

A non-clinical method to simultaneously estimate  
thermal conductivity, volumetric specific heat, and perfusion  
of *in-vivo* tissue

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# A non-clinical method to simultaneously estimate thermal conductivity, volumetric specific heat, and perfusion of *in-vivo* tissue

Marie Madden

Abstract

Many medical therapies, such as thermal tumor detection and hypothermia cancer treatments, utilize heat transfer mechanisms of the body. The focus of this work is the development and experimental validation of a method to simultaneously estimate thermal conductivity, volumetric specific heat, and perfusion of *in-vivo* tissue. The heat transfer through the tissue was modeled using a modified Pennes' equation. Using a least-squares parameter estimation method with regularization, the thermal properties could be estimated from the temperature response to the known applied heat flux.

The method was tested experimentally using a new agar-water tissue phantom designed for this purpose. A total of 40 tests were performed. The results of the experiments show that conductivity can be successfully estimated for perfused tissue phantoms. The values returned for volumetric specific heat are lower than expected, while the estimated values of perfusion are far greater than expected. It is believed that the mathematical model is incorrectly accounting between these two terms. Both terms were treated as heat sinks, so it is conceivable that it is not discriminating between them correctly.

Although the method can estimate all three parameters simultaneously, but it seems that the mathematical model is not accurately describing the system. In the future, improvements to the model could be made to allow the method to function accurately.

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## Table of Contents

|   |    |
|---|----|
| 1. Introduction .....                                     | 1  |
| 1.1. Importance of work.....                              | 2  |
| 1.2. Physiological heat transfer mechanisms.....          | 2  |
| 1.3. Objectives.....                                      | 4  |
| 1.4. Organization of thesis.....                          | 5  |
| 2. Literature review.....                                 | 6  |
| 2.1. Mathematical models.....                             | 6  |
| 2.2. Parameter estimation.....                            | 9  |
| 2.3. Measurement methods and probes.....                  | 10 |
| 2.4. Phantoms.....  | 11 |
| 2.5. Parameter value estimates.....                       | 12 |
| 3. Bioheat model.....                                     | 13 |
| 3.1. Pennes' equation.....                                | 13 |
| 3.2. Finite Difference model.....                         | 14 |
| 3.2.1. Introduction in one dimension.....                 | 14 |
| 3.2.2. Solving finite difference in one dimension.....    | 16 |
| 3.2.3. Finite difference in two dimensions.....           | 18 |
| 3.3. Temperature variations in parameters.....            | 20 |
| 3.4. Finite difference mesh of the test stand volume..... | 23 |
| 4. Parameter estimation problem .....                     | 27 |
| 4.1. Solving a parameter estimation problem.....          | 27 |
| 4.2. Code Development.....                                | 34 |
| 4.3. Numerical Studies.....                               | 36 |
| 4.4. Least Squares with Regularization.....               | 39 |
| 5. Experimental test stand.....                           | 41 |
| 5.1. Phantom theory and development.....                  | 41 |
| 5.2. Phantom construction.....                            | 43 |
| 5.3. Perfusion dye tests.....                             | 44 |
| 5.3.1. Channel size.....                                  | 44 |
| 5.3.2. Dye test procedure.....                            | 45 |
| 5.3.3. Perfusion results.....                             | 47 |
| 5.3.4. Statistical analysis.....                          | 48 |
| 5.3.5. Dye test conclusions.....                          | 49 |
| 5.4. Further work.....                                    | 50 |
| 5.5. Future uses.....                                     | 52 |
| 5.6. Test stand using the perfused agar phantom.....      | 52 |
| 5.7. Experimental test stand.....                         | 53 |
| 6. Performing experiments.....                            | 57 |
| 6.1. Data acquisition.....                                | 57 |
| 6.2. Experimental procedure.....                          | 59 |
| 6.2.1. Unperfused experiments.....                        | 60 |
| 6.2.2. Orbitron experiments.....                          | 61 |
| 6.2.3. IV drip experiments.....                           | 62 |
| 6.2.4. Orbitron IV drip experiments.....                  | 64 |

|  |     |
|--|-----|
| 7. Evaluation and results.....                 | 67  |
| 7.1. Unperfused results.....                   | 68  |
| 7.2. Orbitron results.....                     | 70  |
| 7.3. IV drip results.....                      | 72  |
| 7.4. Orbitron IV drip results.....             | 74  |
| 7.5. Data comparison of each parameter.....    | 75  |
| 7.5.1. Thermal conductivity.....               | 75  |
| 7.5.2. Volumetric specific heat.....           | 76  |
| 7.5.3. Perfusion.....                          | 77  |
| 8. Summary and Conclusions.....                | 81  |
| Appendices                                     |     |
| A. 2D matrix construction .....                | 86  |
| B. Matlab code.....                            | 87  |
| C. Parameter estimation result statistics..... | 105 |
| D. Specifications of equipment.....            | 109 |
| E. Works cited.....                            | 110 |

