Global Optimization of the Nonconvex Containership Design Problem Using the Reformulation-Linearization Technique

by

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(ABSTRACT)

The containership design problem involves optimizing a nonconvex objective function over a design space that is restricted by a set of constraints defined in terms of nonconvex functions. An application of standard nonlinear optimization methods to such a problem can at best attain a local optimum that need not be a global optimum. This thesis investigates the application of alternative modeling, approximation, and global optimization techniques for developing a multidisciplinary approach to the containership design problem.

The problem involves five design variables, which prioritized according to their relative importance in the model are: design draft, depth at side, speed, overall length, and maximum beam. Five constraints are imposed on the design, viz., an equality constraint to enforce the balance between the weight and the displacement, a linear inequality constraint on the length to depth ratio that is implied by the lightship weight formulation for the design to be acceptable, an inequality constraint on the metacentric height to ensure that the design satisfies the Coast Guard wind heel criterion, an inequality on the freeboard to ensure the minimum required freeboard governed by the code of federal regulations for freeboard (46 CFR 42), and an inequality constraint on the rolling period to ensure that the design satisfies the minimum required rolling period criterion. The objective function employed is the required freight rate, expressed in dollars per metric ton per nautical mile in order to recover annualized construction and operational costs. The model also accommodates various practical issues in a manner suitable to improve its representability. For example, it takes into account the discrete container stowage issue. The carrying capacity (number of containers) is expressed as a continuous function of the principal dimensions by using a linear response surface fit that in turn makes the objective function continuous. The weight-displacement balance is maintained by including draft as a design variable and imposing an equality constraint on the weight and displacement rather than introducing an internal loop to calculate draft at each iteration. This speeds up the optimization process. Also, the weight is formulated independent of the draft to ensure independence of the weight and the displacement, which simplifies the optimization process. The time for loading and unloading containers at a given port is a function of the number of cranes available. The number of cranes is formulated as a

function of the length of the ship, and the resulting expression is made continuous through a linear response surface fit.

To solve this problem, we design two approaches based on employing a sequence of polynomial programming approximations, each within two alternative branch-and-bound frameworks. In the first approach, we construct a polynomial programming approximation to the containership design problem using the Response Surface Methodology (RSM) and solve this model to global optimality using the software package BARON (Branch-and-Reduce Optimization Navigator – see Sahinidis, 1996), although the Reformulation-Linearization Technique (RLT)-based procedure of Sherali and Tuncbilek (1992, 1997) offers a viable alternative (BARON itself incorporates some elements of the latter approach). The resulting solution is refined by the application of a local search method. This procedure is integrated into two alternative branch-and-bound frameworks. The motivation is that the solution of the nonconvex polynomial approximations is likely to yield solutions in the near vicinity of the true underlying global optimum, and hence, the application of a local search method initiated at such a solution has a greater prospect of detecting such a global optimum.

In the second approach, we utilize a continuous-space branch-and-bound procedure based on linear programming (LP) relaxations. These relaxations are generated through an approximation scheme that first utilizes RSM to derive polynomial approximations to the objective function and the constraints, and then applies the RLT to obtain an LP relaxation. The initial stage of this lower bounding step generates a tight, nonconvex polynomial programming relaxation for the problem, and the subsequent step constructs an LP relaxation to the resulting polynomial program via a suitable RLT procedure. The underlying motivation for these two steps is to generate a tight outer approximation of the convex envelope of the objective function over the convex hull of the feasible region. The solution obtained using the polynomial approximations is treated as a lower bound. A local search method is applied to this solution to compute an upper bound. This bounding step is then integrated into two alternative branch-and-bound frameworks. The node partitioning schemes are especially designed so that the gaps resulting from these two levels of approximations are induced to approach zero in the limit, thereby ensuring convergence to a (near) global optimum.

A comparison of the containership design obtained from the designed algorithmic approaches with that obtained from the application of the nonlinear optimization methods as in previous research, exhibits a significant improvement in the design parameters translating to a significant amount of annual cost savings. For a typical containership of the size pertaining to a test case addressed in this work, having a gross weight of 90,000 metric tons, an annual transportation capacity of 99,000 containers corresponding to an annual deadweight of 1,188,000 metric tons, and logging 119,000 nautical miles annually, the improvement in the prescribed design translates to an annual estimated savings of $ 1,862,784 (or approximately $ 1.86 million) and an estimated 27 % increase in the return on investment over the life of the ship.

The main contribution of this research is that it develops a detailed formulation and a more precise model of the containership design problem, along with suitable response surface and global optimization methodologies for prescribing an improved modeling and algorithmic approach for the highly nonconvex containership design problem.

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Contents

List of Symbols

Symbol	Description	Units
Aac	Annual average cost	Dollars
Abc	Annual building cost	Dollars
A_BT	Transverse area of bulbous bow at the position where the still-water surface intersects the stem	$Meter^2$
Accoc	Accommodation costs	Dollars
Afc	Annual fuel cost	Dollars
Aoc	Annual operating cost	Dollars
B	Beam, maximum	Meters
BHP	Break horsepower	
Cb	Block coefficient	
Cf	Coefficient to convert long tons to metric tons = 0.9843	
Chld	Cargo handling cost per year	Dollars
Chull	Cost of hull engineering material	Dollars/ metric ton
Cm	Midship section coefficient	
CN	Cubic number = $\frac{Loa \times B \times D}{100}$	$Meter^3$
Cof	Cost of outfit material	Dollars/ metric ton
Cp	Prismatic coefficient	
Cr	Capital recovery factor	
Csh	Cost of hull steel material	Dollars/ metric ton
Cwp	Waterplane coefficient	
C11	Blending coefficient applied to Basis ship11	
C14	Blending coefficient applied to Basis ship14	
C22	Blending coefficient applied to Basis ship22	
D	Depth, at side	Meters
DBH	Double bottom height	Meters
Disp	Displacement, volume	$Meter^3$
Disp1	Displacement, weight	Metric tons
Dst	Range	Nautical miles
EHP	Effective horsepower	Newton-Nautical mile
Fcost	Cost of bunker C fuel oil	Dollars/ Metric ton
Freeboard	Freeboard	Meters

Symbol	Description	Units
Freeboard_min	Minimum required freeboard	Meters
GM	Metacentric height	Meters
GM_min	Minimum metacentric height	Meters
H	The vertical distance from the center of the projected lateral area of the portion of the vessel and deck cargo above the waterline to the center of the underwater lateral area or to the one-half draft point.	Meters
H_B	Position of center of area of bulbous bow above base	Meters
HCH	Hatch coaming height	Meters
InsHM	Insurance, hull & Machinery	Dollars
InsPI	Insurance, Protection & Indemnity	Dollars
Ir	Interest rate in percentage	
KG	Vertical center of gravity, total	Meters
KGf	Vertical center of gravity, fuel	Meters
KGL	Vertical center of gravity, lightship	Meters
KGm	Vertical center of gravity, machinery	Meters
KGmisc	Vertical center of gravity, miscellaneous	Meters
KGs	Vertical center of gravity, hull steel	Meters
KGto	Vertical center of gravity, total outfit weight	Meters
LA	Projected lateral area of the portion of the vessel and deck cargo above the waterline.	Meter^2
LAH	The product of the projected lateral area of the portion of the vessel and deck cargo above the waterline and the vertical distance from the center of the above to the center of the underwater lateral area or to the one-half draft point.	Meter^3
Lbp	Length between perpendiculars. In this work, it is approximated as Loa/ 1.05	Meters
Lc	Labor cost per hour	Dollars/ hour
Lcb	Longitudinal center of buoyancy forward of 0.5 L expressed as percentage of length	
Lhe	Labor cost, hull engineering	Dollars
Lhs	Labor cost, steel hull	Dollars
Lm	Labor cost, machinery	Dollars
Lo	Labor cost, outfit	Dollars
Loa	Length, overall	Meters
Lwl	Length, waterline	Meters
MArm_TEUd	Moment-arm of TEUs on deck	Meters
MArm_TEUbd	Moment-arm of TEUs below deck	Meters

Symbol	Description	Units
Marmfactor_TEUbd	Moment arm factor for TEUs below deck	
Mathe	Material cost, hull engineering	Dollars
Matm	Material cost, machinery	Dollars
Mato	Material cost, outfit	Dollars
Mats	Material cost, steel hull	Dollars
Mhhe	Man-hours, hull engineering	
Mhm	Man-hours, machinery	
Mho	Man-hours, outfit	
Mhs	Man-hours, steel hull	
Miscc	Miscellaneous cost	Dollars
Moment_TEUbd	Moment of TEUs below deck	Newton-meter
Moment_TEUd	Moment of TEUs on deck	Newton-meter
Mrh	Cost of maintenance and repair, hull	Dollars
Mrm	Cost of maintenance and repair, machinery	Dollars
Ncrane_f	Number of cranes, continuous	
Ncrane_i	Number of cranes, integer	
Ncrew	Number of crew	
Nt	Number of round trips per year	
Ntb	Number of tiers below deck	
Ntd	Number of tiers on deck	
Ot	On-hire time per year	Days
Ovhc	Overhead costs	Dollars
Owc	Cost to owner	Dollars
Owe	Owner's expenses	Dollars
Port	Port expenses	Dollars
Pr	Yard's profit	Dollars
Pwt	Port waiting time	Days
RFR	Required freight rate	Dollars/ metric ton/ nautical mile
ROI	Return on investment expressed in percentage	
Rt	Resistance	Newtons
Rtt	Round trip time	Days
SFC	Specific fuel consumption	Gms/ SHP/ hr
SHP	Maximum continuous shaft horsepower	
Sl	Life of the ship	Years
Ss	Cost of stores and supplies	Dollars
St	Time at sea per round trip	Days
T	Draft, design	Meters
TEU	Twenty-foot equivalent units	
TEU_f	Total number of TEUs, continuous	
TEU_i	Total number of TEUs, integer	

Symbol	Description	Units
TEUbd_f	Number of TEUs below deck, continuous	
TEUbd_i	Number of TEUs below deck, integer	
TEUd_f	Number of TEUs on deck, continuous	
TEUd_i	Number of TEUs on deck, integer	
Troll	Rolling period	Seconds
Troll_min	Minimum rolling period	Seconds
TSLU	TEUs loaded/ unloaded per day per crane	
Vk	Speed	Knots
Wage	Wages	Dollars
Wcargo	Weight, cargo	Metric tons
Wcp	Weight, crew and provisions	Metric tons
Wf	Weight, fuel	Metric tons
Wfw	Weight, fresh water	Metric tons
Whc	Weight, miscellaneous when machinery is idle	Metric tons
Whe	Weight, hull engineering	Metric tons
WL	Weight, lightship	Metric tons
Wlo	Weight, luboil for diesel	Metric tons
Wm	Weight, machinery	Metric tons
Wmisc	Weight, total miscellaneous	Metric tons
Wo	Weight, outfit	Metric tons
Wpc	Weight per TEU	Metric tons
Ws	Weight, hull steel	Metric tons
Wto	Weight, total outfit	Metric tons
Wt_TEUbd	Weight, TEUs below deck	Metric tons
Wt_TEUd	Weight, TEUs on deck	Metric tons
Ybc	Yard's building price	Dollars
Ytc	Yard's total cost	Dollars

User- Defined Parameters

The following parameters pertaining to the shipyard and the port characteristics are user-defined as constants and are fixed during the optimization process.

Symbol	Description	Value defined in this work
Chargerate	Rate charged by the ship owner to ship cargo in dollars/ ton/ nautical mile	0.0064
Chull	Cost of hull engineering material in dollars/ ton	3500
Cof	Cost of outfit material in dollars / ton	1500
Csh	Cost of hull steel material in dollars / ton	400
DBH	Double bottom height in meters	1.83

Symbol	Description	Value defined in this work
Dst	Range in nautical miles	7000
Fcost	Cost of Bunker C fuel oil in dollars / ton	80
HCH	Hatch coaming height in meters	1.83
Ir	Interest rate in percentage/ 100	0.08
Lc	Labor cost in dollars / hour. For simplicity we assume the same labor cost for hull steel, hull engineering, outfit and machinery.	20
Ot	On-hire time per year in days	350
Pwt	Port waiting time in days	2
SFC	Specific fuel consumption in gms/ shp/ hour	120
Sl	Ship life in years	20
TSLU	TEUs loaded/ Unloaded per day per crane	1440
Wpc	Weight per TEU in metric tons	12

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Chapter 1

Introduction

1.1 Ship Design: The Problem Domain

Ship design has traditionally been an iterative process in which various aspects of the design pertaining to power, strength, stability, weight, and space balance have been performed in sequence to arrive at a variety of feasible designs. This iterative process of working from mission requirements to a detailed design can be conceived as moving along a Design Spiral [39], as illustrated in Figure 1.1. In this process, the steps progress from basic design that encompasses the conceptual and preliminary designs, to contract design, and finally to the detailed design phase.

A brief description of the various phases of this design process is as follows:

(a) Concept Design: Concept design translates the mission requirements into naval architectural and engineering characteristics. It involves technical feasibility studies to determine fundamental elements of the proposed ship such as length, beam, depth, draft, fullness, power, and other characteristics, all of which meet the required speed, range, cargo cubic, and deadweight. It includes preliminary lightship weight estimates that are usually derived from curves, formulas, or experience. Alternative designs are analyzed in parametric studies during this phase to determine the most economical design solution.

(b) Preliminary Design: Preliminary design refines the major ship characteristics affecting cost and performance. Factors such as length, beam, horsepower, and deadweight are not expected to change after this phase. Its completion provides a precise definition of a vessel that will meet the mission requirements.

(c) Contract Design: Contract design develops plans and specifications suitable for shipyard bidding and contract award. It involves one or more loops around the design spiral thereby refining the preliminary design. Important features of contract design include weight and center of gravity estimates that take into account the weight and location of each major item of the ship. This fixes the overall volumes and areas of cargo, machinery, stores, fuel oil, fresh water, living and utility spaces and their interrelationship to other features such as cargo handling equipment, and machinery components.

(d) Detail Design: Detail design is carried out by the shipyard to further develop the contract plans as required to prepare shop drawings used for an actual construction of the vessel. This phase provides installation and construction

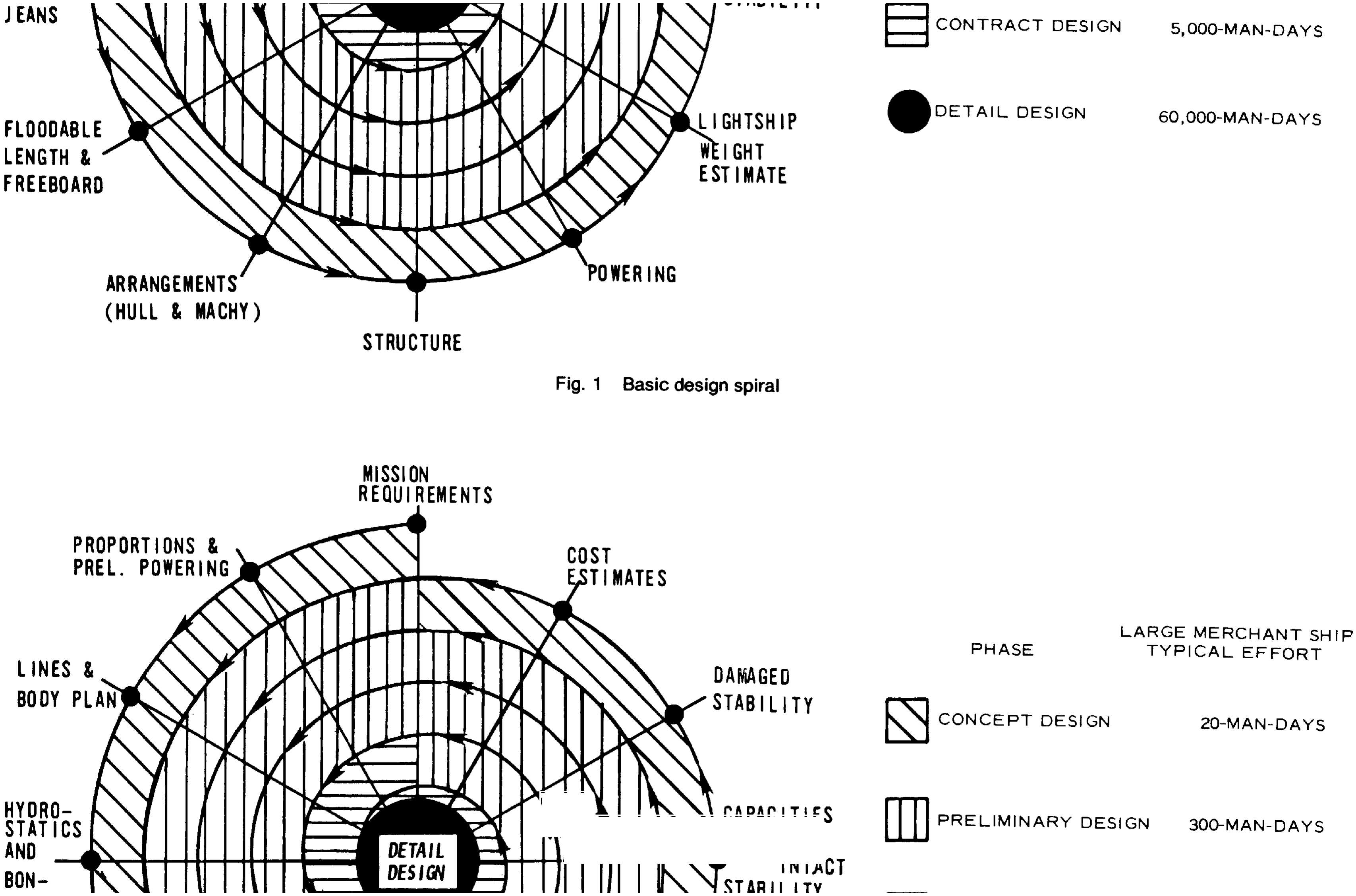


Figure 1.1 Ship Design Spiral [39].

instructions to ship fitters, welders, outfitters, metal workers, machinery vendors, pipe fitters, etc.

Although the design spiral approach has been refined considerably with the advent of various computer programs, due to its fixed nature, it does not provide the designer with an overview of the design process, and is hence not efficient in allowing the designer to compare alternative designs and to identify an optimum design.

This need to make efficient utilization of resources to meet functional requirements has motivated several research efforts to develop an optimization approach to design a ship that would enable the designer to perform an effective synthesis of all design requirements and would provide a useful mechanism for exploring competitive designs.

In the context of an optimization approach for containership design, several problem formulations have been explored and various methodologies pertaining to nonlinear programming problems have been implemented. A summary of these research efforts, and a discussion on the various nonlinear programming methodologies that have been used for ship design is presented in Chapter 2.

1.2 Motivation

The multidisciplinary containership design problem defined in Ganesan and Hughes (1999) as part of the overall ship design tool developed by Neu et al. (2000), involves optimizing a nonconvex objective function over a design space that is restricted by a set of constraints defined in terms of nonconvex functions. An application of standard nonlinear optimization methods to such a problem ensures the attainment of a local optimum that need not be a global optimum. This thesis investigates the application of alternative modeling, approximation, and global optimization techniques for developing a multidisciplinary approach to this containership design problem. A detailed description of the problem formulation adopted from Ganesan and Hughes (1999) is given in Section 1.3. A statement of this optimization problem is provided in Section 1.4.

1.3 Overall Design Process

Neu et al. (2000) develop a prototype tool for multidisciplinary design optimization of ships. Using a containership design as a test case, various aspects of the problem formulation are constructed as modules and linked together as per the flow-chart shown in Figure 1.2. The modules relate to the geometry, hydrostatics, resistance, propulsion, lightship weight, cargo, total weight and economics of the problem. Constraints are imposed on the metacentric height required by the Coast Guard wind heel criterion, the minimum required freeboard, and the rolling period. The modules are linked together with a nonlinear optimization tool (*Design Optimization Tools* provided by Vanderplaats Research and Development, Inc.), allowing the designer to identify different objective functions, limits on the design variables, and constraints on the design, and thereby providing a framework to compare alternative designs in order to identify an optimum or best design. As part of the above effort, Ganesan and Hughes (1999) define and investigate a multidisciplinary design optimization (MDO) model for containership design. This work provides details regarding the construction of the lightship weight, cargo, total weight and the economics modules. It includes a discussion on results of the optimization process, the effect of some key parameters on the design, and a study on the effect of three specific hull forms on the design. Results employing the three different techniques in *DOT* (as mentioned in Section 2.2) are derived and compared. It is shown that once a good solution is identified in the design space, the sequential linear programming algorithm is the most consistent method for converging to a local optimum. Furthermore, the objective function is found to be relatively flat in the vicinity of the optimum. This indicates that while applying secondary (perhaps subjective) criteria, the designer is not confined to a severely restricted design space and has some freedom in designing the optimum ship.

The following paragraphs provide an overview of the design process. A more detailed development of the model is presented in Chapter 3.

We examine the problem from the ship owner's perspective and assume a continuous demand in the market for the cargo being transported, and could attempt to minimize the required freight rate or maximize the return on investment. In this work we minimize the required freight rate. The required freight rate, expressed in dollars per ton per mile, is simply the amount the owner must charge the customer in order to break-even. A more precise definition from [29] is: "The required freight for a given rate of utilization produces net profits which exactly cover the operating costs inclusive of calculated interest on the invested capital".

The hull geometry manipulation is performed as a weighted average of two or three user-defined basis hulls. Accordingly, there are only one or two design variables controlling the hull shape, because the remaining design variable can be expressed in terms of the other variable(s) based on the fact that they must sum to one.

Resistance is estimated using the Holtrop-Mennen regression method [16]. For power prediction, the propulsion module simply assumes an overall propulsive efficiency of 65 percent. The fuel rate is fixed at 120 gms/shp/hr, which is a reasonable figure for low-speed diesels [40].

The discrete container stowage issue has also been addressed in this work. The cargo module calculates the number of containers that can be stowed in the holds by employing a stowage factor on the total available hold volume. The cargo weight is now simply a product of the number of containers and the weight per container, where the latter is user-defined as twelve tons per TEU (Twenty-foot Equivalent Units), which is the standard weight of a TEU.

Furthermore, the lightship and miscellaneous weights are based on a series of regression formulas. These, along with the fuel weight and the centers of gravity of the weight components are brought together in the total weight and center of gravity module.

The economic module computes annual figures for the building, operating, and fuel costs. It also calculates the port and cruising times and the number of round trips made by the ship annually, which are needed to determine the cargo handling, port, and fuel costs.

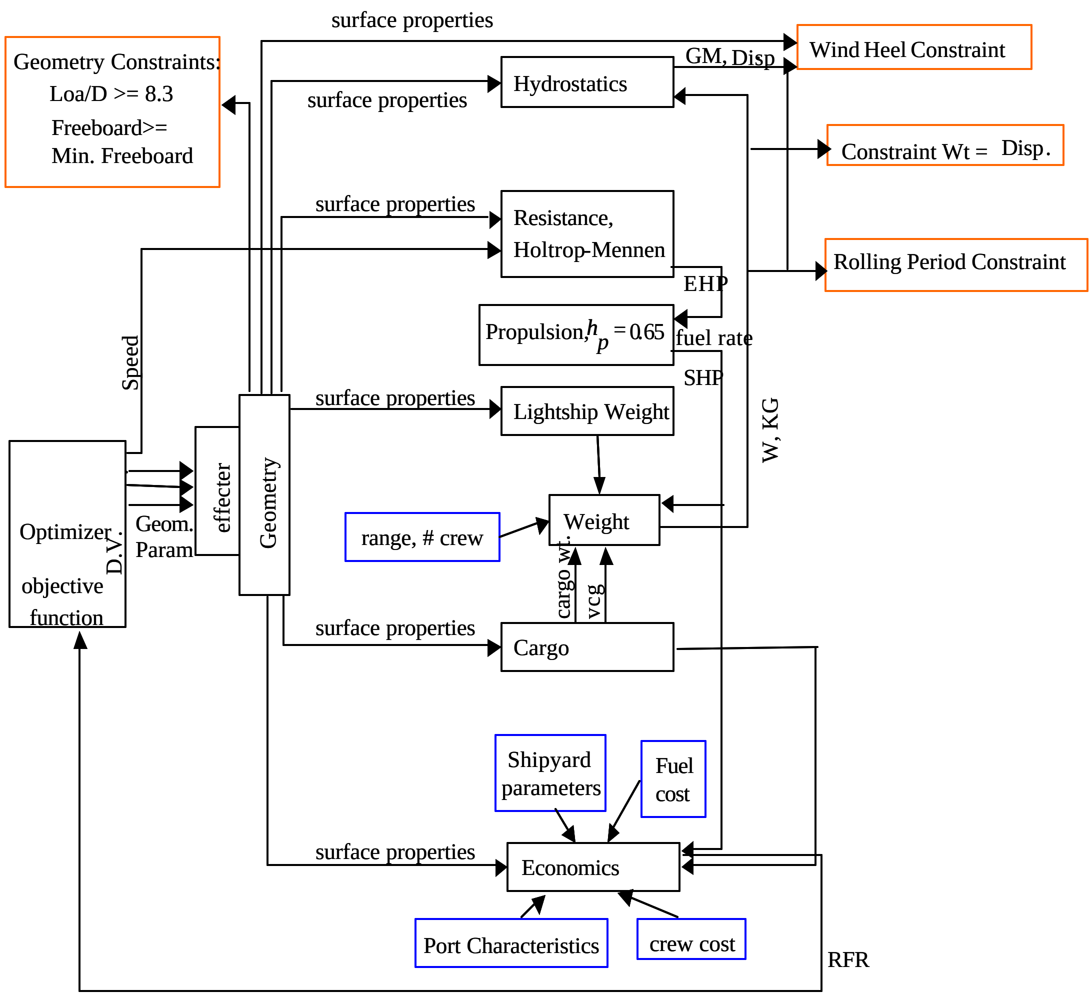


Figure 1.2 Flowchart for Multidisciplinary Design Optimization.

