Chapter 2

Receiver Performance Analysis

The spectrum-sliced WDM system model used in this thesis is shown in Figure 2.1 [10]. In this model, the receiver is the part that interests us most and its performance under different conditions will be analyzed throughout this thesis. The receiver consists of an optical bandpass filter, which is followed by an ideal square law detector and an integrator, which measures the energy in the decision interval. As we are considering only ON-OFF Keying (OOK) modulation in our system model, the energy is compared at the decision circuit to a threshold [18,19].

The received signal (in units of current) at the input of the decision circuit can be expressed as [18]

$$I = \frac{1}{2T}\int_0^T \left[x^2(t) + y^2(t) + \tilde{x}^2(t) + \tilde{y}^2(t)\right]dt + I_n \tag{2.1}$$

where $x(t), y(t), \tilde{x}(t),$ and $\tilde{y}(t)$ are independent, identically distributed baseband Gaussian processes with optical bandwidth $B_0/2$, having zero-mean, and each having a variance σ^2 equal to the photocurrent contributed by each of the two orthogonal polarizations. Note that the four terms within the integral correspond to two orthogonal phases and two orthogonal polarizations. Also, I_n represents the thermal noise current introduced in the electrical portion of the receiver. Shot noise terms are neglected since they are negligible when compared to the inherent signal fluctuation noise, and also negligible compared to the ASE noise for the optical preamplifier receiver. For the optical preamplifier, in the on-state, $x, y, \tilde{x},$ and $\tilde{y}$ are the components of the noise-like signal plus the corresponding components of the preamplifier ASE. In the off-state these are just the components of the preamplifier ASE noise. We assume that the preamplifier gain is sufficiently high that the electrical thermal noise may be neglected. For simplicity purposes the integral in (2.1) may be approximated by sums using a Karhunen-Loeve expansion giving us [18]

$$I = \frac{1}{2m}\sum_{i=1}^{m}\left[x_i^2 + y_i^2 + \tilde{x}_i^2 + \tilde{y}_i^2\right] \tag{2.2}$$

with $m = B_0 T$, and where the terms in the summation are independent. Thus, the summation is the square of 4*m* independent Gaussian variables and is thus chi-square distributed with 4*m* degrees of freedom and having a Moment Generating Function given by [21]

$$M(s) = \left[1 - 2\sigma^2 s\right]^{-2m} \tag{2.3}$$

In the following sub-sections the receiver performance will be analyzed using the Saddlepoint approximation (which is described more in detail in Appendix A) and the Gaussian approximation.

2.1. **Use of Saddlepoint Approximation for Receiver Performance Analysis**

The following analysis assumes the use of a filter with rectangular spectra and with no interchannel interference. First the ON-state is considered and then subsequently the OFF-case. It should be noted that the ON-state is the equivalent of transmitting a '1', and the OFF-state is the equivalent of transmitting a '0'.

2.1.1. ON-Case

The MGF in the ON-case can be expressed as

$$M_{ON}(s) = \left[1 - 2\sigma_{ON}^2 s\right]^{-2m} \tag{2.4}$$

where σ_{ON}^2 is the variance of the received signal in the ON-state. Since in the ON-case both signal and noise are present, the MGF can be expressed in the following form

$$M_{ON}(s) = \left[1 - 2\left(\sigma_s^2 + \sigma_n^2\right)s\right]^{-2m} \tag{2.5}$$

The variances of the data signal and the noise are given by [18]

$$\sigma_s^2 = \overline{N}_p \eta q R_b G \tag{2.6}$$

$$\sigma_n^2 = n_{sp} \eta q (G-1) B_0 \tag{2.7}$$

where:

η = quantum efficiency of the photodetector

q = electron charge

n_{sp} = spontaneous emission factor

G = gain of the optical preamplifier

If G is sufficiently large, we obtain

$$\frac{\sigma_s^2}{\sigma_n^2} = \frac{\overline{N}_p}{n_{sp}} \frac{R_b}{B_0} = \frac{\overline{N}_p}{m n_{sp}} \tag{2.8}$$

By taking $n_{sp} = 2$, we get

$$\frac{\sigma_s^2}{\sigma_n^2} = \frac{\overline{N}_p}{2m} \tag{2.9}$$

By using the relation $x = 2m/\overline{N}_p$ we get the ratio between the variance of the signal and the variance of the noise as

$$\frac{\sigma_s^2}{\sigma_n^2} = \frac{1}{x} \tag{2.10}$$

The above relation will be important when normalizing the saddlepoint approximation equations.

Now applying the Saddlepoint Approximation, which is described in Appendix A, we get the 'phase' function for the ON-case as [19,22]:

$$\phi_{ON}(s) = \ln\left[1 - 2\left(\sigma_s^2 + \sigma_n^2\right)s\right]^{-2m} - s\alpha - \ln\left|s\sigma_n^2\right| \qquad (2.11)$$

To normalize the above equation we need to multiply and divide each element by σ_n^2, thus getting:

$$\phi_{ON}(s) = \ln\left[1 - 2\left(\frac{\sigma_s^2}{\sigma_n^2} + 1\right)s\sigma_n^2\right]^{-2m} - \left(\sigma_n^2 s\right)\left(\frac{\alpha}{\sigma_n^2}\right) - \ln\left|s\sigma_n^2\right| \qquad (2.12)$$

By making use of the following relations

$$s' = \sigma_n^2 s \qquad (2.13)$$

$$\alpha' = \frac{\alpha}{\sigma_n^2} \qquad (2.14)$$

we get

$$\phi_{ON}(s) = \ln\left[1 - 2\left(\frac{1}{x} + 1\right)s\right]^{-2m} - s\alpha - \ln|s| \qquad (2.15)$$

where s and α here (and henceforth) denote the normalized variables denoted by s' and α' in (2.13) and (2.14).

The first derivative of equation (2.15) is given by:

$$\phi'_{ON}(s) = -\frac{1}{s} - \alpha - \frac{n\lambda_1}{1-\lambda_1 s} \tag{2.16}$$

where:

$$\lambda_1 = 2\left(\frac{1}{x} + 1\right) \tag{2.17}$$

and

$$n = -2m \tag{2.18}$$

The second derivative of equation (2.15) is given by:

$$\phi''_{ON}(s) = \frac{1}{s^2} - \frac{n\lambda_1^2}{(1-\lambda_1 s)^2} \tag{2.18}$$

The Bit Error Rate (BER) contribution of the ON-case depends on the negative root of the following equation [19,22]:

$$\phi'_{ON}(s) = 0 \tag{2.19}$$

The above equation can be transformed into a 2nd-degree polynomial equation of the form:

$$a_1 s^2 + b_1 s + c_1 = 0 \tag{2.20}$$

where

$$a_1 = \alpha\lambda_1$$
$$b_1 = (\lambda_1 - \alpha - n\lambda_1)$$
$$c_1 = -1$$

This polynomial has two distinct real roots, one of which is positive and one of which is negative. For purposes of calculating the Bit Error Rate (BER) the negative root denoted as s_{ON} is taken in the ON-case. The negative root is taken [23], as it minimizes the 'phase' function given by (2.15), the positive root gives the maximum.

