A RANDOM COEFFICIENT ANALYSIS OF THE UNITED STATES GASOLINE MARKET FROM 1960-1995

by

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ABSTRACT

This study uses a random coefficient estimation procedure to analyze the U.S. gasoline market from 1960-1995 with three main objectives: (1) provide an empirical methodology that can estimate a gasoline demand function capable of performing well in prediction; (2) evaluate the elasticities of the models presented to determine which model is more accurate at capturing supply shocks that impacted gasoline demand; and (3) evaluate the behavior of the elasticities of the beta coefficients.

This research will show that the variation from historical economic patterns was a result of supply shocks. I argue that when the OLS model of the gasoline market developed by William H. Greene is used supply shocks are not well captured because the coefficients are fixed. If the random coefficient model developed by P.A.V.B. Swamy is introduced, the coefficients vary over time, and thereby, enable supply shocks to be included in the model and more accurate forecasts are produced, as well as, meaningful time patterns in the beta coefficients.

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A=Full Sample
B=Partial Sample

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Section I: Introduction

A considerable body of economic research indicates that short-run effects of oil price fluctuations (spikes and dips) have figured prominently in the United States economic activity since World War II. Rising oil prices have preceded eight of the nine post-WWII recessions. However, an acceleration of U.S. economic activity did not seem to follow the oil price declines that occurred from the early 1980's to the late 1990's. Escalating oil prices also had less of an impact on economic activity during the last fifteen years than they did in the 35 years following WWII. There are three main objectives. The first objective is to provide an empirical methodology that can estimate a gasoline demand function capable of performing well in prediction as compared to the well-used ordinary least squares (OLS) forecasting technique in econometric literature. The second objective will compare the elasticities of the five models presented and show at least one of these models is more accurate at capturing the effects of events which impacted the demand for gasoline from 1960 through 1995 in the United States. The final objective evaluates the elasticities of own price, income, the price of new cars, and the price of old cars in both normal periods and during periods of disruption (i.e. Six Day War, Arab Oil Embargo, Iran Revolution, Iran/Iraq war and invasion of Kuwait by Iraq).

The economics profession increasingly recognizes that the classical regression assumption of constant coefficients is dubious and acknowledges a greater degree of variability than can be captured by classic autoregressive error models.[1] The trend of trying to fix classically estimated models to improve prediction has been increasing over the past decade.

Unlike the classical linear regression model, second generation Random Coefficient Models(RCM) have focused on relaxing the following assumptions frequently made by researchers in econometrics:

> (I) the true functional forms of the systematic components of economic relationships (whether linear or nonlinear) are known
> (II) excluded variables are proxied through the use of an additive error term and, therefore, have means equal to zero and are independent of the included explanatory variables

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[1] Swamy, P.A.V.B, Conway, Roger K., and LeBlanc, Michael R. "The Stochastic Coefficients Approach to Econometric Modeling Part I: A Critique of Fixed Coefficients Models." Journal of Agricultural Economics Research Vol. 40 Spring 1988, pp. 2-10

(III) variables are not subject to measurement error[2]

These models attempt to deal with the problem that arises because aggregate time series models are unlikely to have constant coefficients and are based on assumptions that:

- there is no mis-measurement
- the functional form is correct and there are no omitted variables[3]

Economic theories cannot be tested unless we know how to estimate the true economic relationships. According to Swamy and Tavlas:

> The generalized RCM corresponds to the underlying true economic relationship if each of its coefficients is interpreted as the sum of three parts: (i) a direct effect of the true value of an explanatory variable on the true value of an explained variable; (ii) an indirect effect (or omitted-variable bias) due to the fact that the true value of the explanatory variable affects the true values of excluded variables and the latter values, in turn, affect the true value of the explained variable; and (iii) an effect of mis-measuring the explanatory variable.[4]

It is plausible that the lack of predicative power of the OLS model is due, at least in part, to measurement errors in the data, the effect of omitted variables, and variation in the value of the coefficients of the model over time. This analysis introduces random coefficients in an attempt to address these issues.

Section 2 will discuss the data and the methodology.

Section 3 will review the literature.

Section 4 will develop the empirical models used in this thesis.

Section 5 will discuss the empirical results of all the models presented in this thesis.

Section 6 will offer conclusions of the implications of this work.

Section 7 presents recommendations for further study.

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[2] Swamy, P.A.V.B. and Tavlas, George S. Random Coefficient Models. A Companion to Theoretical Econometrics. Edited by Badi H. Baltagi. Chapter 19 pp 410-428

[3] Ibid.

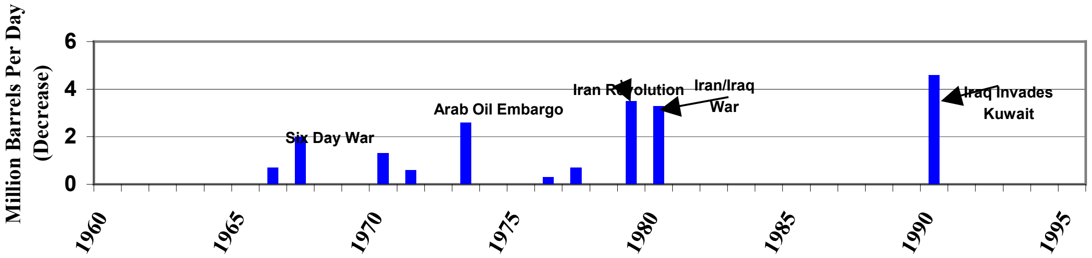
[4] Ibid.

Section II: Data and Methodology

For the purpose of comparison, the data in this study is from Econometric Analysis, 4th edition, by William H. Greene (2000).[5] The data series used in the estimations are: a price index of gasoline (Pg), per capita disposable income (Y), a price index of new cars (Pnc), a price index of used cars (Puc) and United States population in millions (Pop). These components will be used to estimate the total United States gasoline consumption as total expenditure divided by the consumer price index (G). The data is annual and covers the years 1960-1995. To test for out-of-sample performance an ex post forecast was done for 1991-1995 for each of the models used. One concomitant variable will be used to assist in explaining the demand for U.S. gasoline. The concomitant variable is the growth rate of gasoline price $(\frac{Pg_t - Pg_{(t-1)}}{Pg_{(t-1)}})$.[6]

The model chosen will address five major events which caused the decrease of the world oil supply by approximately 2 million barrels per day and in turn had an impact on the United States Gasoline Market as demonstrated in *Chart 1.0*.[7]

Chart 1.0: Major Disruptions of World Oil Supply



These events, or supply shocks, affected the U.S. economy's ability to produce.

> A favorable supply shock, such as an increase in labor productivity or a technological breakthrough, raises the level of output produced by a fixed amount of input—that is, for any given price more can be produced. An adverse supply shock, such as an increase in oil prices, lowers the output potential for existing resources. Capital, because it becomes more expensive to

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[5] See Appendix B for data used in models taken from Econometric Analysis Fourth Edition by William H. Greene.

[6] A concomitant variable is a variable that is not included in the equation determining the relationship between the independent and dependent variables. It is used to capture the effect of the correlation between the independent variables on their coefficients. For a complete explanation see Swamy, P.A.V.B. and Travlas, George, "Random Coefficient Models", (1999).

[7] Energy Information Administration, *Annual Energy Review 1997*, DOE/EIA- 0384(97). (Washington, DC, July 1998), Table 5.22.

> operate, and labor, because each unit of labor has, in essence, less capital with which to produce output, are both less productive. At any price, less can be produced with given inputs.[8]

Adverse supply shocks destroy some of the economy's existing capital stock or reduce labor productivity. The infamous oil and food price shocks of the 1970's are a good example of this. Major disruptions in oil prices adversely affect the economic outlook. As a result, there is a temporary fall in spending on large-ticket items, such as automobiles.

> The mid 1970's saw changes in fossil fuel prices become more frequent and pronounced. Prior to the oil embargo of 1973-1974, the composite real price per million BTU of crude oil had declined to a post World War II low of $1.03 in both 1968 and 1969. The real price of oil declined between 1959 and 1970 and then rose sharply in 1974 and again in 1979 through 1981. Thereafter, in the face of shrinking demand and excess production, price trends reversed sharply. In 1990, the Iraqi invasion of Kuwait contributed to an increase in crude oil prices to $3.69 per million Btu. In 1991 the ability of producers to supply replacement oil, coupled with a worldwide economic recession depressed petroleum demand, leading to a decrease in crude oil prices to $2.93 per million Btu. In 1995, the real price of oil was $2.35 per million Btu. The output and pricing polices of the Organization of Petroleum Exporting Countries (OPEC) was a major factor during much of the 1973-1995 period.[9]

Gasoline has become the most thoroughly studied petroleum product. The mobility afforded by the private automobile has revolutionized transportation in the industrialized world. With few competitive substitutes, gasoline should remain a heavily sought after commodity. With increasing prosperity and travel, petroleum dependence and vulnerability to disruption is always present. Prediction errors of a large magnitude are not trivial when forecasting U.S. gasoline demand.

> To the contrary, both private and public policy decisions depend on the accuracy of such forecasts. Because of the importance of gasoline as the major refinery product, refinery expansion plans and retail marketing strategies are conditioned on such forecasts. Similarly, public policy decisions regarding auto efficiency standards, auto pollution controls, and oil import policy depend on gasoline demand forecasts. [10]

A sizable body of literature has focused on the negative relationship between changes in oil prices and GDP growth.

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[8] Rosenbaum, Mary S. and William C. Hunter. Supply Shocks and Household Demand for Motor Fuel. *Economic Review*, (March/April), 1-11.

[9] Annual Energy Review 1995, July 1996. Energy Information Administration, p.23. This paper can be obtained from the internet at http://tonto.eia.doc.gov/ftproot/mutifuel/038495.pdf.

[10] Baltagi, Badi H; Griffin, James M. U.S. Gasoline Demand: What Next. Energy Journal vol 5 January 1984, p 129-40.

> A change in the price of crude oil can affect output growth through various channels. As the price of oil rises, the price of alternative energy resources increases causing a reduction in energy use. In addition to this direct effect, a substitution effect could take place, causing changes in the utilization of other production inputs. Resources that are complementary to energy would tend to be employed less intensively while the use of substitutes would increase. In addition, the subsequent increase in the aggregate price level can have a negative impact on output through reductions in real money balances. Production could also be affected in an indirect way by the existence of wage and price rigidities. [11]

Important demand side effects can also result from oil price increases. Changes in demand composition can arise due to variations in relative prices and income distribution.

> Bresnahan and Ramey (1993) find out that the oil price shocks of the 1970's induced changes in the composition of demand for autos, which interacted with short-run rigidities in physical and engineering capital to lower capacity utilization in this sector. [12]

Studies of gasoline demand arrive at conflicting conclusions. This is expected since the studies surveyed are based on different models, types of data, time periods, and different functional forms and econometric techniques. However, the use of second-generation RCM models in this area is limited.

Section III: Literature Review

Given the voluminous nature of studies done on the demand for gasoline it would be impossible to focus on all of the them. Bresnahan and Ramey (1993) documented that the oil shocks of 1974 and 1980 caused a significant shift in the mix of demand for different size classes of automobiles. Wykoff (1973) analyzed automobile demand by separating the automobile market into new and used cars. He treated used cars as homogenous and found that new and used cars have distinct price and income elasticities. He found that new car income elasticities are around one and used car demand is income inelastic.[13]

Ohta and Griliches (1986) examine the energy crises of 1973 and 1979 to see if the associated increases in gasoline prices changed consumer evaluations of the relative qualities of used cars in the U.S. They found

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[11] Herrera, Ana Maria. 2000. Inventories, Oil Shocks, and Aggregate Economic Behavior. Ph.D. diss., University of California, San Diego, p2.

[12] Bresnahan, Timothy F., and Valerie A Ramey (1993). Empirical Studies of Capacity Utilization: Segment Shifts and Capacity Utilization in the U.S. Automobile Industry. *Aea Papers and Proceedings*, May, 213-218.

[13] Wykoff, Frank C. A User Cost Approach to New Automobile Purchases. The Review of Economic Studies, Volume 40, Issue 3 (Jul., 1973), 377-390.

when gasoline costs are taken into account, the estimated valuations for relative qualities become more stable over time. Greenlees (1980) looked at automobile sales of three distinct subgroups; four, six, and eight cylinder cars. He concluded that a ten percent increase in fuel price is estimated to yield an increase of over eight percent in the proportion of small (four and six cylinder) cars purchased. Thus, this adjustment of the automobile stock should be a significant component of the total response to rising gasoline prices.

Dahl and Sterner (1991) surveyed over 100 studies on analyzing gasoline demand. They break the studies into 10 model types.[14] An average of both price and income elasticities in all of the studies are used to derive overall average elasticities. The price elasticity for the short run and long run is -.26 and -.86 respectively. The income elasticities for the short run and the long run are .48 and 1.21. Dahl and Sterner find the long-run estimates of elasticities across surveys vary widely and suggest there is strong evidence gasoline consumption is very responsive to prices and income.

Using a random coefficients model, Mehta, Naraimham and Swamy (1997) formulate a demand function for gasoline whose consumption is technologically related to the stock of automobiles owned by individuals. They estimated an income and price elasticity of .8675 and -.0441.

Section IV: Empirical Models

In order to determine if there is a relationship between the demand for gasoline and an oil shock , an econometric model must be representative of the whole economy. The price level and availability of gasoline directly impact the purchase price of automobiles and are key determinants of overall vehicle usage. Consumers are faced with the decision of whether to buy a more gas efficient automobile than the vehicle they currently own. If a decision is reached to purchase a new automobile the consumer must now choose whether to purchase a gas efficient foreign car, a sport-utility vehicle, a truck, etc. However, purchases could be postponed if there is uncertainty on the direction of short-term oil prices until the market is reflective of current events and prices stabilize. The models presented in this research attempt to

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[14] For more detail , consult Sterner, Thomas and Carol Dahl. Analyzing gasoline demand elasticities: a survey, Energy Economics, July 1991, p.203-210.

answer why, during the time period 1960-1995, as different shocks occurred in the economy their impact on the gasoline market was different in terms of output and substitution effects.

Greene proposes the following OLS model to estimate demand in the United States Gasoline market.

4.1 Model 1

$$\log(\frac{G}{POP})_t = \beta_0 + \beta_1 \log(Pg)_t + \beta_2 \log(Yd)_t + \beta_3 \log(Pnc)_t + \beta_4 \log(Puc)_t + \varepsilon_t \quad \textbf{(1.0)}$$

where :

- $\varepsilon_t \sim IN(O, \sigma_\varepsilon^2)$
- $(\frac{G}{POP})$ total U.S. gasoline consumption
- Pg is the price index of gasoline
- Yd is per capita disposable income
- Pnc is price index of new cars
- Puc is the price index of used cars.

Equation (1) imposes a number of classic restrictions [15]

- β_0, β_1, β_2, β_3, β_4 are constants
- excluded explanatory variables are proxied through the use of an error term and therefore, these excluded variables are assumed to have means equal to zero and to be independent of the included explanatory variables
- the true functional form is known (whether linear or nonlinear)
- the variables are not subject to measurement error

Four other models will be explored to develop a more accurate demand function. The second model involves the introduction of an AR(1) error process. First order autocorrelation occurs when the disturbance term is proportional to the disturbance term in the previous time period, plus a spherical disturbance: $\varepsilon_t = \rho\varepsilon_{t-1} + V_t$ (t=1,2,…,n) where V_t is a random error. Therefore, the residual of the observation for period t is a function of the residual of the previous period (t-1) plus a random error.[16]

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[15] George Hondroyiannis, P.A.V.B. Swamy and George S. Tavas. The Time-Varying Performance of the Long Run Demand For Money In the United States. Economic Inquiry vol 39, no 1 Jan 2001, 111-123.

[16] Freund, Rudolf J, and Minton, Paul D. Regression Methods A Tool for Data Analysis. (New York: Marcel Dekker, Inc, 1979), 99-103.

4.2 Model 2

$$y_t = \beta_0(1-\rho) + \sum_{i=1}^{4} \beta_i (X_{it} - \rho X_{it-1}) + \rho y_{t-1} + V_t \quad \textbf{(2.0)}$$

where:

- ρ is the first order autoregressive parameter (AR)
- y_{t-1} is the value of the time series in the previous section
- V_t is a random error term uncorrelated over time, typically called white noise
- X_i are the explanatory variables
- t and t-1 index time

The last three models will be permutations of random coefficient models.[17] The third model is considered a first generation RCM and attempts to deal with the problem that arises when *equation (1)* does not have constant coefficients. Thus, the constant coefficient assumption is going to be relaxed. “This class of models seeks to account for the process generating the coefficients of the regression model, but does not address specification issues related to functional form, omitted variables, and measurement errors.” (Swamy and Tavlas 2000)

4.3 Model 3

$$y_t = \beta_{0t} + \sum_{i=4}^{4} \beta_{it} X_{it} \qquad \textbf{(3.0)}$$

and

$$\beta_{ot} = \Pi_{00} + U_{0t}$$
$$\beta_{1t} = \Pi_{01} + U_{1t}$$
$$\beta_{2t} = \Pi_{02} + U_{2t}$$
$$\beta_{3t} = \Pi_{03} + U_{3t}$$
$$\beta_{4t} = \Pi_{04} + U_{4t}$$

where:

- $U_{it} \sim IN(0, \sigma_u)$
- β_{it} is the coefficient on the ith independent variable
- X_{it} is the ith independent variable of 4 variables
- t serves as an index of time
- Y_t is total U.S. gasoline consumption
- Π_{01} is the coefficient on the price index of gasoline x_{1t}

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[17] See Appendix A for an overview of random coefficients models .

- Π_{02} is the coefficient on per capita disposable income x_{2t}
- Π_{03} is the coefficient on the price index of new cars x_{3t}
- Π_{04} is the coefficient on the price index of used cars x_{4t}

By substituting the values of β_{it} into *equation (3.0)*, *equation (3.1)* is obtained.

$$Y_t = (\Pi_{00} + U_{0t}) + (\Pi_{01} + U_{1t})X_{1t} + (\Pi_{02} + U_{2t})X_{2t} + (\Pi_{03} + U_{3t})X_{3t} + (\Pi_{04} + U_{4t})X_{4t} \quad \textbf{(3.1)}$$

By isolating the error terms in equation (3.1) we have

$$Y_t = \Pi_{00} + \Pi_{01}X_{1t} + \Pi_{02}X_{2t} + \Pi_{03}X_{3t} + \Pi_{04}X_{4t} + U_{0t} + \sum U_{it}X_{it} \quad \textbf{(3.2)}$$

This equation shows the variations in the coefficients of *equation (3.2)* are random and the U_{it} are assumed to be uncorrelated with each other. Finally, the error structure embedded in *equation (3.2)* is heteroskedastic. Specifically, the variance of the error at each sample point is a linear combination of the squares of the explanatory variables. According to Swamy and Tavlas this implies *equation (3.2)* is estimated using a feasible generalized least squares estimation procedure that accounts for the heteroskedastic nature of the error process.

Since t indexes time, a natural extension of *equation (3.2)* would be to incorporate serial correlation in the process, determining the coefficients. *Equation (4.0)* differs from *equation (3.2)* because the assumption is made that the process generating the coefficients is autoregressive as follows:

- $U_t = \rho U_{t-1} + V_t$

and $V_t \sim IN(0, \sigma_t^2)$

where:

- ρ is a matrix describing the autoregressive process generating the coefficients and has eigen values less than 1 in absolute value
- V_t is a vector of the V_{it}, spherical disturbances
- U_t is a vector of the U_{it}

4.4 Model 4

The model equation is

$$Y_t = \Pi_{00} + \sum \Pi_{0i}X_{it} + \sum U_{it}X_{it} + U_{0t} \quad \textbf{(4.0)}$$

where:

- $U_{0t} \sim IN(o, \sigma_t^2)$

The last equation presented in this thesis is a second-generation random coefficient model developed by Swamy and Tavalas (1995 and 2000). A second-generation random coefficient model modifies the first-generation model to relax all of the restrictions mentioned in *equation 1*. The β's are estimated using a concomitant variable. A concomitant may be assumed to be a variable that is not included in the equation used to estimate gasoline demand, but helps with the correlations between the β's and the included explanatory variables. One must choose a concomitant variable only after examining the implications for the estimates of the direct effect portion of the coefficients. The direct effect is the real economic relationship of the explanatory variables on the explained variables. The concomitant chosen should have a high degree of explanatory power. The signs and statistical significance of the direct effect estimates should remain virtually unchanged as various sets of concomitants, other than the determinants of the direct effect, are put into the model.[18] The concomitant variable used is the growth rate of gasoline price P_{gt}, $(\frac{Pg_t - Pg_{(t-1)}}{Pg_{(t-1)}})$.