The weight-displacement balance is maintained through an equality constraint in the optimization model. This has the advantage of speeding the design process over the approach of enforcing this balance through an internal loop used to calculate the draft at each iteration.

1.4 Concept of the Model

The design variables for the problem are as follows:

	Design Variable	Conventional Notation	Mathematical Symbol
1	Length, overall in meters	Loa	x_1
2	Beam, maximum in meters	B	x_2
3	Depth, at side in meters	D	x_3
4	Draft, design in meters	T	x_4
5	Speed in knots	Vk	x_5
6	Blending coefficient	C	*

where C is the design variable controlling the shape of the hull form. As mentioned above, the hull geometry manipulation is carried out as a weighted average of some user defined basis hull forms. The design variables controlling the geometry are the weighting factors or the blending coefficients of the basis hull forms, the length, the beam, and the depth. When using two basis hulls [20], the vector of Fastship net points, H, defining the hull is given by

$$H = c_1 H_1 + c_2 H_2$$

such that

$$c_1 + c_2 = 1,$$

where c_1 and c_2 are the nonnegative weighting factors of the basis hull forms, and where H_1 and H_2 are the net point vectors defining the basis hulls.

When using two basis hull forms, we have only one design variable (denoted as C in this work) for the weighting factors from the fact that they should sum to one. Once we have the resulting hull form, its hydrostatic properties are calculated by integration.

In the mathematical statement of the problem, the hydrostatic module has not been included. Consequently, various hydrostatic parameters are assigned values as functions of the design variables. This is for simplicity only. These hydrostatic parameters are as follows:

Symbol	Description
A_BT	Transverse area of bulbous bow at the position where the still-water surface intersects the stem (Meter^2)
AT	Immersed part of the transverse area of the transom at zero speed (Meters)

Symbol	Description
H_B	Position of center of area of bulbous bow above base (Meters)
Cb	Block coefficient
Cm	Midship section coefficient
Cstern	Coefficient that accounts for the specific shape of the afterbody
Cp	Prismatic coefficient
Cwp	Waterplane coefficient
KM	Vertical distance from the keel to the transverse metacenter (Meters)
S	Projected wetted surface area (Meter^2)

As mentioned earlier, the objective can be to minimize the required freight rate or to maximize the return on investment. In this work we minimize the required freight rate.
These objective functions are developed in greater detail in Section 3.5.

The constraints on the design are as follows (they are developed in detail in Section 3.6).

1. An equality constraint to enforce the balance between weight and displacement.
2. An inequality constraint on the Length to Depth ratio that is required for the design to be acceptable.
3. An inequality constraint on the metacentric height to ensure that it satisfies the Coast Guard wind heel criterion.
4. An inequality constraint on the freeboard to ensure the minimum required freeboard.
5. An inequality constraint on the rolling period to ensure that it satisfies the minimum required rolling period criterion.

In the mathematical statement of the problem, there are parameters that are defined as functions of the design variables based on the range of numerical values these design variables take. These parameters are:

Symbol	Description	Function of:
C4	An intermediate coefficient occurring in resistance	$C4(x_1, x_4)$
C6	An intermediate coefficient occurring in resistance	$C6(x_1, x_5)$
C7	An intermediate coefficient occurring in resistance	$C7(x_1, x_2)$
ha	Half angle of entrance, the angle of the waterline at the bow in degrees with reference to the centerplane, neglecting the local shape at the stem; estimated by Holthrop	$ha(x_1, x_5)$
Ss	Annual cost of stores and supplies (Dollars)	$Ss(x_1, x_2, x_3, x_4, x_5)$

In addition to the objective function and the constraints, the mathematical statement of the problem also includes certain important parameters in the design. These parameters are as follows:

Symbol	Description
Aac	Annual average cost (Dollars)
Abc	Annual building cost (Dollars)
Afc	Annual fuel cost (Dollars)
Aoc	Annual operating cost (Dollars)
KG	Vertical center of gravity, total (Meters)
KGf	Vertical center of gravity, fuel (Meters)
KGL	Vertical center of gravity, lightship (Meters)
KGm	Vertical center of gravity, machinery (Meters)
KGmisc	Vertical center of gravity, miscellaneous (Meters)
KGs	Vertical center of gravity, hull steel (Meters)
ROI	Return on investment expressed in percentage
Rt	Resistance (Newtons)
SHP	Maximum continuous shaft horsepower
Wcargo	Weight, cargo (Metric tons)
Wf	Weight, fuel (Metric tons)
WL	Weight, lightship (Metric tons)
Wmisc	Weight, total miscellaneous (Metric tons)
WT	Weight, total (Metric tons)

1.5 Contributions of this Research

This research contributes toward the containership design problem, addressing the derivation of detailed functional forms of important design issues and the development of effective global optimization methodologies to solve this problem, based on the Reformulation-Linearization Technique (RLT). Together, these two aspects are intended to provide a more precise model and algorithmic approach for the nonconvex containership design problem.

The design issues for which we develop detailed functional forms as part of our contribution to the existing literature are summarized in the following paragraphs. This discussion is followed by an outline of the methodology developed and to be tested against solutions obtained by an application of standard nonlinear optimization methods.

In all previous attempts to formulate the containership design problem using a multidisciplinary perspective, although the principal dimensions of the ship are treated as design variables, the carrying capacity (number of containers) is not modeled as a function of these design variables. Doing so is essential because, during the optimization process, the principal dimensions of the ship change, and hence, so does its carrying capacity. The model employed here includes a treatment of this issue as follows:

- The number of containers below deck is expressed as a function of the length, the beam and the depth of the ship. This has been done by discretizing the space available for container stowage in the lengthwise, the beamwise and the depthwise directions and employing a "stowage factor" to calculate the total volume available for container stowage. The stowage factor accounts for the geometry of the hull form and the space occupied by the container cell guides. The number of containers below deck also depends on the block coefficient when calculated by employing the stowage factor on the total available hull volume. This dependence has been adequately represented by expressing the stowage factor explicitly as a function of the block coefficient.

- The number of containers above deck is expressed as a function of the length and the beam of the ship. This is done by discretizing the space available for container stowage in the lengthwise, and the beamwise directions and accounting for the geometry of the surface area by a "stowage factor" for number of containers above deck. The product of this quantity and the number of tiers above deck gives the total number of containers that can be stowed above deck. Also, data from Panamax and post Panamax ships indicate that the number of tiers above deck is not fixed. Moreover, it need not be an integer since it is averaged over the total available area on deck. This is accounted for by expressing the number of tiers as a function of the beam based on interpolation.

The formulations of the lightship weight, and the fuel and miscellaneous weights do not involve draft. This is done with an intention to ensure that the total weight has no dependence on the draft and hence on the displacement. This independence of the weight and the displacement somewhat simplifies the optimization model in which the weight-displacement balance is maintained by imposing a suitable equality constraint.

The time for loading and unloading containers at a given port is a function of the number of cranes available. The number of cranes is formulated as a function of the length of the ship, and the resulting expression is made continuous through a linear response surface fit.

The lightship weight and the number of containers above and below deck are validated against data pertaining to ships that reasonably cover the range of interest to the designer.

To solve this problem, we design two approaches based on employing a sequence of polynomial programming approximations, each within two alternative branch-and-bound frameworks. A description of these two methods to be developed, and tested is as follows.

In the first approach, we construct a polynomial programming approximation to the containership design problem using the Response Surface Methodology (RSM) and solve this model to global optimality using the software package BARON (Branch-and-Reduce Optimization Navigator – see Sahinidis, 1996), although the Reformulation-Linearization Technique (RLT)-based procedure of Sherali and Tuncbilek (1992, 1997) offers a viable alternative (BARON itself incorporates some elements of the latter approach). The resulting solution is refined by the application of a local search method. This procedure is integrated into two alternative branch-and-bound frameworks. The motivation is that the solution of the nonconvex polynomial approximations is likely to yield solutions in the near vicinity of the true underlying global optimum, and hence, the application of a local search method initiated at such a solution has a greater prospect of detecting such a global optimum.

In the second approach, we utilize a continuous-space branch-and-bound procedure based on linear programming (LP) relaxations. These relaxations are generated through an approximation scheme that first utilizes RSM to derive polynomial approximations to the objective function and the constraints, and then applies the RLT to obtain an LP relaxation. The initial stage of this lower bounding step generates a tight, nonconvex polynomial programming relaxation for the problem, and the subsequent step constructs an LP relaxation to the resulting polynomial program via a suitable RLT procedure. The underlying motivation for these two steps is to generate a tight outer approximation of the convex envelope of the objective function over the convex hull of the feasible region. The solution obtained using the polynomial approximations is treated as a lower bound. A local search method is applied to this solution to compute an upper bound. This bounding step is then integrated into two alternative branch-and-bound frameworks. The node partitioning schemes are especially designed so that the gaps resulting from these two levels of approximations are induced to approach zero in the limit, thereby ensuring convergence to a (near) global optimum.

1.6 Organization of this Thesis

The remainder of this thesis is organized as follows:

In Chapter 2, we provide a literature review on research spanning from 1967 pertaining to containership design. This review includes details of the various formulations adopted and the different methodologies employed to solve the problem. We then present a section discussing the nonlinear optimization methodologies available in *Design Optimization Tools (DOT)* developed by Vanderplaats Research and Development, Inc. This tool was employed by Neu et al. (2000) in their work on containership design. In Section 2.3, we present an overview of global optimization algorithms in contrast with standard nonlinear optimization tools. We next discuss the Reformulation-Linearization Technique (RLT), an effective methodology to solve various classes of problems to global optimality.

In Chapter 3, we develop a detailed model formulation of the problem. Section 3.1 provides the reader with an overview of some terminology and definitions pertaining to naval architecture that are used in the formulation of the model. Section 3.2 provides details of the formulation of the lightship weight, the fuel and miscellaneous weights, and their centers of gravity. It also provides a validation of the model formulated. Section 3.3 presents a detailed description of the formulation of the number of containers above and below deck as functions of the principal dimensions and the block coefficient of the ship, and as functions of the length and the beam of the ship respectively. This section also details the calculation of the center of gravity of the total number of containers, followed by a validation of the formulation of the number of containers. Section 3.4 describes the formulation of annual figures for the building cost, the operating cost, and the fuel cost. Section 3.5 explains the measures of merit used to evaluate the design, followed by Section 3.6 that deals with the formulation of the constraints imposed on the design.

In Chapter 4, we outline the two methodologies that we develop to solve the problem. Section 4.1 provides an overview of the Design of Experiments (DOE). Section 4.3 provides a brief introduction to the Response Surface Methodology (RSM), and describes the mechanism and tools employed for constructing the fifth order polynomial approximations for the objective and the constraint functions. In Section 4.3, we provide a detailed explanation of the development of the Polynomial Programming Approximations (PPA) algorithms.

In Chapter 5, we present computational results pertaining to applying the proposed pseudo-global optimization approaches to solve the containership design problem. Section 5.1 presents results for Algorithm PPA1. Section 5.2 presents results for Algorithm PPA2. Thereafter, Section 5.3 presents results for the nonlinear programming approaches, followed by a comparison of results obtained from the pseudo-global optimization approaches and the nonlinear optimization methods as given in Section 5.4. Breakdowns of the weights and the various costs pertaining to the optimal design are also provided in Section 5.4.

In Chapter 6, we provide an overview of warship design as a possible extension of the modeling and algorithmic approach developed in this work. This chapter begins by providing in Section 6.1, a brief introduction to warship design. Thereafter, Section 6.2 presents a somewhat more detailed explanation of the design considerations introduced in Section 6.1. In Section 6.3, we review the existing literature on research efforts pertaining to warship design. Finally, in Section 6.4, we briefly discuss certain modeling and algorithmic considerations specifically pertaining to warship design.

In Chapter 7, we draw some conclusions from this work, and recommend some issues to be considered for future research.

Chapter 2

Review of Literature and Methodologies Used for Ship Design Optimization

2.1 Literature Review

Ship design, in general, is a multifaceted activity requiring the efficient utilization of resources to meet some functional requirements. A slender hull form is preferred for minimizing resistance, whereas maximizing cargo volume results in a fuller hull form. An optimization approach to such a design problem is essential because it allows us to perform an effective synthesis of all design requirements and provides a useful mechanism for exploring competitive designs.

Chryssosstomidis (1967)

Chryssosstomidis (1967) proposes an optimization approach to containership design in which the carrying capacity (number of containers) and the speed are fixed during the design process. The measure of merit used for the system is the least cost (annual average cost). A random search optimization technique is employed, that computes values of the objective function for thousands of sample designs satisfying a given set of owner's requirements, each corresponding to a set of values of the independent variables as determined by an exponential random search transformation, until a near least-cost design is achieved.

Erichsen (1971)

Erichsen (1971) presents a mathematical model for containership design optimization that considers operating costs incurred at container port terminals and for land transportation, in addition to the annual average cost. The objective function used is the net present value index. Here again, the carrying capacity (number of containers) and the speed are fixed during the design process. Results obtained by using a direct search technique and by employing geometric programming are compared. When using the direct search technique, it is found that when the initial step width is small and the cargo inflow is limited, the search seems to end in the best possible design. However, no integer restriction is imposed on the number of ships and cranes. It is seen that in most cases the search ends when the step width becomes less than a third of the initial values. The objective function is found to be flat in the neighborhood of the optimum point and further reductions of the stepwidths do not improve the value of the objective function by more than 0.2 percent. It is found that geometric programming is applicable to the ship design optimization problem and leads to competitive designs.

Keane et al. (1990)

Keane et al. (1990) describe an integrated computational approach to ship concept design using optimization techniques. Their method incorporates accepted naval architectural tools, a sophisticated database handler and several optimization procedures. It employs modular construction and permits the designer to modify or include a variety of analytical methods and data to suit different problems. It also allows the designer to specify different objective functions and constraints on the design. The approach is applied to the preliminary design of a frigate hull and several optimization procedures are outlined and their merits, failures, suitability, etc., are discussed in context to ship concept design. The example is studied using dual-parameter optimization (beam to draft ratio, and the length to displaced volume ratio) and five-parameter optimization (the foregoing two, and the flare, the prismatic coefficient, and the longitudinal center of buoyancy). The objective is to minimize resistance, which is calculated by the Holtrop-Mennen regression method. Constraints are imposed on the stability, the maximum value of the length to depth ratio, and on the minimum enclosed volume to guarantee payload. Wide limits are also imposed on the length at waterline, the beam at waterline, the draft, the depth, and the midship section coefficient. In both the two-dimensional and the five-dimensional cases, most optimization procedures converged to a local optimum. The most rapid optimization was achieved in the five-dimensional case by the Hooke and Jeeves direct search algorithm with a one pass external penalty function.

Sen (1991)

Sen (1991) advocates the Multiple Criteria Decision Making (MCDM) approach as a more effective approach to marine design than the traditional iterative approach where various aspects of the design are treated in a set sequence to arrive at a range of feasible designs. The three basic approaches to the solution of MCDM problems: weighting methods, prioritizing methods and efficient solution methods are combined into a generalized goal programming approach using normalized objective functions for numerical stability. A brief description of these approaches is as follows:

a. Weighting Methods: The weighting methods attempt to weight all problem objectives or attributes with weights to express them in terms of a single measure. However, since objectives or attributes are often incommensurate, the problem becomes one of deciding credible weights, although appropriate scaling can assist the process.

b. Prioritizing Methods: These methods attempt to avoid the problem stated above by ranking the objectives or the attributes according to their perceived importance. This prioritization of objectives and attributes is intended to simplify the decision process.

c. Efficient Solution Methods: These methods attempt to obtain the total set of all 'efficient' or 'Pareto optimal' solutions. Such efficient solutions are also referred to as non-dominated solutions because within this solution set, an improvement with respect to any single criterion is only possible at the expense of worsening at least one other criterion. This method has the drawback that generating all efficient solutions is usually a prohibitive, if not impossible, task.

The preliminary design of a two-deck Ro-Ro cargo ship is presented as a test case. The problem is formulated using eleven design variables, viz., length, beam, draft, depth, block coefficient, water-plane area coefficient, prismatic coefficient, speed, propeller diameter, propeller pitch and weight of ballast water. The goal constraints imposed on the design are the required ship capacity, the minimum required freight rate, the balance between displacement and weight, and the weight of ballast water. The problem is solved using a generalized goal programming approach employing a software system that includes the sequential linear programming approach with first-order and second-order terms in the Taylor series expansion for goals and constraints, a sequential quadratic programming formulation, and Powell's direct search method for smaller problems. Results that are indicative of the generalized goal programming approach to be a more generalized form of the conventional optimization approach rather than a replacement are presented. Ray and Sha (1994) argue that, in this model, the identification of the weights associated with different goals is difficult since the goals are interrelated. Also, solving the problem using the generalized goal programming approach cannot ensure the attainment of a global optimum.

Ray and Sha (1994)

Ray and Sha (1994) also investigate a multicriteria optimization approach to containership design, treating the number of containers and the speed as fixed parameters in the design process. The objective function used is a weighted average of the building cost, power, and steel weight of the ship. Both approaches to Multiple Criteria Decision Making (MCDM), viz., Multi-Objective Decision Making (MODM) and Multi-Attribute Decision Making (MADM) environments have been explored. A brief description of these approaches and the methodologies that could be applied to solve problems using these approaches are as follows:

Multiobjective Decision Making (MODM)

MODM problems are those in which the development of all alternatives is implied to be part of the solution process. Thus design synthesis is a typical MODM problem. MADM problems, on the other hand, deal with the selection of an alternative from among the different design alternatives available. In actual practice this classification is well suited to the two facets of problem solving. MODM methods are used in the design process, whereas MADM techniques are used in the evaluation and selection phases. Methodologies that could be applied to solve a MODM problem are:

A. Explicit Methods for Optimization

A number of optimization methodologies for different classes of problems have been developed to date. Apart from linear optimization methods, the nonlinear optimization methods used can be broadly classified into three major categories:

1. Method of Successive Linear Approximation,
2. Random Search with Shrinkage, and
3. Direct Search Technique.

Method of Successive Linear Approximation

This nonlinear optimization method starts with a feasible point. The functions are expanded by Taylor's series about the initial point and approximated to only the linear terms by dropping the nonlinear terms. The constraints and objective functions are also linearized in a similar manner and the problem is solved as a linear problem over a trust region. This trust region is appropriately managed while solving a sequence of such LP approximations until termination is attained.

Random Search with Shrinkage

This method consists of a random search for the minimum with iterative shrinkage. Random variables are generated from the following relation:

$$x_i = l_i + r_i \times (u_i - l_i) \tag{2.1}$$

where

x_i = *i*th variable

u_i = upper limit of variable x_i

l_i = lower limit of variable x_i

r_i = random variable between 0 and 1.

Any generated point that violates an equality constraint is discarded before evaluating the other functions. The iterative process stops when the range of each variable is reduced within some predefined limits.

Direct Search Technique

The direct search technique starts with a given point as an initial guess and proceeds to generate a sequence of points $P_1, P_2, P_3, ..., P_n$, that converge to a point P^* at which the function is minimized. The steps taken can be characterized as

$$\begin{aligned} \text{Step} &= P_{i+1} - P_i \\ &= l_i \times r_i \end{aligned} \tag{2.2}$$

where

l_i = step size

r_i = step direction.

The Hooke and Jeeves method and the Fletcher and Powell method are two procedures that fall in this category. Various other methods of this kind basically differ from each other in the manner of choosing the step size and the step direction.

The drawbacks of these nonlinear optimization methods are as follows:

1. The method of successive linear approximations has inexpensive iterations, but the actual behavior of nonlinear relations is lost due to the linearizations and the method can be slow for highly nonlinear problems.
2. The random search with shrinkage method is a time-consuming process where the actual optimum may quite often be missed during the shrinkage process. However, this method can be applied for multimodal functions.

The direct search techniques base their calculation on local search and global moves. The Hooke and Jeeves method, though an unconstrained method, has been applied by Keane et al. with the help of penalty functions. This is a popular method in ship design, but has the following drawbacks:

1. The step size for each variable is required as an input, or is set to a common step size for all the variables. However, as the range of the variables is quite varied, choosing step sizes beforehand or having a common stepsize will adversely affect the optimization process.
2. Most of the direct search techniques are designed for unimodal functions and hence the optimum found by such techniques for multimodal functions should be checked with different initial guesses to establish whether the optimum solution obtained is a global one.

B. The Goal Programming Approach

Goal Programming is an approach to solve problems involving multicriteria decisions. In goal programming it is required to specify aspiration levels of the objectives, the key aim being to minimize the "deviational variables" that equate the difference between the aspired value of the goal and the achieved value.

The drawbacks of the goal programming approach are:

1. The choice of the positive or negative deviational variables for any goal that is to be minimized is to be decided by the user. For example, if there is a rigid requirement that underachievement of the first goal is not allowed, then the positive deviational variable is to be minimized.

2. The deviational variables are in the same units as the objective function. Hence, multiplying them with different cardinal weights and summing them to form the objective function is not desirable as the deviational variables are incommensurate.

The nature of the ship design problem requires that the optimization tool used in an MODM approach should preferably have the following characteristics:

1. The technique should be capable of handling multiobjective, multivariable, constrained nonlinear problems.
2. The method should decide the step size and the direction implicitly.
3. The weights associated with the different objectives should be based on MADM principles.
4. The optimization model should be capable of incorporating qualitative comparisons as required by the MADM principles.

The modified Marquart Levenberg algorithm has been used as a tool in the MODM environment. This tool is based on a direct search technique and is capable of minimizing the sum of several nonlinear objective functions in multidimensional space, and at the same time, provides a scope to incorporate the requirements stated earlier arising out of the MADM environment. The stepsizes and directions are computed based on an implicitly generated Jacobian matrix.

Multiattribute Decision Making (MADM)

Multiattribute decision-making deals with the methodology of selection from among different design alternatives. The design alternatives have some specified desired level of achievement of an attribute/ objective (which may not necessarily be quantifiable) associated with them. Four methods are available for assessing the cardinal weights in the MADM environment:

a. The Weighted Least-Square Method,
b. The Eigenvector Method,
c. The Entropy Method, and
d. The Linmap Method.

Among these four methods, the entropy and linmap methods both require the decision matrix to be a part of the input. However, at the design stage, the requirement is to use the weights to find the best alternative, and not choose the best one from an enumeration of a set of alternatives. Therefore, these methods cannot be used in conjunction with the MODM environment that seeks to determine appropriate weights to associate with the objective functions.

The weighted least-square and eigenvector methods are based on a so-called fundamental scale concept in the MADM environment. They can be used only if the information about the relative importance of each objective/ attribute over another is known. This is represented in a square $(N)\times(N)$ matrix (A), termed as the pairwise comparison matrix. The element in the ith row and the jth column is A_{ij} ($= w_i / w_j$), and represents the relative weight of w_i over w_j.

Weighted Least Square Method

The weighted least square method has the advantage that it only involves the solution of a set of simultaneous linear algebraic equations. Consider A_{ij} to be the elements of a pairwise comparison matrix. It is desired to find the weights w_i $\forall$ i such that

$$\sum_{i=1}^{n}\sum_{j=1}^{n}(w_j A_{ij} - w_i)^2 \text{ is minimized.} \tag{2.3}$$

The weights are found out by solving a constrained optimization problem where the constraints are defined as

$$\sum_{i=1}^{n} w_i \quad = \quad 1.0. \tag{2.4}$$

Eigenvector Method

The eigenvector method of assessing weights has been proposed by Saaty [18]. The method is based on the principle of consistency of the pairwise comparison matrix. In any matrix, small perturbations in the coefficients of the matrix imply small perturbations in the eigenvalues. If A' is defined as the decision maker's estimate of A and W' denotes the corresponding weight vector, then W' is determined as the eigenvector corresponding to the maximum eigenvalue λ_{max} of A', according to

$$A' \, W' \quad = \quad \lambda_{max} \, W'. \tag{2.5}$$

Two cases have been implemented in these two environments. In the first case, the designer identifies building cost as the objective of the highest importance, while power and weight objectives have equal importance compared with one another. In the second case, the designer identifies all three objectives as equally important for the design. The model clearly shows that the proper selection of weights is key to obtaining the best solution. The Analytical Hierarchy Process (AHP) is employed to calculate the weights. The weights for the MADM environment are calculated by the eigenvector and the weighted least-squares method. Weights obtained by both these methods are in close agreement. The MODM tool based on the Modified Marquart Levenberg algorithm is found to be ideal because the system suitably scales the system variables and has a provision for generating the Jacobian matrix, thereby deciding the stepsize and direction.

Ray et al. (1995)

Ray et al. (1995) attempt to find a global optimum solution to a containership design. The model incorporates and employs simulated annealing as a global optimization tool. A decision system handler and accepted naval architectural methods are used to formulate the problem. The objective function used is a weighted average of the building cost, power, and steel weight of the ship. The carrying capacity (number of containers) and the speed are fixed during the design process. Two global optimization methods are applied:

a. Multistart method with
   1. the Hooke and Jeeves method using an external penalty function, and
   2. the Rosenbrock method for nonlinear constrained multivariable optimization;

   and

b. Simulated annealing and random perturbation followed by Hooke and Jeeves (a local optimization method).

A brief description of these methods is as follows:

Multistart Method

For a multistart process, each random search point generated acts as an initial starting point for the local optimization process. This is carried out for the prescribed number of iterations and the minimum function value is chosen. The local optimum solutions are evaluated at the expense of a larger computational time as compared to pure random search.

The random search points are generated using the relation

$$x_i = x_{\text{lower limit}} + (x_{\text{upperlimit}} - x_{\text{lower limit}}) \times R_i \qquad (2.6)$$

where

R_i is a random number between 0 and 1.