The BER contribution of the ON-case can be expressed by the following equation [20]:

$$Pe_{ON} = \frac{\exp[\phi(s_{ON})]}{\sqrt{2\pi\phi''(s_{ON})}} \tag{2.21}$$

2.1.2. OFF-Case

The OFF-case is very similar to the ON-case, but only the noise is present, thus the MGF is of the form:

$$M_{OFF}(s) = \left[1 - 2\sigma_{OFF}^2 s\right]^{-2m} = \left[1 - 2\sigma_n^2 s\right]^{-2m} \tag{2.22}$$

Applying the saddlepoint approximation we obtain:

$$\phi_{OFF}(s) = \ln\left[1 - 2\sigma_n^2 s\right]^{-2m} - s\alpha - \ln\left|s\sigma_n^2\right| \tag{2.23}$$

The normalized form of the equation above is

$$\phi_{OFF}(s) = \ln\left[1 - 2s\right]^{-2m} - s\alpha - \ln|s| \tag{2.24}$$

The first derivative of this function is:

$$\phi'_{OFF}(s) = -\frac{1}{s} - \alpha - \frac{2n}{1-2s} \tag{2.25}$$

And for the second derivative we get:

$$\phi''_{OFF}(s) = \frac{1}{s^2} - \frac{4n}{(1-2s)^2} \tag{2.26}$$

The Bit Error Rate (BER) contribution of the OFF-case depends on the positive root of the equation:

$$\phi'_{OFF}(s) = 0 \tag{2.27}$$

The above equation can be transformed into a 2nd-degree polynomial equation of the form:

$$(2\alpha)s^2 + (2 - \alpha - 2n)s - 1 = 0 \tag{2.28}$$

This polynomial has two distinct real roots, one of which is positive and one of which is negative. For purposes of calculating the BER the positive root which is denoted as s_{OFF} is taken in the OFF-case. The positive root is taken here as it minimizes the phase function (2.24); the negative root is not taken here because it maximizes the phase function.

The BER contribution of the OFF-case can be expressed by the following equation [22]:

$$q_{-}(\alpha) = \frac{\exp[\phi(s_{OFF})]}{\sqrt{2\pi\phi''(s_{OFF})}} \tag{2.29}$$

2.1.3. Results

After calculating all the parameters for the ON and OFF-cases, the Bit Error Rate (BER) contributions of both cases were calculated according to equations (2.21) and (2.29) and then averaged to obtain the overall BER for the system. As we wanted to evaluate the performance of the receiver in terms of the receiver sensitivity N_p and $m = B_0T$, the BER was initially fixed as 10^{-9}. The BER equation is dependent on three different parameters $(m, \overline{N}_p, \text{and}\, \alpha)$; thus the following method was used to obtain a curve of N_p as a function of m:

- A value for m was fixed.
- A range for the threshold α was set.

- An arbitrary value for $\overline{N}_p$ was chosen.
- The BER was calculated.
- If the calculated BER vector had a minimum at 10^{-9}, then the value of $\overline{N}_p$ was chosen as a match for *m*.
- If the calculated BER vector had a minimum lower than 10^{-9}, then the chosen value of $\overline{N}_p$ was lowered until getting the minimum BER vector value at 10^{-9}.
- If the calculated BER vector had a minimum higher than 10^{-9}, then the chosen value of $\overline{N}_p$ was increased until getting the minimum BER vector value at 10^{-9}.

The results are shown in Figure 2.2. When compared to the results obtained by Arya and Jacobs in [18] using the actual chi-square distributions, the results obtained by the Saddlepoint Approximation are validated, as they are identical to those obtained by the exact calculation. The method described above was also used to evaluate the receiver performance for a fixed BER of 10^{-6}. These results are shown in Figure 2.3, where they are compared to the results obtained for a BER of 10^{-9}. As can be seen, when the BER is fixed at 10^{-6} a smaller number of photons per bit is required.

2.2. The Gaussian Approximation

The Gaussian distribution is often used to obtain an approximate expression for the error probability for optical receivers. It is a simple approximation and does not require a lot of computational power.

The error probability when using the Gaussian Approximation is given by the following expression [22,24]:

$$\begin{aligned} P_e &= \frac{1}{\sqrt{2\pi}} \int_Q^{\infty} \exp\left(-x^2/2\right) dx \\ &\approx \frac{1}{Q\sqrt{2\pi}} \exp\left(-Q^2/2\right) \end{aligned} \tag{2.30}$$

where

$$Q = \frac{\mu_1 - \mu_0}{\sigma_1 + \sigma_0} \tag{2.31}$$

μ_1 and μ_0 are the means for the ON and OFF cases and σ_1 and σ_0 are the standard deviations. These can be easily extracted with the use of the Moment Generating Function of the received signal.

The MGF for the ON-case is given by (2.15), with its use we can extract the mean and the standard deviation [21,25]

$$\mu_1 = \mu_{ON} = M'_{ON}(0) \tag{2.32}$$

$$\therefore \mu_1 = 4m\left(\frac{1}{x} + 1\right) \tag{2.33}$$

$$\sigma_1^2 = \sigma_{ON}^2 = M''_{ON}(0) - \left[M'_{ON}(0)\right]^2 \tag{2.34}$$

$$\therefore \sigma_1 = \sqrt{8m}\left(\frac{1}{x} + 1\right) \tag{2.35}$$

For the OFF-case, the mean and standard deviation are calculated in the same way and are given by:

$$\mu_0 = 4m \tag{2.36}$$

$$\sigma_0 = \sqrt{8m} \tag{2.37}$$

With the use of these parameters we can now obtain a complete expression for the error probability. The Q factor defined by (2.31) is given by

$$Q = \frac{\sqrt{2m}\left(1/x\right)}{\left(2 + 1/x\right)} \tag{2.38}$$

By replacing *x* with $2m / \overline{N}_p$ we obtain:

$$Q = \frac{\left(\overline{N}_p / \sqrt{2m} \right)}{\left(2 + \overline{N}_p / 2m \right)} \tag{2.39}$$

By solving equation (2.39) for $\overline{N}_p$ the following expression is obtained

$$\overline{N}_p = \frac{4mQ}{\sqrt{2m} - Q} \tag{2.40}$$

It is seen from equation (2.40) that the Gaussian Approximation gives an error floor; that is, for a given value of m, the maximum *Q* that may be achieved is $Q = \sqrt{2m}$.

