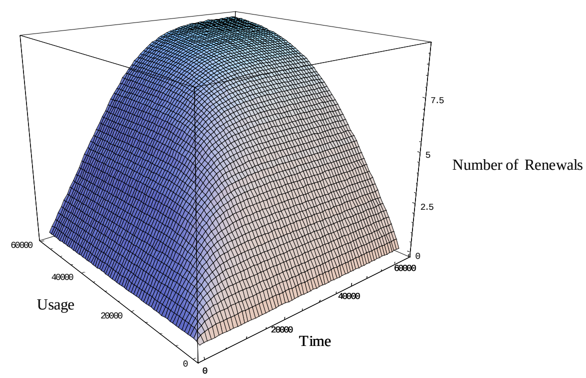

Figure 4-24 RENEWAL FUNCTION (CASE 3, LISF)

The last three cases represent the effects of varying the level of the mean value for *t* and α. Variations in the mean level produce comparable behavior across all parameter sets. The representative combinations are shown in Figure 4-25 through Figure 4-27. As the separation between the means increases, the location of the steepest increase shifts toward the axis corresponding to the larger mean. Also, the increase in the number of renewals near the axes increases as the separation between the means increase. For case 4 and 5, the increase is noted along the usage axis; case 6, the time axis.

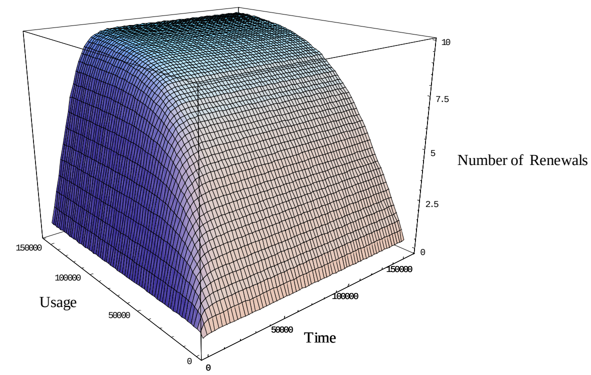

Figure 4-25 RENEWAL FUNCTION (CASE 4, LISF)

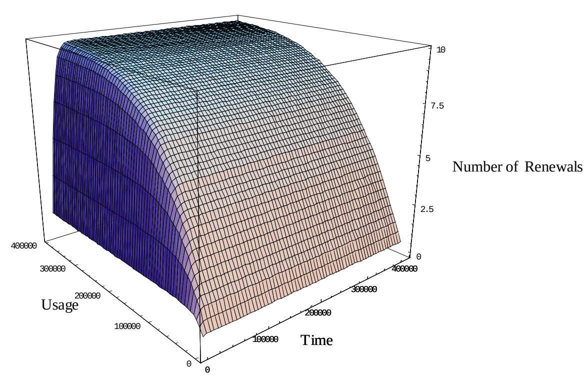

Figure 4-26 RENEWAL FUNCTION (CASE 5, LISF)

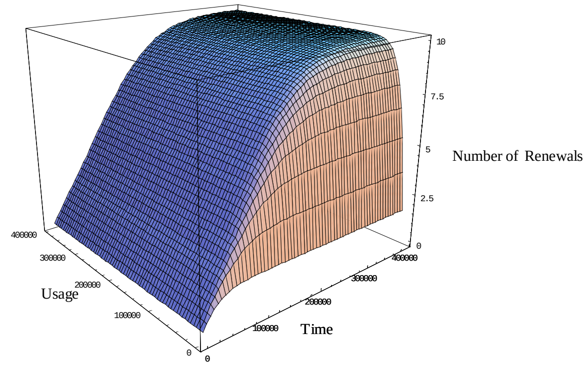

Figure 4-27 RENEWAL FUNCTION (CASE 6, LISF)

4.2.4 Logistic Stochastic Functional Relationship

The renewal function for the logistic stochastic functional relationship (Figure 4-28) demonstrates gradually increasing behavior as time progresses. However, as usage increases there are periods of gradual increase followed by periods of steep increase. For early usage values there is a sharp increase in the number of renewals as time increases. This can be attributed to the high likelihood of extremely early failures in both time and use.

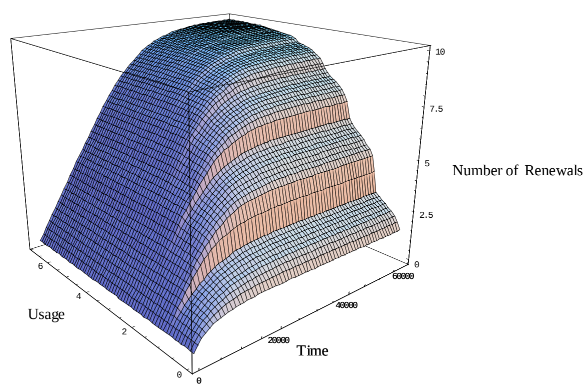

Figure 4-28 RENEWAL FUNCTION (CASE 1, LOSF)

Figure 4-29 represents the renewal function for the second logistic function parameter set. Here, the mean value for time is increased, while the mean value for α is decreased. The resulting behavior is similar to case 1, although the number of renewals increases more quickly as time advances.

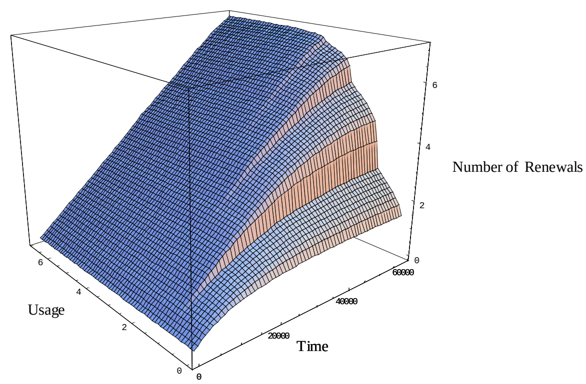

Figure 4-29 RENEWAL FUNCTION (CASE 2, LOSF)

4.2.5 Model Comparison

The renewal function behavior exhibited for each of the models is reasonable given the construction of the data generation procedure. In general, each model illustrates gradually increasing behavior as time and use progress. However, for the bivariate normal and the logistic stochastic functional relationship, relatively large increases in the number of renewals occur at discrete locations. Also, the number of renewals accumulates more quickly in the bivariate normal model than in the other models.

With exception to observations near the axes, the behaviors of the bivariate exponential and the linear stochastic functional relationship are closely related. The linear model tends to display greater immediate increases across comparable parameter sets. The similarities between the linear relationship and the bivariate exponential are a result of the generation procedure selected for the linear model. It is unclear that the behavior seen here is representative of the linear relationship.

In all model instances, the results from the univariate projection method verify the results obtained in the bivariate estimation process. Also, after reviewing the work of Hunter [1974], it is determined that the univariate counting process used here is different from that used by Hunter. The univariate counting process of Hunter accumulates the number of renewals experienced until a particular observation point. The method developed here accumulates the number of renewals experienced for a single variable, while holding the other constant. The difference between the two methods is subtle, but important.

4.3 Bivariate Availability Estimation

As is the case with the renewal estimation, the availability estimation results can be limited to a group of representative cases. The cases reviewed in the renewal estimation section are analyzed for the availability estimation. The cases are reiterated in Table 4-5.

	Bivariate Exponential Model (BVE) (λ, η, ρ)	Bivariate Normal Model (BVN) $(\mu_T, \mu_U, \sigma_T, \sigma_U, \rho)$	Linear Stochastic Function (LISF) (λ, c, β)	Logistic Stochastic Function (LOSF) (λ, c, β)
1	(0.00033, 0.00031, 0.2)	(3000, 3250, 600, 650, 0.2)	(0.00033, 1, 0)	(0.00033, 416, 1511)
2	(0.00033, 0.00031, 0.5)	(3000, 3250, 600, 650, 0.5)	(0.00033, 1, 250)	(0.00014, 1087, 1511)
3	(0.00033, 0.00031, 0.8)	(3000, 3250, 650, 600, 0.8)	(0.00033, 1, 500)	----
4	(0.00033, 0.00014, 0.5)	(3000, 7400, 600, 1480, 0.5)	(0.00033, 0.4, 250)	----
5	(0.00033, 0.000054, 0.5)	(3000, 18500, 600, 3700, 0.5)	(0.00033, 0.167, 250)	----
6	(0.00005, 0.00031, 0.5)	(20000, 3250, 4000, 600, 0.5)	(0.00005, 6.67, 250)	----

Table 4-5 REPRESENTATIVE PARAMETER SETS

4.3.1 Bivariate Exponential Model

The general behavior of the availability function is shown in Figure 4-30 and Figure 4-31, methods 1 and 2, respectively. There is a sharp decrease in the availability near the origin; the decrease is followed by an immediate increase. The increase is consistent throughout the plane and the function reaches steady state (method 1). The availability function displays the anticipated behavior, with exception to the magnitude of the availability values. While the availability is expected to be relatively high given the moderate repair rate, the values reported seem unusually high. One last observation is that the bulk of the availability function activity occurs along the 45° diagonal of the plane, but the observations span a wide area along the diagonal.

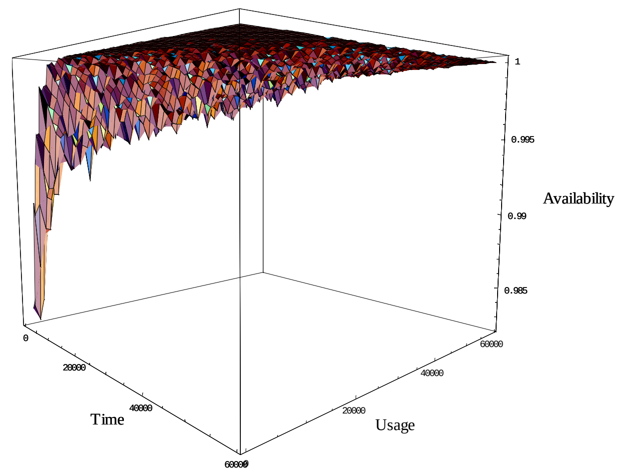

Figure 4-30 AVAILABILITY FUNCTION (METHOD 1, CASE 1, BVE)

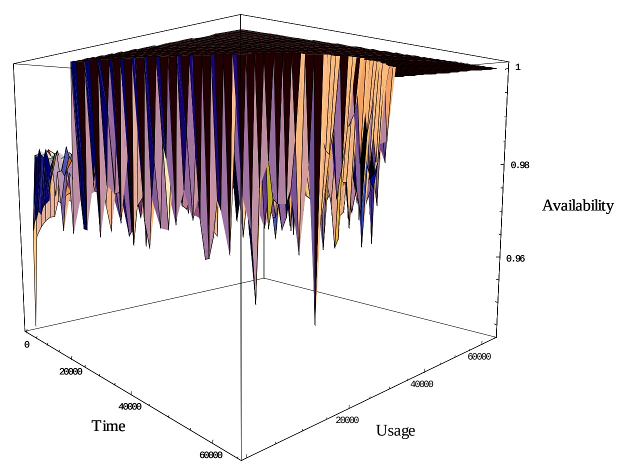

Figure 4-31 AVAILABILITY FUNCTION (METHOD 2, CASE 1, BVE)

Recall, there are two methods of estimating the availability function for each bivariate model. At this point it is important to comment on the disparities between the methods. Figure 4-30 and Figure 4-31 represent the availability statistics reported by method 1 and method 2, respectively. For the bivariate exponential model, the two methods of estimation produce similar results near the origin, but not throughout the plane, particularly near high time, low usage values. This behavior may be attributed to the small number of observations that pass through those points. The inconsistency exists in all experimental cases.

The shape of the availability function is not greatly affected by modifications in the degree of correlation. The increase from $\rho = 0.2$ to 0.5 and $\rho = 0.5$ to 0.8, results in a nominal decrease in the availability throughout the plane. Figure 4-32 through Figure 4-35 show the availability function for case 2 and case 3, $\rho = 0.5$ and $\rho = 0.8$, respectively.

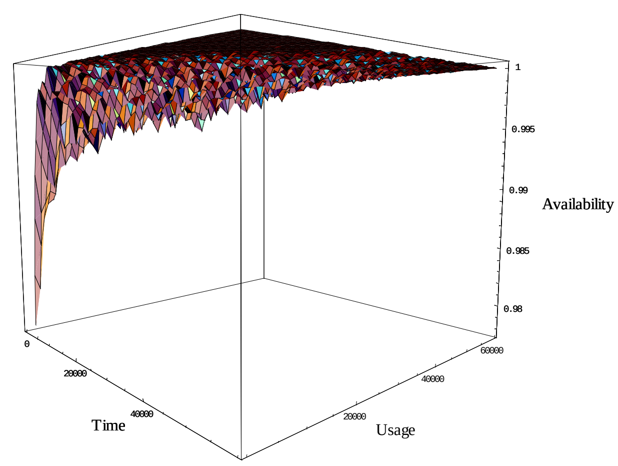

Figure 4-32 AVAILABILITY FUNCTION (METHOD 1, CASE 2, BVE)

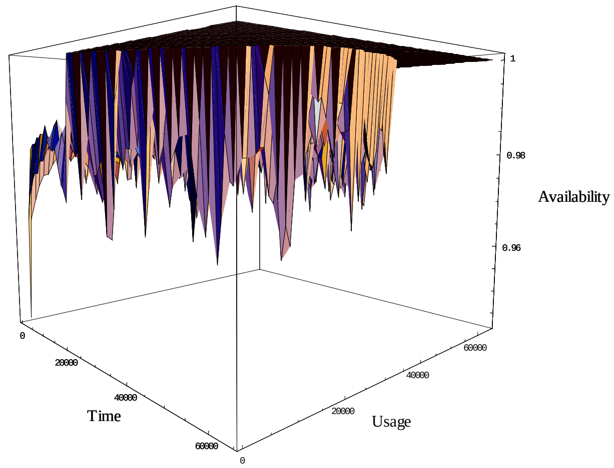

Figure 4-33 AVAILABILITY FUNCTION (METHOD 2, CASE 2, BVE)

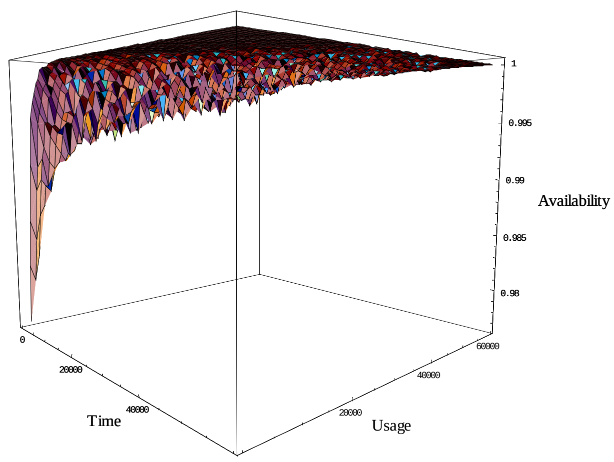

Figure 4-34 AVAILABILITY FUNCTION (METHOD 1, CASE 2, BVE)

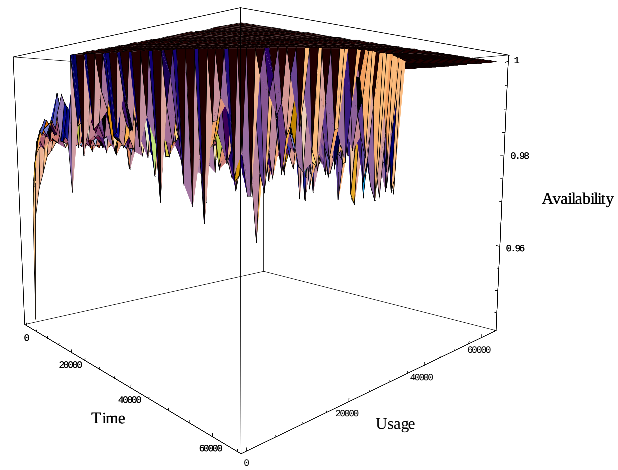

Figure 4-35 AVAILABILITY FUNCTION (METHOD 2, CASE 2, BVE)

The various combinations of failure rate levels exhibit the expected outcomes. For instance, the function shifts away from the axis with the smallest mean, since the failures and repairs associated with the larger mean absorb more time (usage). The estimates for availability tend to decrease, as the means grow farther apart, this is likely a result of the observation grid size. Recall, the grid size is based on the largest mean value, so the smaller mean will incur a larger number of repairs between observations. The results are illustrated in Figure 4-36 through Figure 4-41.

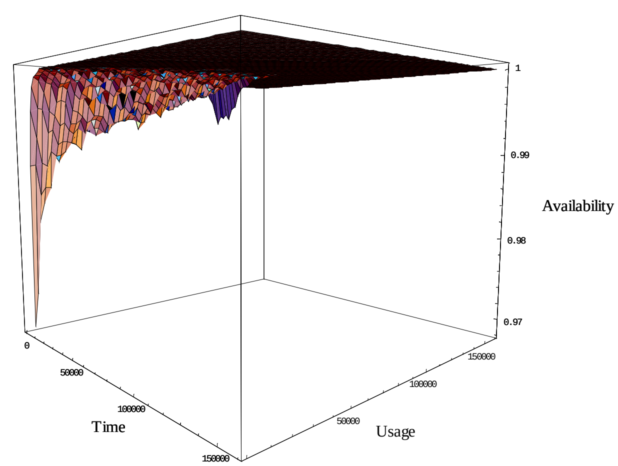

Figure 4-36 AVAILABILITY FUNCTION (METHOD 1, CASE 4, BVE)

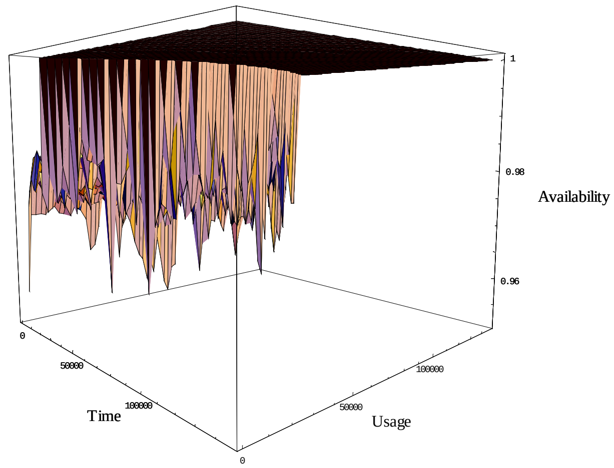

Figure 4-37 AVAILABILITY FUNCTION (METHOD 2, CASE 4, BVE)

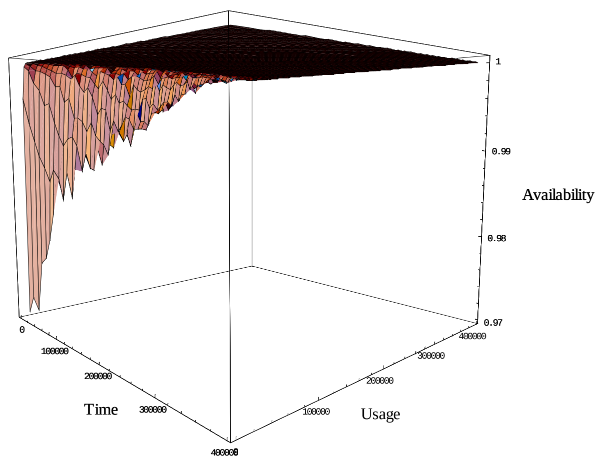

Figure 4-38 AVAILABILITY FUNCTION (METHOD 1, CASE 5, BVE)

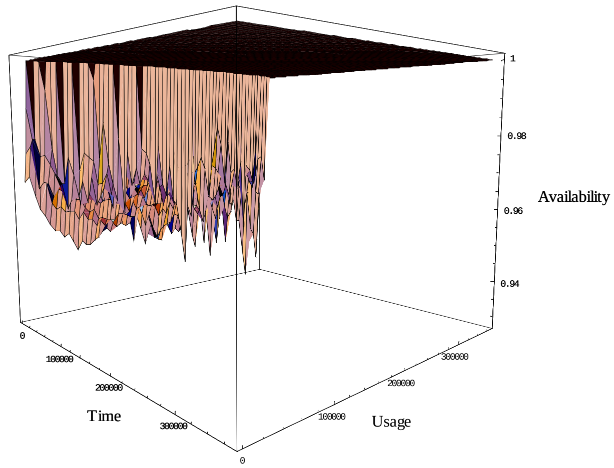

Figure 4-39 AVAILABILITY FUNCTION (METHOD 2, CASE 5, BVE)

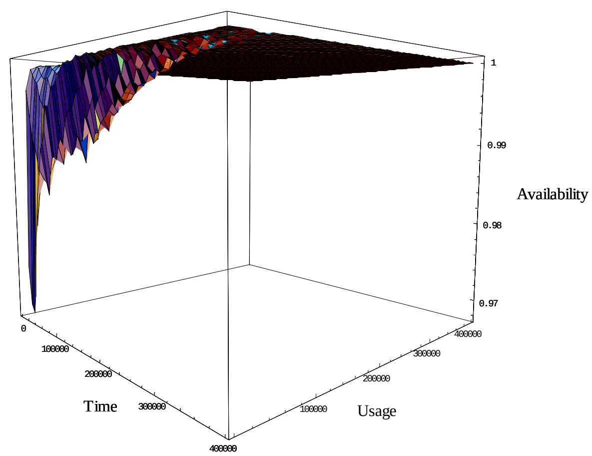

Figure 4-40 AVAILABILITY FUNCTION (METHOD 1, CASE 6, BVE)

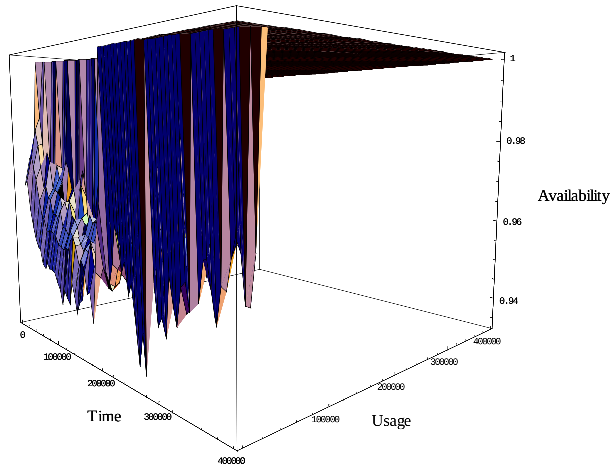

Figure 4-41 AVAILABILITY FUNCTION (METHOD 2, CASE 6, BVE)

Additional experiments were attempted to verify the findings from the original simulation runs. The initial sample population of 100,000 machines was increased to 500,000 machines in order to see if an inadequate sample size caused the high availability. The increased sample size produced a smoother curve (Figure 4-42), but the availability estimation was unchanged. The final test was a decrease in the observation grid interval. The resulting function values and the shape of the curve did not change significantly, but the curve became much less jagged. This behavior is shown in Figure 4-43. In both test cases, the function characteristics did not vary from those found in the initial comparable analysis, so no additional runs were performed.

