

Effect of Non-Rectangular Spectra on Receiver Sensitivity

So far the receiver performance analysis assumed ideal rectangular spectra for all of the filters in our system. This chapter analyzes the effect of non-rectangular filter shapes on the receiver sensitivity. Unlike the case of the rectangular spectra analyzed in Chapter 2, the channel seen by the signal and the noise is different now. While the signal path contains two optical filters, one used to spectrum-slice at the transmitter and another used at the receiver to reject out-of-band ASE noise in the preamplifier and also to select the appropriate WDM channel, the noise path contains only one filter due to the fact that the main source of noise is the optical preamplifier. The method used to analyze the effect of non-rectangular spectra on the receiver sensitivity is similar to that used by Arya in [7]. There is however, one significant difference; while Arya made the assumption that when the data signal and noise are added the resulting signal could also be approximated as being chi-square distributed, we did not make such an assumption. Even though the noise and the data signal each have a chi-square distribution, the sum will not be chi-square distributed. This distribution is not needed when applying the Saddlepoint Approximation which only requires the MGF of the received signal to evaluate the receiver's performance.

3.1. Butterworth Filter Approximation

In WDM applications, Fiber Fabry-Perot (FFP) filters are extensively used because of their simple construction and ready availability. The problem with FFPs is that their passband modeling for system simulation purposes is often hampered by their multiple passband resonances. In the limit of high reflectivity, a single resonant passband of the FFP filter may be assumed to be approximately Lorentzian in shape [7].

The optical filters used in this analysis are modeled as general Butterworth filters. We assumed this because the first order Butterworth filter is equivalent to a Lorentzian

passband. And also because it facilitates the analysis when dealing with the effect of varying the filter response on receiver sensitivity.

The magnitude squared transfer function of a Butterworth filter can be expressed by

$$|H_0(f)|^2 = \frac{1}{1 + (f/f_0)^{2N}} = \frac{1}{1 + (2f/B_0)^{2N}} \quad (3.1)$$

In the equation we have:

f - difference of frequency from the operating optical frequency (Hz)

f_0 - the 3dB baseband bandwidth of the filter (Hz)

B_0 - 3dB bandpass bandwidth of the optical filter (Hz)

N - order of the Butterworth filter

Figure 3.1 shows the normalized frequency response of the Butterworth filter for varying N .

3.2. Mathematical Formulation

The system with the non-rectangular spectra filter is shown in Figure 3.2. If we define P_0 as the single-sided, single-polarization, power spectral density of the source, scaled by the attenuation of the fiber, the following equation can be written [7]:

$$P_0 B_{0,eqv,N} = \bar{N}_p h \nu R_b \quad (3.2)$$

in which $B_{0,eqv,N}$ is the equivalent rectangular bandwidth of the transmitter optical filter which allows the same amount of power to pass as the Butterworth filter. This equivalent bandwidth can be calculated from

$$B_{0,eqv,N} = \frac{1}{|H_0(0)|^2} \int_{-\infty}^{\infty} |H_0(f)|^2 df \quad (3.3)$$

Using equation (3.1), we obtain

$$B_{0,eqv,N} = C_{eqv,N} B_0 \quad (3.4)$$

where

$$C_{eqv,N} = \frac{\pi}{2N \sin(\pi / 2N)} \quad (3.5)$$

Using (3.2) and (3.4), we obtain the following expression for P_0

$$P_0 = \frac{\bar{N}_p h \nu R_b}{C_{eqv,N} B_0} \quad (3.6)$$

As the signal channel is given by the cascade of two optical filters, which are assumed to be centered identically, the optical power spectrum of the signal at the photodetector can be expressed by

$$P_s(f) = P_0 G |H_0(f)|^2 |H_0(f)|^2 \quad (3.7)$$

The G term is the power gain of the optical preamplifier. Using (3.7), the average signal power at the photodetector is obtained as

$$P_s = 2\sigma_{sN}^2 = 2 \int_{-\infty}^{\infty} P_s(f) df \quad (3.8)$$

In (3.8), σ_{sN}^2 is the signal power per polarization when a 1 is transmitted and the subscript N refers to the filter order.