Simulated Annealing

Simulated annealing is a stochastic optimization method based on seeking iterative improvements along with ‘controlled’ deteriorations of the objective function in order to escape local minima. The heuristic is based on an analogy between the problems of combinatorial optimization and statistical mechanics. The general steps for the simulated annealing method are as follows:

1. A random initial solution configuration S_0 is chosen.

2. The function value is evaluated at this configuration S_0 as $f(S_0)$.
3. An initial annealing temperature is selected.
4. A perturbation scheme is applied to the system configuration S_0 so that it switches over to a new configuration S_k.
5. The function is evaluated at this configuration as $f(S_k)$.
6. The difference in the objective function value is computed as $f = f(S_k) - f(S_0)$.
7. If the difference is greater than or equal to zero, a random variable Y between 0 and 1 is compared to the probability of acceptance $P_{acc} = e^{-f/T}$. If Y is greater than P_{acc}, the solution S_k is abandoned and the control is passed to Step 4; otherwise the solution S_k is accepted and it replaces S_0. The acceptance of a solution relates to a successful iteration.
8. If the prescribed number of successful iterations at the current temperature has not been realized, the temperature is lowered. If the final annealing temperature is not reached, the control is passed on to Step 4.

The process stops when the final annealing temperature has been reached or it fails to locate better solutions within the maximum prescribed number of iterations at a particular temperature.

Since the initial temperature should be large enough so that almost all candidate solutions are accepted (even the ones that lead to a large increase in the objective function value) with a probability P_h of 0.9 or more, the initial temperature is determined from

$$P_h = e^{-f_{max}/T_0} \tag{2.7}$$

where f_{max} is the largest change in the objective function value observable between any two system configurations.

Similarly, the minimum annealing temperature is found as

$$P_t = e^{-f_{min}/T_f} \tag{2.8}$$

where f_{min} is the tolerance in the objective function evaluation and P_t is assumed to be 0.5 or lower.

The implementation incorporates a learning process where the system is allowed to run for say 100 iterations and the information regarding f_{min} and f_{max} is stored. Assuming values of P_h as 0.9 and P_t as 0.05, the initial and the final annealing temperatures are computed. The sequence of temperatures is assumed to follow a geometric progression.

Simulated Annealing with Random Perturbation Followed by a Nonlinear Optimization Method.

In this method, the solutions are generated by first deriving a random solution and then allowing a nonlinear local optimization tool to obtain a local minimum from this random point taken as the initial guess. A local optimization tool based on the Hooke and Jeeves method with an external penalty function has been used in such a procedure. In this manner, the solution produced by simulated annealing is polished to a local optimum solution.

The Analytical Hierarchy Process (AHP)

The Analytical Hierarchy Process deals with the study of how to derive relative scales using judgment and data from a standard scale. Subsequently, arithmetic operations are performed on such a scale to arrive at the relative weights to attach to the different objective functions.

The solutions obtained by the two multistart methods are quite different from each other and hence the authors conclude that the use of a nonlinear multivariable constrained optimization tool for such a problem can only provide a local optimum solution that may be quite far from the global optimum. Simulated Annealing successfully identifies the main cluster of points that is very close to the reported global optimum predicted by pure random methods and genetic algorithms. The main cluster of points identified by simulated annealing provides the designer the region of safe and economic design.

Neu et al. (2000)

Neu et al. (2000) develop a prototype tool with a multidisciplinary optimization approach to ship design. This tool employs modular construction of the various disciplines of problem formulation and links them to a geometric manipulation scheme and an optimization tool. A containership is used as a test case and the problem formulation treats the carrying capacity (number of containers) and the speed as variables in the design process. In this work, the weight-displacement equality is enforced through a decomposition approach that speeds up the design process. The user can choose to minimize the required freight rate or to maximize the return on investment. Constraints are imposed on the metacentric height required by the Coast Guard wind heel criterion, and on the minimum required freeboard and the rolling period. The optimization is carried out using the *Design Optimization Tools (DOT)* program obtained from Vanderplaats Research and Development, Inc. As part of this work, Ganesan and Hughes (1999) define and investigate a multidisciplinary design optimization model for containerships. This work provides the details regarding the construction of the lightship weight, cargo, total weight and economics modules. Results employing the three different techniques in DOT are derived and compared. It is shown that once a good solution is identified in the design space, the sequential linear programming algorithm is the most consistent method for converging to a local optimum. Furthermore, the objective function is found to be relatively flat in the vicinity of the optimum. This indicates that while applying secondary (perhaps subjective) criteria, the designer is not confined to a severely restricted design space and has some freedom in designing the optimum ship.

2.2 Nonlinear Optimization Methodologies Used in *DOT*

2.2.1 Modified Method of Feasible Directions (MMFD)

We seek to find the set of design variables, X_i, i=1,…,N, comprising the vector X that will

Minimize F(X)

subject to

$$g_j(X) \leq 0 \quad \forall \; j = 1,\ldots,M$$
$$X_i^L \leq X_i \leq X_i^U \quad \forall \; i = 1,\ldots,N.$$

We provide an initial X vector, X^0, and update the design according to the following optimization process:

1. Start, q = 0, $X = X^0$
2. q = q+1
3. Evaluate $F(X^{q-1})$ and $g(X^{q-1})$, $\forall\; j = 1,\ldots,M$
4. Identify the set of critical constraints, J_i
5. Calculate $\nabla F(X^{q-1})$ and $\nabla g(X^{q-1})$ $\forall\; j \in J$
6. Determine a search direction, S^q
7. Perform a one-dimensional search to find α
8. Set $X^q = X^{q-1} + \alpha S^q$ (2.9)
9. Check for convergence to the optimum. If satisfied, exit. Otherwise go to Step 2.

The critical parts of the optimization task consists of:

1. Finding a search direction, S^q, which will reduce the objective function value while not violating any constraints.
2. Finding the scalar parameter α that will minimize $F(X^{q-1} + \alpha S^q)$ subject to the constraints.
3. Test for convergence to an optimum or to some termination criterion.

2.2.2 Sequential Linear Programming (SLP)

The SLP procedure employs a first-order Taylor series approximation to the objective and constraint functions. The modified method of feasible directions is used to solve the resulting linear problem.

The steps involved in the general algorithm are:

1. For the current value of the design variables, retain the most critical constraints during the cycle. Typically we retain ($5\times$Number of design variables) constraints.
2. Create a first order Taylor series expansion of the objective function and the retained constraints with respect to the design variables.
3. Define move limits on the design variables. Typically, during one cycle, the design variables are allowed to change 20 – 40 %.
4. Solve the linearized approximate problem.
5. Check for convergence. If not satisfied, repeat from step 1.

The Taylor series expansion created by DOT as mentioned in Step 1 is of the form:

$$F(X) = F(X^{q-1}) + \nabla F(X^{q-1})^T (X - X^{q-1}) \tag{2.10}$$

and

$$g_j(X) = g_j(X^{q-1}) + \nabla g_j (X^{q-1})^T (X - X^{q-1}) \qquad \forall j \in J \tag{2.11}$$

where J is the index set of retained constraints.

We can rewrite these equations as:

$$F(X) = F^0 + \nabla F(X^{q-1})^T X \tag{2.12}$$

$$g_j(X) = g_j^0 + \nabla g_j (X^{q-1})^T X \qquad \forall j \in J \tag{2.13}$$

where

$$F^0 = F(X^{q-1}) - \nabla F(X^{q-1})^T X^{q-1} \tag{2.14}$$

and

$$g_j^0 = g_j(X^{q-1}) - \nabla g_j (X^{q-1})^T X^{q-1} \qquad \forall j \in J. \tag{2.15}$$

We now solve the linear approximate problem

Minimize $F(X)$

subject to

$$g_j(X) \leq 0 \qquad \forall j \in J$$

$$X_i^L \leq X_i \leq X_i^U \qquad \forall\ i = 1,\ldots,N$$

where

$$X_i^L = X_i^{q\text{-}1} - D\left|X_i^{q-1}\right| \tag{2.16}$$

and

$$X_i^U = X_i^{q\text{-}1} + D\left|X_i^{q-1}\right|. \tag{2.17}$$

The multiplier D in (2.16) and (2.17) is set by the parameter RMVLMZ in DOT. Depending on the progress of the optimization, this parameter is sequentially revised.

The convergence criteria for the SLP method are the same as those for the other methods. If there are constraint violations, we continue until the constraints are satisfied or the maximum number of iterations is reached.

2.2.3 Sequential Quadratic Programming (SQP)

Unlike SLP that minimizes a linearized objective, this approach creates a quadratic approximation of the objective function. The linearized constraints are used along with this to create a direction finding subproblem of the form

$$\text{Minimize}\quad Q(S) = F^0 + \nabla F^T S + \frac{1}{2} S^T B S \tag{2.18}$$

subject to

$$(\nabla g_j)^T S + g_j^0 \leq 0 \qquad \forall\ j = 1,\ldots,M.$$

The subproblem is solved using the modified method of feasible directions. The matrix B is a positive definite matrix that is initially the identity matrix. On subsequent iterations, B is updated to approach the Hessian of the Lagrangian function. Once the subproblem is solved, we calculate the Lagrange multipliers l_j j = 1,…,M. We now search in direction S by minimizing an l_1 penalty function composed as follows, in order to find a stepsize a.

$$\text{Minimize}\quad \Phi = F(X) + \sum_{j=1}^{M} u_j \max\left[0, g_j(X)\right] \tag{2.19}$$

where

$$X = X^{q\text{-}1} + a\,S$$

$$u_j = \left|l_j\right| \qquad \forall\ j = 1,\ldots,M \text{ first iteration}$$

$$u_j = \max\left[\left|?_j\right|, \frac{1}{2}\left(u_j^1 + \left|?_j\right|\right)\right] \qquad \forall\ j = 1,\ldots,M \text{ subsequent iterations}$$

and u_j^1 = u_j from the previous iteration.

After this one-dimensional search is complete, the B matrix is updated using the BFGS formula. The next iteration begins.

The termination criteria used in DOT for the three methods described above are:

A: Relative convergence criteria (0.001) met for 2 consecutive iterations

B: Absolute convergence criteria (0.0001) met for 2 consecutive iterations

C: Search vector norm is less than the specified tolerance (0.0001).

2.3 Global Optimization and the Reformulation Linearization Technique

2.3.1 Global Optimization

Global optimization methods have been used extensively in the areas of structural optimization, engineering design, VLSI chip design and database problems, nuclear and mechanical design, chemical engineering design and control, process allocation problems, etc. Experience with nonlinear optimization methodologies has shown that no single method exists that is best suited for the solution of all nonlinear problems. One cannot even hope that such a method will ever be found since problems vary widely in size and structure. One limitation of the available nonlinear tools is their incapability to attain a global optimum in all cases and their dependence on the proper choice of an initial guess. Methods to provide good initial guesses such that the function is roughly unimodal in the region of investigation or a method that guarantees the attainment of global optimum irrespective of an initial guess are some of the fastest growing and challenging research issues in mathematical programming and engineering science.

2.3.2 The Reformulation-Linearization Technique

Most deterministic approaches for nonconvex global optimization focus on constructing convex relaxations, and then embedding these lower bounding problems within a branch-and-bound procedure. Consequently, considerable research effort has been devoted to developing exact or tight approximations to convex envelopes for nonconvex functions.

Sherali and Adams (1990, 1994) develop a Reformulation Linearization Technique (RLT) that is designed to generate a hierarchy of linear programming relaxations for mixed 0-1 linear and polynomial integer programming problems. These relaxations span the spectrum from the usual continuous relaxation to the actual convex hull representation of the original feasible solution space. Typically, first or second levels of RLT are used to generate tight LP relaxations that are embedded within a general branch-and-bound procedure. This methodology has been applied to solve many different types of discrete optimization problems as illustrated in Sherali and Adams (1996). Moreover, Sherali and Tuncbilek (1992) have extended this unique approach to the continuous problem domain, in particular, to the global optimization of nonconvex polynomial programming problems. Sherali and Alameddine (1991) further utilize RLT to solve the class of bilinear programming problems. The LP relaxation constructed via RLT gives a close approximation to the convex hull of feasible points of the original bilinear problem. In some cases, it is proven that the approximations are exact. Sherali and Tuncbilek (1995) use RLT to devise a powerful algorithm for solving more general nonconvex quadratic programming problems. In addition to the LP relaxation, other kinds of RLT based cuts are generated so that the resulting convex relaxation yields very tight lower bounds. Various classes of RLT based valid inequalities are also presented in Sherali and Tuncbilek (1997) for univariate and multivariate polynomial programming problems. Sherali and Wang (1998) develop and implement a global optimization algorithm to solve a class of nonconvex factorable programming problems that involve the optimization of a general nonconvex factorable objective function over a feasible region that is restricted by a set of constraints also defined in terms of nonconvex factorable functions. The algorithm involves a branch-and-bound approach based on LP relaxations generated through various approximation schemes that utilize, for example, the Mean-Value Theorem and Chebyshev interpolation polynomials, coordinated with the RLT. The algorithm is tested using a collection of engineering design problems and it is shown that for certain cases, by finding global optimal solutions, this algorithm identifies better solutions than those previously reported in the existing literature.

Chapter 3

Model Formulation

This chapter provides detailed discussions on the formulation of the weight module, the cargo module, and the economic or the cost module. It also presents a discussion on the formulation of the measures of merit used to evaluate the design, and the formulation of the constraints imposed on the design.

Section 3.2 includes details of the formulation of the lightship weight, the fuel and miscellaneous weights, and calculations of their centers of gravity. Section 3.3 details the mathematical formulation of the number of containers that can be stowed in the holds and on-deck of a ship as functions of the ship's principal dimensions and its block coefficient. It also details the formulation of the centers of gravity of the containers. Section 3.4 provides a description of the formulation of uniform annual figures for the building cost, the operating cost, and the fuel cost. It also describes the formulation of the time taken by a ship to complete a round-trip and hence the number of round-trips made by the ship annually that is required to calculate the operating costs. Section 3.5 defines the measures of merit used to evaluate the design, followed by Section 3.6 that explains the formulation of the constraints imposed on the design.

It is worth pointing out that the formulation of the weight module does not involve draft. This is done in order to ensure that the weight has no dependence on the draft and hence on the displacement. This independence of the weight on the displacement is essential to avoid complications during the optimization process.

Before going into the details of the model formulation, it is essential to have a basic understanding of the terminology used in ship design. Section 3.1.1 presents an overview of the requisite naval architectural terms used in this formulation, along with their definitions. Section 3.1.2 introduces the fundamental characteristics of the ship hull form, and the properties relating to hydrostatic curves.

3.1 Terminology and Definitions [39], [40]

3.1.1 Definitions

Beam, maximum: The maximum molded width of the ship measured to the outside of the hull frame angle of channel but inside of the shell plating.

Beam, molded: The maximum breadth of the hull measured between the inboard surfaces of the side shell plating of flush-plated ships, or between the inboard surfaces of the inside strakes of lap seam-plated vessels.

Deadweight: The carrying capacity of a ship at any draft and water density. This includes the weights of the cargo, fuel, lubricating oil, fresh water in tanks, stores, passengers and baggage, crew, and their effects.

Depth, maximum: The molded distance between the ship's baseline and the underside of the deck plating of the uppermost continuous deck, measured at the side of the ship.

Depth, molded: The vertical distance from the molded baseline to the top of the freeboard deck beam at side, measured at mid-length of the ship.

Displacement, light: The weight of the ship including hull, machinery, outfit, equipment and liquids in the machinery.

Displacement, loaded: The displacement of a ship when floating at her greatest allowable draft. It is equal to the weight of water displaced and is the sum of the "light displacement" and the "deadweight".

Draft: The depth of the ship below the waterline measured vertically to the lowest part of the hull, propellers, or some other reference point. When measured to the lowest projecting portion of the vessel, it is called the *extreme draft*; when measured at the bow, it is called *forward draft*, and when measured at the stern, it is called the *after draft*. The average of the forward draft and the after draft is called the *mean draft*. The mean draft when in full load condition is called the *load draft*.

Draft, design: The full-load (100 percent consumable) departure draft used for calculation purposes during the ship's design process.

Freeboard: The distance from the waterline to the upper surface of the freeboard at side.

Keel: The principal fore-and–aft component of a ship's framing, located along the centerline of the bottom and connected to the stem and stern frames.

Knot: A unit of speed, equaling one nautical mile per hour; the international nautical mile is 1852 m (6076 ft.).

Length, overall: The extreme length of a ship measured from the foremost point of the stem to the aftermost (latter) part of the stern.

Length between perpendiculars: The length of a ship between the fore and after perpendiculars. The forward perpendicular is a vertical line at the intersection of the fore side of the stern and the summer load line. The after perpendicular is a vertical line at the intersection of the summer load line and either the after side of the rudder post or the sternpost, or the centerline of the rudder stock if there is no rudder post or sternpost.

Metacenter: The center of buoyancy of a listed ship is not on the vertical centerline plane. The intersection of a vertical line drawn through the center of buoyancy of a slightly listed vessel intersects the centerline plane at a point called the metacenter.

Metacentric height: The metacentric height is defined as the distance from the metacenter to the center of gravity of a ship. If the center of gravity is below the metacenter, the vessel is stable.

Speed, service: The service speed is defined as the predicted average speed at which the ship (at design draft) is expected to operate over its entire life at sea. The prediction takes into account such factors as: the environment, fouling, corrosion, and any other items that tend to reduce a ship's speed.

Speed, trial: The trial speed is defined as the speed of the ship while operating at the design draft, 100 percent of the normal continuous rating (NCR) of the main propulsion plant, and ideal sea trial conditions.

Stability: The stability of a ship is her tendency to remain upright or the ability to return to her normal upright position when heeled by the action of waves, wind, etc.

Trim: The trim is the difference between the draft forward and the draft aft. If the draft forward is the greater, the vessel is said to "trim by the head". If the draft aft is the greater, she is said to "trim by the stern". To trim a ship is to adjust the location of cargo, fuel, etc. so as to result in the desired drafts forward and aft.

Waterline: The waterline is the line of the water's edge when the ship is afloat. Technically, it is the intersection of any horizontal plane with the molded form.

3.1.2 Ship Hull Form [40]

Ship hull form refers to the shape of the hull, especially the part of the hull that is under water in normal operating conditions. Many of the calculations that a naval architect must make in order to design a ship are influenced by hull form. We define the particular geometric properties of the immersed form of the hull that are essential to ensure that the ship is operated in a safe and efficient manner. The properties in question are called hull form characteristics or hydrostatic properties because they pertain to the underwater form of the hull. When these properties are displayed in graphical form, the set of curves is referred to as the hydrostatic curves or curves of form. Since the drafts of a ship can vary considerably with changes in loading, the hydrostatic properties are calculated and plotted for a series of drafts.

Fundamental Hull Form Characteristics

All of the hydrostatic properties to be calculated are derived from the following four fundamental characteristics of the immersed hull form at each even keel waterline.

A. Properties of the Waterplane

Four properties of each waterplane are required:

1. *Area of waterplane (A_w)* : The waterplane area is required to determine the change in mean draft when small weights are loaded or displaced. Units: Meters^2.

2. *Center of floatation (CF)*: The center of flotation is the centroid of the waterplane, also called the center of area or center of gravity of the waterplane. It is required for the calculation of changes in draft at bow and stern as a result of loading, discharging, or shifting weights aboard ship. Its longitudinal position with respect to the midship section is called the longitudinal center of flotation (LCF). Units: Meters from the reference plane.

3. *Longitudinal moment of inertia (I_L)*: This property of the waterplane is its second moment of area about a transverse axis passing through the center of floatation. Units: Meters^4.

4. *Transverse moment of inertia (I_T)*: This is the second moment of area of the waterplane about its centerline. Units: Meters^4.

B. Properties of the Immersed Volume of the Hull

The quantities associated with the immersed volume that are usually determined are as follows:

1. *Volume of displacement (Disp)*: This is the immersed volume itself, called the volume of displacement because it is a measure of the volume of fluid displaced by the floating ship. Units: Meters^3.

2. *Longitudinal center of buoyancy (LCB)*: This is the distance of the center of buoyancy from a specified transverse reference plane, usually the midship section. Alternatively, LCB may be measured from the forward perpendicular (FP) or the aft perpendicular (AP) as long as the reference axis is clearly stated. Units: Meters.

3. *Vertical center of buoyancy (KB)*: This is the height of the center of buoyancy above the baseline or keel. Units: Meters.

C. Properties of the Stations

The last of the fundamental hull form characteristics required to prepare the hydrostatic curves are the immersed station areas (A_s). When plotted against ship length, the immersed station areas form a sectional area curve, whose shape represents the "fullness" or the "fineness" of the ship form, an important consideration in ship resistance and powering. Units: Meters^2.

The Coefficients of Form

1. *Block Coefficient (C_b)*: Overall, or "whole hull" fullness is characterized by the block coefficient, defined as the ratio of the immersed volume of the hull (Disp) to the volume of a rectangular block whose length, width, and height are equal to the length, beam, and draft of the ship, respectively. Figure 3.1 depicts the block coefficient graphically.

$$C_b = \frac{\text{Disp}}{\text{Lwl} \times \text{B} \times \text{T}} \tag{3.1}$$

where

Disp	=	Displacement, volume
Lwl	=	Length, waterline
B	=	Beam, maximum
T	=	Draft, design.

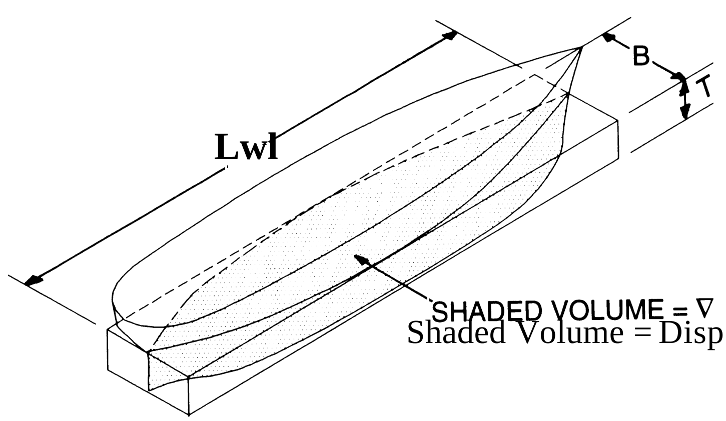


Figure 3.1 Block Coefficient

The block coefficients of typical ships vary from as low as 0.45 for a high-speed combatant ship such as a destroyer or a fast frigate, to as high as 0.85 or more for a large crude-tanker.

2. *Midship Section Coefficient (C_m)*: The fullness of the midship portion of a hull is described by the midship section coefficient, which is the ratio of the immersed section area to the area of its circumscribing rectangle. Figure 3.2 depicts the midship section coefficient graphically.

$$C_m = \frac{A_m}{B \times T} \tag{3.2}$$

where

A_m	=	Immersed midship section area
B	=	Beam, maximum
T	=	Draft, design.

The midship section coefficient varies from 0.75 or less for a fine-hulled destroyer, to 0.995 for a large merchant ship.

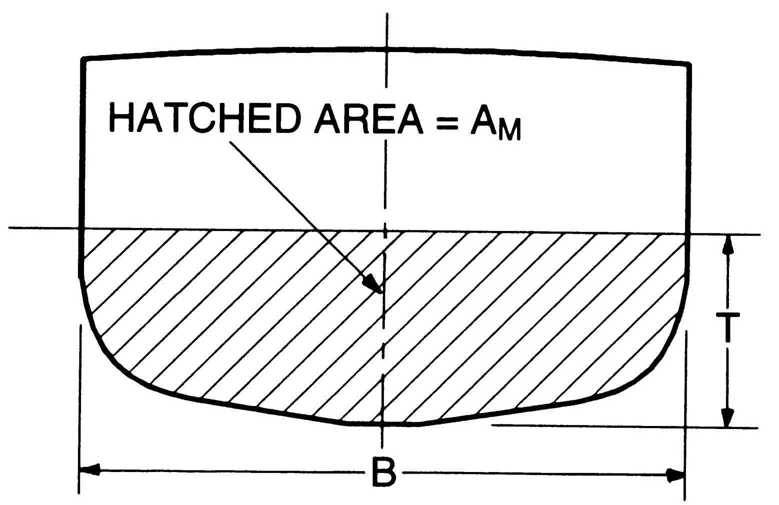


Figure 3.2 Midship Section Coefficient

3. *Prismatic Coefficient (C_p):* The coefficient that describes the fineness of the ends (bow and stern) of a hull without being influenced by its midship fineness is the prismatic coefficient, also called the longitudinal coefficient. It is denoted as the ratio of the volume of displacement (Disp) to the volume of a prism whose cross

section is shaped like the immersed midship section, and whose length is the length of the ship. Figure 3.3 depicts the prismatic coefficient graphically.

$$C_p = \frac{Disp}{A_m \times Lwl} \tag{3.3}$$

where

Disp	=	Displacement, volume
A_m	=	Immersed midship section area
Lwl	=	Length, waterline.

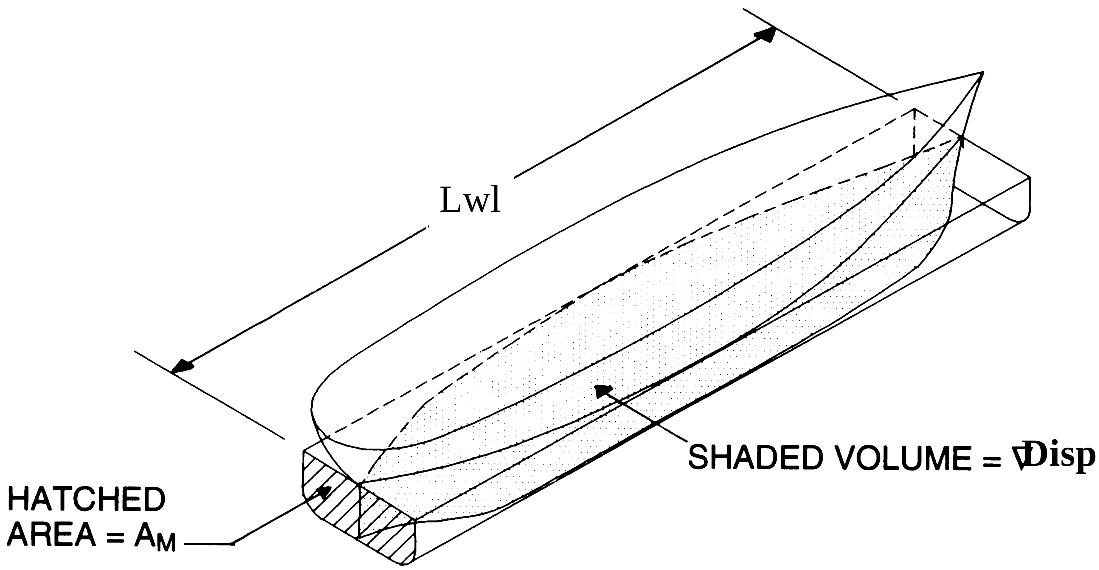


Figure 3.3 Prismatic Coefficient

The prismatic section coefficient ranges from 0.57 for a high-speed, fine-ended ship to 0.85 for a large tanker.