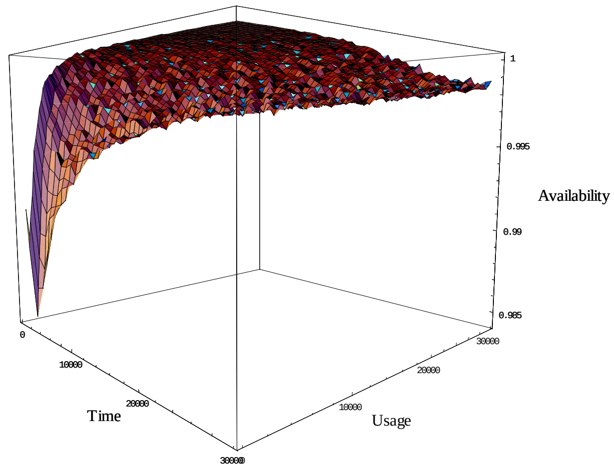

Figure 4-42 AVAILABILITY FUNCTION (N = 500,000, CASE 2, BVE)

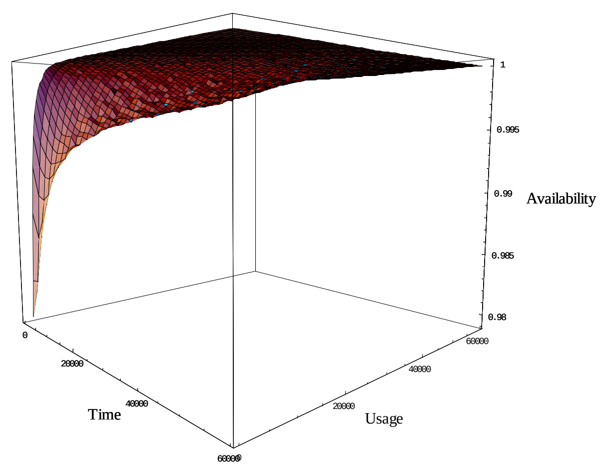

Figure 4-43 AVAILABILITY FUNCTION (DECREASED GRID, CASE 2, BVE)

4.3.2 Bivariate Normal Model

In the first bivariate normal case, Figure 4-44 and Figure 4-45 there are several observations that can be made about the availability function. First, notice there is an initial transient period that begins to stabilize as the machine life increases in *T* and *U*. Additionally, the availability function is concentrated along the 45° diagonal of the plane. The minimum value for availability corresponds to an observation location approximately equivalent to (μ_T, μ_U). This can be attributed to the high probability of experiencing failure at values close to the means for time and usage. Another important observation is the appropriately high availability this is due to the size of the repair interval. Each failure is followed by a modest amount of repair time and usage (1/10 of the failure rate); therefore, the population is expected to be in a functioning state for a high percentage of the observation plane.

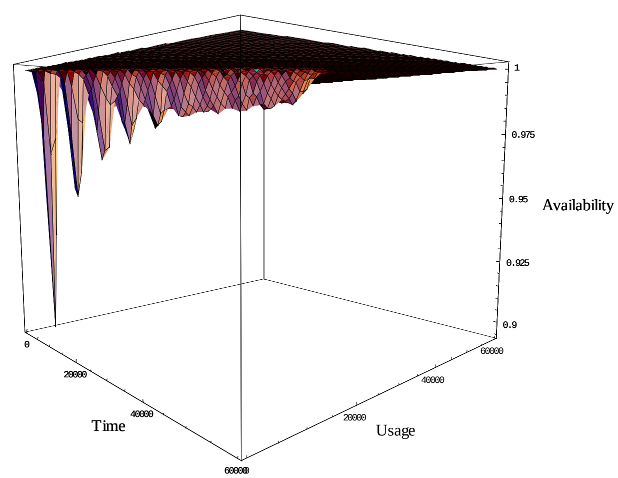

Figure 4-44 AVAILABILITY FUNCTION (METHOD 1, CASE 1, BVN)

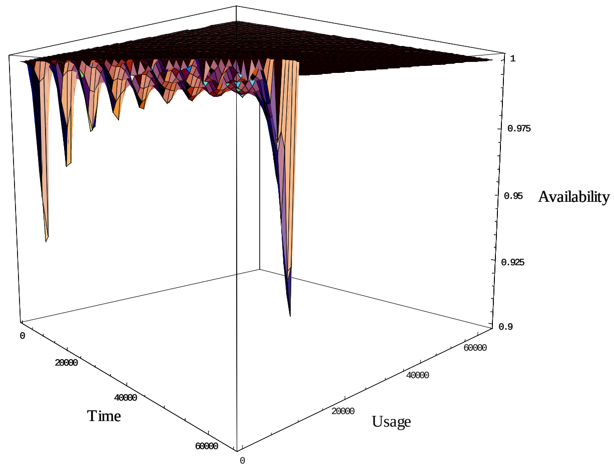

Figure 4-45 AVAILABILITY FUNCTION (METHOD 2, CASE 1, BVN)

Figure 4-44 and Figure 4-45 correspond to the availability statistics (case 1) reported by method 1 and method 2, respectively. In this case and all other bivariate normal cases, the two methods produce comparable results; however, two differences are instantly obvious. First, method 2 generally reports higher availability. The second difference is the sharp decrease in the availability near points late in the time-usage horizon. Both of these distinctions can be attributed to the size of the population at the particular observation points. In particular, the sharp decrease in availability can is associated with the relatively small sample population that passes through late observation points. These inconsistencies appear in all cases of the bivariate normal model.

Figure 4-46 through Figure 4-49 depict the effects of the two additional degrees of correlation on system availability. It is important to note that, the variation in correlation produces the same changes in behavioral characteristics for all parameter sets. At each level of correlation, the same general trends exist; however, as the correlation increases, the curve becomes more concentrated along the diagonal (i.e. the function is less disperse). The differences in the density function after a change in correlation confirm this behavior.

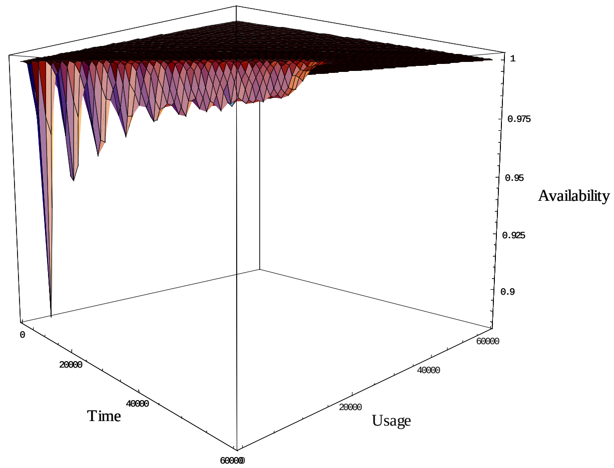

Figure 4-46 AVAILABILITY FUNCTION (METHOD 1, CASE 2, BVN)

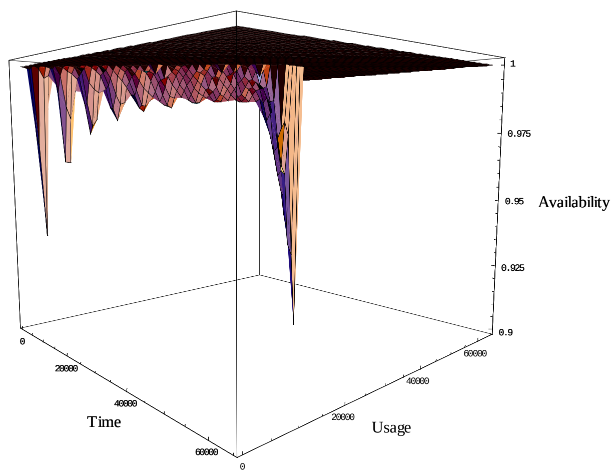

Figure 4-47 AVAILABILITY FUNCTION (METHOD 2, CASE 2, BVN)

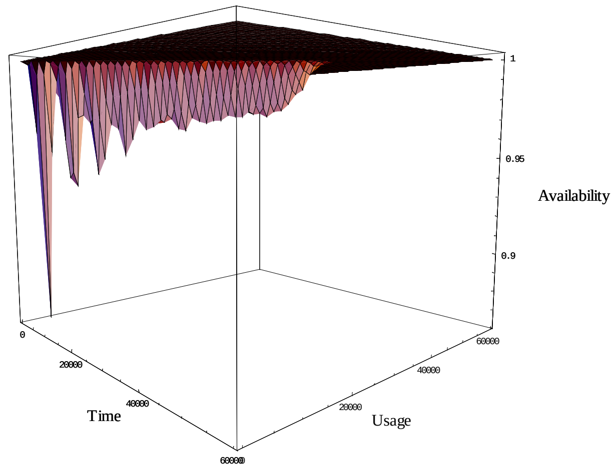

Figure 4-48 AVAILABILITY FUNCTION (METHOD 1, CASE 3, BVN)

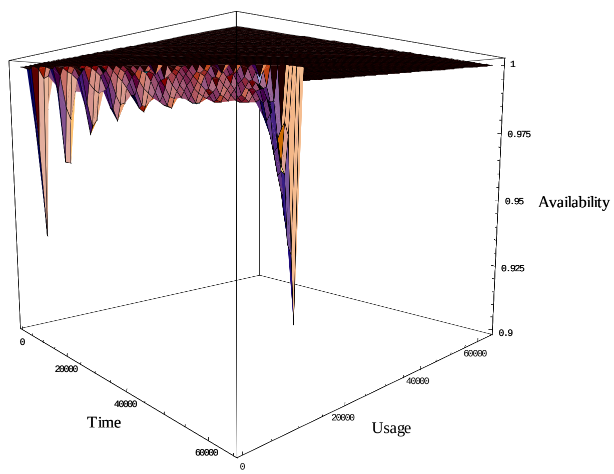

Figure 4-49 AVAILABILITY FUNCTION (METHOD 2, CASE 3, BVN)

Figure 4-50 through Figure 4-55, cases 4-6, represent the remaining parameter combinations that provide meaningful insights about the effects combining low, med, and high values for the mean on the time and usage distributions. The examples provided correspond to ρ = 0.5; charts for the remaining correlations are in Appendix B. The observations made in cases 1-3 are appropriate for cases 4-6. The only significant difference is the location of the availability function in the plane. For example, in case 5

(Figure 4-52 and Figure 4-53) (μ_T = 3,000, μ_U = 18,500), the diagram follows the same pattern as the previous cases, but the curve is shifted toward lower time values paired with higher usage values. A comparable shift occurs in Case 4 (μ_T = 3,000, μ_U = 7,400), only the degree of the shift is decreased. An analogous shift occurs in case 6, (Figure 4-54 and Figure 4-55) (μ_T = 20,000, μ_U = 3,250); however, the function is shifted toward higher time values paired with lower usage values. The result is intuitive considering the differences between the magnitude of the mean values of time and usage.

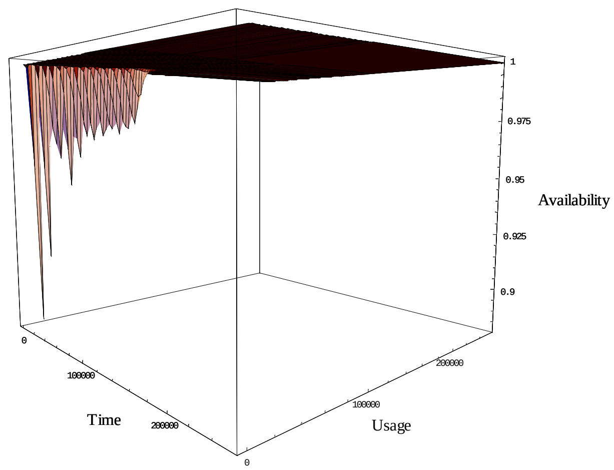

Figure 4-50 AVAILABILITY FUNCTION (METHOD 1, CASE 4, BVN)

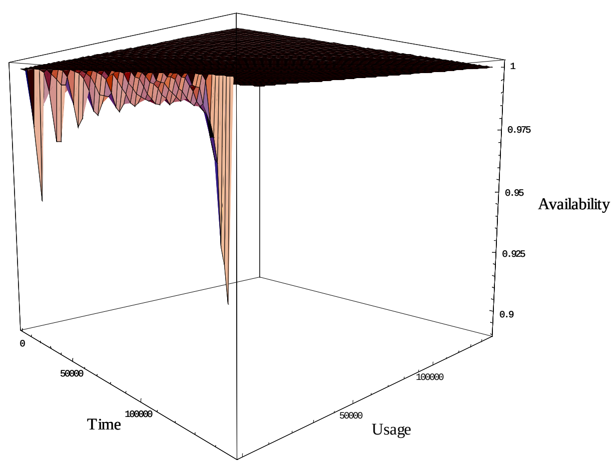

Figure 4-51 AVAILABILITY FUNCTION (METHOD 2, CASE 4, BVN)

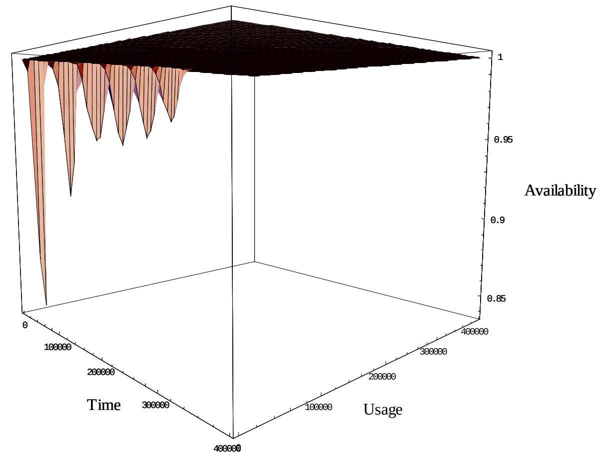

Figure 4-52 AVAILABILITY FUNCTION (METHOD 1, CASE 5, BVN)

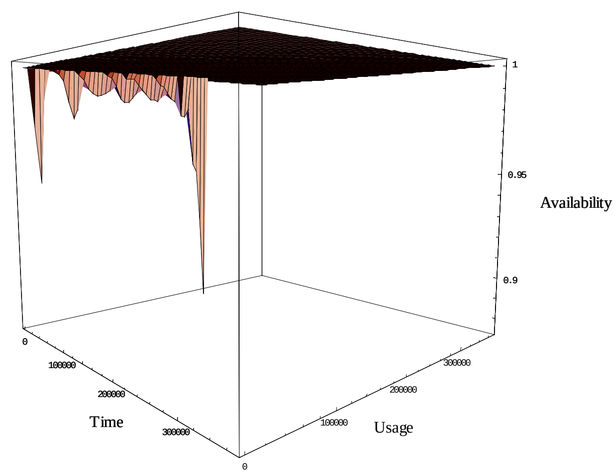

Figure 4-53 AVAILABILITY FUNCTION (METHOD 2, CASE 5, BVN)

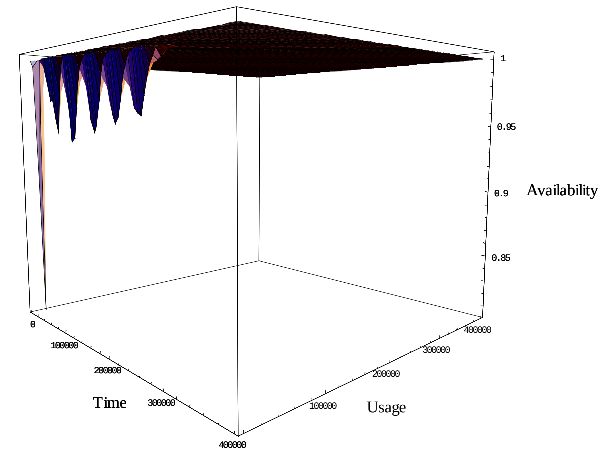

Figure 4-54 AVAILABILITY FUNCTION (METHOD 1, CASE 6, BVN)

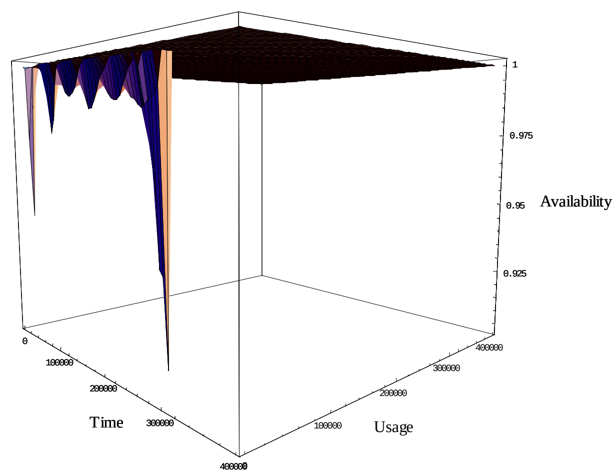

Figure 4-55 AVAILABILITY FUNCTION (METHOD 2, CASE 6, BVN)

Further analysis of the representative plots led to additional simulations runs in order to confirm the previous results. In particular, the availability values reported were higher than expected so two modifications were implemented. First, the sample population was increased from 100,000 to 500,000. The resulting curve (Figure 4-56) was smoother and more full, but the availability estimates were unchanged. The second modification entailed decreasing the observation grid in order to better estimate the availability function. The limitation in doing this was a decrease in the amount of time and usage observed. As a consequence only the early transient behavior, consistent with all other cases, is apparent. The smaller observation interval resulted in an overall increase in the availability approximations. The increase was not anticipated; however, is explained because the larger observation interval captures more instances of failure and repair. Figure 4-57 represents the availability function for case 2 with a decreased observation interval.

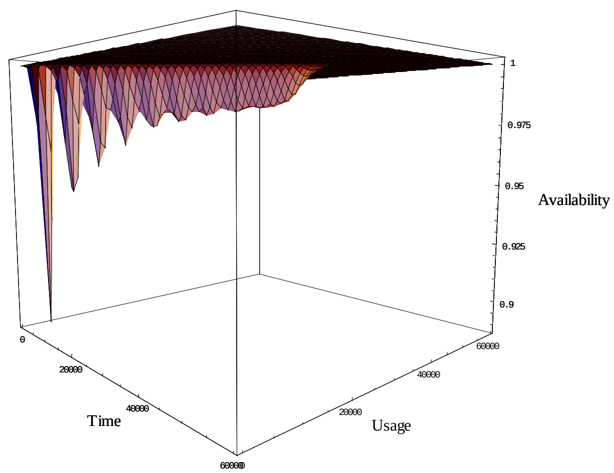

Figure 4-56 AVAILABILITY FUNCTION (N = 500,000, CASE 2, BVN)

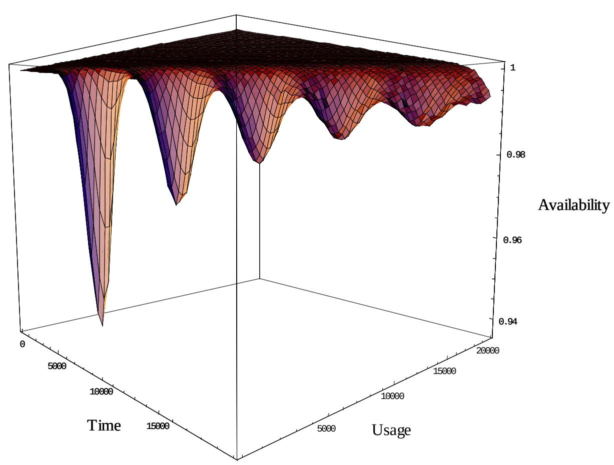

Figure 4-57 AVAILABILITY FUNCTION (DECREASED GRID, CASE 2, BVN)

4.3.3 Linear Stochastic Functional Relationship

The results for the availability function are not surprising given the underlying assumption that t and α are generated from the exponential distribution. The function demonstrates an early decrease in availability caused by predominately low failure times and small repair durations. As time and usage progress the availability continuously increases and reaches a steady state. The function is concentrated slightly off the 45° diagonal (toward lower usage values) and is not dispersed throughout the plane. The general characteristics of the availability function across parameter sets with similar means are presented in Figure 4-58 and Figure 4-59.

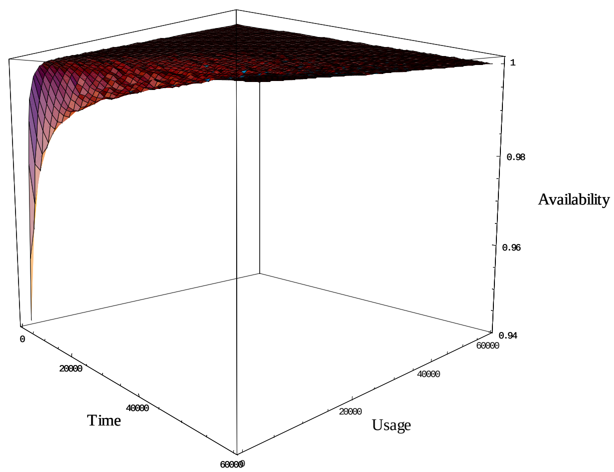

Figure 4-58 AVAILABILITY FUNCTION (METHOD 1, CASE 1, LISF)

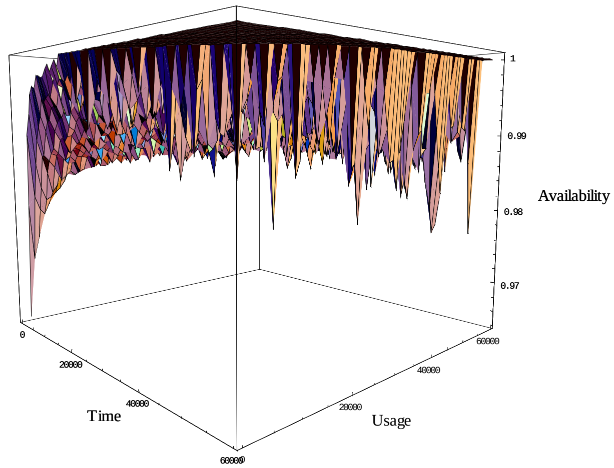

Figure 4-59 AVAILABILITY FUNCTION (METHOD 2, CASE 1, LISF)

As the constant, β, increases the general shape of the function does not change. However, the height of the valleys varies between parameter sets signifying a decrease in availability. An increase in β, from 0 to 500 produces the most significant effect on the availability function—a decrease in availability. Figure 4-60 through Figure 4-63 demonstrate the relationship for similar means and varied levels of β.