By evaluating the integral [26] in the equation we obtain

$$P_s = 2P_0 G C_{sN} B_0 \quad (3.9)$$

where C_{sN} shows the effect of the filter order on the signal power and is given by

$$C_{sN} = \frac{(2N-1)\pi}{4N^2 \sin(\pi/2N)} \quad (3.10)$$

By using (3.8), (3.9) and (3.10) the following expression for the signal power per polarization σ_{sN}^2 is obtained

$$\sigma_{sN}^2 = P_0 G C_{sN} B_0 \quad (3.11)$$

For the noise, the single-sided, single polarization PSD at the output of the preamplifier is given by

$$N_0 = n_{sp} h \nu (G-1) \quad (3.12)$$

The noise power spectrum at the photodetector can be expressed as

$$P_n(f) = N_0 |H_0(f)|^2 \quad (3.13)$$

And the total noise power including both polarizations can be expressed as

$$P_n = 2\sigma_{nN}^2 = 2 \int_{-\infty}^{\infty} P_n(f) df \quad (3.14)$$

with σ_{nN}^2 being the noise power in one polarization.

By evaluating the integral in (3.14) and using (3.1), the noise power is obtained as

$$P_n = 2N_0 C_{nN} B_0 \quad (3.15)$$

where

$$C_{nN} = \frac{\pi}{2N \sin(\pi/2N)} \quad (3.16)$$

With the use of (3.14), (3.15), and (3.16) the following expression for the noise power is obtained

$$\sigma_{nN}^2 = N_0 C_{nN} B_0 \quad (3.17)$$

3.3. Transmission Analysis

To analyze the effect of non-rectangular filter shapes on the receiver sensitivity using the Saddlepoint approximation we need to include the order of the Butterworth filter in the Moment Generating Function (MGF). The Butterworth filter influences directly the number of degrees of freedom for the signal and the noise components as will be shown next.

It is assumed that the energy of the signal and noise is individually chi-square distributed with degrees-of-freedom obtained by matching the first two moments but that of their sum is not. This differs from the assumption in [7] as that the signal and noise are chi-square distributed and so is their sum. Indeed, a prime purpose of this section is to investigate the differences between these two assumptions.

We will begin the analysis with the signal transmission and will then obtain results in a similar fashion for the noise.

3.3.1. Signal Transmission

The variance of the spectrum-sliced signal alone can be expressed by [7,27]

$$\text{var } I_s = \frac{2}{T} \left[\int_{-\infty}^{\infty} |P_s(f)|^2 df \right] \quad (3.18)$$

By substituting (3.1) , (3.7) into (3.18) and using the definition of m we obtain

$$\text{var } I_s = \frac{2m}{T^2} P_0^2 K_{sN} \quad (3.19)$$

where the constant K_{sN} is given by

$$K_{sN} = \frac{(6N-1)(4N-1)(2N-1)\pi}{96N^4 \sin(\pi/2N)} \quad (3.20)$$

The variance of I can be written as a function of m in the following form [7,18]:

$$\text{var } I_{(s/n)} = \frac{2\sigma_{(s/n)N}^4}{m_{(s/n)}} \quad (3.21)$$

By using (3.11), (3.19) and (3.21), an expression for m_{sN} (the number of degrees of freedom for the signal component) is obtained as

$$m_{sN} = \frac{m}{a} \quad (3.22)$$

with the constant a being defined as

$$a = \frac{(6N-1)(4N-1)\sin(\pi/2N)}{6\pi(2N-1)} \quad (3.23)$$

3.3.2. Noise Transmission

The variance of the noise component can be expressed by [7,27]

$$\text{var } I_n = \frac{2}{T} \left[\int_{-\infty}^{\infty} |P_n(f)|^2 df \right] \quad (3.24)$$

