Relative Accuracy and Precision of Differentially Corrected GPS on a Moving Vehicle

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Abstract

Differential corrections provide a method to improve the real-time accuracy and precision of GPS, but there are several sources of differential corrections and each have an associated accuracy and precision.

In dynamic applications, the speed and heading of the rover may also have an effect on the accuracy and precision reported by the GPS receiver. These factors may have more of an effect on one differential correction method than another.

An experiment was designed to test the differential correction methods under dynamic conditions. No corrections, OmniStar HP corrections, and RT2 corrections from a local base station were tested at several speeds and headings. The experiment was designed to determine what relationship, if any, exists between these factors and positional accuracy and precision of the differential correction sources. The results of the experiment will help designers choose the most effective solution for their positioning needs.

The experiment showed that local RT2 corrections offered the most precision under dynamic conditions. The precision of OmniStar HP was close to that of RT2 corrections. The system with no corrections was the least precise of the three tested. The speed and direction of the vehicle were not observed to have a significant affect on the precision of the systems tested.

The type of differential corrections used was not seen to have any influence on relative accuracy. The speed and direction of the vehicle did have an influence on the relative accuracy of the systems.

Table of Contents

List of Figures

List of Tables

Chapter 1 – Introduction

When testing an unmanned vehicle, it is important to quantify how well a certain task is performed. In order to record all the necessary experimental data, instrumentation may need to be added to the vehicle. There is no set of standard instruments that will satisfy all requirements for testing unmanned vehicles. The amount and type of instrumentation will change depending on the type of vehicle being tested and the nature of the experiment. With unmanned vehicles, however, many experiments make use of position and orientation data.

Waypoint navigation is one common application example in which position and orientation information is used. Often, experimenters want to know if a vehicle can move between given points and what path the vehicle takes between the points. Another example comes from obstacle-avoidance. Experimenters may want to know what path a vehicle takes when trying to avoid an obstacle. By looking at the speed of the vehicle and the path taken by the vehicle, an evaluation of the obstacle-avoidance algorithm can be made.

1.1 Project Overview

In June of 2004, Virginia Tech was awarded an RDECOM contract to develop the JOint Unmanned Systems, Test, Experimentation, and Research (JOUSTER) site. The purpose of the JOUSTER site was to provide a dedicated facility to help expedite the development of unmanned systems. One area of research defined in the contract was the development of the technology and instrumentation needed to reliably and accurately track the position and orientation of ground and air vehicles.

The accuracy and precision of the positioning tracking system is of great importance. One method to provide position information was with a GPS receiver. When investigating the performance of GPS, several questions arose. The level of accuracy and precision provided by a GPS receiver on a moving vehicle was unknown. Among the questions that arose was: How does the speed and the direction of travel influence the accuracy or precision? Several differential correction methods exist to improve GPS accuracy and precision. How do vehicle speed and direction influence these differentially corrected GPS? Which method of differential corrections provides the best results?

To help answer these questions we developed an experiment to determine the accuracy and precision of differential correction methods, which falls into another area of research defined by the RDECOM contract.

The focus of this document is to provide a comparison of several differential correction methods and investigate how they are affected by the speed and direction of the vehicle.

1.2 Technical Challenges

There are a variety of challenges associated with testing any type of GPS receiver. In order to collect performance data about the accuracy and precision of the position information, a known reference position is needed. For a static test, the reference position is usually a single point surveyed over a long period of time. It is much harder to establish a reference for a dynamic test. Even if the location of one or more points is known, it is difficult to ensure that the vehicle passes through those points.

When testing a GPS receiver, many of the variables affecting the position data cannot be controlled. The GPS satellites are constantly moving through space. The geometry of the GPS constellation is always changing. The atmospheric conditions of the Earth are changing from one instant to the next. If you multiple GPS receivers are tested simultaneously, the receivers may not acquire the same satellite signals.

There are technical challenges associated with testing the precision and accuracy of a GPS receiver. Testing more than one receiver adds increased complexity, as does conducting a dynamic test.

Chapter 2 – Literature Review

This chapter provides background information about some of the terminology used in this document and an overview of the equipment used to test differential corrections.

2.1 Accuracy and Precision

Throughout this document, the terms accuracy and precision are frequently used to describe the performance of GPS systems. This section defines accuracy and precision as used in this document.

Accuracy refers to how close a position measurement is to the actual position. Precision refers to the repeatability of the position measurement. The difference between accuracy and precision is shown in Figure 2.1. In the figure, the center of the target represents the actual position and the crosses mark the measured positions.

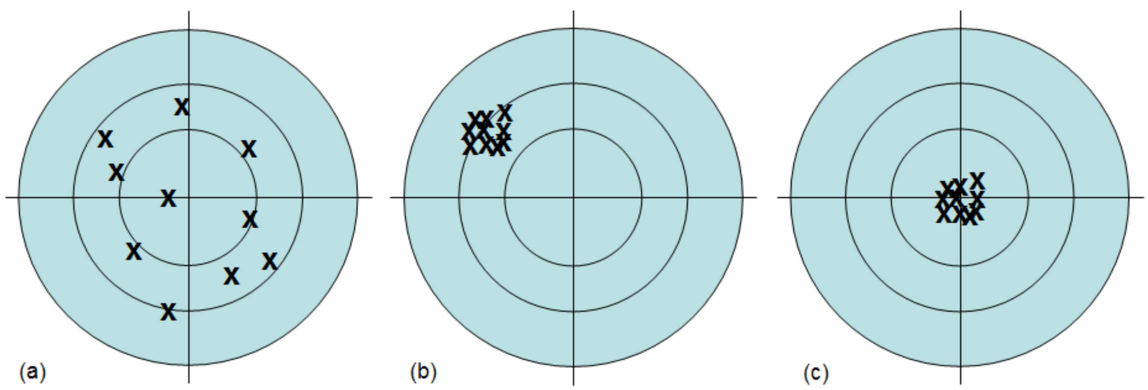

Figure 2.1: Accuracy and Precision (a) accurate but not precise (b) precise but not accurate (c) accurate and precise

Accuracy can be measured against an absolute or a relative position. Absolute positions are usually referenced to the Earth in terms of a latitude, longitude, and elevation. Absolute positions represent the exact location of the point on Earth. One way to obtain an absolute position is to record the position of a point over a 24 hour period with a GPS receiver. Over the course of the 24 hours, the GPS satellites will go through all the possible constellation configurations and the average position will provide a good measure of the absolute position. When an absolute position is not known, it is useful to report relative GPS accuracy. Relative accuracy uses a relative position for the basis of the accuracy analysis.

A relative position is one which is measured relative to another point. Relative position is useful for measuring distance. Let us assume that you have two GPS receivers and you set them some distance apart with no knowledge of the absolute position of either GPS receiver. You can calculate the distance between the receivers by taking the difference in position reported by each receiver. You could also find the distance between the two receivers with a measuring tape. If you assume that the measuring tape is more accurate than the GPS positions, you can calculate the accuracy of the GPS receivers relative to the measuring tape. This is the basic idea behind relative accuracy as used in this document.

2.2 GPS Overview

This section is intended to serves as a brief overview of the GPS system. For more information, readers should consult the references.

2.2.1 How GPS Works

The Global Positioning System (GPS) is a satellite-based navigation system. Satellites broadcast timing and satellite location information that can be used to determine the location of a receiver on earth. There are currently two GPS systems in operation: NAVSTAR operated by the United States and GLONASS operated by Russia. A European Union system called GALILEO is currently in the works. Although this discussion is limited to the NAVSTAR, all three systems work in the same way.

The system can be broken into three main parts: the space segment, the user segment, and the control segment. The three segments are shown in Figure 2.2.

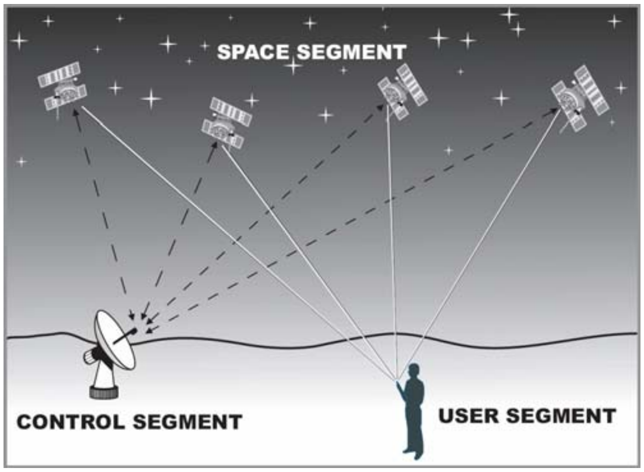

Figure 2.2: GPS Segments [2] Used Courtesy of Garmin Ltd. or its subsidiaries. Copyright Garmin Ltd. or its subsidiaries.

The Space Segment: The space segment consists of 24 operational satellites orbiting 11,000 miles above the surface of the earth. These satellites broadcast a very accurate timestamp, as well as their own location in space. Each satellite features an atomic clock accurate to one-billionth of a second, which is synchronized with the clocks in the other satellites to make sure they are broadcasting the exact same timestamp. At any time, between five and eight NAVSTAR satellites are visible from any location on earth [2]. The satellite orbits are shown in Figure 2.3.

Figure 2.3: Satellite Constellation [2] Used Courtesy of Garmin Ltd. or its subsidiaries. Copyright Garmin Ltd. or its subsidiaries.

The User Segment: The user segment consists of the GPS receivers that receive the satellite signals. A receiver's distance from a satellite can be calculated based on the time-of-flight of the satellite signal. By combining the information from multiple satellites, the GPS receiver can calculate its own position. A minimum of four satellites are needed to calculate position.

The Control Segment: The control segment is a group of ground stations that control and operate the GPS satellites. These stations closely monitor the satellite locations and update the orbit data.

2.2.2 Sources of GPS Errors

Provided in this section are seven sources of error which contribute to GPS position inaccuracy. Most error occurs in measuring the distance from the receiver to the satellites. These are called range errors.

Satellite/Receiver Clock Error: GPS relies on the ability to accurately measure the time-of-flight of satellite signals. Any error in the time measurement translates to a range error. For example, a clock error of one-millionth of a second translates to an error of 300 meters. Most clock error can be contributed to the receiver's inability to accurately measure time. However, the atomic clocks in the satellites can fall out of synch, which can increase the error [2].

Ephemeris Error: GPS satellites do not follow an exact orbit. Any difference between the orbit reported by the satellite and the actual orbit of the satellite will translate to a range error. Ephemeris error plays a small role in the overall error of GPS because the predicted and actual orbits are usually close [3].

Atmospheric Error: Although constant in a vacuum, the speed of light varies depending on the medium through which it is traveling. The satellite signals traveling through the earth's atmosphere are no exception. Calculating the distance between a satellite and receiver is complicated by the non-constant speed of light [3].

Multipath Error: When calculating range, the satellite signals are assumed to travel directly from the satellite to the receiver. In the real world however, signals bounce off objects in the local environment such as the ground or buildings before reaching the receiver. If the reflected signals are strong enough, they can confuse the receiver. A cause of multipath error is shown in Figure 2.4.

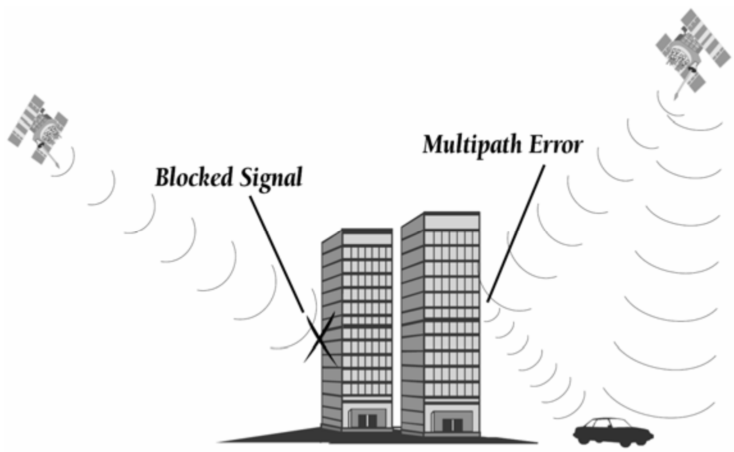

Figure 2.4: Multipath Error [2] Used Courtesy of Garmin Ltd. or its subsidiaries. Copyright Garmin Ltd. or its subsidiaries.

Receiver Error: The GPS receiver itself can introduce noise when measuring the satellite signals. For good receivers this error is negligible.

Geometric Dilution of Precision: The position error that results from range errors depends on the configuration of the satellites with respect to the receiver. For the same range error, the position error at the receiver will be greater if the satellites are close together. The configuration of the satellites is quantified by the Geometric Dilution of Precision (GDOP) and can be though of as the ratio of position error to range error [3]. An example of poor GDOP and good GDOP is shown in Figure 2.5.

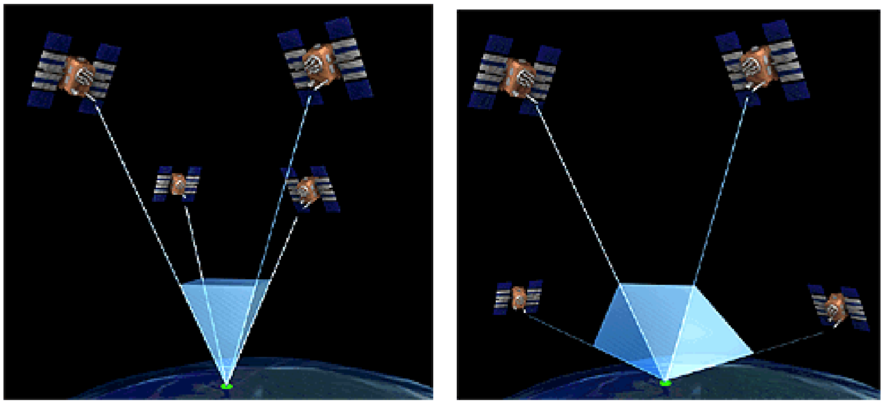

Figure 2.5: GDOP (a) An example of a poor GDOP (b) An example of a good GDOP [3] Used Courtesy of Topcon Positioning Systems. Copyright Topcon Positioning Systems

Selective Availability: In the early days of GPS, the United Stated military intentionally derogated the signal to prevent military adversaries from using the highly accurate positioning system. Although they have stopped degrading the system, the US military reserves the right to do so at any time.

