Lunar Mission Analysis for a Wallops Flight Facility Launch

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(ABSTRACT)

Recently there is an increase in interest in the Moon as a destination for space missions. This increased interest is in the composition and geography of the Moon as well as using the Moon to travel beyond the Earth to other planets in the solar system. This thesis explores the mechanics behind a lunar mission and the costs and benefits of different approaches. To constrain this problem, the launch criteria are those of Wallops Flight Facility (WFF), which has expressed interest in launching small spacecraft to the Moon for exploration and study of the lunar surface. The flight from the Earth to the Moon and subsequent lunar orbits, referred to hereafter as the mission, is broken up into three different phases: first the launch and parking orbit around the Earth, second the transfer orbit, and finally the lunar capture and orbit.

A launch from WFF constrains the direction of the launch and the possible initial parking orbits. Recently WFF has been offered the use of a Taurus XL launch vehicle whose specifications will be used for all other limitations of the launch and initial parking orbit. The orbit investigated in this part of the mission is a simple circular orbit with limited disturbances. These disturbances are only a major factor for long duration orbits and don't affect the parking orbit significantly.

The transfer orbit from the Earth to the Moon is the most complex and interesting part of the mission. To fully describe the dynamics of the Earth-Moon system a three-body model is used. The model is a restricted three-body problem keeping the Earth and Moon orbiting circularly around the system barycenter. This model allows the spacecraft to experience the influence of the Earth and Moon during the entire transfer orbit, making the simulation more closely related to what will actually happen rather than what a patched conic solution would give. This trajectory is examined using Newtonian, Lagrangian, and Hamiltonian mechanics along with using a rotating and non-rotating frame of reference for the equations of motion. The objective of the transfer orbit is to reduce the time and fuel cost of the mission as well as allow for various insertion angles to the Moon.

The final phase of the mission is the lunar orbit and the analysis also uses a simple two body model similar to the parking orbit. The analysis investigates how the orbits around the Moon evolve and decay and explores more than just circular orbits, but orbits with different eccentricities. The non-uniform lunar gravity field is investigated to accurately model the lunar orbit. These factors give a proper simulation of what happens to the craft for the duration of the lunar orbit. Tracking the changes in the orbit gives a description of where it will be and how much of the lunar surface it can observe without any active changes to the orbit. The analysis allows for either pursuing a long duration sustained orbit or a more interesting orbit that covers more of the lunar surface.

These three phases are numerically simulated using MATLAB, which is a focus of this thesis. In all parts of the mission the simulations are refined and optimized to reduce the time of the simulation. Also this refinement gives a more accurate portrayal of what would really happen in orbit. This reduction in time is necessary to allow for many different orbits and scenarios to be investigated without using an unreasonable amount of time.

Dedication

This is dedicated to my sister Alexis, you will always be in the hearts and minds of your friends and family.

Acknowledgments

First I thank my parents for supporting me my entire life and continuing to do so; if it was not for them I would not have accomplished nearly as much as I have, in fact I don't think I would have accomplished anything at all. Secondly I thank Dr. Hall for putting up with my lack of concern for appropriate schedules or time lines to get my work done, and for giving me the opportunity to work on our high altitude balloon project; it was truly the most fun I have had in my entire college career. Finally I would like to thank Dr. Schaub for being the most intimidating professor I think myself or any of his other students will ever meet. Because of him I gained my deep interest and understanding of orbital mechanics and that is why I decided to focus on dynamics in graduate school. None of us will ever forget anything we learned in astromechanics and we will also always wonder what you kept in that fanny pack. I hope that Virginia Tech will be able to fully design, build, and launch a lunar spacecraft in the years to come. I offer the best of luck toward the future teams of aerospace engineers at Virginia Tech who will be working on this project and want this thesis to be a valuable resource. In fact hopefully this thesis will provide information and solutions to anyone who is designing a lunar mission in the future. The programs used in this thesis as well as other code developed during my research is available at `<http://drop.io/WFF_lunar_mission_code>`

Contents

List of Symbols

Roman:

A_k	Spherical Harmonic Gravitational Potential Field Coefficient
a	Semi-major Axis
B_k	Spherical Harmonic Gravitational Potential Field Coefficient
C_k	Spherical Harmonic Gravitational Potential Field Coefficient
C_{nm}	Gravitational Potential Field Coefficient
E	Eccentric Anomaly Angle
e	Eccentricity
f	True Anomaly Angle
G	Universal Gravitational Constant
H	Hamiltonian
h	Angular Momentum
i	Inclination Angle
J_k	Spherical Harmonic Gravitational Potential Field Coefficient
L	Lagrangian
M_0	Mean Anomaly Angle
M	Mass
m	Mass, Order
m_1	Mass of the Earth
m_2	Mass of the Moon
m_3	Mass of the Spacecraft
n	Degree
P_{nm}	Normalized Legendre Functions
p	Semilatus Rectum, Momentum
r	Orbital Radius
r_{32}	Distance from Spacecraft to Moon
r_{31}	Distance from Spacecraft to Earth
r_a	Apolune Radius
r_0	Initial Transfer Perilune Radius
r_p	Post Transfer Perilune Radius
S_{nm}	Gravitational Potential Field Coefficient
T	Kinetic Energy

t	Time
V	Potential Energy
V_E	Earth Orbital Velocity
V_T	Transfer Orbit Velocity
x, y, z	Cartesian Coordinates

Greek:

ΔV	Velocity Change
θ	Summation of Argument of Periapsis and True Anomaly Angle
λ	Longitude
μ	Gravitational Constant, Non-dimensionalized Mass Quantity
ξ_i	Relative Distance from Spacecraft to m_i
ρ	Non-dimensional Relative Distance
τ	Non-dimensional Time
ϕ	Latitude
Ω	Ascending Node
ω	Argument of Periapsis, System Rotation Rate

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Chapter 1

Introduction

1.1   Lunar Mission Analysis

The problem of analyzing a lunar mission is that of timing. In order to reach the Moon the launch and subsequent burn into the transfer orbit has to make the craft reach the lunar orbit where the Moon is positioned to allow for a capture in the Moon's gravity. This problem can be further complicated if the launch is to insert the craft directly into a transfer orbit from the surface of the Earth, leading to a more limited launch window than a launch into a parking orbit. In fact there are many variables in the problem. Thus, we make the following assumptions for the mission:

1. Restricted Three-Body Problem: The Earth and Moon are the only gravitational influences on the spacecraft and these two bodies orbit circularly around their barycenter.

2. Constant Lunar Orbit Radius: The Moon will be a fixed distance from the Earth, taken to be 384400 km.

3. No Disturbance Forces: There is no accounting for disturbances from any other forces other than gravity.

4. Negligible Craft Mass: As in the classical three-body problem, the craft has negligible mass.

The restricted three-body problem is used because it allows for the dynamical influences of the two major bodies in the system to be taken into account for the entire mission, while keeping the motion of those bodies relatively simple and easy to simulate. Both bodies are moving on circular orbits which is close to the actuality of the Earth-Moon system with the lunar orbit's eccentricity being around 0.055 [8] and the Earth wobbling around the barycenter of the system due to the Moon's motion. This nearly circular orbit is also why the radius can be assumed a constant value.

The disturbance forces from non-gravitational sources are ignored because their influence can be considered negligible on the short duration for most components of the mission. The only long term portion of the mission is the lunar orbits, which will be under strong influence of lunar gravity. Finally the mass of the craft is ignored due to the fact that it would have no effect on the orbit of the Earth or Moon, and the craft is assumed to be a point mass because the configuration and shape of the craft is unknown.

1.1.1   Launch & Parking Orbit

The launch is modeled starting from Wallops Flight Facility, which constrains the launch and the possible initial orbits. To further constrain the launch conditions it is modeled using a Taurus XL rocket. These constraints give possible maximum orbit altitudes and payload masses, as well as constraining the maximum possible ΔV that can be delivered. This part of the mission will involve the parking orbit as well. This parking orbit will allow the spacecraft to stay around the Earth until it reaches the desired location to perform the final burn from the launch vehicle achieving velocity to enter the transfer orbit. This part of the mission is modeled using a simple two-body point mass analysis. The two-body analysis is done because of the low orbit around the Earth and the short time period the craft will be in this

orbit. Any influences will be insignificant as shown in Figure (1.1), where the semi-major axis is experiencing less than a 2.5 km drift during the first few orbits.
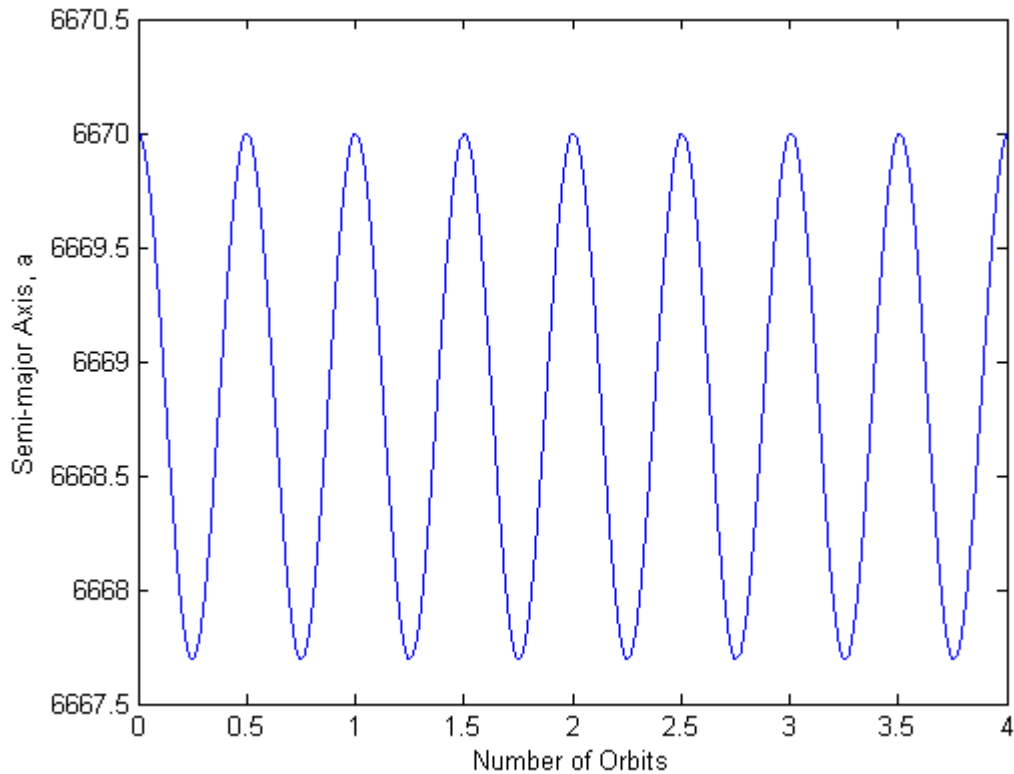
Figure 1.1: Influence of J_2 Perturbation on Earth Orbiting Satellite

1.1.2   Transfer Orbit

The purpose of the analysis of the transfer orbit is to reach the Moon with as low of a ΔV and time of flight as possible. In this situation the cost outweighs the time because the mission is not manned and is a good candidate for low energy trajectory methods. To reach the Moon using the least amount of fuel, it is best to use the natural dynamics of the Earth-Moon system. The natural dynamics allow the craft to reach the Moon and then using a small burn, to be captured by the Moon's gravity. The dynamics of this part of the mission are modeled using the restricted three-body problem discussed above. The transfer

orbit is where the bulk of the work in this thesis has been done. These equations of motion have been analyzed using Newtonian, Lagrangian, and Hamiltonian dynamics as well as using rotating and inertial frames of reference. The different methods of analysis allow for a detailed understanding of how the craft will go from the Earth to the Moon given any set of initial conditions.

It is also important to approach the Moon from outside the Earth-Moon plane. Non-Earth-Moon plane approaches allow for injection into different kinds of lunar orbits without changing the inclination of the orbit after reaching the Moon. Launching from WFF limits the types of lunar inclinations possible. Because of this a few ways of changing the inclination are investigated:

1. Inclination Change In Parking Orbit: Changing the inclination while around the Earth will make the craft reach the Moon with a different insertion angle. Unfortunately plane changes in the parking orbit are costly in terms of ΔV.

2. Performing A Burn During Transfer: Performing a burn during the transfer orbit can cause the spacecraft to reach the Moon at almost any desired inclination. Burning during the transfer orbit can also be the most cost effective, but it is difficult to calculate the optimal burn.

3. Inclination Change During Lunar Orbit: Because of the Moon's weak gravity field, and the relatively slow speed of anything in the Moon's orbit, an inclination change around the Moon requires a small ΔV.

Between the three choices above the latter two choices are the most cost effective ways to change the injection angle at the Moon. The first choice is too expensive for it to be a feasible choice. The second option is investigated in the analysis of the mission and is compared to the final option.

1.1.3 Lunar Orbit

With the lunar orbit there are two objectives: duration and coverage. The duration of the orbit is determined by the gravitational influences of the Moon which cause the spacecraft's orbit to change with time. These influences can cause the craft to crash into the surface of the Moon or drift into an undesirable orbit [3]. The duration of the lunar orbit can be increased with burns and other maneuvers.

The coverage of the orbit is the amount of lunar surface the spacecraft can view during the mission lifetime. The coverage can be increased by either allowing the natural dynamics of the system to cause the orbit to change around the Moon or by changing the orbit with burns to increase coverage. Both of these desired characteristics of the lunar orbit will involve modeling the Moon's gravity field in a more complex way than a simple point mass. This modeling uses data collected from previous lunar missions to map the gravity field.

1.2 Reasons for Investigation

There is current interest in returning to the Moon and furthering solar system exploration. For this reason NASA Wallops has looked toward Virginia Tech to design and test a lunar mission with the possibility of a spacecraft launch using a Taurus XL rocket. Currently this mission is being designed and tested in segments by senior design groups at Virginia Tech. This investigation is designed to aid the future lunar mission in giving the designers a collection of data and an accurate analysis of what can be done with a launch from WFF using a Taurus XL rocket. The students can use the work done here to aid their mission design and analysis of the future real lunar mission [10].

1.3 Previous Work

Previous work has been done for this mission by the Academic Research Team for the Establishment of a lunar Magnetic field Investigation System (ARTEMIS) senior design group at Virginia Tech during the year 2005. This group designed a small satellite that would orbit the Moon collecting magnetic field data. This thesis adapts some of the work done by that group to aid in the lunar orbit analysis. This group did no work on the actual trajectory to the Moon as their craft would be attached to a larger mission to the Moon and would detach upon arrival. Further work was continued on this mission in 2007 when NASA released its interest in a student designed lunar mission which was discussed above. The author of this thesis worked on that project and began developing the work done in this document. This work continued with the Magnetic field Investigation of Luna (MIL) senior design project from 2007 to 2008. The author of this thesis continued the orbital analysis for that project as well, and is using his findings during those project for this work [3] [4].

1.4 Orbital Mechanics

As discussed in above sections this thesis focuses on orbital mechanics. The Earth orbital analysis deals with simple two-body dynamics, the transfer orbit goes into a more complicated restricted three-body problem, and finally the lunar orbit deals with non-uniform gravity fields. In each of these stages the type of analysis is done using the optimal method to keep the computations as simple as possible, while still capturing the dynamics of the system properly. This section presents more detail about the types of analysis done.

1.4.1 Two-Body Analysis

A simple two-body system involves one larger mass and a smaller mass that orbits around it. The two-body system analysis is completed for both the parking and lunar orbits, which

are close to their attracting body. For the Earth orbit both masses are modeled as point masses neglecting the non-uniformity of the Earth. The assumption of neglecting the non-uniformity is done, as explained above, because of the short duration of the parking orbit. Because of the interest in the lunar orbit's duration the effects of the Moon's non-uniform gravitational field must be taken into account.

This analysis will be done in either an Earth centered frame for the initial parking orbit or a Moon centered frame for the lunar orbit. To observe the final lunar orbit the orbit elements will be tracked over time showing any variations and possible collisions or other catastrophic events.

1.4.2   Three-Body Analysis

For the transfer orbit the only way to truly map the complex motion of the craft is to use a three body analysis. A three-body analysis not only allows for a more accurate simulation, but permits the calculating of out-of-plane trajectories that would be difficult to analyze using a patched conic two-body analysis. The three-body problem uses two massive objects and a smaller negligible mass. To further simplify this problem the two larger masses are kept orbiting circularly around their barycenter, or system center of mass.

For this analysis the masses are treated as point masses as the non-uniform gravitational fields have a negligible effect on the spacecraft trajectory. The negligibility of the non-uniform gravity fields is because the craft is either far away from the bodies or only near them for a short time. The frame used for this analysis is centered at the barycenter and is done with either a rotating or non rotating frame. Because the Earth and Moon are locked 180° apart the rotating frame makes a logical choice for evaluation.

1.4.3   Dynamics

The dynamics are analyzed using Newtonian, Lagrangian, and Hamiltonian methods. These methods give confirmation of the dynamical systems because their results are all similar, and shows which produce a more accurate model. Furthermore, because numerical integration is used heavily in the simulations it also compares all three to see which is computationally faster. The Newtonian system uses the basic Cartesian frame and the gravitational forces for the analysis and integrates velocities and accelerations in the numerical integrator. The Lagrangian dynamics calculate the kinetic and potential energy of the system to find the Lagrangian and is performed in cylindrical coordinates. Finally the Hamiltonian dynamics use the momentum and are also done in cylindrical coordinates. These more in-depth dynamical systems are only used for the transfer orbit as the simpler parking and lunar orbit do not need that depth of analysis, because the orbits are well defined with the two-body simulation and do not need the added complexity of the three-body simulation.

1.5   Thesis Layout

This thesis is organized in six chapters that discuss the lunar mission and related topics. The second chapter provides details of the analytical work done to derive the equations used in the simulations and other work. Chapter Three discusses the simulations and calculations done to evaluate the system. This chapter contains the majority of the results discovered. The fourth chapter discusses the applications of this data. Chapter Five describes any future work that needs to be done in this area of research. The final chapter is a summary of the thesis and conclusions derived from the work.

Chapter 2

Analytical Methods

2.1 Available Methods

This chapter discusses the available methods to analyze the three major portions of the mission. The first part of the mission is the phase near the Earth, comprising of the launch and parking orbit. The first phase is a simple analysis of a two-body problem, and an investigation of non-uniform Earth gravity modeling. The next part of the mission is the transfer orbit, which is analyzed using a restricted three-body problem. The transfer orbit involves the analysis of equations of motion using different either Newtonian, Lagrangian, or Hamiltonian techniques. The final part of the mission is the lunar capture and orbit.