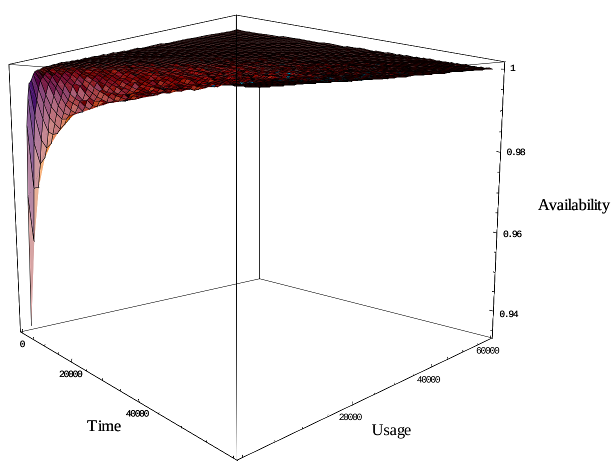

Figure 4-60 AVAILABILITY FUNCTION (METHOD 1, CASE 2, LISF)

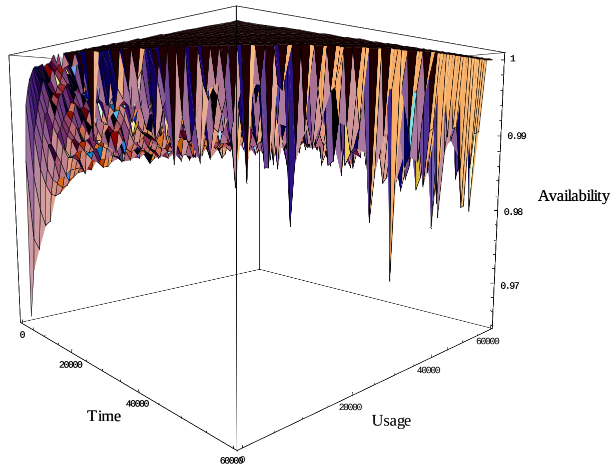

Figure 4-61 AVAILABILITY FUNCTION (METHOD 2, CASE 2, LISF)

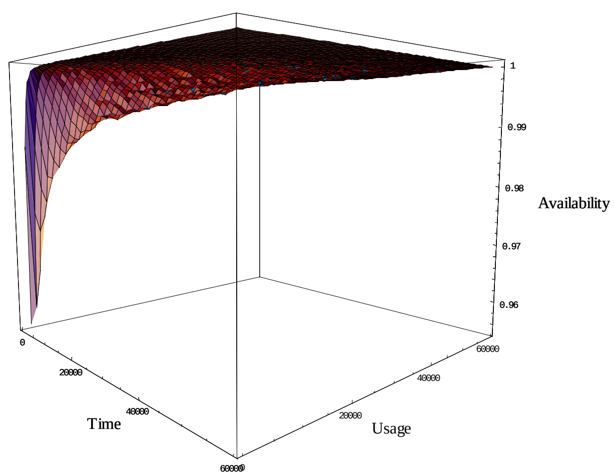

Figure 4-62 AVAILABILITY FUNCTION (METHOD 1, CASE 3, LISF)

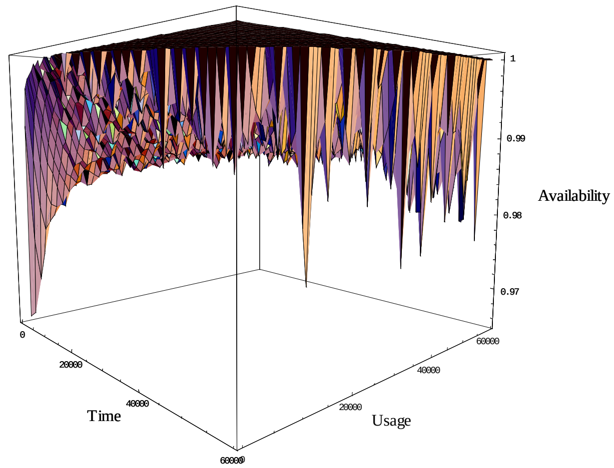

Figure 4-63 AVAILABILITY FUNCTION (METHOD 2, CASE 3, LISF)

The parameter sets including variations in the level of the mean values display behavior analogous to the previous cases; however, the function location is shifted in the plane. Figure 4-64 through Figure 4-67 represent cases with a (low mean time, medium mean usage) and a (low mean time, high mean usage), so the curves are closer to the usage axis indicating more activity for low times paired with higher usages. The opposite occurs in Case 6, Figure 4-68 through Figure 4-69, where the mean on time is much higher than the mean on use.

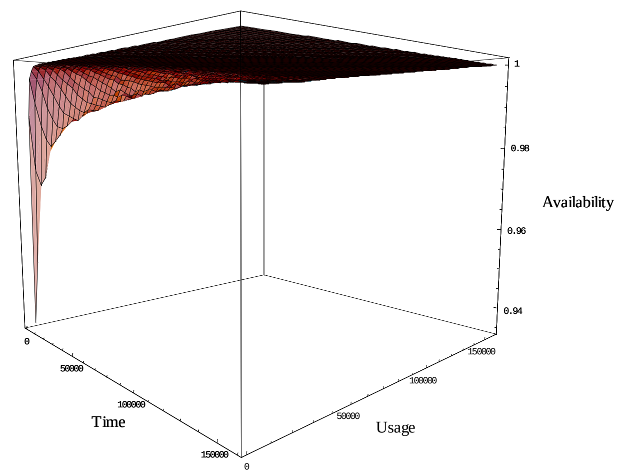

Figure 4-64 AVAILABILITY FUNCTION (METHOD 1, CASE 5, LISF)

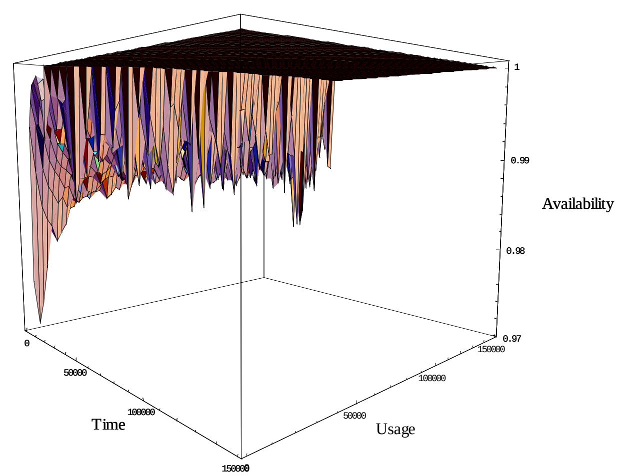

Figure 4-65 AVAILABILITY FUNCTION (METHOD 2, CASE 4, LISF)

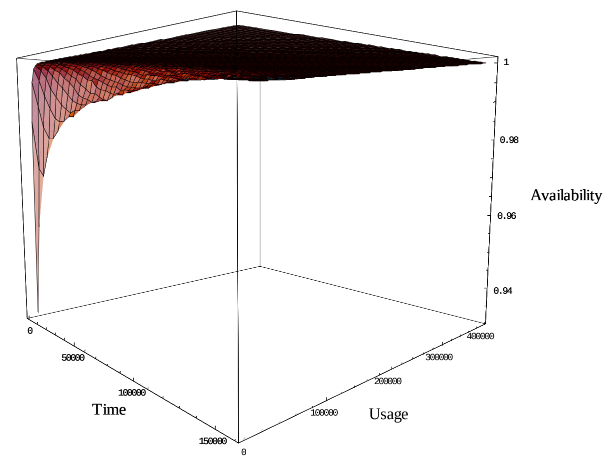

Figure 4-66 AVAILABILITY FUNCTION (METHOD 1, CASE 5, LISF)

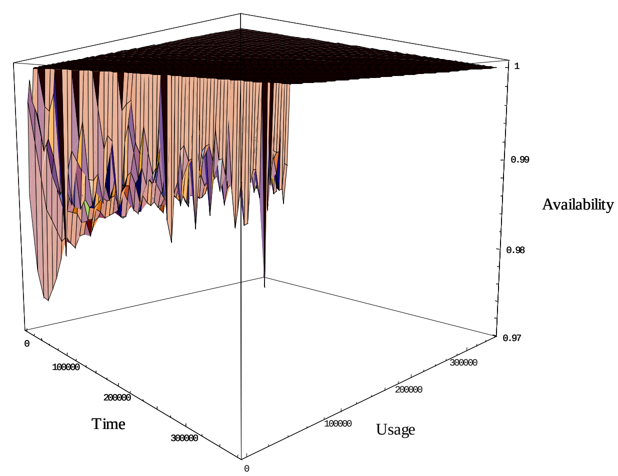

Figure 4-67 AVAILABILITY FUNCTION (METHOD 2, CASE 5, LISF)

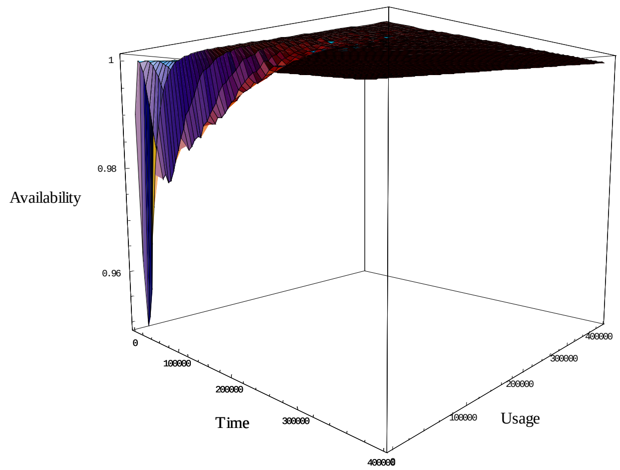

Figure 4-68 AVAILABILITY FUNCTION (METHOD 1, CASE 6, LISF)

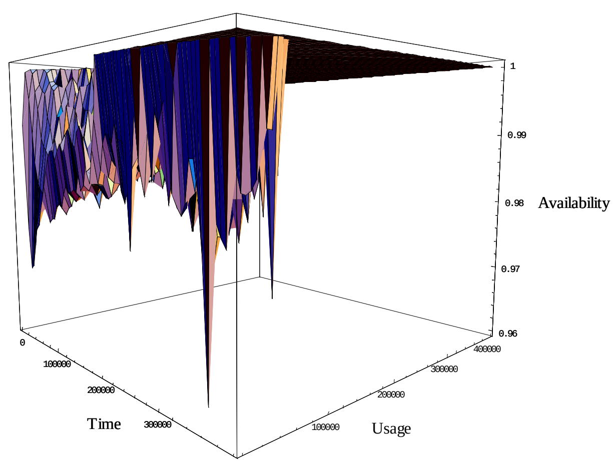

Figure 4-69 AVAILABILITY FUNCTION (METHOD 2, CASE 6, LISF)

The final two cases represent the additional tests run to verify the previous results. Few differences were realized as a result of increasing the sample size (Figure 4-70). The function is less jagged throughout the plane; otherwise, the behavior is analogous to case 2, N = 100,000 (Figure 4-60). The decreased observation grid provided a smoother curve, with slightly lower availability values. Figure 4-71 represents case 2 with a decreased observation interval.

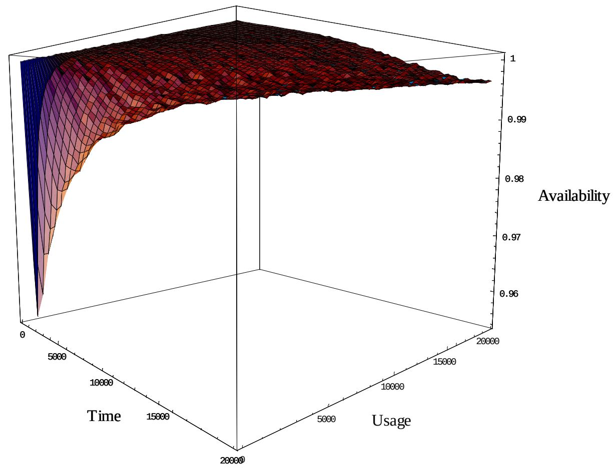

Figure 4-70 AVAILABILITY FUNCTION (N = 500,000 CASE 2, LISF)

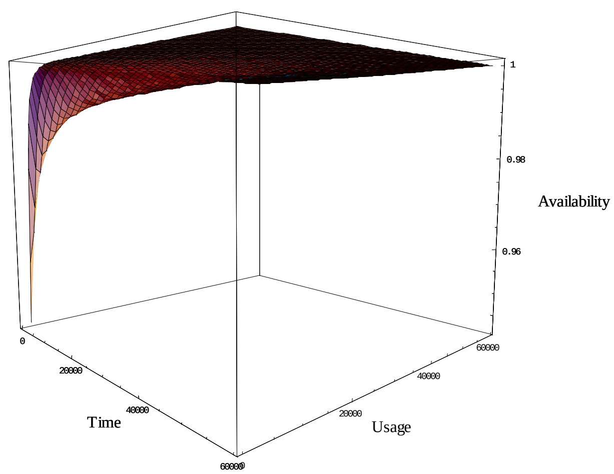

Figure 4-71 AVAILABILITY FUNCTION (DECREASED GRID, CASE 2, LISF)

4.3.4 Logistic Stochastic Functional Relationship

For both cases, the availability function for the logistic model demonstrates transient behavior throughout the plane (Figure 4-72 through Figure 4-75). In case 1, methods 1 and 2 (Figure 4-72 and Figure 4-73), show an immediate decrease in availability followed by a sharp increase. As time and use progress, the magnitude of each decrease and increase lessens, but the pattern is repeated throughout the plane. Case

2 (Figure 4-74 and Figure 4-75) differs from case 1 in that only two replications of the pattern exist, after which the system appears to reach steady state. The function is concentrated in the area of low time and use values. This is consistent with the behavior of the function.

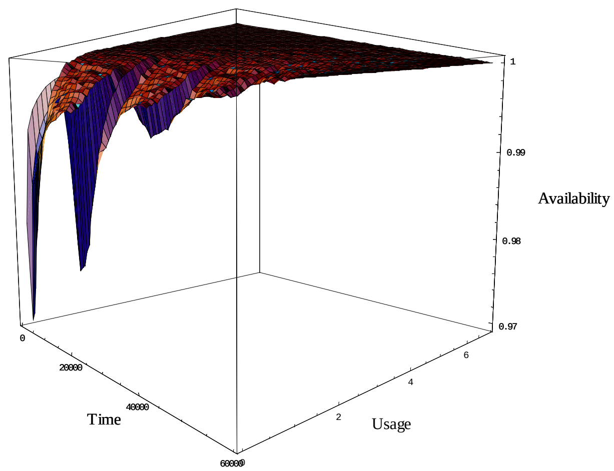

Figure 4-72 AVAILABILITY FUNCTION (METHOD 1, CASE 1, LOSF)

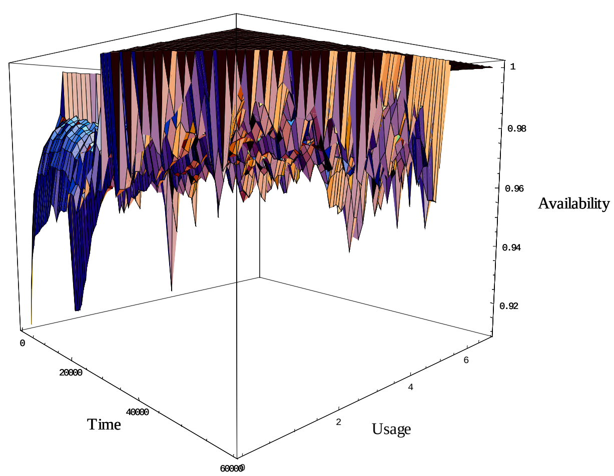

Figure 4-73 AVAILABILITY FUNCTION (METHOD 2, CASE 1, LOSF)

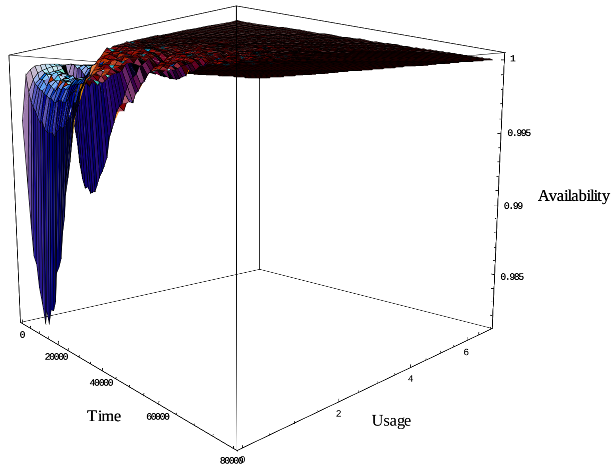

Figure 4-74 AVAILABILITY FUNCTION (METHOD 1, CASE 2, LOSF)

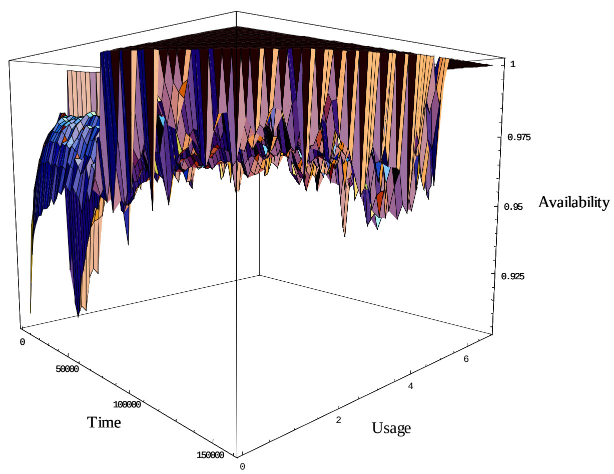

Figure 4-75 AVAILABILITY FUNCTION (METHOD 2, CASE 2, LOSF)

4.3.5 Model Comparison

In all model instances, the availability estimate appears to be overestimated, but the shape of the function represents the behavior of the model appropriately. For time and use values with a high probability of occurring, there are significant reductions in the availability function. All models exhibit this behavior; however, it is clear to see that the dispersion across observations effects the magnitude of the decrease. For instance, observations for the bivariate exponential and linear stochastic model are very similar, except a higher proportion of early failures occur with the linear stochastic model. This causes a more significant initial decrease in the availability function.

In all respects, the bivariate exponential and linear stochastic models are well correlated; each experiences an early decrease in availability, but eventually proceeds to steady state. The bivariate normal, however, exhibits oscillating behavior until very late time and usage values where it reaches a steady state. The logistic model exhibits a higher degree of transient behavior than any of the other models. The model does not appear to reach steady state. It is important to note that in order to verify the bivariate availability estimations, the univariate projection method was implemented. The results for all cases of the bivariate estimation procedures are consistent with the univariate projection, but are not included.

4.4 Summary

The research presented here is a cursory attempt at generating and analyzing bivariate failure models. A reasonably acceptable method of failure/repair generation was implemented for the correlated models and stochastic models; however, alternative methods of generation exist that may provide useful results. Using the failure/repair data, renewal function and availability function estimates were obtained.

The results provide a general idea of the behavior associated with the identified failure models. It is important to note that alternative methodologies exist, but the processes implemented here offer reasonable estimations of the two measures of system effectiveness studied — the bivariate renewal function and the bivariate availability function. Also, the behavior shown in all cases is consistent with the behavior experienced when the values are calculated by holding one variable constant (i.e. the univariate projection). Modifications of the methodologies presented here may provide additional insight on bivariate failure processes. Further study of bivariate failure models is necessary in order to more fully understand the bivariate renewal and availability functions.

Chapter 5 – Future Directions

Several opportunities exist for further study of bivariate failure processes. In particular, it is worth pursuing alternative methods of generating the stochastic functional relationships. The assumption that *t* is exponential limited the usefulness of the results for the functional models. It will be beneficial to explore methods of generation using the joint distribution function.

Another useful area of study is the method of estimating the bivariate availability function. The two methods utilized here, provide reasonable estimates for availability; however, alternative methods should be developed to further verify the results. There are still unanswered questions concerning the appropriate size of the sample population. An alternate method might consider accumulation of the area corresponding to the amount of time and use between failures and the duration (time and use) of repair activities.

The work presented here considers only instances of corrective maintenance. It will be beneficial to include preventive maintenance in future studies of bivariate availability. This information will aid in maintenance planning of systems that experience failure according to a bivariate process.

References

Baggs, G.E. and H.N. Nagaraja (1996), Reliability Properties of Order Statistics From Bivariate Exponential Distributions, *Communications in Statistics—Stochastic Models*, **12**, pp. 611-631.

Banks, J., J.S. Carson II, and B.L. Nelson (1999), *Discrete-Event System Simulation*, Prentice Hall, Inc.

Barlow, R.E. and F. Proschan (1965), *Mathematical Theory of Reliability*, John Wiley and Sons, Inc.

Barlow, R.E. and F. Proschan (1975), Statistical Theory of Reliability and Life Testing: Probability Models, Holt, Rinehart, and Winston, Inc.

Barr, D.R. and N.L. Slezak (1972), A Comparison of Multivariate Normal Generators, *Communications of the ACM*, **15**, pp. 1048-1049.

Birnbaum, Z.W. and S.C. Saunders (1969), A New Family of Life Distributions, *Journal of Applied Probability*, 6, pp. 319-327.

Block, H.W. and A.P. Basu (1974), A Continuous Bivariate Exponential Extension, *Journal of the American Statistical Association*, **69**, pp. 1031-1037.

Devroye, L. (1986), *Non-Uniform Random Variate Generation*, Springer-Verlag.

Downton, F. (1970), Bivariate Exponential Distributions in Reliability Theory, *Journal of the Royal Statistics Society, Series B*, **32**, pp. 408-417.

Eliashberg, J., N.D. Singpurwalla, and S.P. Wilson (1997), *Calculating the Reserve for a Time and Usage Indexed Warranty, Management Science*, **43**(7), pp. 966-975.

Gertsbakh, I.B. (1989), *Statistical Reliability Theory*, Marcel Dekker.

Goldman, A.S. and T.B. Slattery, (1964), *Maintainability*, John Wiley and Sons, Inc.

Hunter, J.J. (1974a), Renewal Theory in Two Dimensions: Asymptotic Results, *Advances in Applied Probability*, **6**, pp. 546-562.

Hunter, J.J. (1974b), Renewal Theory in Two Dimensions: Basic Results, *Advances in Applied Probability*, **6**, pp. 376-391.

Johnson, M. E. (1987), *Statistical Simulation*, John Wiley and Sons, Inc.

Law, A.M. and W.D. Kelton (1991), *Simulation and Modeling Analysis*, Mc-Graw Hill, Inc.

Lemoine, A.J. and M.L. Wenocur. (1985), On Failure Modeling, *Naval Research Logistics Quarterly*, **32**, pp. 497-508.

Lie, C.H., C.L. Hwang, and F.A. Tillman (1977), Availability of Maintained Systems: A-State-of-the-Art-Survey, *AIIE Transactions*, **9**(3), pp. 247-259.

Marshall, A.W. and I. Olkin (1976a), A Multivariate Exponential Distribution, *Journal of the American Statistical Association*, **62**, pp. 30-44.

Marshall, A.W. and I. Olkin (1976b), A Generalized Bivariate Exponential Distribution, *Journal of Applied Probability*, **4**, pp. 291-302

Mercer, A. (1961), Some Simple Wear-dependent Renewal Processes, *Journal of the Royal Statistical Society, Series B*, **23**, pp. 368-376.

Nachlas, J.A. (1998), *Introduction to Reliability Theory*, unpublished manuscript, Virginia Tech

Scheuer, E.M. and D.S. Stoller (1962), On the Generation of Normal Random Vectors, *Technometrics*, **4**, pp. 278-281.

Singurwalla, N.D. and S. Wilson (1993), The Warranty Problem: Its Statistical and Game Theoretic Aspects, *SIAM Reviews*, 35, pp. 17-42.