2.2.3 Correcting GPS Errors

Over the years, many different techniques have been developed to improve the accuracy of GPS. The section discusses five of the different methods used.

Atmospheric Modeling: Some receivers use a model of the atmosphere to predict the amount of delay the satellite signals experience when they pass through the atmosphere. By using models, the error due to atmospheric delay can be reduced by about 50% [3].

Dual Frequency: GPS satellites broadcast signals simultaneously at two separate frequencies. When passing through earth's atmosphere, the satellite signals will slow down at a rate proportional to the frequency of the signal; the lower the frequency, the more the atmosphere slows it down. By measuring the difference in arrival time of these two signals, the GPS receiver can calculate and correct for the amount of delay induced by the atmosphere.

Differential GPS: Differential GPS (DGPS) relies on a GPS receiver placed at a precisely surveyed location. Since the location of the reference GPS is known and the position of the satellite is known, the distance between the two can be calculated. This distance is then compared to the distance based on the time-of-flight measurement. Any difference between the calculated and measured distance can be attributed to errors in the satellite signal; this difference is called the "differential correction."

The differential corrections are then broadcast to DGPS receivers. The receivers take the differential corrections into account when calculating position. Using differential corrections, GPS accuracy is greatly improved. The closer the DGPS receiver is to a reference station, the better the accuracy.

Differential corrections can be broadcasted in several different ways, including satellite-based corrections and local corrections. In the satellite-based systems, differential corrections are collected from many different base stations spread over a large area. The corrections from the base stations are transmitted to a satellite then to the receiver. OmniStar and WAAS are two examples of satellite-based corrections.

Differential corrections can also be broadcast directly from the base station to local areas. The local corrections require a radio link between the base station and the rover. A schematic of differential GPS operation is shown in Figure 2.6.

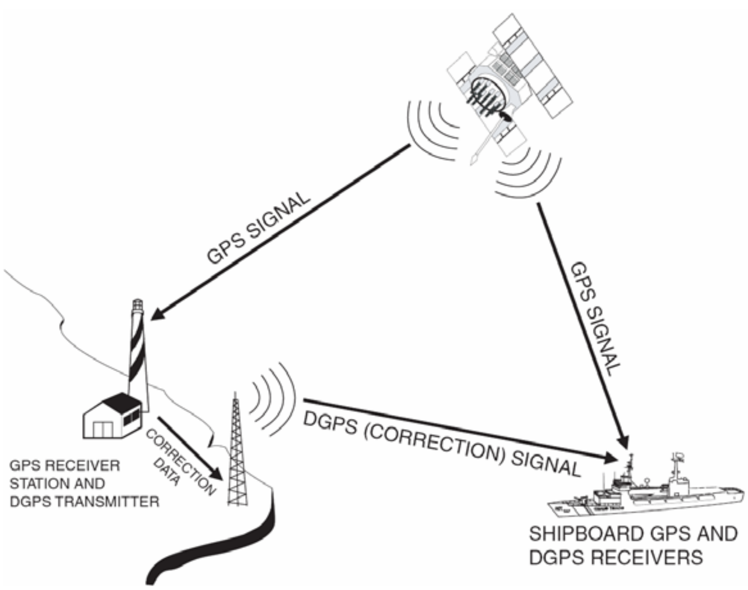

Figure 2.6: Differentially Corrected GPS [7] Used Courtesy of Garmin Ltd. or its subsidiaries. Copyright Garmin Ltd. or its subsidiaries.

Control segment monitoring: The control segment is constantly monitoring the position and time reported by the GPS satellites. When a satellite orbit changes or the atomic clock falls out of synch, the control segment updates the satellite with the new data. The precise orbit of each satellite orbit is recorded and published [2].

Post-processing: In addition to broadcasting, reference stations also record the differential corrections. Using special software, the recorded differential corrections can be applied to the data recorded by the GPS receiver. To improve the solution even further, accurate ephemeris data can be applied. The most accurate GPS solution comes as a result of combining differential corrections with accurate ephemeris data.

Post-processing is an attractive solution because you do not need a DGPS enabled receiver to get accurate results. The downside is that it cannot be done in real-time. When real-time accuracy is needed, DGPS gives the best results.

2.3 INS Overview

This section describes how an inertial navigation system (INS) operates. An INS is composed of two main pieces of equipment: an IMU and a GPS receiver. This section will provide a basic explanation of an IMU and discuss how the IMU can be combined with a GPS receiver to provide increased positional accuracy and precision.

2.3.1 Inertial Measurement Unit

An inertial measurement unit (IMU) is a device used to measure the inertial forces generated by the movement of an object. Accelerometers and gyroscopes are used to measure the linear acceleration and angular velocity of the object about three perpendicular axes. The axes are shown in Figure 2.7. For high-end systems, an acceleration bias error on the order of several milli-gs and an angular velocity bias error of one degree-per-hour are not uncommon.

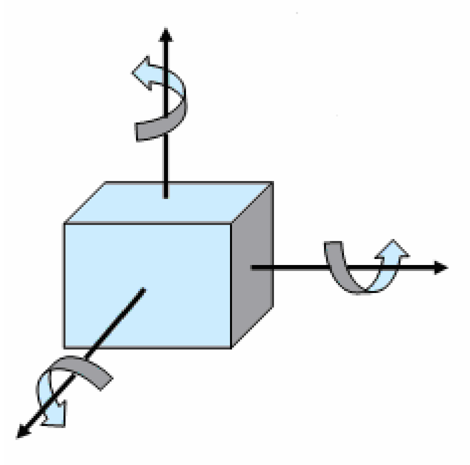

Figure 2.7: The Three Orthogonal IMU Axes

2.3.2 Inertial Navigation System

This section provides an overview of how an inertial navigation system (INS) operates. First, the basic principal of an INS is described, and then some more complicated aspects of the system are discussed. Finally, the errors inherent in the system are considered.

The Basic Idea: Linear acceleration and angular velocity data can be used to determine the change in position and orientation of an object. Integrating the acceleration twice will give a change in position. Integrating the angular velocity once will give a change in orientation. If the initial position and orientation is known, the IMU data can be used to estimate absolute position. This is the basic idea behind an inertial navigation system (INS).

M**ore Complicated Aspects:** Even when the IMU is sitting stationary on the surface of the Earth, it is experiencing acceleration. Most notably, the IMU would experience acceleration towards the center of mass of the Earth due to gravity. Another acceleration experience by the stationary IMU is the centripetal acceleration due to the rotation of the earth about its axis. The direction of the accelerations due to gravity (a_g) and the rotation of the Earth (a_c) are shown in Figure 2.8 below.

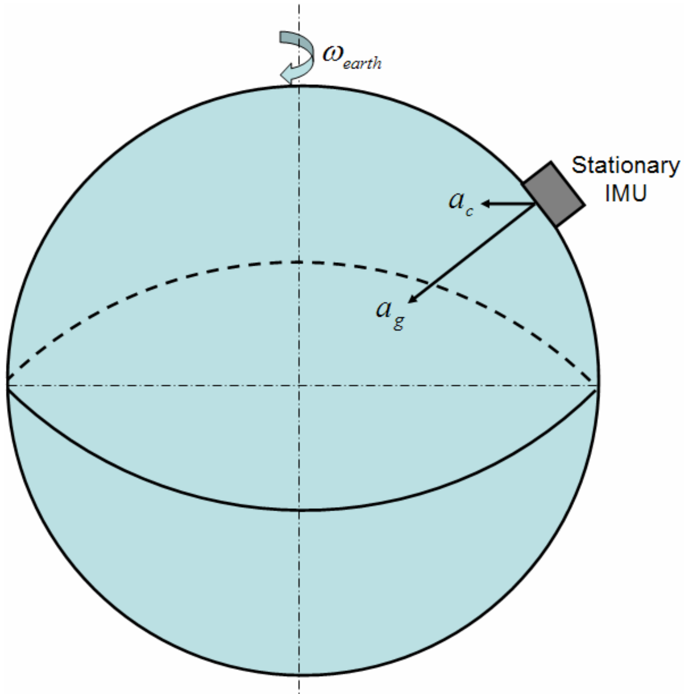

Figure 2.8: Gravity and Centripetal Acceleration Vectors

If we were to integrate the acceleration readings from a stationary IMU sitting on the surface of the Earth, the result would show that the IMU is moving. The IMU is, in fact moving, moving through space.

If the IMU begins to move relative to the Earth, two other accelerations come into play. In order to change from zero velocity to a non-zero velocity, the IMU must be acted upon by a force and, consequently, acceleration. This is the acceleration that most people are familiar with, the acceleration that pushes your back against the seat when you depress the gas pedal in a car. When moving within a rotating reference frame such as the Earth, Coriolis acceleration is also present. The Coriolis acceleration is a function of linear velocity of the IMU and the angular velocity of the Earth. The Coriolis acceleration is best demonstrated with an example.

Let us say you get in a car and drive from the North Pole due south towards the Equator at a constant speed. The North Pole is located at the axis of rotation of the Earth so, when you begin, you will have no tangential velocity due to the rotation of the Earth. As soon as you move away from the North Pole you will be at some non-zero distance away from the axis of rotation of the Earth and will therefore have some tangential velocity. The Coriolis acceleration accounts for the change in tangential velocity.

The Coriolis acceleration is small compared to gravity and the centripetal acceleration cause by the Earth's rotation. Table 2.1 provides a comparison of the magnitudes of these three accelerations. The value for gravity was calculated at sea level, at the Equator. The centripetal acceleration was calculated at the Equator, where it would be the largest. The Coriolis acceleration was calculated at the North Pole, where the surface velocity equals the radial velocity. A radial velocity of 15 *m/s* (33.5 mph) was used for the calculation. Each calculation can be found in Appendix A.

Table 2.1: Acceleration Comparison

	Acceleration (m/s^2)
Gravity	9.7982
Centripetal	0.03373
Coriolis	0.002182

For navigation computations, the only acceleration of interest is the motion induced acceleration in the desired reference frame. If all other accelerations are removed, integrating the remaining acceleration will give the movement of the IMU relative to the Earth. Commercial inertial navigation systems estimate and try to remove the acceleration cause by the mass of the earth, the rotation of the Earth, and the Coriolis effect. In order to estimate these accelerations, the INS must have some knowledge about the location of the IMU relative to the Earth.

The acceleration due to gravity decreases as elevation increases. Knowledge of the elevation of the IMU helps the INS remove the effects of gravity. The centripetal acceleration is greatest at the Equator (farthest from the axis of rotation) and decreases to zero at the poles (at the axis of rotation). Information about the latitude of the IMU helps the INS remove the effects of centripetal acceleration. The latitude, along with the speed of the IMU, help the INS remove the effects of Coriolis acceleration.

Errors: No matter how accurate the accelerometers and gyroscopes in the IMU, there will be some bias error present in the measurements. Any error will be compounded as the measurements are integrated by the INS. As the integration goes on there will be more error and the error will begin to grow rapidly. This is the major downfall of inertial navigation systems. Depending on the accuracy of the sensors in the IMU, the change in position and orientation reported by an INS might only be useful for a few seconds to a few minutes. Any longer and the data reported by the INS could be extremely inaccurate.

2.3.3 INS with GPS Aiding

Combining an INS with a GPS receiver can yield accurate and precise position information. The INS can use the latitude and elevation data from the GPS to help estimate the effects of gravity, the Earth's centripetal acceleration, and the Coriolis acceleration. In turn, the INS can be used to extrapolate position and orientation data in the absence of GPS signals. Because of the inherent error of inertial navigation systems, the extrapolated position may only be accurate for a few seconds to a few minutes. However, once a new GPS signal is received, the INS position is reset and the integration begins again. By combining the data from the INS and GPS using a Kalman filter, the combined system can offer a more accurate and precise position than either of the systems can produce separately.

2.4 UTM Overview

This section provides an overview of the UTM coordinate system. Only the basics of the UTM are covered, as it is a complicated map projection. For more information, readers should consult the references.

When given the coordinates of two points on earth, calculating the distance and bearing between them is often complicated. Using an angular coordinate system such as latitude and longitude can be difficult because degrees of longitude do not exhibit a constant distance relationship. Only at the equator is the distance covered by a degree of longitude equal to that covered by a degree of latitude. Moving away from the equator, the distance covered by a degree of longitude gets smaller. This problem can be avoided by using a Cartesian coordinate system.

Calculating the distance and bearing between two points in a 3-D Cartesian coordinate system is relatively simple. However, if we assume the Earth is flat, the calculations fall into the two-dimensional, plane geometry case and become even easier. Because of the size of the Earth, this assumption is reasonable when working over small distances.

The Universal Transverse Mercator (UTM) system allows us to take advantage of both Cartesian coordinates and the flat-earth assumption. The UTM system subdivides the Earth into narrow longitudinal zones and projects each zone individually. By keeping the projection area small, the UTM system is very accurate over short distances. The derivation of the UTM system is summarized in Figure 2.9.

The method used to derive the UTM projections is rather complicated. However, most GPS units are capable of reporting locations in this format. For most purposes, it is only necessary to understand what the UTM coordinates mean, not how they were calculated.

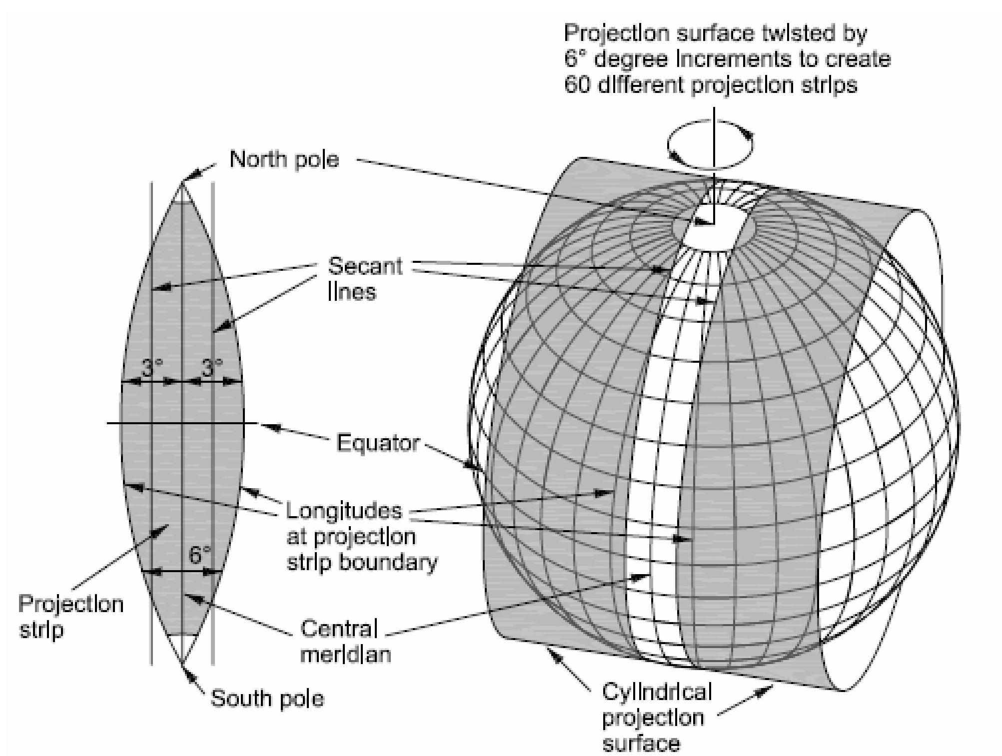

Figure 2.9: Derivation of the UTM System [4] Used Courtesy of Fred Touche. Copyright Fred Touche.