2.1.1 Launch & Parking Orbit

The spacecraft will be launched with a Taurus XL rocket as discussed in previous chapters. This rocket is capable of placing a payload of over 1000 kg into a 1000 km circular orbit from WFF. Using this data for the launch constrains the possible height and orientation of the desired circular parking orbit. Launches from WFF are limited between inclination angles of 38° and 55°, which means that the craft is not aligned with the lunar plane [7]. The Moon's

Table 2.1: Orbit Elements for Earth Parking Orbit

	Min.	Max.
a	6570 km	7370 km
e	0	0
i	38°	55°
ω	0°	0°
Ω	0°	180°
M_0	0°	360°

inclination varies between roughly 18° and 23° due to it being inclined to the ecliptic plane, and not the equatorial plane [8].

With the specified launch site and the circular orbit, the orbit elements for the initial parking orbit are shown in Table 2.1. The semi-major axis is limited by the Taurus XL rocket, the inclination is limited by WFF's location, and the mean anomaly angle is the location on the orbit and open to any value. The value of the eccentricity and the argument of periapsis are zero because of the circular orbit. The longitude of the ascending node can vary between 0° and 180°, which determines the parking orbit's frame orientation compared to the Earth frame. The longitude of the ascending node is determined by the launch angle.

The motion of this simple orbit can be described using the following equations of motion [9]

$$\ddot{\mathbf{r}} = \frac{\mu}{r^3}\mathbf{r} \tag{2.1}$$

where μ is the gravitational parameter for the body being orbited. In this case the value of μ for Earth is about 3.986×10^5 km^3/s^2. Eq. (2.1) can be written in scalar form as

$$\ddot{x} = \frac{\mu}{r^3}x \tag{2.2}$$

$$\ddot{y} = \frac{\mu}{r^3}y \tag{2.3}$$

$$\ddot{z} = \frac{\mu}{r^3}z \tag{2.4}$$

where x, y, and z are the Cartesian coordinates. These coordinates are useful for comparing results between the Earth orbit and the transfer orbit. The transfer orbit's Cartesian coordinates can be described in an Earth-fixed frame; therefore the difference between any points on the transfer orbit and any point on the parking orbit can be compared. The comparison derives a ΔV required to achieve the transition between the parking orbit and transfer orbit. This ΔV is described by

$$\Delta V_{total} = |V_{E,x} - V_{T,x}| + |V_{E,y} - V_{T,y}| + |V_{E,z} - V_{T,z}| \tag{2.5}$$

where the E denotes an orbital velocity around the Earth, and the T denotes a velocity while on the transfer orbit. The minimum ΔV's can be found by comparing velocities of the transfer orbit to the Earth orbits.

To get the desired Earth orbital velocities in the proper Cartesian coordinates, they must be derived from the orbit elements that the launch is capable of achieving. These calculations require some numerical analysis to derive the eccentric anomaly angle from the mean anomaly angle using Newton's method. Once the eccentric anomaly angle has been found it can be used to get the true anomaly [9]:

$$f = 2\tan^{-1}\left(\sqrt{\frac{1+e}{1-e}} \tan\frac{E}{2}\right) \tag{2.6}$$

Using the true anomaly the scalar value of the radius can be found from [9]

$$r = \frac{a\left(1 - e^2\right)}{\left(1 + e\cos f\right)} \tag{2.7}$$

The true anomaly angle can also be used along with the argument of periapsis to find the angle θ [9]:

$$\theta = \omega + f \tag{2.8}$$

The angle θ along with the other orbit element angles can be seen in Figure (2.1). With these angles the position vector can be determined by [9]

$$\mathbf{r} = r \begin{pmatrix} \cos\Omega\cos\theta - \sin\Omega\sin\theta\cos i \\ \sin\Omega\cos\theta - \cos\Omega\sin\theta\cos i \\ \sin\theta\sin i \end{pmatrix} \tag{2.9}$$
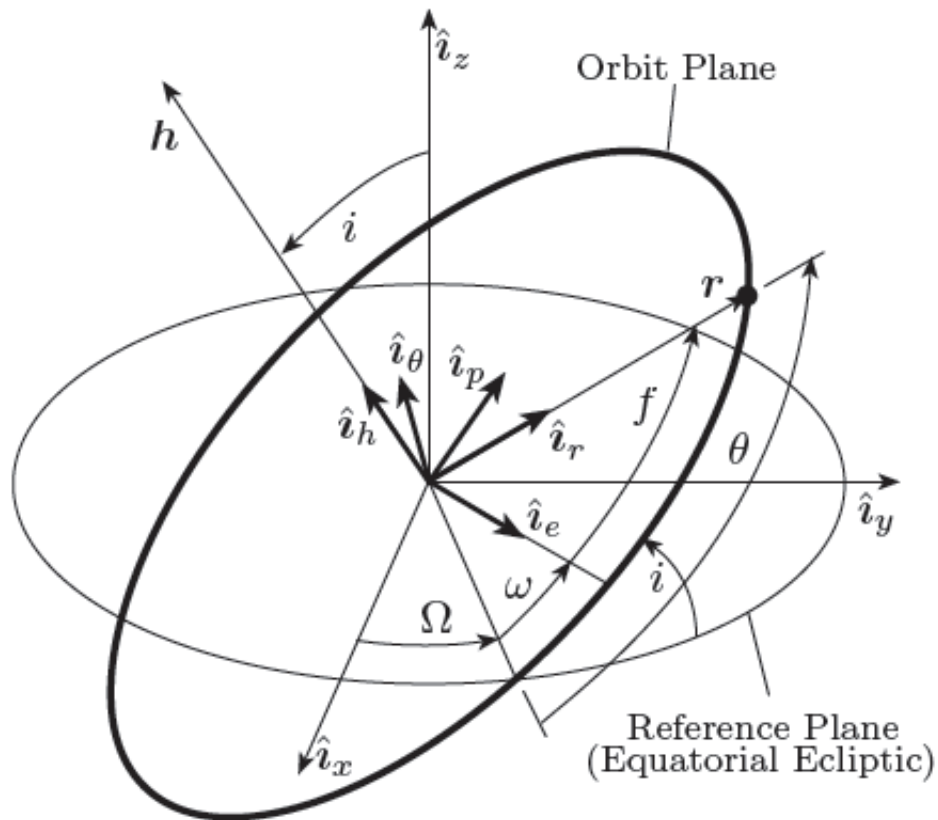

Figure 2.1: Three Orbit Element Angles and Frame Vectors [9]

To obtain the velocity vector the angular momentum must be found using [9]

$$p = a\left(1 - e^2\right) \tag{2.10}$$

$$h = \sqrt{\mu p} \tag{2.11}$$

Using the value of h and μ along with the orbit element angles the velocity vector can be expressed as [9]

$$\dot{\mathbf{r}} = \frac{-\mu}{h} \begin{pmatrix} \cos\Omega\left(\sin\theta + e\sin\omega\right) + \sin\Omega\left(\cos\theta + e\cos\omega\right)\cos i \\ \sin\Omega\left(\sin\theta + e\sin\omega\right) - \cos\Omega\left(\cos\theta + e\cos\omega\right)\cos i \\ -\left(\cos\theta + e\cos\omega\right)\sin i \end{pmatrix} \tag{2.12}$$

Equations (2.9) and (2.12) give the position and velocity vectors of the near Earth orbits for a simple two-body analysis; derived from the orbit elements. These vectors are used to find the feasible transfer orbit trajectories and calculate the necessary ΔV.

Gravity Modeling

Although the Earth is assumed to be a point mass in the simulations, it should be noted how the non-uniform gravity effects can be investigated. The assumption uses spherical harmonics; the Earth shape deviations are expressed in spherical coordinates and use expansions of sinusoidal shapes to write out the gravitational potential terms. Some samples of the spherical harmonics of the gravity potential are shown in Figure (2.2). To use these spherical harmonics the weights of all the different sinusoidal terms have to be known, or at least estimated using a model.

To enhance the Earth's gravity model data the EGM96 model can be used. NASA's EGM96 model uses normalized Legendre functions which need to be converted to the associated Legendre functions in order to compare the EGM96 model to the (A_k, B_k, C_k) and J_k coefficients. The equation for the EGM96 model potential function is [2]

$$V = \frac{GM}{r} \left(1 + \sum_{n=2}^{nmax} \left(\frac{a}{r}\right)^n \sum_{m=0}^{n} \bar{P}_{nm}\left(\sin\phi\right)\left[\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda\right] \right) \tag{2.13}$$
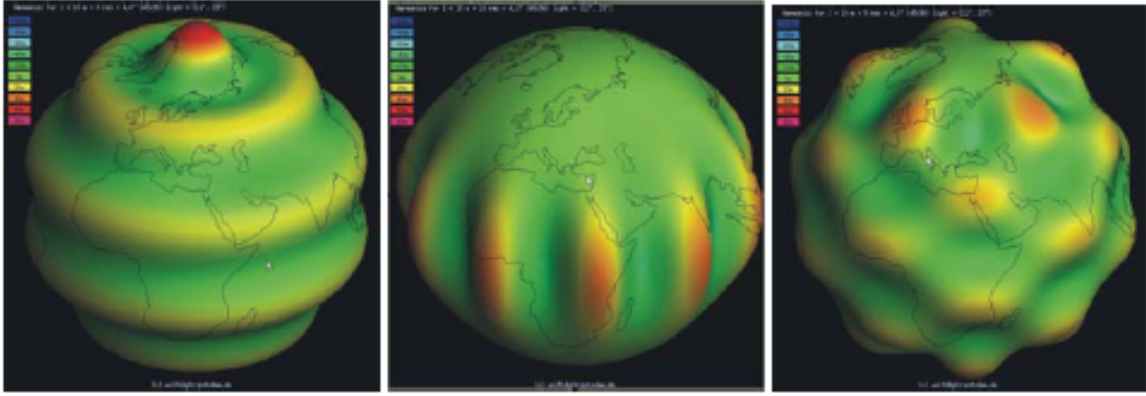
Figure 2.2: Sample Spherical Harmonics of the Gravity Potential [6]

where GM is the gravity constant times the mass of the Earth, which has been referred to previously as μ and is supplied in the EGM96 'readme' more accurately as 3986004.415E+8 m^3/s^2. The value of r is the current orbital radius of the craft, *nmax* is the highest degree of the EGM96 model that is desired, and a is the Earth's equatorial radius supplied as 6378136.3 m. The angles ϕ and λ are the latitude and longitude of the radius vector. The coefficients C_{nm} and S_{nm} are supplied in the EGM96 model's data file. The normalized Legendre functions, P_{nm}, are related to the associated Legendre functions through [9]

$$\bar{P}_{nm}(x) = \sqrt{(2n+1)\frac{(n-m)!}{(n+m)!}} P_n^m(x) \tag{2.14}$$

The associated Legendre functions are related to the Legendre polynomials through [9]

$$P_k^j(\nu) = \left(1 - \nu^2\right)^{j/2} \frac{d^j}{d\nu^j}\left(P_k(\nu)\right) \tag{2.15}$$

To relate the EGM96 coefficients to the (A_k, B_k, C_k) and J_k coefficients, the gravity potential function is expressed as [9]

$$V = -\frac{GM}{r} - \sum_{k=1}^{\infty} \frac{1}{r^{k+1}}\left(A_k P_k(\sin\phi) + \sum_{j=1}^{k} P_k^j(\sin\phi)\left(B_k^j \cos j\theta + C_k^j \sin j\theta\right)\right) \tag{2.16}$$

Expanding this equation results in

$$V = -\frac{GM}{r} - \sum_{k=1}^{\infty} \frac{1}{r^{k+1}} A_k P_k (\sin \phi) - \sum_{k=1}^{\infty} \frac{1}{r^{k+1}} \sum_{j=1}^{k} P_k^j (\sin \phi) \left(B_k^j \cos j\theta + C_k^j \sin j\theta \right) \quad (2.17)$$

where θ is the same as λ from Eq. (2.13). Referring to Eq. (2.13) and expanding the first summation of the m sum, the result is

$$\begin{aligned} V &= \frac{GM}{r} \left(1 + \sum_{n=2}^{nmax} \left(\frac{a}{r} \right)^n \sum_{m=1}^{n} \bar{P}_{nm} (\sin \phi) \left[\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda \right] \right) \\ &+ \frac{GM}{r} \left(\bar{P}_{n0} (\sin \phi) \left[\bar{C}_{n0} + 0 \right] \right) \end{aligned} \quad (2.18)$$

Rearranging this expression for the potential gives

$$\begin{aligned} V &= \frac{GM}{r} + \frac{GM}{r} \left(\sum_{n=2}^{nmax} \left(\frac{a}{r} \right)^n \bar{P}_{n0} (\sin \phi) \bar{C}_{n0} \right) \\ &+ \frac{GM}{r} \left(\sum_{n=2}^{nmax} \left(\frac{a}{r} \right)^n \sum_{m=1}^{n} \bar{P}_{nm} (\sin \phi) \left[\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda \right] \right) \end{aligned} \quad (2.19)$$

Comparing Eqs. (2.19) and (2.17), the two equations have three comparable terms. The first equation is the negative of the second equation because of the way the potential is defined, so the negative sign is dropped on the second equation. Comparing the first term in both equations, they are both GM/r, and the second term is the A_k term. Comparing and reducing these expressions leads to

$$\frac{GM}{r} \sum_{n=2}^{nmax} \left(\frac{a}{r} \right)^n \bar{P}_{n0} (\sin \phi) \bar{C}_{n0} = \sum_{k=1}^{\infty} \frac{1}{r^{k+1}} \left(A_k P_k (\sin \phi) \right) \quad (2.20)$$

$$\frac{GM}{r} \sum_{n=2}^{nmax} \left(\frac{a}{r} \right)^n \bar{P}_{n0} (\sin \phi) \bar{C}_{n0} = \sum_{n=2}^{nmax} \frac{1}{r^{n+1}} \left(A_n P_n (\sin \phi) \right) \quad (2.21)$$

$$\frac{GM}{r} \left(\frac{a}{r} \right)^n \bar{P}_{n0} (\sin \phi) \bar{C}_{n0} = \frac{1}{r^{n+1}} \left(A_n P_n (\sin \phi) \right) \quad (2.22)$$

From this point the normalized Legendre function must be converted to a Legendre polynomial using Eqs. (2.14) and (2.15). Substituting into these equations an order value of zero results in

$$\bar{P}_{n0}\left(\sin\phi\right) = \sqrt{(2n+1)}P_n\left(\sin\phi\right) \tag{2.23}$$

Using this equation in Eq. (2.22) the reduction continus as

$$\frac{GM}{r}\left(\frac{a}{r}\right)^n \sqrt{(2n+1)}P_n\left(\sin\phi\right)\bar{C}_{n0} = \frac{1}{r^{n+1}}\left(A_n P_n\left(\sin\phi\right)\right) \tag{2.24}$$

$$\frac{GM}{r}\left(\frac{a}{r}\right)^n \sqrt{(2n+1)}\bar{C}_{n0} = \frac{1}{r^{n+1}}A_n \tag{2.25}$$

$$GM\left(\frac{a^n}{r^{n+1}}\right)\sqrt{(2n+1)}\bar{C}_{n0} = \frac{1}{r^{n+1}}A_n \tag{2.26}$$

$$A_n = GMa^n\sqrt{(2n+1)}\bar{C}_{n0} \tag{2.27}$$

This equation relates the A_k coefficients to the C_{n0} coefficients in the EGM96 model. To get the J_k values the A_k values are used as

$$J_n = -\frac{A_n}{r_{eq}^n}\frac{1}{GM} \tag{2.28}$$

where r_{eq} is equal to a from the previous equations. Inserting Eq. (2.27) into Eq. (2.28) we get

$$J_n = -\sqrt{2n+1}\bar{C}_{n0} \tag{2.29}$$

This simple equation allows us to get the J_k values from the EGM96 model. As a quick check the value of J_2 is known as $1082.63\cdot10^{-6}$, and the value of $\bar{C}_{20}$ is about $-4.8417\cdot10^{-4}$. When used in Eq. (2.29) with $n = 2$ the values are the same. Finding the relation of the B_k and C_k terms is similar to the procedure used to find the A_k terms. Comparing the first

part of the third terms of Eqs. (2.19) and (2.17)

$$\frac{GM}{r} \sum_{n=2}^{nmax} \left(\frac{a}{r}\right)^n \sum_{m=1}^{n} \bar{P}_{nm}(\sin\phi)\,\bar{C}_{nm}\cos m\lambda = \sum_{k=1}^{\infty} \frac{1}{r^{k+1}} \sum_{j=1}^{k} P_k^j(\sin\phi)\,B_k^j\cos j\theta \qquad (2.30)$$

$$\frac{GM}{r} \sum_{n=2}^{nmax} \left(\frac{a}{r}\right)^n \sum_{m=1}^{n} \bar{P}_{nm}(\sin\phi)\,\bar{C}_{nm}\cos m\lambda = \sum_{n=2}^{nmax} \frac{1}{r^{n+1}} \sum_{m=1}^{n} P_n^m(\sin\phi)\,B_n^m\cos m\lambda \quad (2.31)$$

Using Eq. (2.14) the normalized Legendre function can be written as an associated Legendre function

$$\frac{GM}{r}\left(\frac{a}{r}\right)^n \sqrt{(2n+1)\frac{(n-m)!}{(n+m)!}}P_n^m(\sin\phi)\,\bar{C}_{nm}\cos m\lambda = \frac{1}{r^{n+1}}P_n^m(\sin\phi)\,B_n^m\cos m\lambda \quad (2.32)$$

$$B_n^m = GMa^n\sqrt{(2n+1)\frac{(n-m)!}{(n+m)!}}\,\bar{C}_{nm} \qquad (2.33)$$

Performing the exact same procedure as above but using the second part of the third terms of Eqs. (2.19) and (2.17), the relation for C_k terms can be found as

$$C_n^m = GMa^n\sqrt{(2n+1)\frac{(n-m)!}{(n+m)!}}\,\bar{S}_{nm} \qquad (2.34)$$

These gravity terms are helpful in determining the actual precession of a spacecraft that will spend prolonged periods of time around the Earth. Due to the fact that this mission entails a trip to the Moon and not an extended Earth orbit, the results of the non-uniform Earth gravity modeling are seen as negligible.