Singurwalla, N.D. and S. Wilson (1998), Failure Models Indexed by Two Scales, *Advances in Applied Probability*, **30**, pp.1058-1072.

Yang, S. (1999), *A Bivariate Renewal Process and Its Applications in Maintenance Policies*, Ph.D. Dissertation, Department of Industrial and Systems Engineering, Virginia Tech.

Yang, S. (2000), Bivariate Failure Modeling, Proceedings: *Annual Reliability and Maintainability Symposium*, pp. 281-287

Appendix A – Simulation Code

In this Appendix, the simulation code used in Chapter 3 is introduced. The simulation code is used to generate failure/repair data and is used to collect statistics regarding reliability and availability. The first four sections of this Appendix represent the simulation code used to generate failure and repair data for the four bivariate failure models. The last six sections include the code used to record the statistics necessary to estimate reliability and availability. The estimation procedures included here reflect the bivariate normal model. The same code is used for the other three models by substituting the appropriate alternate generation method (Appendix A.1-A.3).

A.1 Linear Dependence Data Generation Procedure

```
%This is the subroutine that generates the linear dependence data.

%%%%VARIABLE DEFINITION%%%%
%meam=mean on the failure distribution (time)%
%meanalpha=mean on the distribution of alpha (failure)%
%beta=constant for functional relationship%
%cmmean=mean on the repair distribution (time)%
%cmmeanalpha=mean on the distribution of alpha (repair)%
%xt(j)=time elapsed since the last failure (time)%
%xu(j)=usage elapsed since the last failure (use)%
%cmxt(j)=time to complete repair (j)%
%cmxu(j)=usage to complete repair (j)%
%failuretimes=vector of failure times and usages%
%cmtimes=vector of repair times and usages%
%parameters=matrix of failure model parameters%
%cmparameters=matrix of repair model parameters%

parameters=[3000 1 0; 3000 1 250; 3000 1 500; 3000 2.5 0; 3000 2.5 250;
3000 2.5 500; 8000 .4 0; 8000 .4 250; 8000 .4 500; 8000 1 0; 8000 1
250; 8000 1 500; 3000 6 0; 3000 6 250; 3000 6 500; 8000 2 0; 8000 2
250; 8000 2 500; 20000 .15 0; 20000 .15 250; 20000 .15 500; 20000 .35
0; 20000 .35 250; 20000 .35 500; 20000 1 0; 20000 1 250; 20000 1 500];
cmparameters=parameters.*0.1;



%this loop sets the program to read the 27 parameters sets%
for h=1:1:27

   mean=parameters(h,1);
   meanalpha=parameters(h,2);
   beta=parameters(h,3);
```

```
   cmmean=cmparameters(h,1);
   cmmeanalpha=parameters(h,2);
   cmbeta=cmparameters(h,3);

   %this loop generates 10 failure/repair vectors (not cumulative)%
   for j=1:1:10

      z1=rand;
      z2=rand;

      xt(j)=(-mean)*log(1-z1);
      at=xt(j);
      alpha=(-meanalpha)*log(1-z2);
      xu(j)=(alpha*at)+beta;

      first=xt(j);
      second=xu(j);

      rowvector =[first,second];
      bigvector(j,:)=rowvector;
      failuretimes=bigvector;

      cmz1=rand;
      cmz2=rand;
      cmxt(j)=(-cmmean)*log(1-cmz1);
      cmt=cmxt(j);

      cmalpha=(-cmmeanalpha)*log(1-cmz2);
      cmxu(j)=(cmalpha*cmt)+cmbeta;

      cmfirst=cmxt(j);
      cmsecond=cmxu(j);

      cmrowvector =[cmfirst,cmsecond];
      cmbigvector(j,:)=cmrowvector;
      cmtimes=cmbigvector;
   end
end
```

A.2 Logistic Dependence Data Generation Procedure

```
%This is the subroutine that generates the linear dependence data.

%%%%VARIABLE DEFINITION%%%%
%meam=mean on the failure distribution (time)%
%meanalpha=mean on the distribution of alpha (failure)%
%beta=constant for functional relationship%
%cmmean=mean on the repair distribution (time)%
%cmmeanalpha=mean on the distribution of alpha (repair)%
%xt(j)=time elapsed since the last failure (time)%
%xu(j)=usage elapsed since the last failure (use)%
%cmxt(j)=time to complete repair (j)%
```

```
%cmxu(j)=usage to complete repair (j)%
%failuretimes=vector of failure times and usages%
%cmtimes=vector of repair times and usages%
%parameters=matrix of failure model parameters%
%cmparameters=matrix of repair model parameters%

parameters=[3000 0.0024 1511; 8000 0.0092 1511]
cmparameters=parameters.*0.1;

%this loop sets the program to read the 27 parameters sets%
for h=1:1:27

   mean=parameters(h,1);
   meanalpha=parameters(h,2);
   beta=parameters(h,3);

   cmmean=cmparameters(h,1);
   cmmeanalpha=parameters(h,2);
   cmbeta=cmparameters(h,3);

   %this loop generates 10 failure/repair vectors (not cumulative)%
   for j=1:1:10

      z1=rand;
      z2=rand;

      xt(j)=(-mean)*log(1-z1);
      at=xt(j);
      alpha=(-meanalpha)*log(1-z2);
      top=exp(alpha*at)-1;
      bottom=exp(alpha*at)+beta;
      xu(j)=top/bottom;


      first=xt(j);
      second=xu(j);

      rowvector =[first,second];
      bigvector(j,:)=rowvector;
      failuretimes=bigvector;

      cmz1=rand;
      cmz2=rand;
      cmxt(j)=(-cmmean)*log(1-cmz1);
      cmt=cmxt(j);

      cmalpha=(-cmmeanalpha)*log(1-cmz2);
      cmxu(j)=(cmalpha*cmt)+cmbeta;

      cmfirst=cmxt(j);
      cmsecond=cmxu(j);

      cmrowvector =[cmfirst,cmsecond];
      cmbigvector(j,:)=cmrowvector;
      cmtimes=cmbigvector;
   end
end
```

A.3 Bivariate Exponential Data Generation Procedure

```
%This is the subroutine that generates the bivariate exponential data.

%%%%VARIABLE DEFINITION%%%%
%mutime=mean on the failure distribution (time)%
%muuse=mean on the failure distribution (use)%
%rhovec=correlation coefficient%
%cmmutime=mean on the repair distribution (time)%
%cmmuuse=mean on the repair distribution (use)%
%xt(j)=time elapsed since the last failure (time)%
%xu(j)=usage elapsed since the last failure (use)%
%cmxt(j)=time to complete repair (j)%
%cmxu(j)=usage to complete repair (j)%
%failuretimes=vector of failure times and usages%
%cmtimes=vector of repair times and usages%
%parameters=matrix of failure model parameters%
%cmparameters=matrix of repair model parameters%

parameters=[3000 3250 0.2; 3000 3250 0.5; 3000 3250 0.8; 3000 7400 0.2;
3000 7400 0.5; 3000 7400 0.8; 8000 3250 0.2; 8000 3250 0.5; 8000 3250
0.8; 8000 7400 0.2; 8000 7400 0.5; 8000 7400 0.8; 3000 18500 0.2; 3000
18500 0.5; 3000 18500 0.8; 8000 18500 0.2; 8000 18500 0.5; 8000 18500
0.8; 20000 3250 0.2; 20000 3250 0.5; 20000 3250 0.8; 20000 7400 0.2;
20000 7400 0.5; 20000 7400 0.8; 20000 18500 0.2; 20000 18500 0.5; 20000
18500 0.8];
cmparameters=parameters.*0.1;



%this loop sets the program to read the 27 parameters sets%
for h=1:1:27

   mutime=parameters(h,1);
   muuse=parameters(h,2);
   rhovec=parameters(h,3);

   cmmutime=cmparameters(h,1);
   cmmuuse=cmparameters(h,2);

   %this loop generates 10 failure/repair vectors (not cumulative)%
   for j=1:1:10

      z1=rand;
      z2=rand;
      xt(j)=(-mutime)*log(1-z1);
      t=xt(j);
      c1=-1-rhovec+(2*rhovec*exp((-1/mutime)*t));
      c2=rhovec-(2*rhovec*exp((-1/mutime)*t));
      a2=(-c1-sqrt((c1.^2)-(4*c2*z2)))/(2*c2);
      xu(j)=(-muuse)*log(a2);

      first=xt(j);
      second=xu(j);

      rowvector =[first,second];
```

```
      bigvector(j,:)=rowvector;
      failuretimes=bigvector;

      cmz1=rand;
      cmz2=rand;
      cmxt(j)=(-cmmutime)*log(1-cmz1);
      cmt=cmxt(j);
      cmc1=-1-rhovec+(2*rhovec*exp((-1/cmmutime)*cmt));
      cmc2=rhovec-(2*rhovec*exp((-1/cmmutime)*cmt));
      cma2=(-cmc1-sqrt((cmc1.^2)-(4*cmc2*cmz2)))/(2*cmc2);
      cmxu(j)=(-cmmuuse)*log(cma2);

      cmfirst=cmxt(j);
      cmsecond=cmxu(j);

      cmrowvector =[cmfirst,cmsecond];
      cmbigvector(j,:)=cmrowvector;
      cmtimes=cmbigvector;
   end
end
```

A.4 Bivariate Normal Data Generation Procedure

```
%This is the subroutine that generates the bivariate normal data.

%%%%VARIABLE DEFINITION%%%%
%mutime=mean on the failure distribution (time)%
%muuse=mean on the failure distribution (use)%
%sigtime=standard deviation on failure (time)%
%siguse=standard deviation on failure (use)%
%rhovec=correlation coefficient%
%cmmutime=mean on the repair distribution (time)%
%cmmuuse=mean on the repair distribution (use)%
%cmsigtime=standard deviation on repair (time)%
%cmsiguse=standard deviation on repair (use)%
%xt(j)=time elapsed since the last failure (time)%
%xu(j)=usage elapsed since the last failure (use)%
%cmxt(j)=time to complete repair (j)%
%cmxu(j)=usage to complete repair (j)%
%failuretimes=vector of failure times and usages%
%cmtimes=vector of repair times and usages%
%parameters=matrix of failure model parameters%
%cmparameters=matrix of repair model parameters%

parameters=[3000 3250 600 650 0.2; 3000 3250 600 650 0.5; 3000 3250 600
650 0.8; 3000 7400 600 1480 0.2; 3000 7400 600 1480 0.5; 3000 7400 600
1480 0.8; 8000 3250 1600 650 0.2; 8000 3250 1600 650 0.5; 8000 3250
1600 650 0.8; 8000 7400 1600 1480 0.2; 8000 7400 1600 1480 0.5; 8000
7400 1600 1480 0.8; 3000 18500 600 3700 0.2; 3000 18500 600 3700 0.5;
3000 18500 600 3700 0.8; 8000 18500 1600 3700 0.2; 8000 18500 1600 3700
0.5; 8000 18500 1600 3700 0.8; 20000 3250 4000 650 0.2; 20000 3250 4000
650 0.5; 20000 3250 4000 650 0.8; 20000 7400 4000 1480 0.2; 20000 7400
4000 1480 0.5; 20000 7400 4000 1480 0.8; 20000 18500 4000 3700 0.2;
20000 18500 4000 3700 0.5; 20000 18500 4000 3700 0.8];
cmparameters=parameters.*0.1;
```

```
%this loop sets the program to read the 27 parameters sets%
for h=1:1:27

   mutime=parameters(h,1);
   muuse=parameters(h,2);
   sigtime=parameters(h,3);
   siguse=parameters(h,4);
   rhovec=parameters(h,5);

   cmmutime=cmparameters(h,1);
   cmmuuse=cmparameters(h,2);
   cmsigtime=cmparameters(h,3);
   cmsiguse=cmparameters(h,4);

   squarerho=sqrt(1-rhovec.^2);

   %this loop generates 10 failure/repair vectors (not cumulative)%

   for j=1:1:10

      z1=randn;
      z2=randn;

      cmz1=randn;
      cmz2=randn;

      xt(j)=mutime+(sigtime*z1);
      squarerho=sqrt(1-rhovec.^2);
      xu(j)=muuse+(siguse*(rhovec*z1+squarerho*z2));
      first=xt(j);
      second=xu(j);

      rowvector =[first,second];
      bigvector(j,:)=rowvector;
      failuretimes=bigvector;

      cmxt(j)=cmmutime+(cmsigtime*cmz1);
      cmxu(j)=cmmuuse+(cmsiguse*(rhovec*cmz1+squarerho*cmz2));
      cmfirst=cmxt(j);
      cmsecond=cmxu(j);

      cmrowvector =[cmfirst,cmsecond];
      cmbigvector(j,:)=cmrowvector;
      cmtimes=cmbigvector;
   end
end
```

A.5 Bivariate Renewal Estimation

```
%This program estimates the bivariate renewal function (method 1).  The
program is shown for the bivariate normal distribution; for alternate
distribution, substitute alternate generation procedure.%

%%%%VARIABLE DEFINITIONS%%%%
%parameters=failure model parameters%
%renewmat=renewal matrix%
%locationt=matrix location of failure (time)%
%locationu=matrix location of failure (use)%
%wheret(q)=vector of failure (time) matrix locations%
%whereu(q)=vector of failure (use) matrix locations%
%matfailloct=matrix location of failure(m) (time)%
%matfaillocu=matrix location of failure(m)(use)%

parameters=[3000 3250 600 650 0.2; 3000 3250 600 650 0.5; 3000 3250 600
650 0.8; 3000 7400 600 1480 0.2; 3000 7400 600 1480 0.5; 3000 7400 600
1480 0.8; 8000 3250 1600 650 0.2; 8000 3250 1600 650 0.5; 8000 3250
1600 650 0.8; 8000 7400 1600 1480 0.2; 8000 7400 1600 1480 0.5; 8000
7400 1600 1480 0.8; 3000 18500 600 3700 0.2; 3000 18500 600 3700 0.5;
3000 18500 600 3700 0.8; 8000 18500 1600 3700 0.2; 8000 18500 1600 3700
0.5; 8000 18500 1600 3700 0.8; 20000 3250 4000 650 0.2; 20000 3250 4000
650 0.5; 20000 3250 4000 650 0.8; 20000 7400 4000 1480 0.2; 20000 7400
4000 1480 0.5; 20000 7400 4000 1480 0.8; 20000 18500 4000 3700 0.2;
20000 18500 4000 3700 0.5; 20000 18500 4000 3700 0.8];

%the observation grid is input here and varies based on the mean on the
failure distribution (time)%
renewmat=dlmread('renewmat.csv',',');

%initialize placeholder for failure location vector (time and use)%
wheret(11)=203;
whereu(11)=203;

for h=1:1:27

   mutime=parameters(h,1);
   muuse=parameters(h,2);
   sigtime=parameters(h,3);
   siguse=parameters(h,4);
   rhovec=parameters(h,5);

   cmmutime=cmparameters(h,1);
   cmmuuse=cmparameters(h,2);
   cmsigtime=cmparameters(h,3);
   cmsiguse=cmparameters(h,4);

   squarerho=sqrt(1-rhovec.^2);

   %this loop generates data for 10000 machines%

   for p=1:1:1000

      %this part of the program generates 10 random failure vectors for
```

```
each machine

for j=1:1:10

   z1=randn;
   z2=randn;

   cmz1=randn;
   cmz2=randn;

   xt(j)=mutime+(sigtime*z1);
   squarerho=sqrt(1-rhovec.^2);
   xu(j)=muuse+(siguse*(rhovec*z1+squarerho*z2));
   first=xt(j);
   second=xu(j);

   rowvector =[first,second];
   bigvector(j,:)=rowvector;

   failuretimes=bigvector;

   cmxt(j)=cmmutime+(cmsigtime*cmz1);
   cmxu(j)=cmmuuse+(cmsiguse*(rhovec*cmz1+squarerho*cmz2));
   cmfirst=cmxt(j);
   cmsecond=cmxu(j);

   cmrowvector =[cmfirst,cmsecond];
   cmbigvector(j,:)=cmrowvector;
   cmtimes=cmbigvector;

   N=length(failuretimes(:,1));

   N=length(failuretimes(:,1));
   sumvector(1,:)=failuretimes(1,:);
   dummy=failuretimes(1,:);

   %this loop generates the cumulative failure vector%

   for k=2:1:N
      sumvector(k,:)=dummy + failuretimes (k,:);
      dummy=sumvector(k,:);
   end
end

locationt=2;
locationu=2;

for q=1:1:10

   %this matrix steps through each observation time to find the
   matrix location of failure(q), beginning with the matrix
   location of failure(q-1)%

   for t=locationt:1:201
      if sumvector(q,1)<=renewmat(1,t)
         locationt=t;
         wheret(q)=locationt;
```

```
            break
         end
      end

      %this matrix steps through each observation use to find the
      matrix location of failure(q), beginning with the matrix
      location of failure(q-1)%

      for s=locationu:1:201
         if sumvector(q,2)<=renewmat(s,1)
            locationu=s;
            whereu(q)=locationu;
            break
         end
      end
   end

   for m=1:1:10
      matfailloct=wheret(m);
      matfaillocu=whereu(m);

      %this loop updates the renewal matrix to reflect the
      cumulative number of renewals.%

      for t=matfailloct:1:201
         for s=matfaillocu:1:201
            renewmat(s,t)=renewmat(s,t)+1;
         end
      end
   end
 end
end
```

A.6 Univariate Projection Method

```
%%%%VARIABLE DEFINITIONS%%%%
%parameters=failure model parameters%
%renewalvectort=vector of cumulative renewals (time)%
%renewalvectoru=vector of cumulative renewals (use)%
%locationt=matrix location of failure (time)%
%locationu=matrix location of failure (use)%
%wheret(q)=vector of failure (time) matrix locations%
%whereu(q)=vector of failure (use) matrix locations%
%matfailloct=matrix location of failure(m) (time)%
%matfaillocu=matrix location of failure(m)(use)%

parameters=[3000 3250 600 650 0.2; 3000 3250 600 650 0.5; 3000 3250 600
650 0.8; 3000 7400 600 1480 0.2; 3000 7400 600 1480 0.5; 3000 7400 600
1480 0.8; 8000 3250 1600 650 0.2; 8000 3250 1600 650 0.5; 8000 3250
1600 650 0.8; 8000 7400 1600 1480 0.2; 8000 7400 1600 1480 0.5; 8000
7400 1600 1480 0.8; 3000 18500 600 3700 0.2; 3000 18500 600 3700 0.5;
3000 18500 600 3700 0.8; 8000 18500 1600 3700 0.2; 8000 18500 1600 3700
0.5; 8000 18500 1600 3700 0.8; 20000 3250 4000 650 0.2; 20000 3250 4000
650 0.5; 20000 3250 4000 650 0.8; 20000 7400 4000 1480 0.2; 20000 7400
```

```
4000 1480 0.5; 20000 7400 4000 1480 0.8; 20000 18500 4000 3700 0.2;
20000 18500 4000 3700 0.5; 20000 18500 4000 3700 0.8];

%the observation grid is input here and varies based on the mean on the
failure distribution (time)%
renewmat=dlmread('renewmat.csv',',');

%initialize placeholder for failure location vector (time and use)%
wheret(11)=203;
whereu(11)=203;

for h=1:1:27
   renewalvectort=zeros([201,1]);
   renewalvectoru=zeros([201,1]);
   mutime=parameters(h,1);
   muuse=parameters(h,2);
   sigtime=parameters(h,3);
   siguse=parameters(h,4);
   rhovec=parameters(h,5);

   cmmutime=cmparameters(h,1);
   cmmuuse=cmparameters(h,2);
   cmsigtime=cmparameters(h,3);
   cmsiguse=cmparameters(h,4);

   squarerho=sqrt(1-rhovec.^2);

   %this loop generates data for 10000 machines%

   for p=1:1:1000

      %this part of the program generates 10 random failure vectors for
      each machine

      for j=1:1:10

         z1=randn;
         z2=randn;

         cmz1=randn;
         cmz2=randn;

         xt(j)=mutime+(sigtime*z1);
         squarerho=sqrt(1-rhovec.^2);
         xu(j)=muuse+(siguse*(rhovec*z1+squarerho*z2));
         first=xt(j);
         second=xu(j);

         rowvector =[first,second];
         bigvector(j,:)=rowvector;

         failuretimes=bigvector;

         cmxt(j)=cmmutime+(cmsigtime*cmz1);
         cmxu(j)=cmmuuse+(cmsiguse*(rhovec*cmz1+squarerho*cmz2));
         cmfirst=cmxt(j);
         cmsecond=cmxu(j);
```

```
   cmrowvector =[cmfirst,cmsecond];
   cmbigvector(j,:)=cmrowvector;
   cmtimes=cmbigvector;

   N=length(failuretimes(:,1));

   N=length(failuretimes(:,1));
   sumvector(1,:)=failuretimes(1,:);
   dummy=failuretimes(1,:);

   %this loop generates the cumulative failure vector%

   for k=2:1:N
      sumvector(k,:)=dummy + failuretimes (k,:);
      dummy=sumvector(k,:);
   end
end

locationt=2;
locationu=2;

for q=1:1:10

   %this matrix steps through each observation time to find the
   matrix location of failure(q), beginning with the matrix
   location of failure(q-1)%

   for t=locationt:1:201
      if sumvector(q,1)<=renewmat(1,t)
         locationt=t;
         wheret(q)=locationt;
         break
      end
   end

   %this matrix steps through each observation use to find the
   matrix location of failure(q), beginning with the matrix
   location of failure(q-1)%

   for s=locationu:1:201
      if sumvector(q,2)<=renewmat(s,1)
         locationu=s;
         whereu(q)=locationu;
         break
      end
   end
end

for m=1:1:10
   matfailloct=wheret(m);
   matfaillocu=whereu(m);

   %this loop updates the renewal matrix to reflect the
   cumulative number of renewals.%
```

```
         for t=matfailloct:1:201
            renewalvectort(t,1)=renewalvectort(t,1)+1;
         end

         for s=matfaillocu:1:201
           renewalvectoru(s,1)=renewalvectoru(s,1)+1;
         end

         end
      end
   end
end
```