The location within a zone is measured in meters north and east of a reference point. The reference point is along the equator, at the central meridian of the zone. To avoid negative easting values, the central meridian is assigned a false-easting of 500000 meters. Locations to the west have an easting of 500000 meters minus the distance west of the central meridian. Locations to the east have an easting of 500000 meters plus the distance east of the central meridian. Within the northern hemisphere, the northing is measured in meters north of the equator. If working within the southern hemisphere, a false-northing is added [4]. The coordinate system within the Northern Hemisphere is summarized in Figure 2.10.

Like all projections, there are some errors associated with the UTM representation. However, the UTM system was designed in such a way that these errors are minimized. The UTM has a maximum error of 0.04% [5].

Although the UTM system is difficult to derive, the simplicity of the results and minimal error make it the ideal coordinate system to use when working with unmanned vehicles.

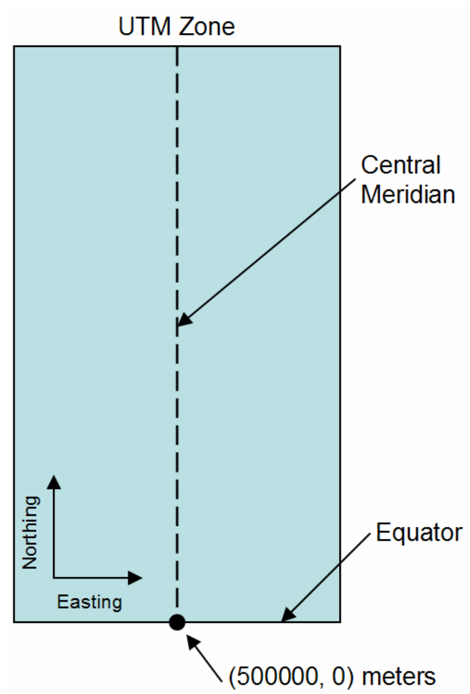

Figure 2.10: UTM Coordinates

2.5 Comparison of Differential Correction Sources

Several sources of differential corrections exist. Each source has an associated availability, cost, and performance. This section presents the results of a dynamic comparison of differential correction sources.

Kansas State University Agricultural Research Center: The performances of three differently corrected sources were tested under dynamic conditions to evaluate suitability for precision agriculture. The differential correction sources tested included the U.S. Coast Guard Beacon (Beacon), Wide-Area Augmentation System (WAAS), and OmniStar.

Data from the differential correction sources was collected by a rover traveling between 5 and 15 km/h. A Real Time Kinematic (RTK) GPS was used as the standard for accuracy. The rover can be seen in Figure 2.11.

Figure 2.11: Kansas State University DGPS rover [2]

The results of the dynamic test indicated that the Beacon source was not affected by ground speed. The WAAS and OmniStar, however, had a decrease in precision as ground speed increased. In addition, the relative accuracy of the WAAS and OmniStar position decreased as ground speed increased [1].

Chapter 3 – Differential Correction Experiment

This chapter provides an overview of the experiment that was conducted to test the relative positional accuracy and precision of various differentially corrected GPS methods. Specifically, the experiment was designed to compare differential correction methods under conditions of varying speed and direction.

3.1 Introduction

When selecting a GPS receiver, users are typically interested in the accuracy of the system. Over the years, several different techniques have been developed to improve the accuracy and precision of GPS. Atmospheric models, dual-frequency satellite signals, differential corrections, and post-processing are some of the techniques used to improve accuracy and precision.

Differential corrections are an attractive way to improve the real-time accuracy and precision of GPS, but there are several sources of differential corrections and each one has an associated accuracy and precision. The sources of differential corrections include: Coast Guard Beacon, Wide-Area Augmentation Service (WAAS), OmniStar VBS, OmniStar HP, local RT2, and local RT20. The Coast Guard Beacon and WAAS are free services. OmniStar is a subscription service. The local RT corrections are free, but require additional equipment and proper configuration of the systems.

The Coast Guard Beacon and WAAS corrections have a limited coverage area. They currently cover most but not all of the United States. OmniStar VBS and HP corrections cover not only the entire United States, but also much of the world. Local RT2 and RT20 corrections could be set up to provide local corrections anyplace in the world.

In dynamic applications, the speed and heading of the rover may also have an effect on the accuracy reported by the GPS receiver. These factors may have more of an affect on one differential correction method than another. This experiment proposes to determine what relationship, if any, exists between these factors and positional accuracy.

The results of this experiment will help designers choose the most effective solution for their positioning needs.

3.2 Experimental Variables

There are many factors that influence the position reported by a GPS receiver. Some of the factors include: satellite clock, receiver clock, satellite ephemeris, atmospheric conditions, signal multipath, satellite positions, and satellite geometry. These factors can be broken into two categories: variables affecting satellite signal and variables affecting position computation.

For this experiment, the variables of interest were differential correction method, rover speed, and rover heading. The range over which each variable was tested is presented in Table 3.1. Two sources of differential corrections were tested against a system with no corrections. OmniStar HP is a high-precision, satellite-based, subscription correction service. Local RT2 corrections are generated at a local base station by a dual-frequency GPS receiver. Three rover speeds were tested: 3 *m/s* (6.7 mph), 7.5 *m/s* (16.7 mph), and 12.5 *m/s* (28 mph). This speed range encompasses the typical speeds achieved by most current unmanned vehicles. Two headings were tested, 180 degrees apart. The headings were chosen such that they were not aligned with a cardinal direction of the local UTM coordinate system.

Table 3.1: Experimental Variables for Differential Correction Test

Experimental Variable	Range
Differential Correction Method	None OmniStar HP Local RT2
Rover Speed	3 m/s 7.5 m/s 12.5 m/s
Rover Heading	Northeast Southwest

An attempt was made to hold all other variables constant. Several variables however, could not be controlled. The location of the GPS satellites and the atmospheric conditions change over time. One way to reduce the effects of the satellite positions and atmospheric conditions is to collect all the data at the same time. By doing so, the error introduced by the satellites and atmosphere would be constant across all the data.

Since there was only one rover, it was not possible to test all speed and heading combinations at once. There were however enough GPS receivers to test all the differential correction modes at the same time. To ensure each GPS unit received the same satellite signals, only one GPS antenna was used. The signal from the antenna was split four ways and sent to the three GPS receivers. The fourth splitter connection went to another GPS receiver. This GPS receiver was used to seed the INS that was used to collect the truth data.

Multiple runs had to be conducted to collect an assortment of rover speed and heading combinations. Each combination of the variables listed in Table 3.1 was tested multiple times in a random order. This was done to spread any time-dependant error across all the data.

3.3 Assumptions

Several assumptions were made during the design of this experiment. These assumptions dealt with the equipment performance, rover performance, differential correction accuracy, and experimental timeframe. This section presents those assumptions and provides some insight into why they were made.

It was assumed that the input to all rover GPS receivers was the same. Such an assumption required that the signal splitter delivered the same signal to each receiver and that the electronics at each receiver read the signal the same way. The splitter was not tested, but it was assumed to output the same signal to each receiver. Each GPS unit was assumed to read the input signal in the same way because each receiver uses the *NovAtel* OEM4 GPS card.

An INS system was used to collect the truth data for the experiment. By using the INS to collect truth data, we are assuming that the INS position data is more accurate than GPS position. For this assumption to be reasonable the INS must not be sampled over a long period of time or the accrued error will become large. During the experiment the INS was sampled over one-second intervals; by sampling over a short time period, the amount of error accrued is assumed to be negligible. The manufacturer's specifications for the IMU used in the experiment (*NovAtel* IMU-G2-H62) report an angular velocity bias of 10 degrees per hour and acceleration bias of 3 milli-g's.

Variation in the speed and heading of the rover during a given run were assumed to be insignificant. To the best ability of the driver, the rover was generally held at the same speed and heading throughout a run.

With differential corrections, the distance from the rover to the base station affects the accuracy and precision of the rover GPS receiver. If the rover is close to the base station, the atmospheric conditions and satellite geometries seen by the rover and base station are essentially the same and the differential corrections will be accurate. As the distance between the rover and base station increases, the accuracy of the differential corrections decreases. During the course of the experiment, the distance between the rover and base station was always changing. However, since the test site was located close (between 100 and 200 meters) to the base station, it was assumed that the rover would see the same atmospheric conditions and satellite configurations as the base station throughout the entire test site. Any error introduced by the distance between the base station and rover was considered negligible.

During the course of the experiment, the satellite configuration and atmospheric conditions were constantly changing. Although these effects would change the error in the GPS signal, they were ignored because the rover speed and heading combinations were performed in a random order. It is assumed that any error influenced by time would be spread across all heading and speed combinations.

3.4 Base Station Instrumentation

A Mobile Instrumented Platform (MIP) was used as the base station for RT2 corrections. The MIP is an instrumented trailer than can be quickly and easily erected at remote testing sites.

Atop the MIP was a *NovAtel* GPS-600-LB antenna connected to a *NovAtel* FlexPak-G2L GPS receiver. The location of the base station must be known before differential corrections can be generated. The position of the base station was determined by averaging the base station FlexPak GPS reading for 20 minutes. The 20 minute averaging period was established after a discussion with *NovAtel* technical support. After it averaged its own position, the FlexPak was used to generate the RT2 corrections. The corrections were then broadcast over an 802.11 wireless connection. The MIP is shown in Figure 3.1. The MIP instrumentation is shown in Figure 3.2.

Figure 3.1: MIP Station

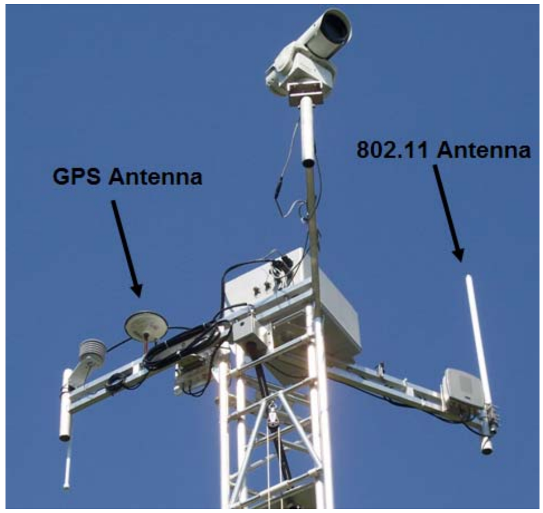

Figure 3.2: MIP Station Instrumentation

3.5 Rover Instrumentation

The signal from a single GPS antenna (*NovAtel* GPS-600-LB) on the rover was run through a splitter (*GPS Networking, Inc.* LDCBS1X4) and sent to four NovAtel OEM4 GPS receivers. A different receiver was used for each of the three differential correction methods; a fourth receiver was used in conjunction with an IMU to provide truth data. The *NovAtel* equipment used on the rover is presented in Table 3.2. Although different models of GPS receivers were used, they are all based on the *NovAtel* OEM4 GPS card.

Table 3.2: NovAtel Equipment used in Experiment

Differential Correction Method	Model
None	ProPak-LB
OmniStar HP	ProPak-LB+
Local RT2	FlexPak-G2L
Truth Data	ProPak-LB+ IMU-G2-H62

The GPS antenna was mounted to the roof of a Ford Excursion. The Excursion functioned as the rover during the experiment. The signal from the antenna was then split and sent to the four GPS receivers. Three of the GPS receivers were located inside the rover, while the fourth receiver and IMU were mounted to the roof. The receiver and IMU were mounted to the roof to make the IMU-to-antenna offset easier to measure; the precise position of the IMU relative to the GPS antenna must be known. The configuration of the rover is shown in Figure 3.3.

While the OmniStar HP corrections were obtained directly through the GPS antenna, the local RT2 corrections had to be relayed a different way. A laptop with an 802.11b wireless card was used to read the local RT2 corrections broadcasted by the MIP station. The corrections were then sent to the RT2-enabled GPS receiver over an RS-232 serial.

Data from each of the four GPS receivers was transmitted over an RS-232 serial connection. A laptop was used to record the data from the receivers, with each receiver connected to its own serial port. One serial port was used for both sending corrections to and receiving data from the RT2-enabled receiver.

3.6 GPS Receiver Configuration

The rover GPS receivers were configured to output *NovAtel*-style messages at one second intervals over an 115200 bps RS-232 serial connection. The MIP receiver was configured to output RTCM-style corrections at one second intervals. The configuration of each GPS receiver is summarized in Table 3.3.

Table 3.3: GPS Receiver Configuration

	None	OmniStar HP	Local RT2	IMU	MIP
Input Messages	.	.	RTCM1 RTCM3 RTCM1819 RTCM22	.	.
Output Messages	Position	Position	Position	Velocity Attitude	RTCM1 RTCM3 RTCM1819 RTCM22
Message Frequency (Hz)	1	1	1	1	1
Data Rate (bps)	115200	115200	115200	115200	115200

Figure 3.3: Rover Instrumentation (a) A Ford Excursion acted as the rover. (b) Three of the GPS receivers were located in the back seat. (c) The GPS antenna, fourth GPS receiver, and IMU were located on the roof.