2.1.2   Transfer Orbit

This section discusses the trajectory that takes place from the instant the spacecraft leaves its Earth parking orbit until it reaches the point where it will perform its burn to capture around the Moon. This section covers the analytical analysis of the transfer orbit and various

techniques to derive the equations of motion for this trajectory. The three methods used to solve for the equations of motion are the Newtonian approach, Lagrangian approach, and Hamiltonian approach. Each method results in different sets of equations of motion, but all three generate nearly identical results. The orbit shown in Figure (2.3) is an hourglass like trajectory that is an ideal orbit to reach the Moon. It uses a low ΔV and is easy to numerically integrate multiple cases due to reaching periselenium with only a y-component in the velocity when looking at the planar case [9].
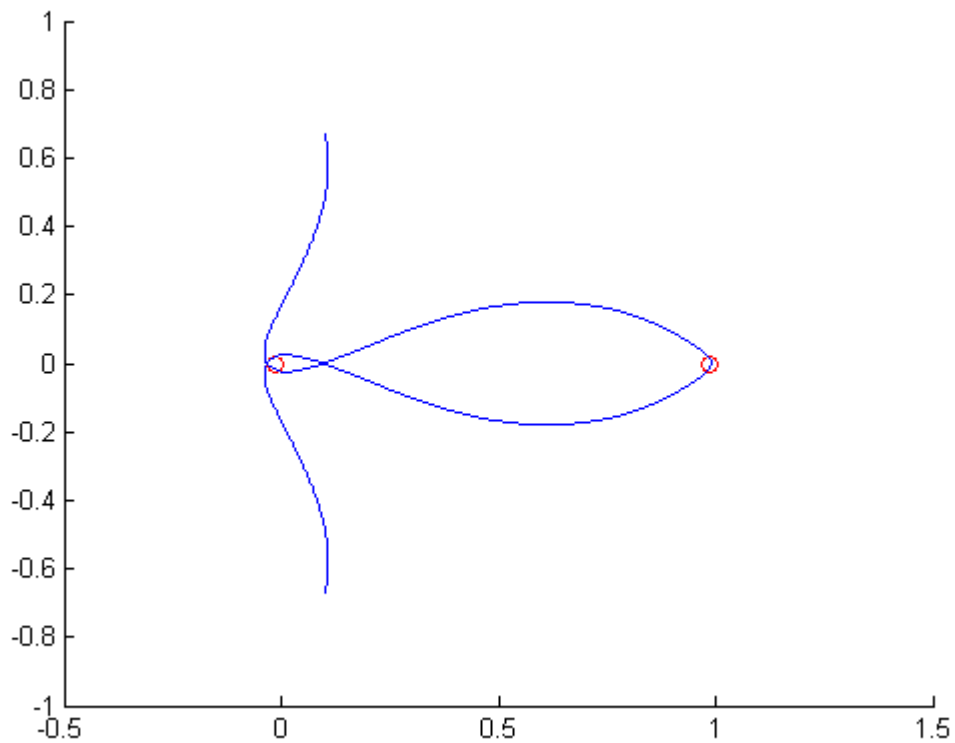

Figure 2.3: Hourglass Type Orbit Going From Earth to the Moon

Newtonian

The equations required to integrate for the inertial frame require almost no derivations, the main equation is [9]

$$m_i\ddot{\mathbf{r}}_i = G\sum_{j=1}^{3} \frac{m_i m_j}{r_{ij}^3}\mathbf{r}_{ij} \tag{2.35}$$

with the three bodies being the Earth, Moon, and the spacecraft. Dividing out the spacecraft mass, m_3, leads to the motion of the spacecraft, $\ddot{r}_3$, being dominated by the other two bodies [9]

$$\ddot{\mathbf{r}} = \frac{Gm_1}{r_{31}^3}\mathbf{r}_{31} + \frac{Gm_2}{r_{32}^3}\mathbf{r}_{32} \tag{2.36}$$

where m_1 is the Earth, m_2 is the Moon, and m_3 is the spacecraft. The radii r_{31} and r_{32} are the distances from the spacecraft to the Earth and the spacecraft to the Moon respectively. This equation can be integrated numerically to solve for the motion of the spacecraft.

In the restricted three-body problem, the Earth and Moon will rotate around their center of mass at a constant rate. The restricted three-body problem is the perfect scenario to use a rotating reference frame. To solve the restricted three-body problem we start with the position vector of the mass. This vector is expressed in F frame components as [9]

$$\mathbf{r} = r_x\hat{\mathbf{e}}_r + r_y\hat{\mathbf{e}}_\theta + r_z\hat{\mathbf{e}}_3 \tag{2.37}$$

The F frame is a rotating frame that keeps the $\mathbf{e}_r$ vector pointing from the center of mass to the Moon of the rotating Earth/Moon system. Taking two inertial derivatives of $\mathbf{r}$ results in

$$\ddot{\mathbf{r}} = \left(\ddot{r}_x - 2\dot{r}_y\omega - r_x\omega^2\right)\hat{\mathbf{e}}_r + \left(\ddot{r}_y + 2\dot{r}_x\omega - r_y\omega^2\right)\hat{\mathbf{e}}_\theta + \ddot{r}_z\hat{\mathbf{e}}_3 \tag{2.38}$$

$$\mathbf{F} = -G\begin{pmatrix} \frac{m_1}{\xi_1^3}\left(r_x - r_1\right) + \frac{m_2}{\xi_2^3}\left(r_x - r_2\right) \\ \left(\frac{m_1}{\xi_1^3} + \frac{m_2}{\xi_2^3}\right)r_y \\ \left(\frac{m_1}{\xi_1^3} + \frac{m_2}{\xi_2^3}\right)r_z \end{pmatrix} \tag{2.39}$$

where the relative distances ξ_i of m to m_i are given by

$$\xi_i = \sqrt{(r_x - r_i)^2 + r_y^2 + r_z^2} \tag{2.40}$$

Combining Eqs. (2.38) and (2.39), the equations of motion for the mass m can be found. These equations are three scalar, coupled differential equations. Only the x and y equations of motion need to be considered if simple planar motion is desired, but to view the spacecraft's motion in all directions all three equations will be used. Those equations are

$$\ddot{r}_x - 2\omega\dot{r}_y - \omega^2 r_x + G\left(\frac{m_1}{\xi_1^3}(r_x - r_1) + \frac{m_2}{\xi_2^3}(r_x - r_2)\right) = 0 \tag{2.41}$$

$$\ddot{r}_y + 2\omega\dot{r}_x - \omega^2 r_y + G\left(\frac{m_1}{\xi_1^3} + \frac{m_2}{\xi_2^3}\right) r_y = 0 \tag{2.42}$$

$$\ddot{r}_z + G\left(\frac{m_1}{\xi_1^3} + \frac{m_2}{\xi_2^3}\right) r_z = 0 \tag{2.43}$$

To simplify the calculations, Eqs. (2.41) through (2.43) are expressed in a non-dimensional form. A non-dimensional time variable τ is used, where [9]

$$\tau = \omega t \tag{2.44}$$

Taking derivatives with respect to this new time variable is denoted with a $\circ$ symbol instead of the $\cdot$ symbol. Relating the non-dimensional time derivative $\overset{\circ}{x}$ to the inertial time derivative $\dot{x}$ is done through [9]

$$\dot{x} = \frac{dx}{dt} = \frac{dx}{d\tau}\frac{d\tau}{dt} = \overset{\circ}{x}\,\omega \tag{2.45}$$

$$\ddot{x} = \omega\frac{d\,\overset{\circ}{x}}{dt} = \omega\frac{d\,\overset{\circ}{x}}{d\tau}\frac{d\tau}{dt} = \overset{\circ\circ}{x}\,\omega^2 \tag{2.46}$$

The distances are non-dimensionalized by dividing them all by the constant distance be-

tween m_1 and m_2 which is r_{12}. The non-dimensionalized mass quantity is a scalar parameter μ which is related to the masses by [9]

$$\mu = \frac{m_2}{m_1 + m_2} \tag{2.47}$$

Using all of the above non-dimensionalized quantities, Eqs. (2.41) through (2.43) can be rewritten as [9]

$$\ddot{x} - 2\,\dot{y} = x - (1 - \mu)\frac{x - x_1}{\rho_1^3} - \mu\frac{x - x_2}{\rho_2^3} \tag{2.48}$$

$$\ddot{y} + 2\,\dot{x} = \left(1 - \frac{1 - \mu}{\rho_1^3} - \frac{\mu}{\rho_2^3}\right)y \tag{2.49}$$

$$\ddot{z} = -\left(\frac{1 - \mu}{\rho_1^3} + \frac{\mu}{\rho_2^3}\right)z \tag{2.50}$$

where the non-dimensional relative distance ρ_i is defined as [9]

$$\rho_i = \sqrt{(x - x_i)^2 + y^2 + z^2} \tag{2.51}$$

Equations (2.48)-(2.50) are numerically integrated to solve for the orbit expressed in the rotating frame. To convert the radii to dimensionalized non-rotating components

$$\mathbf{i}_x = \cos\theta\mathbf{e}_r - \sin\theta\mathbf{e}_\theta \tag{2.52}$$

$$\mathbf{i}_y = \sin\theta\mathbf{e}_r + \cos\theta\mathbf{e}_\theta \tag{2.53}$$

$$\mathbf{i}_z = \mathbf{i}_z \tag{2.54}$$

where θ is found using ω

$$\omega^2 = \frac{G(m_1 + m_2)}{r_{12}^3} \tag{2.55}$$

These values use a relative distance between the masses m_1 and m_2 of 1, so they must

be multiplied by the orbital radius of the Moon r_{12}. To convert the resulting velocities to dimensionalized non-rotating values, they must be multiplied by ω and r_{12}. Doing the conversion allows for the creation of plots in the inertial frame as well as allowing us to find the initial required velocity in dimensional units.

Lagrangian

Deriving the equations of motion using the Lagrangian approach simplifies some of the derivations and necessitates the use of cylindrical coordinates to simplify the algebra. First the Lagrangian has to be found which is expressed as [5]

$$L = T - V \tag{2.56}$$

where V and T are the potential and kinetic energy respectively. Using the Lagrangian the equations of motion can be found by the following equation [5]

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0 \tag{2.57}$$

To find these equations of motion the potential and kinetic energy must be found for a three-body system like the one shown in Figure (2.4). The potential energy is due solely to the gravitational influences of the three bodies in the system. Each of the bodies is under the gravitational influence of the other two, so the combination of all of that energy gives the system potential energy as shown in Eq. (2.58). This potential energy is negative because it is an attractive force between all of the bodies as shown below

$$V = -\left(\frac{\mu_2}{r_{12}} + \frac{\mu_3}{r_{13}}\right)m_1 - \left(\frac{\mu_1}{r_{12}} + \frac{\mu_3}{r_{23}}\right)m_2 - \left(\frac{\mu_1}{r_{13}} + \frac{\mu_2}{r_{23}}\right)m_3 \tag{2.58}$$
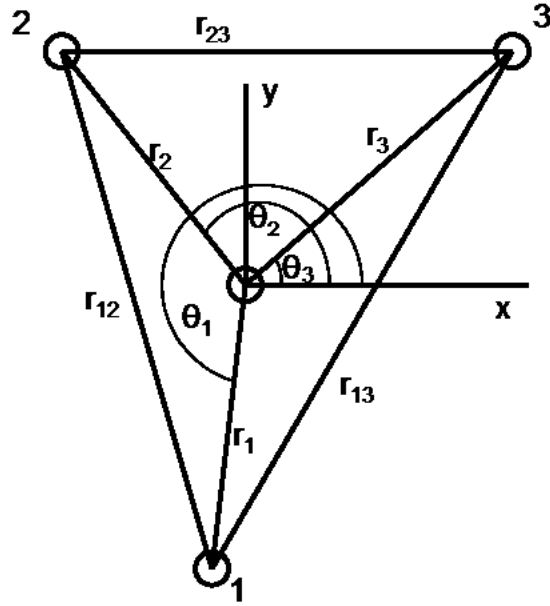
Figure 2.4: Three-Body System Showing Angles and Distances for Calculations

The kinetic energy is due simply to the motion of the bodies and is expressed as

$$T = \frac{1}{2}m_1\left(\dot{r}_1^2 + r_1^2\dot{\theta}_1^2\right) + \frac{1}{2}m_2\left(\dot{r}_2^2 + r_2^2\dot{\theta}_2^2\right) + \frac{1}{2}m_3\left(\dot{r}_3^2 + r_3^2\dot{\theta}_3^2\right) \tag{2.59}$$

where all the variables used in the equation are shown in Figure (2.4), and μ is the gravitational parameter for the body of interest, $\mu_i = Gm_i$. For the calculations the value of the radii between each mass is expressed in terms of r and θ with the following

$$r_{12} = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos\left(\theta_1 - \theta_2\right)} \tag{2.60}$$

$$r_{13} = \sqrt{r_1^2 + r_3^2 - 2r_1r_3\cos\left(\theta_1 - \theta_3\right)} \tag{2.61}$$

$$r_{23} = \sqrt{r_2^2 + r_3^2 - 2r_2r_3\cos\left(\theta_2 - \theta_3\right)} \tag{2.62}$$

Now the Lagrangian can be expressed solely in r_i and θ_i terms. This is necessary in order to work out the partial derivatives in Eq. (2.57). Using Eqs. (2.56) through (2.62), the

equations of motion for the spacecraft radial change and angular change using Lagrangian dynamics are found as

$$\ddot{r}_3 = r_3\dot{\theta}_3^{\ 2} - \left(\mu_1 r_{13}^{\left(\frac{-3}{2}\right)}\left(r_3 - r_1\cos\left(\theta_1 - \theta_3\right)\right) + \mu_2 r_{23}^{\left(\frac{-3}{2}\right)}\left(r_3 - r_2\cos\left(\theta_2 - \theta_3\right)\right)\right) \quad (2.63)$$

$$\ddot{\theta}_3 = \frac{-2\dot{r}_3}{r_3}\dot{\theta}_3 + \left(\mu_1 r_{13}^{\left(\frac{-3}{2}\right)}\left(\frac{r_1}{r_3}\sin\left(\theta_1 - \theta_3\right)\right) + \mu_2 r_{23}^{\left(\frac{-3}{2}\right)}\left(\frac{r_2}{r_3}\sin\left(\theta_2 - \theta_3\right)\right)\right) \quad (2.64)$$

It is not necessary to find the equations of motion for the Earth or the Moon because their motion is constrained to be a circular orbit around their system center of mass. This motion can be easily simulated without using complicated equations. Also the spacecraft equations of motion do not depend on the velocity or acceleration of the Earth or Moon, only their position.

Hamiltonian

To calculate the equations of motion using the Hamiltonian the momenta have to be found by taking the derivative of the Lagrangian that would be found using Eq. (2.56). The derivative of the Lagrangian is shown as [5]

$$p_i = \frac{\partial L}{\partial \dot{q}_i} \quad (2.65)$$

This form allows for the writing of $\dot{q}$ in terms of the generalized coordinates, q, and the generalized momenta, p. The new form is shown as

$$\dot{r}_i = \frac{p_{ri}}{m_i} \quad (2.66)$$

$$\dot{\theta}_i = \frac{p_{\theta i}}{m_i r_i^2} \quad (2.67)$$

Equations (2.66)-(2.67) are the first set of equations used to describe the motion. Using these equations the kinetic energy expressed in Eq. (2.59) can be written in terms of the

momenta:

$$T = \frac{1}{2}\left(\frac{p_{r1}^2}{m_1} + \frac{p_{\theta 1}^2}{m_1 r_1^2}\right) + \frac{1}{2}\left(\frac{p_{r2}^2}{m_2} + \frac{p_{\theta 2}^2}{m_2 r_2^2}\right) + \frac{1}{2}\left(\frac{p_{r3}^2}{m_3} + \frac{p_{\theta 3}^2}{m_3 r_3^2}\right) \tag{2.68}$$

The Hamiltonian can be formed using the following [5]

$$H = T + V \tag{2.69}$$

Equations (2.68)-(2.69) allow the Hamiltonian to be expressed in terms of only the generalized coordinates and the generalized momenta. From the Hamiltonian the remaining equations of motion for the system can be found using [5]

$$\dot{p}_i = -\frac{\partial H}{\partial q_i} \tag{2.70}$$

From Eq. (2.70) these equations of motion for the spacecraft are found as

$$\dot{p}_{r3} = \frac{\dot{p}_{\theta 3}^2}{m_3 r_3^3} - m_3\left(\mu_1 r_{13}^{\left(\frac{-3}{2}\right)}(r_3 - r_1\cos(\theta_1 - \theta_3)) + \mu_2 r_{23}^{\left(\frac{-3}{2}\right)}(r_3 - r_2\cos(\theta_2 - \theta_3))\right) \tag{2.71}$$

$$\dot{p}_{\theta 3} = m_3\left(\mu_1 r_{13}^{\left(\frac{-3}{2}\right)}(r_1 r_3\sin(\theta_1 - \theta_3)) + \mu_2 r_{23}^{\left(\frac{-3}{2}\right)}(r_2 r_3\sin(\theta_2 - \theta_3))\right) \tag{2.72}$$

Similar equations can be derived for the Earth and Moon, but as with the Lagrangian derivation, those derivations are not needed for the simulation.

2.1.3 Lunar Orbit

The lunar orbit has many of the same basic properties as the parking orbit around Earth. For the purposes of this mission, the main points of interest are to extend duration and coverage as much as possible. The duration of the orbit is determined by the amount of time it takes the natural dynamics of the system to cause the craft to crash into the surface of the Moon or move into an undesirable orbit. If the lunar orbit were treated as a simple two-body

problem with no outside disturbances, the craft would stay in a perfect orbit indefinitely. To get a better picture of what will really happen the non-uniform gravity field of the Moon must be modeled. The lunar gravity modeling can be done much as the Earth's gravity field was modeled above.

Using spherical harmonics, as discussed above for the Earth parking orbit, the gravity field of the Moon can be modeled and differential equations of the rate of change of the orbital elements can be found. These equations were found and modeled by the ARTEMIS team to evaluate the duration of a lunar mission using a polar orbit [3]. This team used a simplified model that only includes the five largest contributers to the gravity field from the coefficients available. These contributers are the J_2, J_3, J_5, C_{22}, and C_{31} coefficients and were used to find the first order variation of the orbit elements. The gravity effects from these coefficients can be grouped together as either short, medium, or long term effects. Any of the elements that only undergo short term effects only see variations that average out over about a month. Because the short term effects average out, the medium and long term effects cause the real changes to the orbital lifetime [3].