## Nomenclature

|          |                                    |
|----------|------------------------------------|
| $\alpha$ | regularization term                |
| $\beta$  | inverse estimation parameter       |
| $c$      | volumetric specific heat, $J/m^3K$ |
| $C_p$    | specific Heat, $J/kg/K$            |
| $g$      | heat generation, $W/m^2$           |
| $h_b$    | convective heat, $W/m^2$           |
| $h_m$    | metabolic heat, $W/m^2$            |
| $I$      | identity matrix                    |
| $k$      | thermal conductivity, $W/mK$       |
| $\rho$   | density, $kg/m^3$                  |
| $q_0$    | heat flux at boundary, $W/m^2$     |
| $T$      | temperature, $K$                   |
| $t$      | time, $s$                          |
| $V$      | perfusion flow, $mL/mL/s$          |
| $V$      | volume, $m^3$                      |
| $\omega$ | perfusion term, $W/m^3K$           |
| $X$      | sensitivity matrix                 |

## Super and subscripts

|       |                    |
|-------|--------------------|
| $b$   | blood              |
| $E$   | east               |
| $i$   | current node       |
| $i+1$ | eastern node       |
| $i-1$ | western node       |
| $N$   | north              |
| $p$   | current time step  |
| $p-1$ | previous time step |
| $S$   | south              |
| $W$   | west               |

## List of Figures

|             |  |    |
|-------------|--|----|
| Figure 3.1  | Discretization of volume for finite difference method. The boundary conditions are appropriate for this research and represent the heat entering the volume and the semi-infinite temperature at the opposite boundary<br>.....  | 15 |
| Figure 3.2  | Matrix of finite difference simultaneous equations. This matrix represents a three node system at a single moment in time. A similar matrix must be solved for each time step.....   | 17 |
| Figure 3.3  | Two dimensional finite difference control volume discretizations. Depth and radius are represented by z and r. The cylindrical nature of this control volume is discussed later.....   | 18 |
| Figure 3.4  | Each arrow represents a matrix that is solved for the temperature response. The red arrows show horizontal sweeps of the total temperature matrix while blue show use of the line by line method in the vertical direction...  | 19 |
| Figure 3.5  | Cylindrical nature of the 2D control volume. This assumes that the heat applied to the area moves out in all directions the same way.....  | 20 |
| Figure 3.6  | The black curve shows the change in temperature of the volume in response to a thermal event. The green line is the equilibrium temperature while the red line shows the temperature at which perfusion increases. The temperature response changes with the initiation of the higher perfusion..... | 21 |
| Figure 3.7  | Finding thermal conductivity approximation for water. $R^2$ values were calculated using Microsoft Excel.....  | 22 |
| Figure 3.8  | Finding volumetric specific heat approximation for water. Three line fits were tested to see that the quadratic approximation was necessary.....   | 23 |
| Figure 3.9  | Plot of the number of nodes (i.e. 20 x nodes and 20 y nodes) versus the temperature at the first time step. The temperature is selected from the same measurement point for each mesh. As the number of nodes increase, the temperatures become the same.....  | 24 |
| Figure 3.10 | Plot of the finite difference program total processing time versus the number of nodes. As the mesh becomes finer it takes far longer for the program to run.....  | 25 |
| Figure 3.11 | a) varying the number of nodes in the x direction and b) y direction. Changes in the number of horizontal nodes affect the temperature response more than the number of vertical nodes does.....   | 25 |