3.7 Test Site

The experiment was performed on the Virginia Tech campus. Plantation Road is a generally flat, straight road that runs in the general Southwest-Northeast direction. An aerial photo of the test site, along with the location of the MIP is shown in Figure 3.4 (a). A site survey of the 802.11 wireless network was performed to ensure the rover could receive local RT2 corrections over the entire test site. The wireless site survey was performed with the aid of software that recorded the signal strength and the position of the rover. In Figure 3.4 (b), the green sections represent a good connection, whereas the purple sections represent a poor connection.

Two markers were established along the road, denoted by the “1” and “2” in Figure 3.4. These markers served to indicate to the driver of the rover the start and stop of the course. Figure 3.5 is a photograph taken from southwest end of the test site, looking towards the MIP.

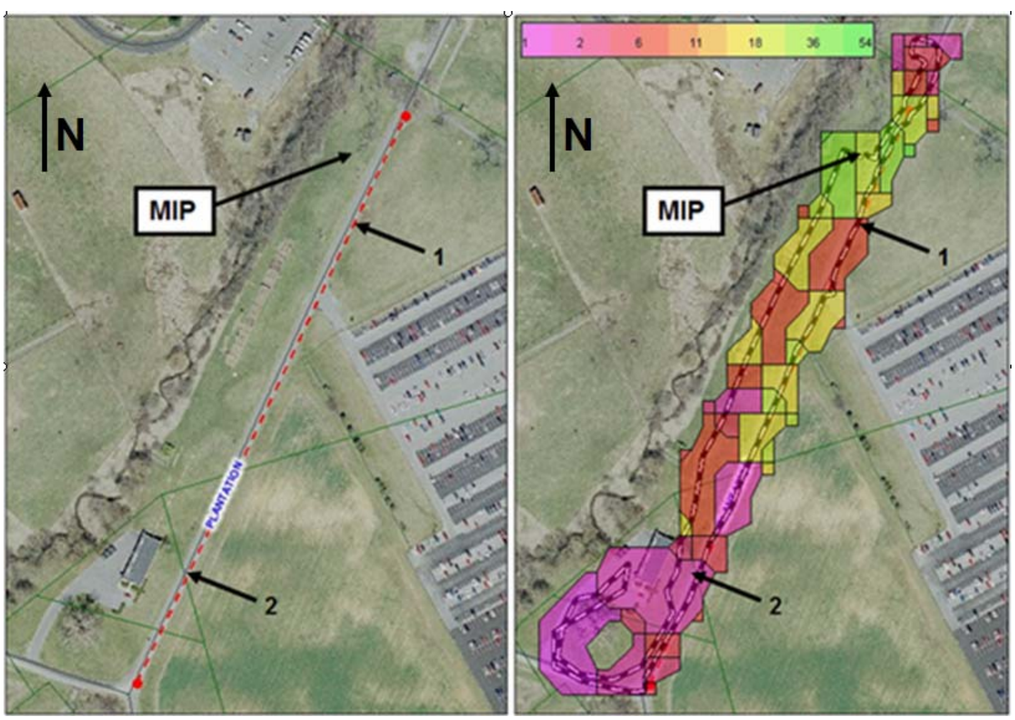

Figure 3.4: Test Site Aerial View (a) Plantation Road was chosen as the site of the experiment. (b) A site survey of the wireless network indicated the rover could receive RT2 corrections from the MIP. Aerial Image Copyright Commonwealth of Virginia [6].

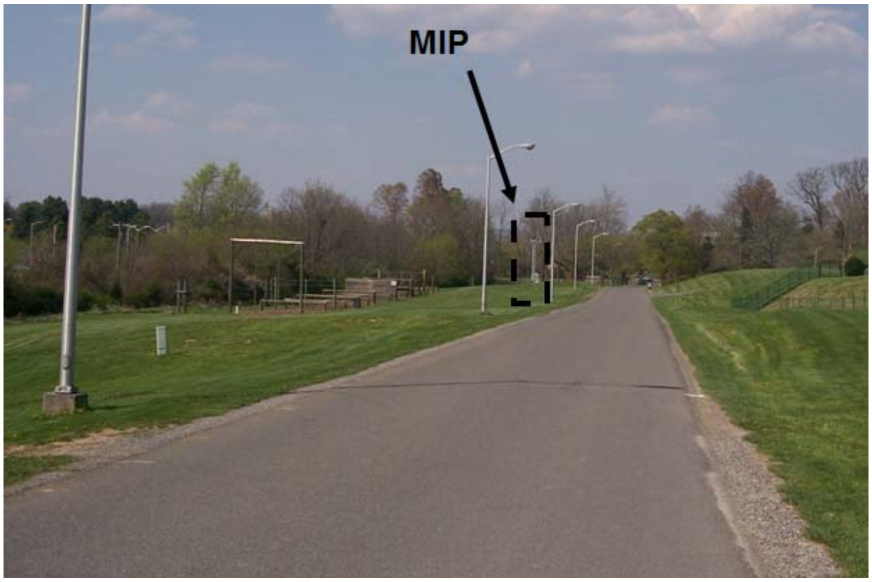

Figure 3.5: Test Site Ground View

3.8 Procedure

Prior to any data collection, all GPS receivers were allowed sufficient time to reach the proper state. For the receivers at the rover, this meant each unit was allowed to acquire the proper correction mode. The correction mode was verified by checking the position data from the GPS receivers. For the receiver at the MIP, this meant that the unit was allowed to complete position averaging. The MIP GPS receiver was commanded to average its position for 20 minutes, per a recommendation from *NovAtel* technical support.

Each combination of speed and direction was driven multiple times in a random order. The order was generated before the experiment with the aid of a random number generator. When driving away from the MIP (Southwest), the driver of the rover was instructed to be up to speed by the time the he passed the first marker. The driver was to maintain the speed until passing the second marker. When driving the opposite direction, the driver was to maintain speed between the second and first marker.

Data from each of the four rover GPS receivers was recorded with a laptop. Data collection continued even while the rover was between runs. To help sift through the data later, the GPS time tag was recorded each time a marker was passed. All data was collected between 1:00 pm and 4:30 pm EST on April 20, 2005.

Chapter 4 – Results and Analysis

This chapter discusses the results and analysis of the data collected during the Differential Correction Experiment discussed in Chapter 3.

4.1 Data Preparation

Before any of the data analysis began, all the position data was converted to the UTM coordinate system. The UTM system is a Cartesian coordinate system that can be used to measure positions in meters East and North of a reference point.

During the course of the experiment, the data collection did not stop between consecutive runs. While this made the testing run more quickly, date irrelevant to the experiment was collected. For example, in Figure 4.1 it can be seen that the rover position was recorded while the rover was turning around.

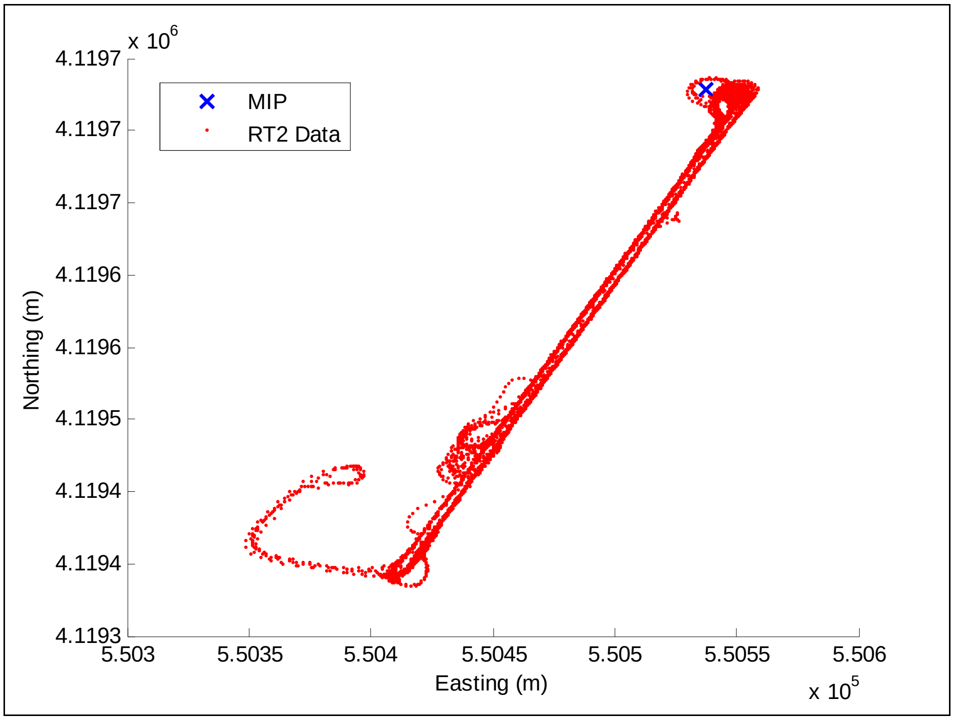

Figure 4.1: Experimental Data

Per the design of the experiment, the time tag of the beginning and end of each run was recorded. Most of the irrelevant data was removed be keeping only the data between the start and stop time tags. The start and stop time tags worked well to prune the data, but they only offered a rough estimate of the beginning and end of a run.

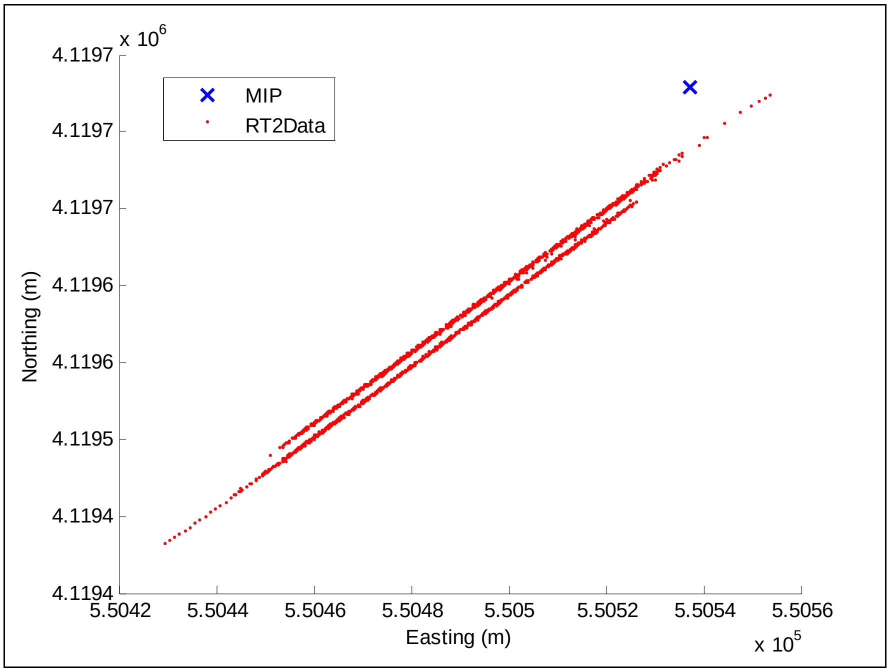

Figure 4.2: Trigger Trimmed Data

To make sure all the data generally reflected the same start and end location, only the data within a certain distance range was kept. The data was trimmed to a 100 meter run. The distance limits and the final data set are shown in Figure 4.3 and Figure 4.4, respectively.