Since the fineness of the ends (C_p) and the middle (C_m), and the overall fineness (C_b) of a hull have been independently defined, the following relation holds, without loss of generality:

$$C_b = C_p \times C_m . \tag{3.4}$$

4. *Waterplane Coefficient (C_W):* Waterplane fullness or fineness is quantified by defining the waterplane coefficient (C_W), which is the ratio of the area of the waterplane (A_W) to the area of its circumscribing rectangle. Figure 3.4 depicts the waterplane coefficient graphically. Typical values of the waterplane coefficient vary from 0.67 to 0.92.

$$C_W = \frac{A_W}{Lwl \times B} \tag{3.5}$$

where

A_W = Area of the waterplane
Lwl = Length, waterline
B = Beam, maximum.

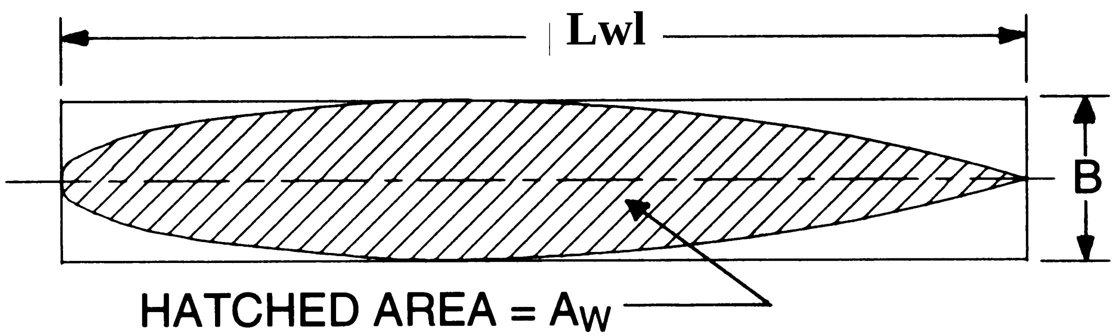


Figure 3.4 Waterplane Coefficient

Properties Included in Hydrostatic Curves

There are fourteen hydrostatic properties that are included in hydrostatic curves. They are listed below. The first six are always included in hydrostatic curves provided to ship personnel.

1. *Displacement* (Δ): In the U.S. measurement system, displacement is the weight of the ship and its contents. It is equal to the volume of displacement times the weight density of the water in which the ship floats. Units: Metric tons.

$$\Delta = \rho \times g \times Disp \quad \text{(U.S. units)} \tag{3.6}$$

where

Δ	=	Displacement, weight
ρ	=	Mass density of water
g	=	Acceleration of gravity
Disp	=	Displacement, volume.

2. *Longitudinal center of buoyancy (LCB):* The longitudinal center of buoyancy is defined in Section 3.1.2, B. It is stated again here for convenient reference.

The longitudinal center of buoyancy is the distance of the center of buoyancy from a specified transverse reference plane, usually the midship section. Alternatively, LCB may be measured from the forward perpendicular (FP) or the aft perpendicular (AP) as long as the reference axis is clearly stated. Units: Meters.

3. *Height of transverse metacenter above keel (KM):* This quantity is defined as follows:

$$KM = KB + BM \tag{3.7}$$

where

KB	=	Height of center of buoyancy above keel
BM	=	Transverse metacentric radius.

The height of transverse metacenter above keel is required to calculate the initial transverse stability.

4. *Tons per inch immersion (TPI):* The tons per inch immersion is defined as follows:

$$TPI = \frac{A_W}{420} \quad \text{(U.S. units)} \tag{3.8}$$

where

A_w	=	Area of the waterplane.

5. *Longitudinal center of floatation (LCF):* The longitudinal center of floatation is the centroid of the waterplane about amidships (amidships is exactly halfway between the forward and the after perpendiculars).

6. *Moment to change trim one inch (MTI):* The moment to change trim one inch is defined as follows:

$$MTI = \frac{\Delta \times BM_L}{12 \times Lwl} \tag{3.9}$$

where

Δ = Displacement, weight

BM_L = Longitudinal metacentric radius

Lwl = Length, waterline.

7. *Change in displacement per inch ($d\Delta PI$):* The change in displacement per inch is defined as:

$$d\Delta PI = \frac{TPI \times LCF}{Lwl} \tag{3.10}$$

where

TPI = Tons per inch immersion

LCF = Longitudinal center of floatation

Lwl = Length, waterline.

8. *Vertical center of buoyancy (KB):* The vertical center of buoyancy is defined above in Section 3.1.2., B. It is stated here again for convenient reference.

   The vertical center of buoyancy is the height of the center of buoyancy above the baseline or keel.

9. *Height of longitudinal metacenter above keel (KM_L):* The height of longitudinal metacenter above keel is defined as follows:

$$KM_L = KB + BM_L . \tag{3.11}$$

10-13 *Coefficients of form:* The coefficients of form, namely the block coefficient, the midship section coefficient, the prismatic coefficient, and the waterplane coefficient are defined in Section 3.1.2, C and illustrated graphically in Figures 3.1, 3.2, 3.3 and 3.4 respectively. They are stated here again for convenient reference.

10. *Block Coefficient (C_b)*: The block coefficient is defined as the ratio of the immersed volume of the hull (Disp) to the volume of a rectangular block whose

length, width, and height are equal to the length, beam, draft of the ship, respectively.

11. *Midship Section Coefficient (C_m)*: The midship section coefficient is defined as the ratio of the immersed section area to the area of its circumscribing rectangle.

12. *Prismatic Coefficient (Cp)*: The prismatic coefficient is defined as the ratio of the volume of displacement (∇) to the volume of a prism whose cross section is shaped like the immersed midship section, and whose length is the length of the ship.

13. *Waterplane Coefficient (C_w)*: The waterplane coefficient is defined as the ratio of the area of the waterplane (A_w) to the area of its circumscribing rectangle.

14. ***Wetted Surface (WS)*:** The wetted surface is the total area of the shell of the ship that is wetted by the water in which it floats for any given draft.

3.2 Summary of the Weight Module

The lightship weight and its center of gravity have been calculated using the empirical relations given in [1], [11] and [29]. In this summary, the numbers alongside the heading of each section indicate the source of the formulas used in that section. All weights are in metric tons, and all centers of gravity are in meters.

Weights have been broadly classified into three categories:
1. Lightship
2. Cargo
3. Fuel and miscellaneous.

The lightship, and the fuel and miscellaneous weights are explained here. The cargo weight is simply the product of the number of twenty-foot equivalent units (TEUs) and the weight per unit. The derivation of the number of TEUs is explained in Section 3.3.

The lightship weight is composed of:
1. Hull steel weight
2. Outfit and hull engineering weight
3. Machinery weight.

3.2.1 Lightship Weight

Hull Steel Weight [29]

$$Ws = 5905.98 \times \left(\frac{CN}{1000}\right)^{1.003} \times (1 + 0.49532 \times Cb) \times \left[1 + 0.000928 \times \left(\frac{Loa}{D} - 8.3\right)^{1.691}\right] \tag{3.12}$$

where

CN = Cubic number = $\frac{Loa \times B \times D}{100}$
Loa = Length, overall
B = Beam, maximum
D = Depth, at side
Cb = Block coefficient = $\frac{Disp}{Lwl \times B \times T}$
Disp = Displacement, volume
Lwl = Length, waterline
T = Draft, design.

Vertical Center of Gravity for Hull Steel [1]

$$KGs\ (\%\ D) = \left[48 + 0.15(0.85 - Cb)\left(\frac{Loa}{D}\right)^2\right] \times \frac{Ds}{D} \tag{3.13}$$

where
Ds = Depth increased to take account of the shear and hatchway volume.
We approximate the ratio: Ds / D as 1.008. Note that this is expressed as a percentage of D.

Outfit and Hull Engineering Weight [29]

$$Wo = 1727.20 \times \left(\frac{CN}{1000}\right)^{0.724} \tag{3.14}$$

and

$$Whe = 856.44 \times \left(\frac{CN}{1000}\right)^{0.724}. \tag{3.15}$$

Therefore, the Total Outfitting Weight is given by

Wto = Wo + Whe. (3.16)

<u>Vertical Center of Gravity for Total Outfit Weight</u> [39]

$$\textbf{KGto} = (1.005 - 0.000689 \times \text{Loa})\text{D}. \tag{3.17}$$

<u>Propulsion Machinery Weight</u> [29]

$$\text{Wm} = 93.448 \times \left(\frac{\text{BHP}}{1000}\right)^{0.775} \tag{3.18}$$

where
BHP = Brake horsepower.

<u>Vertical Center of Gravity for Machinery [29]</u>

$$\textbf{KGm} = 0.47 \times \text{D}. \tag{3.19}$$

Therefore, the Total Lightship Weight is given by

$$\textbf{WL} = (\text{Ws} + \text{Wto} + \text{Wm}) \times 1.03 \tag{3.20}$$

where 1.03 denotes a 3 percent margin. The mathematical form of this expression is explained in Appendix H.

$$\textbf{KGL} = \left[\frac{(\text{Ws} \times \text{KGs}) + (\text{Wto} \times \text{KGto}) + (\text{Wm} \times \text{Kgm})}{(\text{WL}/\ 1.03)}\right] + 0.3. \tag{3.21}$$

From (3.21) it can be seen that we provide a margin of 0.3 meters to the center of gravity of the total lightship weight. The mathematical form of this expression is given in Appendix I.

3.2.2 Fuel and Miscellaneous Weights

<u>Fuel</u> [39]

$$\textbf{Wf} = \frac{\text{SHP} \times \text{Dst} \times 1.1 \times \text{SFC}}{\text{Vk} \times 10^6} \tag{3.22}$$

where

SHP	=	Maximum continuous shaft horsepower
Dst	=	Range in nautical miles defined by the user (in this work, it is defined as 7000 nautical miles)
SFC	=	Specific fuel consumption defined by the user (in this work, it is defined as 120 Gms/ SHP/ hr)
Vk	=	speed in knots.

The mathematical form of this expression is presented in Appendix J.

Center of Gravity of Fuel

$$\mathbf{KGf} = \mathrm{Wf} \times \left(\frac{6.1}{3385}\right). \tag{3.23}$$

The mathematical form of this expression is detailed in Appendix K.

Miscellaneous

Crew and Provisions: [11]

$$\mathrm{Wcp} = [(6 \times \mathrm{Dst}) + 50] \times \mathrm{Cf} \tag{3.24}$$

where

Cf	=	Coefficient to convert long tons to metric tons = 0.9843.

Freshwater: [11]

$$\mathrm{Wfw} = 280 \times \mathrm{Cf}\,. \tag{3.25}$$

Luboil for diesel: [11]

$$\mathrm{Wlo} = 50 \times \mathrm{Cf}\,. \tag{3.26}$$

Miscellaneous weights when machinery is idle: [11]

$$\mathrm{Whc} = 5 \times \mathrm{Pwt} \times \mathrm{Cf} \tag{3.27}$$

where

Pwt	=	Port waiting time in days defined by the user (in this work, it is defined as 2 days).

Therefore, the Total Miscellaneous Weight is given by

$$\mathbf{Wmisc} = \mathrm{Wcp} + \mathrm{Wfw} + \mathrm{Wlo} + \mathrm{Whc}. \tag{3.28}$$

In this work, the value of the miscellaneous weights evaluates to 41724.5 metric tons.

Vertical Center of Gravity of Miscellaneous Weight **[39]**

$$\mathbf{KGmisc} = 0.5 \times x_3 . \quad (3.29)$$

3.3 Validation of the Weight Module

Once we have formulated a model, it is essential to validate it against benchmarks to check the accuracy of the model. Table 3.1 displays the weight data on seven ships and provides a comparison of the weights obtained by using the expressions in section 3.3 with the actual values for all components of the lightship weight.

From Table 3.1, it is seen that the model for lightship weight is 0.4 to 15.2 percent accurate.

3.4 Summary of the Cargo Module

In all previous optimization approaches to containership design [8], [11], [25], and [26]; the carrying capacity (number of containers) is fixed during the design process. The number of containers a ship can carry is a function of the principal dimensions and the block coefficient of the ship. As these dimensions change during the optimization process, so do the number of containers.

The cargo module calculates the number of containers that can be stowed in the holds and on deck by employing stowage factors on the total available hold volume and the surface area on the deck, respectively. It then calculates the vertical center of gravity of the cargo.

The number of rows and columns of containers in a hold is an integer value. This makes the carrying capacity (number of containers), and hence the objective function, as discontinuous functions of the principal dimensions. This problem has been solved by using a linear response surface fit to the number of TEUs (Twenty-foot Equivalent Units) below and above deck.

3.4.1 TEUs Below Deck

We discretize the space available for container stowage in the lengthwise, beamwise and the depthwise directions. The product of the number of TEUs in each direction and a “Stowage factor” gives us the total number of containers that can be stowed. The stowage factor accounts for the geometry of the hull form and the space available for container stowage after accounting for the space occupied by the container cell guides. It

Table 3.1 Validation of the Weight Module[6].

(a): Specifications of the Ships Used for the Validation.

Ships	Loa	B	D	T	Vk
1	204.12	27.43	16.15	8.84	22.50
2	263.09	32.23	20.12	9.02	23.75
3	249.94	30.48	18.29	10.06	21.50
4	219.61	28.96	16.47	9.75	23.00
5	213.51	25.91	15.39	8.38	22.25
6	247.88	27.43	16.15	8.84	21.00
7	185.93	23.77	16.61	9.14	19.50

(b): Validation of the Weight Module.

Ships	Hull Steel Weight (Metric Tons)		Total Outfit Weight (Metric Tons)		Machinery Weight (Metric Tons)		Lightship Weight (Metric Tons)	
	Actual Value	Modeled Value	Actual Value	Modeled Value	Actual Value	Modeled Value	Actual Value	Modeled Value (% Diff.)
1	6949	7190	2631	2402	1077	1344	10657	11264 (5.7)
2	13004	13720	3139	3803	1829	1861	19891	19966 (0.4)
3	10037	11148	3637	3285	1280	1326	8666	8380 (3.3)
4	6827	6809	1747	2299	965	1213	11652	10631 (8.8)
5	10057	8859	3759	2764	1057	1111	14873	13117 (11.8)
6	4500	5825	1808	2065	843	842	10606	8995 (15.2)
7	16031	16159	4419	4274	1697	1079	23284	22157 (4.8)

is found that the number of containers below deck shows a dependence on the block coefficient when it is calculated by employing the stowage factor on the total available hull volume. The dependence has been accurately represented by expressing the stowage factor explicitly as a function of the block coefficient. The resulting expression for the TEUs below deck is:

$$\text{TEUbd\_i} = \left\lfloor \frac{\text{Loa}}{1.05 \times 6.1} \right\rfloor \times \left\lfloor \frac{\text{B} - (2 \times \text{DBH})}{2.44} \right\rfloor \times \left\lfloor \frac{\text{D} - \text{DBH}}{2.44} \right\rfloor \times (0.8479 \times \text{Cb} - 0.0918) \tag{3.30}$$

where

- $\frac{\text{Loa}}{1.05}$ = The approximation for the length between perpendiculars (Lbp) that has been followed throughout this work
- 6.1 = Length of a TEU in meters, equivalent to twenty feet
- 2.44 = Width and the depth of a TEU in meters, equivalent to eight feet
- DBH = Double bottom height is defined by the user (in this work, it is defined as 1.83 meters)
- TEUbd = Number of TEUs below deck
- STWbd = Stowage factor for TEUs below deck averaged from data on twelve ships [3]

and where the notation $\lfloor \ \rfloor$ indicates that the number within is rounded down to an integer if it is fractional.

The number of TEUs given by (3.30), and hence the objective function, are discontinuous functions of the principal dimensions. We solve this problem using a linear response surface fit, using the method of least squares, that makes the number of TEUs and hence the objective function continuous. The resulting expression for the continuous number of TEUs is as follows.

$$\text{TEUbd\_f} = ((0.0196 \times \text{Loa} \times \text{B} \times \text{D}) - 148.6129) \times (0.8479 \times \text{Cb} - 0.0918)\,. \tag{3.31}$$

3.4.2 TEUs Above Deck

We discretize the space available for container stowage in the lengthwise and beamwise directions. The geometry of the surface area is accounted for by a "stowage factor" for TEUs above deck. The product of the number of TEUs that can be stowed in these two directions, the number of tiers above deck, and the stowage factor gives us the total number of TEUs that can be stowed above deck.

$$\text{TEUd\_i} = \left\lfloor \frac{\text{Loa}}{1.05 \times 6.1} \right\rfloor \times \left\lfloor \frac{\text{B}}{2.44} \right\rfloor \times \text{Ntd} \times \text{STWd} \qquad (3.32)$$

where

- TEUd_i = Number of TEUs above deck
- Ntd = Number of tiers above deck
- STWd = Stowage factor for TEUs above deck obtained from a single ship as we have only one source of data for the number of tiers and the total number of TEUs above deck which are required to calculate STWd : 0.7534.

We could allocate a fixed value for the number of tiers on deck. However, data from Panamax and post Panamax ships [7] indicate otherwise. A Panamax containership, measuring 292 meters overall by 32.2 meters wide, can have a stated capacity of 4000 TEUs carrying 5-high on deck for the aft hatches. The forward hatches taper down to 4-high for visibility purposes. The post Panamax containerships, that are almost all that are built nowadays, have capacities of 5200 TEUs, carrying containers 5-high on deck and 10-deep below deck with an overall length of 292 meters and a width of 40 meters. Using the same length and 43 meters wide, 10-deep below deck and 5 to 6-high on deck, the rated capacity is 6600 TEUs. Furthermore, the fact that the containers need not be evenly distributed over the surface area on deck implies that the number of tiers on deck need not be integral.

To account for these factors we chose to express the number of tiers as a function of the beam using interpolation. The following expression has been formulated based on the above data.

Table 3.2: Number of Tiers on Deck as a Function of Beam

Beam	Ntd
$B \leq 32.2$	4.0
$32.2 < B \leq 40.0$	$4.0 + \frac{(B - 32.2)}{(40.0 - 32.2)}$
$40.0 < B \leq 43.0$	$5.0 + \frac{(B - 40.0)}{(43.0 - 40.0)}$
$43.0 < B$	6.0

Again the number of TEUs as expressed in (3.32) is a discontinuous function of the length and the beam of the ship. The continuous expression obtained through a linear response surface fit is as follows:

$$\text{TEUd\_f} = (\text{Loa} \times \text{B} \times 0.050117 \times \text{Ntd}) - 82.6702\,. \qquad (3.33)$$

3.4.3 Center of Gravity of Containers

Containers below deck

To account for the geometry of the hull form, the moment of the containers below deck has to be expressed as a function of the block coefficient. A parameter, "MArmfactor_TEUbd", is defined as the value with which the height of the number of tiers below deck must be multiplied in order to derive its center of gravity.
From this definition, we obtain the information given in Table 3.2.

Table 3.3: Moment-Arm Factor for TEUs Below Deck as a Function of the Block Coefficient

Cb	MArmfactor_TEUbd
1.0 (Rectangular block)	0.5
0.5 (Triangular block)	0.6667

Without loss of generality, it can be stated that:

$$\text{Marmfactor\_TEUbd} = 0.5 + \frac{(1-\text{Cb})}{3}. \qquad (3.34)$$

The following equations express the moment arm, weight, and the moment of the TEUs below deck respectively.

$$\text{MArm\_TEUbd} = \text{DBH} + (\text{Ntb} \times 2.44 \times \text{MArmfactor\_TEUbd}) \qquad (3.35)$$

where
Ntb = number of tiers below deck,

$$\text{Wt\_TEUbd} = \text{TEUbd} \times \text{Wpc} \qquad (3.36)$$

where
Wpc = weight per TEU. This is defined by the user (in this work this is taken as 12 metric tons),

and

$$\text{Moment\_TEUbd} = \text{Marm\_TEUbd} \times \text{Wt\_TEUbd}. \qquad (3.37)$$

Containers above deck

The following equations express the moment arm, weight, and the moment of the TEUs above deck, respectively.

$$\text{MArm\_TEUd} = D + HCH + \left(\frac{Ntd \times 2.44}{2}\right) \tag{3.38}$$

where
HCH = Hatch coaming height defined by the user (in this work, it is defined as 1.83 meters),

$$\text{Wt\_TEUd} = \text{TEUd} \times \text{Wpc}, \tag{3.39}$$

and

$$\text{Moment\_TEUd} = \text{MArm\_TEUd} \times \text{Wt\_TEUd}. \tag{3.40}$$

The mathematical forms of the cargo weight and its center of gravity are detailed in Appendix L and the mathematical forms of the total weight and its center of gravity are provided in Appendixes M and N respectively.

3.5 Validation of the Cargo Module

Table 3.4 provides a comparison of the actual number of containers and the number of containers obtained by using the discrete and continuous expressions described in section 3.4. It can be seen that the formulated expressions for the number of containers above deck and below deck are fairly accurate representations for the actual number of containers.

3.6 Summary of the Cost Module

The total cost is comprised of building and operating costs. The building cost of the ship can be converted to uniform annual amounts using a capital recovery factor. The operating and fuel costs are calculated on an annual basis. All costs are in U.S. dollars.

3.6.1 Annual Building Costs [2], [11], [29]

Building costs are broken down into steel hull, outfit, and hull engineering and machinery costs. Each of these constitutes costs for material and labor. Material costs are a function of the weight. Labor costs are functions of the man-hours that in turn, are functions of the weight.

Throughout this module we use a labor cost of 20 dollars per hour. Overheads are calculated as 70 percent of the labor costs. Furthermore, since these formulas were developed in 1962, we use the following approximations to accommodate the present day trends:

- reduce man-hours by sixty percent to account for automation;

Table 3.4 Validation of the Cargo Module [5].

	Loa	B	D	Cb	TEUbd, Actual	TEUbd, Discrete	TEUbd, Cont	TEUd, Actual	TEUd, Discrete	TEUd, Cont
1	204.12	27.43	16.48	.5521	706	629	624.68	720	770	759.14
2	263.09	32.23	20.12	.5524	1332	1188	1203.27	1160	1204	1192.21
3	249.94	30.48	18.29	.5941	784	963	1063.78	858	1057	1062.73
4	219.61	28.96	16.48	.5889	738	831	776.62	590	845	873.55
5	249.94	30.48	18.29	.6576	850	1089	1202..82	936	1057	1062.73
6	259.00	32.21	18.91	.6120	1296	1315	1257.16	1224	1175	1171.61
7	213.51	25.91	15.39	.6129	644	635	650.42	534	745	749.07
8	247.88	27.43	16.48	.6279	1186	904	902.18	927	944	939.62
9	185.93	23.77	16.61	.6285	612	614	569.11	474	589	581.81
10	185.93	23.77	16.61	.6431	519	631	585.09	288	589	581.81
11	177.60	23.77	15.54	.7040	546	545	574.43	288	549	552.04
12	289.56	32.21	21.49	.7505	2412	2156	2058.31	1370	1322	1319.62

- increase material costs by 40 percent .

Steel Hull

$$\textit{Man-hours:}\ \ \mathrm{Mhs} = \mathrm{Cmhs} \times \left(\frac{\mathrm{Ws}}{1000}\right)^{0.85} \tag{3.41}$$

where
Cmhs is a coefficient depending on the effectiveness of the yard. We assume a fixed value of $3160 = 7900 \times 0.4$.

$$\textit{Labor:}\ \ \mathrm{Lhs} = \mathrm{Mhs} \times \mathrm{Lc} \tag{3.42}$$

where
Lc = unit labor cost defined by the user (in this work, it is defined as 20 dollars/ hour).

$$\textit{Material:}\ \ \mathrm{Mats} = \mathrm{Csh} \times \mathrm{Ws} \tag{3.43}$$

where
Csh = cost of hull steel material defined by the user (in this work it is defined as 400 dollars/ metric ton).

Outfit

$$\textit{Man-hours:}\ \ \mathrm{Mho} = \mathrm{Co} \times \left(\frac{\mathrm{Wo}}{100}\right)^{0.9} \tag{3.44}$$

where
Co is a coefficient with a fixed value of $8000 = 20000 \times 0.4$.

$$\textit{Labor:}\ \ \mathrm{Lo} = \mathrm{Mho} \times \mathrm{Lc}\,. \tag{3.45}$$

$$\textit{Material:}\ \ \mathrm{Mato} = \mathrm{Cof} \times \mathrm{Wo} \tag{3.46}$$

where
Cof = cost of outfit material defined by the user (in this work it is defined as 1500 dollars/ metric ton).

Hull Engineering

$$\textit{Man-hours:}\ \ \mathrm{Mhhe} = \mathrm{Chhe} \times \left(\frac{\mathrm{Whe}}{100}\right)^{0.75} \tag{3.47}$$

where
Chhe is a coefficient with a fixed value of $20400 = 51000 \times 0.4$.

Labor: $\text{Lhe} = \text{Mhhe} \times \text{Lc}$. (3.48)
Material: $\text{Mathe} = \text{Chull} \times \text{Whe}$ (3.49)

where
Chull = cost of hull engineering material defined by the user (in this work it is defined as 3500 dollars/ metric ton).

Machinery

Man-hours: $$\text{Mhm} = \text{Chm} \times \left(\frac{\text{SHP}}{1000}\right)^{0.6} \quad (3.50)$$

where
Chm has a fixed value of $6773.33 = 25400 \times 0.4 / 1.5$.