As it can be seen the error probability expression depends only on two parameters, *m* and $\overline{N}_p$. The relation between these two parameters is plotted in Figure 2.4 for Bit Error Rates of 10^{-6} and 10^{-9}. As it can be seen, both plots have the same characteristic. They both have high values of $\overline{N}_p$ for low values of *m*. $\overline{N}_p$ decreases for increasing value of *m* until it reaches a minimum and starts to increase again. These minimum values are the optimum values for receiver operation. For a BER of 10^{-6}, the optimum values are approximately $m = 50$ and $\overline{N}_p = 181.7$, and for a BER of 10^{-9}, the optimum values are approximately $m = 70$ and $\overline{N}_p = 288.2$.

The Saddlepoint approximation and the Gaussian approximation are compared in Figure 2.5 and 2.6 for Bit Error Rates of 10^{-6} and 10^{-9}. As it can be seen, the saddlepoint approximation gives better results while the Gaussian approximation gives much more conservative results, especially for lower values of *m*.

Although the Gaussian approximation does not give very good results it is often used in the calculation of receiver performance due mainly to its simplicity and low computational complexity. The saddlepoint approximation has a little higher computational complexity mainly due to the necessity of finding the optimum threshold while calculating the BER.

The optimum value of m is evaluated for different error probabilities. The results are shown in Figure 2.7 where the results were calculated with the aid of the Saddlepoint Approximation. In Figure 2.8 these results are compared to results obtained with the Gaussian Approximation. At these optimum values of m there are corresponding minimum values of the number of photons per bit. These minimum values are shown in Figures 2.9 and 2.10. As it can be seen, both the optimum m and the minimum sensitivity vary essentially linearly with log P_e where both the optimum m and the minimum N_p are smaller using the saddlepoint than the Gaussian approximation. Note also that the saddlepoint results are essentially identical to the exact calculations in [18]. This validates the saddlepopint method, and allows us to then use it in Chapter 4 for situations where the exact calculation is much more difficult.

With these results described above it is possible to answer the following question [18]: for a given source noise bandwidth, what is the total transmission capacity at the optimum value of m. The total transmission capacity T_{cap} of the SS-WDM system is given by

$$\begin{aligned} T_{cap} &= NR_b = \frac{B_{ss} R_b}{MB_0} \\ &= \frac{B_{SS}}{mM} \end{aligned} \tag{2.41}$$

in which N is the total number of channels in the system, R_b is the bit rate per channel, B_{SS} is the available gain bandwidth of the spectrum-sliced source, and M is the ratio of the channel spacing to the optical bandwidth B_0 . The transmission capacity is evaluated for different error probabilities, and the results when the Saddlepoint Approximation is used are shown in Figure 2.11. It should be noted that B_{SS}, the available gain bandwidth of the spectrum-sliced source is fixed at 35 nm, and M is 3. These results indicate that the spectrum-sliced OOK system with an optical preamplifier has a transmission capacity of approximately 45 Gb/s when operated at a BER of 10^{-9}. The results obtained for the transmission capacity with the Saddlepoint Approximation are compared to those obtained with the Gaussian Approximation, this comparison is shown in Figure 2.12. As it can be seen the Gaussian Approximation gives very pessimistic results. The transmission capacity obtained with the Gaussian Approximation is approximately 20 Gb/s when the system operates with a BER of 10^{-9}, which is considerably lower than that obtained with the Saddlepoint Approximation.