The differential equations listed in Eqs. (2.73)-(2.80) are the elements that influenced the orbit lifetime. These elements are grouped by coefficient and then each element has all coefficient contributers summed together to get the total effect. Only some of the equations are shown due to the increased complexity of the higher order terms, which although easily computed by a computer are rather difficult to express analytically. Below are the equations pertaining to the J_2 coefficient [3]

$$\dot{e}_{J2} = 0 \tag{2.73}$$

$$\dot{i}_{J2} = 0 \tag{2.74}$$

$$\dot{\Omega}_{J2} = -\frac{3}{2}\frac{nr_{moon}^2}{a^2\left(1-e^2\right)^2}J_2\cos i \tag{2.75}$$

$$\dot{\omega}_{J2} = \frac{3}{4}\frac{nr_{moon}^2}{a^2\left(1-e^2\right)^2}J_2\left(4-5\sin^2 i\right) \tag{2.76}$$

where $n = \sqrt{\mu/a^3}$ and $r_{moon} = 1739 km$ is the radius of the Moon. Equations (2.77)-(2.80) describe how the orbit elements are influenced by the J_3 coefficient [3]

$$\dot{e}_{J3} = \frac{3}{2}\frac{nr_{moon}^3}{a^3\left(1-e^2\right)^2}J_3\sin i\left(\frac{5}{4}\sin^2 i - 1\right)\cos\omega \tag{2.77}$$

$$\dot{i}_{J3} = -\frac{3}{2}\frac{nr_{moon}^3 e}{a^3\left(1-e^2\right)^3}J_3\cos i\left(\frac{5}{4}\sin^2 i - 1\right)\cos\omega \tag{2.78}$$

$$\dot{\Omega}_{J3} = -\frac{3}{2}\frac{nr_{moon}^3 e}{a^3\left(1-e^2\right)^3}J_3\cot i\left(\frac{15}{4}\sin^2 i - 1\right)\sin\omega \tag{2.79}$$

$$\dot{\omega}_{J3} = -\frac{3}{2}\frac{nr_{moon}^3}{a^3\left(1-e^2\right)^3}J_3\left(\frac{1+4e^2}{e}\sin i\left(\frac{5}{4}\sin^2 i - 1\right)\right)$$

$$+ \frac{3}{2}\frac{nr_{moon}^3}{a^3\left(1-e^2\right)^3}J_3\left(\frac{e\cos^2 i}{\sin i}\left(\frac{15}{4}\sin^2 i - 1\right)\right)\sin\omega \tag{2.80}$$

The remaining equations for the rate of change of the orbit elements are omitted from the report, but follow the same pattern of increase as shown by the Legendre polynomials discussed in the gravity modeling section of the parking orbit discussion [3].

2.1.4 Plane Changes

Since this mission will be launched from WFF, the inclination of the parking orbit is constrained between 38° and 55°; placing the craft on a different plane than the lunar orbit plane [7]. Although the craft can be drawn toward the Moon if the initial transfer orbit ΔV is performed at the right time, the resulting inclination at the Moon may not be desired. In order to correct the inclination the craft will have to undergo a plane change to place it in the desired orbital plane around the Moon. The inclination is the most important of the orbit elements describing the lunar orbit because it controls what portion of the Moon's surface the spacecraft will orbit over. Figure (2.5) shows a simple diagram of different orbits and how they would cover the surface of a sphere with different inclinations.

There are three different methods to perform this plane change, each of which are listed in this section. First the plane change can occur around the Earth. The craft can launch
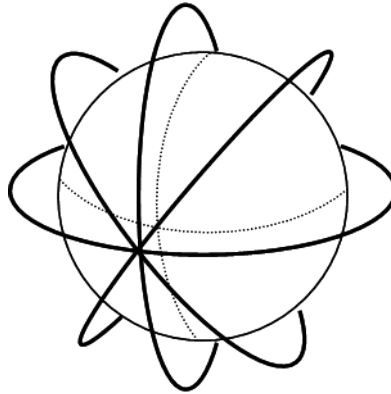
Figure 2.5: Differently Inclined Orbits Around a Spherical Planet

from WFF into an orbit with the closest inclination to the lunar orbit plane, and then the orbit plane can be adjusted. The plane change is done by performing a burn perpendicular to the orbital plane, which results in a ΔV cost of [8]

$$\Delta V = 2V \sin \frac{\theta}{2} \tag{2.81}$$

where θ is the angle of change and V is the current velocity and equivalent final velocity of the orbit being changed. Because this ΔV is larger for a faster moving spacecraft, it is advantageous to perform this kind of a plane change while moving slower. The cost leads to the second option for a plane change, performing the same type of maneuver but when in orbit around the Moon. Because of the Moon's weaker gravity field, the spacecraft would orbit the Moon at a much slower speed than around the Earth. This option allows for a 180° plane change be possible, meaning if the entire surface of the Moon needs to be passed over, it can be accomplished with a relatively lower amount of fuel.

If continuous change of the inclination angle is not needed, and the spacecraft will stay in one orbit throughout its entire lifetime, than the plane change can be performed during the transfer orbit. This option opens up a variety of possibilities for the ΔV burn. Because the transfer orbit is being modeled with a three-body system the ΔV change can be in any direction and of any magnitude, some of which may lead to the desired orbit and others may

push the craft completely off course. This option is investigated with a numerical simulation and requires no analytical analysis other than comparing the results to other methods.

2.2 Methods Used

This section discusses the benefits and drawbacks of each of the above methods discussed and explains which method is chosen for each part of the mission. Although numerical simulations were performed using all of these methods only a few were used for extensive simulations of the mission. The main factors used to weigh each of the methods were speed of numerical analysis and accuracy of the results.

2.2.1 Launch & Parking Orbit

For the launch and parking orbit only two methods were discussed, using the gravity field modeling or using a simple point based gravity analysis for the Earth. When presented with these two options the solution for this mission is simple, to not use the spherical harmonic model and to opt for the simpler point mass model. If the spacecraft was going to spend extended orbital periods around the Earth then the non-uniform gravity field of the Earth would begin to alter its course significantly, but for this mission the craft will only spend a fraction of one orbit around the Earth before performing its burn to push it into the transfer orbit.

2.2.2 Transfer Orbit

This part of the mission has the most methods available for the analysis. The three possibilities for the dynamic analysis are Newtonian, Lagrangian, and Hamiltonian. Each type of analysis leads to almost identical results for any given initial conditions. The speeds of the simulations are given in Table (2.2). These speeds show that using the Lagrangian is the

Table 2.2: Numerical Simulation Speeds

Method	Time (sec)
Newtonian	3.2
Lagrangian	1.7
Hamiltonian	2.5

best approach for calculation. The increased computational speed is because with the assumptions made in the system the Lagrangian performs the fewest calculations per iteration of the numerical integrator.

Simplicity is an additional criterion that sets the three methods apart. Because the speeds of calculation for a single orbit are all generally small, even when performed on a desktop computer, the final weighing factor for the methods is whether or not they can be easily used. In this regard the Newtonian method is the easiest to use. The ease of use is because of the simple conversion from Cartesian coordinates to orbit elements, and the simple method of using a rotating or inertial reference frame. It is for this reason that the bulk of the major calculations are done with the Newtonian calculations.

2.2.3   Lunar Orbit

For the lunar orbit only one method is discussed in detail, to use a two-body system and a simplified non-uniform gravity field for the disturbances on the spacecraft orbit. The Moon's gravity field is modeled using only the major contributers because of the significant computation time it can take if all of the coefficients are used to find the gravitational influences. Using the five major contributers gives a reasonable estimation of the actual orbit, which can be used as a good estimate of the lifetime of the craft.

Chapter 3

Calculations, Simulations, and Results

3.1 Launch & Parking Orbit

As stated in above chapters the Taurus XL rocket is used as the launch vehicle to constrain this mission. The Taurus XL rocket and launching from WFF will limit the initial inclination choices and the maximum altitude of the parking orbit. These limitations are discussed below in the Launch Constraints section. From the launch the spacecraft will coast into a circular parking orbit around the Earth. From this parking orbit the spacecraft will perform a burn into its transfer orbit. The timing of the burn is crucial to minimizing the total ΔV of the mission.

Using a two-body analysis of the spacecraft and the Earth, the ΔV required to push the spacecraft into the transfer orbit and capture around the Moon can be found. The speed and motion of the spacecraft is found around the Earth with the two-body problem, and the necessary speed to move into the transfer orbit is found by solving Lambert's problem. The necessary speed to slow down around the Moon depends on the initial conditions of the transfer orbit and how the craft will enter the lunar sphere of influence. These calculations are done to show the importance of the positioning in the parking orbit when the burn is

performed.

These basic calculations are not used for the transfer orbit or lunar orbit but are used to generate the contour plot shown in Figure (3.1). This figure shows the total ΔV that the craft will have to be able to perform to push itself into the transfer orbit and then be captured around the Moon. The plot generated in Figure (3.1) shows the ΔV requirements for a 400 km parking orbit launched on January 1st, 2015. As can be seen the values for the ΔV can vary greatly and the timing of this burn is crucial to make sure this cost is not greater than the spacecraft's budget. This contour plot also shows that if the lunar transfer orbit duration is important it can be improved by increasing the ΔV. Because this mission does not depend on speed, taking the lowest ΔV possible is advisable. Shown in Figure (3.2) is another contour plot generated for a 1000 km parking orbit launched on the same date January 1st, 2015. A launch like this would use a smaller spacecraft reaching the limits of the Taurus rocket's range [7].

This code takes a user input of a desired launch date and processes a simple Earth-Moon model to the correct date. This model uses lunar ephemeris data for the Moon's position in 2006 and then extrapolates the position of the Moon to a desired date [1]. The altitude of the parking orbit can be changed in the code to any value which should be constrained by the Taurus rocket. The launch is simulated as a simple Hohmann transfer from the surface of the Earth to the parking orbit altitude. This gives a simplified idea of the time the flight will take. The duration of in the parking orbit is stepped through in loops and used to find the position of the spacecraft. The code then uses loops to solve Lambert's problem for the ΔV requirements and time of flight to the Moon. This Lambert's problem solver takes in the position of the spacecraft and Moon as well as the desired time of flight. This analysis is not as accurate as the transfer orbit analysis that is done in the following sections, but still illustrates the point of the timing of the burn.
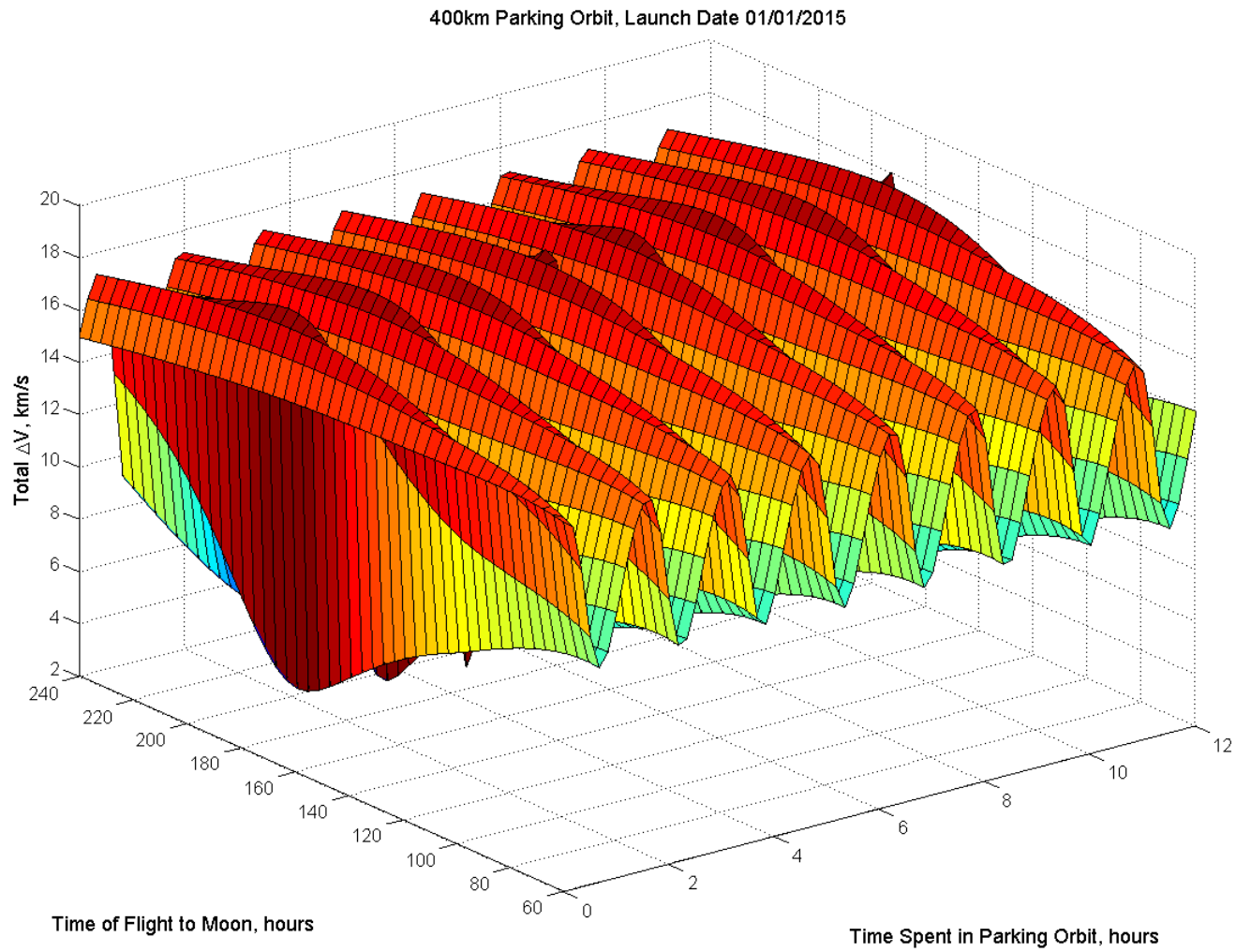
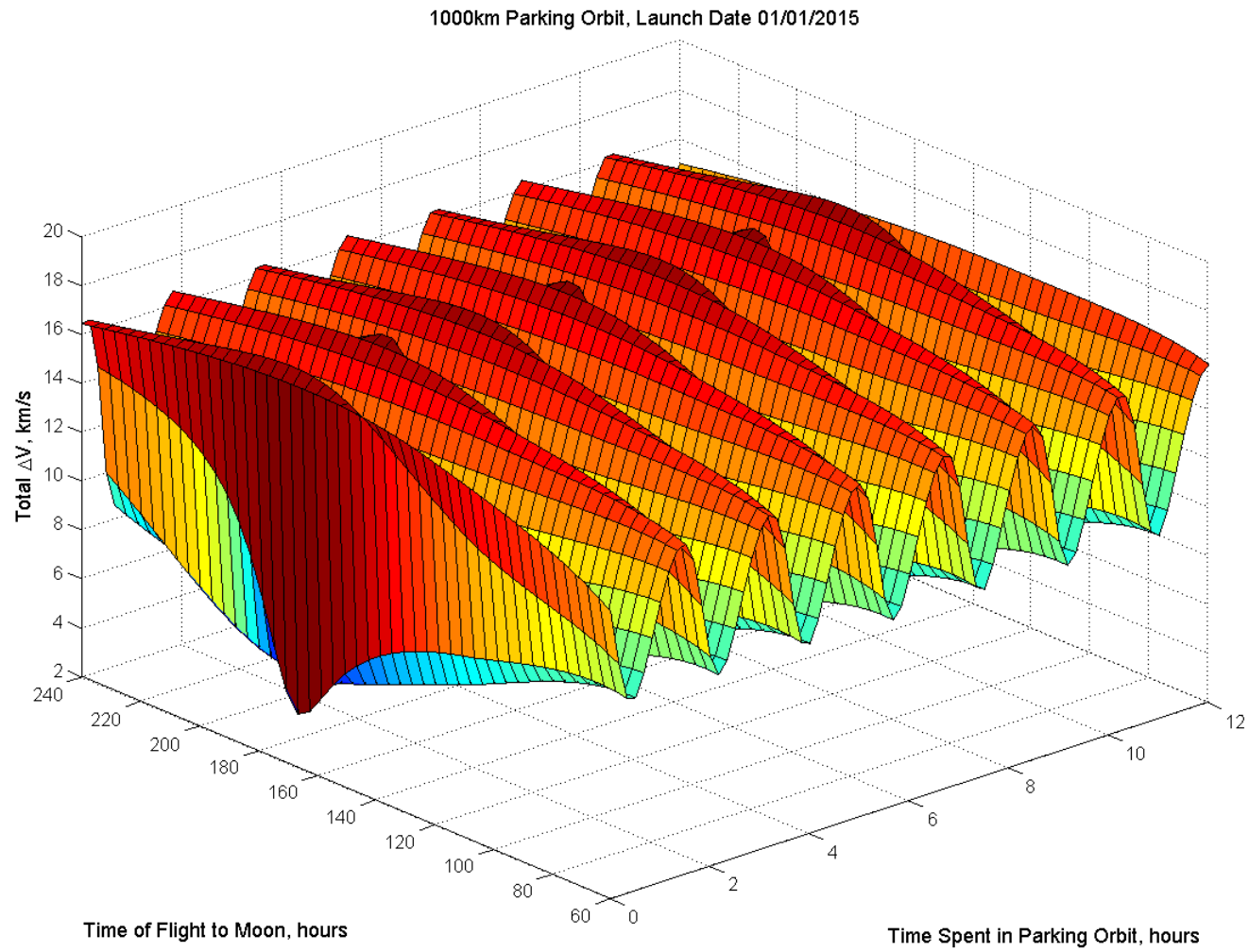Figure 3.1: Contour Plot of ΔV Requirements for Lunar Mission from a 400km Parking Orbit

Figure 3.2: Contour Plot of ΔV Requirements for Lunar Mission from a 1000km Parking Orbit

3.1.1   Launch Constraints

The launch constraints that the Taurus XL rocket gives are shown in Figures (3.3)-(3.6). Figure (3.3) shows the possible launch angles from three locations in the United States with WFF being one of them. This angle range is what limits the inclination angle of the parking orbit. Figures (3.4)-(3.6) show the maximum altitude for a given payload for different configurations of the Taurus rocket. The Taurus XL rocket is the 3110 configuration with the highest possible altitudes and payload masses. These figures give the constraints on the launch of the spacecraft [7].