By substituting (3.1), (3.13) into (3.18) and using the definition of m we obtain

$$\text{var } I_n = \frac{2m}{T^2} N_0^2 K_{nN} \quad (3.25)$$

where the constant K_{nN} is given by

$$K_{nN} = \frac{(2N-1)\pi}{4N^2 \sin(\pi/2N)} \quad (3.26)$$

By using (3.11), (3.19) and (3.21), an expression for m_{nN} (the number of degrees of freedom for the noise component) is obtained as

$$m_{nN} = \frac{m}{b} \quad (3.27)$$

with the constant b being defined as

$$b = \frac{(2N-1) \sin(\pi/2N)}{\pi} \quad (3.28)$$

3.3.3. Evaluation of Filter Influence

Initially, the transmission of "1" is analyzed, in this case both signal and noise are present. Assuming that the received signal $z(t)$ has two components, the signal component $z_s(t)$ and the noise component $z_n(t)$. The received signal then will have an energy in the following form:

$$E = \frac{1}{T} \int_0^T z^2(t) dt = \frac{1}{T} \left(\int_0^T z_s^2(t) dt + \int_0^T z_n^2(t) dt + 2 \int_0^T z_s(t) z_n(t) dt \right) \quad (3.29)$$

With the use of a Karhunen-Loeve expansion (which is explained more completely in Chapter 4), equation (3.29) can be re-written in the form

$$E = \sum_{i=1}^{m_s} z_s^2 + \sum_{i=1}^{m_n} z_n^2 + 2 \sum_{i=1}^{m_s} z_s z_n \quad (3.30)$$

which can be simplified into

$$E = \sum_{i=1}^{m_s} (z_s + z_n)^2 + \sum_{i=m_s+1}^{m_n} z_n^2 \quad (3.31)$$

It should be noted that the Karhunen-Loeve expansion has an infinite number of terms. Replacement by a finite sum of terms, all of which have the same variance, is a good approximation for the rectangular spectra case. It is an assumption, the validity of which we really don't know for the case of Butterworth spectra, although we expect it to be better when N becomes large.

Thus, the MGF for the "1" case can be expressed as:

$$M_1(s) = [1 - 2(\sigma_{sN}^2 + \sigma_{nN}^2)s]^{-2m_s} [1 - 2\sigma_n^2 s]^{-2(m_n - m_s)} \quad (3.32)$$

This equation when normalized is of the form

$$M_1(s) = [1 - 2\left(\frac{1}{x} + 1\right)s]^{-2m_s} [1 - 2s]^{-2(m_n - m_s)} \quad (3.33)$$

The variable x is defined as

$$x = \frac{\sigma_{nN}^2}{\sigma_{sN}^2} = \frac{2m}{N_p} \left[\frac{\pi}{\sin(\pi/2N)(2N-1)} \right] \quad (3.34)$$

In a similar manner, the MGF for the "0" case is also obtained and is given by

$$M_0(s) = [1 - 2\sigma_{nN}^2 s]^{-2m_n} \quad (3.35)$$

The normalized version of this equation is

$$M_0(s) = [1 - 2s]^{-2m_n} \quad (3.36)$$

Thus, we get the MGF for the "1" and "0" cases as a function of (m, \bar{N}_p, N) and for different values of N the procedure to obtain the Bit Error Rate contribution from each case is identical to the method described in Chapter 2.

By adding the contributions of the "1" and "0" case we get the final Bit Error Rate (BER). For a fixed BER of 10^{-9} and $n_{sp} = 2$ we analyzed the receiver performance, and the results are shown in Figure 3.3 which is a function of (m, \bar{N}_p) for different values of N . It should be noted that the case $N=100$ is identical to the rectangular spectra case.

Analyzing the results in Figure 3.2 we can see that the receiver sensitivity degrades when the order of the filter is reduced. The principal explanation for this is that as the order of the filter is reduced the noise power starts increasing relative to the signal power. This is because the noise passes through only one filter whereas the signal passes through two filters.

It should also be noted that while the case $N=1$ has an optimum \bar{N}_p at 1488 photons/bit which is higher than the rectangular spectra case (optimum \bar{N}_p at 230), its required value of m is lower, being at $m=20$ as compared to $m=32.5$. Although this result implies that a lower order of filter will need a lower bandwidth and will result in a larger transmission capacity for a given source bandwidth, one also has to recognize that lower order Butterworth filters have larger tails and may introduce a larger amount of interchannel interference. Thus, a complete analysis will have to take into account both the smaller value of optical bandwidth, and the larger value of interchannel interference, when the $N=1$ is considered for implementation in a spectrum-sliced system. If these results are compared to the ones obtained by Arya in [7], it can be seen that the use of the Saddlepoint Approximation gives results which are similar to the Chi-Square Approximation for higher values of N , but gives results similar to those obtained with the Gaussian Approximation when lower values of N are used. A point to be noted is that when $N=1$, the results obtained are much higher than those obtained even with the Gaussian Approximation. This is probably due to the fact that when we used a Karhunen-Loeve expansion on (3.29) we used only a finite number of terms. Similar results were also obtained for a BER of 10^{-6} and are shown in Figure 3.4.