4.5 Model 5

By adding this additional information to *equation (4.0)* the β's are defined as

$$\beta_{1t} = \Pi_{01} + \Pi_{11} Z_t + U_{1t}$$
$$\beta_{2t} = \Pi_{02} + \Pi_{21} Z_t + U_{2t}$$
$$\beta_{3t} = \Pi_{03} + \Pi_{31} Z_t + U_{3t}$$
$$\beta_{4t} = \Pi_{04} + \Pi_{41} Z_t + U_{4t}$$

where:

- $Z_t = \ln(\frac{Pg_t - Pg_{(t-1)}}{Pg_{(t-1)}})$ =growth rate of gasoline price
- $U_{1t} = \rho_1 U_{1t-1} + V_t$
- $U_{2t} = \rho_2 U_{2t-1} + V_t$
- $U_{3t} = \rho_3 U_{3t-1} + V_t$
- $U_{4t} = \rho_4 U_{4t-1} + V_t$

The estimated equation for model 5 now becomes

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[18] See Swamy and Tavalas, "Random Coefficient Models" (1999) for a detailed discussion on choosing the right concomitant.

$$Y_t = \Pi_{00}(1-\rho_0) + \Pi_{01}(Z_t - \rho_1 Z_{t-1}) + \sum \Pi_{01}(X_{it} - \rho_1 X_{it-1}) + \sum \Pi_{i1}(X_{it} - \rho_1 X_{it-1}) + \sum \rho_i Y_{t-1} + \sum_{j=0}^{4} V_{jt}$$

(5.1)

Equation 5.1 makes two assumptions. First, the coefficients are linear functions of the Z_t variable, called a concomitant, including a constant term with added error terms, which may be serially correlated, captured by the autocorrelation parameter, ρ_1. Second, the explanatory variables of this equation are independent of their coefficient's error terms, given any values of concomitants.

Section V: Empirical Results and Analysis

The optimal forecasting model will be selected by the following criteria:

- correct specification of the model
- time trend of the beta vectors
- lowest root mean squared error (RMSE)

Tables 1 and *2* below contain the results for each of the models. In *table 1,* it is interesting that all the signs of the coefficients across models are the same except for the new car price coefficient, which is negative only in *models 1A* and *3A.* However, according to the t-stats in *table 2* these two coefficients are not statistically different from zero.

Table 1: Comparison of Coefficients[19], R-squared and Durbin Watson Statistics (36 annual observations)

Restriction	Own Price	Income	New Car Price	Used Car Price	R-Squared	Durbin-Watson
Model 1A	-0.0591	1.3734	-0.1268	-0.1187	.9988	.6047
Model 1B	-0.0996	1.4041	0.0648	-0.1874	.9985	.9714
Model 2A	-0.2116	1.0641	0.0980	-0.0335	.9959	.8677
Model 2B	-0.2243	1.0560	0.1872	-0.0480	.9964	.9187
Model 3A	-0.0592	1.3706	-0.1425	-0.1091	N/A	N/A
Model 3B	-0.0988	1.4018	0.0654	-0.1881	N/A	N/A
Model 4A	-0.2212	1.0677	0.1316	-0.0453	N/A	N/A
Model 4B	-0.2372	1.0471	0.2389	-0.0714	N/A	N/A
Model 5A	-0.2005	1.0932	0.1774	-0.0979	N/A	N/A
Model 5B	-0.1919	1.1097	0.3076	-0.1604	N/A	N/A

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[19] Note (A) = full sample and covers the period of 1960-1995. (B)= a partial sample and covers periods 1960-1990 with 1991-1995 being used to forecast. The numbers for models 3-5 refer to the average value of the respective vectors for the period 1960-1995.

Table 2: Standard Errors and T-Statistics[20] [21]

(36 annual observations)

		Level of Significance (%)					Level of Significance (%)			
Restriction	**Own Price**	**T-stat**	**1**	**5**	**10**	**Income**	**T-stat**	**1**	**5**	**10**
Model 1A	.0325	-1.8192			X	.0756	18.1600	X	X	X
Model 1B	.0348	-2.8634	X	X	X	.0724	19.3820	X	X	X
Model 2A	.0347	-6.0930	X	X	X	.1303	8.1663	X	X	X
Model 2B	.0348	-6.4461	X	X	X	.1274	8.2892	X	X	X
Model 3A	.0316	-1.8726			X	.0760	18.0340	X	X	X
Model 3B	.0346	-2.8750	X	X	X	.0722	19.417	X	X	X
Model 4A	.0326	-7.2723	X	X	X	.1308	8.0055	X	X	X
Model 4B	.0367	-6.0257	X	X	X	.1340	7.9666	X	X	X
Model 5A	.0453	-4.9885	X	X	X	.1601	7.0485	X	X	X
Model 5B	.0504	-3.8083	X	X	X	.1410	7.8702	X	X	X
Restriction	**New Car Price**	**T-stat**	**1**	**5**	**10**	**Used Car Price**	**T-stat**	**1**	**5**	**10**
Model 1A	.1270	-.9985				.0813	-1.4595			X
Model 1B	.1385	.4682				.0816	-2.2968		X	X
Model 2A	.1257	.7797				.0651	-.5155			
Model 2B	.1314	1.4252			X	.0655	-.7330			
Model 3A	.1243	-1.1462				.0797	-1.3695			X
Model 3B	.1373	.4764				.0812	-2.3155		X	X
Model 4A	.1284	1.8602				.0620	-1.1524			
Model 4B	.1280	1.0284				.0650	-.6978			
Model 5A	.1409	.7342				.0731	-.4911			
Model 5B	.1573	1.9551			X	.0926	-1.7319			X

Looking at the Durbin Watson statistic for *models 1* and *2* in *table 1*, we are unable to reject the hypothesis of no autocorrelation at the 95% significance level. The r-squared for all models is above 99%. The results for *models 1* and *2,* as displayed in *table 1* and 2, are suspect. *Charts 2.0* and *3.0* are plots of the residuals for *models 1* and *2*. They show that the residuals are serially correlated, and for *model 2*, chart *3.0*, the assumption of a constant variance and a zero mean does not appear to be true.

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[20] IBID

[21] In models 3-5 the standard errors and t-statistics are the average value of the individual time-varying coefficients for the time period 1960-1995. A coefficient is significant when marked by an X. Table values for T-statistics for 31 observations are 1.310(10%), 2.042(5%), and 2.750 (1%). Table values for T-statistics for 36 observations are 1.306(10%), 2.030(5%) and 2.724(1%). Estimated standard errors could be biased, inefficient and inconsistent. Therefore, no importance is attributed to them and the model selected will focus on out of sample model performance.

Chart 2.0: Time Series Plot of Residuals from Regression
(Model 1: OLS Full Sample)

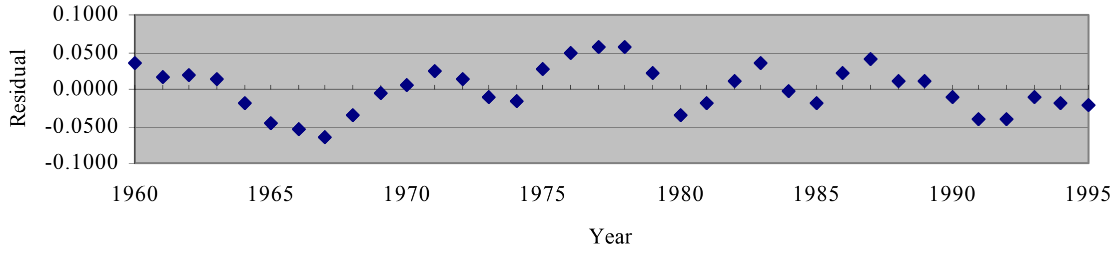
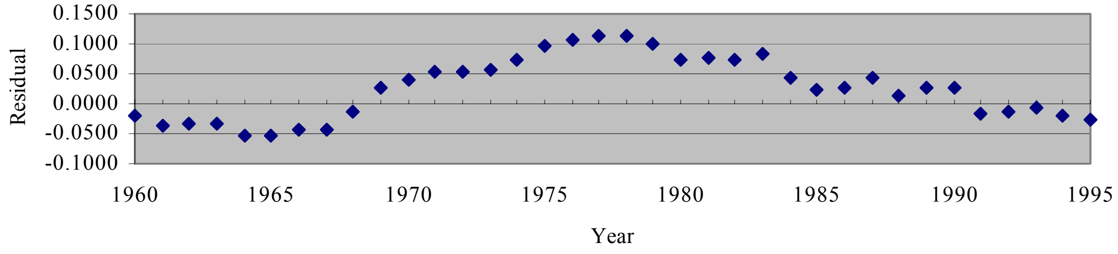


Chart 3.0: Time Series Plot of Residuals from Regression
(Model 2: AR(1) Full Sample)



Therefore, the implications are that *model 2* suffers from heteroskedasticity. The presence of autocorrelation and heteroskedasticity means the results and conclusions drawn for the models are unreliable because serious violations of the assumptions are made when using an ordinary least squares estimator as in *model 1* and a mis-specified error cleanup procedure as in *model 2*.[22]

For random coefficient models the r-squared and Durbin Watson statistic are simply not valid. According to Swamy and Tavlas, within the RCR framework a low r-squared implies that the set of concomitants may not adequately explain the direct effects. However, a high r-squared may result by arbitrarily increasing the number of concomitants even if these concomitants are not relevant for explaining

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[22] See Amemiya, Takeshi. A note on a Heteroscedastic Model. Journal of Econometrics, 1997, p 365-370.

the coefficients. Therefore, traditional validation methods will not yield statistically sound results. Swamy and Tavlas consider a model to be validated if:

- it fits within sample values well
- it fits out-of-sample values well
- it has high explanatory power
- the signs and statistical significance of the estimates of direct effects remain virtually unchanged as one set of concomitants after another is introduced into the model[23]

It is important to see if the signs of the coefficients for the significant variables remain constant across all models in *table 2*. This gives credibility to the direction of the effect a variable has on the demand for gasoline. The coefficient for income in every model is significant at the 1% level. It seems plausible that income level will affect gasoline demand. In fact, one would expect a positive effect of higher income on demand for gasoline. The variable new car price is only significant at the 10% level in *models 4A* and *4B,* but the signs are positive, contrary to expectations. The used car price variable is significant at the 5% level in *models 1B* and *model 3B*. However, at the 10% level the used car price variable is significant for *models 1A, 1B, 3A, 3B*, and *5B*. The sign of used car prices is negative for all models, which makes economic sense because as the price of used cars increases individuals will buy fewer used cars and demand less gasoline, ceteris paribus. The own price is the response of the quantity of gasoline demanded to a given percentage change in the price of gasoline. The own price sign is negative and significant at 99% in all models except *models 1A* and *3A*. This implies there will be a decrease in the quantity demanded as the price of gasoline increases. The Durbin Watson Statistic (.9187) for *model 2B* indicates serial correlation. The presence of a significant time pattern in the error terms violates the assumption that the errors are independent over time.

Results are obtained using OLS, an AR(1) error cleanup, and random coefficient modeling procedures with and without the inclusion of a concomitant variable. Using data from the time period 1960-1995, the forecasting accuracy over the out-of-sample time period 1990-1995 is examined for each model.

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[23] Swamy, P.A.V.B. and Tavlas, George S. Random Coefficient Models. A Companion to Theoretical Econometrics. Edited by Badi H. Baltagi. Chapter 19 pp 410-428

The actual and forecasted values of domestic gasoline consumption from 1990-1995 are presented in t*able 3,* along with the root mean square error, and the Theil U statistic.

Table 3: Actual and Forecast Values, and Accuracy Statistics

Year	Actual Data	Model 1	Model 2	Model 3	Model 4	Model 5
1991	4.6784	4.7515	4.7260	4.7513	4.7162	4.7323
1992	4.6978	4.7677	4.7426	4.7674	4.7311	4.7441
1993	4.7103	4.7577	4.7521	4.7574	4.7409	4.7495
1994	4.7124	4.7682	4.7696	4.7678	4.7582	4.7655
1995	4.7287	4.785	4.7926	4.7845	4.7792	4.7814

Model	r-squared	RMSE	MAPE[24]	Theil U Statistic
1B	.9985	.0158	.0027	.0007
2B	.9964	.0089	.0015	.0004
3B	.9766	.0160	.0027	.0007
4B	.9788	.0077	.0014	.0003
5B	.9866	.0074	.0010	.0003

As expected, the use of the RCM with the inclusion of the concomitant variable, model 5B, provides forecasts closest to the actual value of gasoline consumption. The Theil U statistic was examined because unlike r-squared, it is not bounded by 0 and 1. Large values are indicative of poor forecasting performance. As shown in *table 3* the Theil U statistic for each model is very small indicating that each model produces fairly accurate forecast. However, based on the previous discussion, *models 1* and *2* seem mis-specified and are not considered to produce reliable forecasts.

According to t*able 3, models 4* and *5* have the smallest root mean squared errors and are very close to one another. Therefore, it could be asked whether these models were statistically different from one another. According to P.A.V.B. Swamy, there is really no test statistic to determine this. *Chart 7.0* shows the forecast of all 5 models benchmarked to 1991 and suggests model 5 is superior.

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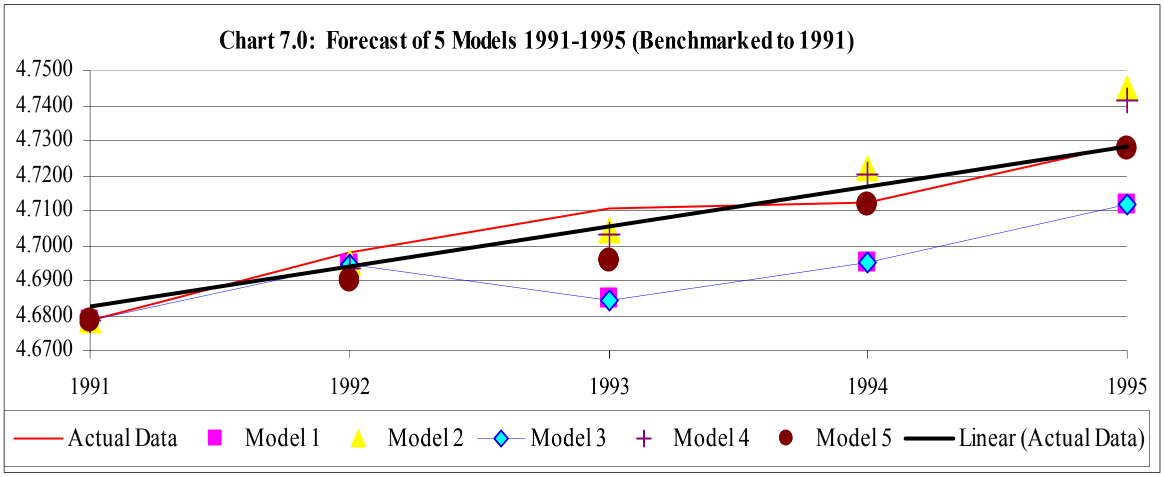
[24] According to Peter Kennedy, A Guide to Econometrics 4th edition the mean absolute percentage error (mape) is the average of the absolute values of the percentage errors. MAPE is more appropriate when the cost of the forecast error is more closely related to the percentage error than to the numerical size of the error.



Note that *model 5* gets closer to the actual data in 1994 and 1995 than any of the other models. *Models 1* and *3* under predict in the later years, while *models 2* and *4* over-predict.

Section VI: Beta Vectors

The beta vector consists of random values of each coefficient that are the sum of the direct effect, indirect effect, and the mis-measurement effect as discussed in *appendix a*. The beta vector gives the elasticities of U.S. gasoline demand with respect to the own price (β_1), income (β_2), the price of new cars (β_3), and the price of used cars (β_4), over the time period 1960-1995. The time profiles of the coefficient values on β_1, β_2, β_3, and β_4 are presented in a*ppendix C* and *D* for *models 3-5*.

The time profiles of the coefficients of models 3-5 are examined over the period 1960-1995. The time profiles in *appendix C* reveal oil price shocks of the 1970's were large and disruptive.

> The first shock, between 1973 and 1974, sent oil prices from about $3.60 per barrel (1972 average) to more than $10 per barrel in 1974. This episode also included an oil embargo limiting U.S. gasoline supplies and resulting temporarily in informal rationing and other non-price restraints on gasoline purchases. The second shock, a series of price increases during 1979 and 1980, raised the per barrel cost from $12.46 in 1978 to more than $35.00 in 1981.[25]

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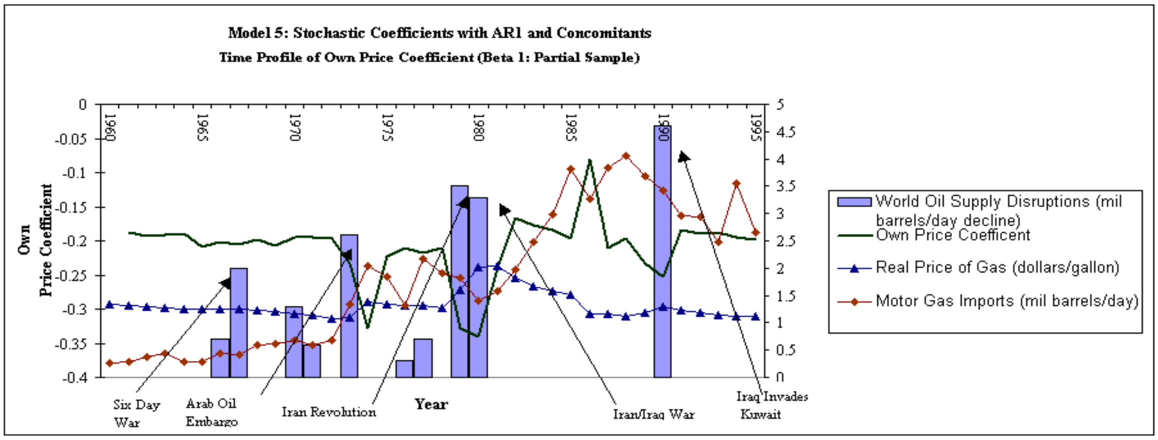
[25] Rosenbaum, Mary S. and William C., Hunter (1991). Supply Shocks and Household Demand for Motor Fuel. *Economic Review*, (March/April),1-11.

Appendix C contains the graphs of the time profiles of the coefficients for *models 3-5*. For discussion purposes, only *model 5*'s charts are included in the body of the paper. *Charts 4.1B, 5.1B*, and *6.1B* display the time profiles of β_1, the coefficient on own price for *models 3-5*. As expected, $\beta_1 < 0$. During the 1970's at each shock β_1 first becomes slightly more negative and then rebounds to its pre-shock level in *chart 4.1B*, and more so in *chart 6.1B*, suggesting that while own price rose, the negative effect on gasoline consumption was initially reinforced to some extent by β_1 becoming a larger number (still negative, but larger in absolute value). *Chart 5.1B* shows no relationship between β_1 and the oil shocks. In *chart 6.1B* the graph of the profile seems to suggest that the oil shock caused by Iraq invading Kuwait had a similar but smaller effect on β_1.

Chart 6.1B: Time Profile of Beta 1 for model 5 (partial sample). [26]



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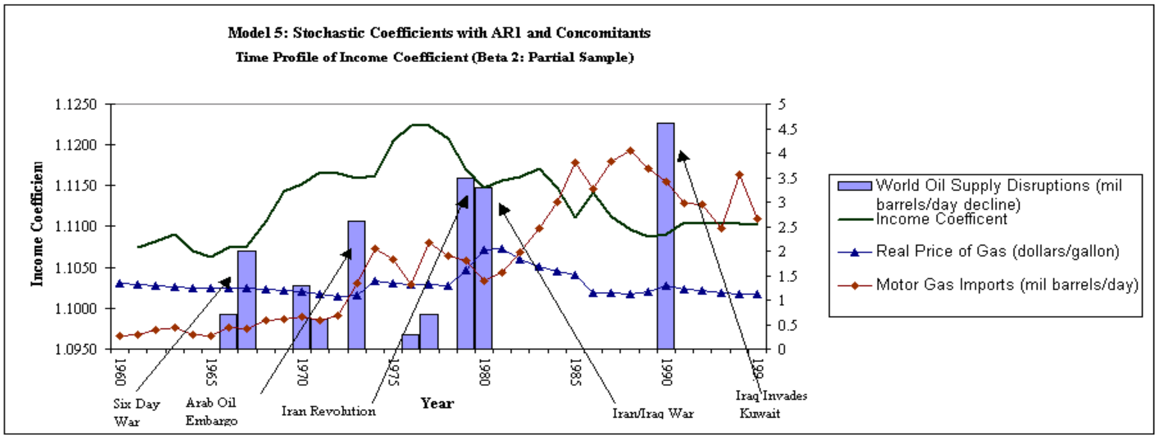
[26] U.S. retail price of gasoline series obtained from Energy Information Administration, Annual Energy Review 1997, DOE table 5.22. 1972-1997 price of leaded gasoline was used. 1978-1995 price for all types of gasoline used. Motor gasoline imports obtained from the Energy Information Administration/Annual Energy Review 2000, table 5.3 petroleum imports by type, 1949-2000 measured in thousands of barrels per day.