Labor: $\text{Lm} = \text{Mhm} \times \text{Lc}$. (3.51)

Material: $$\text{Matm} = \text{Cmm} \times \left(\frac{\text{SHP}}{1000}\right)^{0.6} \quad (3.52)$$

where
Cmm is a coefficient with a fixed value of $388666.67 = 583000 / 1.5$.

Miscellaneous Costs

This is taken as 10 percent of the subtotal of the material costs.

$$\text{Miscc} = 0.10 \times (\text{Mats} + \text{Mato} + \text{Mathe} + \text{Matm}). \quad (3.53)$$

Accommodation Costs

This cost factor is approximated as a function of the number of crews.

$$\text{Accoc} = 180{,}000 \times \text{Ncrew}^{0.56} \quad (3.54)$$

where
Ncrew = number of crews.

$$\text{Ncrew} = \text{Cst} \times \left[\text{Cdk} \times \left(\frac{\text{CN}}{1000}\right)^{1/6} + \text{Ceng} \times \left(\frac{\text{SHP}}{1000}\right)^{1/5}\right] \quad (3.55)$$

where
Cst = coefficient for stewards department = 1.25
Cdk = coefficient for the deck department = 15.4
Ceng = coefficient for the engine department = 10.

Overhead Costs

This is 70 percent of the total labor cost.
$$Ovhc = 0.70 \times (Lhs + Lo + Lhe + Lm). \quad (3.56)$$

Yard's total

Ytc: This is the sum of all the above cost components.

Yard's Profit

This is 5 percent of the Yard's total.
$$Pr = 0.05 \times Ytc. \quad (3.57)$$

Yard's building price

This is the sum of the Yard's total and profit.
$$Ybc = Ytc + Pr . \quad (3.58)$$

Owner's expenses

This is 5 percent of the yard's building price.
$$Owe = 0.05 \times Ybc. \quad (3.59)$$

Cost to owner

This is the sum of the Yard's building price and the owner's expenses.
$$Owc = Ybc + Owe. \quad (3.60)$$

For computing uniform annual amounts, we use a capital recovery factor that is defined as:
$$Cr = \left[\frac{(1 + Ir)^{Sl} \times Ir}{(1 + Ir)^{Sl} - 1} \right] \quad (3.61)$$

where
Ir = interest rate in percentage defined by the user. In this work it is defined as

0.08.

Sl = ship life in years defined by the user. In this work it is defined as 20 years.

Therefore, Annual Building Cost is given by

$$\textbf{Abc} = Owc \times Cr. \tag{3.62}$$

The mathematical form of this expression is given in Appendix O.

Before we proceed to compute the operating costs, we need to calculate the number of round-trips made annually by the ship. This is required to determine the cargo handling, port, and fuel costs.

3.6.2 Round-trip Time [19]

The total time for a round-trip is comprised of the times required for loading and unloading cargo, the waiting time in port, and the time spent at sea.

Time for loading/ unloading per round trip

$$Lut = 4\times \frac{TEU\_f}{TSLU \times Ncrane\_f} \tag{3.63}$$

where

TEU_f	=	total number of TEUs (this is a continuous function of the length, the beam and the depth of the ship)
TSLU	=	TEUs loaded/ unloaded per day per crane defined by the user (in this work, it is defined as 1440)
Ncrane_f	=	number of cranes, this is approximated as a continuous function as defined below.

A single crane can handle 1440 TEUs per day [7]. The total number of TEUs loaded/ unloaded per day can be computed as: $TSLU \times Ncrane$. The number of cranes can be computed based on the relationship that one crane can be accommodated every 135 feet over 75 percent of the ship's length.

$$Ncrane\_i = \left\lfloor \frac{0.75 \times Loa}{41.175} \right\rfloor + 1 \tag{3.64}$$

where

Ncrane_i = number of cranes, integer.

The notation $\lfloor \ \rfloor$ indicates that the number within is rounded down to an integer if it is fractional. This number is seen to be a discontinuous function of Loa that induces a discontinuity in the objective function. The following equation obtained through a least squares fit, makes it continuous for use in (3.63):

$$\text{Ncrane\_f} = (0.0187 \times \text{Loa}) + 0.3572. \tag{3.65}$$

Waiting time in port

The waiting time in port is assumed to be constant for a given route. It is the time spent in port waiting for the availability of cranes and for maintenance. When analyzing different routes, the waiting time will be a function of the number of port calls. In this model we assume a waiting time of two days per round-trip.

Time at sea

The time at sea per round-trip in days is given by
Therefore, the total round-trip time in days can be calculated as:

$$\textbf{Rtt} = \text{Lut} + \text{Pwt} + \text{St}. \tag{3.67}$$

Hence, the number of trips per year is:

$$\textbf{NT} = \frac{\text{Ot}}{\text{Rtt}} \tag{3.68}$$

where

Ot = on-hire time per year in days, as defined by the user (in this work it is defined as 350 days).

3.6.3 Annual Operating Costs [2], [29]

Wages

$$\text{Wage} = (27000 \times 1.4) \times \text{Ncrew}^{4/5}. \tag{3.69}$$

The above relationship accounts for the fact that automation will replace manpower to a large extent, and avoids the use of a flat-rate per man.

Stores and Supplies

The cost of stores and supplies is computed based on the number of crew (Ncrew) that is derived in (3.55).

$$Ss = (80 \times 1.4) \times \left(\frac{Ncrew}{10} \right)^4 \quad \text{if } Ncrew \text{ is less than 50} \tag{3.70}$$

or

$$Ss = [50{,}000 + 4000 \times (Ncrew - 50)] \times 1.4, \text{ otherwise.} \tag{3.71}$$

Insurance

Protection and Indemnity:

$$InsPI = 965 \times 1.4 \times Ncrew. \tag{3.72}$$

Hull and Machinery:

$$InsHM = [10{,}000 + 0.007 \times (Mathe + Mats + Mato + Matm + Lm + Lhs + Lo + Lhe)] \times 1.4 . \tag{3.73}$$

Maintenance and Repair

Hull: $$Mrh = \left[108000 \times \left(\frac{CN}{1000} \right)^{2/3} \times 1.4 \right]. \tag{3.74}$$

Machinery: $$Mrm = \left[10000 \times \left(\frac{SHP}{1000} \right)^{2/3} \times 1.4 \right]. \tag{3.75}$$

Port Expenses

$$Port = \left[20 + 290 \times \left(\frac{CN}{1000} \right) \right] \times Pwt \times NT . \tag{3.76}$$

Cargo handling Costs*

$$Chld = 50 \times Wcargo \times NT \times 2 \tag{3.77}$$

where

Wcargo = weight of cargo in metric tons.

* The cargo handling cost has been excluded in the calculation of the annual operating cost for lack of an adequate formulation of this cost. This is done because, in the above formulation, we assume a rate of 50 dollars/ metric ton of cargo for loading and unloading once. This causes the cargo handling cost and hence the annual operating cost to be unreasonably high. We need more realistic data to formulate this cost adequately.

Therefore Annual Operating Cost:

$$\mathbf{Aoc} = \text{Wage} + \text{Ss} + \text{InsPI} + \text{InsHM} + \text{Port} + \text{Chld}. \tag{3.78}$$

The mathematical form of this expression is presented in Appendix P.

3.6.4 Annual Fuel Cost

$$\mathbf{Afc} = \text{Wf} \times \text{Fcost} \times \text{NT} \tag{3.79}$$

where

Fcost = cost of bunker C fuel oil as defined by the user (in this work, we used 80 dollars/ metric ton).

The mathematical form of this expression is provided in Appendix Q.

Therefore, Annual Average Cost:

$$\mathbf{Aac} = \text{Abc} + \text{Aoc} + \text{Afc}. \tag{3.80}$$

The mathematical form of this expression is given in Appendix R.

3.7 Measures of Merit

3.7.1 Freight Rate

The required freight rate, expressed in dollars per ton per mile, is simply the amount the owner must charge the customer in order to break-even. A more precise definition from [29] is: "The required freight for a given rate of utilization produces net profits which exactly cover the operating costs inclusive of calculated interest on the invested capital". Therefore, the required freight rate, RFR, is given by

$$\mathbf{RFR} = \frac{\text{Aac}}{\text{NT} \times \text{Wcargo} \times \text{Dst}}. \tag{3.81}$$

The mathematical form of this expression is stated in Appendix A.

3.7.2 Return on Investment

The return on investment, expressed in percentage per year, is defined as the ratio given by the difference in the annual gross income and the annual operating costs divided by the invested capital. The annual gross income is calculated by defining a charge rate expressed in dollars per ton. Five percent of the depreciated value of the investment, which is a reasonable value, is included in the gross income to account for the salvage value of the ship, assuming that the ship is in operable condition throughout its expected life-time. The ship life is identified by the user, and is taken as 20 years in this work.

$$\text{ROI} = \frac{\text{annual gross income - annual average cost} + \text{salvage value of the ship}}{\text{invested capital}} \times 100\ \%.$$

Therefore the return on investment, ROI, is given by

$$\textbf{ROI} = \frac{(\text{Chargerate} \times \text{Wcargo} \times \text{NT} \times \text{Dst}) - \text{Aac} + \left[\dfrac{0.05 \times \text{Owc}}{(1+\text{Ir})^{\text{Sl}}}\right]}{\text{Owc}} \times 100\% \quad (3.82)$$

where

Chargerate = The rate charged by the ship owner to ship cargo, as defined by the user. In this work it is defined as 44.8 dollars/ metric ton.

The mathematical form of this expression is provided in Appendix E.

3.8 Constraints on the Design

We impose five constraints on the design. The constraint that the displacement equals the weight is mandatory. The inequality constraint involving the length to draft ratio is required to hold true for (3.12) to be meaningful and is also mandatory. The mandatory constraints are "hard" constraints that the user does not have the freedom to relax. The other three constraints involve the Coast Guard wind heel criterion for metacentric height, the minimum required freeboard from the freeboard tables [40], and a constraint on the rolling period [23]. These are also modeled as "hard" constraints. A description of these constraints is as follows:

1) An equality constraint to enforce the balance between weight and displacement. The mathematical form of this constraint is presented in Appendix B.

2) An inequality constraint on the Length to Depth ratio that is required for (3.11) to hold true. The mathematical form of this constraint is as follows:

$$x_1 - 8.3\, x_3 \geq 0. \quad (3.83)$$

3) An inequality constraint on the metacentric height to ensure that it satisfies the Coast Guard wind heel criterion [5].

$$GM \geq \frac{P \times LAH}{Disp \times \tan(?)} \tag{3.84}$$

where

GM = KM –KG. This is a variable in the design process since KG is a function of the design variables.

P = $0.036 + \left(\frac{Lbp}{1309}\right)^2$ metric tons/ meter ^{2}.

LAH = the product of the projected lateral area of the portion of the vessel and deck cargo above the waterline and the vertical distance from the center of the above to the center of the underwater lateral area or to the one-half draft point. The mathematical form of this parameter is as follows:

$$\frac{0.5 \times \left[(x_3 - x_4)^2 + (Ntd \times 2.4)^2\right]}{(x_3 - x_4) + (Ntd \times 2.4)} \tag{3.85}$$

$$\tan(?) = \frac{(x_3 - x_4) \times 2}{x_2}.$$

The mathematical form of this constraint is detailed in Appendix C.

4) An inequality constraint on the freeboard to ensure that it satisfies the minimum required freeboard criterion from the freeboard Tables [38].

$$(x_3 - x_4) \geq \text{Freeboard\_min}. \tag{3.86}$$

The following expression is obtained by fitting a curve using the method of least squares through the 114 points given in the freeboard tables [39], governed by the code of federal regulations for freeboard (46 CFR 42).

$$\text{Freeboard\_min} = 0.025633 \times x_1^{0.9146}. \tag{3.87}$$

Correction in (3.87) for the block coefficient

If Cb > 0.68, multiply (3.87) by $\frac{Cb + 0.68}{1.36}$.

Correction in (3.87) for the length to draft ratio

If $\frac{x_1}{x_3} < 15$, add $\left(x_3 - \frac{x_1}{15}\right) \times 0.25$ to (3.87).

5) An inequality constraint on the rolling period to ensure that it satisfies the minimum required rolling period criterion.

$$Troll \geq Troll\_min \tag{3.88}$$

where an expression for the rolling period [16] is:

$$Troll = 0.58\sqrt{\frac{x_2{}^2 + \left(4 \times KG^2\right)}{|GM|}}$$

and where,
Troll_min is defined by the user (in this work it is taken as 15 seconds).

A simplified form of this constraint is as follows:

$$x_2{}^2 + (4 \times KG^2) - 668.845 \times |GM| \geq 0. \tag{3.89}$$

The mathematical form of this constraint is presented in Appendix D.

Chapter 4

Reformulation-Linearization Technique and Response Surface Methodology Based Polynomial Programming Approximation Approaches

Consider the containership design problem (CSD), and let us represent it in the following generic form

$$\text{CSD:}\quad \text{Minimize}\quad f(x) \tag{4.1a}$$
$$\text{subject to}\quad g_1(x) = 0 \tag{4.1b}$$
$$g_i(x) \geq 0 \qquad \forall\, i = 2, ..., 4 \tag{4.1c}$$
$$h(x) \geq 0 \tag{4.1d}$$
$$l_j^0 \leq x_j \leq u_j^0 \quad \forall\ j = 1, \ldots, 5. \tag{4.1e}$$

Here, the design variables $x_j \; \forall \; j = 1, ..., 5$ are assumed to be indexed in the order of diminishing priority with respect to their relative importance in the model, as reflected by the evident sensitivity of the objective and constraint functions to their selected values. To obtain these priority indices for the design variables, the effect of each design variable on the objective and constraint functions was studied. This was done by examining the objective and constraint functions as functions of each design variable, defined over its bounding interval, while the other design variables were kept fixed at certain nominal values within their respective ranges. Linear response surface fits were then constructed for the nonlinear objective and constraint functions over each separate dimension of the hyperrectangle of interest defined in Equation (4.3). For functions exhibiting a strict local minimum in the range, the design space was partitioned into two subregions, one on each side of the strict local minimum, and linear response surface fits were constructed for both regions. The slopes of the linear response surface fits were then computed. For functions where partitioned subregions of the design space were constructed, the average absolute value of the slope was used. The average absolute value of the resulting slopes of the objective and constraint functions with respect to a design variable were used to determine the relative importance, or the priority index, of that design variable in the model. Accordingly, the design variables were indexed as follows:

$$x_1 = T,$$
$$x_2 = D,$$
$$x_3 = Vk, \tag{4.2}$$

$x_4 \;=\;$ B, and

$x_5 \;=\;$ L.

The particular form of the nonlinear (and nonconvex) objective function $f(x)$ is given in Appendix A. The particular forms of the nonlinear (and nonconvex) constraint functions $g_i(x)$ for $i = 1,\ldots,4$ as designated in (4.1b) and (4.1c), i.e., the equality constraint on the weight and the displacement, and the inequality constraints on the metacentric height, the rolling period, and the minimum required freeboard, are given respectively in Appendices B, C and D, and in equation (3.86). The particular form of the linear constraint (4.1d) is highlighted separately for use in the sequel, and is given by Equation (3.83). The lower and upper bounds on the variables specified in (4.1e) are stated with a superscript of zero to denote that these are the initial root-node or node-zero interval bounds on the variables in the branch-and-bound enumeration framework that is described below. These initial bound values are given as follows:

$$\begin{aligned} 100 &\le x_1 \le 300 \\ 20 &\le x_2 \le 43 \\ 12 &\le x_3 \le 25 \\ 6 &\le x_4 \le 11 \\ 4 &\le x_5 \le 35. \end{aligned} \tag{4.3}$$

We will now explore two polynomial programming approximation approaches (denoted as PPA1 and PPA2) based on using the Response Surface Methodology (RSM), or curve-fitting procedure. Note that traditional RSM techniques simply consider an interval constrained nonlinear program, and employ only quadratic approximations to the objective function in order to ascertain whether a current solution is a local optimum, or else, to detect a search direction that leads to a new solution, and the procedure is reiterated (see Myers (1995), for example). However, this process is essentially memoryless. Sherali, Joshi, and Tew (1998) describe an enhanced conjugate gradient type of scheme within the context of an RSM-simulation-optimization framework that employs the previously generated information to derive more effective search directions. In the present context, because of the highly nonlinear and nonconvex functions in the problem that appear not only in the defining objective criterion, but also in the various constraints, we will adopt a different strategy. First, as discussed in the next section, we will employ fifth-order polynomial functions to construct approximations for (4.1a), (4.1b) and (4.1c), rather than simply use quadratic approximations. As we shall see in Section 4.2, this provides greatly improved response surface representations for the various underlying functions. Second, by exploiting available modern-day technology to effectively solve such polynomial programming problems (see Sahinidis (1996), and Sherali and Tuncbilek (1992, 1997)), we will design a (pseudo) global optimization scheme for solving Problem CSD.

The remainder of this chapter is organized as follows. Section 4.1 provides a brief introduction to the Design of Experiments. In Section 4.2, we present a brief discussion on Response Surface Methodology, and describe the mechanism and tools that we have employed for constructing fifth-order polynomial approximations for each of the functions in (4.1a), (4.1b) and (4.1c) over a defined hyperrectangle. Details of this approach are given subsequently in Section 4.3.

4.1 Design of Experiments

The Design of Experiments (DOE) theory provides a suitable means for selecting a set of points within the design space and performing analysis based on these points. Several types of experimental designs are available for selecting such points. The four commonly employed designs are:

4.1.1 3^m Full-Factorial Experimental Design

A 3^m full-factorial design is created by specifying a lower bound, a midpoint, and an upper bound for each of the m variables. A 3^3 experimental design is presented in Figure 4.1. As the number of design variables increases, the number of experiments required for the 3^m full-factorial design becomes prohibitively large.

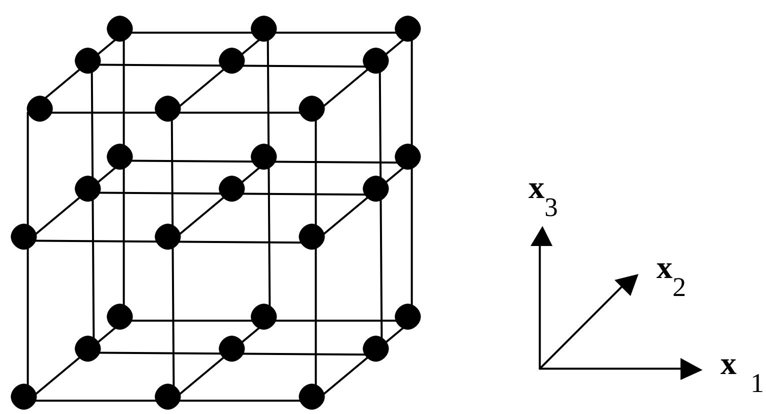


Figure 4.1 A 3^3 Full Factorial Design.

4.1.2 Face-Centered Central-Composite Design

The face-centered central composite design is created by taking a 2^m full-factorial design and adding 2m "star" points on the facets of the hypercube and another point in the middle of the design space. Figure 4.2 displays a face-centered central-composite design. Here too, as the number of design variables increases, the experimental designs become prohibitively large.

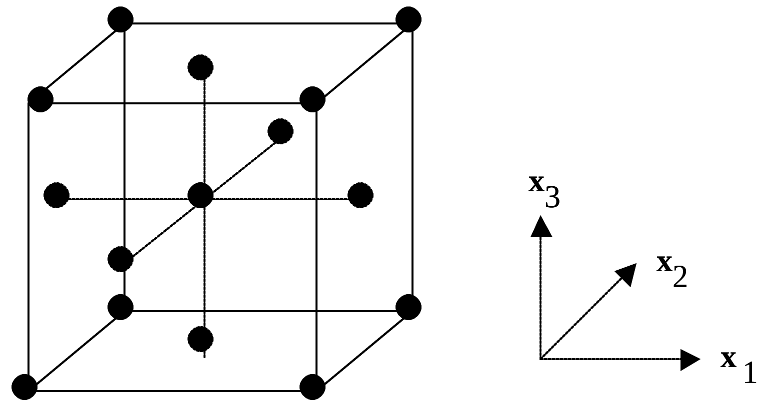


Figure 4.2 A Face-Centered Central Composite Design in Three Dimensions.

4.1.3 Small-Composite Design

The small-composite experimental design allows even fewer experiments. The design is similar to the central-composite design except that a fractional design is used in place of a 2^m full-factorial design. This means that certain vertices of the design space will not have any associated data.

4.1.4 D-Optimal Experimental Design

D-optimal designs permit a relatively greater degree of flexibility in selecting the points for conducting experimental analyses. These designs are also well suited for irregularly shaped feasible regions.

The fractional and composite experimental designs are geared toward rectangular design spaces. On the other hand, D-optimal designs minimize the uncertainty in the estimates of the polynomial coefficients and in the predicted value of the response, by using suitably placed evaluations, over more general design spaces.

4.1.5 General Factorial Design of Experiments and Analysis

To perform a general factorial design analysis, a user first selects a fixed number of "levels" for each variable "factor", and then runs experiments with all possible resulting combinations. If there are l_1 levels for the first variable, l_2 levels for the second variable, and l_k levels for the k^{th} variable, the complete experiment is an $l_1 \times l_2 \times \ldots \times l_k$ factorial design.

Often times, experiments are designed in which each variable is considered at only two levels. These are advantageous in the fact that they require relatively fewer runs, and although they do not fully explore a wide region in the factor space, they tend to at least indicate major trends and prescribe a promising direction for further experimentation. If a greater degree of local exploration is needed, the design may be suitably augmented to form composite designs.

4.2 Response Surface Approximations

In this section, we provide a brief introduction to Response Surface Methodology, and later, describe the mechanism and tools that we have employed for constructing fifth-order polynomial approximations for each of the functions in (4.1a), (4.1b) and (4.1b) over a defined hyperrectangle.

The use of Response Surface Methodology offers the following benefits:

1. They smooth out the numerical noise present in the analyses. This noise can distort gradient information and lead to artificial local minima in the design space when using an optimization approach.

2. They also allow the functional analysis codes to be separated from the optimization routines. This permits flexibility in applying parallel computing technologies.
3. By replacing complex functional relationships with simple polynomials, the designer can more readily obtain information on design trade-offs, sensitivities to certain variables, and insights into highly constrained, nonconvex design spaces.

Although this approach provides all the tools necessary to perform the RSM analyses, it is up to the designer to use these concepts in a structured approach toward deriving good model representations.

In the context of our problem, for the initial root-node, the hyperrectangle bounding the initial variables is given by (4.1d), and for subsequent node subproblems as discussed subsequently, we will be considering suitable partitioned sub-hyperrectangles of this initial region. Given any such hyperrectangle, we wish to determine a polynomial approximation for each of the objective and constraint functions.

Note that in general, a polynomial response surface model of order d, in n variables, is of the form

$$y = c_0 + \sum_{1 \le j_1 \le n} c_{j_1} x_{j_1} + \sum_{1 \le j_1 \le j_2 \le n} c_{j_1 j_2} x_{j_1} x_{j_2} + \; .... \; + \sum_{1 \le j_1 ... \le j_d \le n} c_{j_1 ... j_d} x_{j_1} ... x_{j_d} \,. \tag{4.4}$$

Furthermore, for a function of n variables, a polynomial approximation of order d will have

$$1 \qquad + \qquad \sum_{t=1}^{d} \binom{n+t-1}{t} \equiv \qquad \binom{n+d}{d} \tag{4.5}$$

terms, including a constant value. For n = 5 and d = 5 as in our context for the objective function, the equality constraint on the weight and the displacement, and the inequality constraints on the metacentric height and the rolling period; and for n = 4 and d = 5 for the inequality constraint on the Freeboard, this yields 252 terms and 126 terms, respectively. However, several of these terms will have zero (insignificant) coefficients.

The methodology that we employed to construct such polynomial approximation response surfaces is as follows:

1. Perform a screening using a full factorial experimental design (FFD). We used at least twice the number of points as the number of terms in the model. This results in a full factorial design having 4 levels for n = 5 that yields 1024 points, and having 5 levels for n = 4 that yields 625 points.

2. Apply the geometric constraint on the length to depth ratio as described in (3.83) to eliminate infeasible designs.

3. Perform a regression analysis (we used a forward stepwise regression process with threshold values of 0.25 and 0.10, determined by statistical tests, for terms to enter and leave the model, respectively) to obtain the coefficients of the fitted polynomial response functions of order d =5.

This strategy for determining the polynomial approximation response surfaces was implemented using the *JMP* software from the SAS Institute in Cary, North Carolina.

For the initial root-node, this hyperrectangle is given by (4.1d), and a summary of the response surface approximation is provided in Table 4.1. For the sake of comparison, we provide in Table 4.2 the relevant results based on fitting lower order designs.

Table 4.1 Summary of the Response Surface Approximations for the Initial Root-Node.

Response	DOE Points		Response Terms	R^2
	Initial Full-Factorial Design	Geometrically Feasible Points		
$f(x)$	1024	706	117	0.9941
$g_1(x)$	1024	706	134	0.9984
$g_2(x)$	1024	706	133	0.9971
$g_3(x)$	1024	434	29	0.9998
$g_4(x)$	625	536	150	0.9661

4.3 Proposed Polynomial Programming Approximation (PPA) Algorithms

As alluded above, we will design two approaches for solving the containership design problem CSD, based on employing a sequence of polynomial programming approximations, each within two alternative branch-and-bound frameworks. In the first method, abbreviated PPA1, for each subproblem corresponding to some node k in this branch-and-bound framework, starting with the initial root-node (k=0), we construct a polynomial programming approximation problem as discussed in the foregoing section. Let $\left[l_j^k, u_j^k\right] \; \forall j$ define the bounding intervals for the variables at this node k. The polynomial approximations are constructed over the hyperrectangle defined by these interval bounds. The resulting polynomial program is then solved to global optimality to yield a solution $\bar{x}^k$ of objective value z_k. For this purpose, we employed the software package BARON (Branch-and-Reduce Optimization Navigator - see Sahinidis, 1996), although the RLT-based procedure of Sherali and Tuncbilek (1992, 1997) offers a viable alternative (BARON itself incorporates some elements of the latter approach). The value z_k is assumed to represent a lower bound on the node subproblem. We found this to be usually the case, although this is not necessarily true in theory. (For this reason, we refer to such a process as a *pseudo-global optimization* method.) Using the resulting solution $\bar{x}^k$ as a starting solution, we next apply a nonlinear programming search method to the original problem CSD given by (4.1) in order to identify a local minimum. For this purpose, we employed the software package developed by Neu et al. (2000) as described in Chapter 2. The motivation for this is that if the polynomial approximating problem is a

Table 4.2 Goodness of Fit for the Response Surface Approximations.