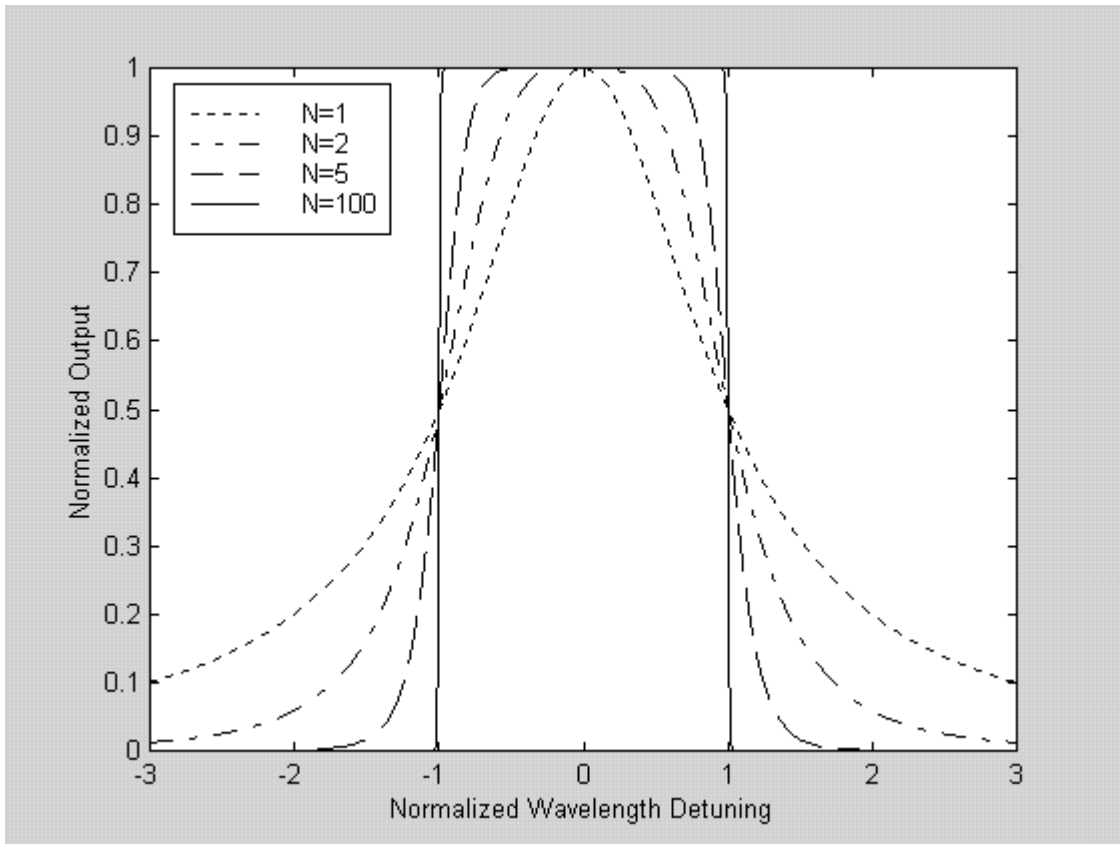


Fig 3.1 Normalized frequency response of Butterworth filter for varying N

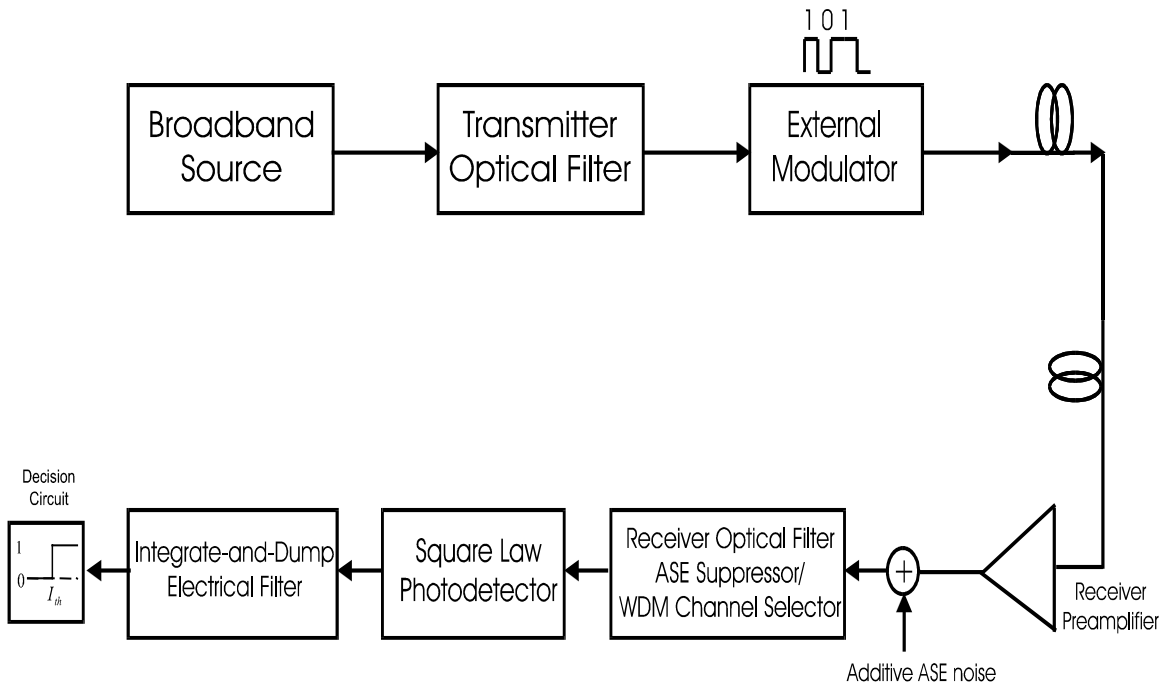


Fig 3.2 Schematic of a spectrum-sliced system to illustrate the difference between the signal and noise paths.