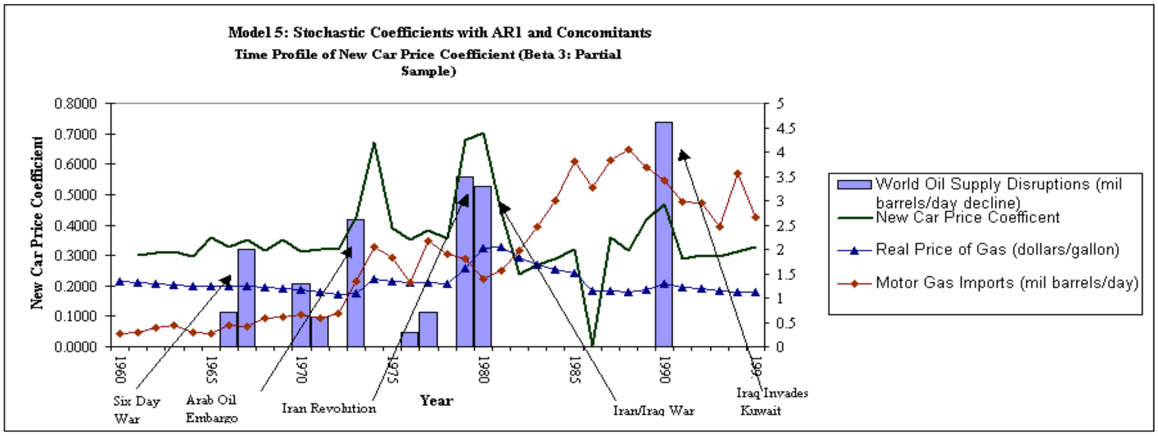
The income elastacities displayed in *charts 4.2B*, *5.2B*, and *6.2B* trends are similar. A positive relationship exists. However, chart 6.2B also shows that the income elasticity was larger in the 1973-1984 period than previously or subsequently.

Chart 6.2B: Time Profile of Beta 2 for model 5 (partial sample).



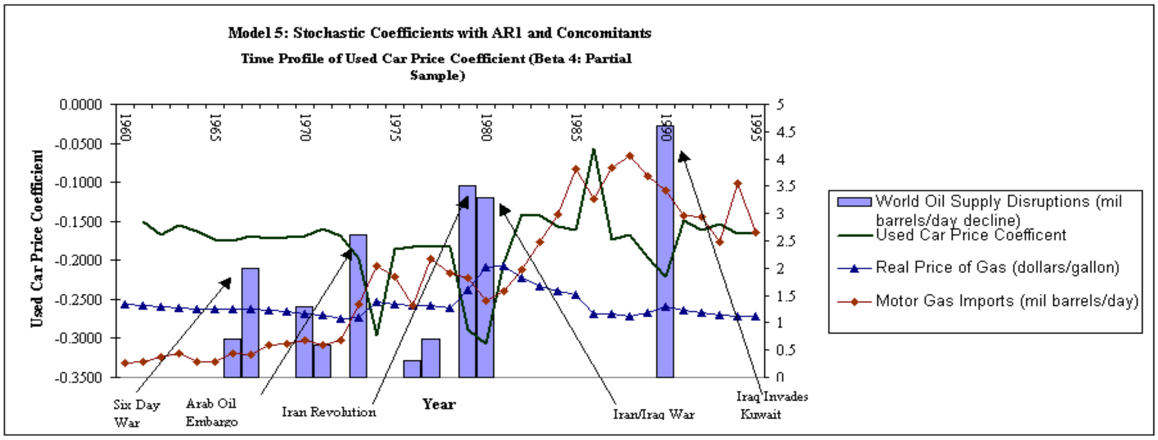
The new car price coefficients in *appendix C* are in *charts 4.3B*, *5.3B* and *6.3B*. The coefficient β_3 is not significantly different from zero except in model 5. When new car prices rise people buy fewer of them and keep older less fuel efficient cars running longer and that may be why the sign is positive on β_3 and is significant at the 10% level in model 5.

Chart 6.3B: Time Profile of Beta 3 for model 5 (partial sample).



The last coefficient's time profile examined is for used car prices. Since used cars and gasoline are complimentary goods, as the price of used cars rises we would expect gasoline consumption to fall, ceteris paribus. Therefore, one would expect this coefficient to have a negative sign. In fact, *models 3*, *4*, and *5* do have a negative sign as shown in *table 1*. In *chart 6.4B* the relationship between oil shocks and β_4 is similar to that between oil shocks and β_1 shown in *chart 6.1B*.

Chart 6.4B: Time Profile of Beta 4 for model 5 (partial sample).



Section VII: Conclusions

The research had three objectives:

- provide an empirical methodology that can estimate a gasoline demand function capable of performing well in prediction
- evaluate the elasticites of the five models presented to determine which model is more accurate at capturing the effects of events which impacted the demand for gasoline
- evaluate the behavior of the elasticites of the beta coefficients

The random coefficient models presented relaxed the restrictions of the OLS model:

- true functional form of the equation is known
- error term captures the effect of all the excluded variables
- the coefficients are fixed and the data does not include measurement errors

Swamy and Tavlas (1999) demonstrate that this modeling approach often provides results that more closely coincide with true economic interpretations of the data. The results discussed in section 5 indicate the forecasted values of the demand for gasoline are more accurate when these restrictions are removed.

The conclusions of this investigation must be interpreted with caution because they were obtained from a small data set of 36 annual observations. To obtain more reliable estimates, a larger number of observations should be used. Nonetheless, this research indicates that the combination of economic analysis and econometric technique of random coefficient models provide a useful and powerful tool in forecasting gasoline demand.

Section VIII: Suggestions for further study

A few suggestions for future empirical research are:

- Develop a random coefficient model that includes supply effects to see if better time trends can be obtained.
- Differentiate new cars into luxury cars, sport utilities and such. It would be interesting to investigate whether consumers care more about performance and comfort of an automobile than for fuel efficiency if income rises.
- Incorporate age structures and other demographic characteristics into a model of gasoline demand . It would be of interest because the United States has seen a substantial aging of the population over the last several decades, as well as the emergence of dual income households.

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Appendix A: Overview of Random Coefficient Models

The random coefficient model specification used here follows the general form developed by Swamy and Tavlas (1995 and 2000).[27] This specification is used to provide forecasts of the real price index of gasoline and a time profile of the coefficients representing the true economic relationships between the explanatory variables and the explained variable. A brief overview of the random coefficient model developed by Swamy and Tavlas [28] is discussed in this section.

Many popular econometric forecast models use either ordinary least squares or generalized least squares regression models. Both these models impose certain restrictions.[29] It has been argued that these restrictions may not be true when using economic time series data. The most notable restriction for the classical linear regression models requires that a constant vector of coefficients exists to explain the dependent and independent variables through the following assumptions:

- the effect of all excluded variables is captured through the use of a normally distributed and independent additive error term with an expected value of zero.
- the observed values of the variables have no measurement error
- the functional form of the model is known.

Due to the very nature of economic times series data the above assumptions are rarely valid and or supported. As a response to this problem, traditional random coefficient models have relaxed the assumption of a constant vector of coefficients. Swamy and Tavlas describe these models as first-generation models and present them as follows:

$$y = x_{t1}\beta_{t1} + \sum_{j=2}^{K} x_{tj}\beta_{tj} = x_t^{'}\beta_t \qquad \textbf{(A.0)}$$

where:

- $x_t^{'}$ is a row vector of K explanatory variables
- β_t is a column vector of K coefficients
- $x_{t1} = 1$ for all t

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[27] See Swamy, P.A.V.B. and George Tavlas "Random Coefficient Models", (1999)

[28] Ibid.

[29] See Kennedy, Peter A Guide to Econometrics 4th edition for detailed discussion on model restrictions.

- x_{tj} for j=2…K are all explanatory variables and t serves as an index for time

Since β_t is changing in this model it is necessary to explain how it is changing by introducing some structure into the process thought to be determining the coefficients. β_t can be defined as

$$\beta_t = \overline{\beta} + \varepsilon_t \qquad \textbf{(A.1)}$$

where: $\overline{\beta}$ in e*quation A.1* is vector of means and ε_t is the vector of errors. This equation states that the variations in all the coefficients in equation *(A.0)* are random. Substituting *equation(A.1)* into equation *(A.0)*

$$y = x_t^{'}\overline{\beta} + \sum_{j=1}^{K} X_{tj}\varepsilon_{tj} \quad \textbf{(A.2)}$$

Since t indexes time, a natural extension of this class model is the incorporation of serial correlation in the process determining the coefficients as follows:

$$\varepsilon_t = \Phi\varepsilon_{t-1} + a_t \qquad \textbf{(A.3)}$$

where:

- Φ in *equation (A.3)* is a matrix defining the autoregressive process generating the coefficients
- a_t is the spherical disturbance

Equations (A.0) – (A.3) are the basic building blocks of first generation RCMs.

Second generation models, discussed by Swamy and Tavalas (1995 and 2000), relax the restrictions concerning:

- direct effects of explanatory variables on the explained variables
- functional forms
- measurement errors
- use of an additive error term

It is important to be aware that if any of these restrictions are violated, in first generation RCMs, specification errors arise. In second generation RCMs each coefficient is the sum of three parts: a direct effect of the true value of a regression on the true value of the dependent variable, omitted variable biases, and the effect of mis-measuring explanatory variables. Second generation RCMs are assumed to describe the true relationship between the explanatory variables and the explained variables with a certain, but unknown, variation in the coefficients. Therefore, these models have different functional forms for different paths of variation in the coefficients. These paths will be unrestricted as long as the variation in the coefficients is unrestricted.

Specification errors may also be present when the observed variables contain measurement errors. In addition, specification errors exist when the assumed functional form of the model is false. It is rare that the true functional form of an equation will be known and little, if any, information nay exist about the omitted variables. Therefore, one can not assume the omitted variables are independent of the observed variables and that they are constant over time. The second generation model developed by Swamy and Tavlas (1999) includes time varying coefficients that are more likely to be of the correct functional form than is the case for first generation RCM's. The second generation model with two independent variables is:

$$y_t = \alpha_{0t} + \alpha_{1t}x_{1t}^{*} + \alpha_{2t}x_{2t}^{*} + \sum_{j=3}^{n_t} \alpha_{jt}x_{jt}^{*} \qquad \textbf{(A.4)}$$

where:

- the α's represent the coefficients
- x_{jt}^{*} represents all omitted variables
- x_{1t}^{*} and x_{2t}^{*} are the true values of the included variables
- n_t are the number of explanatory variables

Swamy and Tavlas (1995 and 2000) demonstrate how *equation (A.4)* coincides with the true equation. *Equation (A.4)* encompasses time-varying coefficients, the true values of variables, the correct functional form, and any omitted variables. However, two fundamental problems are inherent in *equation (A.4)*. Little information may be known about the omitted variables (x_{jt}^{*}) to disprove that they are uncorrelated with the other explanatory variables in *equation (A.4)*.[30] Second, observed variables are likely to contain

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[30] See Pratt, John W. and Robert Schlaifer, "On the Nature of Discovery of Structure", (1984) for comments on the observability of stochastic laws.

measurement errors. The solution to the former problem is to assume that the x_{jt}^{*} are correlated with the other explanatory variables as follows:

$$x_{jt}^{*} = \psi_{0jt} + \psi_{1jt}x_{1t}^{*} + \psi_{2jt}x_{2t}^{*} \qquad \textbf{(A.5)}$$

where:

- ψ_{0jt} represents the portion of the omitted variables after the effects of the included explanatory variables have been removed
- remaining coefficients (Ψ) represent the partial correlations between the excluded and included variables.

Unless *equation (A.5)* is known to be linear the coefficients will not be constants. To fix this problem *equation (A.5)* is substituted into *equation (A.4)***.** The resulting equation is:

$$y_t^{*} = (\alpha_{0t} + \sum_{j=3}^{n_t}\alpha_{jt}\psi_{0jt}) + (\alpha_{1t} + \sum_{j=3}^{n_t}\alpha_{jt}\psi_{1jt})x_t^{*} + (\alpha_{2t} + \sum_{j=3}^{n_t}\alpha_{jt}\psi_{2jt})x_t^{*} \qquad \textbf{(A.6)}$$

The coefficients of *equation (A.6)* represent both the direct effect of x_t^{*} on y_t^{*} $(\alpha_{0t}, \alpha_{1t}, \alpha_{2t})$ and the omitted variable bias . While this has brought us closer to estimation, measurement error still has to be addressed. By substituting the observer counter parts(x_{1t}, x_{2t}) into equation (A.6) equation (A.7) is derived as follows:

$$y_t = \gamma_{0t} + \gamma_{1t}x_{1t} + \gamma_{2t}x_{2t} \qquad \textbf{(A.7)}$$

The coefficients in *equation (A.7)* are defined by Swamy and Tavlas as:

$$\gamma_{0t} = (\gamma_{0t} + \sum_{j=3}^{n_t}\alpha_{jt}\psi_{0jt} + \upsilon_{0t)}$$

$$\gamma_{1t} = (\gamma_{1t} + \sum_{j=3}^{n_t}\alpha_{jt}\psi_{1jt})(1 - \frac{\upsilon_{1t}}{x_{1t}})$$

$$\gamma_{2t} = (\gamma_{2t} + \sum_{j=3}^{n_t}\alpha_{jt}\psi_{2jt})(1 - \frac{\upsilon_{2t}}{x_{2t}})$$

All the coefficients in *equation (A.7)* are derived based on a set of realistic assumptions. These assumptions address the problems that arise due to

- omitted explanatory variables and their correlations with the included explanatory variables
- measurement error's

- unknown functional form

Equation (A.7) is correct in theory, but to implement it empirically some additional information must be provided. It is necessary to separate the direct effects from the other effects contained in the interpretation of the coefficients. The direct effects represent the true elasticity's. To isolate the direct effects, estimates must be obtained for α_{1t} and α_{2t}. The estimation of the direct effects of the explanatory variables on the explained variable involves the use of concomitant variables.[31] Concomitant variables are assumed to explain the variation in the coefficients.

According to Swamy and Tavlas, the coefficients of *equation (A.7)* satisfy stochastic equations

$$\gamma_{kt} = \pi_{k0} + \sum_{j=1}^{p} \pi_{kj} Z_{jt} + \varepsilon_{kt} \qquad \text{(A.8)}$$

where the z_{jt} are the concomitants and ε_{kt} is the error term which satisfies the stochastic equation

$$\varepsilon_{kt} = \phi_{kk} \varepsilon_{kt-1} \quad \text{(A.81)}$$

where: ϕ_{kk} is the autocorrelation parameter.

The assumption is made that the explanatory variables in *equation (A.7)* are independent of the error term (ε_{kt}). By substituting *equation (A.7)* into *equation (A.81)* Swamy and Tavlas arrive at the estimable form:

$$y_t = \pi_{00} + \sum_{j=1}^{p} \pi_{0j} z_{jt} + \pi_{10} X_{1t} + \sum_{j=1}^{p} \pi_{1j} z_{jt} + \pi_{20} x_{2t} + \sum_{j=1}^{p} \pi_{2j} z_{jt} x_{2t} + \varepsilon_{0t} + \varepsilon_{1t} x_{1t} + \varepsilon_{2t} x_{2t} \quad \text{(A.9)}$$

This equation is estimated using a computer program developed by Chang, Swamy, Hallahan, and Tavlas (1999). It is important to be aware that *equation (A.9)* has three error terms, two of which are the products of ε s and the included explanatory variables of *equation (1)*. The sum of these three terms is both heteroskedastic and serially correlated.

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[31] A formal definition of concomitants is provided in footnote 7 in Swamy and Tavlas.

Appendix B: Data on the U.S. Gasoline Market, 36 Yearly Observations, 1960-1995

YEAR	G	PG	Y	PNC	PUC	POP
1960	129.7	0.925	6036	1.045	0.836	180.7
1961	131.3	0.914	6113	1.045	0.869	183.7
1962	137.1	0.919	6271	1.041	0.948	186.5
1963	141.6	0.918	6378	1.035	0.96	189.2
1964	148.8	0.914	6727	1.032	1.001	191.9
1965	155.9	0.949	7027	1.009	0.994	194.3
1966	164.9	0.97	7280	0.991	0.97	196.6
1967	171	1	7513	1	1	198.7
1968	183.4	1.014	7728	1.028	1.028	200.7
1969	195.8	1.047	7891	1.044	1.031	202.7
1970	207.4	1.056	8134	1.076	1.043	205.1
1971	218.3	1.063	8322	1.12	1.102	207.7
1972	226.8	1.076	8562	1.11	1.105	209.9
1973	237.9	1.181	9042	1.111	1.176	211.9
1974	225.8	1.599	8867	1.175	1.226	213.9
1975	232.4	1.708	8944	1.276	1.464	216
1976	241.7	1.779	9175	1.357	1.679	218
1977	249.2	1.882	9381	1.429	1.828	220.2
1978	261.3	1.963	9735	1.538	1.865	222.6
1979	248.9	2.656	9829	1.66	2.01	225.1
1980	226.8	3.691	9722	1.793	2.081	227.7
1981	225.6	4.109	9769	1.902	2.569	230
1982	228.8	3.894	9725	1.976	2.964	232.2
1983	239.6	3.764	9930	2.026	3.297	234.3
1984	244.7	3.707	10421	2.085	3.757	236.3
1985	245.8	3.738	10563	2.152	3.797	238.5
1986	269.4	2.921	10780	2.24	3.632	240.7
1987	276.8	3.038	10859	2.321	3.776	242.8
1988	279.9	3.065	11186	2.368	3.939	245
1989	284.1	3.353	11300	2.414	4.019	247.3
1990	282	3.834	11389	2.451	3.926	249.9
1991	271.8	3.766	11272	2.538	3.942	252.6
1992	280.2	3.751	11466	2.528	4.113	255.4
1993	286.7	3.713	11476	2.663	4.47	258.1
1994	290.2	3.732	11636	2.754	4.73	260.7
1995	297.8	3.789	11934	2.815	5.224	263.2

Appendix C:

Chart 4.1A: Time Profile of Beta 1 for model 3 (full sample).

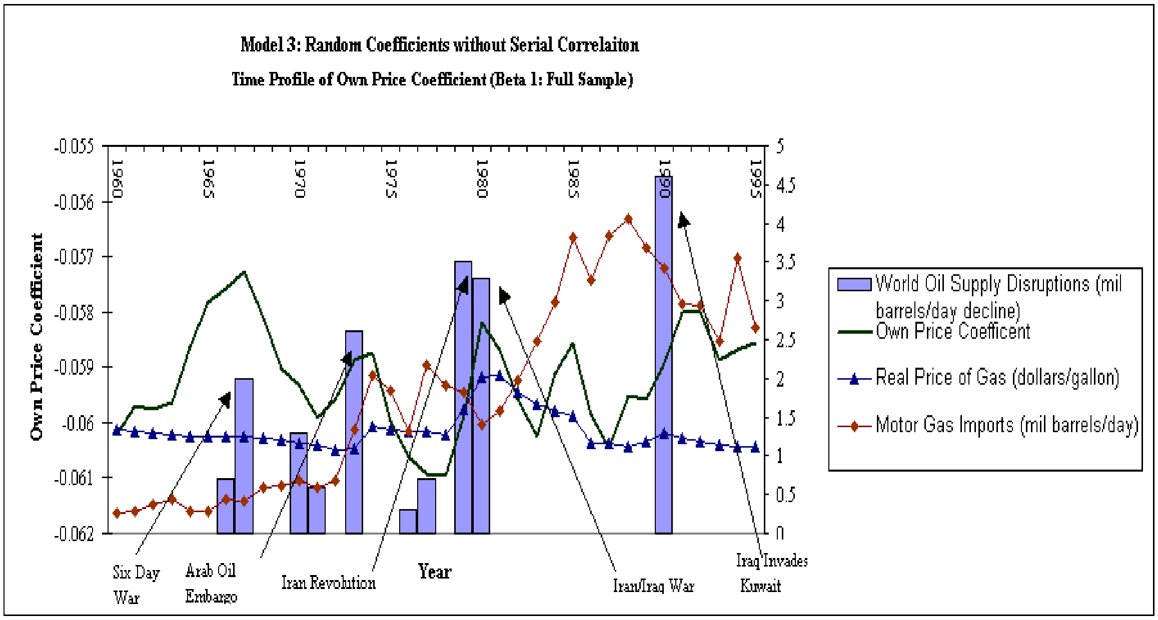


Chart 4.1B: Time Profile of Beta 1 for model 3 (partial sample).