	Order 5			Order 4			Order 3			Order 2		
Function	Response Terms	R^2 Values	DOE Points	Response Terms	R^2 Values	DOE Points	Response Terms	R^2 Values	DOE Points	Response Terms	R^2 Values	DOE Points
$f(x)$	117	0.9941	706	73	0.9759	706	25	0.9264	162	11	0.8587	42
$g_1(x)$	134	0.9984	706	79	0.9942	706	34	0.9942	162	15	0.9886	42
$g_2(x)$	133	0.9971	706	89	0.9932	706	32	0.9957	162	13	0.9899	42
$g_3(x)$	29	0.9998	434	23	0.9998	421	16	0.9999	84	10	0.9999	42
$g_4(x)$	150	0.9661	536	77	0.9046	536	26	0.9176	176	5	0.6296	30

sufficiently reliable representation of the true underlying problem over the subregion of concern, the solution $\bar{x}^k$ is likely to be a near-global optimum to the original problem over this region. Hence, refining $\bar{x}^k$ to a local optimum is likely to yield a good quality solution. The resulting solution x^{k*}, say, is used to update the incumbent solution x^* of objective value z^*, if necessary. If $z_k \geq z^*(1-\varepsilon)$, for some optimality tolerance $0 \leq \varepsilon < 1$ (we used $\varepsilon = 0.01$ in our computations), we fathom this node k. Otherwise, we partition the node k into two subnodes based on splitting the current interval (l_q^k, u_q^k) of a selected variable x_q at a suitable value x_q^k as prescribed below. Toward this end, we first define

$$x^k = \begin{cases} x^* & \text{if } x_j^* \in [l_j^k, u_j^k]\ \forall j, \text{ with } x_j^* \in (l_j^k, u_j^k) \text{ for some } j \\ \bar{x}^k & \text{otherwise}. \end{cases} \tag{4.6}$$

Next, we find the lowest indexed (highest priority) variable x_q for which $l_q^k < x_q^k < u_q^k$. Note that such an x_q must necessarily exist by definition in (4.6) in case $x^k \equiv x^*$. Furthermore, if $x^k = \bar{x}^k$, then such an x_q is likely to exist because at the feasible vertices of the hyperrectangle Ω^k that are included within our response surface designs, the polynomial approximations tend to be exact, whence node k would likely be fathomed in case $\bar{x}^k$ is a vertex of Ω^k. However, whenever such an x_q does not exist, and if there exists an index for which

$$l_q^k < x_q^* < u_q^k \tag{4.7}$$

we let q be the smallest such index. Else, failing this, we select q = 1. For either of the latter two cases, we define

$$x_q^k \equiv \begin{cases} 0.1 l_q^k + 0.9 u_q^k & \text{if } \bar{x}_q^k = u_q^k \\ 0.9 l_q^k + 0.1 u_q^k & \text{if } \bar{x}_q^k = l_q^k. \end{cases} \tag{4.8}$$

Accordingly, we now create two subnodes for node k by replacing the corresponding bounding interval $[l_j^k, u_j^k]$ for x_q by

$$l_q^k \leq x_q \leq x_q^k, \text{ and } x_q^k \leq x_q \leq u_q^k \tag{4.9}$$

in the two respective subproblems. Each of these subproblems is then analyzed similar to the parent node k. The motivation for this node partitioning or branching rule, abbreviated BR1, is that the approximations tend to be exact at the points defined by interval end-points of the variables. Hence, for a node subproblem, if the incumbent solution lies in the corresponding hyperrectangle, we attempt to generate subnodes whose end-points would match with the incumbent solution values, at least with respect to higher priority variables. The motivation for (4.8) is to perturb the current relaxation solution away from the vertex of Ω^k with respect to a critical index so as to encourage either detecting an improved incumbent, or fathoming this solution. Otherwise, the (pseudo) lower bounding mechanism is tightened by creating end-points at the corresponding approximating problem's solution with respect to the higher priority variables. In this overall process, we also maintain a list of active nodes L for which the (pseudo) lower bound is lesser than $z^*(1-\varepsilon)$. At any stage, we *extract* (i.e., select and remove) that node k from L, which has the least lower bound. If L = ϕ, or if we have encountered a total of some N_{max} nodes, we terminate the overall process, and prescribe the current incumbent solution for implementation. (In our computational runs, because of the prototypical nature of our procedure that employs disparate tools for constructing response surface approximations, solving polynomial programming approximations, and performing local searches, each node analysis required an intense manual interaction for implementing the foregoing steps. Hence, we have used N_{max} = 8. This value is further motivated by the fact that usually, only three or so variables are different from their bounds in the root-node approximation solution in the incumbent solution, and so, this at least provides the opportunity to explore the 2^3 = 8 subproblems based on a combination of subintervals, each split at the corresponding variables' non-end-point interval values.

The second node partitioning or branching rule that we explored, abbreviated BR2, follows the same procedure as branching rule BR1, with one difference. Each node k is partitioned into two subnodes based on splitting the current interval (l_q^k, u_q^k) of a selected variable x_q at a suitable value x_q^k as prescribed below. We first define

$$x^k \quad = \quad \begin{cases} x^* \text{ if } x_j^* \in \left[l_j^k, u_j^k\right] \forall\, j, \text{ with } x_j^* \in \left(l_j^k, u_j^k\right) \text{for some } j \\ \bar{x}^k \text{ otherwise ,if } \bar{x}_j^k \in \left(l_j^k\; u_j^k\right) \text{for some } j \\ \hat{x}_j \text{ otherwise , where } \hat{x}_j^k = 0.75\, u_j^k + 0.25\, l_j^k \text{ for } j, \end{cases} \tag{4.10}$$

and

$b_j^k =$ number of times x_j has been selected as the branching variable in the path from the root node to the node k in the enumeration tree thus far, $\forall\, j$.

Letting

$$q \in \arg \operatorname{lex} \min \left\{b_j^k : j=1,...,5\right\} \tag{4.11}$$

We create two subnodes from node k by replacing the corresponding bounding interval $\lfloor l_j^k, u_j^k \rfloor$ for x_q by

$$l_q^k \leq x_q \leq x_q^k, \text{ and } x_q^k \leq x_q \leq u_q^k. \tag{4.12}$$

The motivation for the modification implemented in BR2 via (4.11) is that it avoids selecting the same branching variable if another choice exists that has not been considered (or has not been considered as many times). Figure 4.3 provides a flowchart for this algorithmic framework. Note that the second proposed algorithmic scheme PPA2 follows the same overall procedure as identified in Figure 4.3, with one key difference. For any node k, k $\geq$ 0, having formulated the polynomial programming approximation PPk, in lieu of solving this polynomial program to optimality as in Figure 4.3, we instead construct an RLT-based tight linear programming (LP) relaxation RLT-PPk, say, for the problem PPk, and solve this LP relaxation to derive the solution $\overline{x}^k$ of objective value z_k. The remainder of this algorithm follows the schema of Figure 4.3. Note again that as shown in Sherali and Tuncbilek (1992), the RLT relaxation value matches that of the original polynomial program at the vertices of Ω^k, and therefore, the motivation for the branching index selection process remains the same. The key difference is that the (pseudo) lower bound is now being computed via the solution of a *linear* program RLT-PPk, as opposed to a *polynomial* program PPk. The motivation here is that if RLT-PPk is a tight relaxation of PPk, (which the results in Sherali and Tuncbilek (1997) anticipate to be the case), then we might save on computational effort without impairing the quality of the solution derived at termination of the proposed algorithm.

The derivation of RLT-PPk from PPk involves the following steps:

Reformulation

We first generate implied constraints using distinct products of bounding factors

$$(x_j - l_j) \geq 0, \ (u_j - x_j) \geq 0 \qquad \forall\, j \in N, \tag{4.13}$$

taken δ at a time, where δ is the maximum degree of any polynomial term appearing in PPk. These constraints are of the form

$$F_\delta(J_1, J_2) \equiv \prod_{j \in J_1} (x_j - l_j) \prod_{j \in J_2} (u_j - x_j) \geq 0, \tag{4.14}$$

where J_1 and J_2 contain indices (with possible repetitions) from N, and where $\left| J_1 \bigcup J_2 \right| = \delta.$ The number of distinct constraints of type (4.14) is given by

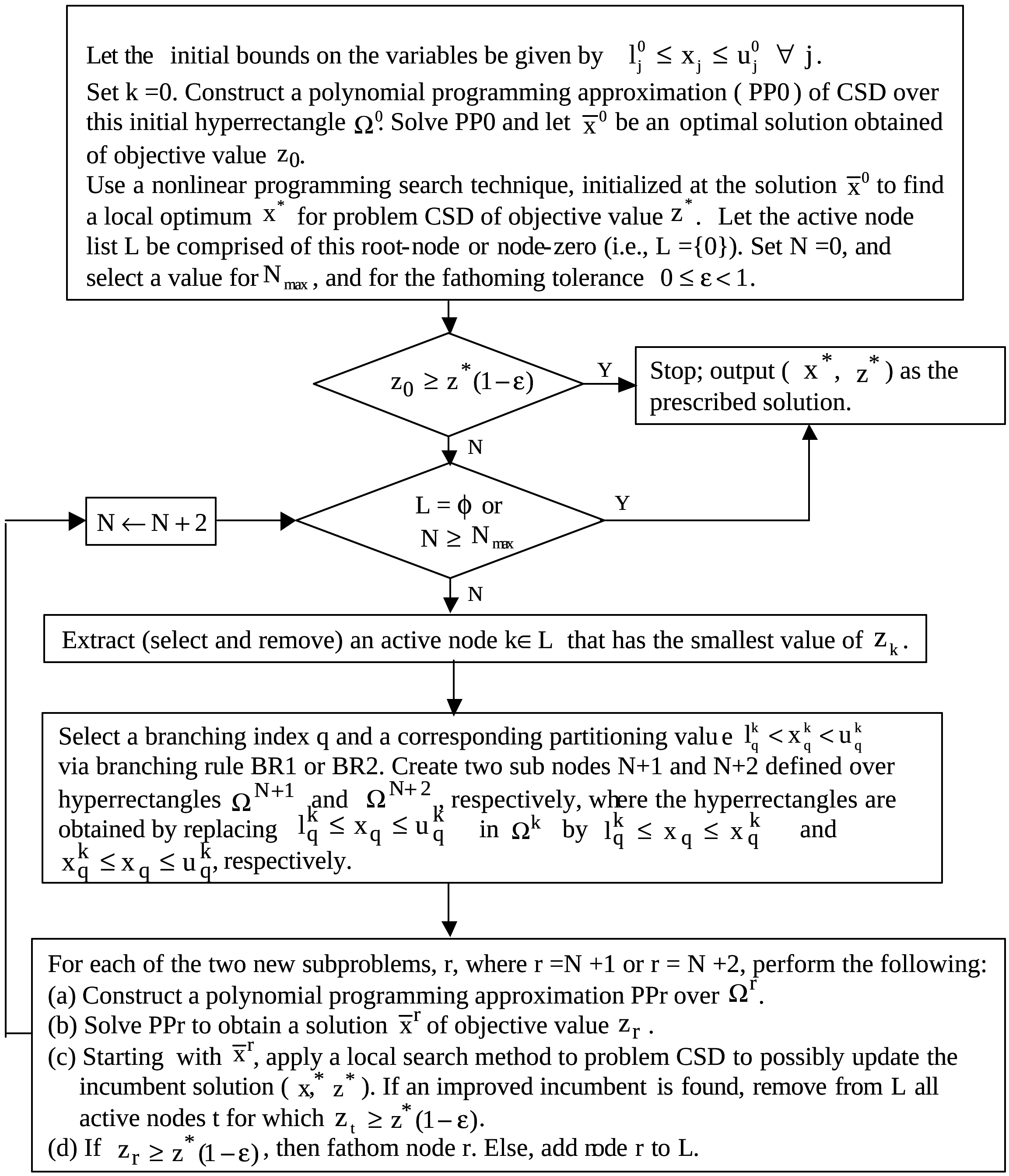


Figure 4.3 Flow–Chart for the Polynomial Programming Approximation Algorithm PPA1.

$$\binom{2n+\delta-1}{\delta}. \tag{4.15}$$

For our problem CSD, we have n = 5 and $\delta = 5$, and so, the corresponding number of constraints in (4.14) is 2002.

Linearization

After including the constraints given by (4.14) in PPk, we substitute

$$X_J = \prod_{j \in J} x_j \qquad \forall\, J, \tag{4.16}$$

where the indices in J are taken from N, with possible repetitions, and are assumed to be sequenced in nondecreasing order, and where $X_{\{j\}} \equiv x_j \; \forall\, j \in N$, and $X_\phi \equiv 1$. The number of X-variables defined here, besides $X_{\{j\}}$, $j \in N$, and X_ϕ is $\binom{n+\delta}{\delta} - (n+1)$. Note that each distinct set J produces one distinct X_J variable.

Chapter 5

Computational Results for the Proposed Pseudo-Global Optimization Approaches

In this chapter, we present computational results pertaining to applying the proposed pseudo-global optimization approaches to the containership design problem CSD as described in Chapter 4. Section 5.1 presents results for Algorithm PPA1. Section 5.2 presents results for Algorithm PPA2. Thereafter, Section 5.3 presents results for the nonlinear programming approaches, followed by a comparison of results obtained from the pseudo-global optimization approaches and the nonlinear optimization methods as given in Section 5.4. Breakdowns of the weights and the various costs pertaining to the optimal design are also provided in Section 5.4.

5.1 Results for Algorithm PPA1

Tables 5.1 and 5.2 detail the stepwise results when applying Algorithm PPA1-BR1 and Algorithm PPA1-BR2, respectively. All symbols are consistent with the descriptions as provided in Chapter 4. As seen from the table, the pseudo-global optimal solutions obtained from Algorithms PPA1-BR1 and PPA1-BR2 are consistent. This solution is as follows:

$$x^* = \begin{bmatrix} 300.00 \\ 43.00 \\ 16.60 \\ 10.50 \\ 18.40 \end{bmatrix}$$

$$z^* = 0.001026.$$

The only constraint active at this solution is the mandatory constraint on the displacement and the weight. When the approach PPA1 terminates, it coincidentally meets both the termination criteria of enumerating $N_{max} = 8$ nodes, as well as having the set of active nodes $L = \phi$.

Some important design parameters corresponding to the optimal containership design obtained from Algorithm PPA1 are delineated in Table 5.3.

Table 5.3 Design Parameters Obtained from Algorithm PPA1.

Containership Design Parameter	Value
Carrying Capacity	5,716 TEUs
Number of Roundtrips made Annually	17.06
Annual Transportation Capacity	97,514.96 TEUs
Dead Weight	1,182,929.0 Metric tons
Gross Weight	1,593,248.8 Metric tons
Return on Investment	80.83 %
Cost to Owner	$ 54,334,781.11

5.2 Results for Algorithm PPA2

Tables 5.4 and 5.5 detail the stepwise results when applying Algorithm PPA2-BR1 and Algorithm PPA2-BR2 respectively. All symbols are consistent with the descriptions as provided in Chapter 4.

As seen from the table, the pseudo-global optimal solutions obtained from Algorithms PPA2-BR1 and PPA2-BR2 are again consistent. This solution is as follows:

$$x^* = \begin{bmatrix} 300.00 \\ 43.00 \\ 16.64 \\ 10.50 \\ 18.62 \end{bmatrix}$$

$$z^* = 0.001026.$$

The only constraint active at this solution is the mandatory constraint on the displacement and the weight. When the approach PPA2 terminates, it coincidentally meets both the termination criteria of enumerating $N_{max} = 8$ nodes, as well as having the set of active nodes $L = \phi$.

Some important design parameters corresponding to the optimal containership design obtained from Algorithm PPA2 are delineated in Table 5.6.

Table 5.6 Design Parameters Obtained from Algorithm PPA2.

Containership Design Parameter	Value
Carrying Capacity	5,713 TEUs
Number of Roundtrips made Annually	17.21
Annual Transportation Capacity	98,450.73 TEUs
Dead Weight	1,193,014.4 Metric tons
Gross Weight	1,593,473.9 Metric tons
Return on Investment	83.55 %
Cost to Owner	$ 54,403,474.10

5.3 Results for the Nonlinear Optimization Algorithms

The previous best-known solution obtained via the nonlinear optimization algorithms included in the software program developed by Neu et al. (2000) as described in Chapter 2 was:

$$x^* = \begin{bmatrix} 211.19 \\ 40.90 \\ 17.00 \\ 10.00 \\ 16.70 \end{bmatrix}$$

$$z^* = 0.001250.$$

The constraints active at this solution are the mandatory constraint on the displacement and the weight. Note that this was the best solution detected by using *three nonlinear programming search algorithms*, viz., Modified Method of Feasible Directions (MMFD), Sequential Linear Programming (SLP) and Sequential Quadratic Programming (SQP) as detailed in Sections 2.2.1, 2.2.2 and 2.2.3 respectively, *each initialized at three points in the design space*, viz., the lower bound (LB), the mid-point (M. Pt.) and the upper bound (UB) of the design space, and optimized until local convergence was obtained. Details of this implementation are provided as Cases 1, 2 and 3 in Table 5.7.

Table 5.7 Solution Details Employing Nonlinear Programming Algorithms.

Case No.	Method	St. Pt.	Loa	B	D	T	Vk	Obj: RFR	IT	TC
Case 1: Using MMFD and Starting from the LB, M.Pt. and UB in the Design Space.										
1.1	MMFD	LB	107.3	29.3	12.2	7.0	4.3	0.005579	9	A, B
1.2	MMFD	M.Pt.	211.9	40.9	17.0	10.0	16.7	0.001254	6	A
1.3	MMFD	UB	299.8	37.2	16.8	10.6	32.9	0.002000	4	A
Case 2: Using SLP and Starting from the LB, M.Pt. and UB in the Design Space.										
2.1	SLP	LB	107.3	29.3	12.2	7.0	4.3	0.005579	9	C
2.2	SLP	M.Pt.	211.9	40.9	17.0	10.0	16.7	0.001254	6	C
2.3	SLP	UB	299.8	37.2	16.8	10.6	32.9	0.002000	4	C
Case 3: Using SQP and Starting from the LB, M.Pt. and UB in the Design Space.										
3.1	SQP	LB	107.3	29.3	12.2	7.0	4.3	0.005579	9	C
3.2	SQP	M.Pt.	211.9	40.9	17.0	10.0	16.7	0.001254	6	C
3.3	SQP	UB	299.8	37.2	16.8	10.6	32.9	0.002000	4	C
Case 4. Starting from the Best Point identified by MMFD										
4.1	MMFD	1.2	211.9	40.9	17.0	10.0	16.7	0.001250	1	C
4.2	SLP	1.2	211.9	40.9	17.0	10.0	16.7	0.001250	1	C
4.3	SQP	1.2	211.9	40.9	17.0	10.0	16.7	0.001250	1	C
Case 5. Starting from the Best Point identified by SLP										
5.1	MMFD	2.2	211.9	40.9	17.0	10.0	16.7	0.001250	1	C
5.2	SLP	2.2	211.9	40.9	17.0	10.0	16.7	0.001250	1	C
5.3	SQP	2.2	211.9	40.9	17.0	10.0	16.7	0.001250	1	C
Case 6. Starting from the Best Point identified by SQP										
6.1	MMFD	3.2	211.9	40.9	17.0	10.0	16.7	0.001250	1	C
6.2	SLP	3.2	211.9	40.9	17.0	10.0	16.7	0.001250	1	C
6.3	SQP	3.2	211.9	40.9	17.0	10.0	16.7	0.001250	1	C

The symbolic abbreviations used in Table 5.7 are explained below:

IT: Number of iterations
TC: Termination Criteria
- A: Relative convergence criteria (0.001) met for 2 consecutive iterations
- B: Absolute convergence criteria (0.0001) met for 2 consecutive iterations
- C: Maximum KKT residual = 0.0000, less than the specified tolerance (0.001)

Cases 4, 5 and 6 in Table 5.7 indicate the results obtained by using the best solution *identified by each of the three methods as a starting point* and optimizing until local

convergence, again, *using each of the three methods*. Thus, the solutions given by cases
4, 5 and 6, which are consistent, is the best solution obtained using this implementation.
The mandatory constraint on the displacement-weight equality is active at this solution
for all cases. Some important design parameters corresponding to the best containership
design obtained from the nonlinear optimization methods are delineated in Table 5.8.

Table 5.8 Design Parameters Obtained from the Nonlinear Optimization Methods.

Containership Design Parameter	Value
Carrying Capacity	3,591 TEUs
Number of Roundtrips made Annually	16.58
Annual Transportation Capacity	59,538.78 TEUs
Dead Weight	704,207.31 Metric tons
Gross Weight	976,494.02 Metric tons
Return on Investment	64.58 %
Cost to Owner	$ 39,968,975.88

5.4 Comparison of the Results Obtained from the Proposed Pseudo-Global Optimization Approaches and the Nonlinear Optimization Methods

Comparing the solution obtained from Algorithms PPA1 and PPA2, and the nonlinear optimization methods, we see a significant improvement in the design when using the pseudo-global optimization approaches. Algorithms PPA1 and PPA2 result in a 17.92 % improvement in the objective function value. Recall that the objective function used is the required freight rate expressed in dollars per metric ton per nautical mile. For a typical containership of the size pertaining to this test case, having a gross weight of 90,000 metric tons, an annual transportation capacity of 99,000 containers corresponding to an annual deadweight of 1,188,000 metric tons, and logging 119,000 nautical miles annually, this translates to an annual estimated savings of $ 1,862,784 (or approximately $ 1.86 million) and an estimated 27 % increase in the return on investment over the life of the ship. The translation of this improvement to some other important design parameters at the prescribed solution is provided in Table 5.9.

Figures 5.1 through 5.5 show the breakdowns for the weights and the costs for the containership design obtained from Algorithm PPA1 (the breakdowns for the containership design obtained from Algorithm PPA2 would be similar since the two designs are almost similar).

Notice that although the objective function value obtained from Algorithms PPA1 and PPA2 are similar, the ship design pertaining to the latter algorithm is slightly larger in the Depth dimension, and somewhat faster. This increase in ship size is not significant

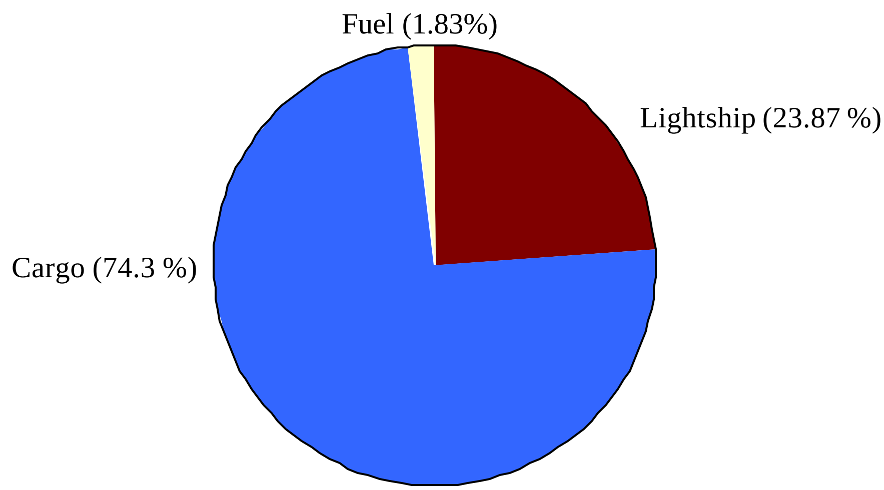


Figure 5.1 Breakdown of Weight.

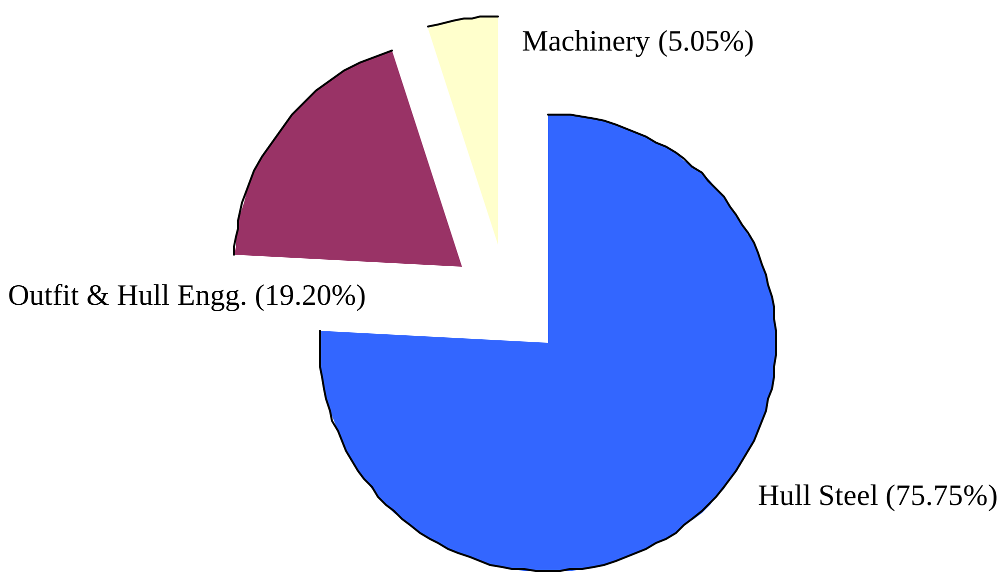


Figure 5.2 Breakdown of Lightship Weight.