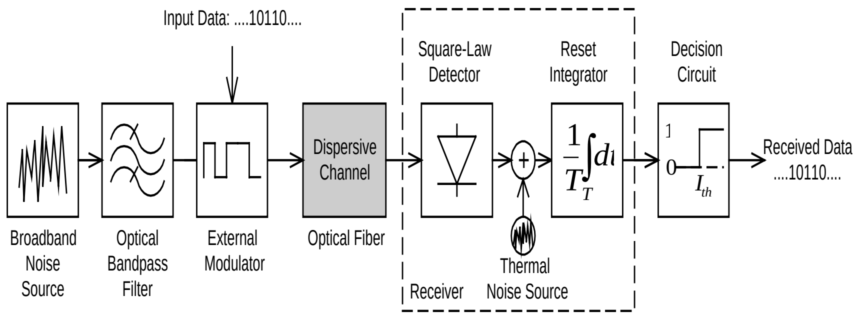

Fig 2.1 *SS-WDM complete system used in calculations*

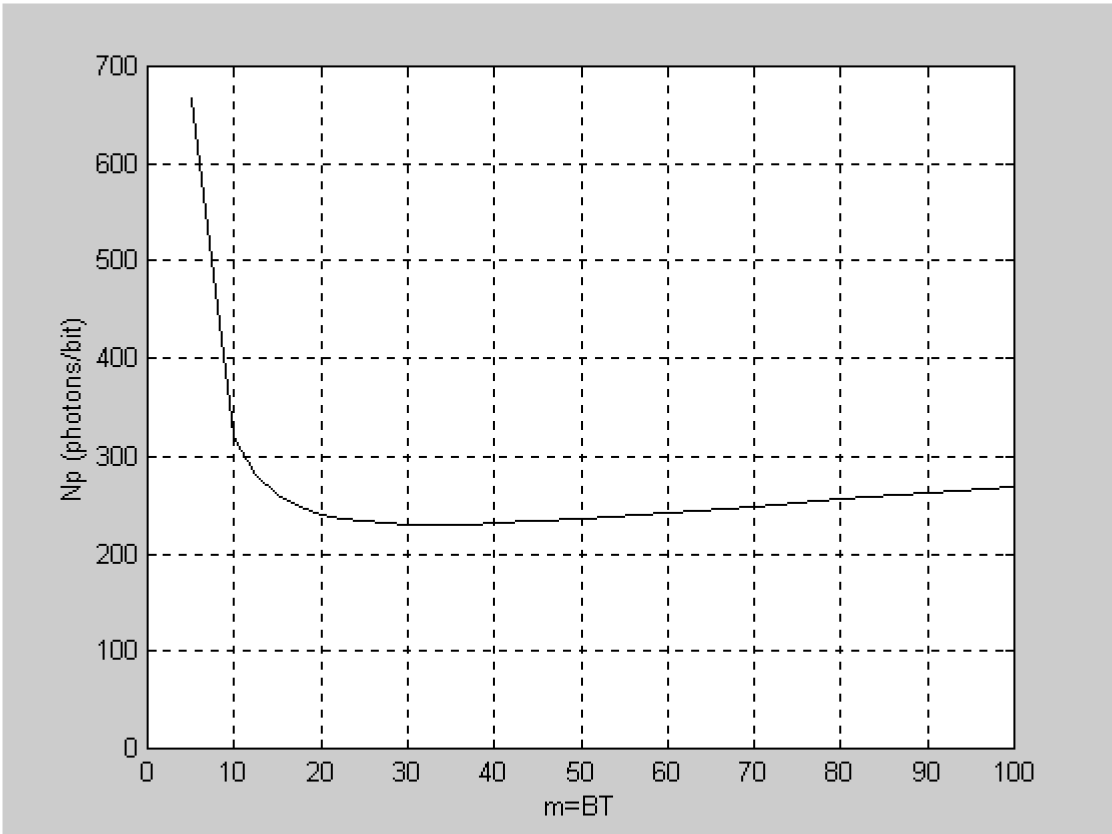

Fig 2.2 *Receiver Sensitivity at Pe=10^{-9} for an optical preamplifier receiver as calculated with the Saddlepoint Approximation*

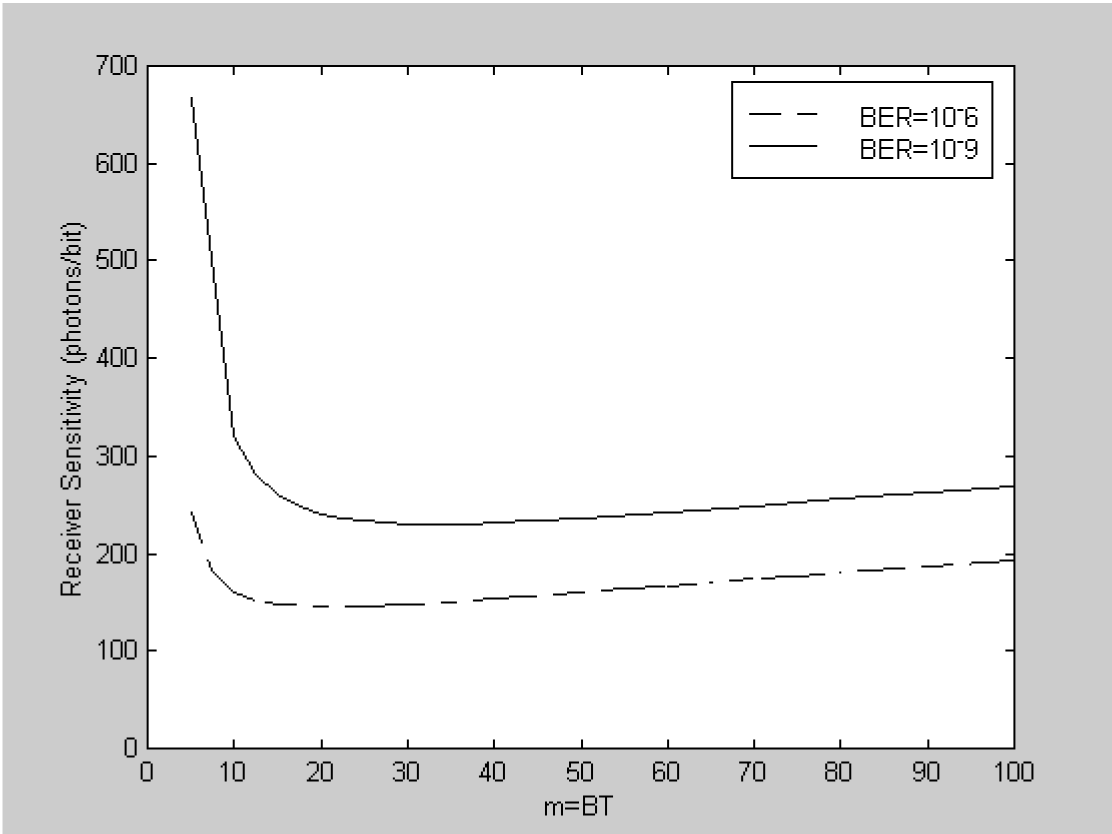

Fig 2.3 *Receiver Sensitivity at Pe=10^{-6} and 10^{-9} for an optical preamplifier receiver as calculated with the Saddlepoint Approximation*