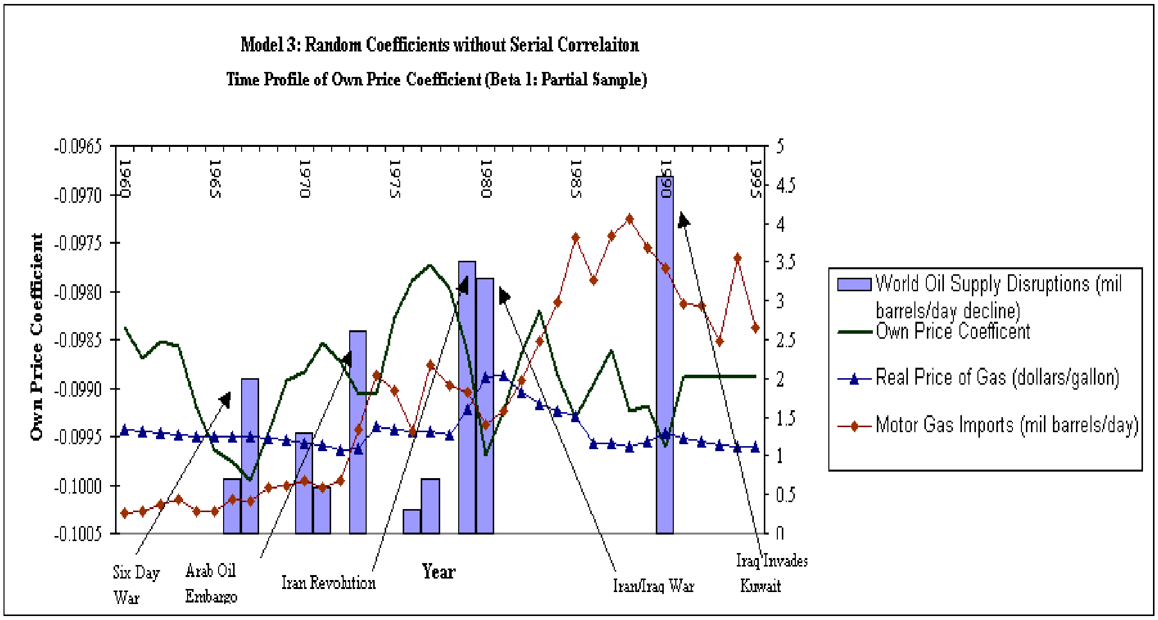


Chart 4.2A: Time Profile of Beta 2 for model 3 (full sample).

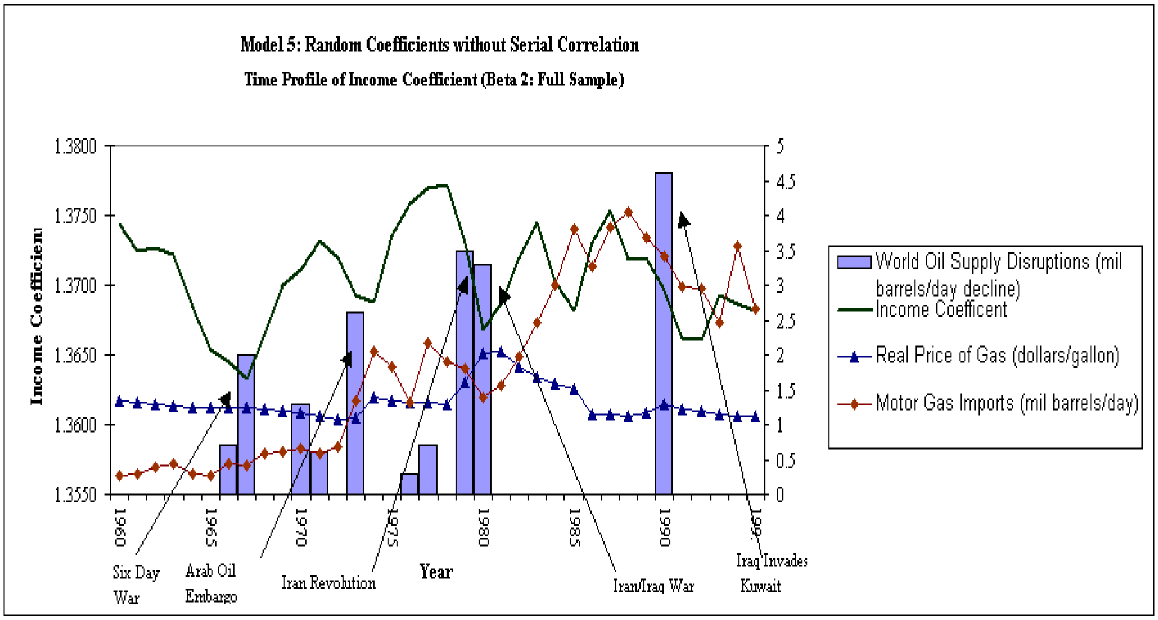


Chart 4.2B: Time Profile of Beta 2 for model 3 (partial sample).

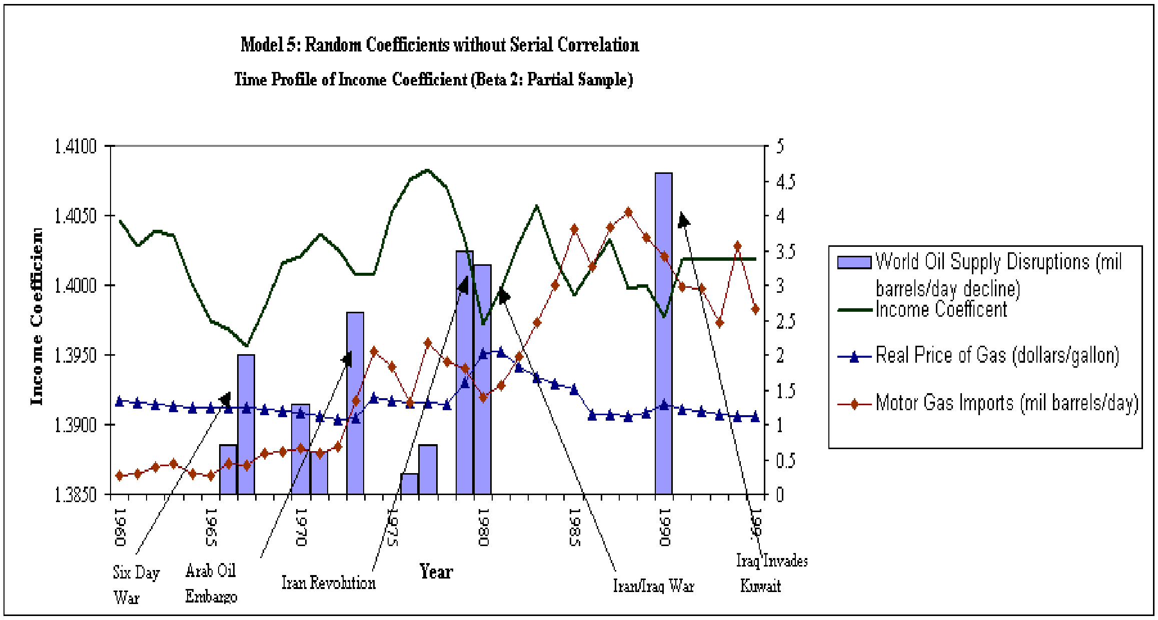


Chart 4.3A: Time Profile of Beta 3 for model 3 (full sample).

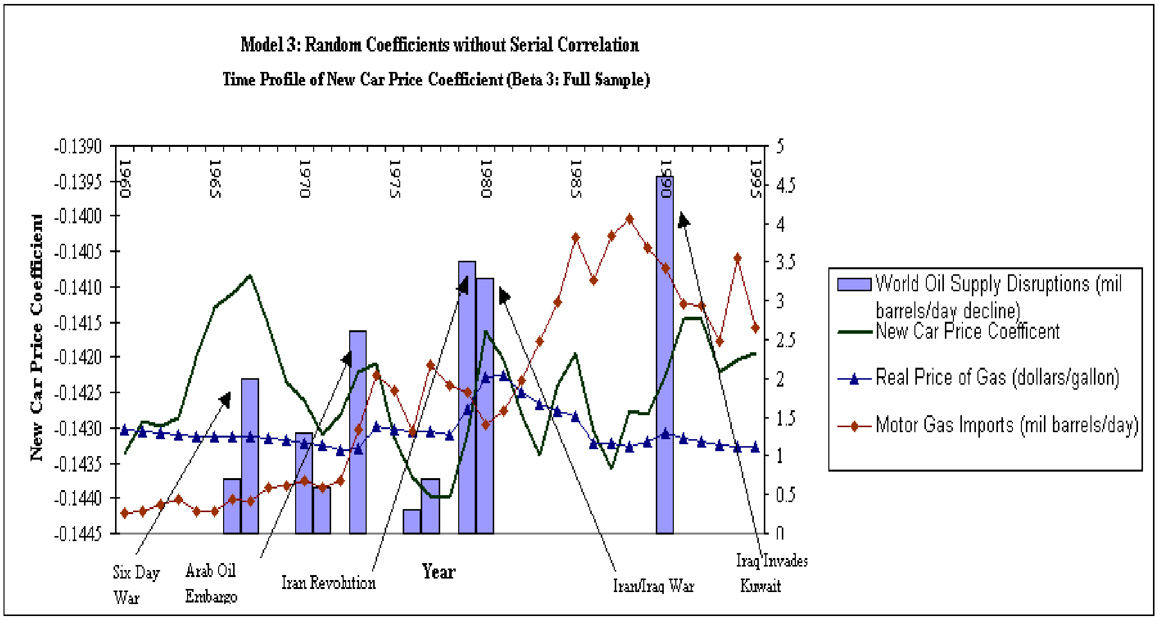


Chart 4.3B: Time Profile of Beta 3 for model 3 (partial sample).

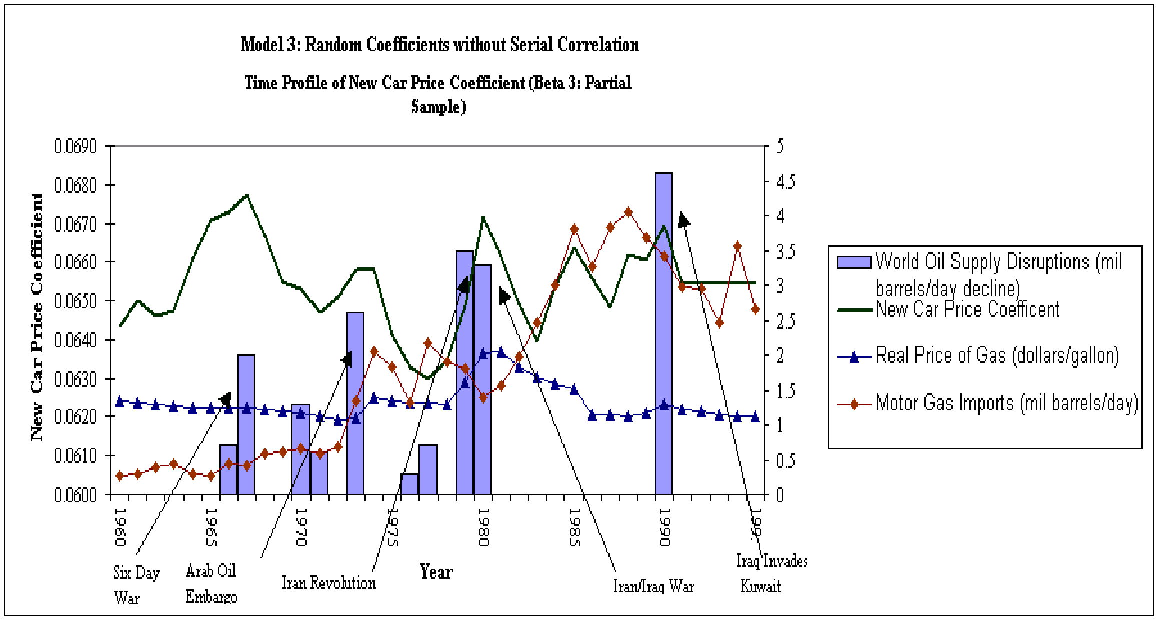


Chart 4.4A: Time Profile of Beta 4 for model 3 (full sample).

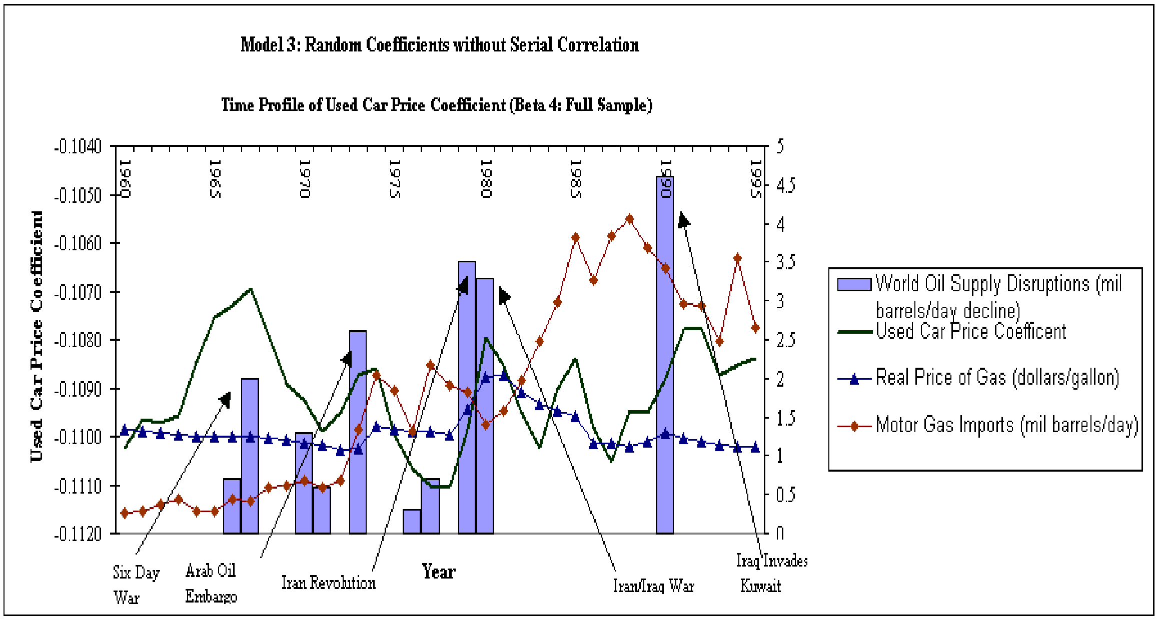


Chart 4.4B: Time Profile of Beta 4 for model 3 (partial sample).

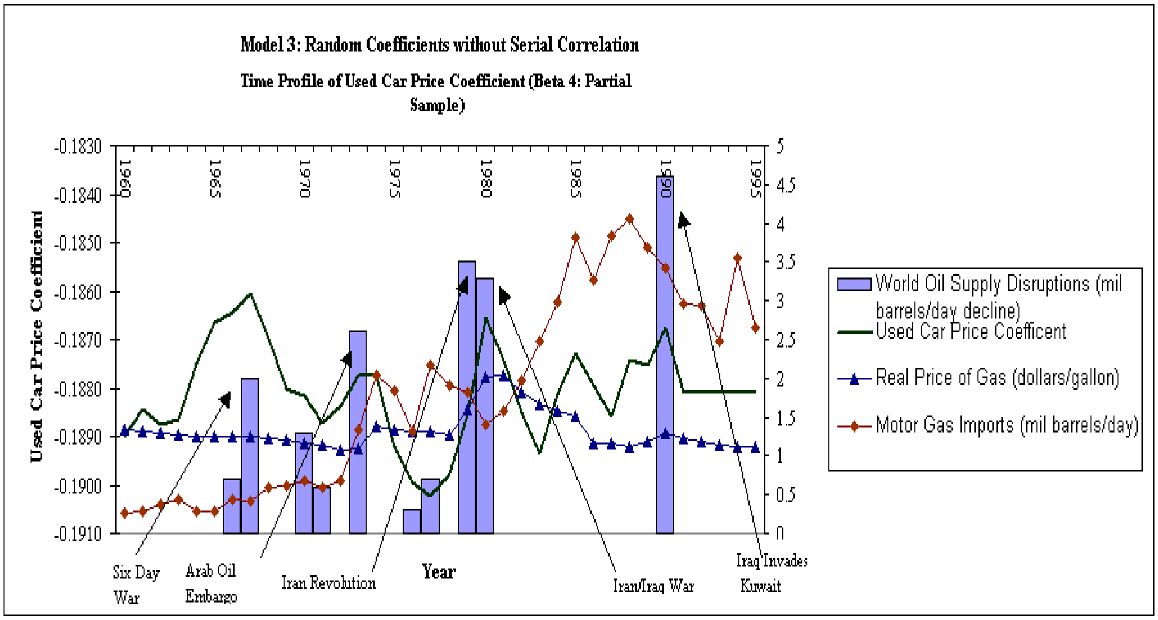


Chart 5.1A: Time Profile of Beta 1 for model 4 (full sample).

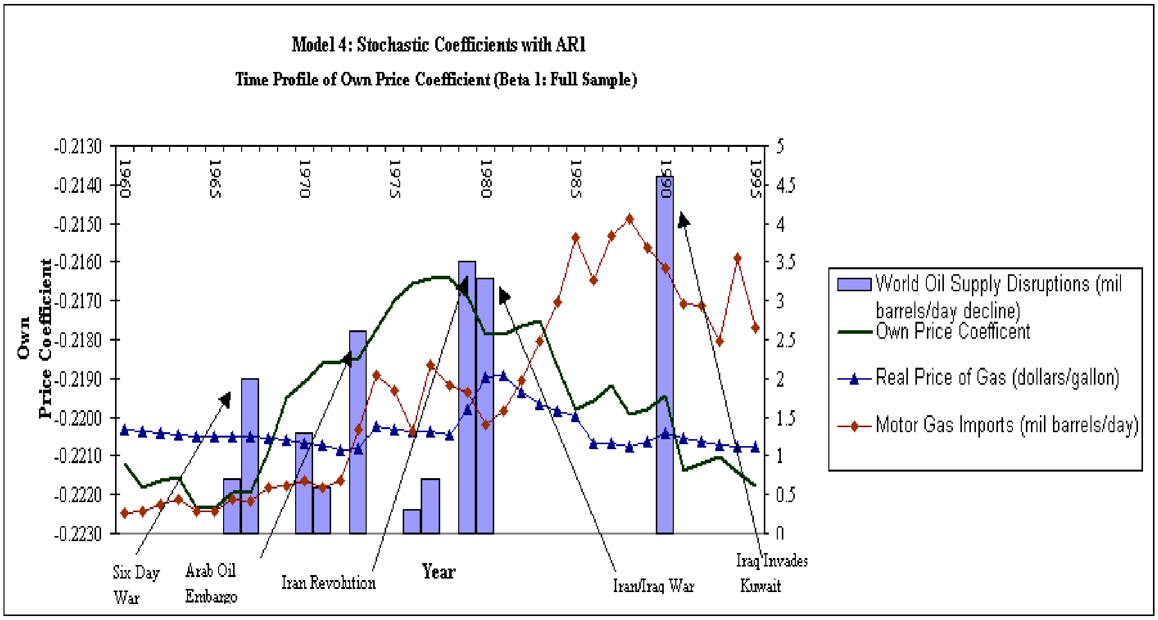


Chart 5.1B: Time Profile of Beta 1 for model 4 (partial sample).