A.7 Bivariate Availability Estimation (Method 1)

```
%This program estimates the bivariate availability function (method 1).
The program is shown for the bivariate normal distribution; for
alternate distribution, substitute alternate generation procedure.%

%%%%VARIABLE DEFINITIONS%%%%
%parameters=failure model parameters%
%cmparameters=repair model parameters%
%workmat=availability matrix%
%failloct=matrix location of failure (time)%
%faillocu=matrix location of failure (use)%
%reploct=matrix location of repair (time)%
%replocu=matrix location of repair(use)%
%failwheret(q)=vector of failure (time) matrix locations%
%failwhereu(q)=vector of failure (use) matrix locations%
%repwheret(q)=vector of repair completion (time) matrix locations%
%repwheres(q)=vector of repair completion (use) matrix locations%
%matlocfailt=matrix location of failure(m) (time)%
%matlocfailu=matrix location of failure(m) (use)%
%matlocrept=matrix location of repair completion(m) (time)%
%matlocrepu=matrix location of repair completions(m) (use)%
%tlast=last time update%
%ulast=last use update%

parameters=[3000 3250 600 650 0.2; 3000 3250 600 650 0.5; 3000 3250 600
650 0.8; 3000 7400 600 1480 0.2; 3000 7400 600 1480 0.5; 3000 7400 600
1480 0.8; 8000 3250 1600 650 0.2; 8000 3250 1600 650 0.5; 8000 3250
1600 650 0.8; 8000 7400 1600 1480 0.2; 8000 7400 1600 1480 0.5; 8000
7400 1600 1480 0.8; 3000 18500 600 3700 0.2; 3000 18500 600 3700 0.5;
3000 18500 600 3700 0.8; 8000 18500 1600 3700 0.2; 8000 18500 1600 3700
0.5; 8000 18500 1600 3700 0.8; 20000 3250 4000 650 0.2; 20000 3250 4000
650 0.5; 20000 3250 4000 650 0.8; 20000 7400 4000 1480 0.2; 20000 7400
4000 1480 0.5; 20000 7400 4000 1480 0.8; 20000 18500 4000 3700 0.2;
20000 18500 4000 3700 0.5; 20000 18500 4000 3700 0.8];
cmparameters=parameters.*0.1;

%the observation grid is input here and varies based on the mean on the
failure distribution (time)%

workmat=dlmread('workmat.csv',',');
workmatd=workmat;
```

```
for h=1:1:27

   mutime=parameters(h,1);
   muuse=parameters(h,2);
   sigtime=parameters(h,3);
   siguse=parameters(h,4);
   rhovec=parameters(h,5);

   cmmutime=cmparameters(h,1);
   cmmuuse=cmparameters(h,2);
   cmsigtime=cmparameters(h,3);
   cmsiguse=cmparameters(h,4);

   squarerho=sqrt(1-rhovec.^2);

   %this loop generates data for 100,000 machines%
   for p=1:1:100000

      %this part of the program generates 10 random failure vectors for
      each machine

      for j=1:1:10

         z1=randn;
         z2=randn;

         cmz1=randn;
         cmz2=randn;

         xt(j)=mutime+(sigtime*z1);
         squarerho=sqrt(1-rhovec.^2);
         xu(j)=muuse+(siguse*(rhovec*z1+squarerho*z2));
         first=xt(j);
         second=xu(j);

         rowvector =[first,second];
         bigvector(j,:)=rowvector;

         failuretimes=bigvector;

         cmxt(j)=cmmutime+(cmsigtime*cmz1);
         cmxu(j)=cmmuuse+(cmsiguse*(rhovec*cmz1+squarerho*cmz2));
         cmfirst=cmxt(j);
         cmsecond=cmxu(j);

         cmrowvector =[cmfirst,cmsecond];
         cmbigvector(j,:)=cmrowvector;
         cmtimes=cmbigvector;

         N=length(failuretimes(:,1));

         failurevector(1,:)=failuretimes(1,:);
         cmvector(1,:)=failuretimes(1,:)+cmtimes(1,:);
         dummycm=cmvector(1,:);

         %this loop generates the cumulative failure/repair vectors%
```

```
   for g=2:1:N
      failurevector(g,:)=failuretimes(g,:)+dummycm;
      cmvector(g,:)=failurevector(g,:)+cmtimes(g,:);
      dummycm=cmvector(g,:);
   end
end

failloct=2;
reploct=2;
faillocu=2;
replocu=2;

for q=1:1:10

   %this loop steps through each observation time to find the
   matrix location of failure(q), beginning with the matrix
   location of repair completion(q-1).%
   for t=reploct:1:201
      if failurevector(q,1)<=workmat(1,t)
         failloct=t;
         failwheret(q)=failloct;
         break
      end
   end

   %this loop steps through each observation time to find the
   matrix location of repair completion(q), beginning with the
   matrix location of failure(q-1).%
   for ts=failloct:1:201
      if cmvector(q,1)<=workmat(1,ts)
         reploct=ts;
         repwheret(q)=reploct;
         break
      end
   end

   %this loop steps through each observation usage to find the
   matrix location of failure(q), beginning with the matrix
   location of repair completion(q-1).%
   for s=replocu:1:201
      if failurevector(q,2)<=workmat(s,1)
         faillocu=s;
         failwhereu(q)=faillocu;
         break
      end
   end

   %this loop steps through each observation use to find the
   matrix location of failure(q), beginning with the matrix
   location of failure(q-1).%
   for st=faillocu:1:201
      if cmvector(q,2)<=workmat(st,1)
         replocu=st;
         repwhereu(q)=replocu;
         break
      end
   end
```

```
      end

      for m=1:1:10

         matlocfailt=failwheret(m);
         matlocfailu=failwhereu(m);
         matlocrept=repwheret(m);
         matlocrepu=repwhereu(m);

         %this loop updates the availability matrix based on the
         failure/repair matrix location vectors found in the previous
         step.%
         for t=matlocfailt:1:matlocrept

            for s=matlocfailu:1:matlocrepu
               workmat(s,t)=workmat(s,t)-1;
               tlast=matlocrept;
               ulast=matlocrepu;
            end
         end
         workmat(ulast,tlast)=workmat(ulast,tlast)+1;
      end
   end
end
```

A.8 Bivariate Availability Estimation (Method 2)

```
%This program estimates the bivariate availability function (method 2).
The program is shown for the bivariate normal distribution; for
alternate distribution, substitute alternate generation procedure.%

%%%%VARIABLE DEFINITIONS%%%%
%parameters=failure model parameters%
%mumax=observation increment%
%renewals=cumulative number of renewals%
%upmat=availability matrix%
%relevantmat=relevant matrix%
%col=matrix location of failure(q)/repair completion(q) (time)%
%row=matrix location of failure(q)/repair completion(q) (usage)%
%lastrow=the last row of the matrix that has been updated%
%lastcol=the last column of the matrix that has been updated%

parameters=[3000 3250 600 650 0.2; 3000 3250 600 650 0.5; 3000 3250 600
650 0.8; 3000 7400 600 1480 0.2; 3000 7400 600 1480 0.5; 3000 7400 600
1480 0.8; 3000 18500 600 3700 0.2; 3000 18500 600 3700 0.5; 3000 18500
600 3700 0.8; 8000 3250 1600 650 0.2; 8000 3250 1600 650 0.5; 8000 3250
1600 650 0.8; 8000 7400 1600 1480 0.2; 8000 7400 1600 1480 0.5; 8000
7400 1600 1480 0.8; 8000 18500 1600 3700 0.2; 8000 18500 1600 3700 0.5;
8000 18500 1600 3700 0.8; 20000 3250 4000 650 0.2; 20000 3250 4000 650
0.5; 20000 3250 4000 650 0.8; 20000 7400 4000 1480 0.2; 20000 7400 4000
1480 0.5; 20000 7400 4000 1480 0.8; 20000 18500 4000 3700 0.2; 20000
18500 4000 3700 0.5; 20000 18500 4000 3700 0.8];
cmparameters=parameters.*0.1;

for h=1:1:27

   mutime=parameters(h,1);
   muuse=parameters(h,2);
   sigtime=parameters(h,3);
   siguse=parameters(h,4);
   rhovec=parameters(h,5);

   cmmutime=cmparameters(h,1);
   cmmuuse=cmparameters(h,2);
   cmsigtime=cmparameters(h,3);
   cmsiguse=cmparameters(h,4);
   squarerho=sqrt(1-rhovec.^2);

   %initialize matrices to zero%
   relevantmat=zeros([201,201]);
   upmat=zeros([201,201]);

   mu=[mutime muuse];
   point=max(mu);
   mumax=point/10

   %generate data for 100,000 machines%

   for p=1:1:100000
```

```
lastrow=1;
lastcol=1;


%this part of the program generates 10 random failure vectors for
each machine

for r=1:1:10

   z1=randn;
   z2=randn;

   cmz1=randn;
   cmz2=randn;

   xt(r)=mutime+(sigtime*z1);
   xu(r)=muuse+(siguse*((rhovec*z1)+(squarerho*z2)));
   first=xt(r);
   second=xu(r);

   rowvector =[first,second];
   bigvector(r,:)=rowvector;
   failuretimes=bigvector;

   cmxt(r)=cmmutime+(cmsigtime*cmz1);
   cmxu(r)=cmmuuse+(cmsiguse*((rhovec*cmz1)+(squarerho*cmz2)));
   cmfirst=cmxt(r);
   cmsecond=cmxu(r);

   cmrowvector =[cmfirst,cmsecond];
   cmbigvector(r,:)=cmrowvector;
   cmtimes=cmbigvector;

   failurevector(1,:)=failuretimes(1,:);
   cmvector(1,:)=failuretimes(1,:)+cmtimes(1,:);
   dummycm=cmvector(1,:);

   N=length(failuretimes(:,1));

   %this loop generates the cumulative failure/repair vectors
   for g=2:1:N
      failurevector(g,:)=failuretimes(g,:)+dummycm;
      cmvector(g,:)=failurevector(g,:)+cmtimes(g,:);
      dummycm=cmvector(g,:);
   end
end


for q=1:1:10

   %here, the matrix location of failure q is identified%
   t=failurevector(q,1);
   u=failurevector(q,2);

   col=floor(t/mumax)+1;
   row=floor(u/mumax)+1;
```

```
        lastrowcount=lastrow+1;
        lastcolcount=lastcol+1;

        %here, the matrix is updated to reflect the number of visits
        to each matrix location and the number of functioning
        machines%
        if row>=lastrowcount & col>=lastcolcount

           for k=lastrowcount:1:row
              for m=lastcolcount:1:col
                 relevantmat(k,m)=relevantmat(k,m)+1;
                 upmat(k,m)=upmat(k,m)+1;
              end
           end
           lastrow=row;
           lastcol=col;
        end

        %here, the matrix location of repair completion q is
        identified%
        t=cmvector(q,1);
        u=cmvector(q,2);

        col=floor(t/mumax)+1;
        row=floor(u/mumax)+1;

        lastrowcount=lastrow+1;
        lastcolcount=lastcol+1;

        %here, the matrix is updated to reflect the number of visits
        to each matrix location is updated (no update is made to the
        availability matrix because the machines are not functioning%

        if row>=lastrowcount & col>=lastcolcount
           for k=lastrowcount:1:row
              for m=lastcolcount:1:col
                 relevantmat(k,m)=relevantmat(k,m)+1;
              end
           end
        end

        lastrow=row;
        lastcol=col;
     end
  end
end
```

Appendix B – Additional Charts

The figures included in this Appendix correspond to those discussed in Chapter 4. The cases represented are identified in Table B-1.

	Bivariate Exponential Model (BVE) (λ, η, ρ)	Bivariate Normal Model (BVN) (μ_t, μ_u, σ_t, σ_u, ρ)	Linear Stochastic Function (LISF) (λ, c, β)
1	(0.00033, 0.00031, 0.2)	(3000, 3250, 600, 650, 0.2)	(0.00033, 1, 0)
2	(0.00033, 0.00031, 0.5)	(3000, 3250, 600, 650, 0.5)	(0.00033, 1, 250)
3	(0.00033, 0.00031, 0.8)	(3000, 3250, 600, 650, 0.8)	(0.00033, 1 500)
4	(0.00033, 0.00014, 0.5)	(3000, 7400 600, 1480, 0.5)	(0.00033, 2.5, 250)
4a	(0.00033, 0.00014, 0.2)	(3000, 7400 600, 1480, 0.2)	(0.00033, 2.5, 0)
4b	(0.00033, 0.00014, 0.8)	(3000, 7400, 600, 1480, 0.8)	(0.00033, 2.5, 500)
5	(0.00033, 0.000054, 0.5)	(3000, 18500 600, 3700, 0.5)	(0.00033, 6, 250)
5a	(0.00033, 0.000054, 0.2)	(3000, 18500 600, 3700, 0.2)	(0.00033, 6, 0)
5b	(0.00033, 0.000054, 0.8)	(3000, 18500, 600, 3700, 0.8)	(0.00033, 6, 500)
6	(0.00005, 0.00031, 0.5)	(20000, 3250, 4000, 650, 0.5)	(0.00005, 0.15, 250)
6a	(0.00005, 0.00031, 0.2)	(20000, 3250, 4000, 650, 0.2)	(0.00005, 0.15, 0)
6b	(0.00005, 0.00031, 0.8)	(20000, 3250, 4000, 650, 0.8)	(0.00005, 0.15, 500)

Table B-1 PARAMETER SETS

B.1 Renewal Function

The figures included here represent the effects of varying the correlation coefficient, ρ, or the constant, β. The results are qualitatively the same as those presented in Chapter 4. Also, the figures related to the univariate projection method are included.

B.1.2 Bivariate Exponential Model

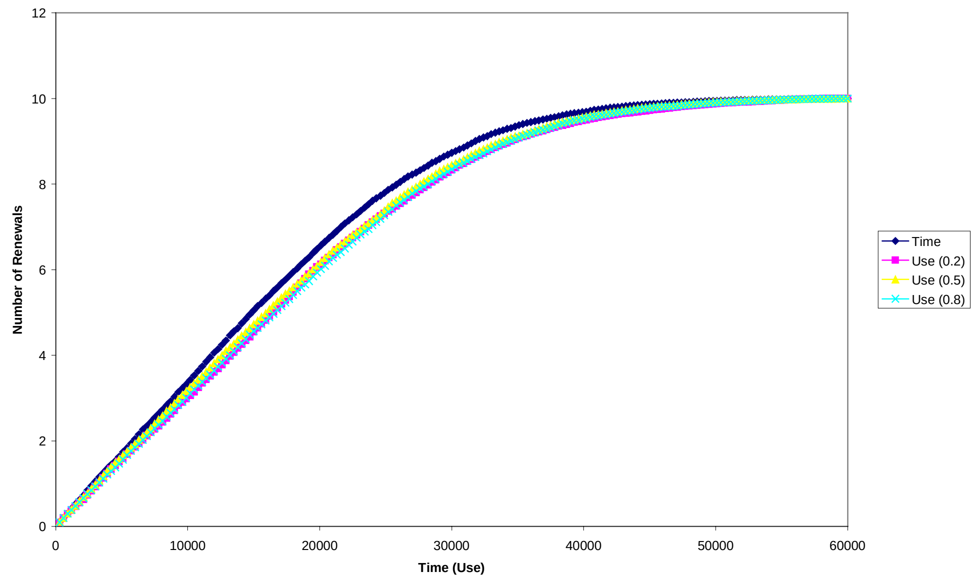

Figure B-1 UNIVARIATE PROJECTION (CASES 1, 2, 3, BVE)

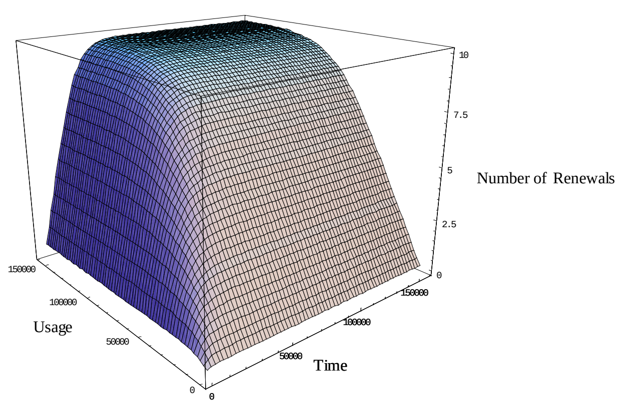

Figure B-2 RENEWAL FUNCTION (CASE 4a, BVE)

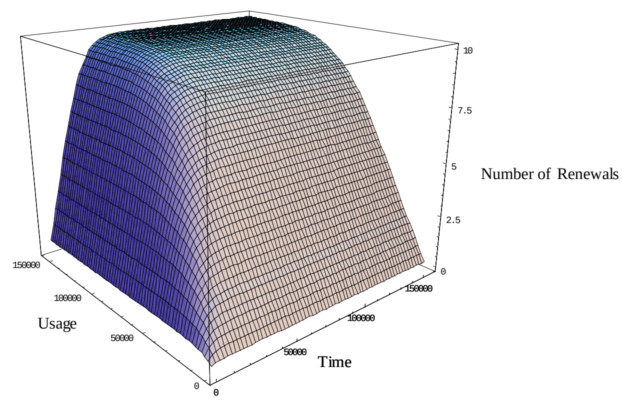

Figure B-3 RENEWAL FUNCTION (CASE 4b, BVE)

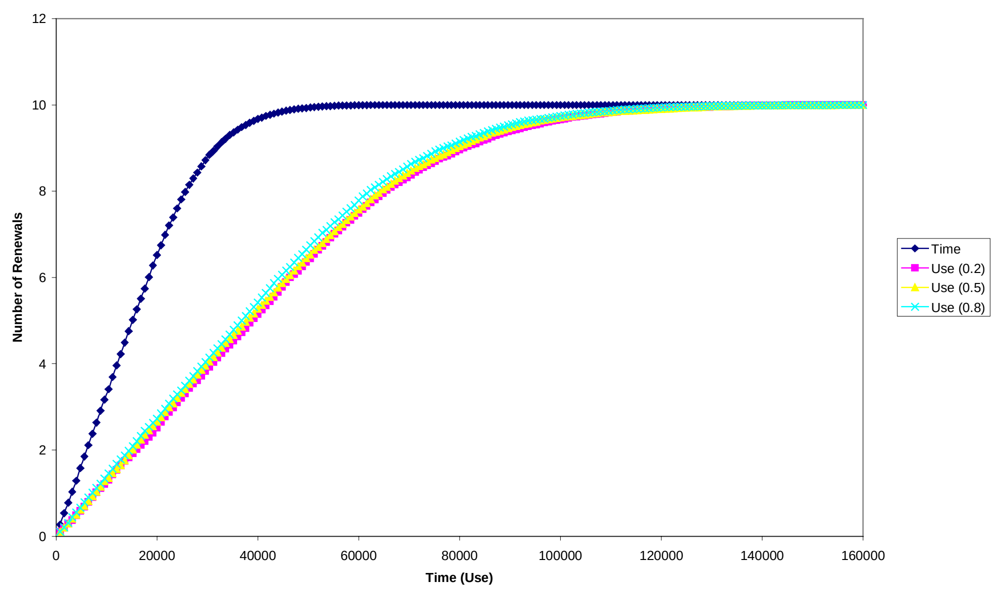

Figure B-4 UNIVARIATE PROJECTION (CASES 4, 4a, 4b, BVE)

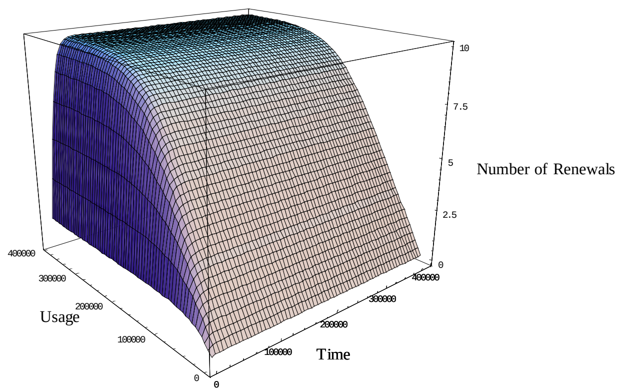

Figure B-5 RENEWAL FUNCTION (CASE 5a, BVE)

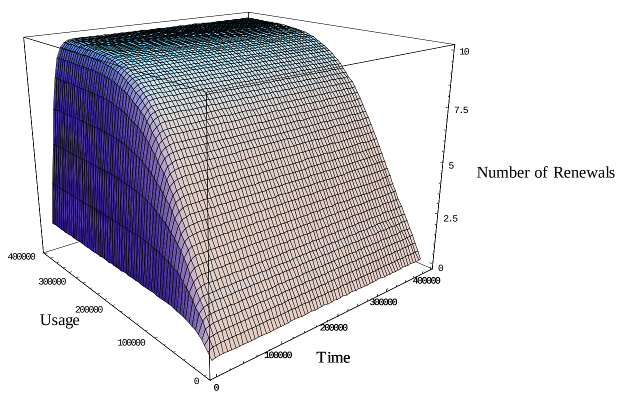

Figure B-6 RENEWAL FUNCTION (CASE 5b, BVE)

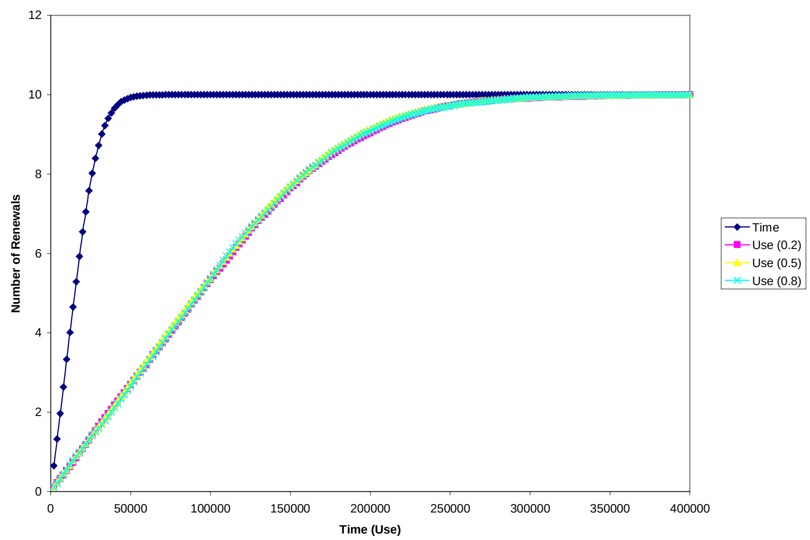

Figure B-7 UNIVARIATE PROJECTION (CASES 5, 5a, 5b, BVE)

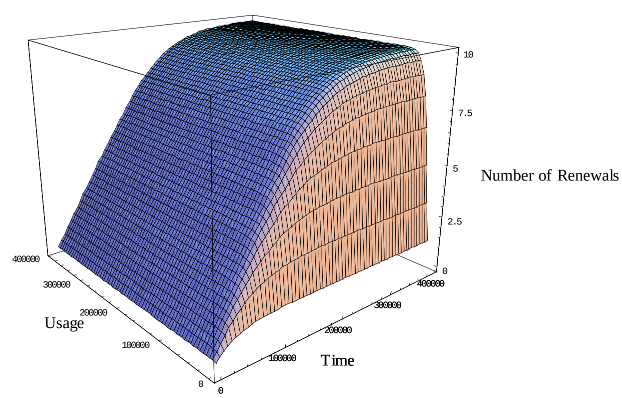

Figure B-8 RENEWAL FUNCTION (CASE 6a, BVE)

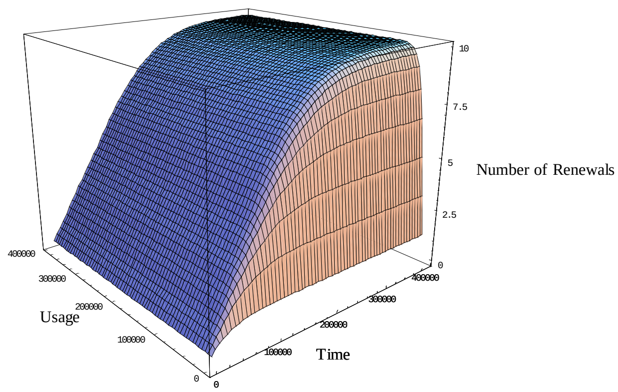

Figure B-9 RENEWAL FUNCTION (CASE 6b, BVE)