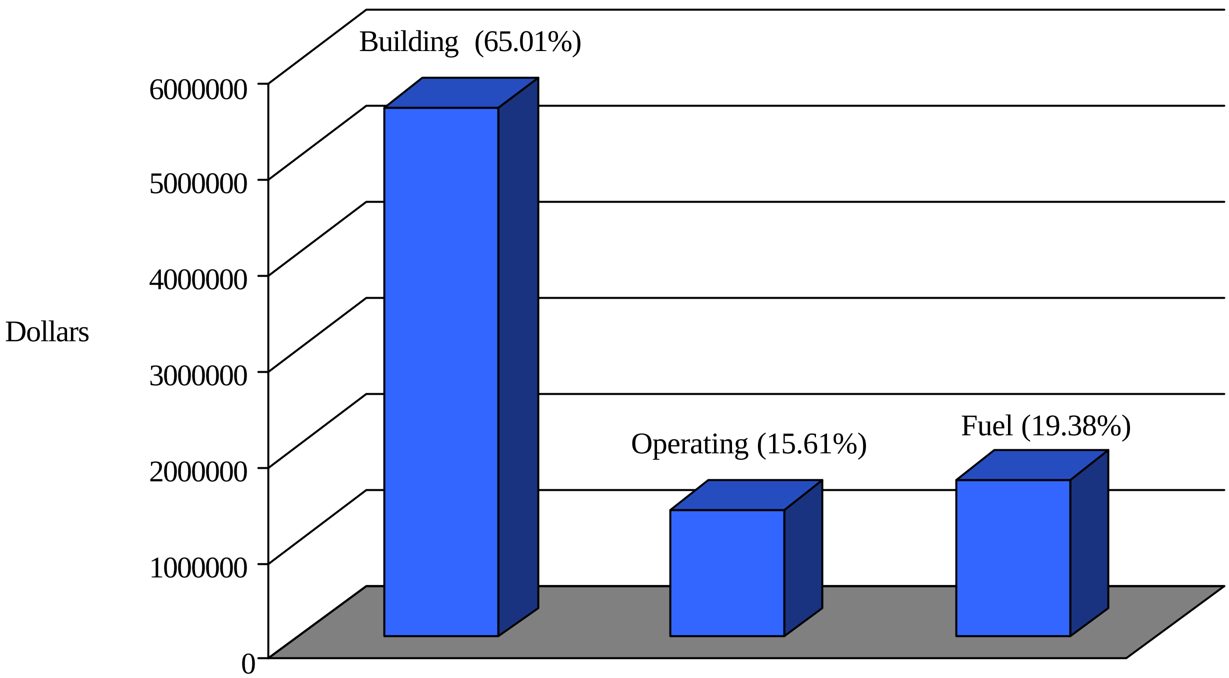


Figure 5.3 Breakdown of Annual Costs.

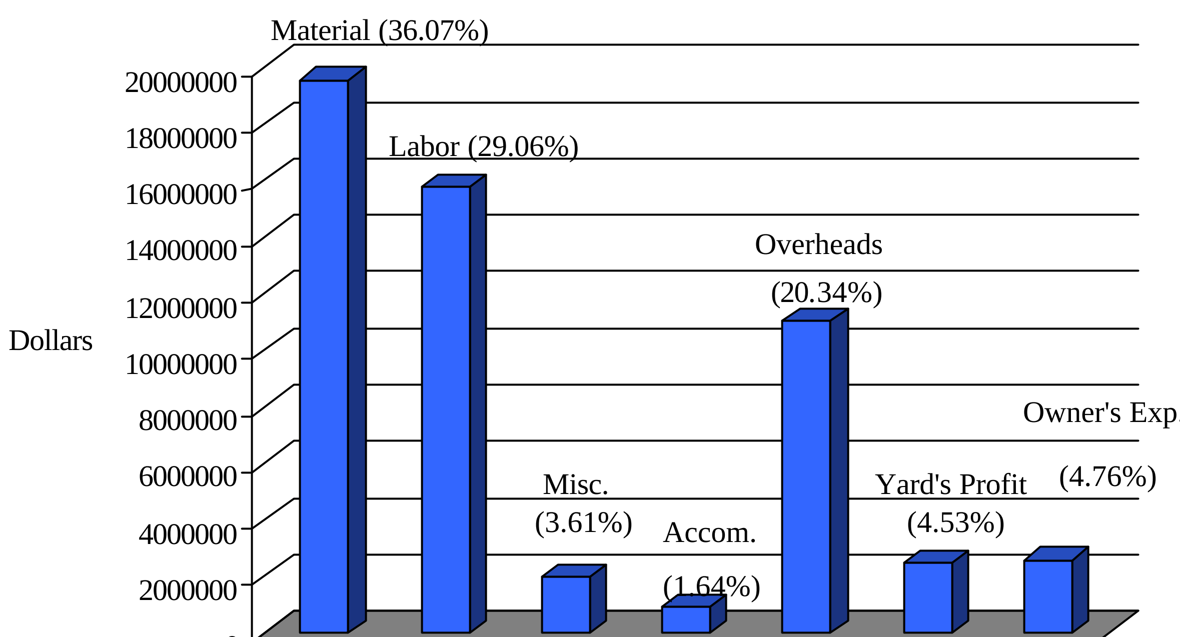


Figure 5.4 Breakdown of Annual Building Costs.

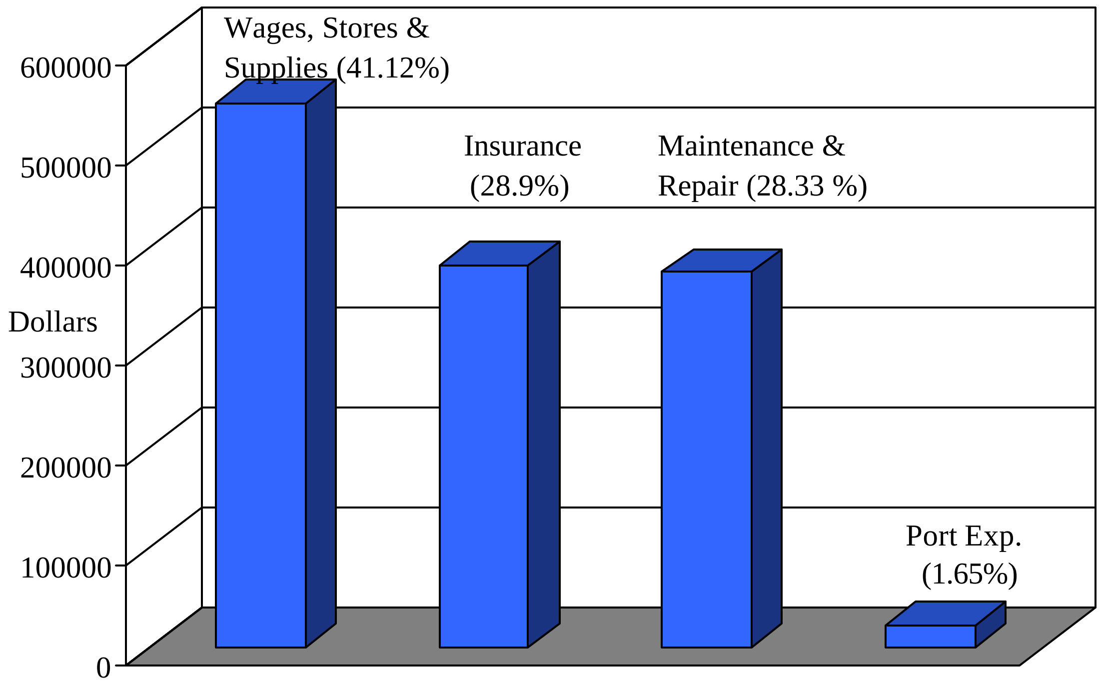


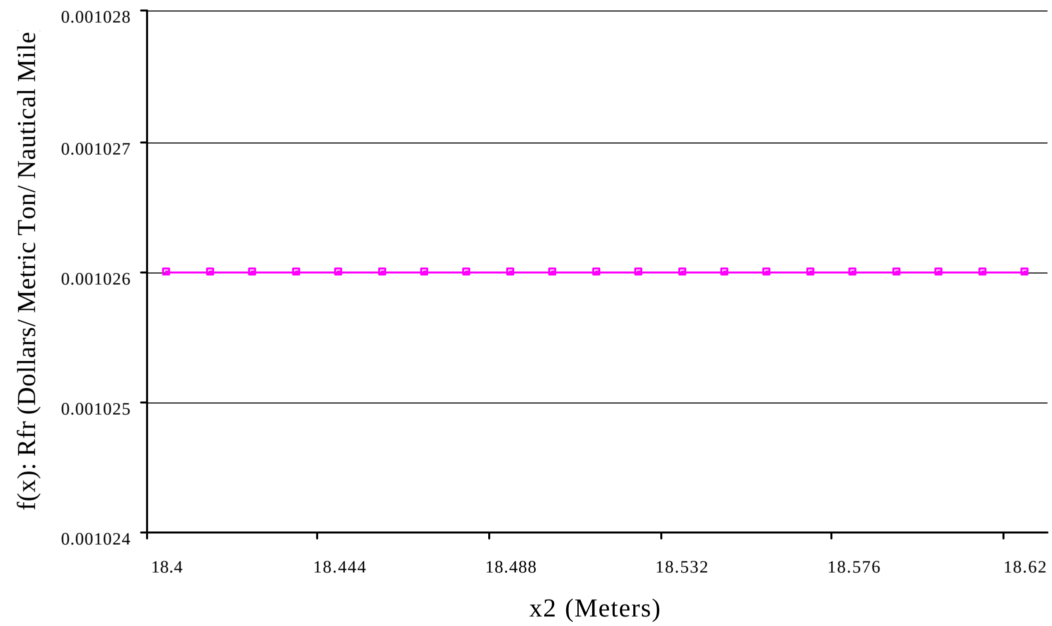
Figure 5.5 Breakdown of Operating Costs.

enough to cause an increase in the carrying capacity, but the increase in speed does enable the ship to make more roundtrips or log more nautical miles annually.

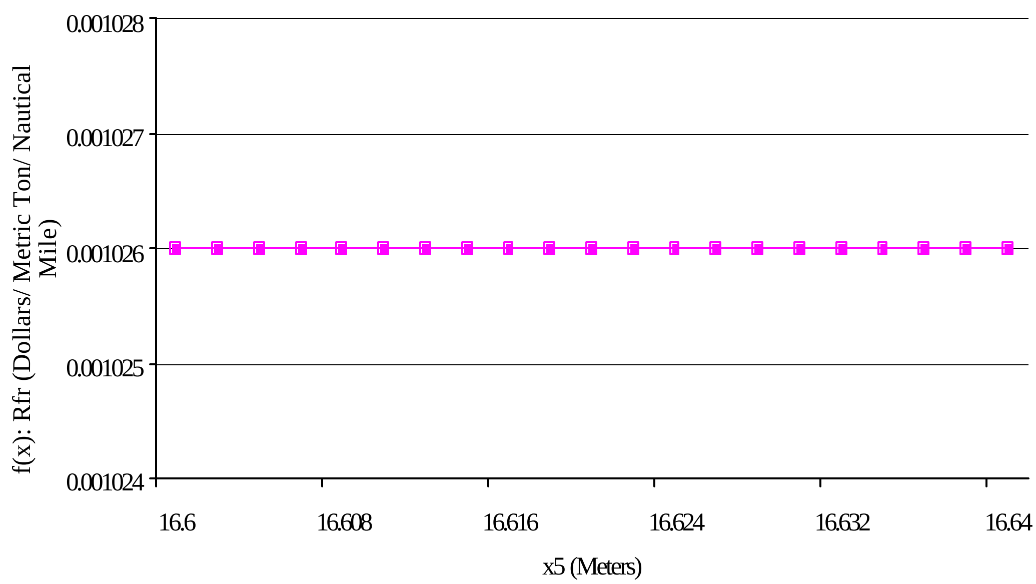
Figure 5.6 provides a representation of the design space between the optimal solutions identified by Algorithms PPA1 and PPA2. It can be seen that the design space is flat in this region, indicating that the designer is not confined to a severely restricted design space and has some nominal freedom in designing the ship.

Table 5.9 Comparison of Design Parameters Pertaining to the Proposed Pseudo-Global Optimization Approaches and the Nonlinear Optimization Methods.

Improvement in Containership Design Parameter	Value	
	PPA1	PPA2
Required Freight Rate (% decrease)	17.92	17.92
Carrying Capacity (increase in TEUs)	2,125	2,122
Number of Roundtrips made Annually/ Annual Nautical Miles Logged (increase)	0.48/ 3,360	0.63/ 4,410
Annual Transportation Capacity (increase in TEUs)	37,976	38,911
Return on Investment (% increase)	25.16	29.37



(a): Representation of the Design Space in the Depth Dimension.



(b): Representation of the Design Space in the Speed Dimension.

Figure 5.6 Representation of the Design Space between the Designs Obtained from Algorithm PPA1 and Algorithm PPA2.

Details of the computational statistics obtained using the proposed pseudo-global optimization approaches are provided in Table 5.10 (a). Table 5.10 (b) summarizes these results.

The symbolic abbreviations used in Table 5.10 are as follows:

IT: Total number of iterations per node in the algorithm enumeration tree for the given algorithm (PPA1 or PPA2) and the given branching strategy (BR1 or BR2).
Time: CPU times in seconds measured by internal time clocks.

* The times are increased estimates of the actual CPU times since the runs were made by connecting remotely from a Sun Ultra 5 workstation with Solaris 8 operating system to a Sun Ultra 1 workstation Solaris 2.5.1 operating system over a T1 network connection.

Table 5.10 (a) Details of the Computational Statistics Pertaining to the Proposed Pseudo-Global Optimization Approaches.

	PPA1								PPA2							
	BR1				BR2				BR1				BR2			
	LB		UB		LB		UB		LB		UB		LB		UB	
	IT	Time	IT	Time	IT	Time	IT	Time	IT	Time*	IT	Time	IT	Time*	IT	Time
0	1081	20	6	71	1081	20	6	71	29	40	6	71	29	40	6	71
1	411	20	4	49	411	20	4	49	21	40	6	71	21	40	6	71
2	388	20	4	49	388	20	4	49	21	40	4	50	21	40	4	50
3	390	20	2	21	390	20	5	60	21	40	5	50	20	40	4	60
4	290	20	5	60	310	20	4	49	20	40	4	48	21	40	5	61
5	380	20	2	22	400	20	4	49	22	40	4	49	24	40	4	49
6	406	20	4	49	390	20	3	33	20	40	3	32	20	40	4	49
7	4431	35	2	20	4941	40	2	20	1	40	2	20	1	40	2	20
8	2981	40	2	20	3040	40	2	20	1	40	2	20	1	40	2	20

Table 5.10 (b) Summary of the Computational Statistics Pertaining to the Proposed Pseudo-Global Optimization Approaches.

Average Time per Node	PPA1	PPA2
Lower Bounding Scheme	24.17	40
Upper Bounding Scheme	42.28	47.89

Table 5.1 Results for Algorithm PPA1-BR1.

Node (k)	$\Omega^k : \begin{bmatrix} l_j^k & u_j^k \end{bmatrix}$	$\bar{x}^k$	z_k	x^*	z^*	L	x^k	(q, x_q^k)	Updated x^*	Updated z^*	Constraints Active at x^*
0	$\begin{bmatrix} 6 & 11 \\ 12 & 25 \\ 4 & 35 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 8.22 \\ 12.00 \\ 4.00 \\ 122.71 \\ 32.38 \end{bmatrix}$	-1.21985×10^{-8}	$\begin{bmatrix} 8.10 \\ 12.00 \\ 4.80 \\ 131.90 \\ 41.00 \end{bmatrix}$	0.003105	{0}	$\begin{bmatrix} 8.10 \\ 12.00 \\ 4.80 \\ 131.90 \\ 41.00 \end{bmatrix}$	(1, 8.10)	$\begin{bmatrix} 8.10 \\ 12.00 \\ 4.80 \\ 131.90 \\ 41.00 \end{bmatrix}$	0.003105	Displ. = Wt.
1	$\begin{bmatrix} 6 & 8.1 \\ 12 & 25 \\ 4 & 35 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 8.04 \\ 13.05 \\ 9.13 \\ 171.09 \\ 33.53 \end{bmatrix}$	0.0034058	$\begin{bmatrix} 8.10 \\ 13.60 \\ 10.10 \\ 174.90 \\ 36.70 \end{bmatrix}$	0.001823				$\begin{bmatrix} 8.10 \\ 13.60 \\ 10.10 \\ 174.90 \\ 36.70 \end{bmatrix}$	0.001823	Displ. = Wt.
2	$\begin{bmatrix} 8.1 & 11 \\ 12 & 25 \\ 4 & 35 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 11.00 \\ 15.52 \\ 17.73 \\ 138.00 \\ 43.00 \end{bmatrix}$	-0.0027008	$\begin{bmatrix} 10.00 \\ 15.50 \\ 16.10 \\ 138.00 \\ 43.00 \end{bmatrix}$	0.001338	{2}	$\begin{bmatrix} 10.00 \\ 15.50 \\ 16.10 \\ 138.00 \\ 43.00 \end{bmatrix}$	(1, 10.0)	$\begin{bmatrix} 10.00 \\ 15.50 \\ 16.10 \\ 138.00 \\ 43.00 \end{bmatrix}$	0.001338	Displ. = Wt, Troll $\geq$ Troll_min
3	$\begin{bmatrix} 8.1 & 10 \\ 12 & 25 \\ 4 & 35 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 10.00 \\ 15.50 \\ 16.10 \\ 138.00 \\ 43.00 \end{bmatrix}$	0.129006	$\begin{bmatrix} 10.00 \\ 15.50 \\ 16.10 \\ 138.00 \\ 43.00 \end{bmatrix}$	0.001338						

Table 5.1 Results for Algorithm PPA1-BR1 (continued).

Node (k)	$\Omega^k : \left[l_j^k \quad u_j^k \right]$	$\bar{x}^k$	z_k	x^*	z^*	L	x^k	(q, x_q^k)	Updated x^*	Updated z^*	Constraints Active at x^*
4	$\begin{bmatrix} 10 & 11 \\ 12 & 25 \\ 4 & 35 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 11.00 \\ 16.58 \\ 4.00 \\ 273.97 \\ 43.00 \end{bmatrix}$	-0.093989	$\begin{bmatrix} 10.50 \\ 16.90 \\ 4.10 \\ 274.20 \\ 43.00 \end{bmatrix}$	0.002809	{4}	$\begin{bmatrix} 10.00 \\ 15.50 \\ 16.10 \\ 138.00 \\ 43.00 \end{bmatrix}$	(2, 15.5)			
5	$\begin{bmatrix} 10 & 11 \\ 12 & 15.5 \\ 4 & 35 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 10.00 \\ 15.50 \\ 16.10 \\ 138.00 \\ 43.00 \end{bmatrix}$	0.719958	$\begin{bmatrix} 10.00 \\ 15.50 \\ 16.10 \\ 138.00 \\ 43.00 \end{bmatrix}$	0.001338						
6	$\begin{bmatrix} 10 & 11 \\ 15.5 & 25 \\ 4 & 35 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 10.50 \\ 16.19 \\ 18.30 \\ 300.00 \\ 43.00 \end{bmatrix}$	0.00070017	$\begin{bmatrix} 10.50 \\ 16.60 \\ 18.40 \\ 300.00 \\ 43.00 \end{bmatrix}$	0.001026	{6}	$\begin{bmatrix} 10.50 \\ 16.60 \\ 18.40 \\ 300.00 \\ 43.00 \end{bmatrix}$	(1, 10.5)	$\begin{bmatrix} 10.50 \\ 16.60 \\ 18.40 \\ 300.00 \\ 43.00 \end{bmatrix}$	0.001026	Displ. = Wt.
7	$\begin{bmatrix} 10 & 10.5 \\ 15.5 & 25 \\ 4 & 35 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 10.50 \\ 16.60 \\ 18.40 \\ 300.00 \\ 43.00 \end{bmatrix}$	0.00166342	$\begin{bmatrix} 10.50 \\ 16.60 \\ 18.40 \\ 300.00 \\ 43.00 \end{bmatrix}$	0.001026						

Table 5.1 Results for Algorithm PPA1-BR1 (concluded).

Node (k)	$\Omega^k : \lfloor l_j^k \quad u_j^k \rfloor$	$\bar{x}^k$	z_k	x^*	z^*	L	x^k	(q, x_q^k)	Updated x^*	Updated z^*	Constraints Active at x^*
8	$\begin{bmatrix} 10.5 & 11 \\ 15.5 & 25 \\ 4 & 35 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 10.50 \\ 16.60 \\ 18.40 \\ 300.00 \\ 43.00 \end{bmatrix}$	0.8645319	$\begin{bmatrix} 10.50 \\ 16.60 \\ 18.40 \\ 300.00 \\ 43.00 \end{bmatrix}$	0.001026	$\{\phi\}$					

Table 5.2 Results for Algorithm PPA1-BR2.

Node (k)	$\Omega^k : \begin{bmatrix} l_j^k & u_j^k \end{bmatrix}$	$\bar{x}^k$	z_k	x^*	z^*	L	x^k	(q, x_q^k)	Updated x^*	Updated z^*	Constraints Active at x^*
0	$\begin{bmatrix} 6 & 11 \\ 12 & 25 \\ 4 & 35 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 8.22 \\ 12.00 \\ 4.00 \\ 122.71 \\ 32.38 \end{bmatrix}$	-1.21985×10^{-8}	$\begin{bmatrix} 8.10 \\ 12.00 \\ 4.80 \\ 131.90 \\ 41.00 \end{bmatrix}$	0.003105	{0}	$\begin{bmatrix} 8.10 \\ 12.00 \\ 4.80 \\ 131.90 \\ 41.00 \end{bmatrix}$	(1, 8.10)	$\begin{bmatrix} 8.10 \\ 12.00 \\ 4.80 \\ 131.90 \\ 41.00 \end{bmatrix}$	0.003105	Displ. = Wt.
1	$\begin{bmatrix} 6 & 8.1 \\ 12 & 25 \\ 4 & 35 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 8.04 \\ 13.05 \\ 9.13 \\ 171.09 \\ 33.53 \end{bmatrix}$	0.0034058	$\begin{bmatrix} 8.10 \\ 13.60 \\ 10.10 \\ 174.90 \\ 36.70 \end{bmatrix}$	0.001823				$\begin{bmatrix} 8.10 \\ 13.60 \\ 10.10 \\ 174.90 \\ 36.70 \end{bmatrix}$	0.001823	Displ. = Wt.
2	$\begin{bmatrix} 8.1 & 11 \\ 12 & 25 \\ 4 & 35 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 11.00 \\ 15.52 \\ 17.73 \\ 138.00 \\ 43.00 \end{bmatrix}$	-0.0027008	$\begin{bmatrix} 10.00 \\ 15.50 \\ 16.10 \\ 138.00 \\ 43.00 \end{bmatrix}$	0.001338	{2}	$\begin{bmatrix} 10.00 \\ 15.50 \\ 16.10 \\ 138.00 \\ 43.00 \end{bmatrix}$	(2, 15.5)	$\begin{bmatrix} 10.00 \\ 15.50 \\ 16.10 \\ 138.00 \\ 43.00 \end{bmatrix}$	0.001338	Displ. = Wt, Troll ≥ Troll_min
3	$\begin{bmatrix} 8.1 & 11 \\ 12 & 15.5 \\ 4 & 35 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 9.60 \\ 12.05 \\ 15.22 \\ 100.00 \\ 43.00 \end{bmatrix}$	0.00241981	$\begin{bmatrix} 9.80 \\ 14.90 \\ 15.60 \\ 132.60 \\ 43.00 \end{bmatrix}$	0.001352						

Table 5.2 Results for Algorithm PPA1-BR2 (continued).

Node (k)	$\Omega^k : [l_j^k \quad u_j^k]$	$\bar{x}^k$	z_k	x^*	z^*	L	x^k	(q, x_q^k)	Updated x^*	Updated z^*	Constraints Active at x^*
4	$\begin{bmatrix} 8.1 & 11 \\ 15.5 & 25 \\ 4 & 35 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 9.39 \\ 15.50 \\ 18.27 \\ 300.00 \\ 36.05 \end{bmatrix}$	0.00099331	$\begin{bmatrix} 9.30 \\ 15.50 \\ 18.30 \\ 300.00 \\ 39.20 \end{bmatrix}$	0.001161	{4}	$\begin{bmatrix} 9.30 \\ 15.50 \\ 18.30 \\ 300.00 \\ 39.20 \end{bmatrix}$	(3, 18.3)	$\begin{bmatrix} 9.30 \\ 15.50 \\ 18.30 \\ 300.00 \\ 39.20 \end{bmatrix}$	0.001161	Displ. = Wt.
5	$\begin{bmatrix} 8.1 & 11 \\ 15.5 & 25 \\ 4 & 18.3 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 8.80 \\ 15.50 \\ 8.45 \\ 128.65 \\ 34.50 \end{bmatrix}$	0.00842600	$\begin{bmatrix} 8.80 \\ 15.50 \\ 8.45 \\ 128.65 \\ 34.50 \end{bmatrix}$	0.002142						
6	$\begin{bmatrix} 8.1 & 11 \\ 15.5 & 25 \\ 18.3 & 35 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 11.00 \\ 16.19 \\ 18.30 \\ 300.00 \\ 43.00 \end{bmatrix}$	-0.0001447	$\begin{bmatrix} 10.50 \\ 16.60 \\ 18.40 \\ 300.00 \\ 43.00 \end{bmatrix}$	0.001026	{6}	$\begin{bmatrix} 10.50 \\ 16.60 \\ 18.40 \\ 300.00 \\ 43.00 \end{bmatrix}$	(1, 10.5)	$\begin{bmatrix} 10.50 \\ 16.60 \\ 18.40 \\ 300.00 \\ 43.00 \end{bmatrix}$	0.001026	Displ. = Wt.
7	$\begin{bmatrix} 8.1 & 10.5 \\ 15.5 & 25 \\ 18.3 & 35 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 10.50 \\ 16.60 \\ 18.40 \\ 300.00 \\ 43.00 \end{bmatrix}$	1.601800	$\begin{bmatrix} 10.50 \\ 16.60 \\ 18.40 \\ 300.00 \\ 43.00 \end{bmatrix}$	0.001026						

Table 5.2 Results for Algorithm PPA1-BR2 (concluded).