3.1.2   Available Parking Orbits

When launching from WFF the available parking orbits are restricted by the launch angle and vehicle. Looking at these available orbits they can be matched up to the necessary velocity to achieve any of the transfer orbits. Comparing the two gives the necessary ΔV as discussed in Chapter 2. To do this comparison properly the velocities have to be expressed in the same Cartesian coordinates, so the available parking orbits should be listed with their position and velocities converted from the orbit elements. Some of these position and velocity numbers are listed in Tables (3.1) and (3.2). The first table shows the values for the parking orbits for an M_0 of zero degrees, which would place it between the Earth and the Moon. This configuration can be seen in Figure (3.7) and explains why the y and z position data is all zeros, as well as the x-velocity data. The data shown in Table (3.2) is for a mean anomaly angle of 90 degrees. These tables were made using a series of nested loops in MATLAB to create a 4-D matrix. This matrix contains the (Semi-major Axis, Inclination, Mean Anomaly, Position or Velocity). The final entry in the matrix would have a magnitude of six, where the first three entries would be the x, y, or z position, and the final three entries would be the velocities. This can be visualized as a three-dimensional matrix where each element contains the position and velocity vector. The tables shown in Table (3.1) and (3.2) are slices of this three-dimensional matrix.

Figure 3.3: Possible Inclination Angles from Various USA Launch Points [7]
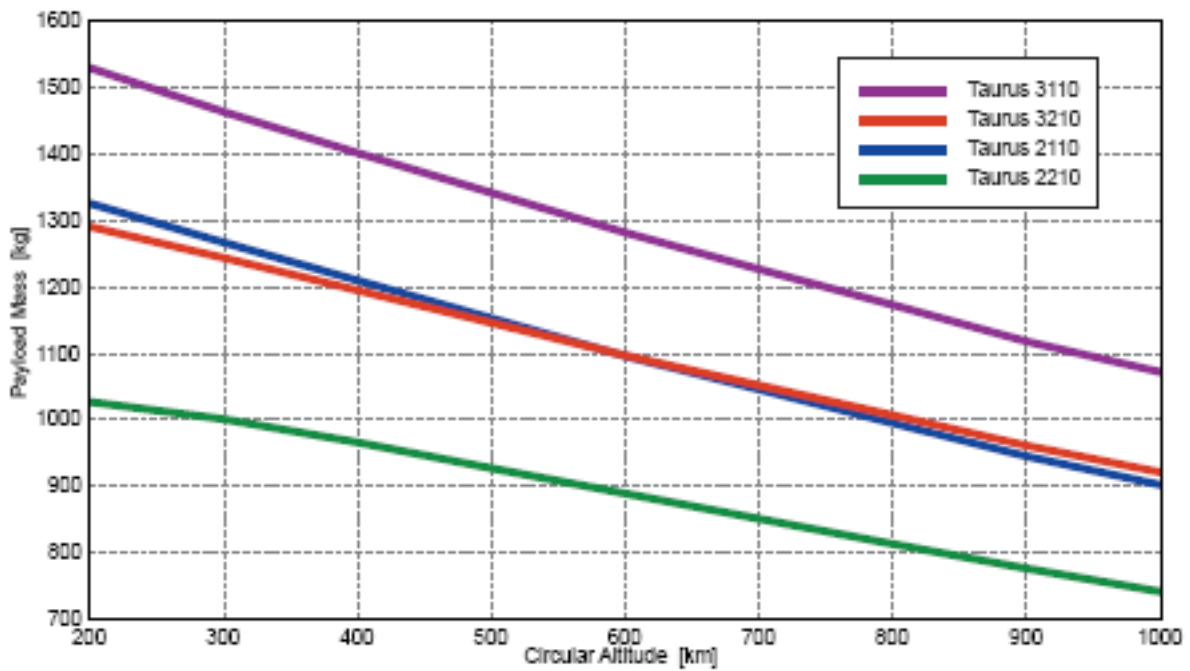

Figure 3.4: Maximum Altitudes and Payloads for the Taurus Rocket Launched From WFF at 40° Inclination [7]
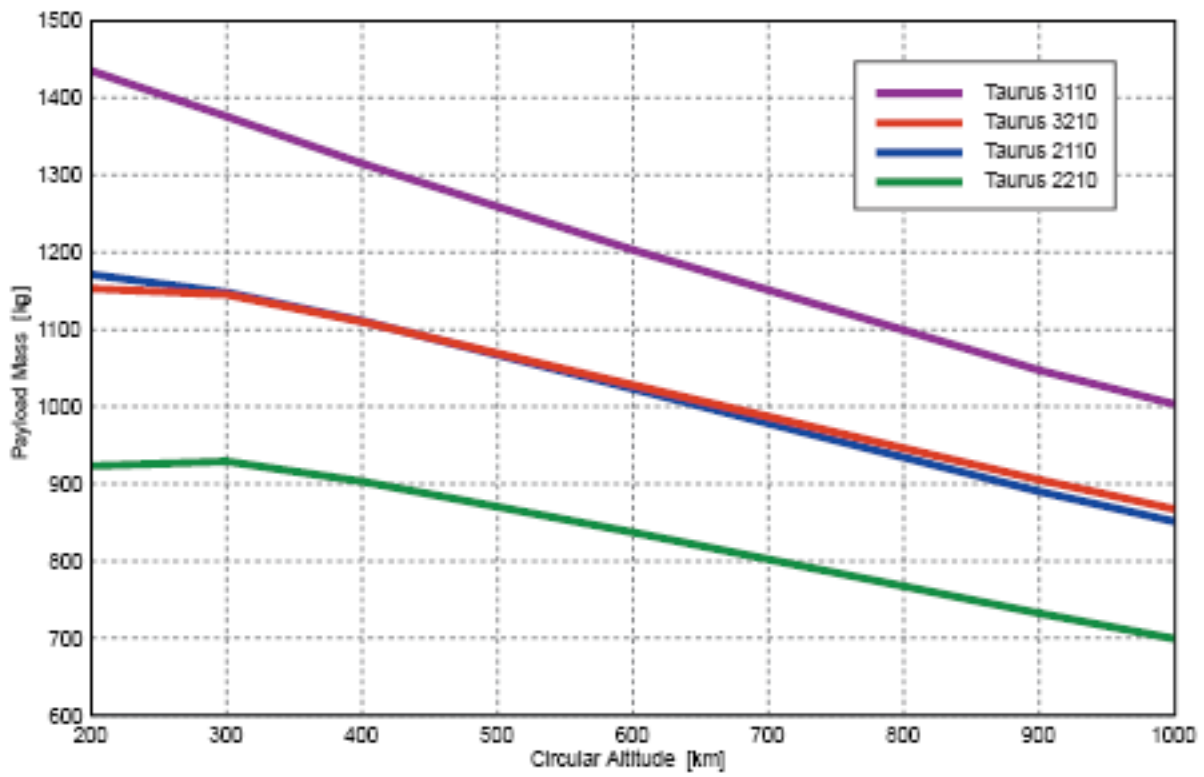
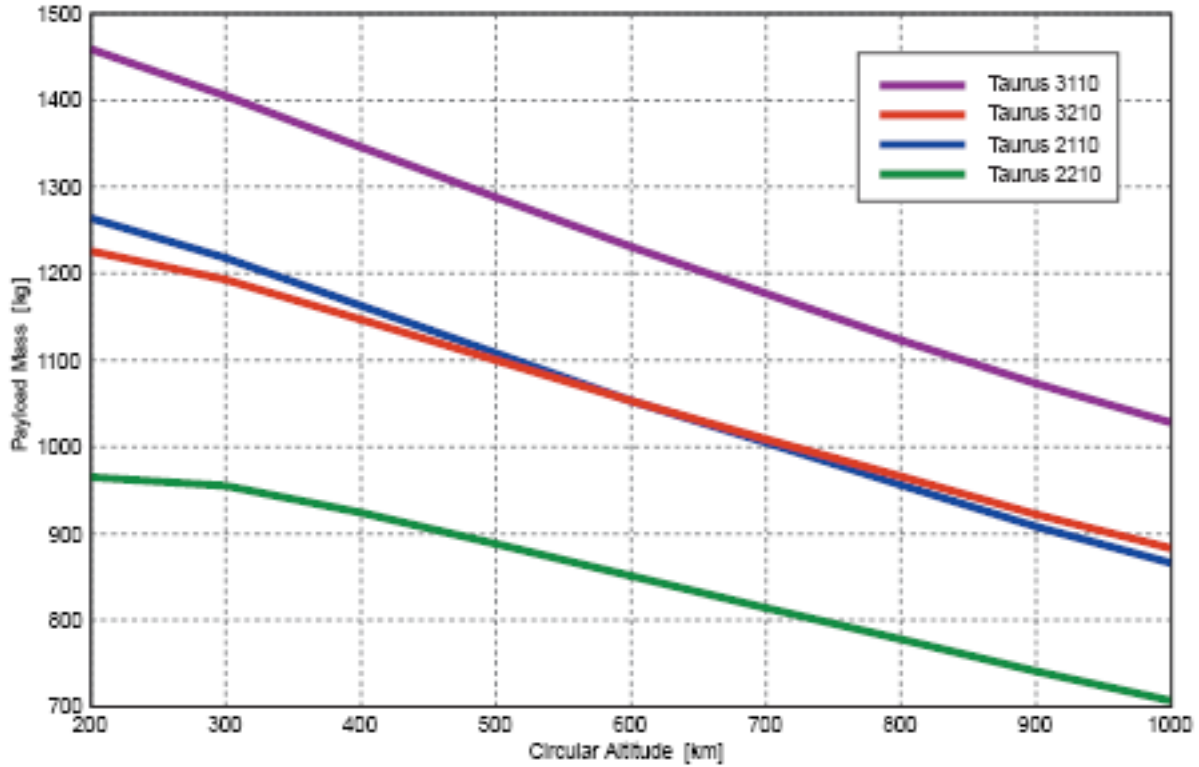Figure 3.5: Maximum Altitudes and Payloads for the Taurus Rocket Launched From WFF at 45° Inclination [7]

Figure 3.6: Maximum Altitudes and Payloads for the Taurus Rocket Launched From WFF at 50° Inclination [7]
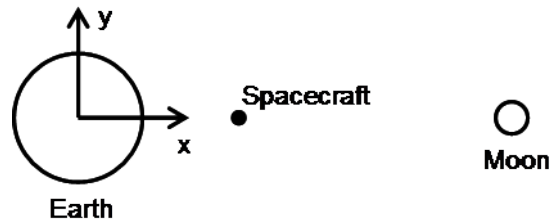

Figure 3.7: Simple Diagram of M_0 for Parking Orbit

Table 3.1: Parking Orbit Position and Velocity Vectors for $M_0 = 0 deg$

		$i(deg)$							⋯		
		38		39		40				55	
		pos(km)	vel(km/s)	pos(km)	vel(km/s)	pos(km)	vel(km/s)			pos(km)	vel(km/s)
	6570	$\begin{pmatrix} 6570 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 6.1379 \\ 4.7954 \end{pmatrix}$	$\begin{pmatrix} 6570 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 6.0532 \\ 4.9018 \end{pmatrix}$	$\begin{pmatrix} 6570 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 5.9668 \\ 5.0067 \end{pmatrix}$			$\begin{pmatrix} 6570 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 4.4676 \\ 6.3804 \end{pmatrix}$
	6620	$\begin{pmatrix} 6620 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 6.1147 \\ 4.7773 \end{pmatrix}$	$\begin{pmatrix} 6620 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 6.0303 \\ 4.8833 \end{pmatrix}$	$\begin{pmatrix} 6620 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 5.9442 \\ 4.9878 \end{pmatrix}$			$\begin{pmatrix} 6620 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 4.4507 \\ 6.3563 \end{pmatrix}$
$a(km)$	6670	$\begin{pmatrix} 6670 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 6.0917 \\ 4.7594 \end{pmatrix}$	$\begin{pmatrix} 6670 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 6.0077 \\ 4.8649 \end{pmatrix}$	$\begin{pmatrix} 6670 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 5.9219 \\ 4.9690 \end{pmatrix}$			$\begin{pmatrix} 6670 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 4.4340 \\ 6.3324 \end{pmatrix}$
	⋮										
	7370	$\begin{pmatrix} 7370 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 5.7952 \\ 4.5277 \end{pmatrix}$	$\begin{pmatrix} 7370 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 5.7153 \\ 4.6281 \end{pmatrix}$	$\begin{pmatrix} 7370 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 5.6336 \\ 4.7272 \end{pmatrix}$			$\begin{pmatrix} 7370 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 4.2182 \\ 6.0242 \end{pmatrix}$

Table 3.2: Parking Orbit Position and Velocity Vectors for $M_0 = 90deg$

			$i(deg)$				
		38		39		40	$\cdots$
		pos(km) vel(km/s)		pos(km) vel(km/s)		pos(km) vel(km/s)	
	6570	$\begin{pmatrix} 4645.7 \\ 3660.9 \\ 2860.2 \end{pmatrix}\begin{pmatrix} -5.5077 \\ 4.3401 \\ 3.3909 \end{pmatrix}$		$\begin{pmatrix} 4645.7 \\ 3610.4 \\ 2923.6 \end{pmatrix}\begin{pmatrix} -5.5077 \\ 4.2803 \\ 3.4661 \end{pmatrix}$		$\begin{pmatrix} 4645.7 \\ 3558.8 \\ 2986.2 \end{pmatrix}\begin{pmatrix} -5.5077 \\ 4.2191 \\ 3.5403 \end{pmatrix}$	
		pos(km) vel(km/s)		pos(km) vel(km/s)		pos(km) vel(km/s)	
	6620	$\begin{pmatrix} 4681.0 \\ 3688.7 \\ 2881.9 \end{pmatrix}\begin{pmatrix} -5.4869 \\ 4.3237 \\ 3.3781 \end{pmatrix}$		$\begin{pmatrix} 4681.0 \\ 3637.9 \\ 2945.9 \end{pmatrix}\begin{pmatrix} -5.4869 \\ 4.2641 \\ 3.4530 \end{pmatrix}$		$\begin{pmatrix} 4681.0 \\ 3585.9 \\ 3008.9 \end{pmatrix}\begin{pmatrix} -5.4869 \\ 4.2032 \\ 3.5269 \end{pmatrix}$	
		pos(km) vel(km/s)		pos(km) vel(km/s)		pos(km) vel(km/s)	
$a(km)$	6670	$\begin{pmatrix} 4716.4 \\ 3716.6 \\ 2903.7 \end{pmatrix}\begin{pmatrix} -5.4663 \\ 4.3075 \\ 3.3654 \end{pmatrix}$		$\begin{pmatrix} 4716.4 \\ 3665.3 \\ 2968.1 \end{pmatrix}\begin{pmatrix} -5.4663 \\ 4.2481 \\ 3.4530 \end{pmatrix}$		$\begin{pmatrix} 4716.4 \\ 3613.0 \\ 3031.6 \end{pmatrix}\begin{pmatrix} -5.4663 \\ 4.1874 \\ 3.5136 \end{pmatrix}$	
	$\vdots$						

3.1.3   Parking Orbit Choice

As discussed above the parking orbit should be chosen so it closely matches the initial velocity requirements of the transfer orbit. This will minimize the required ΔV to change into the transfer orbit. In the next section the transfer orbits will be discussed and this will show how these choices are significant. The possible inclination angles of the parking orbits from WFF are all out of range of the Moon's orbit plane inclination, therefore any mission launched from WFF will not be able to make a simple planar orbit transition from the Earth to the Moon unless the spacecraft performs a plane change while orbiting the Earth. A plane change during Earth orbit is not advisable as the ΔV requirements would be excessively high.

3.2   Transfer Orbit

One of the more heavily investigated parts of the mission is the transfer orbit. This portion of the mission involves the most variables and is analyzed with the most computer simulations to fully investigate the possibilities of a lunar mission from WFF. This section discusses these numerical simulations and shows plots of the trajectories that take the craft from the Earth to the Moon. As discussed in previous sections this analysis is done by solving the restricted three-body problem of the Earth-Moon-Spacecraft system. The spacecraft has a negligible mass and the Earth and Moon stay on circular orbits around their barycenter, remaining 180° apart from one another. The lunar orbital radius is constant and the Earth-Moon system is considered planar.

3.2.1   The Basic Trajectory - Planar

The first investigation into the transfer orbit is to consider a planar orbit from the Earth to the Moon, neglecting the z-axis. This basic trajectory is simulated using Newtonian,

Lagrangian, and Hamiltonian dynamics. The results of this test showed nearly identical orbits for each case, which was to be expected, and these results are shown in Figures (3.8)-(3.10). The orbit shown is an hourglass trajectory similar to the Apollo missions to the Moon. This trajectory is used to show the more interesting dynamics of the three-body system, allowing the craft to depart the Earth and swing by the Moon. Performing the necessary burn at the Moon would allow for capture.

These simulations are performed in MATLAB and use the *ode45* function to numerically integrate the equations of motion. In each case the integration is run for a time of seven days and given more strict tolerances than the standard function. The time is broken up into ten thousand equal time steps for the integration and the initial conditions for each case will be discussed in the sections below for each method. Using the defined time-steps keeps *ode45* from creating jagged edges on the more simply performed portions of the orbit. From running these methods it is seen that the Lagrangian method is faster as explained in previous sections. The increased speed is because the Lagrangian method has fewer overall computations when the planet dynamics are removed from the simulation.