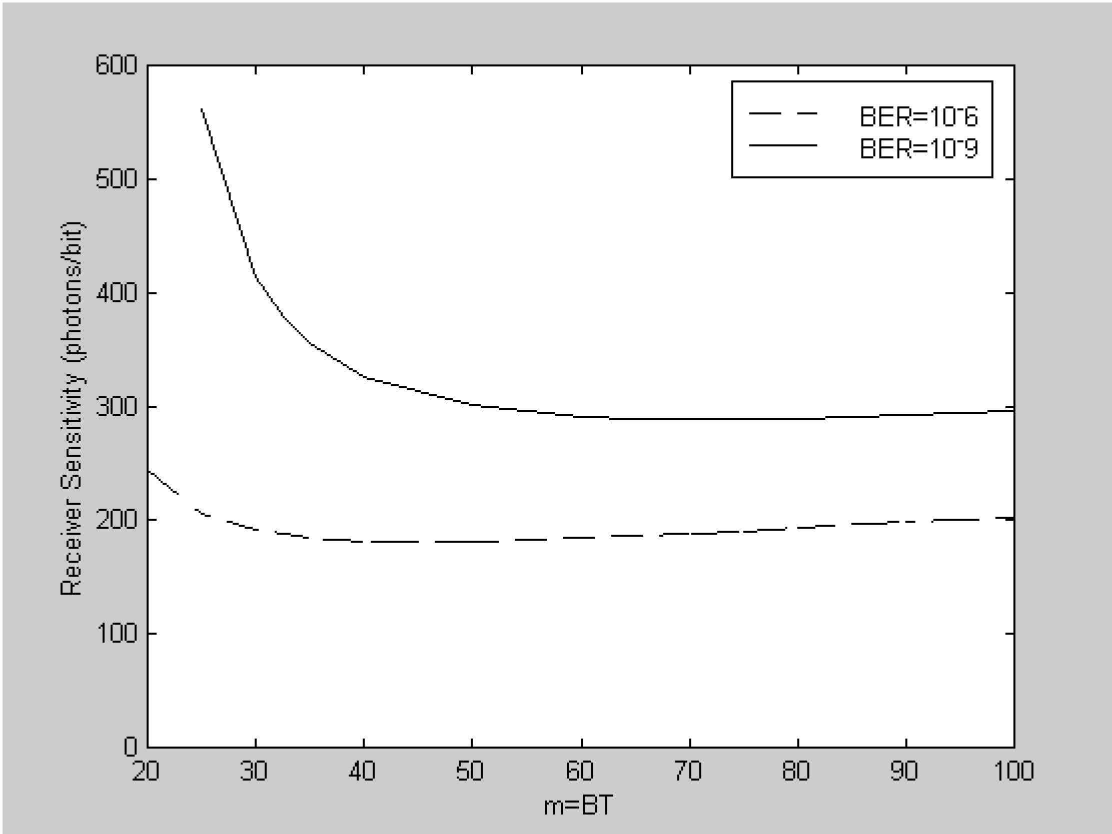

Fig 2.4 *Receiver Sensitivity at Pe=10^{-6} and 10^{-9} for an optical preamplifier receiver as calculated with the Gaussian Approximation*

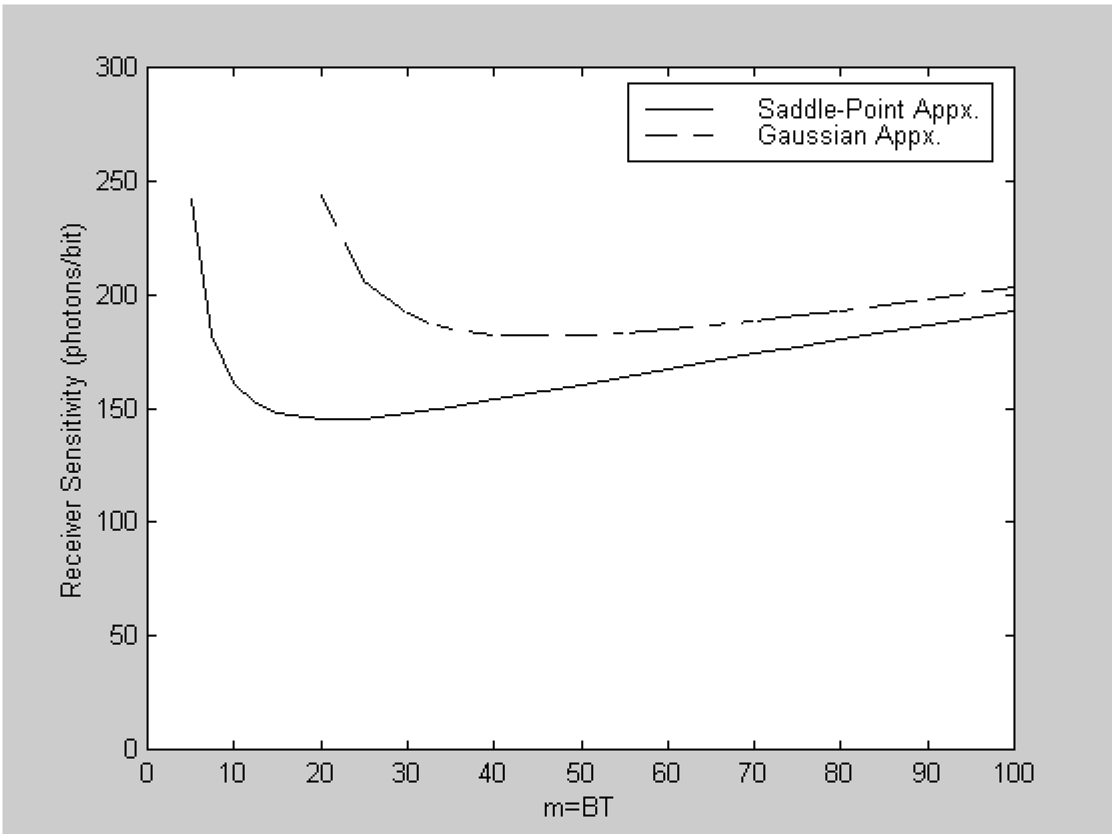

Fig 2.5 *Receiver Sensitivity at Pe=10^{-6} for an optical preamplifier receiver as calculated with the Gaussian Approximation and the Saddlepoint Approximation*

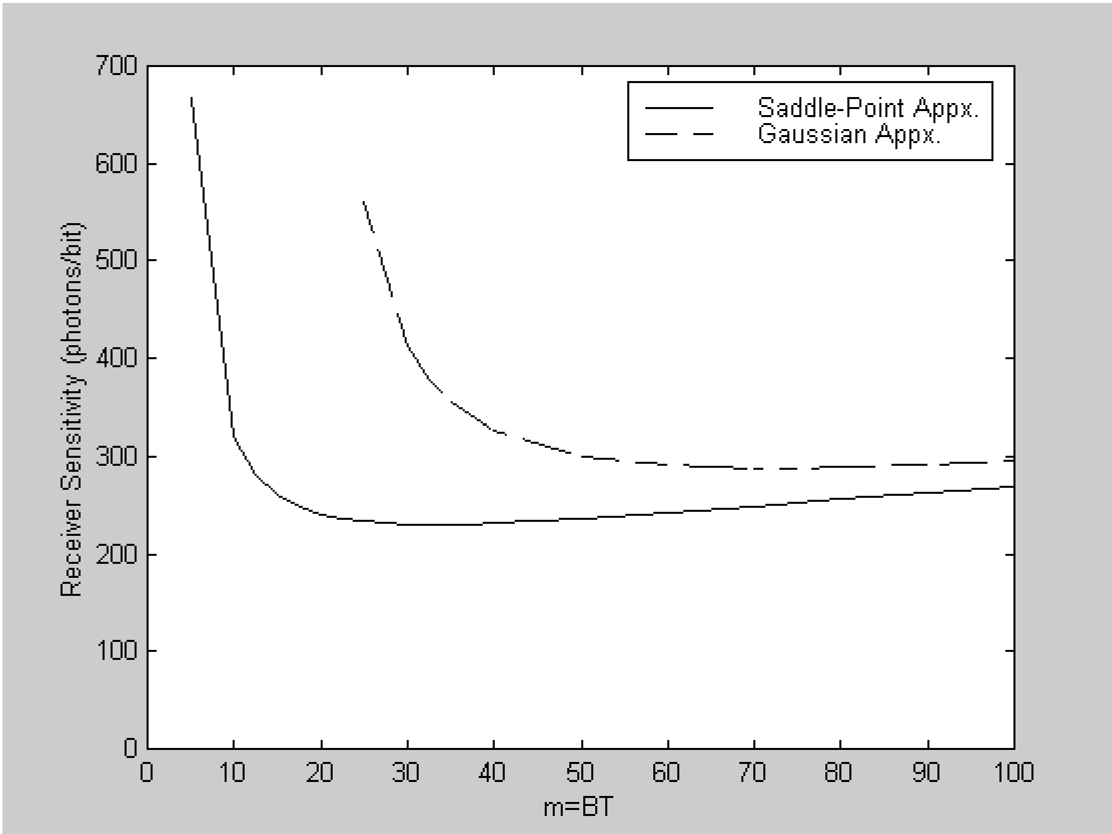

Fig 2.6 *Receiver Sensitivity at Pe=10^{-9} for an optical preamplifier receiver as calculated with the Gaussian Approximation and the Saddlepoint Approximation*