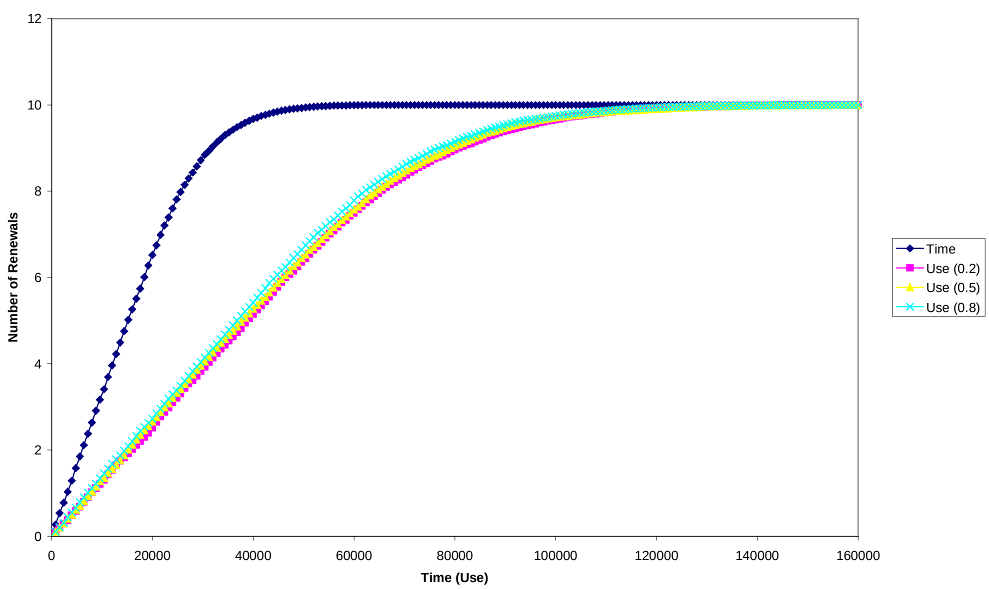

Figure B-10 UNIVARIATE PROJECTION (CASES 6, 6a, 6b, BVE)

B.1.2 Bivariate Normal Model

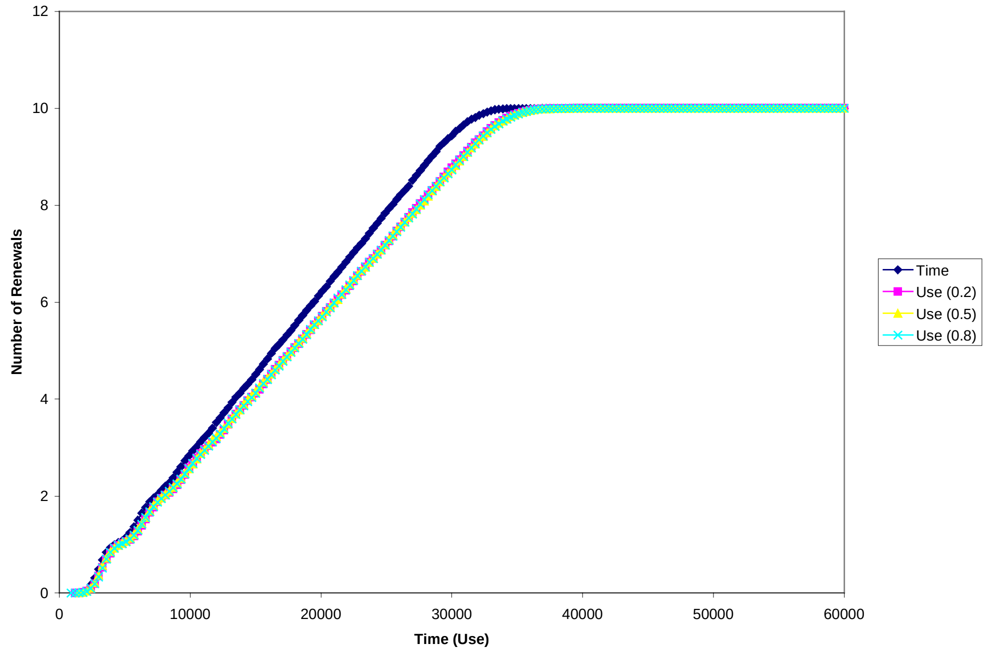

Figure B-11 UNIVARIATE PROJECTION (CASES 1, 2, 3, BVN)

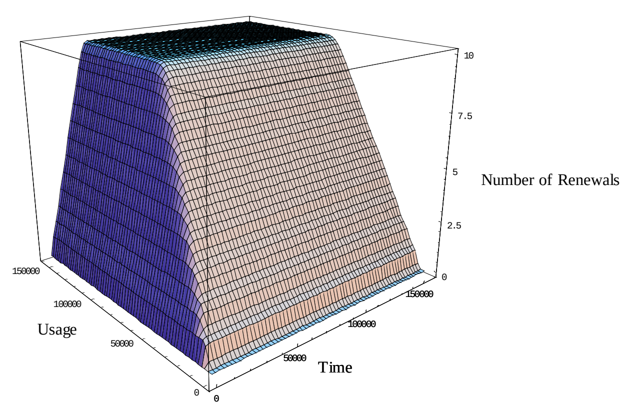

Figure B-12 RENEWAL FUNCTION (CASE 4a, BVN)

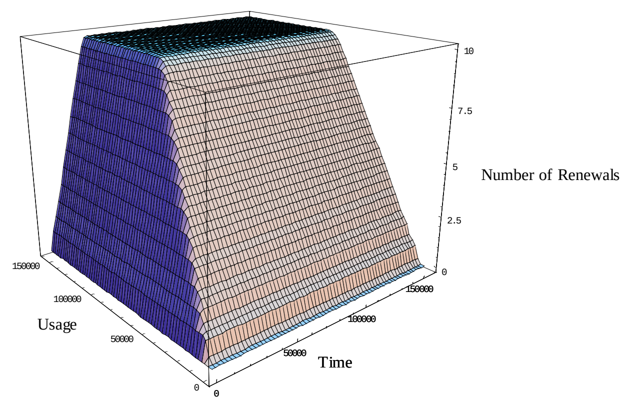

Figure B-13 RENEWAL FUNCTION (CASE 4b, BVN)

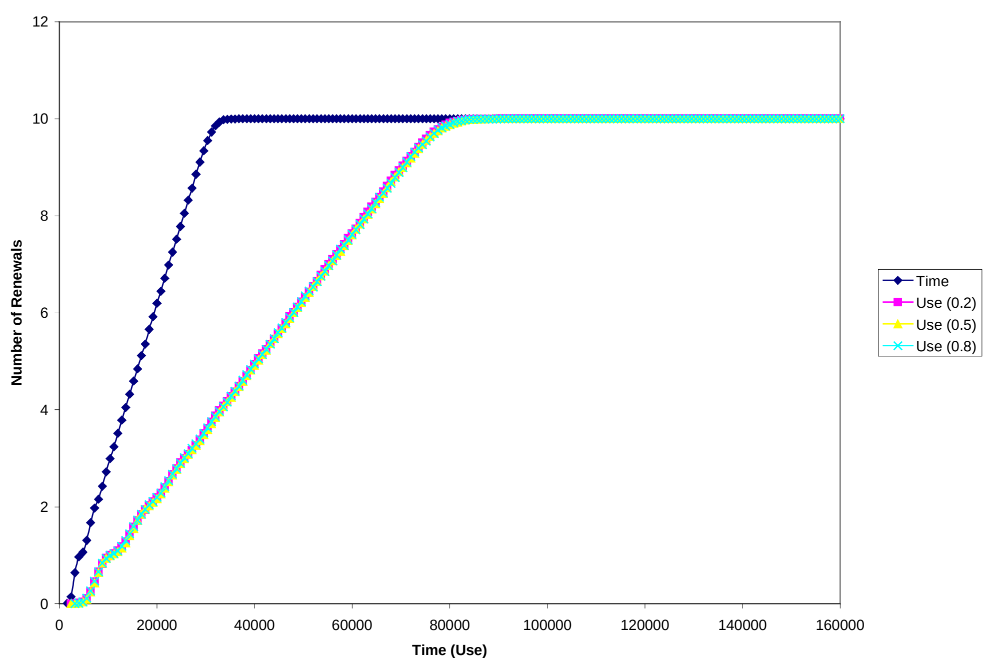

Figure B-14 UNIVARIATE PROJECTION (CASES 4, 4a, 4b, BVN)

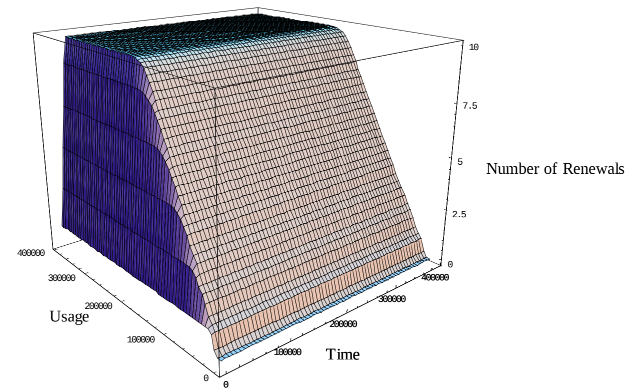

Figure B-15 RENEWAL FUNCTION (CASE 5a, BVN)

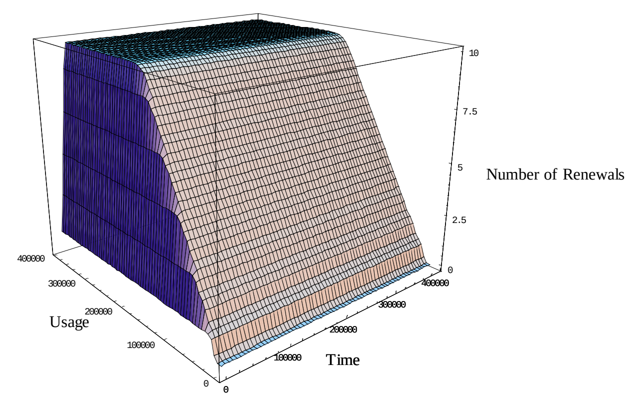

Figure B-16 RENEWAL FUNCTION (CASE 5b, BVN)

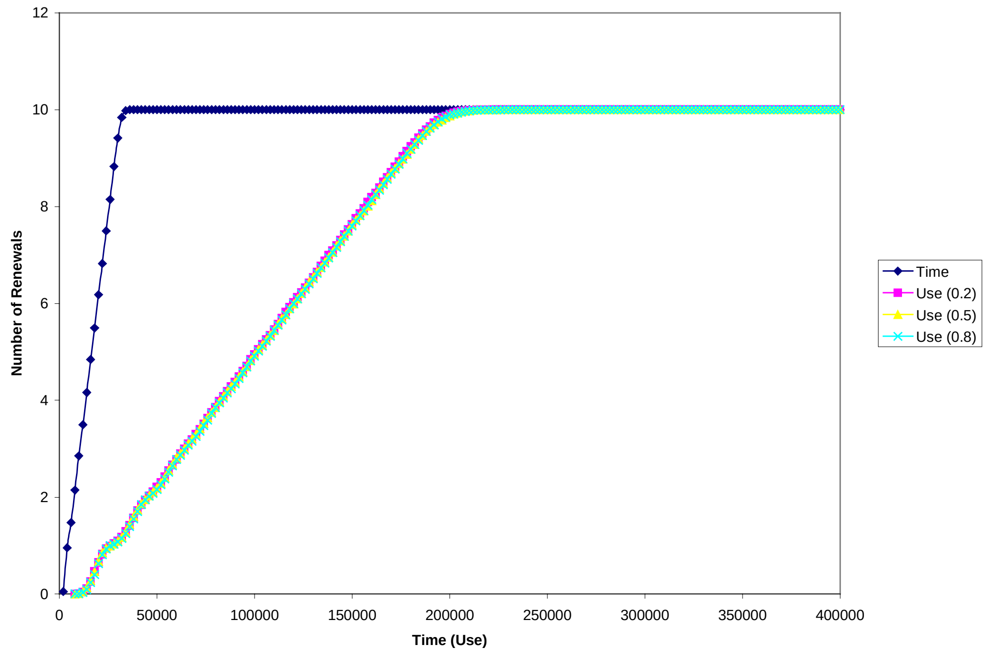

Figure B-17 UNIVARIATE PROJECTION (CASES 5, 5a, 5b, BVN)

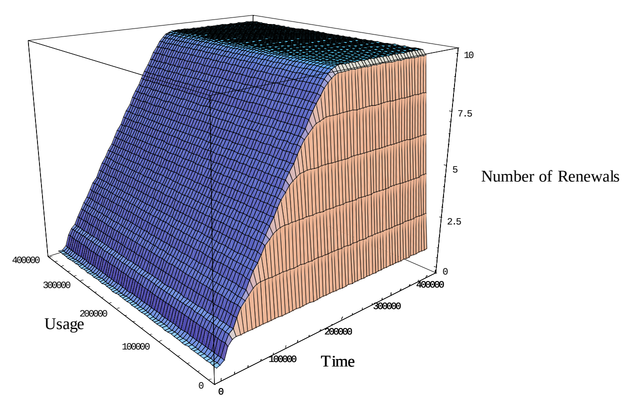

Figure B-18 RENEWAL FUNCTION (CASE 6a, BVN)

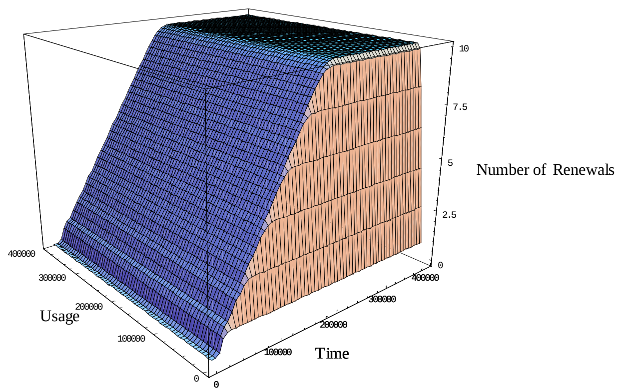

Figure B-19 RENEWAL FUNCTION (CASE 6b, BVN)

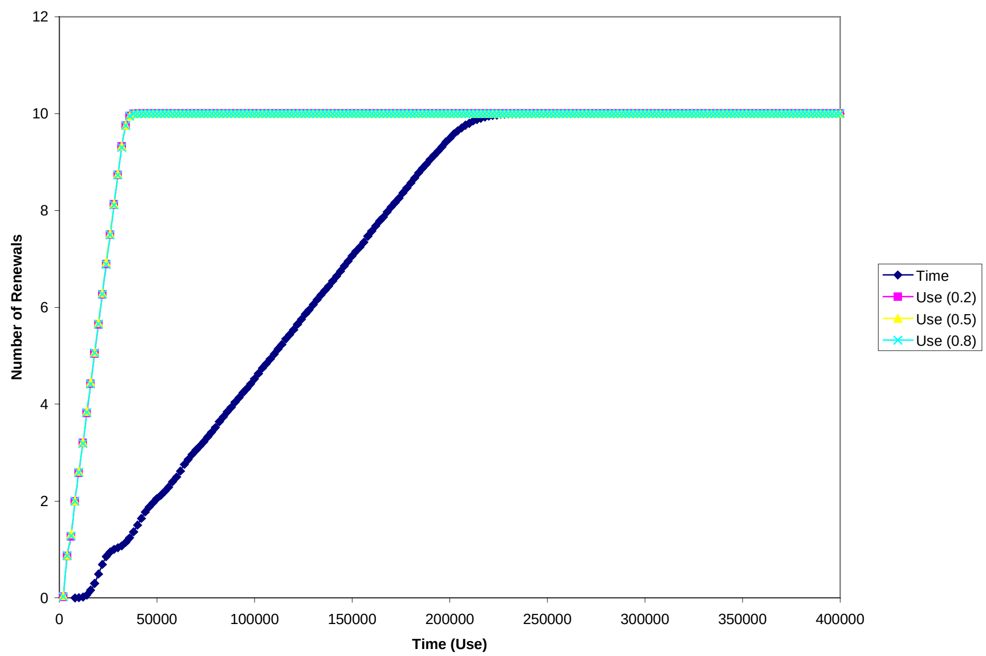

Figure B-20 UNIVARIATE PROJECTION (CASES 6, 6a, 6b, BVN)

B.1.3 Linear Stochastic Functional Relationship

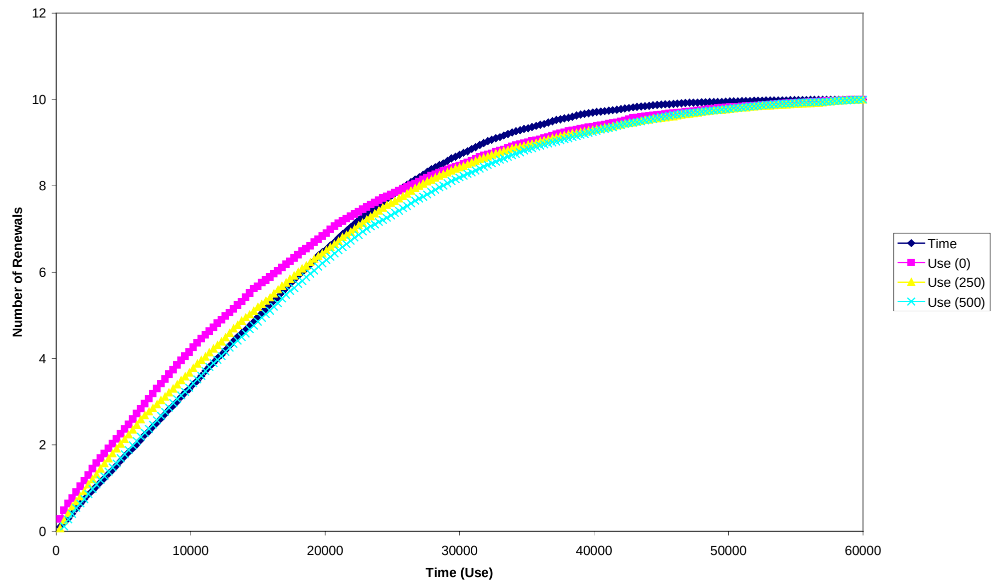

Figure B-21 UNIVARIATE PROJECTION (CASES 1, 2, 3, LISF)

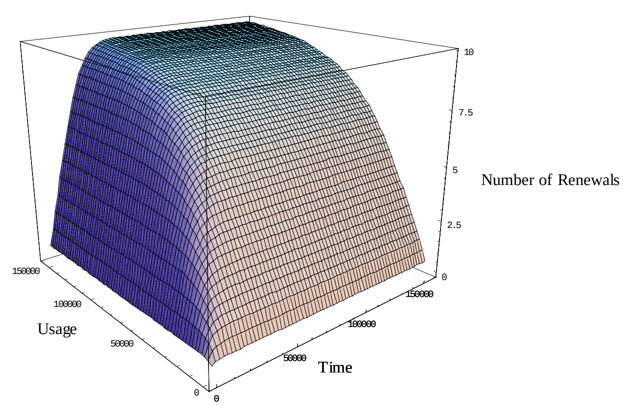

Figure B-22 RENEWAL FUNCTION (CASE 4a, LISF)

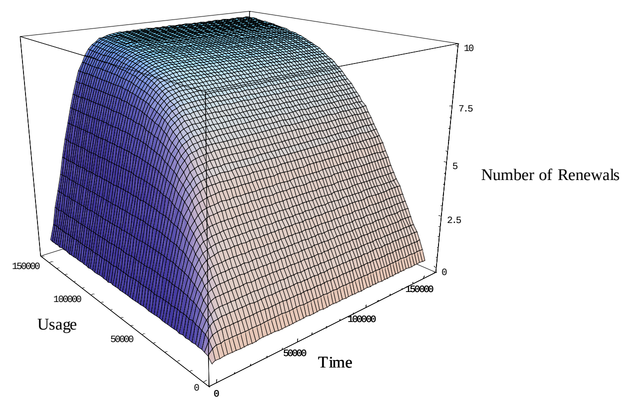

Figure B-23 RENEWAL FUNCTION (CASE 4b, LISF)

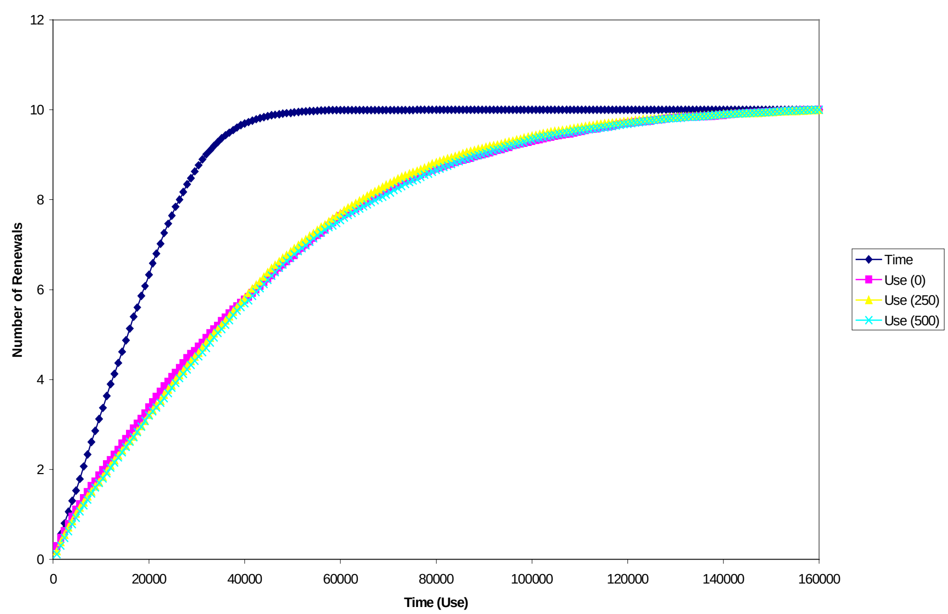

Figure B-24 UNIVARIATE PROJECTION (CASES 4, 4a, 4b, LISF)

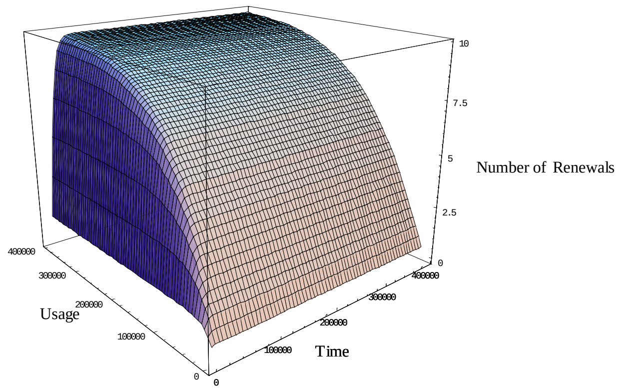

Figure B-25 RENEWAL FUNCTION (CASE 5a, LISF)