Newtonian

Because the motion of the planets is so simple it is easily maintained by using the right initial conditions and neglecting their more complicated equations of motion. Ignoring the planet's equations of motion is not only done in the Newtonian method but all methods. The initial conditions for the simulation place the spacecraft at periselenium where it will be traveling in only the y-direction. This placement and direction of travel make the only variables for the initial conditions the initial velocity and the lunar miss distance. By integrating backwards the required Earth orbit departure velocity can be found and used to formulate the ΔV required to get on this trajectory. The Earth and Moon start on the x-axis positioned at their respective points away from the barycenter. Using the equations of motion for the Newtonian system the trajectory is formed in Figure (3.8). The Earth is located near the

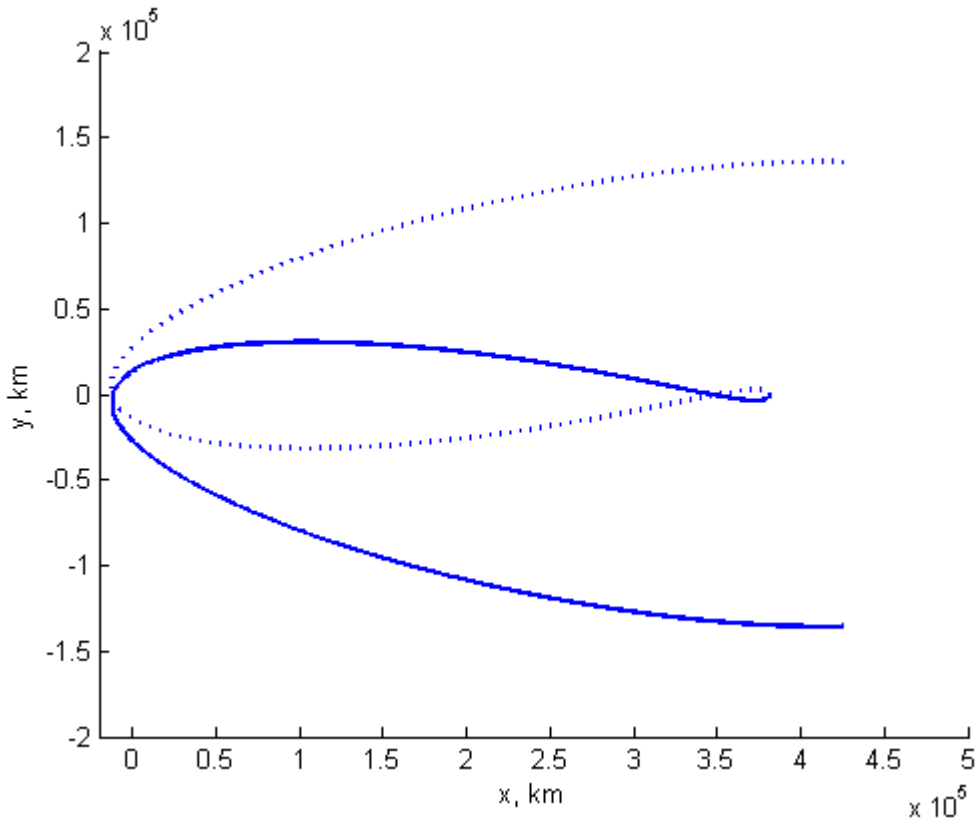origin and the Moon is located near the loop on the right.

Figure 3.8: Earth-Moon Transfer Orbit Calculated Using Newtonian Mechanics

In this simulation the initial velocity is 1.5 in the negative y-direction in non-dimensional units which corresponds to an actual inertial velocity of about 2.55 km/s. The lunar miss distance is equivalent to about 140 km from the Moon's surface. For the Newtonian simulation the numerical integrator's initial conditions are the Cartesian coordinates of the Earth, Moon, and spacecraft as well as the initial velocities. These twelve initial conditions, which is the same number for all the simulations, are easy to express because the initial conditions and results are in Cartesian coordinates.

Lagrangian

In the simulation the Earth starts at 180° and the Moon at 0°. Using the equations of motion for the Lagrangian system a simple Earth-Moon trajectory can be seen in Figure (3.9). This trajectory is identical to the Newtonian simulation and shows that both methods give the same results. The Lagrangian method uses cylindrical coordinates because these coordinates make the derivation of the equations of motion easier. The cylindrical coordinates make the initial conditions simpler to input into the simulation as the initial radii and angles are all necessary for the Cartesian initial conditions used above. The radial rate of change is zero for all the bodies and the angular rate of change is a constant for the planets. Using the same 1.5 velocity in non-dimensional units and converting to an angular rate of change gives the same results as the Newtonian. As stated in previous sections, the Lagrangian method is computationally faster because of the fewer calculations done in each time step. This computational speed is a negligible difference though and not significant because the simulations are not run in large numbers.

Hamiltonian

The final method investigated for the planar trajectory is the Hamiltonian. This method generated the plot shown in Figure (3.10), which is the same as the other plots. The Hamiltonian method uses momenta instead of velocities or angular rates of change. These momenta are only applied to the spacecraft as the planetary motion is kept simple. The initial conditions are derived from the same 1.5 velocity and 140 km miss distance as used in all the other simulations. The Hamiltonian equations of motion are more complicated than the other methods and do not give any real benefit over the simple motion being described. If a phase space analysis was to be investigated the Hamiltonian would be the obvious choice for the dynamics.
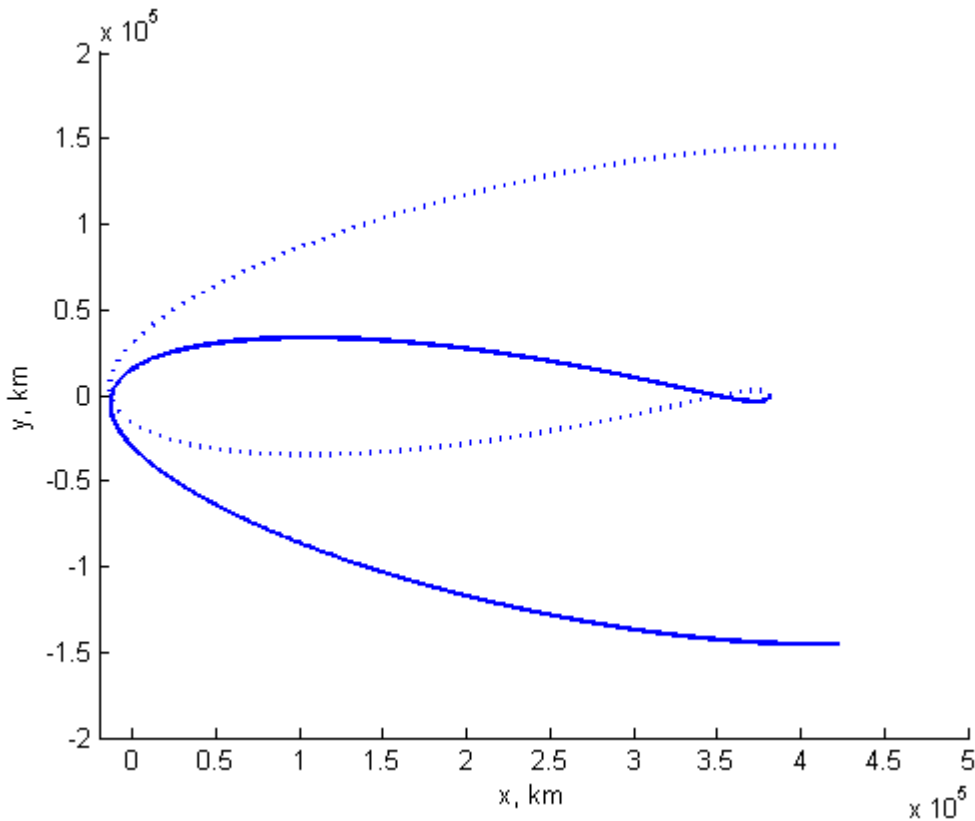
Figure 3.9: Earth-Moon Transfer Orbit Calculated Using Lagrangian Mechanics

Energy Surfaces

Using the energy of the system to limit the motion of the spacecraft creates barriers for the motion. These energy surfaces can be easily shown using a simple computer program to step through the energy levels and plot all possible x and y locations available to the craft at that energy level. This plot is shown in Figure (3.11) where the Earth would be near the origin and the Moon would be at around 0.99 along the x-axis. What this represents is the limit of motion of the craft for any given amount of energy. This is useful for knowing if a spacecraft will leave the Earth-Moon system, but only applies to craft that are in free flight and not under any powered flight. The plot shows the varying energy levels in different colors; if a craft is located in a lower energy zone it cannot make it to a higher energy area.
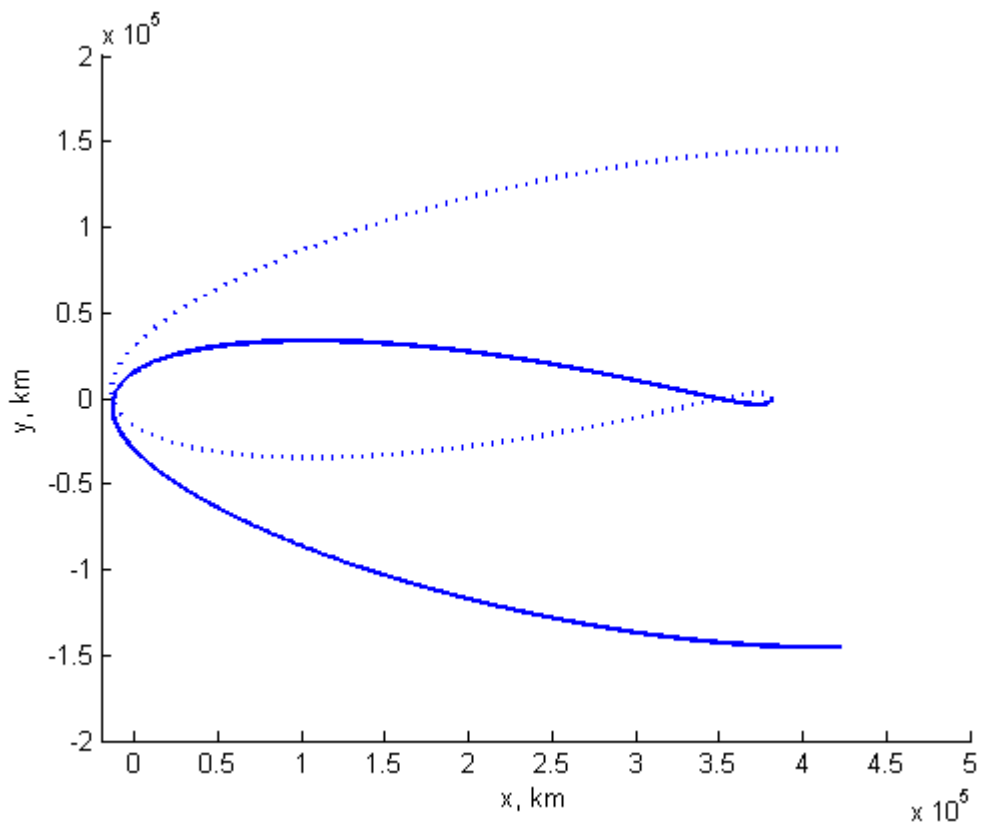
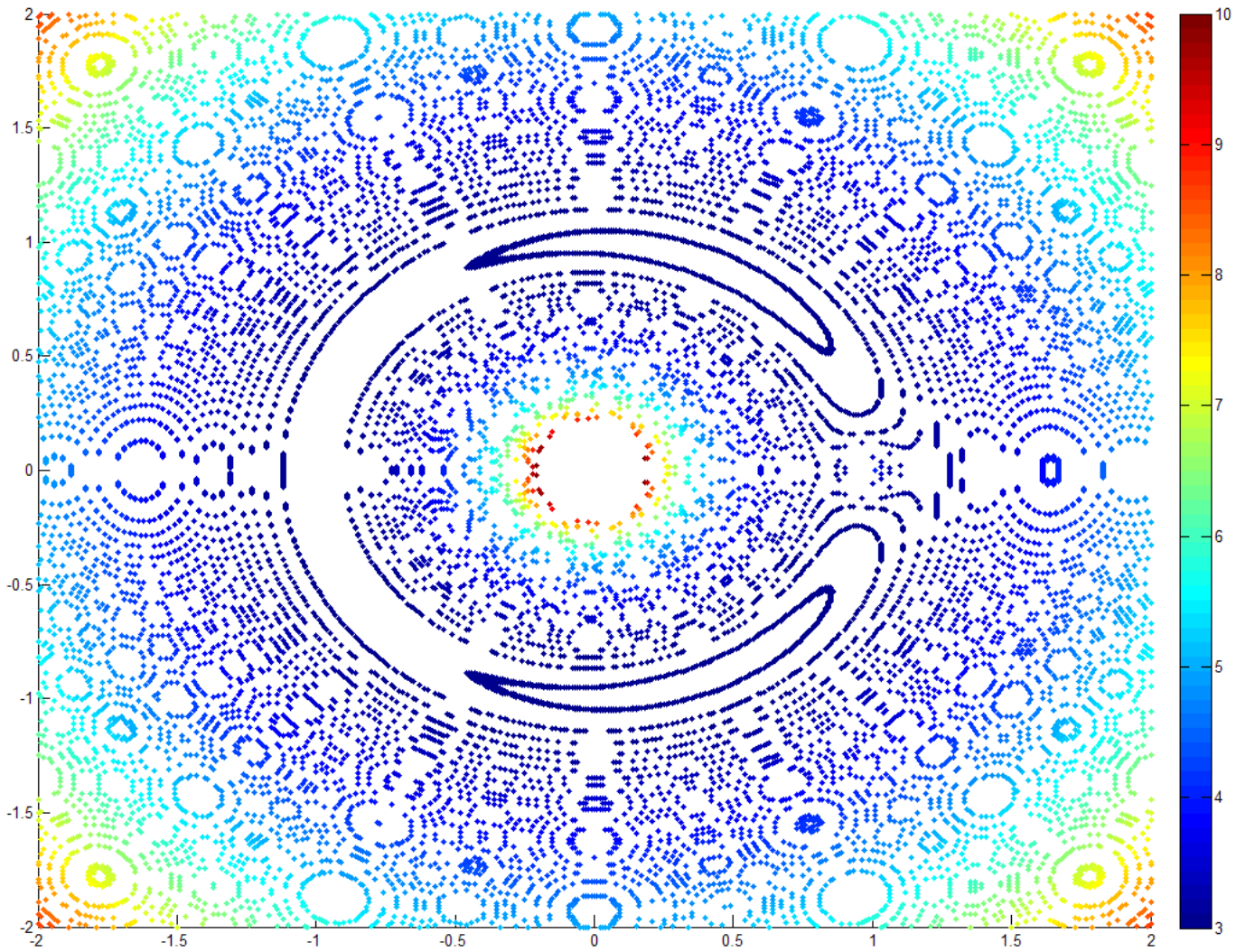Figure 3.10: Earth-Moon Transfer Orbit Calculated Using Hamiltonian Mechanics

Figure 3.11: Energy Contours of the Earth-Moon System

3.2.2   Three Dimensional Trajectories

For the purposes of the mission a three dimensional investigation has to be conducted due to the inclination changes that will have to occur to launch a craft from WFF and have it arrive at the Moon. Because of the reasons stated in previous sections and the fact that the simulation will be in three dimensions the Newtonian equations of motion are used. These equations of motion are integrated in the same way as the planar cases. A successful trajectory for this case is shown in Figure (3.12) where the craft is arriving at the Moon with a slight z-direction velocity. This figure has the z-axis magnified to show the change to the orbit shape.
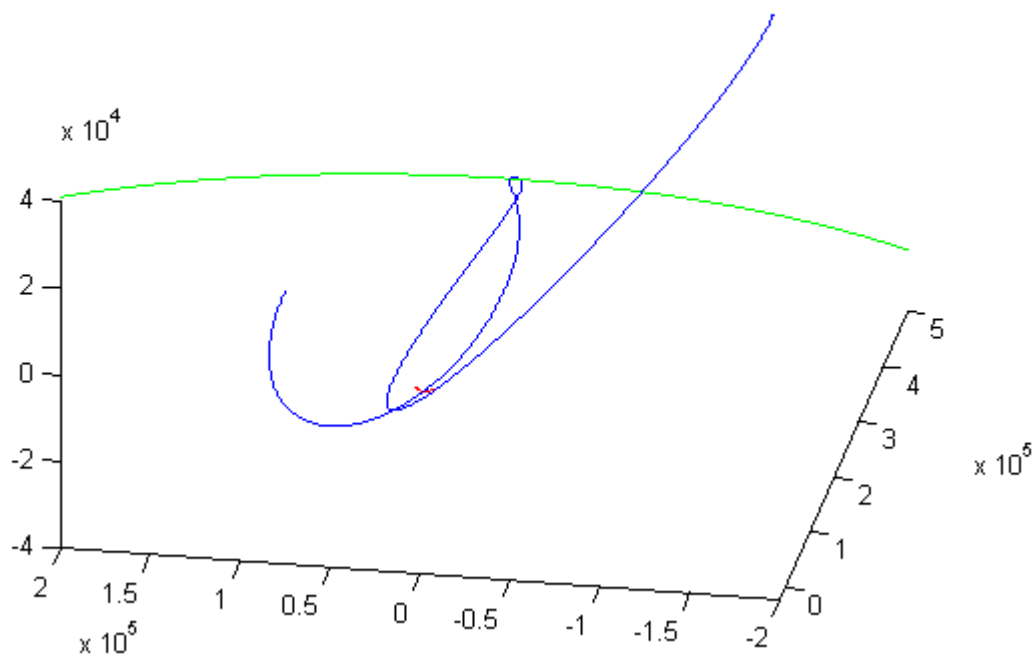

Figure 3.12: Three-Dimensional Earth-Moon Transfer Orbit Calculated Using Newtonian Mechanics

This trajectory is integrated both forward and backward in time from the initial conditions

that are the same as the Newtonian planar case, with a zero z position and a -0.2 z velocity. This trajectory places the craft on an orbit that is inclined roughly 4.5° to the equatorial of the lunar orbit. To compare what Earth parking orbit would best match this case it is necessary to find the velocities of this orbit around the Earth. It is also a good check to see what inclination the craft will have while around the Earth. For the simulation in Figure (3.12) the craft has an inclination of roughly 20.5° which when put into an Earth centered frame would be an inclination of anywhere from 38.5° to 48.5°. The variable Earth centered inclination is due to the fact that the Moon's orbital plane varies its inclination with respect to the equatorial plane of the Earth from roughly 18° to 28°.

The resulting Earth centered inclination means that this orbit would fall into the inclination constraint from launching from WFF. The point where this velocity data is taken is at an altitude of roughly 847 km, placing it within the maximum altitude of the Taurus XL rocket. The position and velocity data at this point is shown in Table (3.3). This data is used to find the ΔV necessary to get the craft from the parking orbit to this transfer orbit. From these calculations the ΔV is roughly 2.62 km/s. So from the lunar ephemeris the Moon's inclination can be determined and used to find the necessary parking orbit inclination for the desired launch date.

Table 3.3: Position and Velocity of Point on Near Earth Orbit

$$
\begin{array}{cc}
\text{Position} & \text{Velocity} \\
\begin{pmatrix} -6618.8 \\ -2420.5 \\ 1555.2 \end{pmatrix} & \begin{pmatrix} 3.7473 \\ -8.1802 \\ 2.5508 \end{pmatrix}
\end{array}
$$

3.2.3   Plane Change & Other Mid-Flight Maneuvers

One method of changing the final inclination angle of the lunar orbit is to perform a mid-flight maneuver to alter the trajectory and the resulting entry angle toward the Moon. The

numerical simulation can be terminated at a point in the middle of the transfer orbit and the position and velocity data from this point can be used to start the simulation again continuing it undisturbed. But if the velocity data is altered it will go on a different trajectory, so the difference between the velocity data will give the ΔV to perform this mid-flight maneuver.