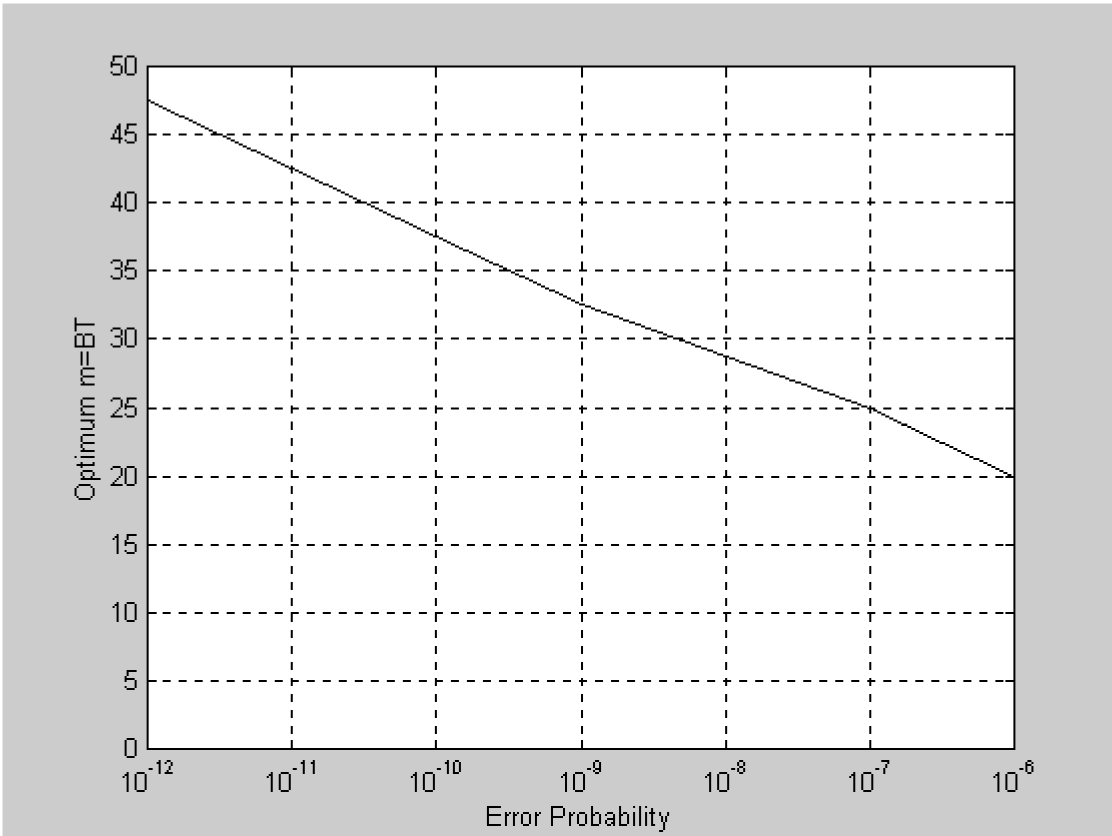

Fig 2.7 *Optimum $m=B_oT$ evaluated at different error probabilities as calculated with the Saddlepoint Approximation*

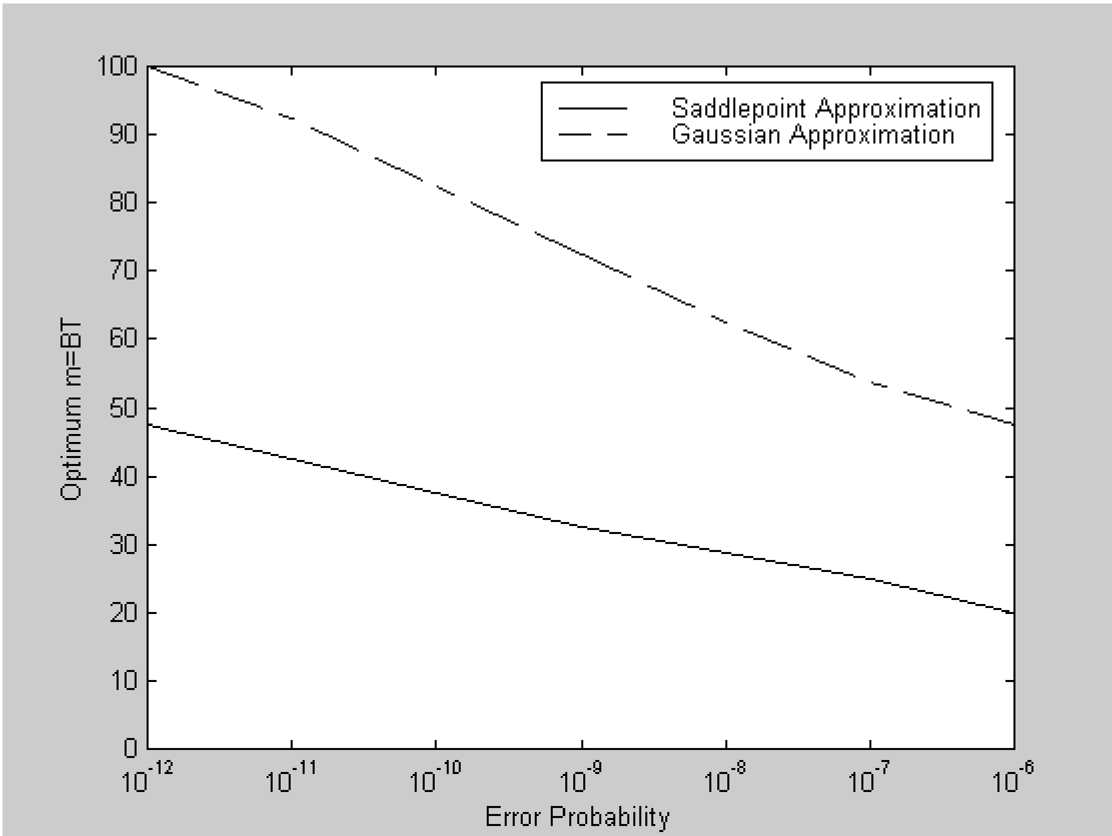

Fig 2.8 *Optimum $m=B_oT$ evaluated at different error probabilities as calculated with the Gaussian Approximation and the Saddlepoint Approximation*