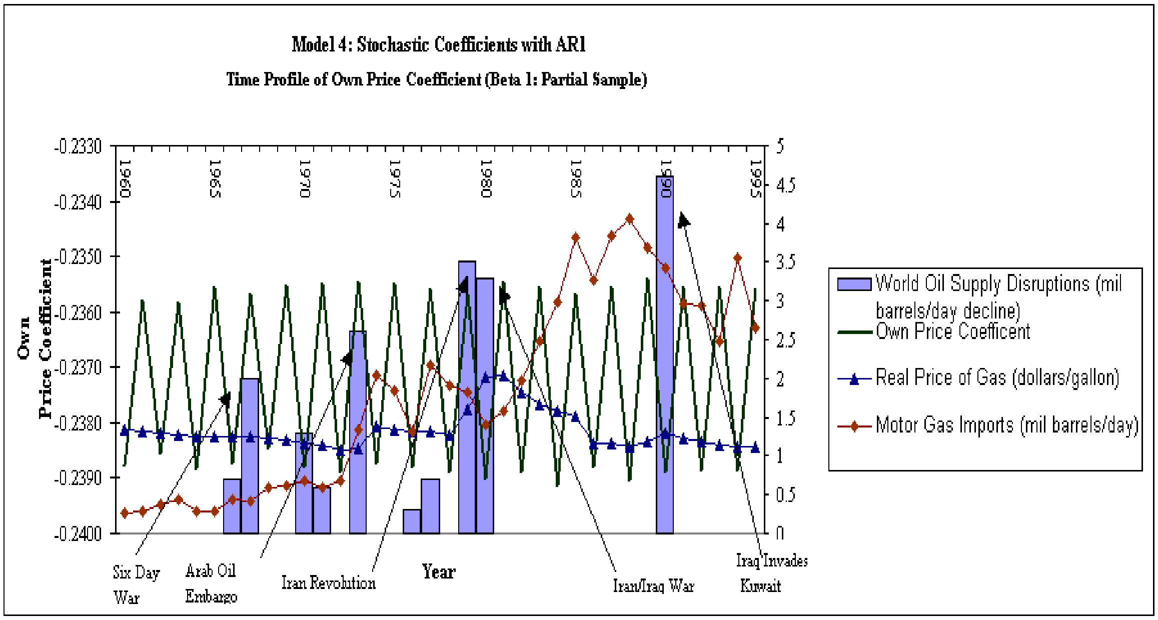


Chart 5.2A: Time Profile of Beta 2 for model 4 (full sample).

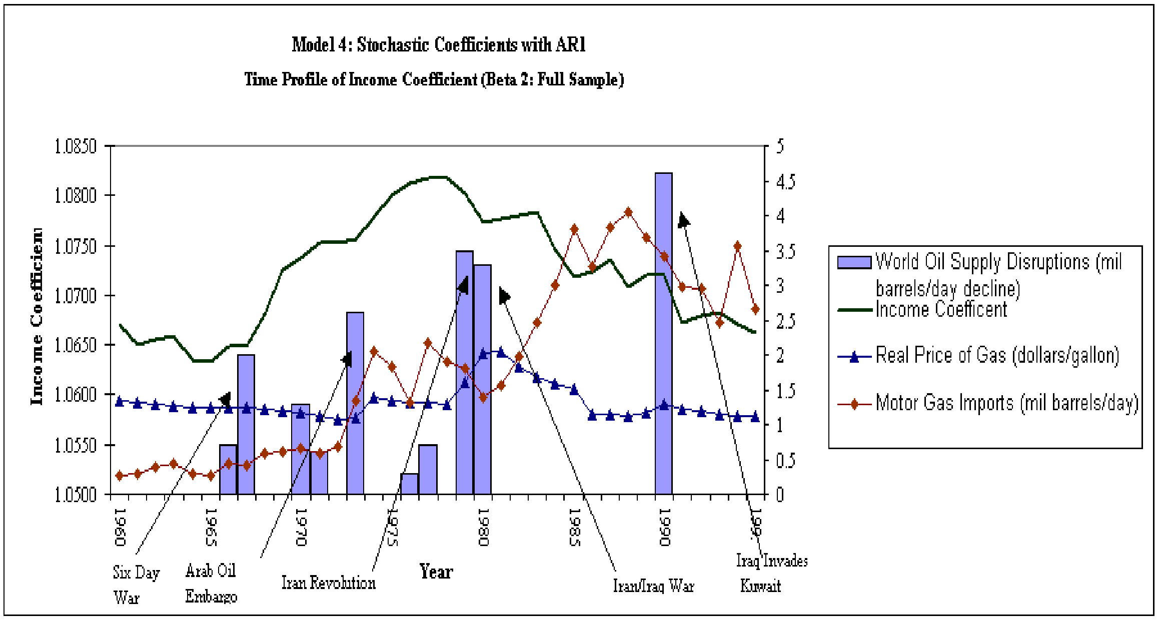


Chart 5.2B: Time Profile of Beta 2 for model 4 (partial sample).

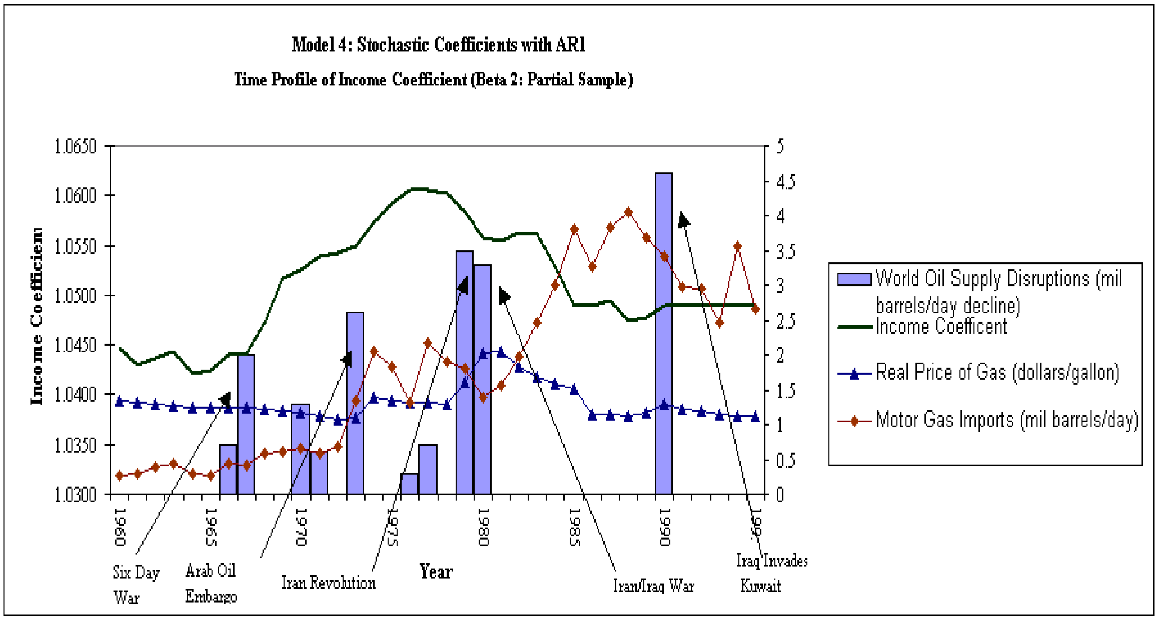


Chart 5.3A: Time Profile of Beta 3 for model 4 (full sample).

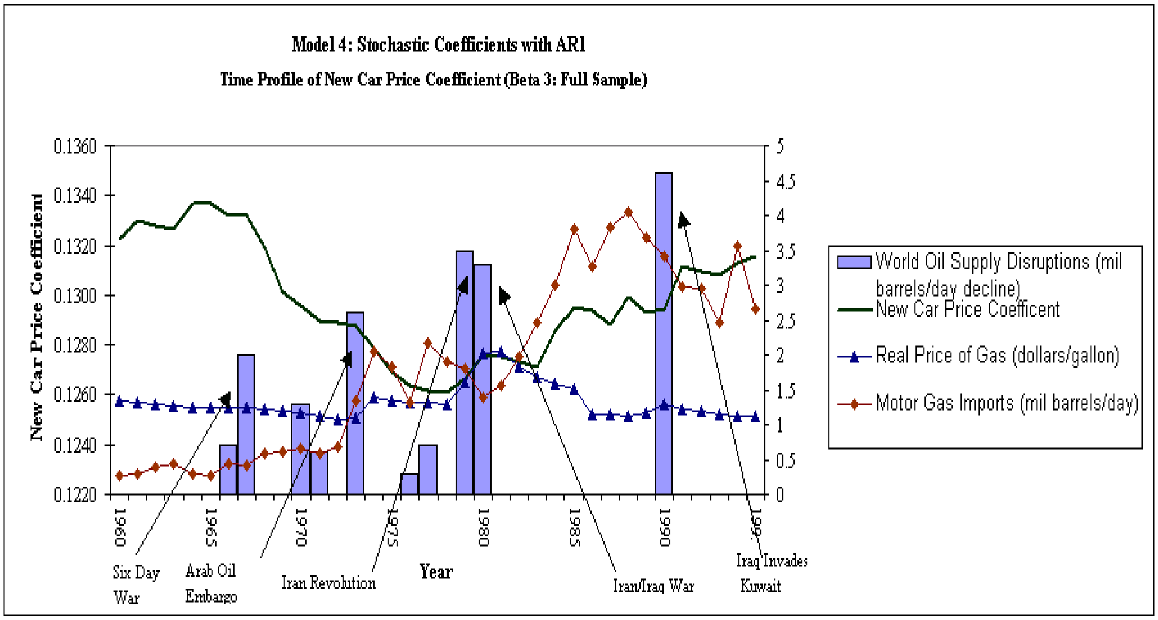


Chart 5.3B: Time Profile of Beta 3 for model 4 (partial sample).

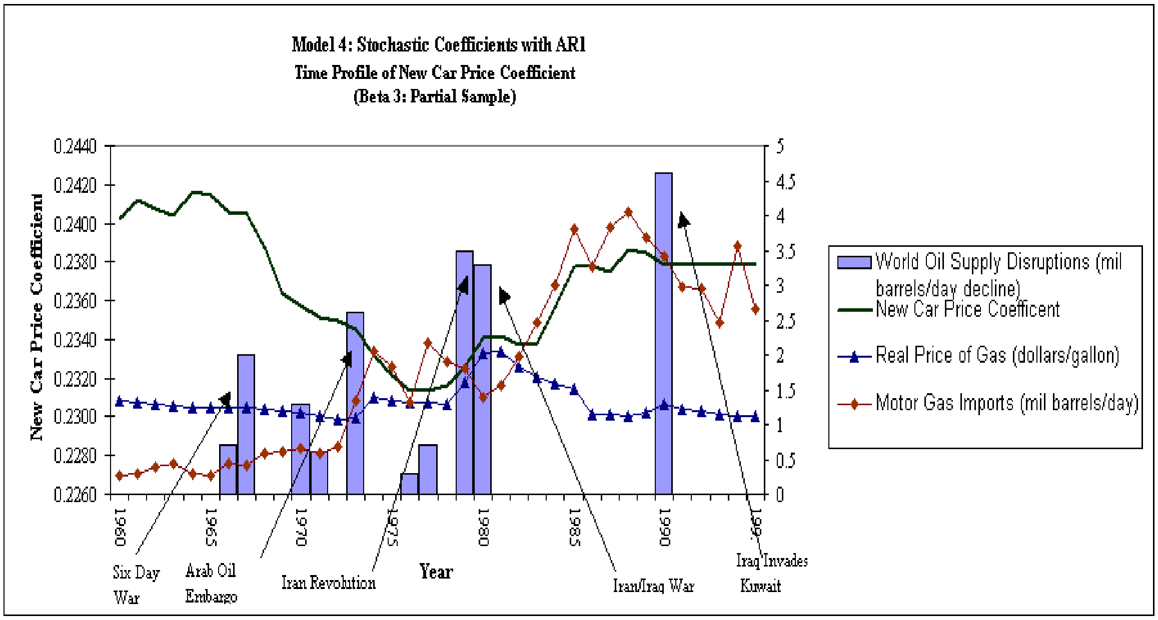


Chart 5.4A: Time Profile of Beta 4 for model 4 (full sample).

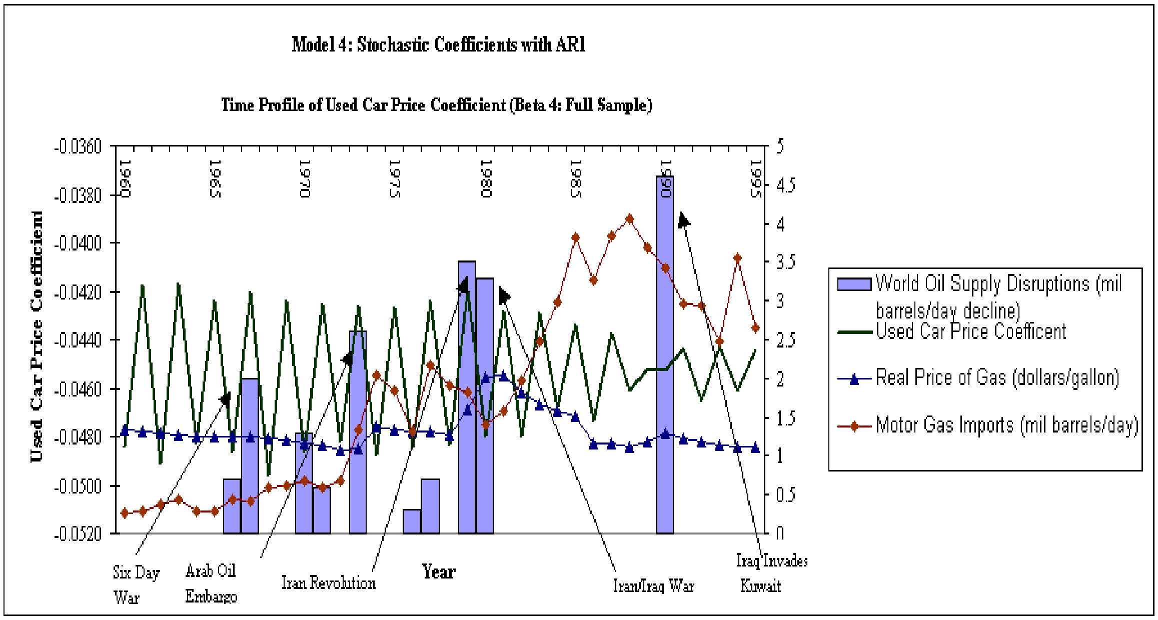


Chart 5.4B: Time Profile of Beta 4 for model 4 (partial sample).

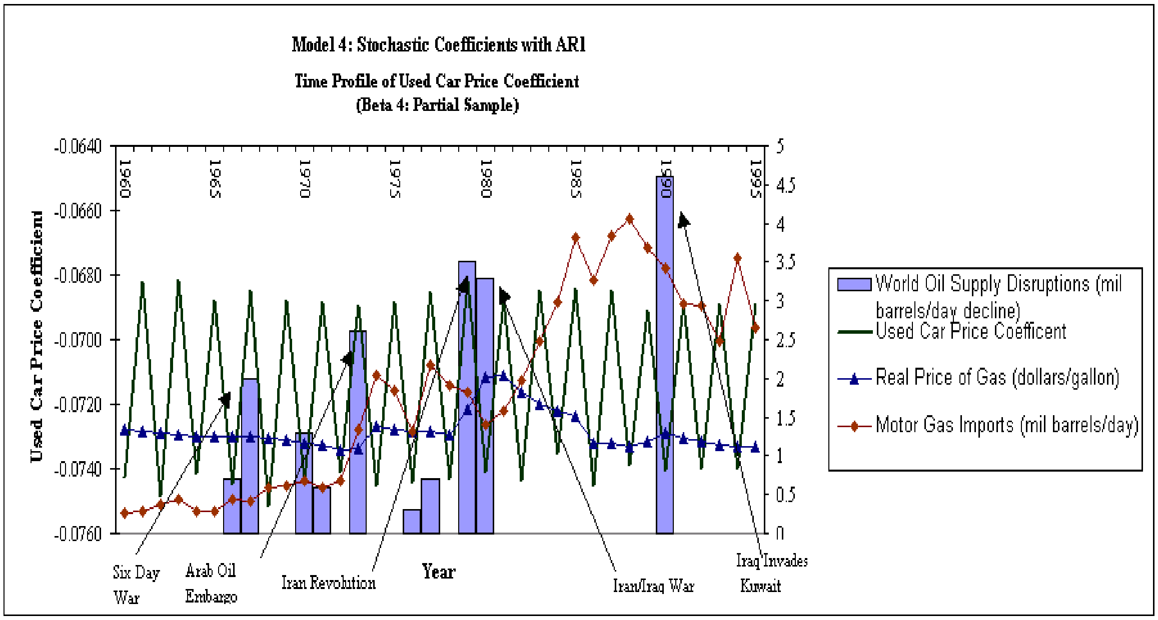


Chart 6.1A: Time Profile of Beta 1 for model 5 (full sample).

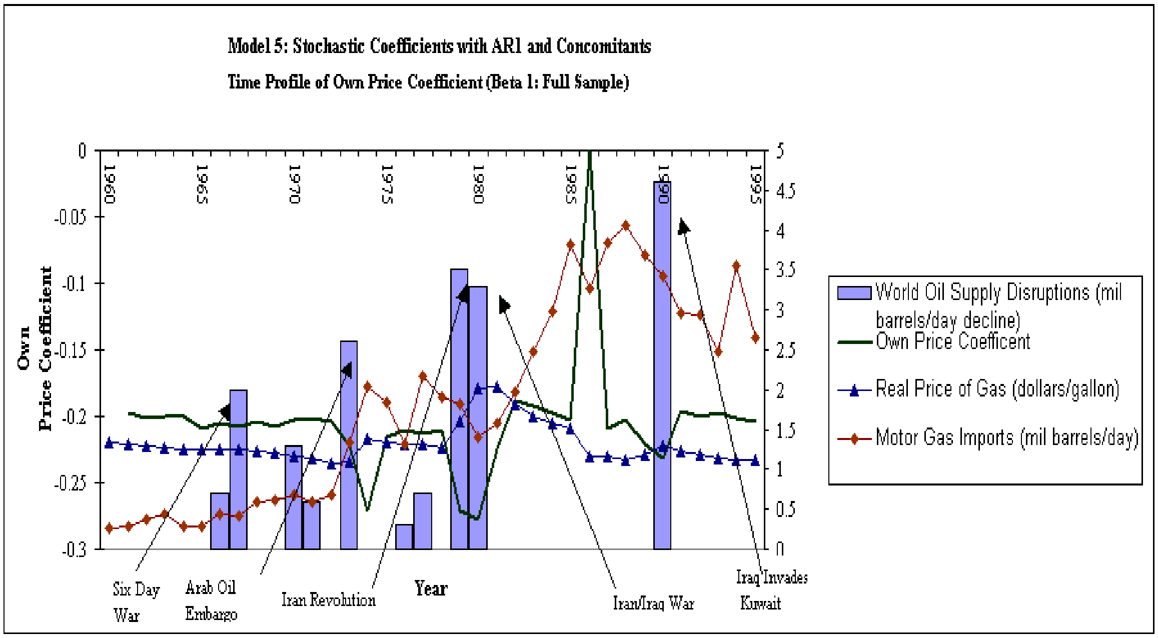


Chart 6.1B: Time Profile of Beta 1 for model 5 (partial sample).

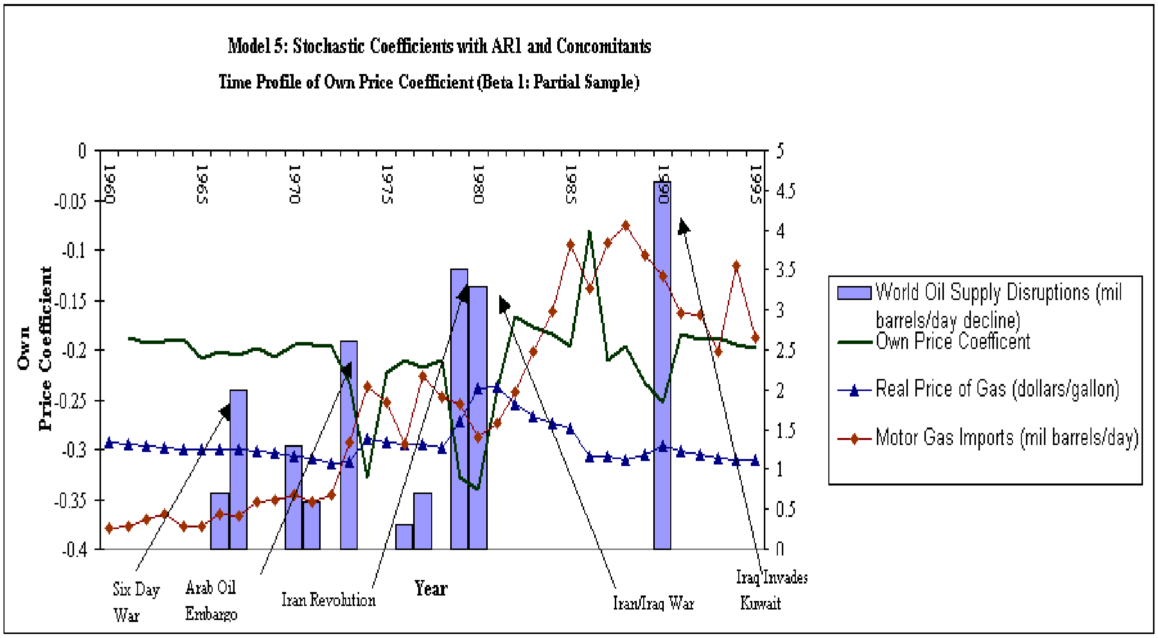


Chart 6.2A: Time Profile of Beta 2 for model 5 (full sample).