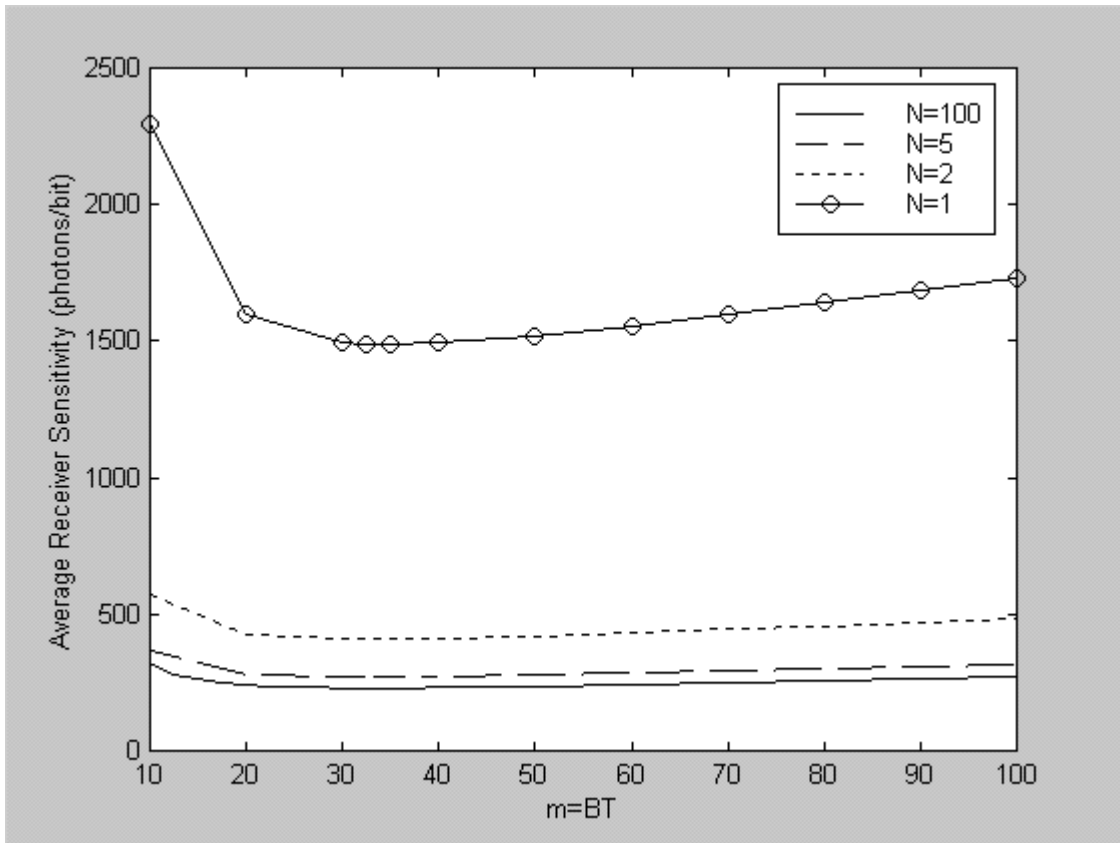


Fig 3.3 Influence of filter order on Receiver Sensitivity with $Pe=10^{-9}$