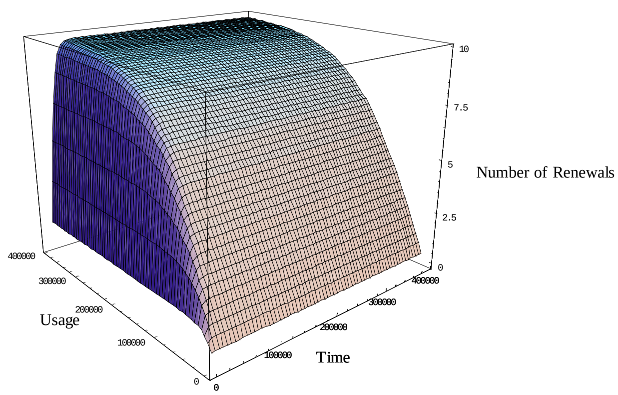

Figure B-26 RENEWAL FUNCTION (CASE 5b, LISF)

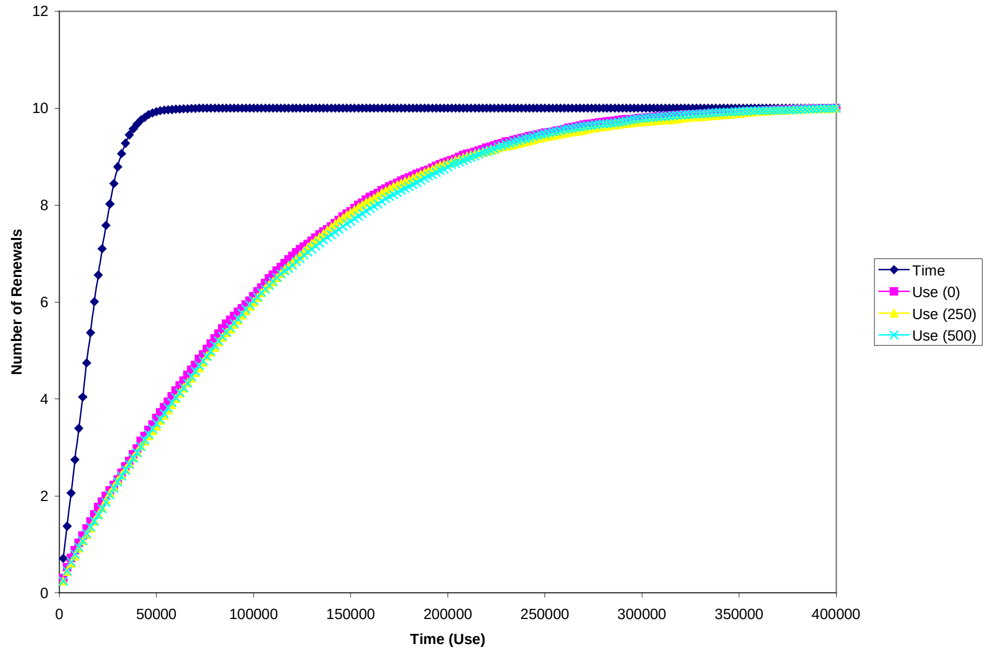

Figure B-27 UNIVARIATE PROJECTION (CASES 5, 5a, 5b, LISF)

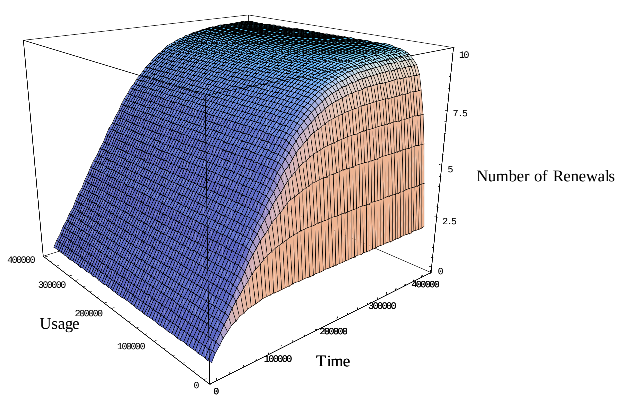

Figure B-28 RENEWAL FUNCTION (CASE 6a, LISF)

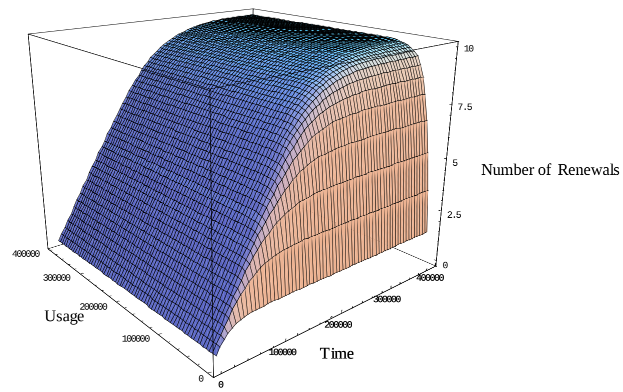

Figure B-29 RENEWAL FUNCTION (CASE 6b, LISF)

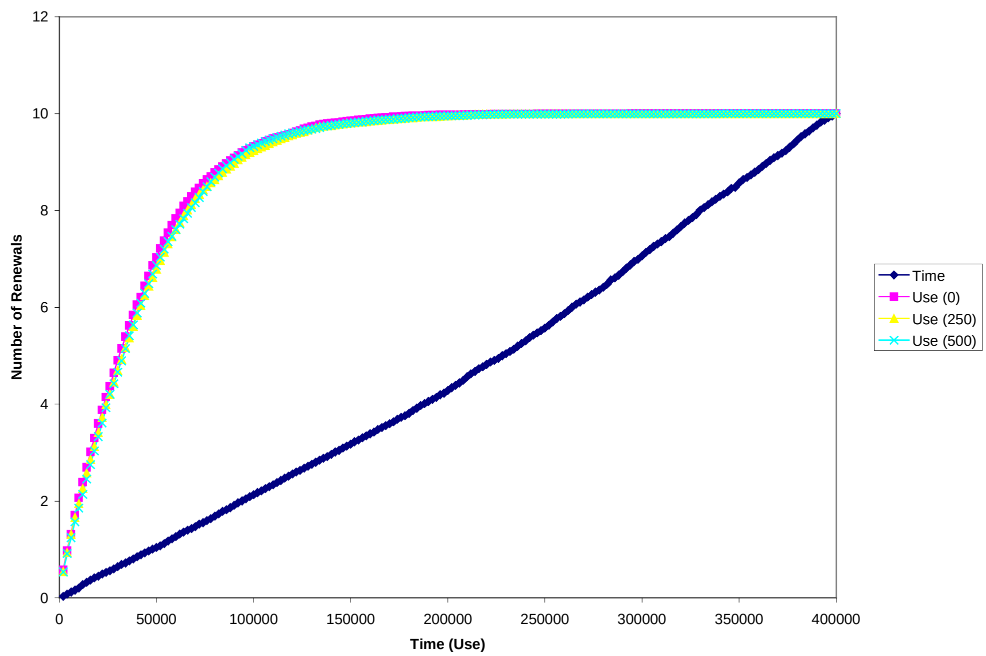

Figure B-30 UNIVARIATE PROJECTION (CASES 6, 6a, 6b, LISF)

B.2 Availability Function

The figures shown here represent the cases that correspond to those mentioned in Chapter 4. The differences are a result of the variations in the correlation coefficient, ρ, or the constant, β.

B.2.1 Bivariate Exponential Model

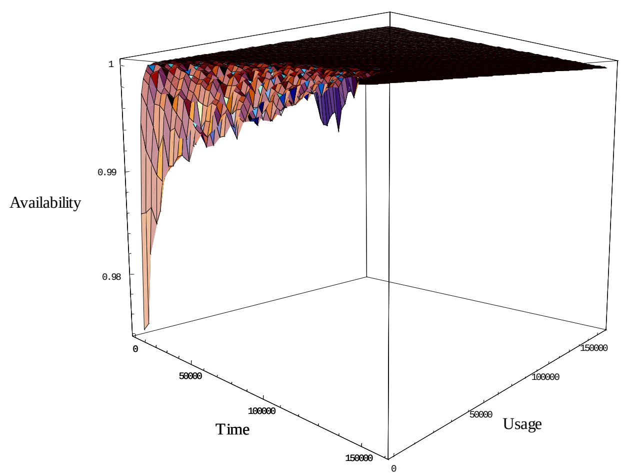

Figure B-31 AVAILABILITY FUNCTION (METHOD 1, CASE 4a, BVE)

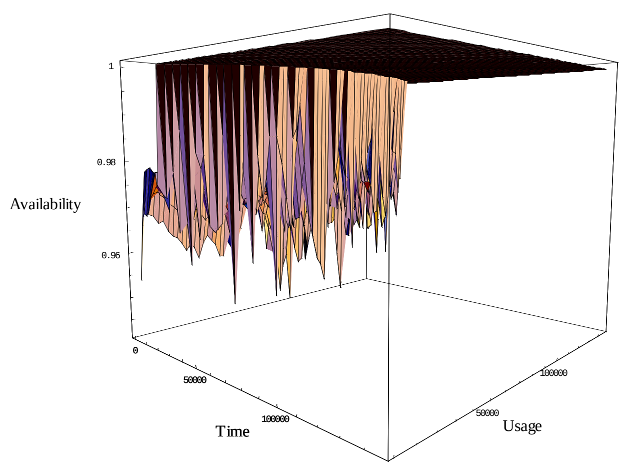

Figure B-32 AVAILABILITY FUNCTION (METHOD 2, CASE 4a, BVE)

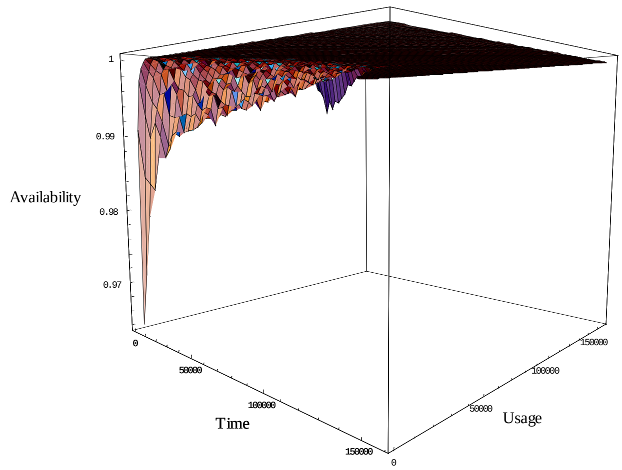

Figure B-33 AVAILABILITY FUNCTION (METHOD 1, CASE 4b, BVE)

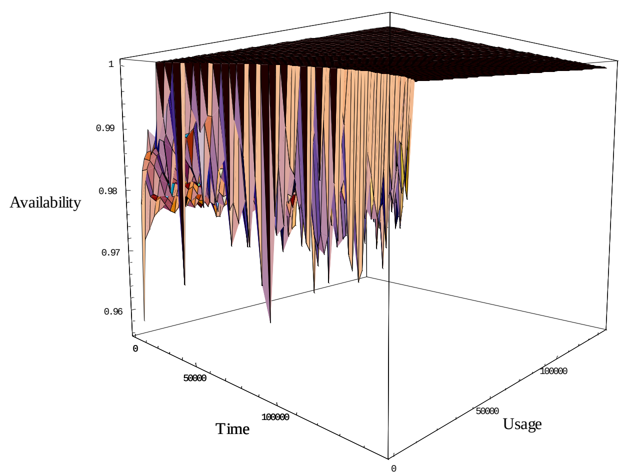

Figure B-34 AVAILABILITY FUNCTION (METHOD 2, CASE 4b, BVE)

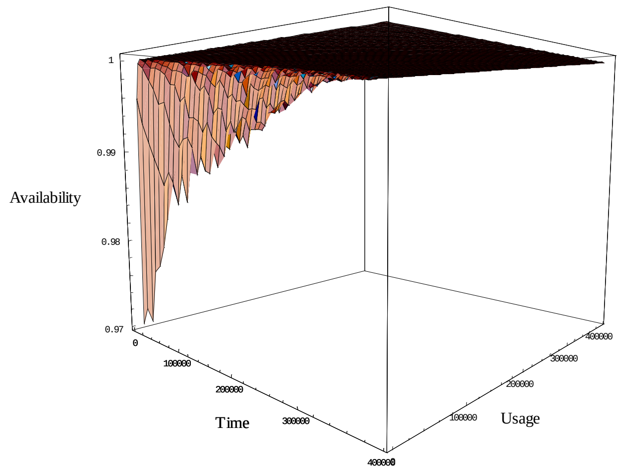

Figure B-35 AVAILABILITY FUNCTION (METHOD 1, CASE 5a, BVE)

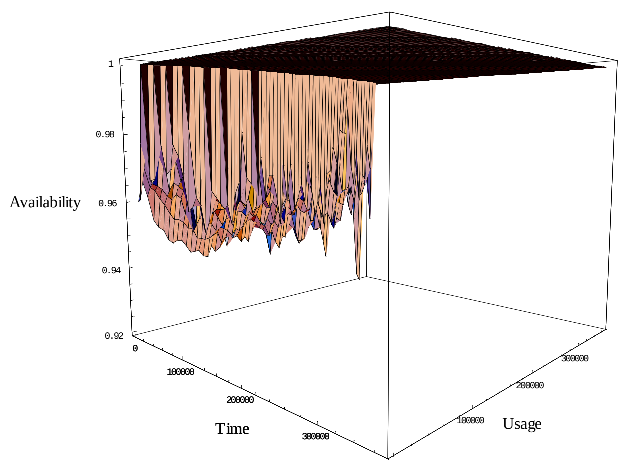

Figure B-36 AVAILABILITY FUNCTION (METHOD 2, CASE 5a, BVE)

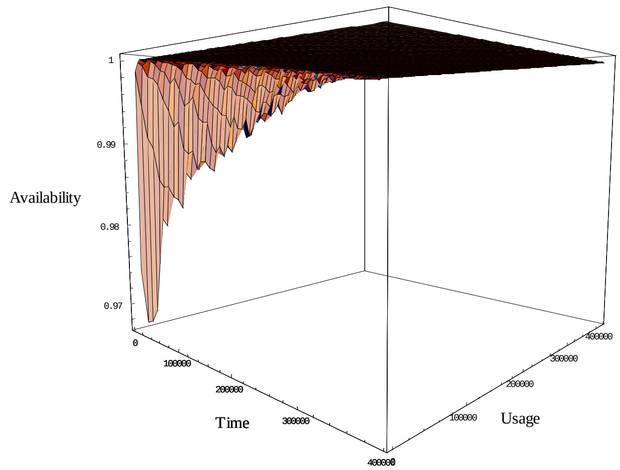

Figure B-37 AVAILABILITY FUNCTION (METHOD 1, CASE 5b, BVE)

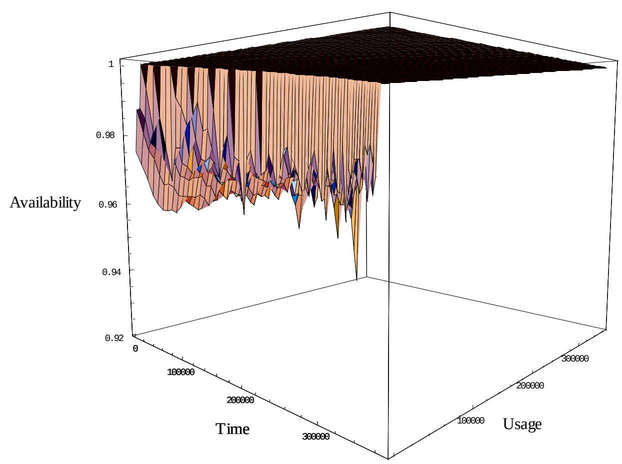

Figure B-38 AVAILABILITY FUNCTION (METHOD 2, CASE 5b, BVE)

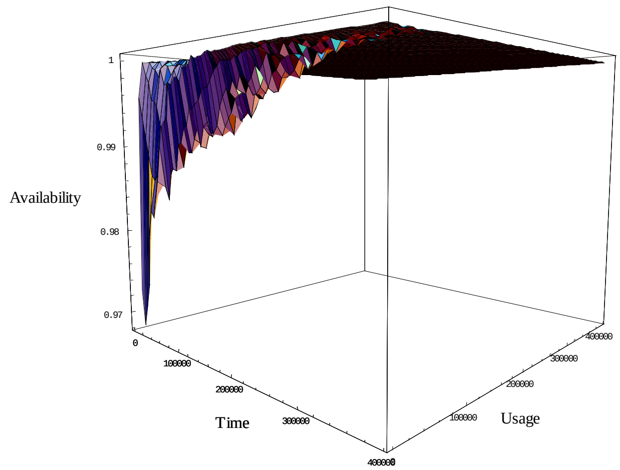

Figure B-39 AVAILABILITY FUNCTION (METHOD 1, CASE 6a, BVE)

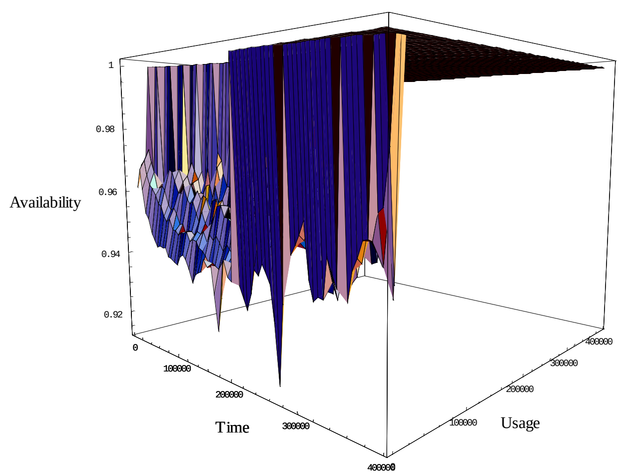

Figure B-40 AVAILABILITY FUNCTION (METHOD 2, CASE 6a, BVE)

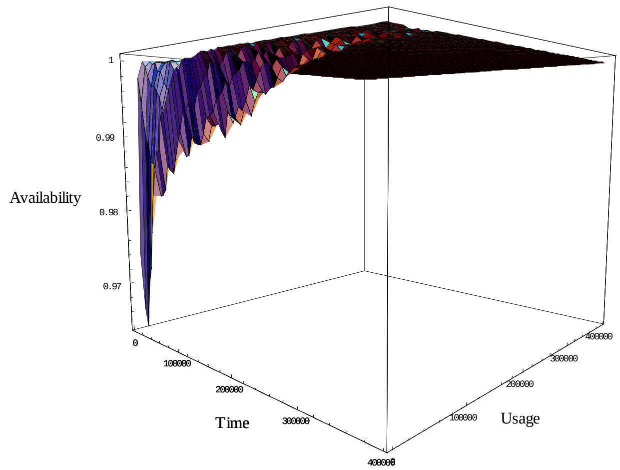

Figure B-41 AVAILABILITY FUNCTION (METHOD 1, CASE 6b, BVE)

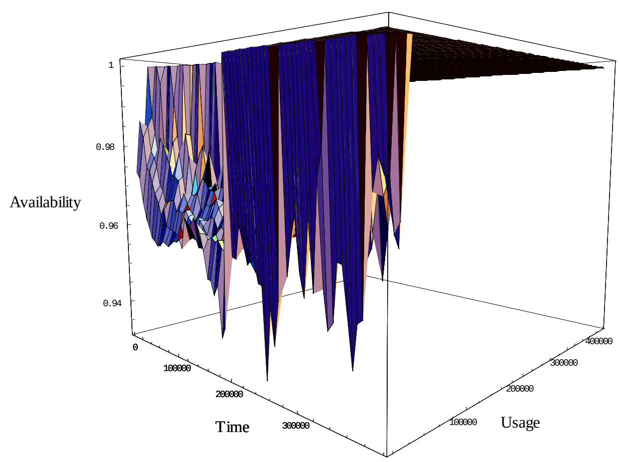

Figure B-42 AVAILABILITY FUNCTION (METHOD 2, CASE 6b, BVE)

B.2.2 Bivariate Normal Model

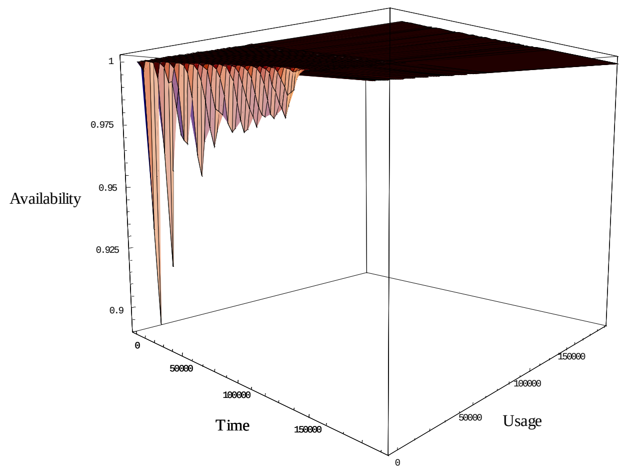

Figure B-43 AVAILABILITY FUNCTION (METHOD 1, CASE 4a, BVN)

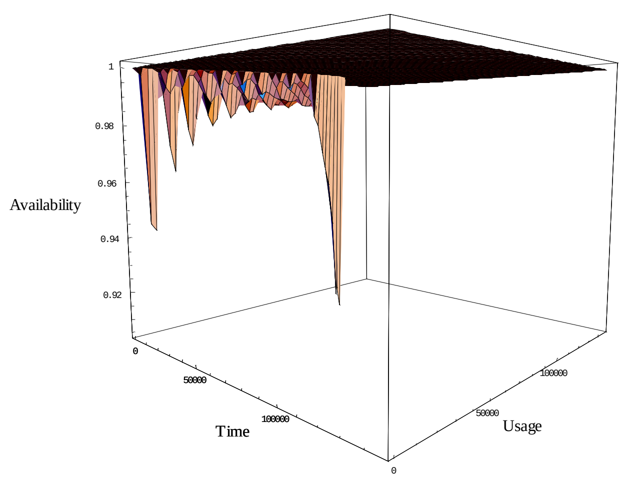

Figure B-44 AVAILABILITY FUNCTION (METHOD 2, CASE 4a, BVN)

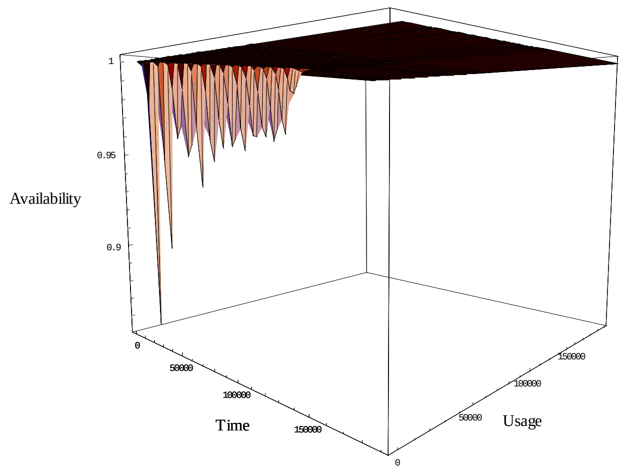

Figure B-45 AVAILABILITY FUNCTION (METHOD 1, CASE 4b, BVN)