|             |  |    |
|-------------|--|----|
| Figure 3.12 | The red line shows the temperature response at the first time step for the 30x 25y mesh. The blue line plots the temperature response for various other node combinations.....   | 26 |
| Figure 4.1  | Flow chart of inverse heat transfer method. The estimate of the parameter of interest is updated during each iteration until a final answer is found..   | 28 |
| Figure 4.2  | Plot of the relationship between measured and calculated temperatures to correct conductivity value.....   | 30 |
| Figure 4.3  | A sensitivity matrix for a system of three parameters of interest and four temperature measurements. Each column represents the change in temperature by the perturbation of the parameter for each parameter.....   | 30 |
| Figure 4.4  | a) a parameter that levels out from four to seven any of which could be the correct answer. b) the sum of squares curve has two local minima. Depending on the initial estimate, a gradient method could easily choose the local minimum rather than the global minimum as the answer..... | 32 |
| Figure 4.5  | Plot of the sum of squares for volumetric specific heat. The curve has a definite minimum and is well suited to the gradient method.....   | 33 |
| Figure 4.6  | Plot of the sum of squares for the perfusion term. This curve has a more equal slope on both sides.....  | 34 |
| Figure 4.7  | Plot of temperature response for time and space for the inverse example. This reflects the ‘measured’ or ‘real’ temperature values that will be given to the inverse method.....   | 37 |
| Figure 4.8  | Successive estimates of a) low and b) high perfusion, c) volumetric specific heat, and d) conductivity.....  | 38 |
| Figure 5.1  | A Boekel Rotator I.....  | 43 |
| Figure 5.2  | a) dye moving through diced agar b) representation of the method used to record the dye measurements.....  | 46 |
| Figure 5.3  | Graph of measured perfusion values from the dye test experiments. The diced agar results are about an order of magnitude higher than those of the mashed.....  | 47 |
| Figure 5.4  | Individual perfusion estimates from literature more closely compare to those found from the dye test.....  | 48 |

|             |  |    |
|-------------|--|----|
| Figure 5.5  | Boxplot of perfusion results strengthens the hypothesis that the large and small piece results are statistically equal.....  | 49 |
| Figure 5.6  | a) Schematic of the drip perfusion test stand. Arrows indicate the direction of water flow from the iv bag out of the agar container. b) Section of the actual test stand with the IV drip entering the bottom of the agar container.....                        | 53 |
| Figure 5.7  | Schematic of the three functioning areas of the probe. The thermal event and measurement of the thermal event happen outside of the tissue volume.....   | 54 |
| Figure 5.8  | a) the electric heater on the inner lid of the test stand b) the copper heat spreader. The black square is the heat flux sensor discussed in the next paragraph.....   | 54 |
| Figure 5.9  | Two wooden skewers cross an empty plastic container. Two thermocouples are attached to each skewer (their locations indicated by the blue marks) and will not move when agar is added. At the bottom of the bowl the drip perfusion tub can be seen.....         | 55 |
| Figure 5.10 | Test stand used for unperfused and orbitron tests. The plumber's putty was used to keep the stand from leaking, especially during the orbitron tests.....  | 56 |
| Figure 6.1  | The TBX 68-T board. Thermocouple and heat flux sensor leads are connected to this board which samples the data and conveys it to the computer DAQ card.....  | 57 |
| Figure 6.2  | LabView front panel. The top graph displays the thermocouple temperatures while the bottom shows heat flux. The VI can be easily modified through the front panel to account for different numbers of thermocouples and heat flux sensors.....                   | 58 |
| Figure 6.3  | Heat flux into the tissue phantom for an unperfused test. Heat flux begins at zero, the battery is attached and it rises to about 350 W/m <sup>2</sup> . The heater is turned off at about 550s and the heat flux falls.....                                     | 60 |
| Figure 6.4  | Unperfused test temperature response. Only the temperature of the two surface thermocouples is raised from the baseline measurement of the constant temperature water bath (blue line). The interior thermocouples do not feel the affect of this heat flux..... | 60 |
| Figure 6.5  | Heat flux measurements from orbitron perfused test. The heat flux in is about 200 W/m <sup>2</sup> . The power is turned off at about 700s and the heat flux begins to drop.....   | 61 |