Because any new velocity can be chosen, and in any direction the choices are infinite. To limit the choice, the maneuver will be performed against the direction of travel to only slow down the spacecraft. This type of maneuver is done to cause the spacecraft to drift toward the Moon instead of orbiting past it. A sample trajectory of this type is shown in Figure (3.13), where a mid-flight correction is made and causes the craft to change direction and drift toward the Moon. This correction occurs at the starred point in the orbit. The Earth is represented by the blue circle and the Moon by the red circle.
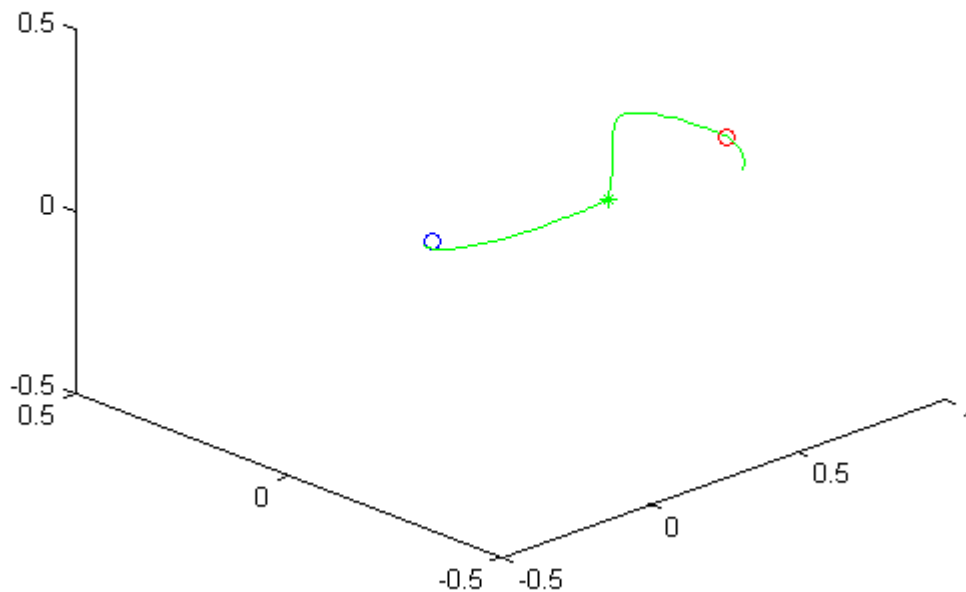
Figure 3.13: Transfer Orbit with Mid-Flight Burn

To choose the location of the maneuver a simple bisection method is used, which tests the Earth miss distance when the orbit is integrated backward through time from the Moon to the Earth. Testing the Earth miss distance ensures that the craft will be close enough to the Earth to be near one of the parking orbit choices from WFF. The ΔV value is chosen as small as possible to still get the craft close enough to the Earth for a successful transfer. After the miss distance is within 1000 km the inclination angle of the resulting near Earth orbit can be calculated and used to determine if this orbit will be a viable WFF launch [7].

This type of plane change allows for any type of final lunar orbit to be acquired without performing a plane change while orbiting the Moon. Because the choice of velocity changes is infinite and can cause any number of changes to the orbit, any final orbit type is possible, although some changes will cost a significantly higher ΔV than a simple plane change around the Moon. The orbit shown in Figure (3.13) uses a ΔV of 1.55 km/s and would have to leave WFF anywhere between 45° and 55° to arrive at a lunar orbit with an inclination of 50°.

3.3   Lunar Orbit

The final stage of the mission is the lunar orbit. The numerical simulations and calculations for this part of the mission focused on determining how long the spacecraft will orbit the Moon to extend duration and coverage of the lunar surface. The first parts of this section discuss the lifetime of the lunar orbit based on the orbit choice, whether it be equatorial, inclined, polar, or some type of non-circular orbit. The final section discusses plane changes to change the inclination and cover a larger surface of the Moon for investigation.

This code was developed by the ARTEMIS senior design team and has been used and slightly modified to meet the needs of this thesis [3]. The code uses a numerical integrator to find the change over time of the orbit elements that are affected by the Moon's gravity field. This change is recorded and the data is plotted to show how the orbit evolves over

time.

3.3.1   Equatorial Orbits

Although most orbits reaching the Moon are inclined in some way, it is interesting to look at the equatorial case. The equatorial case is a simple case where initially the only value of interest is the altitude above the lunar surface. Using the numerical simulation to generate the plot in Figure (3.14), these values can be seen over the time of one year. The other orbit elements start as zero and only truly affect the orbit if they deviate far from zero. Both the eccentricity and the inclination oscillate but do not change significantly, this change is especially true for the inclination as its order of magnitude makes its change negligible. Because the semi-major axis does not change significantly due to the gravity field, the only value that can cause a crash is the eccentricity. The plots in Figure (3.14) show the eccentricity oscillating but not enough to cause an impact as seen in the subplot above.

With a large enough initial orbital radius the spacecraft will never impact the Moon on a circular equatorial orbit. With the semi-major axis of the orbit above 1800 km, the simulation shows a no-impact scenario. Of course this scenario also makes the spacecraft drop down to less than 10 km above the surface of the Moon, which would be dangerously close for any mission.
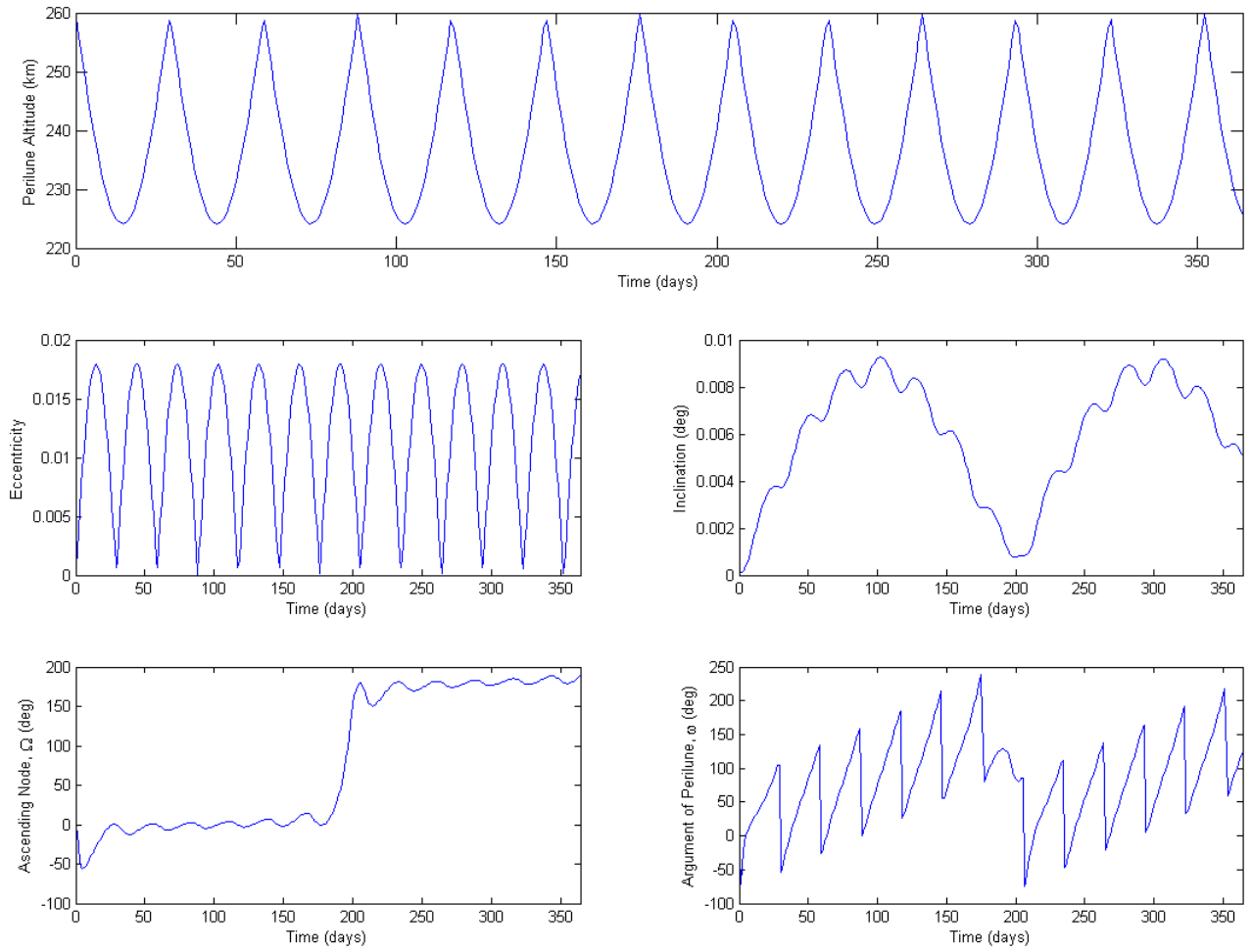
Figure 3.14: Orbital Data for an Initially Zero Inclination Circular Orbit

3.3.2   Polar & Other Inclined Orbits

The more likely scenario for the lunar mission is one that uses an inclined orbit around the Moon. Of these orbits there is one special case where the spacecraft is orbiting the Moon in a polar orbit, to investigate the north and south poles. This type of orbit is important because the poles are the least explored regions of the Moon. Figure (3.15) shows the data for a polar orbit with the same initial conditions as the one discussed above except the inclination which has been set to 90°.

In Figure (3.15) the dynamics of the orbit are no longer oscillatory, and permanent changes are occurring, at least in the short term. For the first year, shown in the figure, the spacecraft is still orbiting and the eccentricity has not changed significantly from a circular orbit. Letting the orbit continue for a two and a half year period shows that the orbit will return to the original conditions. The return to original conditions is seen in Figure (3.16), and illustrates that even though the orbit will return to its original position, it will drop closer to the lunar surface and this drop has to be taken into account for mission planning.

For intermediate inclinations the simulations show similar trends, as the inclination increases the length of time for the orbit to repeat increases. Figure (3.17) shows an orbit with a 45° inclination that begins the return to original conditions before the one year point. This trend does not completely continue as the largest decrease occurs for 60° inclination. Overall the desired inclination has to be checked against the drop in the altitude for the mission. Table (3.4) shows the drop in altitude for various inclination angles.
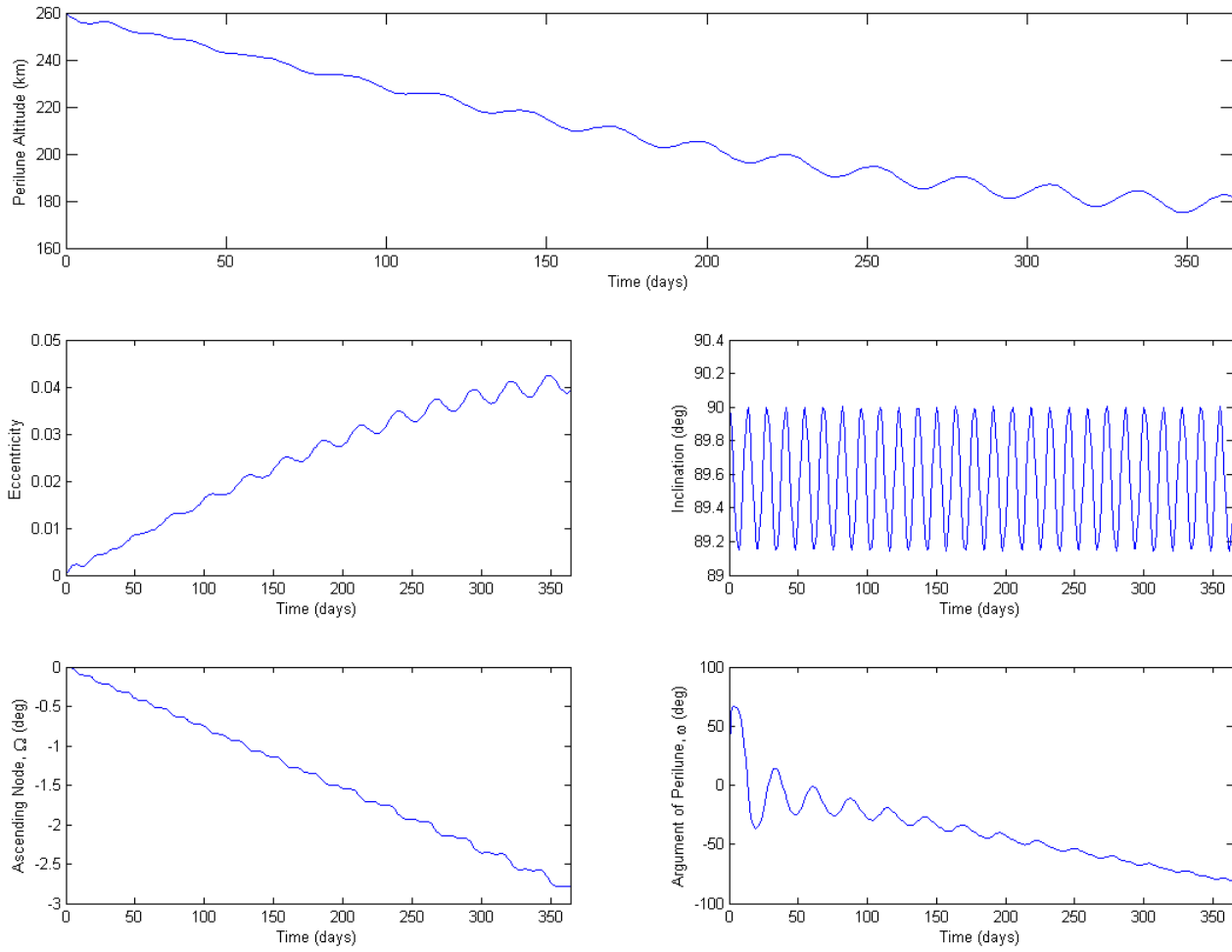
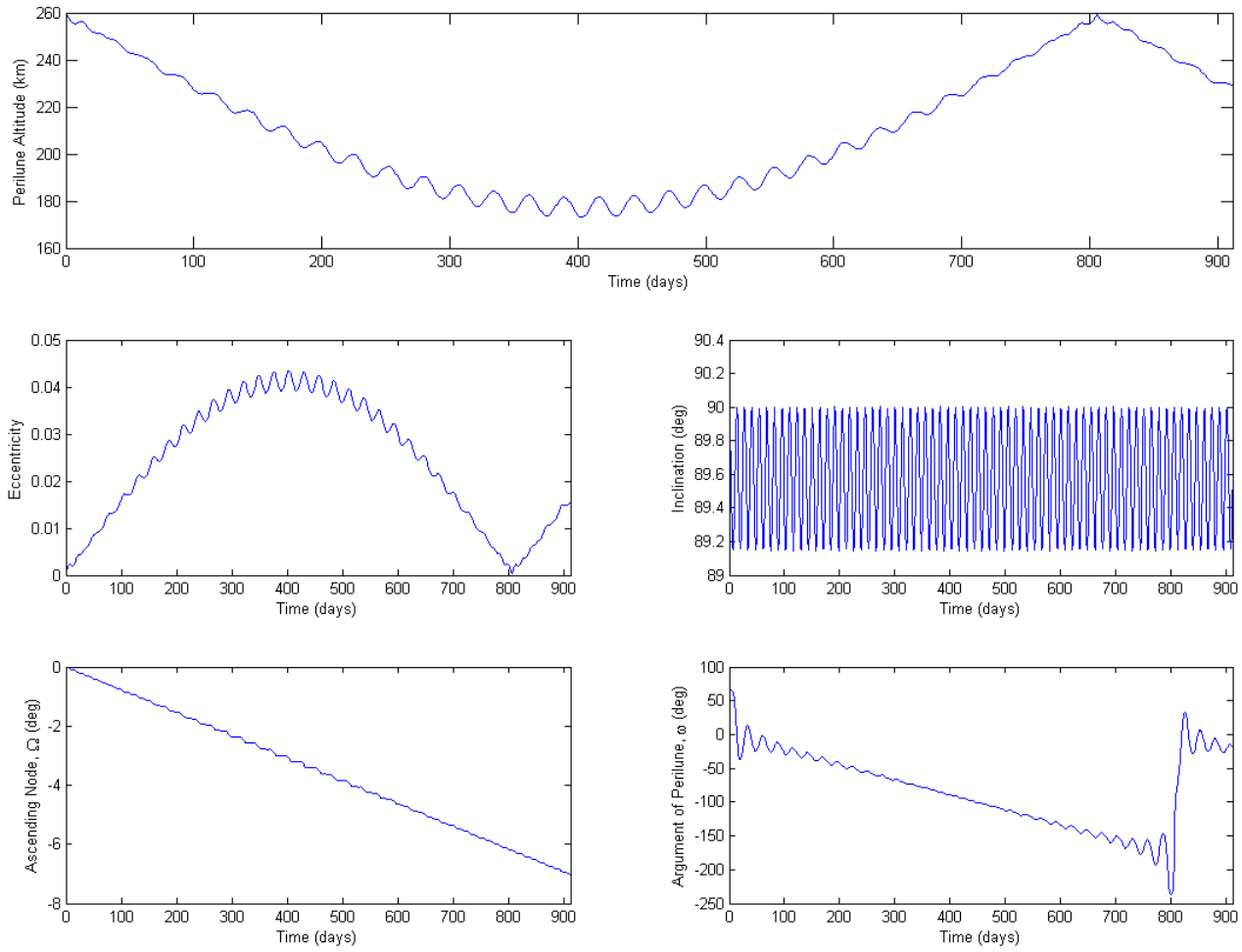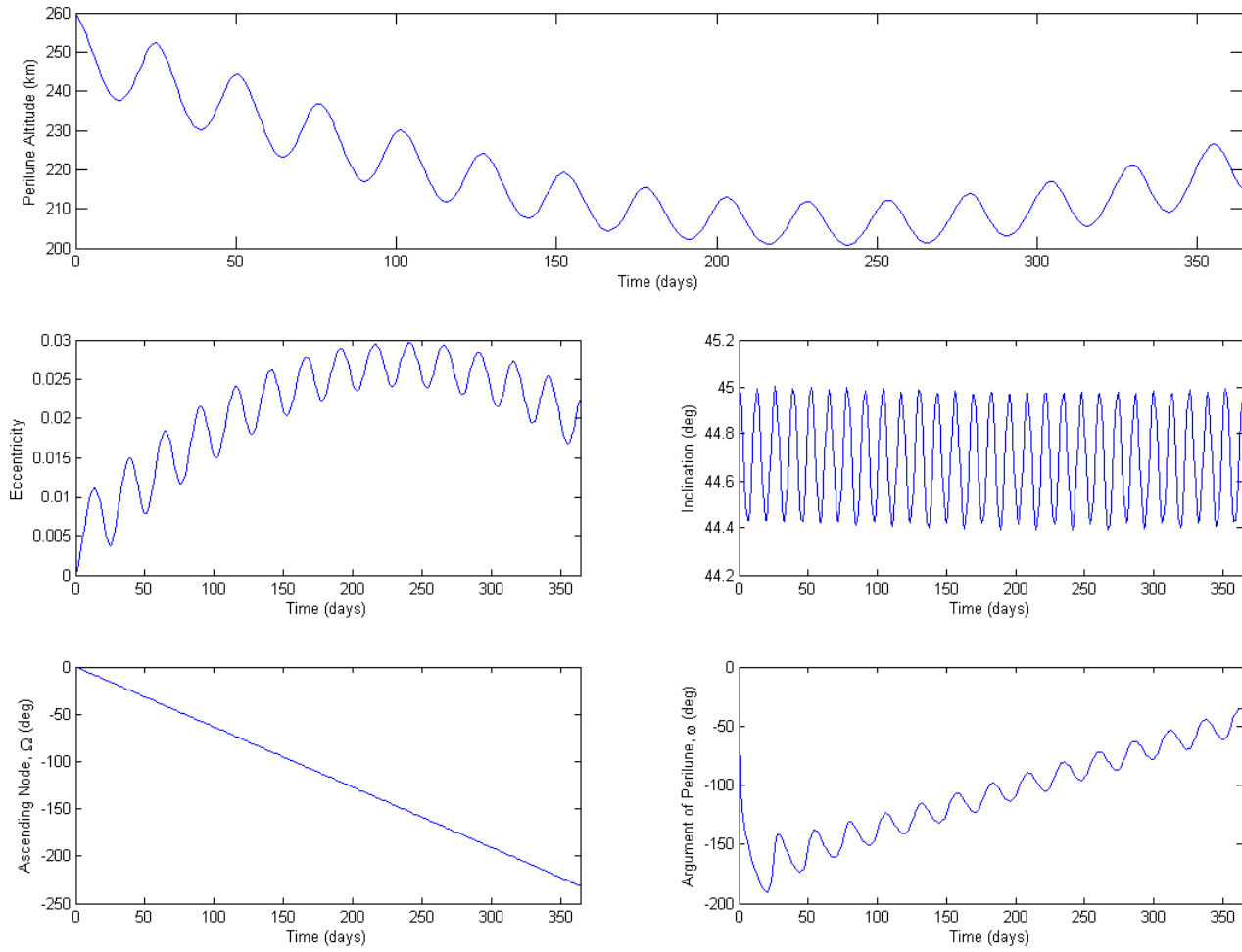Figure 3.15: Orbital Data for an Initially Polar Circular Orbit Over One Year