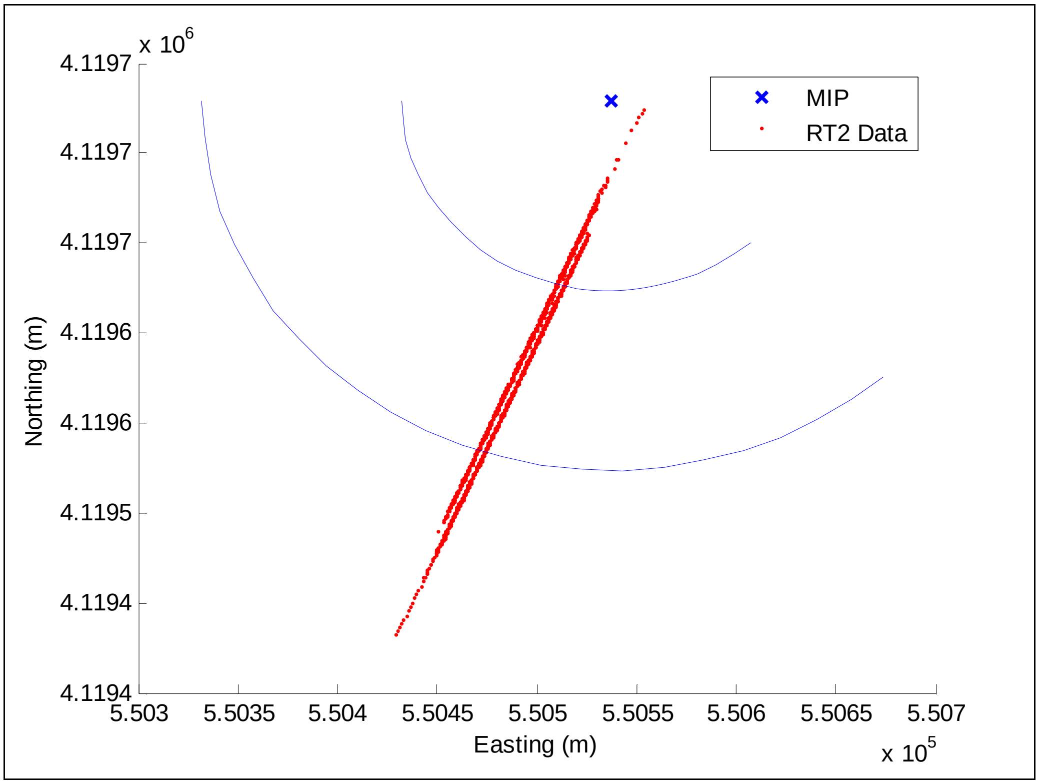

Figure 4.3: Distance Range

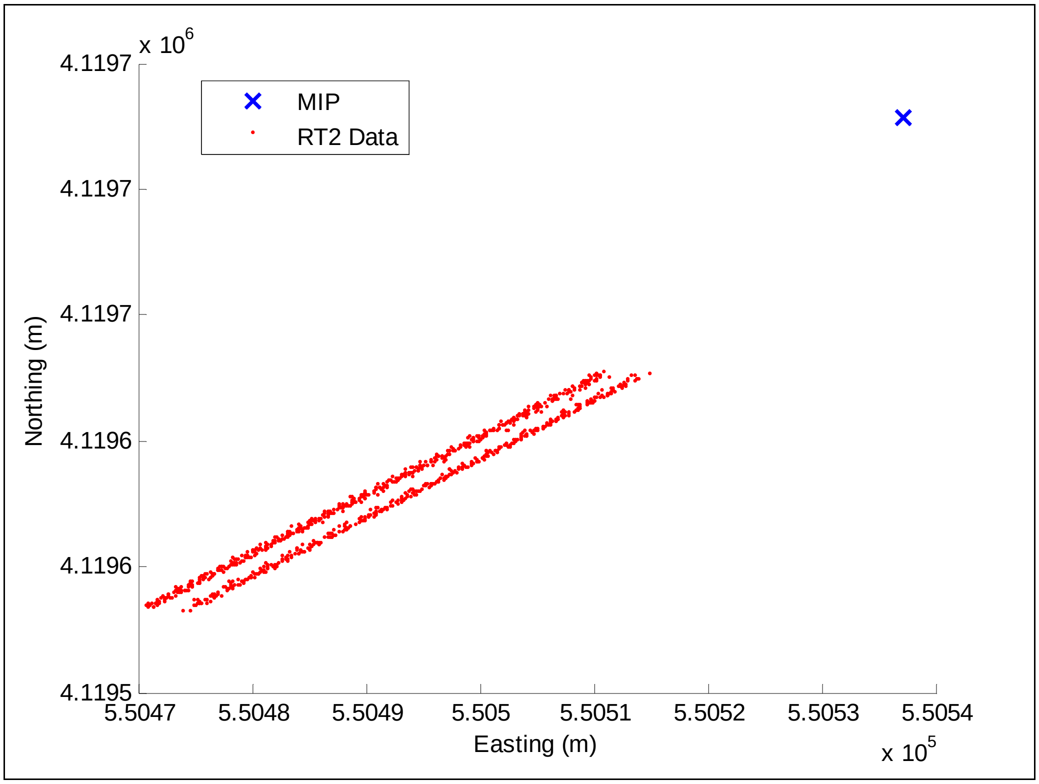

Figure 4.4: Trimmed Data Set

4.2 Data Analysis

There was not an absolute position baseline recorded during the differential correction experiment. As a consequence, no statements can be made about the absolute accuracy of any of the systems compared. Instead, a GPS aided INS was used to provide a relative baseline. By measuring the change in position reported by the INS, a relative measure of accuracy can be provided.

Since no points during a given run were at a known location, an absolute measure of the error could not be made. If it is assumed that the rover did not change speed or heading between INS readings, then the relative change in position of the rover can be calculated. Comparing the vector reported by the INS and the vector calculated between consecutive positions produced a relative error.

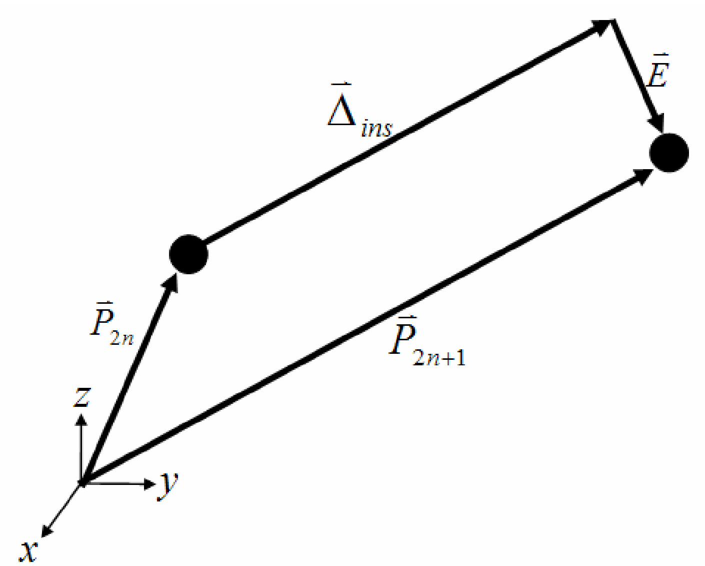

Figure 4.5: Relative Error graphical derivation

The operation shown in Figure 4.5 makes the assumption that there is no error in the position at point P_{2n} and that all the error is in the point P_{2n+1}. While this assumption is not perfect, it is reasonable because we are only using it to make a measure of relative, not absolute error. The derivation of the relative error term is summarized in Equation 4.1.

$$\vec{E} = \vec{P}_{2n+1} - \vec{P}_{2n} - \vec{\Delta}_{ins} \tag{4.1}$$

This method was used to calculate the relative error between non-overlapping sets of consecutive points throughout a given run. This process decimated the data by a factor of two. An example of how the relative error in the Northing measurement was calculated is provided in Figure 4.6. By taking non-overlapping sets, the error in one set does not affect the error in the sets before and after.

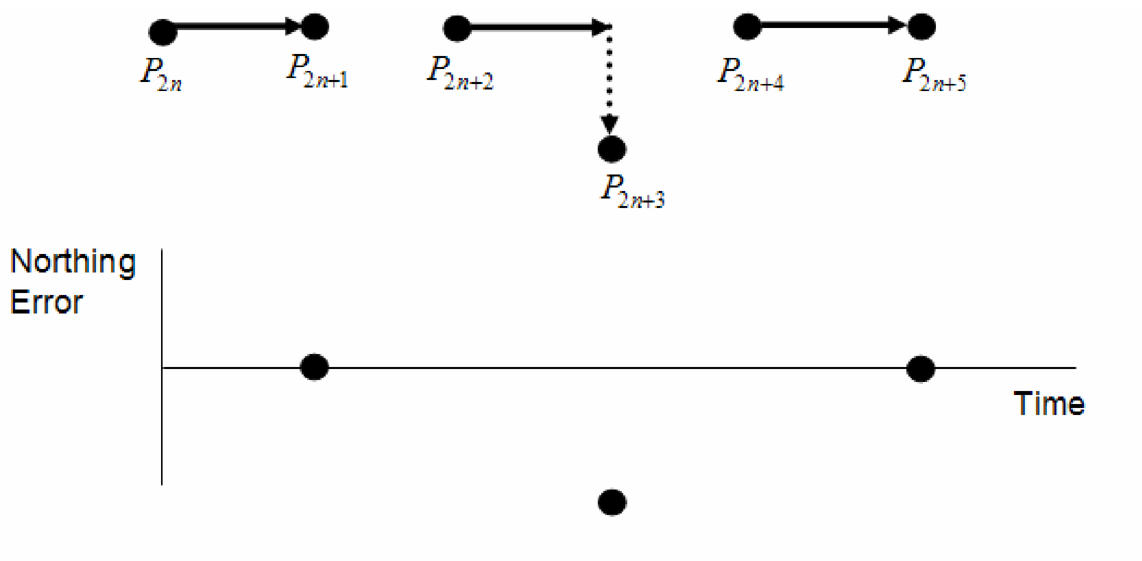

Figure 4.6: Relative Northing error calculation

Once a relative baseline was established for each run, the relative accuracy and precision of each GPS receiver was calculated. An analysis of variance (ANOVA) was performed to determine if the differential correction source, rover speed, or rover direction had any effect on accuracy or precision. A level of 5% was used as the criteria for statistical significance. The relative baseline data and results of the ANOVA are provided in Appendix B.

<u>Precision:</u> ANOVA failed to show that the rover speed or direction had a significant effect on the precision of the position data. The precision of the OmniStar HP and RT2 systems were similar and were both more precise than the system with no corrections. The only significant difference between the precision of the OmniStar HP and RT2 system was observed in the height measurement. The RT2 systems reported a more precise height measurement. The results of the precision analysis are shown in Table 4.1. The table shows the three differential correction sources and six speeds that were tested. The magnitude of the speeds were roughly 3 (m/s), 7.5 (m/s), and 12.5 (m/s). The sign of the speed indicates the direction of travel. Positive speed values signify that the vehicle was traveling away from the MIP. Negative values indicate the vehicle was moving towards the MIP.

Table 4.1: Results of Precision Analysis

DGPS Mode	Average Speed (m/s)	Easting StdDev (m)	Northing StdDev (m)	Height StdDev (m)
None	-12.69	0.0489	0.0603	0.1144
None	-7.67	0.0401	0.0427	0.0968
None	-3.24	0.0376	0.0532	0.0704
None	2.94	0.0588	0.0763	0.1436
None	7.54	0.0385	0.0561	0.0814
None	12.72	0.0501	0.0501	0.1169
OmniStar HP	-12.69	0.0337	0.0515	0.0486
OmniStar HP	-7.67	0.0263	0.0320	0.0236
OmniStar HP	-3.24	0.0254	0.0305	0.0198
OmniStar HP	2.94	0.0323	0.0446	0.0436
OmniStar HP	7.54	0.0290	0.0494	0.0252
OmniStar HP	12.72	0.0385	0.0467	0.0234
RTK	-12.69	0.0341	0.0482	0.0393
RTK	-7.67	0.0253	0.0325	0.0184
RTK	-3.24	0.0248	0.0295	0.0137
RTK	2.94	0.0274	0.0430	0.0130
RTK	7.54	0.0282	0.0494	0.0178
RTK	12.72	0.0384	0.0444	0.0268

The precision of the Easting and Northing measurement are shown in Figure 4.7 and Figure 4.8, respectively. These figures confirm that the speed and heading of the rover do not matter, only the source of differential corrections. There was not a significant difference observed in the Easting or Northing precision between the OmniStar HP and RTK corrections.

The precision of the height measurement is shown in Figure 4.9. This figure shows that the RTK corrections are the most precise, followed by the OmniStar HP corrections and the system with no corrections.