|             |  |    |
|-------------|--|----|
| Figure 6.6  | Temperature response to heat flux from Figure 6.5. Only a one degree change in temperature was seen across the surface thermocouples. The interior thermocouples rose about a half a degree.....   | 61 |
| Figure 6.7  | a) the complete test set up including the hanging IV bag, the data acquisition system, the test stand and the heater power. b) the water collection from the test stand on a scale for measurement.....  | 62 |
| Figure 6.8  | Heat flux in the IV drip experiment. The heat flux into the tissue phantom is about $225 \text{ W/m}^2$ .....  | 63 |
| Figure 6.9  | Test five from the IV drip experimental runs. Here 6V was able to raise the temperature about one and a half degrees Celsius. The lower thermocouples had a temperature rise of about three quarters of a degree. This is slightly more of a temperature rise than the orbitron results..... | 64 |
| Figure 6.10 | Orbitron IV drip test stand. The drip cup collected water leaving the phantom and was measured at the end of the test to find the average flow through the test stand. The hanging IV bag creates the potential for water to flow through the stand.....                                     | 65 |
| Figure 6.11 | The heat flux input for the IV orbitron test.....  | 66 |
| Figure 6.12 | The temperature response to the heat flux from Figure 6.11. The temperature of all four thermocouples rose about equally about a half a degree.....  | 66 |
| Figure 7.1  | Thermocouple data sources for the estimation problem. The two ‘surface’ thermocouples are on the heated interface. The ‘depth’ thermocouples lie along the center of the volume and the ‘interior’ thermocouples have no contact with the heated surface.....                                | 68 |
| Figure 7.2  | The a) conductivity and b) volumetric specific heat estimates for the unperfused tests. The plots show the individual results for the three different data analysis methods for the regularization method.....   | 69 |
| Figure 7.3  | Individual test results from the orbitron experiments. For a) conductivity and c)&d) perfusion the middle temperatures estimate higher parameters than the surface temperatures. b) Volumetric specific heat does not follow this trend affecting the perfusion flow result.....             | 71 |
| Figure 7.4  | Individual test results from the IV drip experiment run. a) conductivity, b) volumetric specific heat, c) perfusion term and in d) the measured perfusion (from water weight over time) is multiplied by 1000 to see if the estimates follow a trend.....                                    | 73 |

|            |   |    |
|------------|---|----|
| Figure 7.5 | Individual results of the orbitron drip tests. a) conductivity, b) volumetric specific heat, c) perfusion term and in d) the measured flow (from water weight over time) is multiplied by 100 to see if there are any trends..... | 75 |
| Figure 7.6 | Average thermal conductivity results for all four test runs. Data are displayed with reference to surface or depth thermocouple combinations. Error bars relate to one standard deviation.....                                    | 76 |
| Figure 7.7 | Average volumetric specific heat results for all four test runs. Data is displayed with reference to surface or depth thermocouple combinations. Error bars relate to one standard deviation.....                                 | 77 |
| Figure 7.8 | Average perfusion term results for all four test runs. Data is displayed with reference to surface or depth combinations. Error bars relate to one standard deviation.....  | 78 |
| Figure 7.9 | Average perfusion flow results for all four test runs. Data is displayed with reference to top, middle, or bottom thermocouple combinations. Error bars relate to one standard deviation.....                                     | 79 |

## List of Tables

|            |   |    |
|------------|---|----|
| Table 2.1  | Perfusion estimates from literature. The middle column shows perfusion in the individuals units. The right hand column shows perfusion in units used in this work.....  | 12 |
| Table 5.1. | Average perfusion values found for tests conducted by Campbell using the cooled air probe. Perfusion increases when both methods are used together.....   | 50 |
| Table 5.2. | Estimated thermal properties of solid agar from Ricketts' test series. Specific heat is higher than generally assumed while thermal conductivity shows good agreement.....  | 51 |
| Table 6.1  | The test matrix for all experiments. The method of each test is shown to the right of the test number for each run group.....   | 59 |
| Table 6.2  | The measured perfusion term through the test stand.....   | 63 |
| Table 6.3  | Perfusion flow rate for the Orbitron IV drip experiments. Values are multiplied by 1000 for comparison to phantom study values.....   | 64 |
| Table 7.1  | Average parameter estimates and variance for unperfused tests. Interestingly, it takes less time on average to complete more runs for surface $a=1e-4$ than for $a=1e-5$ .....  | 69 |
| Table 7.2  | Average parameter estimates and variance for orbitron tests. Both estimates maintain the same order of magnitude.....   | 70 |
| Table 7.3  | Average results for the drip case when $\alpha =10^{-4}$ . Same orders of magnitude for all results though the interior thermocouple estimates seem to be higher due to its lower sensitivity than the other two which are quite similar..... | 72 |
| Table 7.4  | Average results for the drip case when $\alpha =1e-4$ . Similar results were found for all thermocouple combinations.....   | 74 |