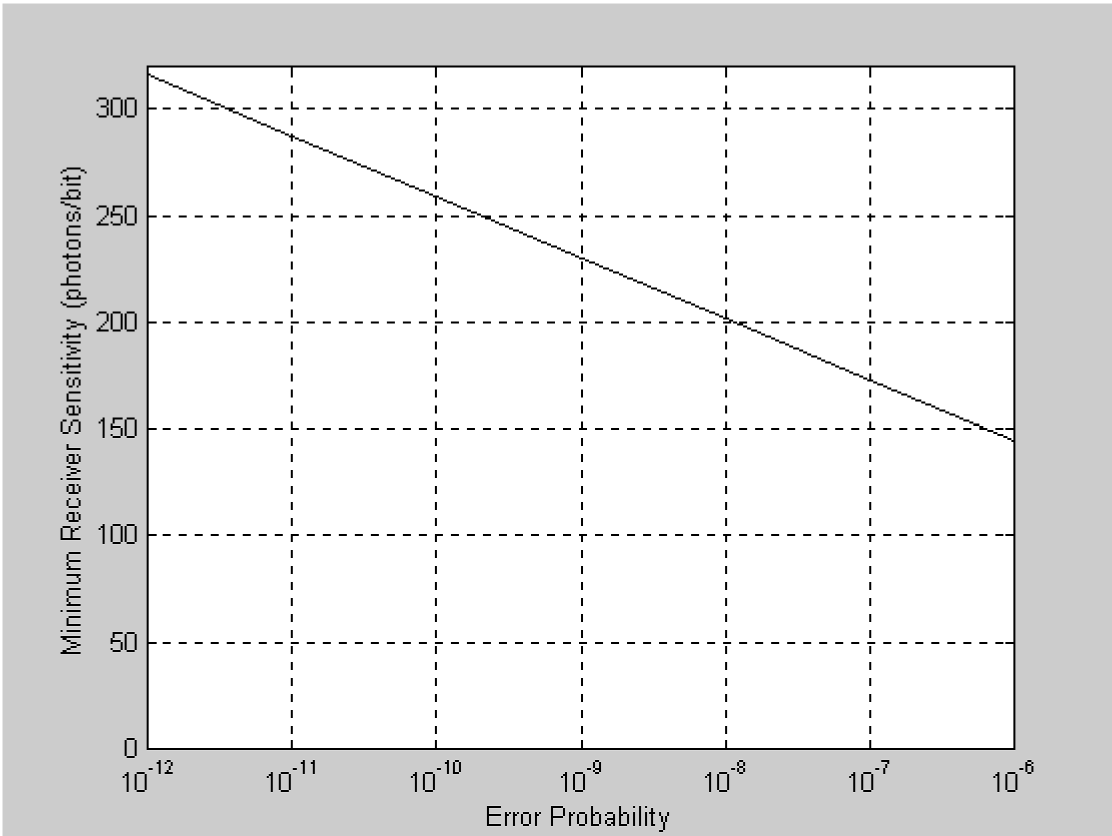

Fig 2.9 *Minimum receiver sensitivity evaluated at different error probabilities as calculated with the Saddlepoint Approximation*

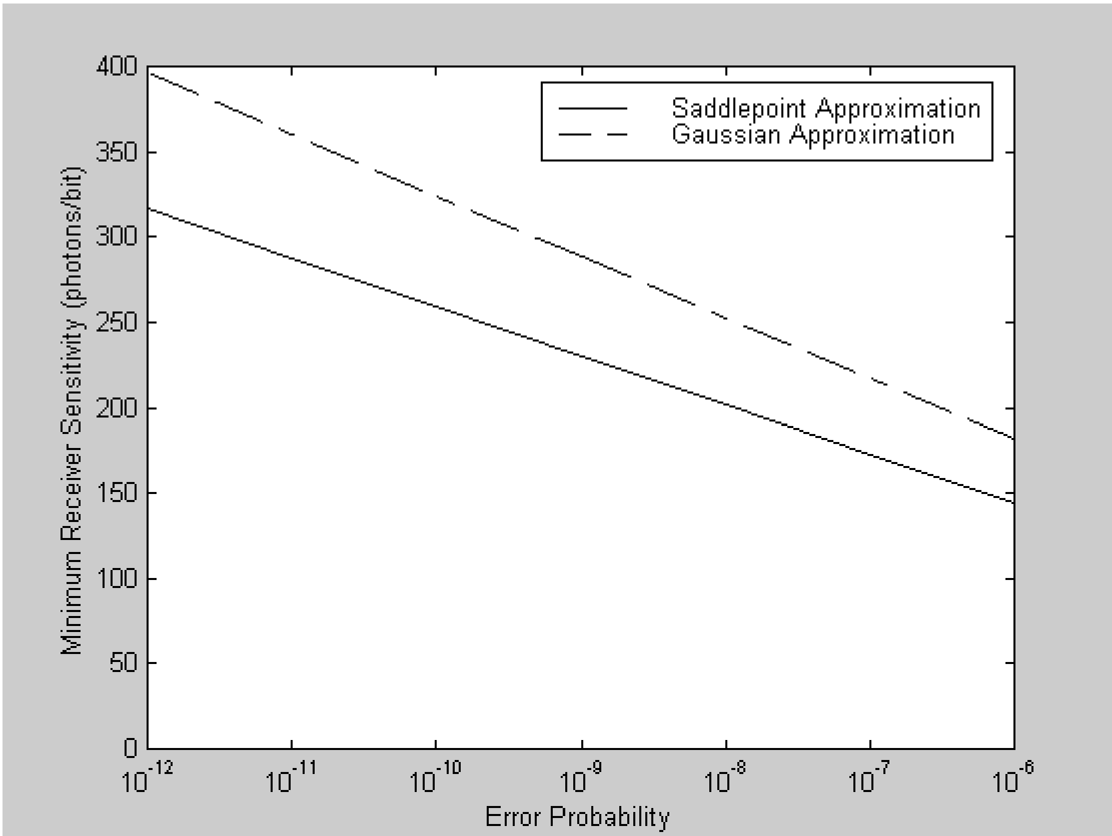

Fig 2.10 *Minimum receiver sensitivity evaluated at different error probabilities as calculated with the Gaussian Approximation and the Saddlepoint Approximation*