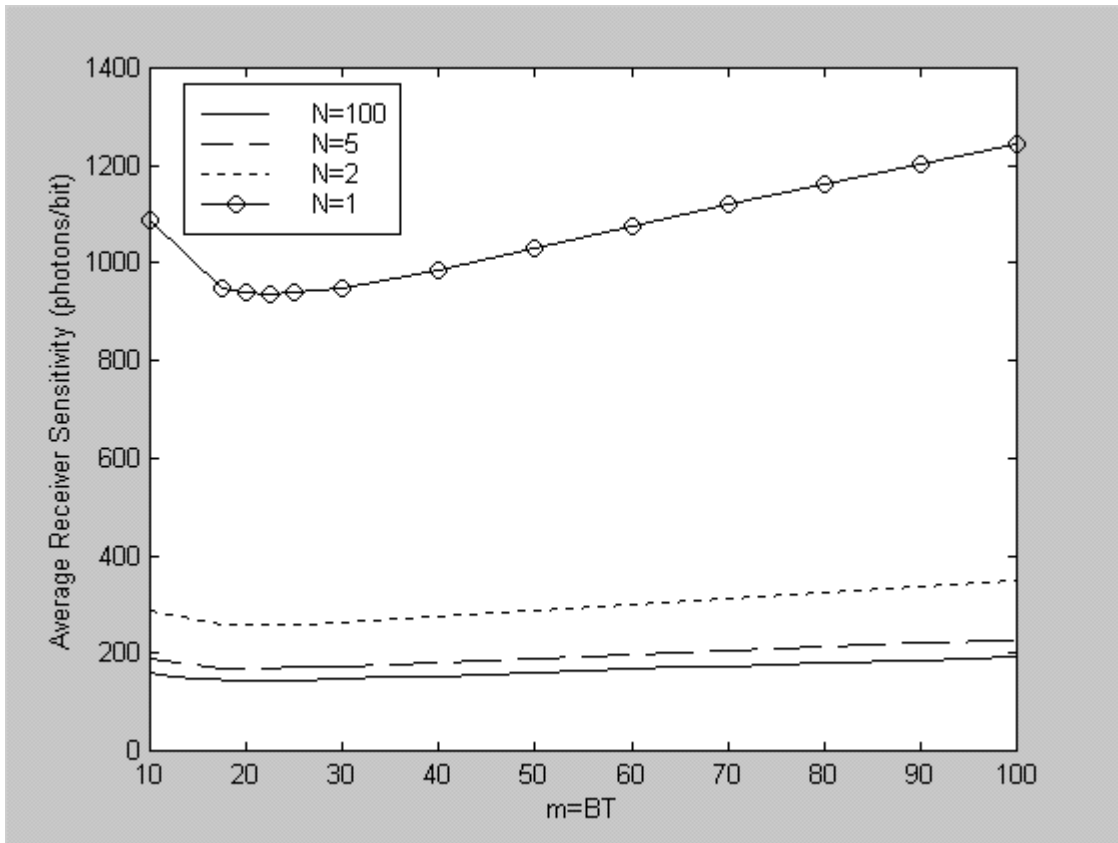


Fig 3.4 Influence of filter order on Receiver Sensitivity with $Pe=10^{-6}$