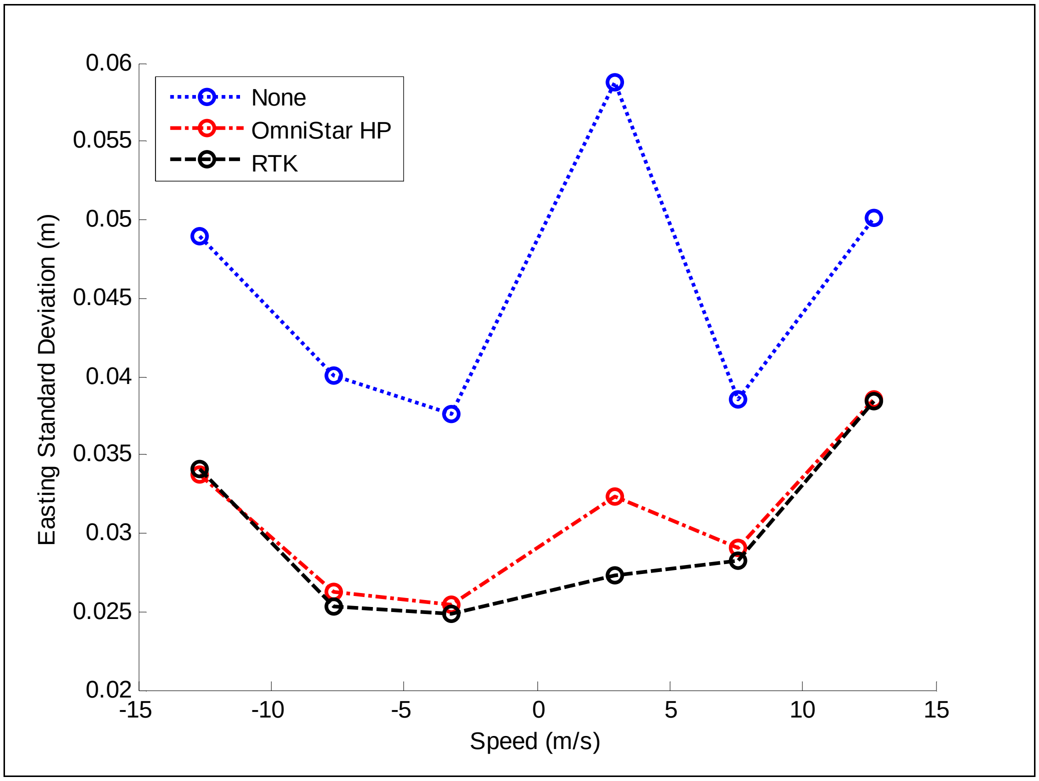

Figure 4.7: Precision of Easting Measurement

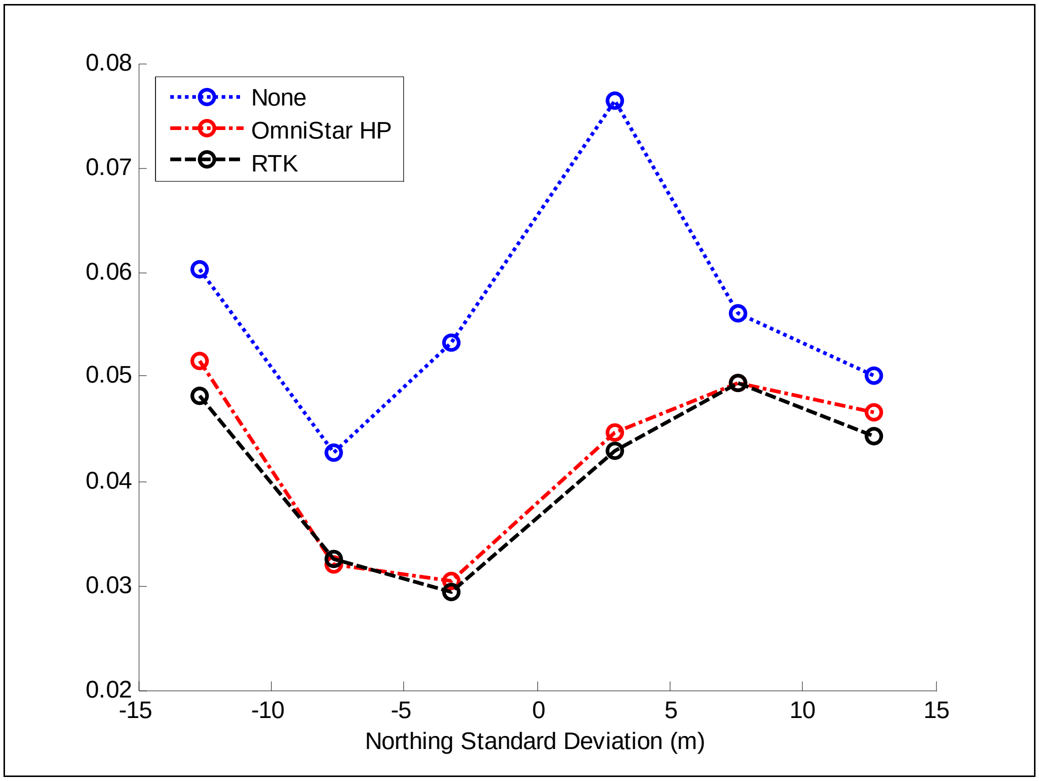

Figure 4.8: Precision of Northing Measurement

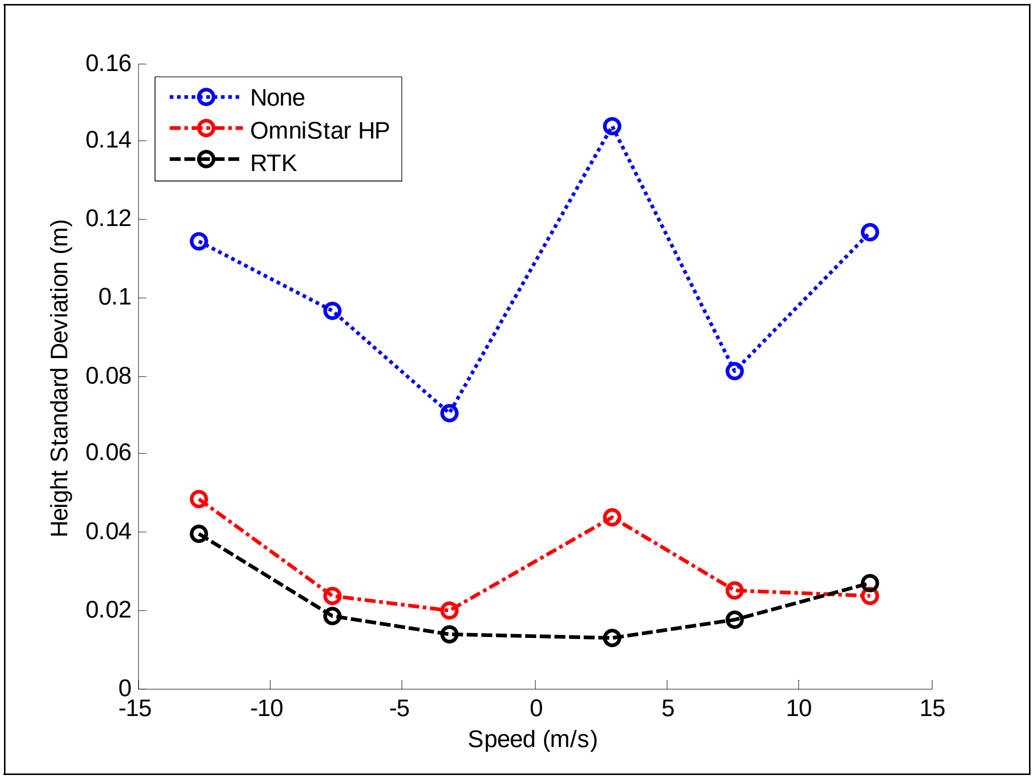

Figure 4.9: Precision of the Height Measurement

Relative Accuracy: ANOVA failed to show that the type of differential corrections used had a significant effect on the relative accuracy of the position data. The accuracies of the Easting and Northing positions were shown to be affected by the speed and direction of the rover. The accuracy of the height measurement was not observed to be affected by correction type, rover speed, or direction.

The accuracy of the Easting measurement was best at low speeds and grew slightly worse as rover speed increased. The direction of the rover had little impact on the magnitude of the Easting position, but affected the sign of the bias error. ANOVA also indicated that there was a secondary interaction between speed and direction which affected the Easting measurement.

The accuracy of the Northing measurement was significantly affected by the direction and speed of the rover. The relationship between speed, direction, and accuracy of the Northing measurement was not clear. The results of the relative accuracy analysis are shown in Table 4.2.

Table 4.2: Results of Relative Accuracy Analysis

DGPS Mode	Average Speed (m/s)	Easting Bias (m)	Northing Bias (m)	Height Bias (m)
None	-12.69	-0.1122	0.0440	0.0019
OmniStar HP	-12.69	-0.1101	0.0498	0.0135
RTK	-12.69	-0.1081	0.0512	0.0180
None	-7.67	-0.0574	0.0070	0.0002
OmniStar HP	-7.67	-0.0585	-0.0010	0.0080
RTK	-7.67	-0.0578	-0.0016	0.0071
None	-3.24	0.0203	0.0040	0.0138
OmniStar HP	-3.24	0.0209	0.0076	0.0065
RTK	-3.24	0.0213	0.0073	0.0090
None	2.94	0.0366	-0.0209	-0.0049
OmniStar HP	2.94	0.0333	-0.0191	-0.0073
RTK	2.94	0.0272	-0.0216	0.0060
None	7.54	0.0619	-0.0019	0.0150
OmniStar HP	7.54	0.0626	-0.0007	0.0052
RTK	7.54	0.0633	-0.0007	0.0026
None	12.72	0.1149	0.0349	0.0035
OmniStar HP	12.72	0.1039	0.0152	0.0160
RTK	12.72	0.1043	0.0169	0.0181

The relative accuracy of the Easting measurement is shown in Figure 4.10. This figure confirms that the source of differential correction does not matter, only the speed and direction of the rover. The amount of error in the Easting measurement appears to grow linearly.

The relative accuracy of the Northing measurement is shown in Figure 4.11. This figure shows that the source of differential corrections does not have a significant impact on the error in the Northing measurement. Unlike the Easting measurement, a clear relationship between the speed, direction, and the amount of Northing error was not seen.

The error in the height measurement is shown in Figure 4.12. The source of differential corrections, speed of the rover, and heading of the rover were not observed to have any significant effect on the error in the height measurement.

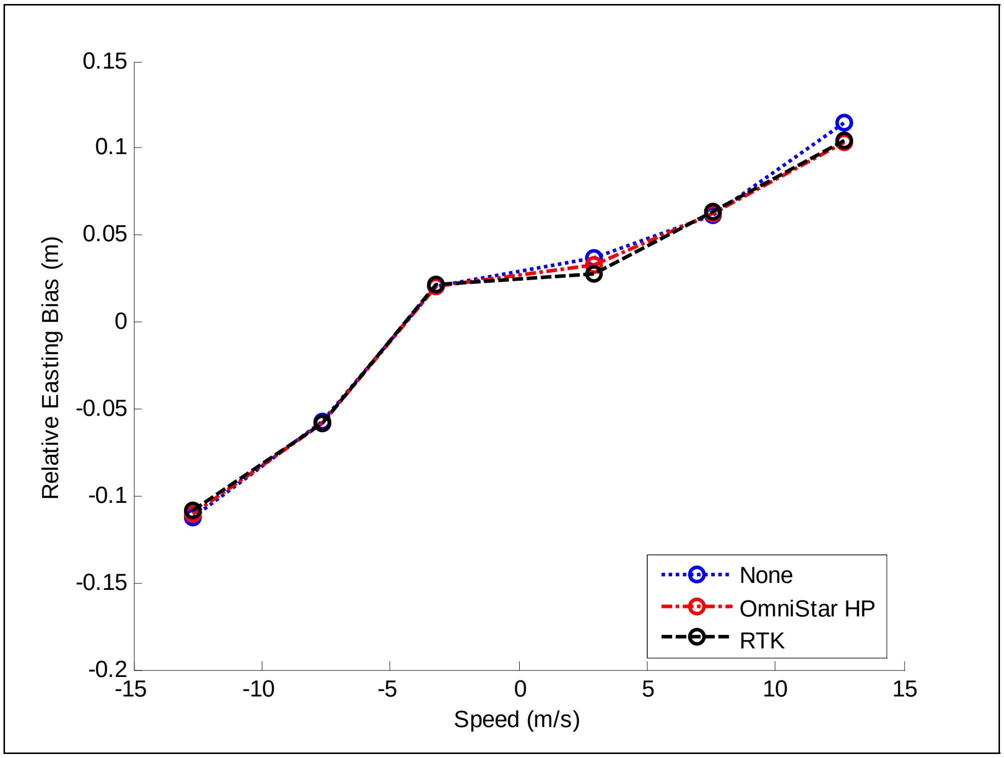

Figure 4.10: Relative Accuracy of Easting Measurement

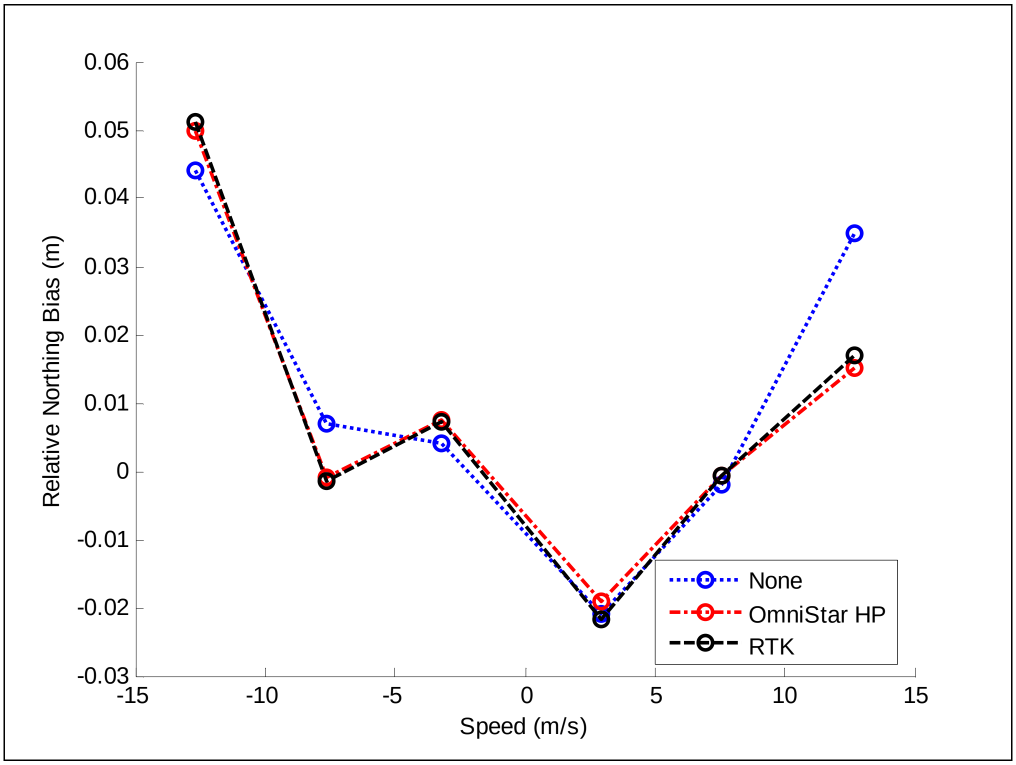

Figure 4.11: Relative Accuracy of Northing Measurement

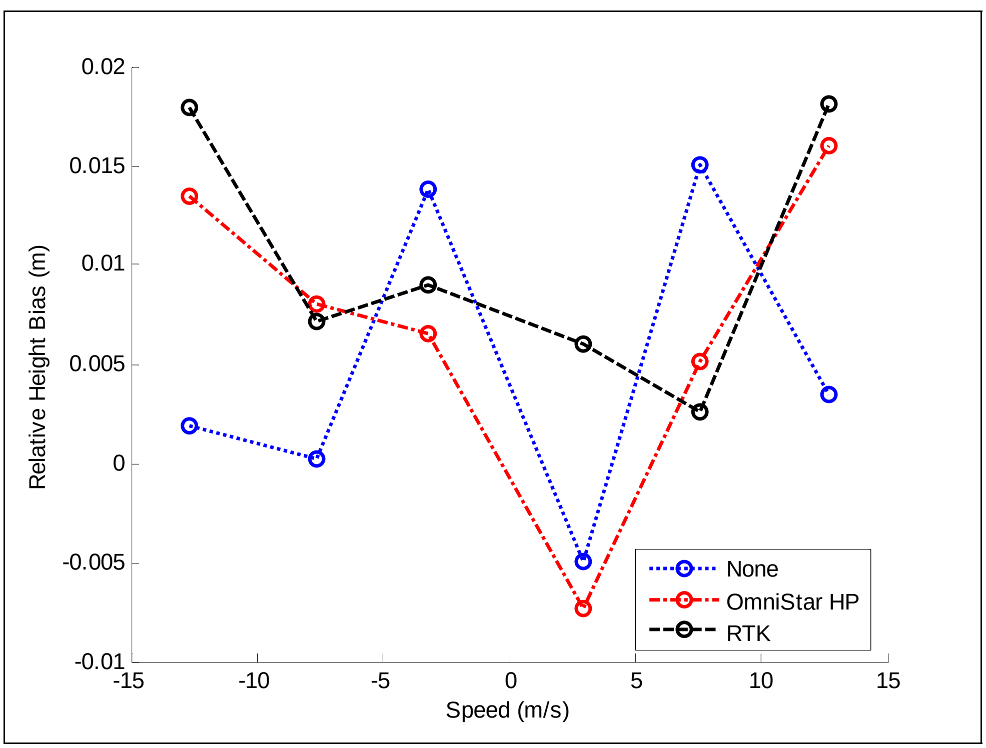

Figure 4.12: Relative Accuracy of Height Measurement

4.3 Interpretation

This section provides an interpretation of the results of the ANOVA. The interpretation is divided into sections for precision and relative accuracy.

Precision: The results of the precision analysis showed that local RT2 corrections were the most precise, followed closely by OmniStar HP. The system with no differential correction was the least precise. These results fall in line with the generally accepted static performance of each system; RT2 corrections are the most precise, followed by OmniStar HP and systems with no corrections. The rover speed and direction had no observed affect on the precision of any of the systems.

Relative Accuracy: The relative accuracy analysis showed that all the systems were affected by the speed and direction of the rover. In general, slower speeds had higher relative accuracy. This conclusion makes sense from an intuitive standpoint. The GPS receivers require some finite amount of time to compute the position of the rover, during which time the rover continues to move. The result is that the position reported by the GPS receiver lags the actual location of the rover. The faster the rover is moving, the more the lag and the less accurate the position. The direction of travel would also affect the accuracy. Two rovers going the same speed in opposite directions would have the same amount of lag in the position computation, but the lag would be in opposite directions.