Node (k)	$\Omega^k : \begin{bmatrix} l_j^k & u_j^k \end{bmatrix}$	$\bar{x}^k$	z_k	x^*	z^*	L	x^k	(q, x_q^k)	Updated x^*	Updated z^*	Constraints Active at x^*
8	$\begin{bmatrix} 10.5 & 11.0 \\ 15.5 & 25 \\ 18.3 & 35 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 10.50 \\ 16.60 \\ 18.40 \\ 300.00 \\ 43.00 \end{bmatrix}$	1.579800	$\begin{bmatrix} 10.50 \\ 16.60 \\ 18.40 \\ 300.00 \\ 43.00 \end{bmatrix}$	0.001026	$\{\phi\}$					

Table 5.4 Results for Algorithm PPA2-BR1.

Node (k)	$\Omega^k : \left[l_j^k \quad u_j^k\right]$	$\overline{x}^k$	z_k	x^*	z^*	L	x^k	(q, x_q^k)	Updated x^*	Updated z^*	Constraints Active at x^*
0	$\begin{bmatrix} 6 & 11 \\ 12 & 25 \\ 4 & 35 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 8.07 \\ 12.05 \\ 19.50 \\ 100.00 \\ 28.67 \end{bmatrix}$	- 0.0119694	$\begin{bmatrix} 7.80 \\ 13.20 \\ 15.00 \\ 123.00 \\ 34.90 \end{bmatrix}$	0.001869	{0}	$\begin{bmatrix} 7.80 \\ 13.20 \\ 15.00 \\ 123.00 \\ 34.90 \end{bmatrix}$	(1, 7.80)	$\begin{bmatrix} 7.80 \\ 13.20 \\ 15.00 \\ 123.00 \\ 34.90 \end{bmatrix}$	0.001869	Displ. = Wt.
1	$\begin{bmatrix} 6 & 7.8 \\ 12 & 25 \\ 4 & 35 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 7.80 \\ 22.08 \\ 35.00 \\ 183.28 \\ 43.00 \end{bmatrix}$	2.42037	$\begin{bmatrix} 7.80 \\ 13.20 \\ 5.90 \\ 154.10 \\ 35.70 \end{bmatrix}$	0.002985				$\begin{bmatrix} 7.80 \\ 13.20 \\ 5.90 \\ 154.10 \\ 35.70 \end{bmatrix}$	0.002985	Displ. = Wt.
2	$\begin{bmatrix} 7.8 & 11 \\ 12 & 25 \\ 4 & 35 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 7.80 \\ 14.44 \\ 19.50 \\ 150.00 \\ 38.68 \end{bmatrix}$	- 0.0126222	$\begin{bmatrix} 8.50 \\ 13.90 \\ 19.10 \\ 150.20 \\ 36.70 \end{bmatrix}$	0.001708	{2}	$\begin{bmatrix} 8.50 \\ 13.90 \\ 19.10 \\ 150.20 \\ 36.70 \end{bmatrix}$	(1, 8.5)	$\begin{bmatrix} 8.50 \\ 13.90 \\ 19.10 \\ 150.20 \\ 36.70 \end{bmatrix}$	0.001708	Displ. = Wt, Troll $\geq$ Troll_min
3	$\begin{bmatrix} 7.8 & 8.5 \\ 12 & 25 \\ 4 & 35 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 8.50 \\ 13.65 \\ 16.30 \\ 115.37 \\ 43.00 \end{bmatrix}$	0.0086186	$\begin{bmatrix} 8.50 \\ 13.30 \\ 16.10 \\ 116.20 \\ 40.05 \end{bmatrix}$	0.001590						

Table 5.4 Results for Algorithm PPA2-BR1 (continued).

Node (k)	$\Omega^k : \lfloor l_j^k \quad u_j^k \rfloor$	$\bar{x}^k$	z_k	x^*	z^*	L	x^k	(q, x_q^k)	Updated x^*	Updated z^*	Constraints Active at x^*
4	$\begin{bmatrix} 8.5 & 11 \\ 12 & 25 \\ 4 & 35 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 11.00 \\ 13.90 \\ 30.06 \\ 300.00 \\ 43.00 \end{bmatrix}$	-0.0045961	$\begin{bmatrix} 10.70 \\ 15.50 \\ 27.60 \\ 300.00 \\ 43.00 \end{bmatrix}$	0.001347	{4}	$\begin{bmatrix} 10.70 \\ 15.50 \\ 27.60 \\ 300.00 \\ 43.00 \end{bmatrix}$	(1, 10.7)	$\begin{bmatrix} 10.70 \\ 15.50 \\ 27.60 \\ 300.00 \\ 43.00 \end{bmatrix}$	0.001347	Displ. = Wt, Fb ≥ Fb_min
5	$\begin{bmatrix} 8.5 & 10.7 \\ 12 & 25 \\ 4 & 35 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 8.50 \\ 15.40 \\ 19.08 \\ 300.00 \\ 43.00 \end{bmatrix}$	0.00064124	$\begin{bmatrix} 10.50 \\ 16.64 \\ 18.62 \\ 300.00 \\ 43.00 \end{bmatrix}$	0.001026	{5}	$\begin{bmatrix} 10.50 \\ 16.64 \\ 18.62 \\ 300.00 \\ 43.00 \end{bmatrix}$	(1, 10.5)	$\begin{bmatrix} 10.50 \\ 16.64 \\ 18.62 \\ 300.00 \\ 43.00 \end{bmatrix}$	0.001026	Displ. = Wt.
6	$\begin{bmatrix} 10.7 & 11 \\ 12 & 25 \\ 4 & 35 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 10.70 \\ 16.15 \\ 18.30 \\ 300.00 \\ 43.00 \end{bmatrix}$	0.0037870	$\begin{bmatrix} 10.70 \\ 17.30 \\ 18.40 \\ 300.00 \\ 43.00 \end{bmatrix}$	0.001034						
7	$\begin{bmatrix} 8.5 & 10.5 \\ 12 & 25 \\ 4 & 35 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 10.50 \\ 16.64 \\ 18.62 \\ 300.00 \\ 43.00 \end{bmatrix}$	0.0095127	$\begin{bmatrix} 10.50 \\ 16.64 \\ 18.62 \\ 300.00 \\ 43.00 \end{bmatrix}$	0.001026						

Table 5.4 Results for Algorithm PPA2-BR1 (concluded).

Node (k)	$\Omega^k : [l_j^k \quad u_j^k]$	$\bar{x}^k$	z_k	x^*	z^*	L	x^k	(q, x_q^k)	Updated x^*	Updated z^*	Constraints Active at x^*
8	$\begin{bmatrix} 10.5 & 10.7 \\ 12 & 25 \\ 4 & 35 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 10.50 \\ 16.64 \\ 18.62 \\ 300.00 \\ 43.00 \end{bmatrix}$	0.0088899	$\begin{bmatrix} 10.50 \\ 16.64 \\ 18.62 \\ 300.00 \\ 43.00 \end{bmatrix}$	0.001026	$\{\phi\}$					

Table 5.5 Results for Algorithm PPA2-BR2.

Node (k)	$\Omega^k : \left[l_j^k \quad u_j^k \right]$	$\overline{x}^k$	z_k	x^*	z^*	L	x^k	(q, x_q^k)	Updated x^*	Updated z^*	Constraints Active at x^*
0	$\begin{bmatrix} 6 & 11 \\ 12 & 25 \\ 4 & 35 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 8.07 \\ 12.05 \\ 19.50 \\ 100.00 \\ 28.67 \end{bmatrix}$	-0.0119694	$\begin{bmatrix} 7.80 \\ 13.20 \\ 15.00 \\ 123.00 \\ 34.90 \end{bmatrix}$	0.001869	{0}	$\begin{bmatrix} 7.80 \\ 13.20 \\ 15.00 \\ 123.00 \\ 34.90 \end{bmatrix}$	(1, 7.80)	$\begin{bmatrix} 7.80 \\ 13.20 \\ 15.00 \\ 123.00 \\ 34.90 \end{bmatrix}$	0.001869	Displ. = Wt.
1	$\begin{bmatrix} 6 & 7.8 \\ 12 & 25 \\ 4 & 35 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 7.80 \\ 22.08 \\ 35.00 \\ 183.28 \\ 43.00 \end{bmatrix}$	2.42037	$\begin{bmatrix} 7.80 \\ 13.20 \\ 5.90 \\ 154.10 \\ 35.70 \end{bmatrix}$	0.002985				$\begin{bmatrix} 7.80 \\ 13.20 \\ 5.90 \\ 154.10 \\ 35.70 \end{bmatrix}$	0.002985	Displ. = Wt.
2	$\begin{bmatrix} 7.8 & 11 \\ 12 & 25 \\ 4 & 35 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 7.80 \\ 14.44 \\ 19.50 \\ 150.00 \\ 38.68 \end{bmatrix}$	-0.0126222	$\begin{bmatrix} 8.50 \\ 13.90 \\ 19.10 \\ 150.20 \\ 36.70 \end{bmatrix}$	0.001708	{2}	$\begin{bmatrix} 8.50 \\ 13.90 \\ 19.10 \\ 150.20 \\ 36.70 \end{bmatrix}$	(2, 13.9)	$\begin{bmatrix} 8.50 \\ 13.90 \\ 19.10 \\ 150.20 \\ 36.70 \end{bmatrix}$	0.001708	Displ. = Wt, Troll ≥ Troll_min
3	$\begin{bmatrix} 7.8 & 11 \\ 12 & 13.9 \\ 4 & 35 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 10.80 \\ 13.65 \\ 16.30 \\ 115.37 \\ 43.00 \end{bmatrix}$	0.0021794	$\begin{bmatrix} 10.50 \\ 15.50 \\ 21.90 \\ 146.40 \\ 42.30 \end{bmatrix}$	0.001857						

Table 5.5 Results for Algorithm PPA2-BR2 (continued).

Node (k)	$\Omega^k : \left[l_j^k \quad u_j^k \right]$	$\bar{x}^k$	z_k	x^*	z^*	L	x^k	(q, x_q^k)	Updated x^*	Updated z^*	Constraints Active at x^*
4	$\begin{bmatrix} 7.8 & 11 \\ 13.9 & 25 \\ 4 & 35 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 11.00 \\ 13.90 \\ 30.06 \\ 300.00 \\ 43.00 \end{bmatrix}$	-0.0045961	$\begin{bmatrix} 10.70 \\ 15.50 \\ 27.60 \\ 300.00 \\ 43.00 \end{bmatrix}$	0.001347	{4}	$\begin{bmatrix} 10.70 \\ 15.50 \\ 27.60 \\ 300.00 \\ 43.00 \end{bmatrix}$	(3, 27.6)	$\begin{bmatrix} 10.70 \\ 15.50 \\ 27.60 \\ 300.00 \\ 43.00 \end{bmatrix}$	0.001347	Displ. = Wt, Fb $\geq$ Fb_min
5	$\begin{bmatrix} 7.8 & 11 \\ 13.9 & 25 \\ 4 & 27.6 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 11.00 \\ 17.42 \\ 18.01 \\ 300.00 \\ 43.00 \end{bmatrix}$	0.0009243	$\begin{bmatrix} 10.50 \\ 16.64 \\ 18.62 \\ 300.00 \\ 43.00 \end{bmatrix}$	0.001026	{5}	$\begin{bmatrix} 10.50 \\ 16.64 \\ 18.62 \\ 300.00 \\ 43.00 \end{bmatrix}$	(1, 10.5)	$\begin{bmatrix} 10.50 \\ 16.64 \\ 18.62 \\ 300.00 \\ 43.00 \end{bmatrix}$	0.001026	Displ. = Wt.
6	$\begin{bmatrix} 7.8 & 11 \\ 13.9 & 25 \\ 27.6 & 35 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 11.00 \\ 13.90 \\ 27.60 \\ 300.00 \\ 41.59 \end{bmatrix}$	1.037870	$\begin{bmatrix} 10.50 \\ 15.30 \\ 27.60 \\ 300.00 \\ 43.00 \end{bmatrix}$	0.001361						
7	$\begin{bmatrix} 7.8 & 10.5 \\ 13.9 & 25 \\ 4 & 27.6 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 10.50 \\ 16.64 \\ 18.62 \\ 300.00 \\ 43.00 \end{bmatrix}$	0.00345918	$\begin{bmatrix} 10.50 \\ 16.64 \\ 18.62 \\ 300.00 \\ 43.00 \end{bmatrix}$	0.001026						

Table 5.5 Results for Algorithm PPA2-BR2 (concluded).

Node (k)	$\Omega^k : [l_j^k \quad u_j^k]$	$\bar{x}^k$	z_k	x^*	z^*	L	x^k	(q, x_q^k)	Updated x^*	Updated z^*	Constraints Active at x^*
8	$\begin{bmatrix} 10.5 & 11 \\ 13.9 & 25 \\ 4 & 27.6 \\ 100 & 300 \\ 20 & 43 \end{bmatrix}$	$\begin{bmatrix} 10.50 \\ 16.64 \\ 18.62 \\ 300.00 \\ 43.00 \end{bmatrix}$	0.0027955	$\begin{bmatrix} 10.50 \\ 16.64 \\ 18.62 \\ 300.00 \\ 43.00 \end{bmatrix}$	0.001026	$\{\phi\}$					

Chapter 6

An Overview of Warship Design

This chapter begins by providing in Section 6.1 below a brief introduction to warship design. Thereafter, Section 6.2 presents a somewhat more detailed explanation of the design considerations introduced in Section 6.1. In Section 6.3, we review the existing literature on research efforts pertaining to warship design. Finally, in Section 6.4, we briefly discuss certain modeling and algorithmic considerations specifically pertaining to warship design.

6.1 Introduction [16], [24]

Warship design is more complicated than the design of all other kinds of ships. Warships are categorized by function as follows:

1. Aircraft carriers,
2. Frigates,
3. Destroyers,
4. Mine countermeasure ships and
5. Submarines.

Frigates and destroyers are mainly used as anti-submersible or anti-aircraft vessels, and their weapon and sensor fittings reflect this. Usually, the main armament in these types of warships consists of some form of missile system deigned to engage the enemy at some distance from the ship. The missile may be guided all the way to the target by sensors in the ship or may be of the "fire and forget" type.

Mine countermeasure ships may be either sweepers or hunters of mines, or a combination of these two functions in one hull. Since mine countermeasure vessels themselves are a target for the mines that they try to destroy, their ship signatures must be extremely low and their hulls very robust.

Submarines have many unique features. They are classified as warships because to date, all submarines have been warships. However, they could be used for alternative commercial applications such as deep ocean research, exploitation of the ocean's resources, rescuing the crews of stricken submarines, or for investigation of shipwrecks. In practice, the use of submarines has been found to be uneconomical for these types of general commercial work. Although submarines are designed to operate primarily at depth, their behavior on the surface must also be considered during their design.

Warships need to carry equipment such as sensors to detect enemy vessels and weapons to defend themselves and to attack others. Warship hulls tend to have greater curvature and consequently, the calculations of their coefficients of form involve greater subdivisions. The two most important considerations in the design of warships as compared to other types of ships are:

1. **Susceptibility**: Susceptibility is defined as the ability to avoid detection, and failing that, to prevent being hit by the enemy weapon.

2. **Vulnerability**: A ship may be quite safe in the intact condition, but likely to suffer extensive damage, or loss, as a result of minor incidents. For example, a ship having no internal subdivisions could operate safely until water entered by some means, whence it would sink. Such a design is unduly vulnerable. Because warships are expected to suffer such damage in action, vulnerability is an important consideration.

6.2 An Overview of Design Requirements [4], [5], [16], [39]

6.2.1 Stealth

A warship can betray its presence by a variety of *signatures*; each must be as low as possible to avoid detection by the enemy and to prevent the triggering of sensitive mines.
These signatures are as follows:

1. **Noise**: Noise emanates from propulsion, running machinery, or the flow of water past the ship. An attacking ship can detect noise by passive sonars without betraying its own presence. Noise levels can be reduced by special propulsion design, by fitting anti-noise mountings to noisy machines, and by applying special coatings to the hull. Creating a smooth hull reduces the risk of turbulence in water.

2. **Radar Cross-Section**: When illuminated by radar, a ship causes a return pulse depending upon its size and geometry. By arranging the structural shape so that the returning pulses are scattered over a wide arc, the signal picked up by the searching ship can be made much weaker. Radar absorbent materials can be applied to the outer skin to absorb much of the incident signal.

3. **Infra-Red Emissions from Areas of Heat**: Although the ship is bound to be warmer than its surroundings, its main heat concentration can be reduced, the idea being to minimize the likelihood of detection due to heat. Cooling the funnel exhaust and pointing it in a direction from which the enemy is less likely to approach is one way of reducing the heat concentration.

4. **Magnetic Fields**: Mines could be triggered by the changes in the local magnetic field caused by the passage of a ship. All steel ships have a degree of in-built magnetism that can be countered by the creation of opposing fields by using

special electric current carrying coils. This treatment is known as “degaussing”. Moreover, the ship needs to detect the strength and direction of the earth’s field in order to similarly ameliorate the effect of the distortion of the earth’s magnetic field by the ship by applying the required corrections.

5. **Pressure Fields**: A change in the pressure field is caused by the movement of the ship through water and this can trigger mines. The effect is usually reduced by reducing the speed of the ship.

It is impossible to eliminate the foregoing signatures completely. The designer should aim at reducing these signature levels, requiring the enemy to deploy more sophisticated sensors and weapons, and thereby increasing the likelihood that the enemy itself would be detected in this process.

6.2.2 Sensors

Sensors require careful siting to ensure a good field of view and to prevent the ship’s signatures or motions from degrading their performance. Search radars must have complete 360° coverage and be placed high on the ship. Hull mounted sonars are usually fitted below the keel forward where they are remote from major noise sources and where the boundary layer is still relatively thin. Some ships carry sonars that can be towed astern to isolate them from the ship’s noises and to enable them to operate at a depth from which they are more likely to detect submarines.

6.2.3 Weapons

A ship’s own weapons can cause problems for it, apart from the usual aspects such as weight, space, and supplies. They require a good covering radius and must provide allowances for the expected missile trajectories. Missiles create an efflux that can harm protective coatings on structures as well as affect the more sensitive equipment on exposed decks. Weapons carry a great deal of explosive material, and precautions must be taken to reduce the risk of premature detonation.

6.2.4 Enemy Weapons

A good warship design adopts a policy of layered defenses. The aim is to detect an enemy and any incoming weapon at the greatest possible range, and engage it with a long-range defense system. If the weapon is not detected in time, or if it penetrates the first line of defense, a medium range system is first used followed by a short-range system. When an aircraft carrier is present in the task force, its aircraft would usually provide the first line of defense. It is in the later stages that decoys may be released. The incoming weapon’s homing system locks onto the decoy and is diverted from the real target, although the resulting explosion may still be uncomfortably close. The shortest-range systems are the "closest in weapon systems". These are essentially rapid-firing guns.

6.2.5 Sustaining Damage

Even the most articulate defense system can be defeated. The ship, under such circumstances, must be able to withstand at least some measure of damage before it is put out of action or before it sinks. The effects on the ship would generally involve a combination of structural damage, fire, flooding, blast, shock, and fragment damage. The ship must be designed to contain these effect within as small a space as possible by "zoning" or separating out vital functions, so that not all capability is lost as a result of a single hit, and by providing extra equipments and protection to vital spaces. An underwater explosion is perhaps the most serious threat to a ship. This could cause whipping and shock.

6.3 Literature Review

Eames and Drummond (1976) describes a versatile tool for the design of small warships (1000-6000 tons) intended for use in the opening phases of the design process. The "Concept Exploration Model" provides an alternative approach to the usual immediate reliance on a "basis ship", enabling the designer to explore a wider range of concepts. A simple algorithm is developed using empirical data from a number of successful small warships. This has been programmed to search a wide range of assumed dimensions in order to determine a hypothetical optimum ship for specified operational objectives. The trends of design behavior in the vicinity of a prescribed best design are clearly illustrated. The major shortcoming of the model is shown to be its inadequate treatment of seakeeping, which requires maintaining the same speed under all sea conditions. However, recent advances in seakeeping theory suggest that the design should support a specified speed in a specified sea state.

Holmes (1980) outlines the experience gained in the ship department of the Ministry of Defense during the first phase of application and development of a Computer Aided Ship Design (CASD) System. He demonstrates the successful application of the system to current warship design studies and proposes areas for improvement and extension. The main areas of improvement are concerned with the program and user interfaces, program response times, and presentation of output design drawing capability. Better methods of design synthesis, improved program performances, and techniques for producing documentation, improved drawing production, efficient data storage and retrieval, and communication of design information are stressed upon. A ship Design System (SDS) is developed to support design work during the phases of ship design and contract definition, and also for major refit design studies. The application of SDS to submarine design and the design of other marine vehicles is also proposed.

6.4 Modeling and Algorithmic Considerations

In this section, we provide a brief overview of the modeling and algorithmic considerations specifically pertaining to warship design. As highlighted in Section 6.1, warship design is more complicated than the design of all other kinds of ships. The problem formulation is somewhat more intricate, requiring the definition of additional design variables and constraints, and an adequate representation of these additional design variables in the objective and constraint functions.

1. **Design Variables**: In addition to the design variables relating to the principal dimensions and the coefficients of form as defined in the containership design problem described in this work, the following three operational objectives need to be considered in the problem formulation for designing warships:

a) Design speed in calm water (in knots):
b) Cruise speed at which the required endurance is to be attained in calm water (in knots):
c) Endurance at the cruise speed with all available fuel used (in nautical miles):

2. **Constraints**: The following constraints could be imposed on the design, in addition to the constraints pertaining to the minimum required freeboard, stability, and the rolling period:

a) Maximum permissible operational load (in metric tons): Operational load is defined in terms of the standard weight-class system used by the Canadian Forces. Essentially, it comprises all armament, ammunition, aircraft, command and control equipment, and any other military payload.

b) Minimum number of watertight bulkheads: A nominal expression for the minimum number of watertight bulkheads is $\left[\frac{\text{Loa}}{(10+(0.04\times\text{Loa}))}\right]-1.$
c) Minimum number of decks below the upper deck: A nominal expression for the minimum number of decks below the upper deck is $\left(\frac{D}{8}\right)-1.$

The following minimum values could be imposed on the following design parameters in addition to the above constraints:

a) Minimum number of propeller shafts: A nominal value for the minimum number of propeller shafts is 2.

b) Minimum number of electrical generators: A nominal value for the minimum number of electrical generators is 4.

3. **Objective Function**: The objective function should be representative of the additional design variables as suggested earlier in this section. One such objective function is the maximum transport effectiveness that is a combination of the specific power, and the operational load ratio or the ratio of the operational load to the total weight of the ship.

The overall solution methodology to identify a solution to the warship design problem would be similar to the methodology proposed in this work.

Chapter 7

Conclusions and Recommendations for Future Research

7.1 Conclusions

The goal of this research was to explore the application of alternative modeling, approximation, and global optimization techniques for developing a multidisciplinary approach to the Containership Design Problem (CSD). This has been accomplished by addressing the derivation of detailed functional forms of important design issues and the development of effective pseudo-global optimization methodologies based on polynomial programming approximations and the Reformulation-Linearization Technique (RLT). Together, these two aspects have provided a more precise model and algorithmic approach to the nonconvex CSD.

A comparison of the design obtained from the proposed algorithmic approaches with that resulting from the application of the nonlinear optimization methods as in previous research, exhibits a significant improvement in the design parameters translating to a significant amount of annual cost savings. For a typical containership of the size pertaining to a test case addressed in Chapter 5, having a gross weight of 90,000 metric tons, an annual transportation capacity of 99,000 containers corresponding to an annual deadweight of 1,188,000 metric tons, and logging 119,000 nautical miles annually, the improvement in the prescribed design translates to an annual estimated savings of $ 1,862,784 (or approximately $ 1.86 million) and an estimated 27 % increase in the return on investment over the life of the ship.

Despite this difference, our experience from applying Algorithms PPA1 and PPA2 reveals that the design space is slightly flat in the vicinity of the pseudo-global optimum (as evident from the representation of the design space in section 5.4), indicating that the designer is not confined to a severely restricted design space and has some nominal freedom in designing the ship.

7.2 Recommendations for Future Research

- Extend the pseudo-global optimization approaches developed herein to the design of other vehicles and structures. One such possible extension, namely, the design of warships, has been briefly reviewed in Chapter 6.

- Automate the process of creating the polynomial programming approximations using the Response Surface Methodology (RSM), and integrate this automated process within the optimization framework described herein. This is necessary to reduce manual interactions during implementation, and even essential for problems in which optimal solutions might tend to lie away from the design variables bounds, causing a relatively larger number of nodes to be enumerated in the branch-and-bound tree. It is worthwhile to point out here that available statistical software do not always allow the user to construct response surface models of orders greater than quadratic, and the ones that do allow greater orders require the additional polynomial terms to be added manually. Hence the automation pertaining to the creation of the polynomial programming approximations may have to be restricted to quadratic response surface models.

- Test the developed methodology on a wider variety of case studies. This has presently not been implemented due to limitations in the executable pertaining to the nonlinear optimization methods. This limitation arises from the fact that the executable accepts only the required freight rate as the objective function. Furthermore, it disallows the imposition of additional constraints, such as a constraint on the maximum allowable displacement.

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