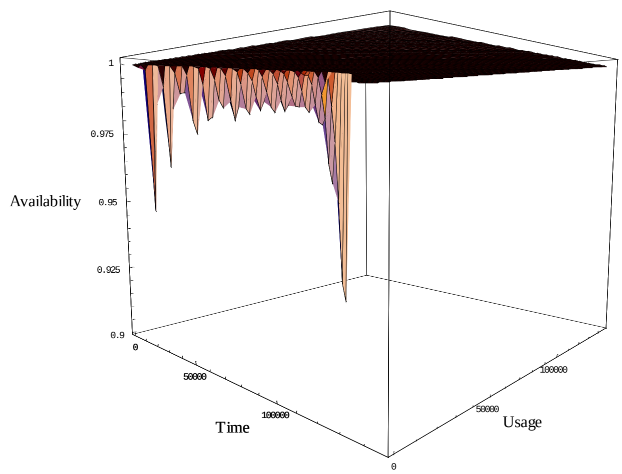

Figure B-46 AVAILABILITY FUNCTION (METHOD 2, CASE 4b, BVN)

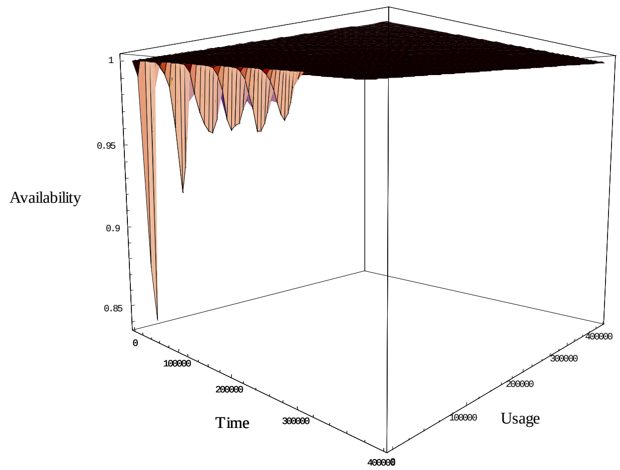

Figure B-47 AVAILABILITY FUNCTION (METHOD 1, CASE 5a, BVN)

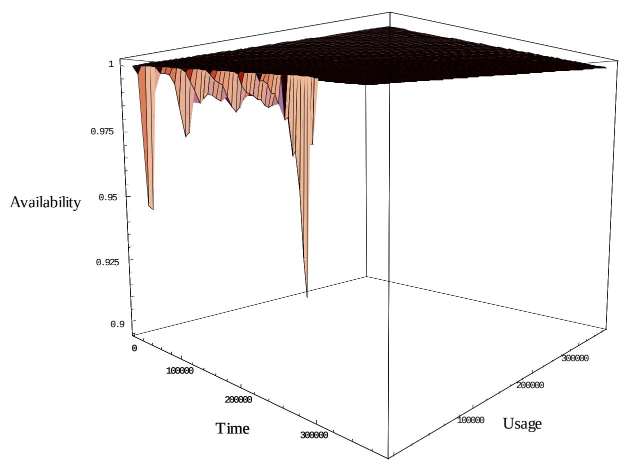

Figure B-48 AVAILABILITY FUNCTION (METHOD 2, CASE 5a, BVN)

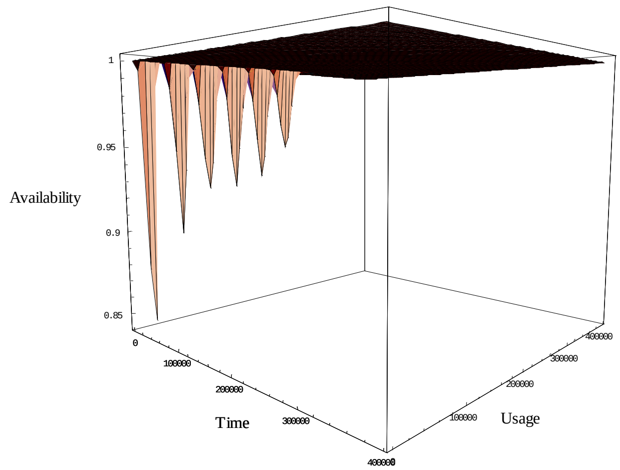

Figure B-49 AVAILABILITY FUNCTION (METHOD 1, CASE 5b, BVN)

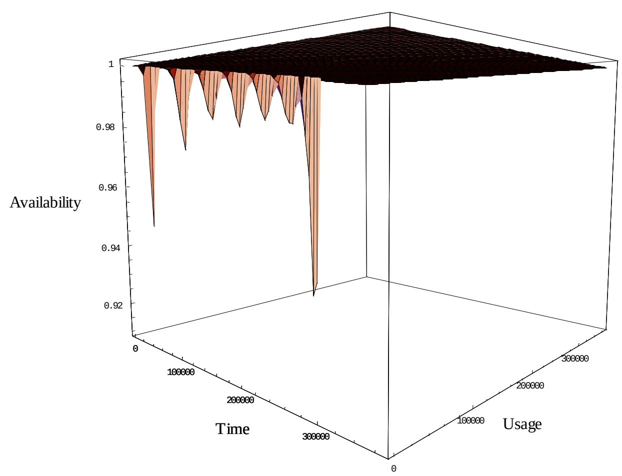

Figure B-50 AVAILABILITY FUNCTION (METHOD 2, CASE 5b, BVN)

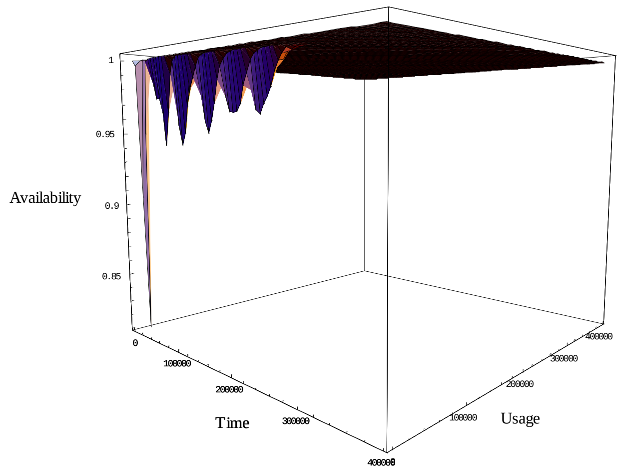

Figure B-50 AVAILABILITY FUNCTION (METHOD 1, CASE 6a, BVN)

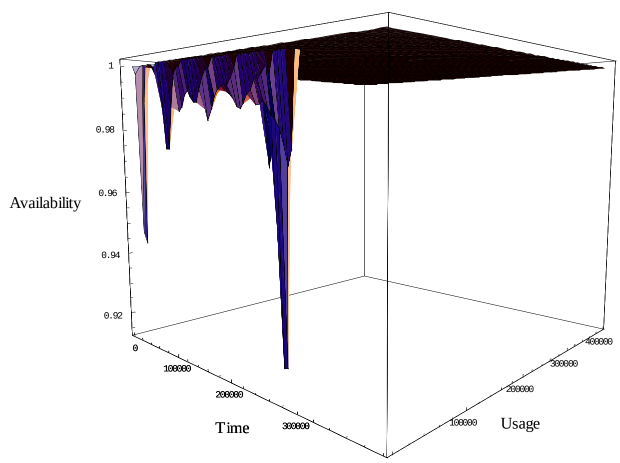

Figure B-51 AVAILABILITY FUNCTION (METHOD 2, CASE 6a, BVN)

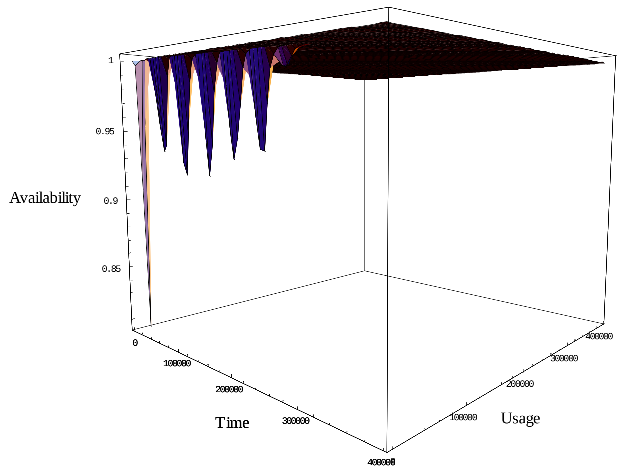

Figure B-52 AVAILABILITY FUNCTION (METHOD 1, CASE 6b, BVN)

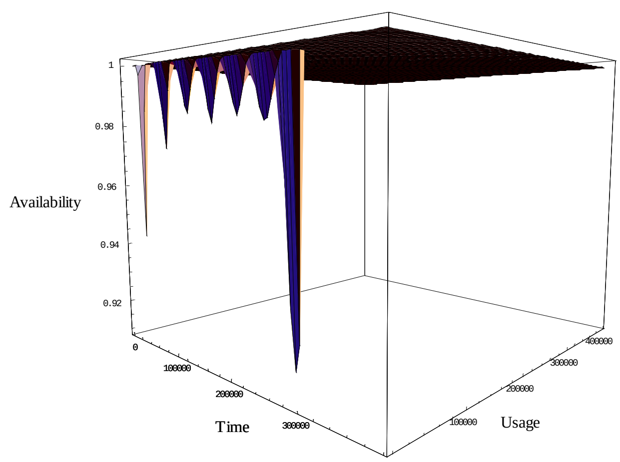

Figure B-53 AVAILABILITY FUNCTION (METHOD 2, CASE 6b, BVN)

B.2.3 Linear Stochastic Function

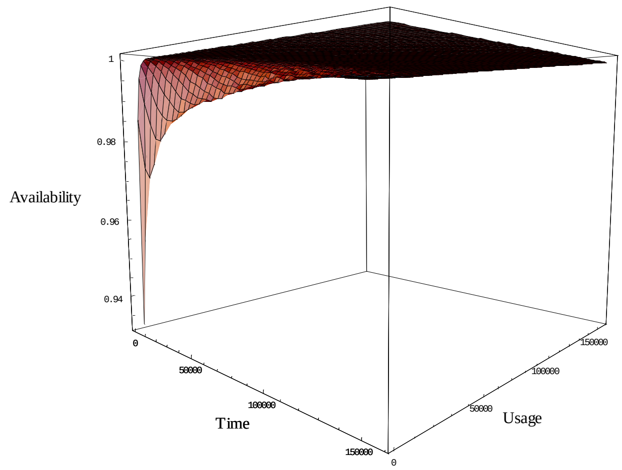

Figure B-54 AVAILABILITY FUNCTION (METHOD 1, CASE 4a, LISF)

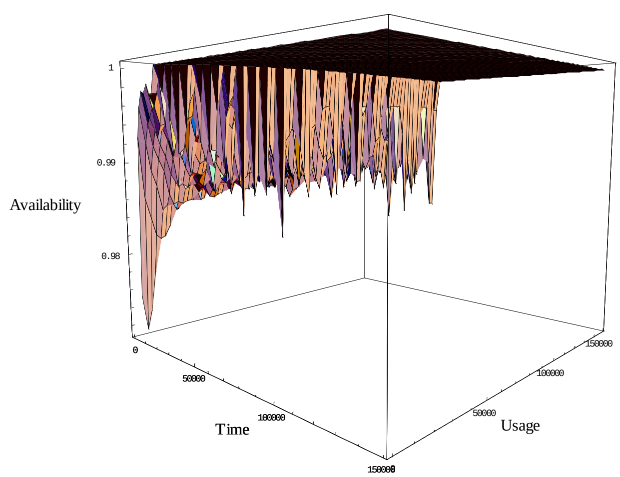

Figure B-55 AVAILABILITY FUNCTION (METHOD 2, CASE 4a, LISF)

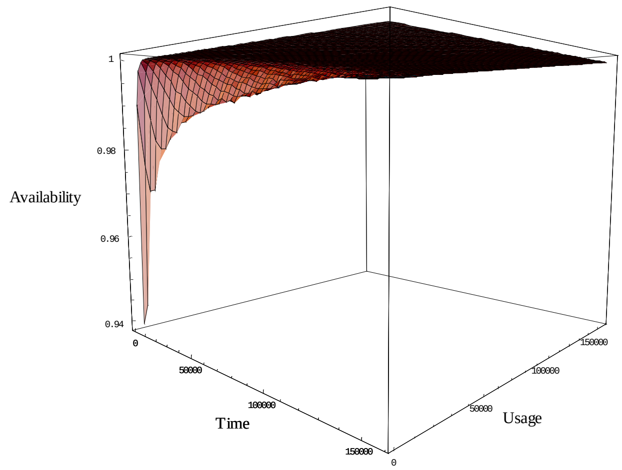

Figure B-56 AVAILABILITY FUNCTION (METHOD 1, CASE 4b, LISF)

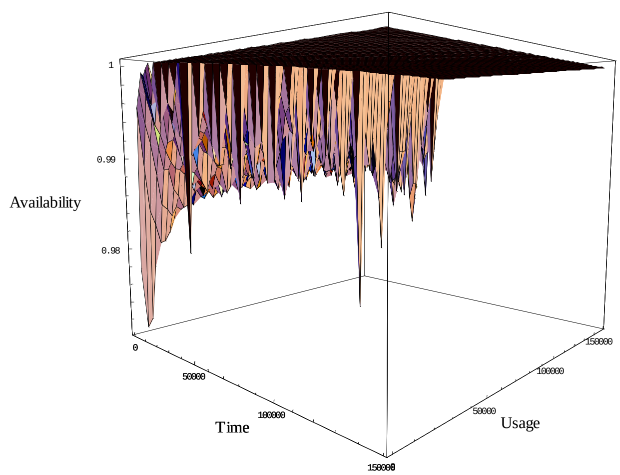

Figure B-57 AVAILABILITY FUNCTION (METHOD 2, CASE 4b, LISF)

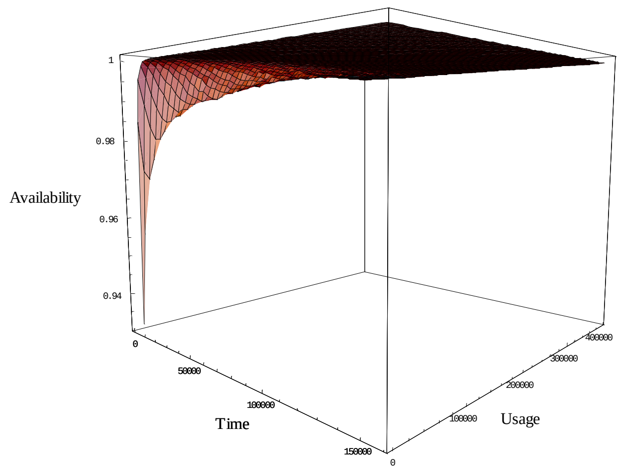

Figure B-58 AVAILABILITY FUNCTION (METHOD 1, CASE 5a, LISF)

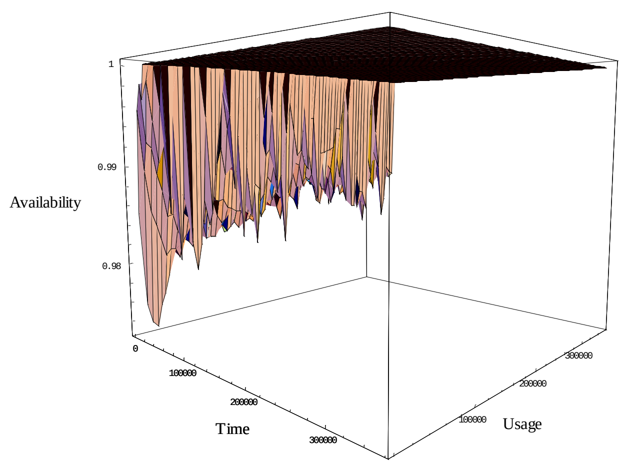

Figure B-59 AVAILABILITY FUNCTION (METHOD 2, CASE 5a, LISF)

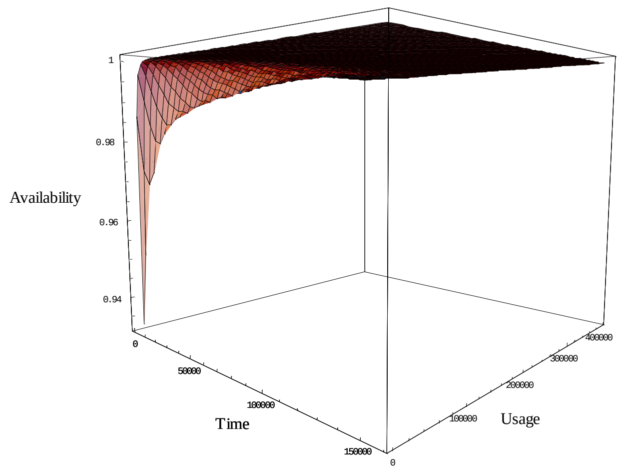

Figure B-60 AVAILABILITY FUNCTION (METHOD 1, CASE 5b, LISF)

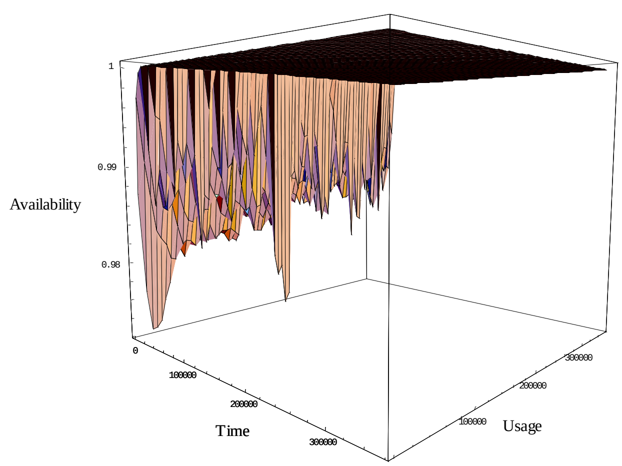

Figure B-61 AVAILABILITY FUNCTION (METHOD 2, CASE 5b, LISF)

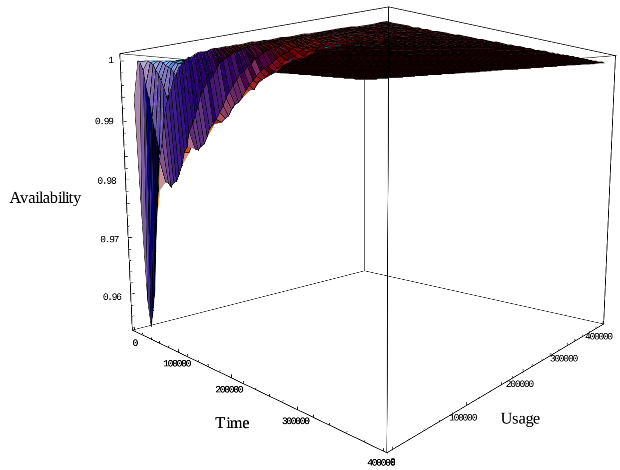

Figure B-62 AVAILABILITY FUNCTION (METHOD 1, CASE 6a, LISF)

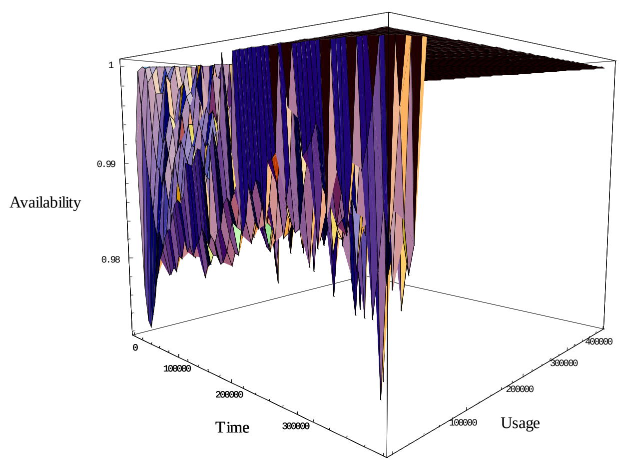

Figure B-63 AVAILABILITY FUNCTION (METHOD 2, CASE 6a, LISF)

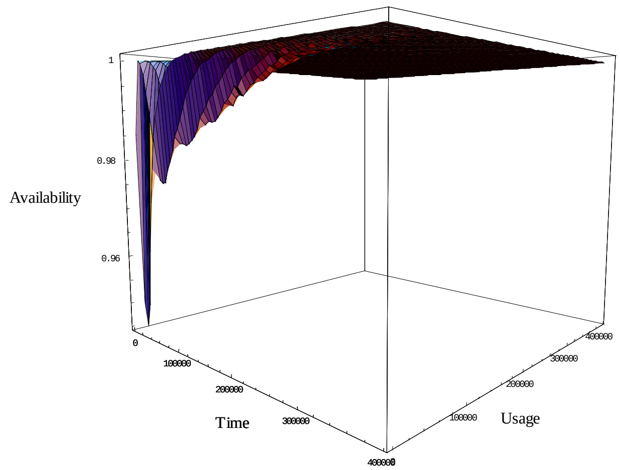

Figure B-64 AVAILABILITY FUNCTION (METHOD 1, CASE 6b, LISF)

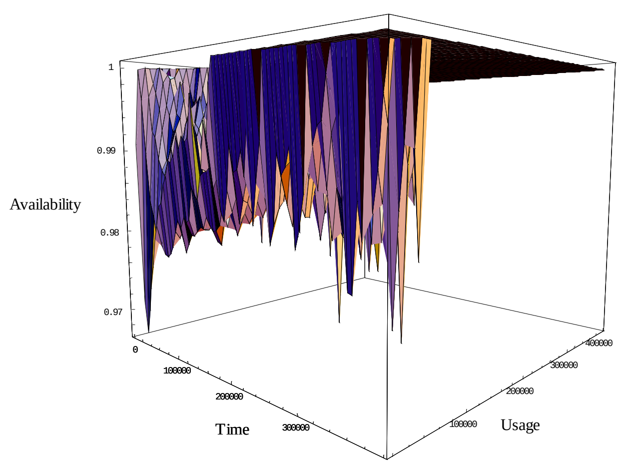

Figure B-65 AVAILABILITY FUNCTION (METHOD 2, CASE 6b, LISF)

VITA

Elise M. Caruso

Elise M. Caruso was born on July 16, 1976 in Corning, New York, to Enrico and Mary Caruso. In May 1999, she received her Bachelor of Science Cum Laude in Industrial and Systems Engineering with a minor in Spanish, from Virginia Polytechnic Institute and State University. While completing her Bachelor's degree, she was inducted into Gamma Beta Phi and Alpha Pi Mu and was a member of the Institute of Industrial Engineers and the Society of Women Engineers. In addition, she held internship positions with Hardee's Food Systems, Inc, General Motors, and The Washington Post. In addition, she was named The Outstanding Senior in the College of Engineering for the '98-'99 academic year. In July 2000 she will receive her Master of Science in Industrial and Systems Engineering with a concentration in Operations Research. She will begin work with Cummins Engine Co. in August 2000. She is currently a member of the Institute of Industrial Engineers and the Institute for Operations Research and Management Science.