The lag in the Easting measurement supports the intuitive influence of speed and direction on positional accuracy. As the speed of the rover increased, the accuracy of the Easting position decreased. In addition, the direction of the rover did not affect the magnitude of the accuracy, only the sign of the bias.

While the lag probably does affect the Northing and height measurements, it was not observed in the results of the experiment. It could be that other errors in the measurements mask the influence of the lag on the Northing and height position readings. Any lag in the height measurement would be hard to identify from this particular experiment because all runs took place on a relatively level road.

Chapter 5 – Differential Correction Experiment Conclusion

This chapter summarizes the results of the Differential Correction Experiment and offers suggestions about future experiments.

5.1 Experiment Findings

The experiment showed that local RT2 corrections, out of the three systems tested, offered the most precision under dynamic conditions. The precision of OmniStar HP was close to that of RT2. The only significant difference between the precision of the OmniStar HP and RT2 systems was observed in the height measurement. The RT2 system reported a more precise height measurement. The system with no corrections was the least precise of the three tested. The speed and direction of the rover were not observed to have a significant effect on the precision of the systems tested.

The type of differential corrections was not seen to have any influence on relative accuracy. Speed and direction did have an influence on the relative accuracy of the systems. The relative accuracy of the Easting measurement was observed to lag the baseline position. The amount of lag increased with speed and the lag was present in both directions of travel. The analysis revealed that the Northing measurement was affected by both rover speed and direction, but a trend in the error was not identified. The height measurement was not observed to be influenced by speed or direction.

5.2 Recommendations

Several lessons were learned during the course of the differential correction experiment that might be useful for future testing. This section details some of the lessons learned.

Relative versus Absolute Accuracy: Future testers should consider measuring the absolute position of the rover. By its very design, the experiment that was run could not identify any absolute bias imparted by the dynamics of the rover.

Testing Site: The test site chosen for the experiment was good for testing some basic interactions between speed and heading. However, for a more complete analysis of the effects of speed and direction, a more elaborate test site may be needed. Testing more than two directions might offer more insight into the influence of direction on accuracy. The elevation change at the Plantation Road test site was minimal. As a result, the influence of vertical speed could not be adequately tested.

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Appendix A – Acceleration Calculations

This appendix presents calculations of gravitational, centripetal, and Coriolis accelerations.

Gravity: The gravitational acceleration of the Earth at sea level is given by:

$$a_g = G\frac{M}{r^2} \qquad \text{(A1)}$$

where *G* is the gravitational constant, *M* is the mass of the Earth, and *r* is the radius of the Earth. The gravitational constant is $6.6726 \cdot 10^{-11}\ N \cdot m^2/kg^2$, the mass of the Earth is $5.9736 \cdot 10^{24}\,kg$, and the equatorial radius is 6378.1 *km*. Substituting these values into Equation A1 yields a gravitational acceleration of $9.7982\,m/s^2$.

$$a_g = G\frac{M}{r^2} = \left(6.6726 \cdot 10^{-11}\ N \cdot m^2/kg^2\right) \cdot \frac{\left(5.9736 \cdot 10^{24}\,kg\right)}{\left(6378.1 \cdot 10^3\,m\right)^2} = 9.7982\frac{m}{s^2}$$

Centripetal Acceleration: The centripetal acceleration due to the Earth's rotation is given by:

$$a_c = \omega^2 r \qquad \text{(A2)}$$

where ω is the angular velocity of the Earth and *r* is the distance from the axis of rotation of the Earth. At sea level at the Equator, the radius of the Earth is 6378.1 *km*. The angular velocity of the Earth can be calculated based on the fact that the Earth completes a rotation once every 24 hours. Substituting this information into Equation A2 yields a centripetal acceleration of $0.03373\,m/s^2$.

$$a_c = \omega^2 r = \left[\left(\frac{2\pi\ rad}{1\,rev}\right)\left(\frac{1\,rev}{24\,hr}\right)\left(\frac{1\,hr}{3600s}\right)\right]^2 \left(6378.1 \cdot 10^3\,m\right) = 0.03373\frac{m}{s^2}$$

Coriolis Acceleration: The Coriolis acceleration is given by:

$$a_{Coriolis} = 2(v_r \times \omega) \tag{A3}$$

where v_r is the radial velocity and ω is the angular velocity of the Earth. At the North Pole, the surface velocity will be equal to the radial velocity. The direction of the velocity and rotation vectors is shown in Figure A.1.

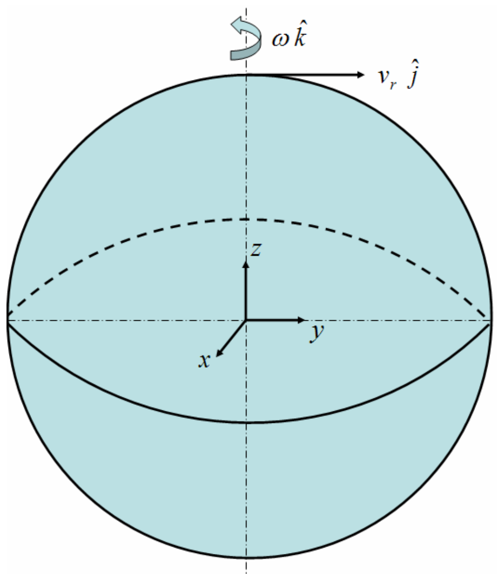

Figure A.1: Coriolis Acceleration

The angular velocity of the Earth can be calculated based on the fact that the Earth completes a rotation once every 24 hours. Using a surface velocity of 15 *m/s* (33.5 mph), the Coriolis acceleration is $0.002182\, m/s^2\, \hat{i}$.

$$a_{Coriolis} = 2(v_r \times \omega) = 2 \cdot \left(\begin{bmatrix} 0 \\ 15\, m/s \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ \left(\frac{2\pi\, rad}{1\, rev}\right)\left(\frac{1\, rev}{24\, hr}\right)\left(\frac{1\, hr}{3600 s}\right) \end{bmatrix} \right) = \begin{bmatrix} 0.002182 \\ 0 \\ 0 \end{bmatrix} \frac{m}{s^2}$$

Appendix B – Data Analysis

This appendix presents the data analysis that was performed on the data from the differential correction experiment. The appendix is divided into two sections: relative accuracy and precision of all runs and the ANOVA.

B.1 Relative Accuracy and Precision

This section provides the relative accuracy and precision of each run in the differential correction experiment. Positive speeds indicate the rover was moving away from the MIP (Southwest). Negative speeds indicate the rover was moving towards the MIP (Northeast).

Table B.1: Relative Accuracy and Precision

DGPS Mode	Start Time	Number of Points	Average Speed (m/s)	Easting Bias (m)	Northing Bias (m)	Height Bias (m)	Easting StdDev (m)	Northing StdDev (m)	Height StdDev (m)
None	321043	20	2.44	0.0140	-0.0387	0.0725	0.1123	0.2170	0.4616
None	321198	19	-2.55	0.0497	0.0200	0.0288	0.0995	0.1925	0.3666
None	321332	4	12.84	0.0828	0.0154	-0.0838	0.0308	0.1106	0.1746
None	322006	21	2.43	0.0242	0.0353	-0.0060	0.0204	0.0410	0.0350
None	322170	7	7.35	0.0829	0.0568	-0.0054	0.0153	0.0373	0.0925
None	322389	4	-12.60	-0.1477	0.0769	-0.0799	0.0548	0.0786	0.2304
None	322498	6	-7.52	0.0860	0.0134	-0.0123	0.0349	0.0502	0.1118
None	322629	6	-7.54	-0.0927	-0.0243	-0.0052	0.0372	0.0313	0.1711
None	322769	18	-2.80	0.0812	0.0136	0.0194	0.0198	0.0442	0.0251
None	322933	6	-7.55	-0.0658	0.0072	-0.0045	0.0242	0.0715	0.1214
None	323043	4	-12.92	-0.0903	-0.0103	0.0437	0.0378	0.0656	0.2365
None	323200	4	12.69	0.1651	-0.0105	-0.0037	0.0444	0.0519	0.0371
None	323256	4	-12.81	0.0021	0.0722	0.0703	0.0263	0.0398	0.0949
None	323312	6	7.63	0.1083	-0.0264	-0.0200	0.0452	0.0276	0.0401
None	323417	7	7.23	0.0268	-0.0342	0.0206	0.0534	0.0303	0.1948
None	323526	4	12.86	0.1085	0.0399	0.1607	0.0428	0.0253	0.2072
None	323684	17	2.84	0.1667	-0.0373	-0.1464	0.1835	0.1433	0.4613
None	324681	6	7.49	0.0787	-0.0366	0.0202	0.0425	0.0623	0.0378
None	324772	4	12.66	0.1513	0.1345	-0.0747	0.0670	0.0255	0.1513

None	324884	17	2.86	0.0354	-0.0878	0.0025	0.0352	0.0624	0.0298
None	325050	14	-3.48	-0.0371	0.0529	0.0130	0.0381	0.0172	0.0153
None	325292	15	-3.42	0.0755	-0.0827	0.0193	0.0195	0.0249	0.0215
None	325395	16	3.05	0.0110	-0.0198	0.0035	0.0163	0.0368	0.0418
None	325499	6	-7.57	-0.0981	-0.0084	0.0546	0.0547	0.0502	0.0775
None	325568	7	7.51	-0.0457	-0.0682	0.0604	0.0771	0.1113	0.1349
None	325656	4	-12.54	-0.1592	-0.0252	-0.0176	0.0300	0.0425	0.1270
None	325702	4	12.79	0.1829	0.0232	-0.0061	0.0317	0.0083	0.0596
None	325811	6	7.91	0.1501	0.0528	0.0271	0.0235	0.0862	0.0774
None	325932	14	3.49	0.0311	0.0559	0.0170	0.0288	0.0625	0.0248
None	326031	7	-7.02	-0.0522	-0.0369	-0.0125	0.0419	0.0479	0.0735
None	326150	6	-7.97	-0.0915	0.0110	0.0266	0.0397	0.0418	0.0428
None	326339	4	-13.07	-0.0517	0.0208	0.0293	0.0913	0.0976	0.1079
None	326621	4	-12.52	-0.1545	0.0987	-0.0299	0.0298	0.0301	0.0279
None	326680	4	12.73	0.0151	0.0089	-0.0119	0.0476	0.0414	0.0375
None	329348	14	-3.48	-0.0518	0.0174	-0.0028	0.0180	0.0296	0.0315
None	329533	4	-12.37	-0.1831	0.0074	-0.0096	0.0432	0.0208	0.0267
None	329643	14	-3.45	-0.0171	-0.0139	0.0076	0.0293	0.0342	0.0164
None	329780	4	12.72	-0.0275	0.0385	-0.0025	0.0927	0.0286	0.0282
None	329918	6	7.97	0.0730	0.0700	0.0165	0.0174	0.0810	0.0310
None	330091	15	3.17	-0.0524	-0.0395	0.0078	0.0273	0.0366	0.0157
None	330179	14	-3.50	0.0417	0.0210	0.0116	0.0387	0.0302	0.0167
None	330278	17	2.89	0.0513	-0.0566	-0.0086	0.0751	0.0548	0.1905
None	330412	15	3.33	0.0480	0.0003	0.0134	0.0306	0.0326	0.0323
None	330554	7	7.01	-0.0151	0.0176	-0.0038	0.0307	0.0366	0.0753
None	330641	7	-7.15	-0.0269	-0.0015	0.0138	0.0385	0.0187	0.0506
None	330707	6	7.77	0.0978	-0.0490	0.0194	0.0414	0.0322	0.0486
None	331014	4	-12.72	-0.1135	0.1112	0.0088	0.0780	0.1075	0.0638
None	331237	6	-8.16	-0.1723	0.0596	0.0102	0.0364	0.0386	0.0519
None	331394	5	-8.55	-0.0033	0.0424	-0.0689	0.0529	0.0342	0.1705
None	331449	4	12.10	0.1489	0.0190	0.0733	0.0308	0.0879	0.0918
None	331609	4	13.06	0.2072	0.0448	-0.0201	0.0628	0.0710	0.2651

OmniStar HP	321043	20	2.44	-0.0002	-0.0116	0.0037	0.0328	0.0489	0.0361
OmniStar HP	321198	19	-2.55	0.0463	0.0353	-0.0085	0.0134	0.0224	0.0428
OmniStar HP	321332	4	12.84	0.0761	-0.0382	0.0107	0.0152	0.0438	0.0214
OmniStar HP	322006	21	2.43	0.0278	0.0394	-0.0146	0.0205	0.0405	0.0182
OmniStar HP	322170	7	7.35	0.0813	0.0521	0.0002	0.0170	0.0461	0.0121
OmniStar HP	322389	4	-12.60	-0.1460	0.0634	0.0116	0.0142	0.0437	0.0438
OmniStar HP	322498	6	-7.52	0.0879	0.0108	0.0097	0.0335	0.0547	0.0357
OmniStar HP	322629	6	-7.54	-0.0872	-0.0268	0.0156	0.0326	0.0315	0.0416
OmniStar HP	322769	18	-2.80	0.0789	0.0167	0.0168	0.0210	0.0452	0.0193
OmniStar HP	322933	6	-7.55	-0.0689	-0.0185	0.0002	0.0215	0.0396	0.0216
OmniStar HP	323043	4	-12.92	-0.0922	0.0466	0.0161	0.0460	0.0690	0.0408
OmniStar HP	323200	4	12.69	0.1628	-0.0033	-0.0041	0.0418	0.0593	0.0104
OmniStar HP	323256	4	-12.81	-0.0116	0.0738	0.0112	0.0059	0.0237	0.0772
OmniStar HP	323312	6	7.63	0.1145	-0.0176	-0.0042	0.0402	0.0198	0.0182
OmniStar HP	323417	7	7.23	0.0241	-0.0420	0.0009	0.0295	0.0380	0.0088
OmniStar HP	323526	4	12.86	0.1295	0.0215	0.0370	0.0134	0.0582	0.0235
OmniStar HP	323684	17	2.84	0.1399	-0.0567	-0.1041	0.0889	0.0831	0.2259
OmniStar HP	324681	6	7.49	0.0922	-0.0156	0.0241	0.0225	0.0670	0.0632
OmniStar HP	324772	4	12.66	0.0919	0.0496	-0.0138	0.0431	0.0169	0.0286
OmniStar HP	324884	17	2.86	0.0407	-0.0806	0.0077	0.0209	0.0395	0.0201
OmniStar HP	325050	14	-3.48	-0.0416	0.0524	0.0115	0.0390	0.0218	0.0161
OmniStar HP	325292	15	-3.42	0.0833	-0.0785	0.0020	0.0240	0.0298	0.0169
OmniStar HP	325395	16	3.05	0.0160	-0.0171	0.0028	0.0160	0.0316	0.0275
OmniStar HP	325499	6	-7.57	-0.1150	-0.0419	0.0076	0.0392	0.0245	0.0344
OmniStar HP	325568	7	7.51	-0.0318	-0.0457	-0.0111	0.0421	0.0471	0.0201
OmniStar HP	325656	4	-12.54	-0.1411	-0.0098	0.0175	0.0054	0.0437	0.0599
OmniStar HP	325702	4	12.79	0.1585	0.0239	0.0312	0.0292	0.0558	0.0278
OmniStar HP	325811	6	7.91	0.1334	0.0253	0.0033	0.0282	0.1005	0.0242
OmniStar HP	325932	14	3.49	0.0282	0.0533	0.0214	0.0334	0.0619	0.0178
OmniStar HP	326031	7	-7.02	-0.0462	-0.0364	-0.0035	0.0202	0.0300	0.0168
OmniStar HP	326150	6	-7.97	-0.1076	0.0083	0.0099	0.0216	0.0411	0.0188
OmniStar HP	326339	4	-13.07	-0.0574	0.0075	0.0128	0.0422	0.0758	0.0513