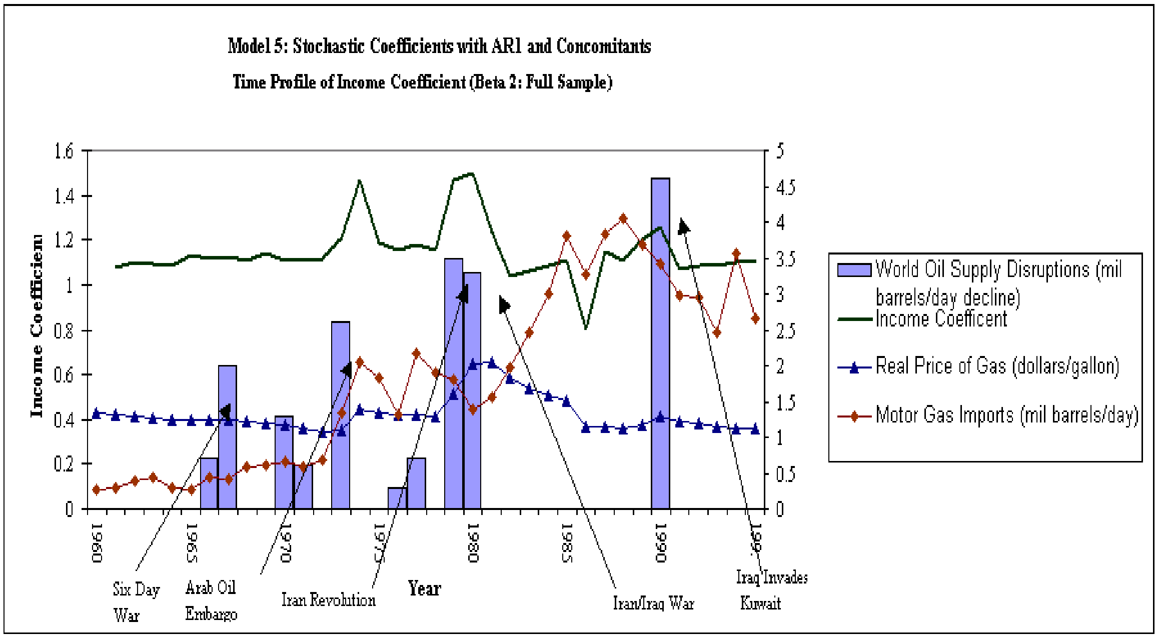


Chart 6.2B: Time Profile of Beta 2 for model 5 (partial sample).

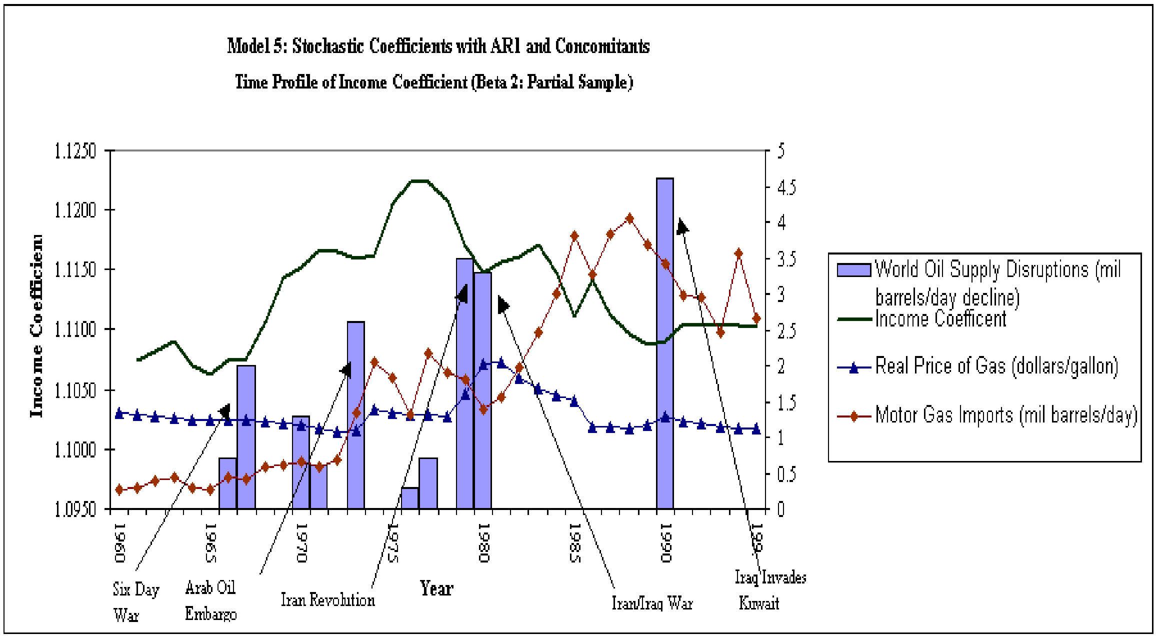


Chart 6.3A: Time Profile of Beta 3 for model 5 (full sample).

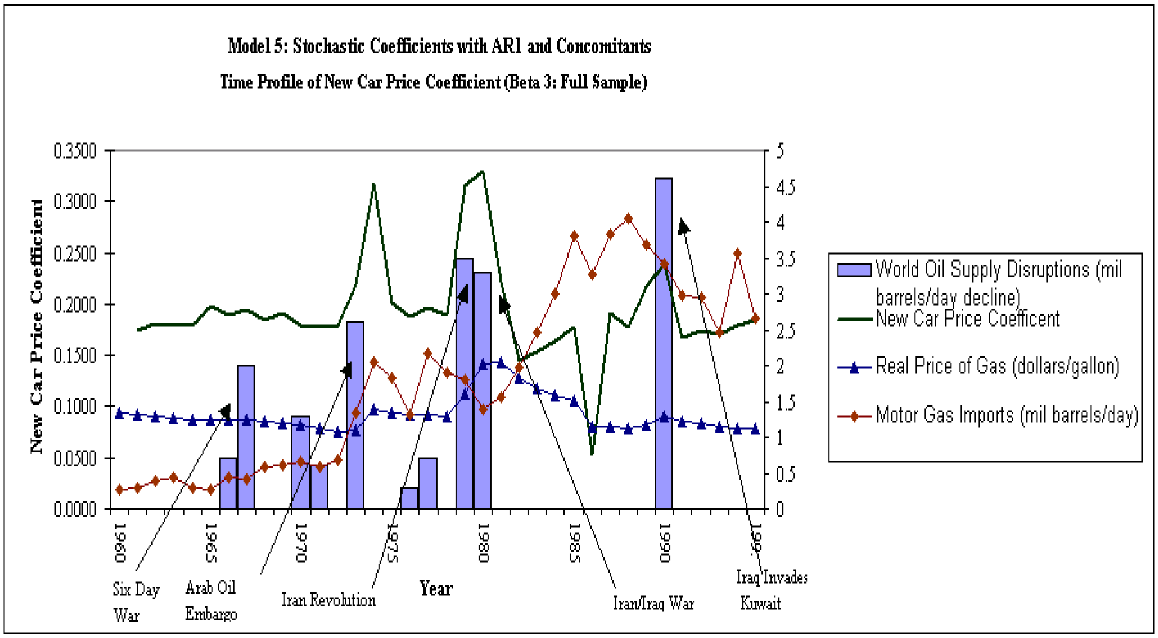


Chart 6.3B: Time Profile of Beta 3 for model 5 (partial sample).

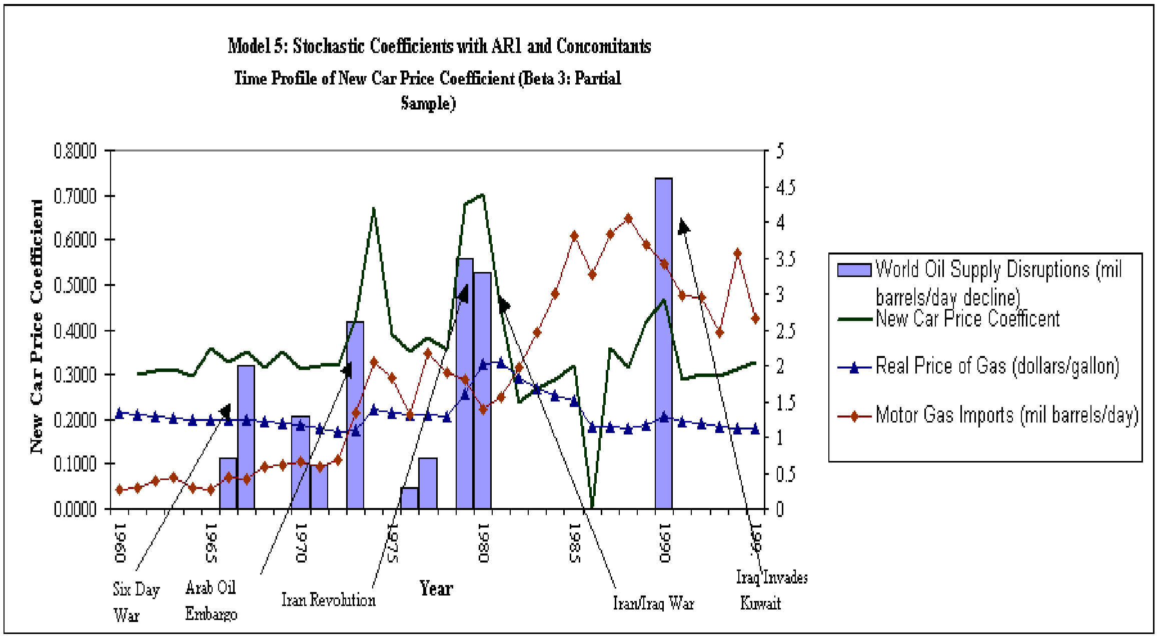


Chart 6.4A: Time Profile of Beta 4 for model 5 (full sample).

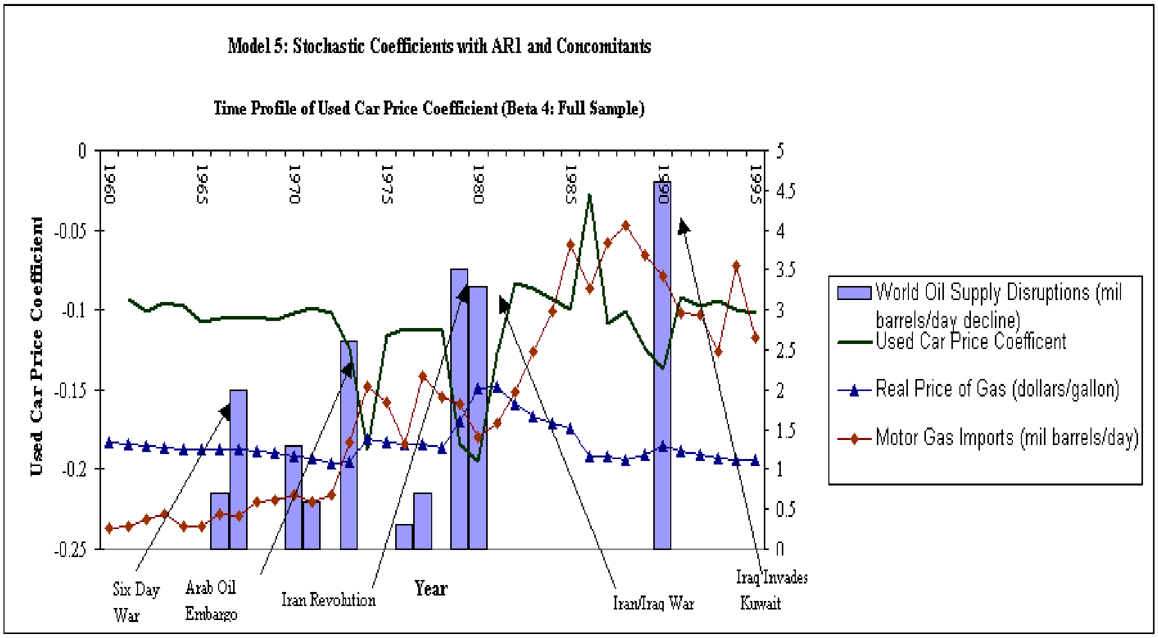
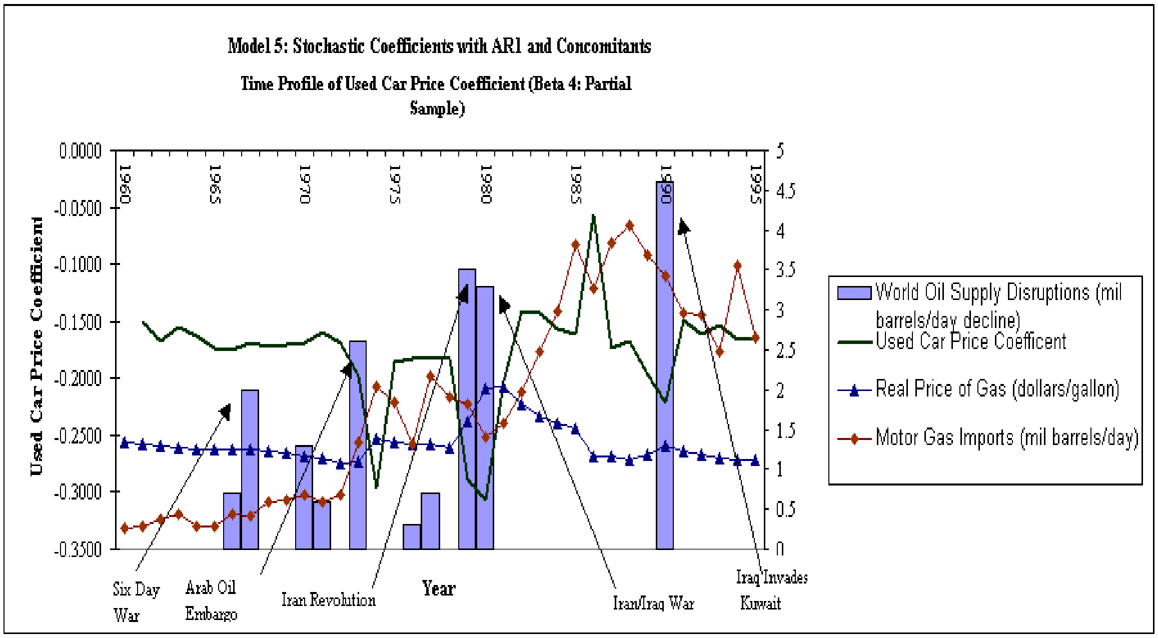


Chart 6.4B: Time Profile of Beta 4 for model 5 (partial sample).



Appendix D: Calculation of Beta Vectors[32]

Table A: Beta Vectors for Model 3 (Full Sample and Partial Sample)								
Full Sample					**Partial Sample**			
			New Car	Used Car			New Car	Used Car
Year	Own Price	Income	Price	Price	Own Price	Income	Price	Price
1960	-0.0602	1.3744	-0.1434	-0.1103	-0.0984	1.4046	0.0644	-0.1890
1961	-0.0597	1.3724	-0.1429	-0.1097	-0.0987	1.4028	0.0650	-0.1884
1962	-0.0598	1.3727	-0.1430	-0.1097	-0.0985	1.4039	0.0646	-0.1888
1963	-0.0596	1.3722	-0.1429	-0.1096	-0.0986	1.4035	0.0648	-0.1887
1964	-0.0586	1.3684	-0.1420	-0.1085	-0.0992	1.4000	0.0661	-0.1875
1965	-0.0578	1.3654	-0.1413	-0.1076	-0.0996	1.3975	0.0670	-0.1867
1966	-0.0576	1.3645	-0.1411	-0.1073	-0.0998	1.3968	0.0673	-0.1864
1967	-0.0573	1.3633	-0.1408	-0.1069	-0.1000	1.3956	0.0677	-0.1861
1968	-0.0581	1.3665	-0.1416	-0.1079	-0.0995	1.3984	0.0667	-0.1870
1969	-0.0590	1.3700	-0.1424	-0.1089	-0.0989	1.4016	0.0655	-0.1880
1970	-0.0593	1.3711	-0.1426	-0.1093	-0.0988	1.4021	0.0653	-0.1882
1971	-0.0599	1.3732	-0.1431	-0.1099	-0.0985	1.4037	0.0647	-0.1887
1972	-0.0596	1.3719	-0.1428	-0.1095	-0.0987	1.4026	0.0651	-0.1884
1973	-0.0589	1.3693	-0.1422	-0.1087	-0.0991	1.4007	0.0658	-0.1877
1974	-0.0587	1.3688	-0.1421	-0.1086	-0.0991	1.4007	0.0658	-0.1877
1975	-0.0600	1.3735	-0.1432	-0.1100	-0.0983	1.4053	0.0641	-0.1892
1976	-0.0606	1.3759	-0.1437	-0.1107	-0.0979	1.4075	0.0633	-0.1899
1977	-0.0609	1.3770	-0.1440	-0.1110	-0.0977	1.4083	0.0630	-0.1902
1978	-0.0610	1.3772	-0.1440	-0.1111	-0.0980	1.4070	0.0635	-0.1898
1979	-0.0599	1.3731	-0.1431	-0.1099	-0.0986	1.4032	0.0649	-0.1885
1980	-0.0582	1.3668	-0.1416	-0.1080	-0.0997	1.3972	0.0672	-0.1866
1981	-0.0587	1.3686	-0.1420	-0.1085	-0.0992	1.3998	0.0662	-0.1874
1982	-0.0596	1.3720	-0.1428	-0.1095	-0.0986	1.4031	0.0649	-0.1885
1983	-0.0602	1.3745	-0.1434	-0.1103	-0.0982	1.4057	0.0640	-0.1894
1984	-0.0591	1.3703	-0.1424	-0.1090	-0.0989	1.4019	0.0654	-0.1881
1985	-0.0586	1.3682	-0.1420	-0.1084	-0.0993	1.3993	0.0663	-0.1873
1986	-0.0599	1.3730	-0.1431	-0.1098	-0.0989	1.4014	0.0656	-0.1880
1987	-0.0605	1.3754	-0.1436	-0.1105	-0.0986	1.4033	0.0648	-0.1886
1988	-0.0595	1.3718	-0.1428	-0.1095	-0.0992	1.3997	0.0662	-0.1874
1989	-0.0596	1.3720	-0.1428	-0.1095	-0.0992	1.4000	0.0661	-0.1875
1990	-0.0589	1.3696	-0.1423	-0.1088	-0.0996	1.3977	0.0670	-0.1867
1991	-0.0580	1.3662	-0.1415	-0.1078	-0.0989	1.4018	0.0654	-0.1881
1992	-0.0580	1.3661	-0.1415	-0.1078	-0.0989	1.4018	0.0654	-0.1881
1993	-0.0589	1.3693	-0.1422	-0.1087	-0.0989	1.4018	0.0654	-0.1881
1994	-0.0587	1.3687	-0.1421	-0.1085	-0.0989	1.4018	0.0654	-0.1881
1995	-0.0586	1.3682	-0.1420	-0.1084	-0.0989	1.4018	0.0654	-0.1881
Averages	-0.0592	1.3706	-0.1425	-0.1091	-0.0989	1.4018	0.0654	-0.1881

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[32] The averages displayed in *Tables A-C* will not match the numbers reported in *Table 1.0* because this is only a partial sample of what goes into Swamy's estimation procedure.