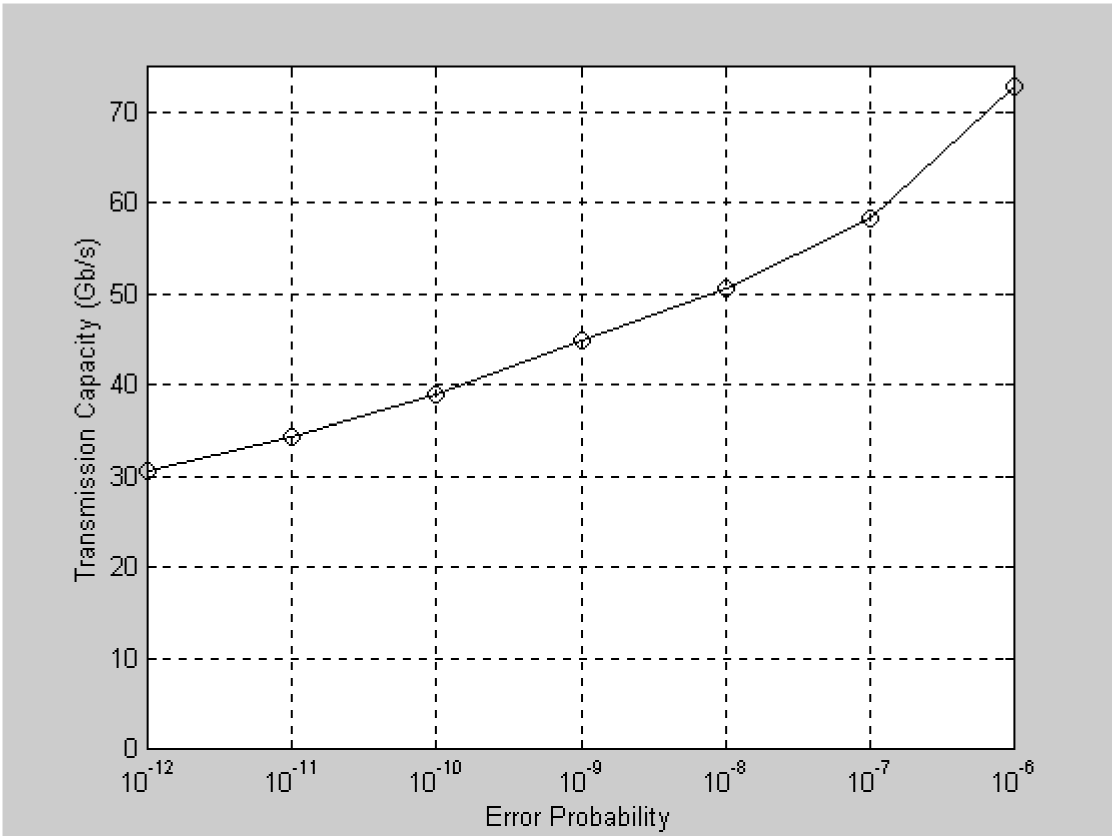

Fig 2.11 *Transmission capacity in Gb/s for an SS-WDM system operating at the optimum calculated with the Saddlepoint Approximation.*

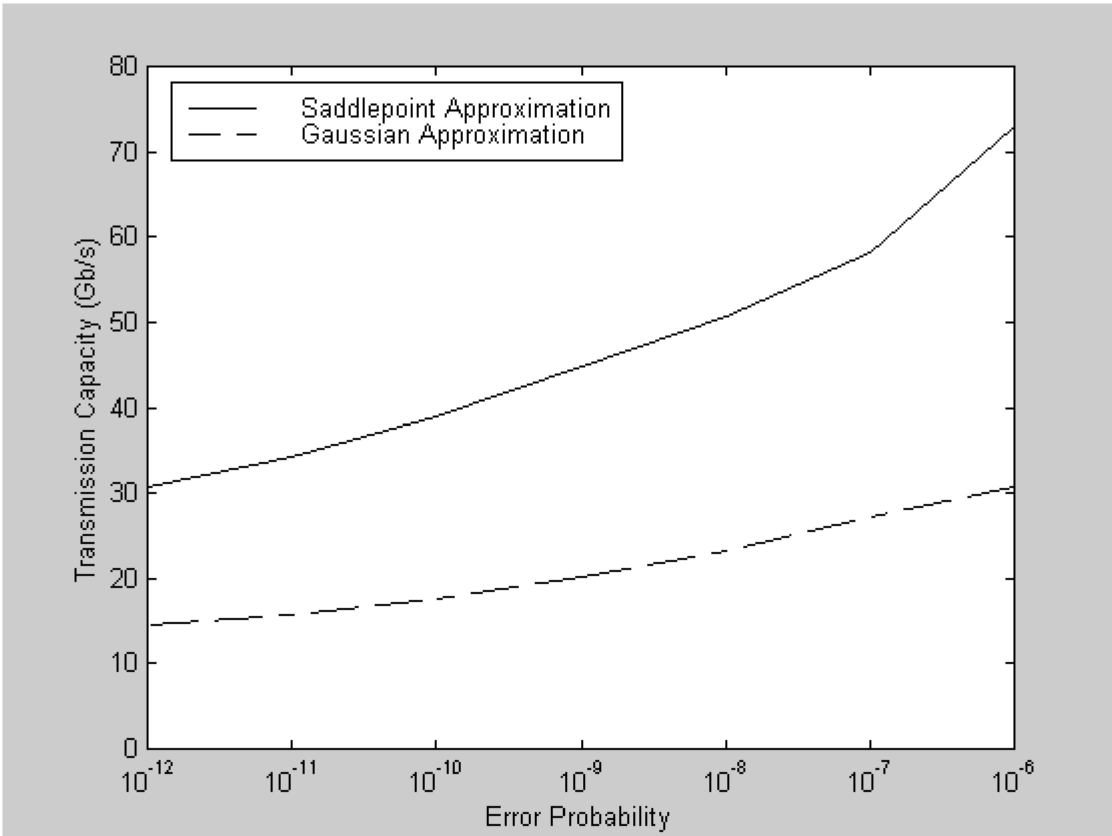

Fig 2.12 *Transmission capacity in Gb/s for an optical preamplifier receiver-based SS-WDM system operating at the optimum calculated with the Saddlepoint and Gaussian Approximations.*