OmniStar HP	326621	4	-12.52	-0.1317	0.1057	-0.0001	0.0094	0.0220	0.0354
OmniStar HP	326680	4	12.73	0.0079	-0.0070	0.0058	0.0497	0.0350	0.0363
OmniStar HP	329348	14	-3.48	-0.0479	0.0179	0.0014	0.0175	0.0293	0.0231
OmniStar HP	329533	4	-12.37	-0.1807	0.0069	0.0207	0.0470	0.0255	0.0351
OmniStar HP	329643	14	-3.45	-0.0138	-0.0135	0.0114	0.0288	0.0315	0.0092
OmniStar HP	329780	4	12.72	-0.0337	0.0352	0.0153	0.0963	0.0281	0.0193
OmniStar HP	329918	6	7.97	0.0776	0.0747	0.0043	0.0153	0.0793	0.0213
OmniStar HP	330091	15	3.17	-0.0525	-0.0412	0.0072	0.0254	0.0321	0.0182
OmniStar HP	330179	14	-3.50	0.0409	0.0226	0.0111	0.0344	0.0335	0.0115
OmniStar HP	330278	17	2.89	0.0490	-0.0619	0.0058	0.0222	0.0330	0.0133
OmniStar HP	330412	15	3.33	0.0505	0.0044	0.0044	0.0308	0.0308	0.0153
OmniStar HP	330554	7	7.01	-0.0243	0.0140	0.0037	0.0276	0.0210	0.0198
OmniStar HP	330641	7	-7.15	-0.0246	-0.0044	0.0138	0.0290	0.0145	0.0145
OmniStar HP	330707	6	7.77	0.0964	-0.0511	0.0253	0.0387	0.0262	0.0387
OmniStar HP	331014	4	-12.72	-0.1197	0.1047	0.0179	0.0997	0.1088	0.0453
OmniStar HP	331237	6	-8.16	-0.1505	0.0690	0.0152	0.0204	0.0406	0.0181
OmniStar HP	331394	5	-8.55	-0.0144	0.0308	0.0037	0.0187	0.0117	0.0110
OmniStar HP	331449	4	12.10	0.1596	0.0237	0.0349	0.0301	0.0797	0.0240
OmniStar HP	331609	4	13.06	0.1824	0.0310	0.0270	0.0278	0.0432	0.0197
RTK	321043	20	2.44	-0.0014	-0.0129	0.0079	0.0327	0.0548	0.0138
RTK	321198	19	-2.55	0.0456	0.0319	0.0008	0.0124	0.0188	0.0114
RTK	321332	4	12.84	0.0752	-0.0351	0.0135	0.0140	0.0452	0.0186
RTK	322006	21	2.43	0.0253	0.0365	-0.0118	0.0190	0.0415	0.0142
RTK	322170	7	7.35	0.0800	0.0494	0.0092	0.0169	0.0458	0.0132
RTK	322389	4	-12.60	-0.1463	0.0639	-0.0013	0.0150	0.0422	0.0342
RTK	322498	6	-7.52	0.0915	0.0125	-0.0015	0.0358	0.0615	0.0160
RTK	322629	6	-7.54	-0.0863	-0.0250	0.0262	0.0287	0.0326	0.0175
RTK	322769	18	-2.80	0.0808	0.0200	0.0191	0.0191	0.0433	0.0176
RTK	322933	6	-7.55	-0.0678	-0.0228	-0.0052	0.0144	0.0395	0.0073
RTK	323043	4	-12.92	-0.0889	0.0527	0.0260	0.0514	0.0758	0.0478
RTK	323200	4	12.69	0.1620	-0.0060	0.0007	0.0405	0.0591	0.0205
RTK	323256	4	-12.81	-0.0085	0.0719	0.0105	0.0112	0.0212	0.0473

RTK	323312	6	7.63	0.1132	-0.0212	-0.0007	0.0407	0.0209	0.0124
RTK	323417	7	7.23	0.0237	-0.0425	0.0042	0.0300	0.0353	0.0128
RTK	323526	4	12.86	0.1308	0.0317	0.0348	0.0181	0.0542	0.0151
RTK	323684	17	2.84	0.0973	-0.0808	0.0191	0.0540	0.0584	0.0151
RTK	324681	6	7.49	0.0916	-0.0206	0.0084	0.0229	0.0736	0.0280
RTK	324772	4	12.66	0.0916	0.0568	0.0011	0.0434	0.0174	0.0352
RTK	324884	17	2.86	0.0378	-0.0790	0.0084	0.0180	0.0419	0.0177
RTK	325050	14	-3.48	-0.0392	0.0513	0.0075	0.0396	0.0211	0.0131
RTK	325292	15	-3.42	0.0778	-0.0807	0.0133	0.0233	0.0292	0.0159
RTK	325395	16	3.05	0.0121	-0.0164	-0.0007	0.0130	0.0321	0.0122
RTK	325499	6	-7.57	-0.1137	-0.0414	0.0029	0.0388	0.0249	0.0363
RTK	325568	7	7.51	-0.0308	-0.0427	-0.0122	0.0433	0.0456	0.0165
RTK	325656	4	-12.54	-0.1400	-0.0034	0.0199	0.0078	0.0493	0.0502
RTK	325702	4	12.79	0.1612	0.0178	0.0215	0.0269	0.0385	0.0380
RTK	325811	6	7.91	0.1381	0.0287	0.0086	0.0225	0.0939	0.0127
RTK	325932	14	3.49	0.0279	0.0561	0.0194	0.0323	0.0625	0.0099
RTK	326031	7	-7.02	-0.0468	-0.0376	0.0013	0.0209	0.0295	0.0198
RTK	326150	6	-7.97	-0.1079	0.0037	0.0070	0.0205	0.0411	0.0198
RTK	326339	4	-13.07	-0.0541	0.0083	0.0348	0.0430	0.0721	0.0476
RTK	326621	4	-12.52	-0.1328	0.1066	0.0011	0.0100	0.0177	0.0339
RTK	326680	4	12.73	0.0070	-0.0047	0.0085	0.0540	0.0336	0.0384
RTK	329348	14	-3.48	-0.0486	0.0176	0.0003	0.0146	0.0286	0.0134
RTK	329533	4	-12.37	-0.1744	0.0008	0.0231	0.0412	0.0165	0.0327
RTK	329643	14	-3.45	-0.0110	-0.0142	0.0133	0.0295	0.0314	0.0098
RTK	329780	4	12.72	-0.0319	0.0363	0.0209	0.0939	0.0271	0.0330
RTK	329918	6	7.97	0.0777	0.0734	-0.0042	0.0140	0.0777	0.0233
RTK	330091	15	3.17	-0.0541	-0.0417	0.0024	0.0255	0.0326	0.0076
RTK	330179	14	-3.50	0.0438	0.0248	0.0086	0.0355	0.0338	0.0146
RTK	330278	17	2.89	0.0500	-0.0599	0.0059	0.0223	0.0332	0.0149
RTK	330412	15	3.33	0.0503	0.0037	0.0036	0.0294	0.0297	0.0119
RTK	330554	7	7.01	-0.0224	0.0163	0.0002	0.0271	0.0256	0.0205
RTK	330641	7	-7.15	-0.0240	-0.0040	0.0130	0.0293	0.0117	0.0153

RTK	330707	6	7.77	0.0982	-0.0472	0.0096	0.0364	0.0256	0.0205
RTK	331014	4	-12.72	-0.1197	0.1086	0.0296	0.0934	0.0909	0.0211
RTK	331237	6	-8.16	-0.1494	0.0692	0.0192	0.0208	0.0405	0.0210
RTK	331394	5	-8.55	-0.0160	0.0312	0.0013	0.0187	0.0112	0.0124
RTK	331449	4	12.10	0.1573	0.0240	0.0356	0.0291	0.0820	0.0243
RTK	331609	4	13.06	0.1853	0.0311	0.0262	0.0260	0.0421	0.0184

B.2 ANOVA

This section presents the results of the analysis of variance (ANOVA) of the data in Table B.1. The analysis was broken into two sections: a comparison of all three correction modes and a comparison of OmniStar HP versus RT2.

<u>Analysis of None, OmniStar HP, and RT2:</u>

```
Analysis of Variance for EastingBias, using Adjusted SS for Tests

Source           DF   Seq SS   Adj SS   Adj MS       F      P
DGPSMode          2  0.00019  0.00019  0.00010    0.03  0.974
Direction         1  0.54795  0.51549  0.51549  139.92  0.000
Speed             2  0.01049  0.02232  0.01116    3.03  0.051
Direction*Speed   2  0.26052  0.26052  0.13026   35.36  0.000
Error           145  0.53419  0.53419  0.00368
Total           152  1.35335

S = 0.0606967   R-Sq = 60.53%   R-Sq(adj) = 58.62%
```

```
Analysis of Variance for NorthingBias, using Adjusted SS for Tests

Source      DF    Seq SS    Adj SS    Adj MS      F      P
DGPSMode     2  0.000301  0.000301  0.000151   0.09  0.914
Direction    1  0.012717  0.012232  0.012232   7.30  0.008
Speed        2  0.052216  0.052216  0.026108  15.58  0.000
Error      147  0.246353  0.246353  0.001676
Total      152  0.311587

S = 0.0409374   R-Sq = 20.94%   R-Sq(adj) = 18.25%
```

```
Analysis of Variance for HeightBias, using Adjusted SS for Tests

Source      DF     Seq SS     Adj SS     Adj MS     F      P
DGPSMode     2  0.0007494  0.0007494  0.0003747  0.43  0.649
Direction    1  0.0002316  0.0002076  0.0002076  0.24  0.625
Speed        2  0.0019186  0.0019186  0.0009593  1.11  0.332
Error      147  0.1270583  0.1270583  0.0008643
Total      152  0.1299579

S = 0.0293997   R-Sq = 2.23%   R-Sq(adj) = 0.00%
```

```
Analysis of Variance for EastingStdDev, using Adjusted SS for Tests

Source      DF     Seq SS     Adj SS     Adj MS     F      P
DGPSMode     2  0.0081778  0.0081778  0.0040889  7.98  0.001
Direction    1  0.0009312  0.0008747  0.0008747  1.71  0.193
Speed        2  0.0023311  0.0023311  0.0011656  2.28  0.106
Error      147  0.0753039  0.0753039  0.0005123
Total      152  0.0867440

S = 0.0226334   R-Sq = 13.19%   R-Sq(adj) = 10.24%
```

```
Analysis of Variance for NorthingStdDev, using Adjusted SS for Tests

Source      DF     Seq SS     Adj SS     Adj MS     F      P
DGPSMode     2  0.0070987  0.0070987  0.0035494  4.15  0.018
Direction    1  0.0030189  0.0029268  0.0029268  3.42  0.066
Speed        2  0.0009629  0.0009629  0.0004815  0.56  0.571
Error      147  0.1256718  0.1256718  0.0008549
Total      152  0.1367523

S = 0.0292388   R-Sq = 8.10%   R-Sq(adj) = 4.98%
```

Analysis of Variance for HeightStdDev, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
DGPSMode	2	0.213940	0.213940	0.106970	25.77	0.000
Direction	1	0.000844	0.000719	0.000719	0.17	0.678
Speed	2	0.007822	0.007822	0.003911	0.94	0.392
Error	147	0.610103	0.610103	0.004150		
Total	152	0.832710				

S = 0.0644233   R-Sq = 26.73%   R-Sq(adj) = 24.24%

Analysis of OmniStar HP and RT2:

Analysis of Variance for EastingStdDev, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Mode	1	0.0000382	0.0000382	0.0000382	0.12	0.734
Direction	1	0.0004031	0.0003687	0.0003687	1.12	0.293
Speed	2	0.0018321	0.0018321	0.0009161	2.78	0.067
Error	97	0.0319344	0.0319344	0.0003292		
Total	101	0.0342077				

S = 0.0181444   R-Sq = 6.65%   R-Sq(adj) = 2.80%

Analysis of Variance for NorthingStdDev, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Mode	1	0.0000427	0.0000427	0.0000427	0.10	0.750
Direction	1	0.0019515	0.0020051	0.0020051	4.81	0.031
Speed	2	0.0015857	0.0015857	0.0007928	1.90	0.155
Error	97	0.0404619	0.0404619	0.0004171		
Total	101	0.0440418				

S = 0.0204238   R-Sq = 8.13%   R-Sq(adj) = 4.34%

Analysis of Variance for HeightStdDev, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Mode	1	0.0022148	0.0022148	0.0022148	4.09	0.046
Direction	1	0.0001524	0.0001698	0.0001698	0.31	0.577
Speed	2	0.0032192	0.0032192	0.0016096	2.97	0.056
Error	97	0.0525837	0.0525837	0.0005421		
Total	101	0.0581701				

S = 0.0232830   R-Sq = 9.60%   R-Sq(adj) = 5.88%