Table B: Beta Vectors for Model 4 (Full Sample and Partial Sample)

Full Sample					**Partial Sample**			
			New Car	Used Car			New Car	Used Car
Year	Own Price	Income	Price	Price	Own Price	Income	Price	Price
1960	-0.2212	1.0670	0.1322	-0.0484	-0.2388	1.0446	0.2403	-0.0742
1961	-0.2218	1.0651	0.1330	-0.0417	-0.2358	1.0429	0.2412	-0.0682
1962	-0.2217	1.0656	0.1328	-0.0491	-0.2386	1.0437	0.2408	-0.0749
1963	-0.2216	1.0659	0.1327	-0.0417	-0.2358	1.0443	0.2404	-0.0682
1964	-0.2223	1.0635	0.1337	-0.0483	-0.2388	1.0421	0.2417	-0.0742
1965	-0.2223	1.0634	0.1337	-0.0424	-0.2356	1.0425	0.2414	-0.0688
1966	-0.2219	1.0648	0.1332	-0.0487	-0.2387	1.0440	0.2406	-0.0745
1967	-0.2219	1.0648	0.1332	-0.0420	-0.2357	1.0442	0.2405	-0.0685
1968	-0.2209	1.0681	0.1319	-0.0496	-0.2385	1.0473	0.2388	-0.0752
1969	-0.2195	1.0725	0.1301	-0.0424	-0.2355	1.0517	0.2363	-0.0688
1970	-0.2191	1.0737	0.1296	-0.0486	-0.2388	1.0526	0.2358	-0.0744
1971	-0.2186	1.0752	0.1290	-0.0425	-0.2355	1.0539	0.2351	-0.0688
1972	-0.2186	1.0753	0.1289	-0.0482	-0.2389	1.0542	0.2349	-0.0741
1973	-0.2185	1.0756	0.1288	-0.0426	-0.2355	1.0549	0.2345	-0.0690
1974	-0.2178	1.0778	0.1279	-0.0488	-0.2387	1.0574	0.2332	-0.0745
1975	-0.2170	1.0801	0.1269	-0.0426	-0.2355	1.0592	0.2321	-0.0688
1976	-0.2166	1.0813	0.1264	-0.0485	-0.2388	1.0605	0.2314	-0.0744
1977	-0.2164	1.0817	0.1262	-0.0424	-0.2356	1.0606	0.2314	-0.0685
1978	-0.2164	1.0817	0.1261	-0.0484	-0.2389	1.0602	0.2316	-0.0743
1979	-0.2169	1.0802	0.1266	-0.0420	-0.2357	1.0584	0.2326	-0.0681
1980	-0.2179	1.0774	0.1276	-0.0480	-0.2390	1.0557	0.2341	-0.0741
1981	-0.2179	1.0776	0.1275	-0.0428	-0.2355	1.0555	0.2342	-0.0687
1982	-0.2176	1.0780	0.1273	-0.0480	-0.2389	1.0563	0.2337	-0.0744
1983	-0.2175	1.0784	0.1271	-0.0428	-0.2355	1.0562	0.2338	-0.0685
1984	-0.2187	1.0746	0.1286	-0.0468	-0.2392	1.0529	0.2356	-0.0735
1985	-0.2198	1.0719	0.1295	-0.0434	-0.2357	1.0490	0.2378	-0.0684
1986	-0.2196	1.0723	0.1294	-0.0474	-0.2388	1.0490	0.2378	-0.0745
1987	-0.2192	1.0736	0.1288	-0.0438	-0.2356	1.0495	0.2375	-0.0685
1988	-0.2200	1.0709	0.1299	-0.0461	-0.2391	1.0474	0.2387	-0.0739
1989	-0.2198	1.0721	0.1293	-0.0452	-0.2354	1.0477	0.2385	-0.0691
1990	-0.2195	1.0721	0.1294	-0.0452	-0.2389	1.0490	0.2378	-0.0740
1991	-0.2214	1.0673	0.1312	-0.0444	-0.2356	1.0490	0.2378	-0.0689
1992	-0.2212	1.0679	0.1309	-0.0465	-0.2389	1.0490	0.2378	-0.0740
1993	-0.2210	1.0682	0.1308	-0.0443	-0.2356	1.0490	0.2378	-0.0689
1994	-0.2214	1.0671	0.1313	-0.0461	-0.2389	1.0490	0.2378	-0.0740
1995	-0.2218	1.0662	0.1316	-0.0444	-0.2356	1.0489	0.2378	-0.0689
Averages	-0.2196	1.0722	0.1298	-0.0454	-0.2372	1.0509	0.2368	-0.0715

Table C: Beta Vectors for Model 5 (Full Sample and Partial Sample)

Full Sample					Partial Sample			
Year	Own Price	Income	New Car Price	Used Car Price	Own Price	Income	New Car Price	Used Car Price
1960								
1961	-0.1975	1.0763	0.1744	-0.0929	-0.1871	1.1074	0.2987	-0.1506
1962	-0.2018	1.0988	0.1817	-0.1013	-0.1937	1.1082	0.3081	-0.1676
1963	-0.2002	1.0910	0.1786	-0.0956	-0.1920	1.1091	0.3119	-0.1553
1964	-0.1997	1.0849	0.1784	-0.0980	-0.1895	1.1070	0.2980	-0.1623
1965	-0.2090	1.1356	0.1983	-0.1076	-0.2093	1.1062	0.3577	-0.1727
1966	-0.2058	1.1176	0.1902	-0.1056	-0.2012	1.1075	0.3290	-0.1743
1967	-0.2074	1.1278	0.1944	-0.1053	-0.2061	1.1076	0.3496	-0.1694
1968	-0.2037	1.1110	0.1846	-0.1040	-0.1975	1.1106	0.3178	-0.1716
1969	-0.2076	1.1368	0.1907	-0.1061	-0.2067	1.1143	0.3507	-0.1704
1970	-0.2027	1.1097	0.1787	-0.1017	-0.1953	1.1152	0.3133	-0.1682
1971	-0.2017	1.1088	0.1768	-0.0988	-0.1952	1.1166	0.3194	-0.1596
1972	-0.2036	1.1154	0.1794	-0.1025	-0.1970	1.1165	0.3185	-0.1693
1973	-0.2218	1.2118	0.2184	-0.1239	-0.2342	1.1159	0.4242	-0.1974
1974	-0.2710	1.4653	0.3176	-0.1880	-0.3280	1.1162	0.6702	-0.2966
1975	-0.2156	1.1847	0.2018	-0.1157	-0.2220	1.1205	0.3914	-0.1854
1976	-0.2101	1.1561	0.1885	-0.1115	-0.2098	1.1224	0.3522	-0.1823
1977	-0.2134	1.1751	0.1956	-0.1127	-0.2177	1.1223	0.3805	-0.1807
1978	-0.2105	1.1574	0.1891	-0.1116	-0.2105	1.1208	0.3550	-0.1824
1979	-0.2707	1.4660	0.3155	-0.1841	-0.3286	1.1169	0.6788	-0.2879
1980	-0.2772	1.4965	0.3291	-0.1952	-0.3399	1.1148	0.7034	-0.3071
1981	-0.2252	1.2320	0.2221	-0.1276	-0.2406	1.1157	0.4419	-0.2032
1982	-0.1883	1.0388	0.1450	-0.0835	-0.1673	1.1161	0.2390	-0.1405
1983	-0.1922	1.0639	0.1535	-0.0872	-0.1768	1.1171	0.2690	-0.1426
1984	-0.1975	1.0828	0.1645	-0.0934	-0.1849	1.1147	0.2878	-0.1560
1985	-0.2020	1.1084	0.1774	-0.0998	-0.1959	1.1111	0.3209	-0.1607
1986	-0.1434	0.8061	0.0532	-0.0269	-0.0804	1.1142	0.0049	-0.0565
1987	-0.2094	1.1466	0.1913	-0.1084	-0.2099	1.1112	0.3588	-0.1740
1988	-0.2030	1.1076	0.1783	-0.1007	-0.1957	1.1097	0.3158	-0.1671
1989	-0.2209	1.2045	0.2171	-0.1239	-0.2326	1.1088	0.4191	-0.1966
1990	-0.2321	1.2565	0.2389	-0.1368	-0.2521	1.1090	0.4674	-0.2218
1991	-0.1961	1.0714	0.1680	-0.0918	-0.1841	1.1105	0.2888	-0.1495
1992	-0.1997	1.0887	0.1743	-0.0977	-0.1898	1.1104	0.2999	-0.1617
1993	-0.1979	1.0820	0.1709	-0.0940	-0.1875	1.1104	0.2981	-0.1529
1994	-0.2019	1.0989	0.1790	-0.0999	-0.1939	1.1103	0.3110	-0.1656
1995	-0.2038	1.1102	0.1843	-0.1016	-0.1989	1.1102	0.3288	-0.1640
Averages	-0.2098	1.1464	0.1937	-0.1096	-0.2100	1.1130	0.3566	-0.1778

Appendix E: Model 5 (Full Sample): Beta and Direct Effect Vectors Tables and Graphs

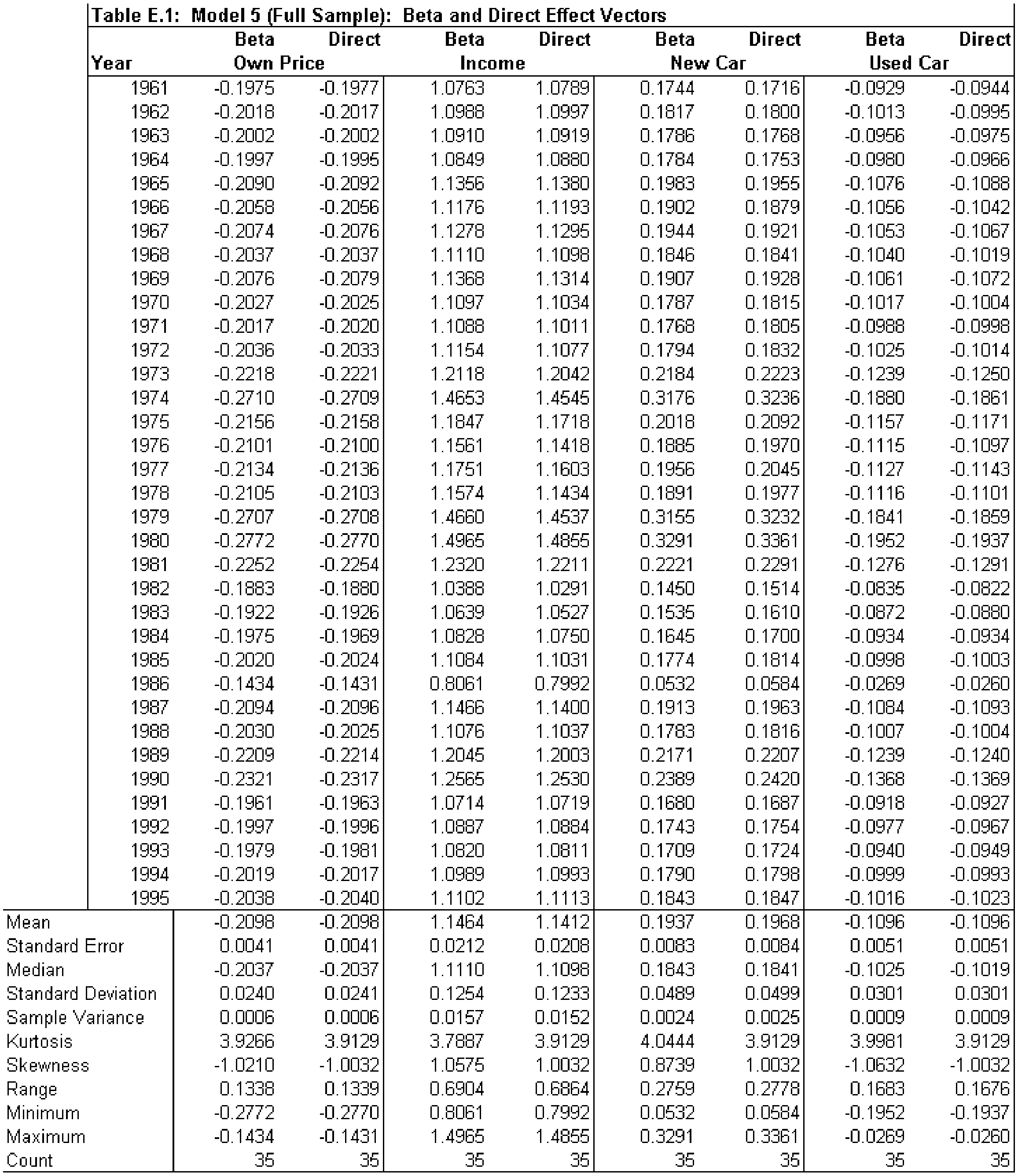


Table E.1: Model 5 (Full Sample): Beta and Direct Effect Vectors

	Beta	Direct	Beta	Direct	Beta	Direct	Beta	Direct
Year	**Own Price**		**Income**		**New Car**		**Used Car**	
1961	-0.1975	-0.1977	1.0763	1.0789	0.1744	0.1716	-0.0929	-0.0944
1962	-0.2018	-0.2017	1.0988	1.0997	0.1817	0.1800	-0.1013	-0.0995
1963	-0.2002	-0.2002	1.0910	1.0919	0.1786	0.1768	-0.0956	-0.0975
1964	-0.1997	-0.1995	1.0849	1.0880	0.1784	0.1753	-0.0980	-0.0966
1965	-0.2090	-0.2092	1.1356	1.1380	0.1983	0.1955	-0.1076	-0.1088
1966	-0.2058	-0.2056	1.1176	1.1193	0.1902	0.1879	-0.1056	-0.1042
1967	-0.2074	-0.2076	1.1278	1.1295	0.1944	0.1921	-0.1053	-0.1067
1968	-0.2037	-0.2037	1.1110	1.1098	0.1846	0.1841	-0.1040	-0.1019
1969	-0.2076	-0.2079	1.1368	1.1314	0.1907	0.1928	-0.1061	-0.1072
1970	-0.2027	-0.2025	1.1097	1.1034	0.1787	0.1815	-0.1017	-0.1004
1971	-0.2017	-0.2020	1.1088	1.1011	0.1768	0.1805	-0.0988	-0.0998
1972	-0.2036	-0.2033	1.1154	1.1077	0.1794	0.1832	-0.1025	-0.1014
1973	-0.2218	-0.2221	1.2118	1.2042	0.2184	0.2223	-0.1239	-0.1250
1974	-0.2710	-0.2709	1.4653	1.4545	0.3176	0.3236	-0.1880	-0.1861
1975	-0.2156	-0.2158	1.1847	1.1718	0.2018	0.2092	-0.1157	-0.1171
1976	-0.2101	-0.2100	1.1561	1.1418	0.1885	0.1970	-0.1115	-0.1097
1977	-0.2134	-0.2136	1.1751	1.1603	0.1956	0.2045	-0.1127	-0.1143
1978	-0.2105	-0.2103	1.1574	1.1434	0.1891	0.1977	-0.1116	-0.1101
1979	-0.2707	-0.2708	1.4660	1.4537	0.3155	0.3232	-0.1841	-0.1859
1980	-0.2772	-0.2770	1.4965	1.4855	0.3291	0.3361	-0.1952	-0.1937
1981	-0.2252	-0.2254	1.2320	1.2211	0.2221	0.2291	-0.1276	-0.1291
1982	-0.1883	-0.1880	1.0388	1.0291	0.1450	0.1514	-0.0835	-0.0822
1983	-0.1922	-0.1926	1.0639	1.0527	0.1535	0.1610	-0.0872	-0.0880
1984	-0.1975	-0.1969	1.0828	1.0750	0.1645	0.1700	-0.0934	-0.0934
1985	-0.2020	-0.2024	1.1084	1.1031	0.1774	0.1814	-0.0998	-0.1003
1986	-0.1434	-0.1431	0.8061	0.7992	0.0532	0.0584	-0.0269	-0.0260
1987	-0.2094	-0.2096	1.1466	1.1400	0.1913	0.1963	-0.1084	-0.1093
1988	-0.2030	-0.2025	1.1076	1.1037	0.1783	0.1816	-0.1007	-0.1004
1989	-0.2209	-0.2214	1.2045	1.2003	0.2171	0.2207	-0.1239	-0.1240
1990	-0.2321	-0.2317	1.2565	1.2530	0.2389	0.2420	-0.1368	-0.1369
1991	-0.1961	-0.1963	1.0714	1.0719	0.1680	0.1687	-0.0918	-0.0927
1992	-0.1997	-0.1996	1.0887	1.0884	0.1743	0.1754	-0.0977	-0.0967
1993	-0.1979	-0.1981	1.0820	1.0811	0.1709	0.1724	-0.0940	-0.0949
1994	-0.2019	-0.2017	1.0989	1.0993	0.1790	0.1798	-0.0999	-0.0993
1995	-0.2038	-0.2040	1.1102	1.1113	0.1843	0.1847	-0.1016	-0.1023
Mean	-0.2098	-0.2098	1.1464	1.1412	0.1937	0.1968	-0.1096	-0.1096
Standard Error	0.0041	0.0041	0.0212	0.0208	0.0083	0.0084	0.0051	0.0051
Median	-0.2037	-0.2037	1.1110	1.1098	0.1843	0.1841	-0.1025	-0.1019
Standard Deviation	0.0240	0.0241	0.1254	0.1233	0.0489	0.0499	0.0301	0.0301
Sample Variance	0.0006	0.0006	0.0157	0.0152	0.0024	0.0025	0.0009	0.0009
Kurtosis	3.9266	3.9129	3.7887	3.9129	4.0444	3.9129	3.9981	3.9129
Skewness	-1.0210	-1.0032	1.0575	1.0032	0.8739	1.0032	-1.0632	-1.0032
Range	0.1338	0.1339	0.6904	0.6864	0.2759	0.2778	0.1683	0.1676
Minimum	-0.2772	-0.2770	0.8061	0.7992	0.0532	0.0584	-0.1952	-0.1937
Maximum	-0.1434	-0.1431	1.4965	1.4855	0.3291	0.3361	-0.0269	-0.0260
Count	35	35	35	35	35	35	35	35

Appendix E continued:

Chart E.1: Time Profile of Own Price: Beta and Direct Effect Vectors (Model 5 Beta 1:Full Sample)