Figure 3.16: Orbital Data for an Initially Polar Circular Orbit Over Two and a Half Years

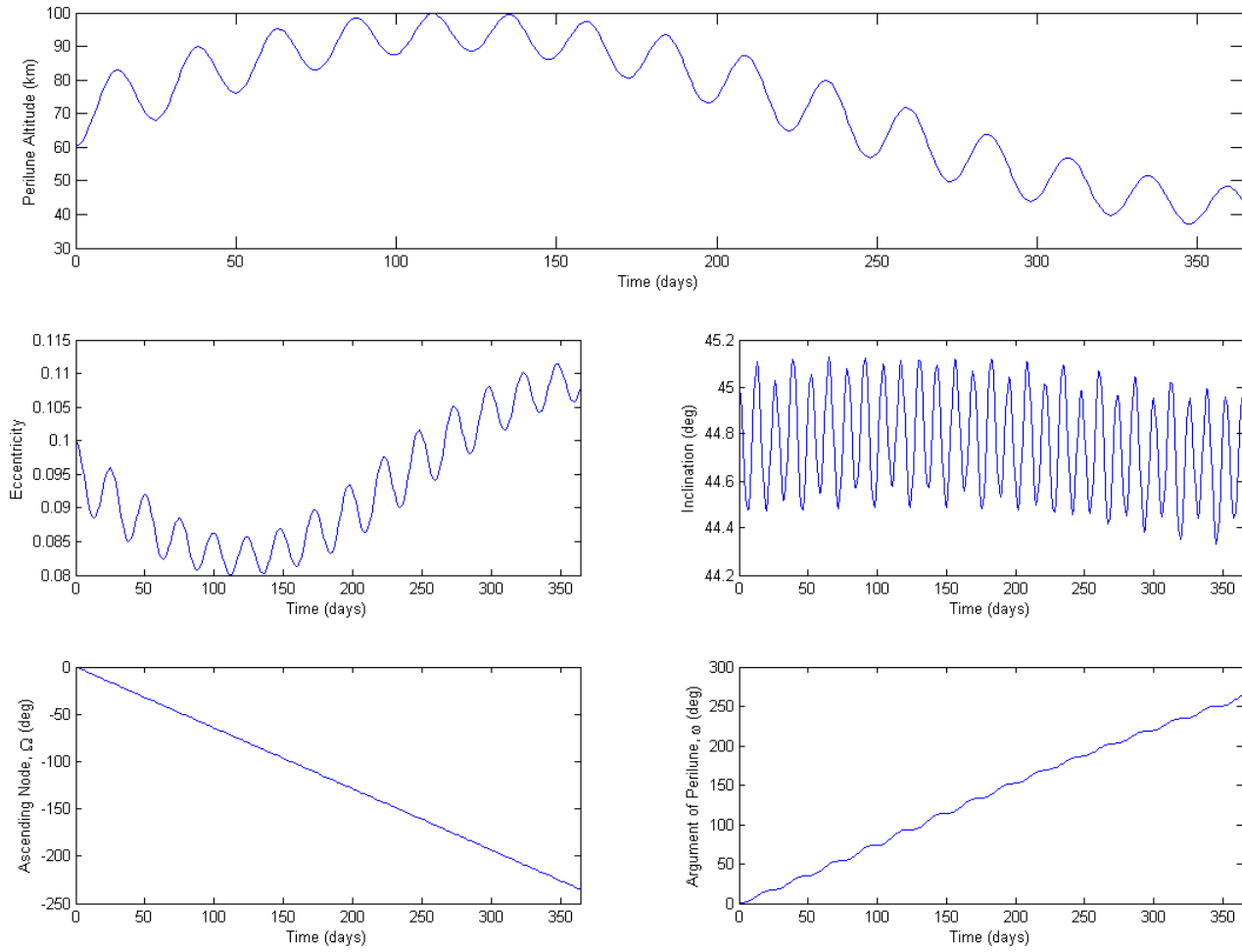Figure 3.17: Orbital Data for an Initially 45° Inclined Circular Orbit

Figure 3.18: Orbital Data for an Initially 45° Inclined Elliptical Orbit

Table 3.4: Minimum Perilune Distance for Given Orbital Inclination Angles, Starting at 260 km

Inclination	Minimum Perilune Distance
0°	224 km
15°	225 km
30°	222 km
45°	200 km
60°	141 km
75°	188 km
90°	173 km

3.3.3   Non-Circular Orbits

Changing from a circular orbit to an elliptical orbit can give the orbit more desirable properties such as a larger view area over certain regions of the Moon. The elliptical orbit makes the argument of perilune and the ascending node important values because they will determine where the apoapsis and periapsis of the orbit are around the Moon. Increasing the eccentricity to 0.1 and using a 45° inclination creates the plot shown in Figure (3.18). The important thing to note is that the perilune altitude is significantly lower as compared to the circular orbits. Also the argument of perilune and ascending node have changed slightly. The change to these orbit elements means that the orbit's coverage of the lunar surface will have changed over the year it is orbiting the Moon.

Table 3.5: ΔV for Lunar Orbit Plane Changes at 200 km

Inclination Change	ΔV
15°	0.42 km/s
30°	0.82 km/s
45°	1.22 km/s
60°	1.59 km/s
75°	1.94 km/s
90°	2.25 km/s

3.3.4   Plane Changes

If a circular orbit is used the natural dynamics of the system under lunar gravity will not induce any significant changes to the orbital plane. If the orbital plane need to be changed a simple plane change can move the plane to any desired inclination. The plane change can have a high ΔV for large changes and the values are shown in Table (3.5) for a craft orbiting at an altitude of 200 km.

Chapter 4

Applications

4.1   Overall Application

This analysis is an overall look at lunar missions being launched from WFF but can be used as a resource for any lunar or space mission. The original intent of this work was for a NASA funded project for Virginia Tech to launch a spacecraft from WFF. It has expanded to encompass that individual mission as well as being applicable to many other lunar missions. This document contains work on lunar trajectories and other analysis including and applicable, but not limited to, three-body dynamics, non-uniform gravity modeling, Lagrangian and Hamiltonian dynamics, as well as any other space missions.

4.2   Future WFF Projects

This thesis is inspired by a project funded by WFF for their Mission Management Initiative for Solar System Exploration (MMI). After the success of the first MMI mission project, Wallops Flight Facility has continued to expect Virginia Tech to design lunar missions that have some component that can be tested and flown on a high altitude balloon. These projects

give students a chance to handle real hardware and gain experience with real engineering work, as well as building an experiment that will actually approach the boundaries of the upper atmosphere. The purpose of these continued projects is to build on the previous missions to grow and expand until the lunar mission has been developed enough, and enough experience has been gained, to build a real lunar craft.

Because of the years that these projects span and the desire to continue to build on previous projects, rather than starting fresh every time, it is important for detailed analysis of the work done to be saved and passed on. The major work of this thesis started with the trajectory analysis for the first MMI project [10]. With saved information and previous work the designing of the space mission can continue for years, and focus can be given to the spacecraft and balloon design. The cataloged previous work means that the entire lunar mission can be given an overall design with some new work being done every year. The WFF projects focus on the balloon mission and, over many years, designing the overall future lunar mission.

The MMI requirements document specifies that any proposal submitted for approval must contain at least a basic lunar trajectory analysis [10]. For future submissions the work in this document can be used as that part of the proposal. Furthermore, the work in this thesis is applicable to a more detailed trajectory analysis for a full lunar mission.

4.3 Senior Design Projects

In conjunction with the WFF projects the senior aerospace engineering students at Virginia Tech will be completing projects that will design spacecraft to travel to the Moon. These senior design projects are considered part of the WFF project but focus more on the hypothetical lunar mission and help provide a basis for the future real lunar mission. As with all basic mission designs, there must be a trajectory analysis. Much of the work done in this thesis can be expanded to any space mission, and can help for any spacecraft trajectory

analysis.

4.4 Virginia Tech Spacecraft

The future Virginia Tech spacecraft will launch from WFF and travel to the Moon using a Taurus XL rocket as discussed in above sections. The purpose of this mission is laid out in the MMI as a mission to further the scientific understanding of the Moon to begin to prepare it for colonization and usage as a jumping point for further exploration of the solar system [10]. This will be a real lunar mission and it will need a detailed trajectory analysis before it will launch. This project will take years to design, build, test, and then finally launch. Currently the mission is in the beginnings of the design phase, with each year a new senior design group adding to and progressing the mission along.

This thesis contains much of the groundwork for such a mission. Not all the information provided in the analytical portions of this thesis is used for the simulations, it is there to apply to different mission goals. As discussed in the next chapter the work in this thesis is general and would need to be built upon to provide adequate information for a specific spacecraft design, but this thesis provides a starting point for that work.

Chapter 5

Future Work

5.1 Specific Spacecraft Influences

This thesis discusses sending an unknown spacecraft from WFF to the Moon. Although this thesis provides the groundwork for getting any craft from WFF to the Moon, the characteristics of a specific spacecraft will alter the mission. These influences can alter any part of the overall mission. Future work would need to investigate how the spacecraft design would alter the launch and parking orbit, the transfer orbit, and the final lunar orbit.

5.1.1 Launch & Parking Orbit

The information presented in this thesis does not go into detail about the specifics of the launch, and focuses more on the parking orbit around the Earth. The launch vehicle is used in this document to constrain the parking orbit. Further work would need to be done calculating the exact capabilities of the Taurus XL rocket with the specific spacecraft. The mass of the craft will constrain the maximum altitude of the launch, and the Taurus XL rocket will constrain the shape and design of the spacecraft [7].

The parking orbit will need to be modified based on the needs of the spacecraft. If the craft needs a longer systems check period or deployment time it may spend a prolonged period of time in the parking orbit. The parking orbit may also be modified based on the desired final lunar orbit inclination. Starting in a certain parking orbit reduces the total ΔV required to get into the desired lunar orbit inclination. These variables mean that further analysis of a specific parking orbit may need to be done in the future for the actual spacecraft and launch.

5.1.2   Transfer Orbit

Much of the focus of this thesis has been on the transfer orbit and it is thoroughly investigated. The specifics of a spacecraft would have minimal effect on the transfer orbit, although if a certain type of mid-flight maneuver is decided on or required because of the craft, it would be advantageous to investigate that specific case in greater detail. The specifics of the spacecraft also apply to how the trajectory will be chosen. This thesis provides the background and analytical tools to solve for many types of orbits.

The unique spacecraft may not have a high ΔV capability or may be able to spend a longer period of time orbiting in the Earth-Moon system before it finally reaches the Moon. These variables would drastically change the analysis of the trajectory and would warrant some more detailed analysis.

5.1.3   Lunar Orbit

The lunar orbit is where the bulk of future work will need to be done based on the spacecraft design. The spacecraft will be investigating the Moon and its orbit around the Moon will be dictated by the desired data from this mission. Furthermore the design of the spacecraft will place the sensors and other equipment, which will greatly influence how far away the craft can be from the surface, how stable the orbit has to be, and other factors that can only

be determined by the sensors. Also the rate of communications with the Earth will dictate how long the spacecraft can remain on the dark side of the Moon out of sight of the Earth. Finally the power requirements of the craft will determine the orbit because of solar panels and the need of energy to power systems.

5.2 Desired Mission Constraints

Depending on the ultimate mission goal, there are parts of the mission that could use further analysis. If the craft will stay on a specific lunar orbit, more work can be done investigating that distinct orbit, rather than the full spectrum of possibilities. There are also many other science goals that can be performed by a craft traveling to the Moon which will be discussed below.

5.2.1 Parking Orbit Science

While the spacecraft is in orbit around the Earth there may be a desire to take readings and accomplish some science goal. A scenario involving a science goal around Earth would require a much more intricate parking orbit investigation, possibly including an analysis of the Earth's non-uniform gravity field on the orbit. Any parking orbit mission goal would place the craft into a prolonged Earth orbit which would be influenced by the Earth's non-uniform gravity field. Also if this science goal involves a lower Earth orbit there may be other disturbances, such as atmospheric drag, which would need to be accounted for.

5.2.2 Transfer Orbit Science

There may be a desire to perform a scientific goal while on transit to the Moon. This goal could involve a much more intricate transfer orbit utilizing a multi-burn scenario, or could use the natural dynamics of the three-body system to swing the craft around the Earth-Moon

system. A detailed analysis of this scenario would be warranted, and can be done using the analytical methods presented in this thesis. Changing the initial conditions of the numerical simulations and altering them with more than one mid-flight burn can create a large variety of orbits that could take the craft through any regions in the Earth-Moon system that are of interest.

5.2.3   Unique Lunar Orbit Scenarios

Finally there may be a different kind of lunar orbit desired than a simple circular or elliptical orbit. Although those types of orbits are not investigated in this thesis they are fully possible, including a halo orbit or possibly situating the craft at the Lagrange point of the Earth-Moon system that rests outside the lunar orbit. These types of orbits can be analyzed using methods presented and can provide some interesting solutions to certain desired science goals.

There is one more scenario that is not discussed in this thesis which is a lunar landing. The mission goal would determine if the craft is going to crash into the Moon or try to perform a controlled descent onto the surface and survive for communications with Earth. If landing is the desired outcome then an in depth analysis of this landing would be crucial for mission success. This analysis would involve not only a dynamics analysis, but a control, and structural analysis as well.

Chapter 6

Conclusions

6.1 Purpose

This thesis has been written to lay the ground work for the dynamic analysis of a future lunar mission from WFF to the Moon. The mission to the Moon would consist of three portions, the parking orbit, transfer orbit, and finally the lunar orbit. This thesis explains how to perform, and shows some calculations for each section of this mission. With the analytical derivations and equations derived a complete analysis of the lunar mission can be done. This thesis shows sample calculations for each portion with the intent that they can be used as a guide to other calculations. Because the future lunar mission's details are not known, it is not possible to perform those simulations, but the simulations will be done similarly with minor changes to the numerical analysis.

6.2 Findings

In doing the research for this thesis many findings about the lunar mission were discovered and are listed here for the future designs with the foremost finding being that the three-body

analysis is crucial to the analysis of the transfer orbit. It is possible to do the total trajectory as a patched conic analysis, but this does not include the influences of the extra body during transit. These influences can create unique solutions to the problem that are impossible to recreate with a simple patched conic two-body analysis.

Next using the three dimensional analysis allows for the investigation of inclination angles and again, more unique orbit scenarios. Using a planar analysis and assuming all the bodies are in one plane makes it so the motion cannot include inclination angles at departure and arrival, which in most cases is crucial for mission analysis, as the inclination angle determines launch locations and coverage of orbits.

Another important aspect of the mission analysis is the gravity modeling that can be done on any part of the mission. It is true that the Earth's gravity field does play a part in influencing the orbits of anything around it. The non-uniform shape of the Earth creates a potential field that can, over time, influence the motion of a spacecraft. But these influences are only noticeable over a long period of time. So although the gravity field may influence the long term motion of an Earth orbiting satellite, it will not play a large role in a craft that will only be in its immediate radius for a few hours.

The Moon's gravity field is used in the lunar orbit simulation because the spacecraft will orbit the Moon for many months and even years. Without the non-uniform gravity field it would appear that the craft could orbit indefinitely, but in reality the long term effects are critical to mission analysis, as they can cause the orbiting craft to drift close to the Moon, and could possibly cause a collision.

These findings are some of the ones present in this thesis that stood out to the author, but are in no way the only gains learned in this research. This overall mission investigation has shown how a complete lunar mission analysis can be done for a launch from WFF, or with some alteration any location. Even though the focus of this research has been the transfer orbit of the Earth-Moon trajectory, all aspects of the mission have been presented here and thoroughly discussed.

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