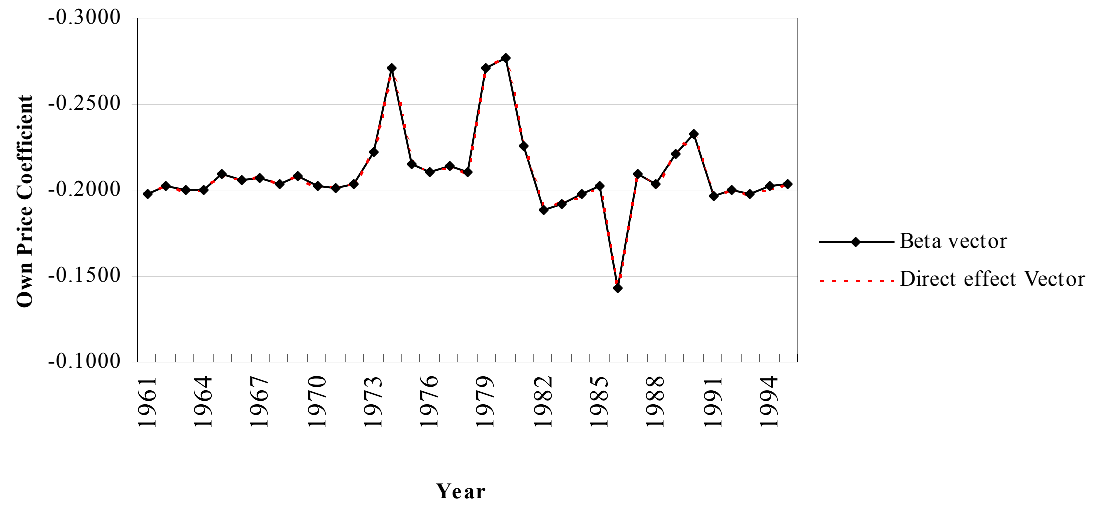
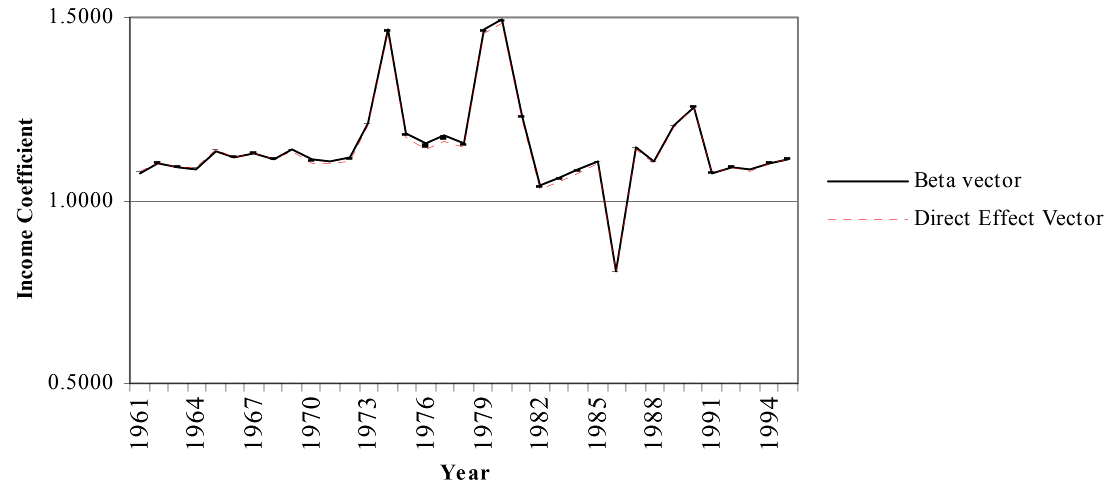
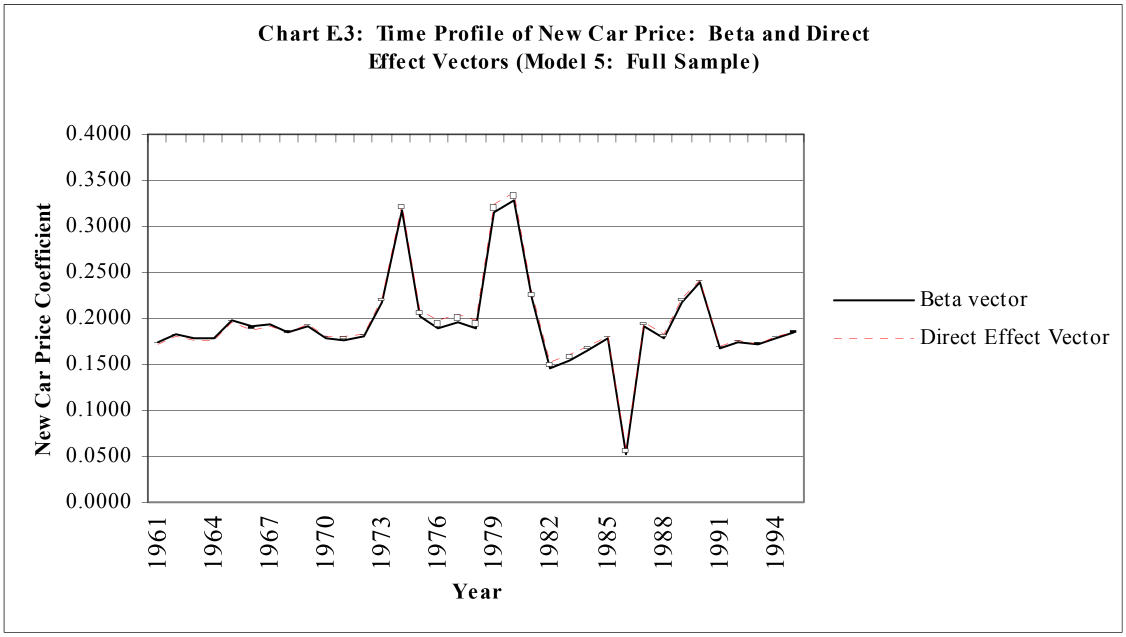
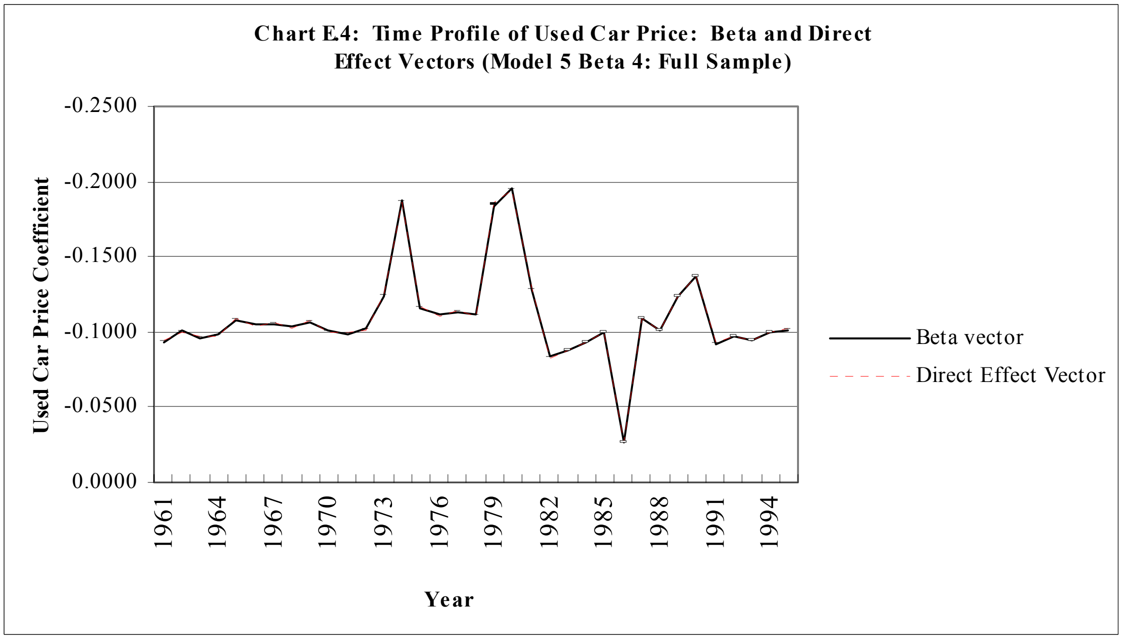


Chart E.2: Time Profile of Income: Beta and Direct Effect Vector (Model 5 Beta 2: Full Sample)



Appendix E continued:



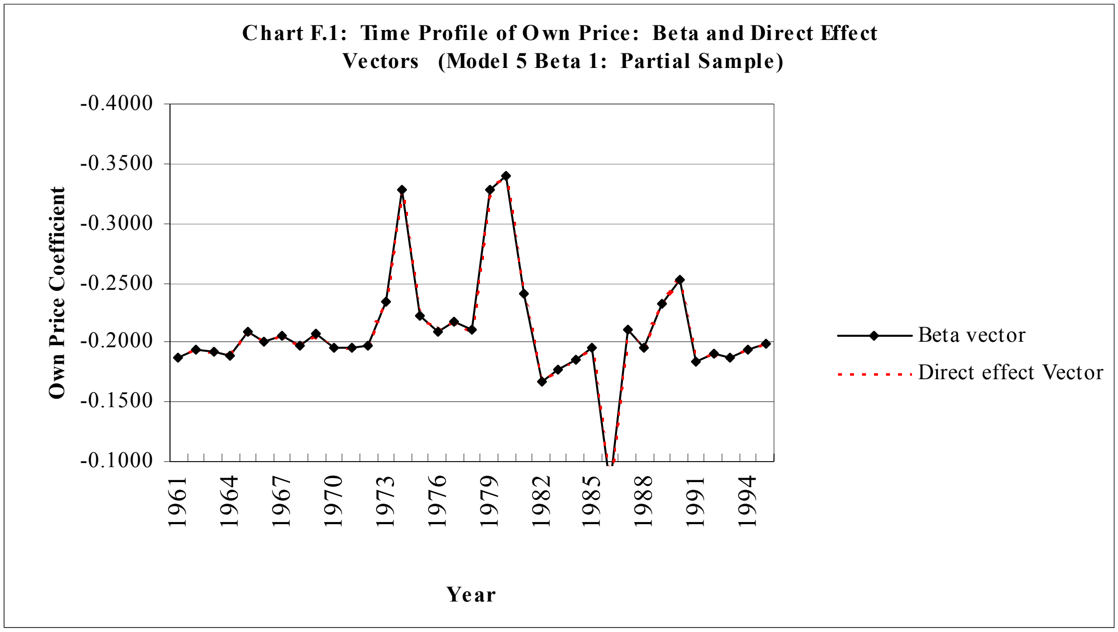


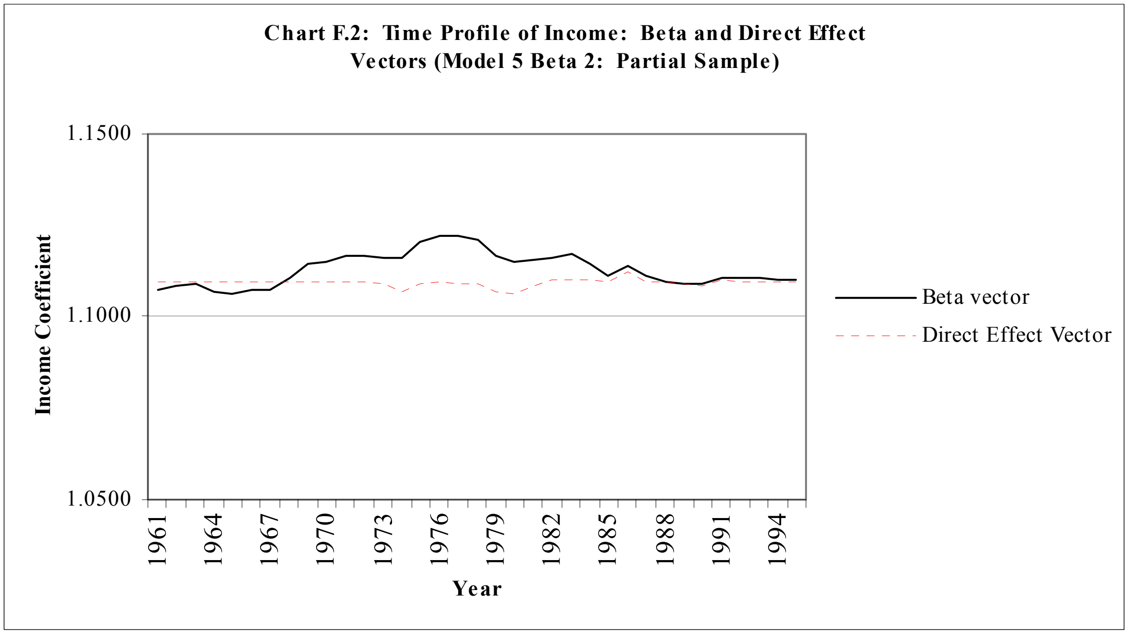
Appendix F: Model 5 (Partial Sample): Beta and Direct Effect Vectors Tables and Graphs

Table E.1: Model 5 (Partial Sample): Beta and Direct Effect Vectors

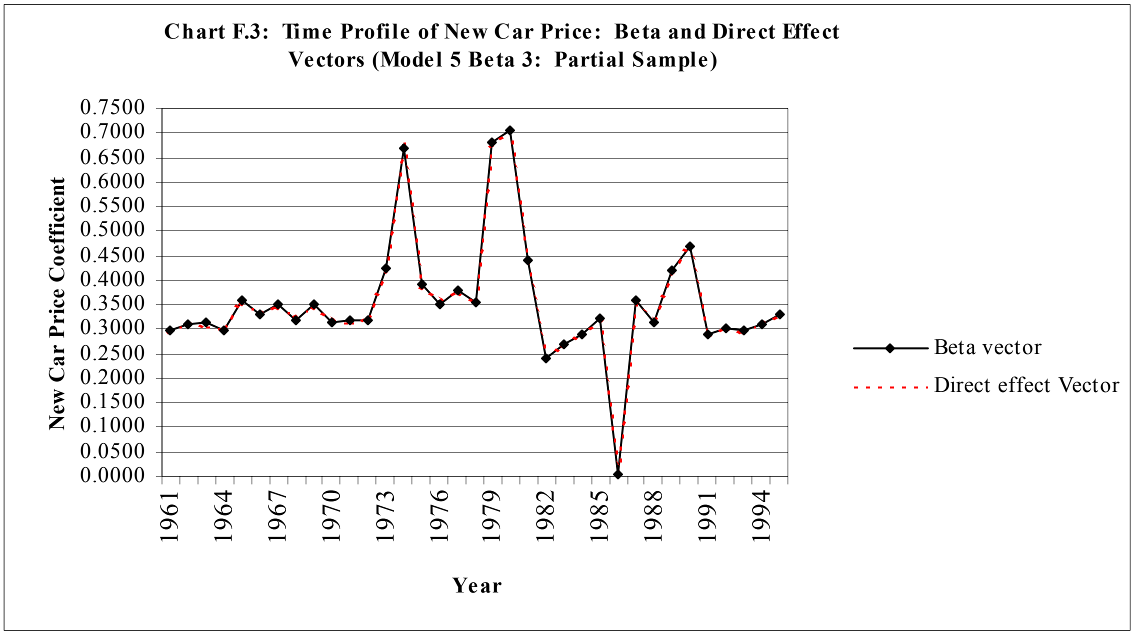
	Beta	Direct	Beta	Direct	Beta	Direct	Beta	Direct
Year	Own Price		Income		New Car		Used Car	
1961	-0.1871	-0.1865	1.1074	1.1098	0.2987	0.2931	-0.1506	-0.1552
1962	-0.1937	-0.1943	1.1082	1.1096	0.3081	0.3142	-0.1676	-0.1628
1963	-0.1920	-0.1914	1.1091	1.1097	0.3119	0.3063	-0.1553	-0.1599
1964	-0.1895	-0.1899	1.1070	1.1097	0.2980	0.3023	-0.1623	-0.1585
1965	-0.2093	-0.2088	1.1062	1.1093	0.3577	0.3532	-0.1727	-0.1767
1966	-0.2012	-0.2017	1.1075	1.1095	0.3290	0.3342	-0.1743	-0.1699
1967	-0.2061	-0.2056	1.1076	1.1094	0.3496	0.3446	-0.1694	-0.1736
1968	-0.1975	-0.1981	1.1106	1.1096	0.3178	0.3245	-0.1716	-0.1664
1969	-0.2067	-0.2063	1.1143	1.1094	0.3507	0.3465	-0.1704	-0.1743
1970	-0.1953	-0.1957	1.1152	1.1096	0.3133	0.3180	-0.1682	-0.1641
1971	-0.1952	-0.1948	1.1166	1.1096	0.3194	0.3156	-0.1596	-0.1633
1972	-0.1970	-0.1974	1.1165	1.1096	0.3185	0.3223	-0.1693	-0.1657
1973	-0.2342	-0.2338	1.1159	1.1088	0.4242	0.4206	-0.1974	-0.2009
1974	-0.3280	-0.3285	1.1162	1.1067	0.6702	0.6752	-0.2966	-0.2922
1975	-0.2220	-0.2216	1.1205	1.1090	0.3914	0.3876	-0.1854	-0.1891
1976	-0.2098	-0.2102	1.1224	1.1093	0.3522	0.3570	-0.1823	-0.1781
1977	-0.2177	-0.2172	1.1223	1.1091	0.3805	0.3759	-0.1807	-0.1849
1978	-0.2105	-0.2109	1.1208	1.1093	0.3550	0.3587	-0.1824	-0.1787
1979	-0.3286	-0.3282	1.1169	1.1067	0.6788	0.6744	-0.2879	-0.2919
1980	-0.3399	-0.3402	1.1148	1.1064	0.7034	0.7068	-0.3071	-0.3036
1981	-0.2406	-0.2402	1.1157	1.1086	0.4419	0.4378	-0.2032	-0.2071
1982	-0.1673	-0.1676	1.1161	1.1102	0.2390	0.2424	-0.1405	-0.1370
1983	-0.1768	-0.1766	1.1171	1.1100	0.2690	0.2664	-0.1426	-0.1456
1984	-0.1849	-0.1850	1.1147	1.1099	0.2878	0.2891	-0.1560	-0.1537
1985	-0.1959	-0.1956	1.1111	1.1096	0.3209	0.3177	-0.1607	-0.1640
1986	-0.0804	-0.0807	1.1142	1.1122	0.0049	0.0084	-0.0565	-0.0531
1987	-0.2099	-0.2096	1.1112	1.1093	0.3588	0.3552	-0.1740	-0.1775
1988	-0.1957	-0.1959	1.1097	1.1096	0.3158	0.3183	-0.1671	-0.1642
1989	-0.2326	-0.2324	1.1088	1.1088	0.4191	0.4166	-0.1966	-0.1995
1990	-0.2521	-0.2523	1.1090	1.1084	0.4674	0.4702	-0.2218	-0.2187
1991	-0.1841	-0.1838	1.1105	1.1099	0.2888	0.2859	-0.1495	-0.1526
1992	-0.1898	-0.1901	1.1104	1.1097	0.2999	0.3028	-0.1617	-0.1586
1993	-0.1875	-0.1873	1.1104	1.1098	0.2981	0.2952	-0.1529	-0.1559
1994	-0.1939	-0.1942	1.1103	1.1096	0.3110	0.3138	-0.1656	-0.1626
1995	-0.1989	-0.1987	1.1102	1.1095	0.3288	0.3260	-0.1640	-0.1670
Mean	-0.2100	-0.2100	1.1130	1.1093	0.3566	0.3565	-0.1778	-0.1779
Standard Error	0.0079	0.0079	0.0008	0.0002	0.0213	0.0212	0.0076	0.0076
Median	-0.1975	-0.1981	1.1112	1.1096	0.3209	0.3245	-0.1693	-0.1664
Standard Deviation	0.0467	0.0466	0.0045	0.0010	0.1257	0.1255	0.0450	0.0450
Sample Variance	0.0022	0.0022	0.0000	0.0000	0.0158	0.0158	0.0020	0.0020
Kurtosis	3.9003	3.9129	-0.7146	3.9129	3.8567	3.9129	3.9925	3.9129
Skewness	-0.9910	-1.0032	0.4535	-1.0032	0.9532	1.0032	-1.1266	-1.0032
Range	0.2596	0.2595	0.0162	0.0058	0.6985	0.6984	0.2505	0.2505
Minimum	-0.3399	-0.3402	1.1062	1.1064	0.0049	0.0084	-0.3071	-0.3036
Maximum	-0.0804	-0.0807	1.1224	1.1122	0.7034	0.7068	-0.0565	-0.0531
Count	35	35	35	35	35	35	35	35

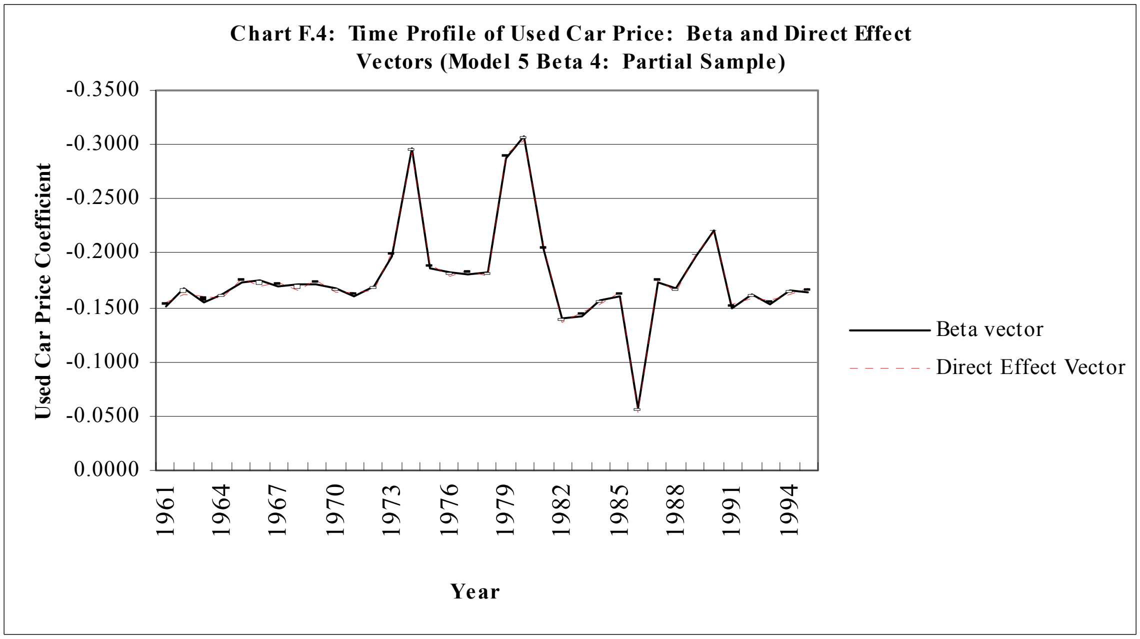
Appendix F continued:





Appendix F continued:





VITA

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