2.0 Research Objectives and Thesis Organization

2.1 Problem Definition

Quantitative feedback of error is a prerequisite for any attempt to interactively modify or match a surface patch. The degree to which a designer is successful in modeling a particular free form shape is contingent upon how successfully he or she can characterize the differences between the original and the new surface. One of the inherent problems associated with error measurement is to put a numerical estimate on the characteristic differences (position, slope, surface normals, curvature, frequency content, etc.) between two surfaces. There are numerous attributes of a surface that can be used to convey useful information to the designer. Defining a single norm to characterize all the differences between any two surfaces is difficult. A more reasonable approach is to define norms that relate a deviation along a particular characteristic to a numerical value. For example one may have a norm that identifies the point with the maximum error value along an isoparametric curve. Typically, a designer would need to refer to some or all of these norms to garner useful information. This information provides designers with the means to compare and contrast various matches for a patch.

Norms can compact to a certain extent, the amount of data that describe the error between two surfaces. To illustrate this statement, consider the earlier example about maximum error along isoparametric curves. Suppose there are *m* points along each of the *n* isoparametric curves where position data is being sampled on each surface. Typically the designer is interested in the maximum error associated with these points. Using this norm, the designer can successfully compact the error data along that curve from *m* error values into a single value. However, in most practical situations, the designer cannot easily interpret the resulting condensation per se, into something meaningful. In such cases, the solution is to use graphical methods to present information effectively, wherever possible. Effective visualization of error related data is readily achieved with the use of graphical methods. In this research, graphical tools have been used to present results of Fourier analysis, among other things.

As can be seen from the above discussion, the problem here is twofold. At a higher level, the problem is one of abstraction. Using suitable norms and graphical methods, the data related to a surface matching operation need to be reduced into meaningful information. The second aspect of this problem relates to the presentation of this information.

2.2 Research Objectives

The objective of this research is to develop error measures for two closely matching surfaces that provide:

- Location of error between two patches;
- Characteristic information about two matching surfaces; and
- A method of data reduction that will allow graphical presentation of information for easy visualization.

The focus is on reduction of data related to error between two surfaces. As mentioned earlier, there are various attributes of a surface that may be used to characterize its differences as compared to another one. The following paragraphs present a number of tools that characterize the difference between two closely matching surfaces. Each of these tools investigates a particular aspect of the matching surface's behavior as compared to the original. Given below is a list of the tools used in this thesis:

- Ordinate intercept plot for vector difference in position values
- Fourier coefficients plot for vector difference in position values
- Ordinate intercept plot of angle between surface normals
- Ordinate intercept plot for difference between radii of curvature values

One of the most important tool is calculating the vector difference of position data on the two patches. A large vector difference at a few points locally or even a small difference at a number of points suggests a poor match to the designer. However, the position vector difference data for two large surface patches will need to be reduced into meaningful information. The next section discusses the data reduction mechanism used in this research.

There are other methods for characterizing the differences between two surfaces. Fourier analysis can be used to determine the frequency content of the error surface. Here, frequency content means the coefficients of the various harmonics. The reader will appreciate that knowing the frequency content of the error surface plays an important role in determining the quality of a match. For example, if the Fourier analysis indicates a preponderance of high frequency components for the error, it can suggest that the designer is trying to match widely dissimilar types of surfaces, such as a plane and a rippled surface.

Comparing the orientation of the surface normals is another way of characterizing the differences between two surfaces. The angle between the normals is calculated for corresponding points on the two surfaces. This data is reduced and presented in a manner similar to the position difference data. This method adds value to the information elicited by the position difference method. For example, angular separation data can tell the designer how much the discrepancy affects the orientation of the normals in that area.

Radius of curvature values at a number of points along isoparametric curves on the surface track the straightness of the curve comprising those points. For two matching surfaces, the radius of curvature values at corresponding points on the two surfaces should be very close in magnitude. The difference between torsion values is another way of characterizing the differences between two surfaces. Data reduction is done using the method discussed earlier for position differences and surface normal differences.

Another important application of these methods is to gain more insight into the characteristics of a surface. Tools developed here can provide the designer with information about characteristics of prospective matching surfaces. The designer can use that information as a guide while designing matching surface patches.

2.3 Proposed Method

Any effort towards measurement of error between two surfaces, or even towards characterizing the differences between two surfaces, is limited by the data reduction mechanism used. This thesis proposes a method for reducing the amount of data related to the error between two closely matched surfaces. The two surfaces, say $P_1(u, v)$ and $P_2(u, v)$ are continuous functions of the parameters u and v. The vector difference between them can be calculated at different points and is also continuous, even though it is being sampled only at a discrete number of points. The magnitude of this vector difference is a scalar quantity. A plot of the scalar error values in the u, v plane is referred to as the "error plot." This is not a true surface in E^3 but a scalar function of error, that is, nevertheless useful in describing error in an organized manner. This description is also useful for comparing points corresponding to the same parameter values on the two surfaces. To differentiate between the description of a surface in E^3 and in the parametric domain, please refer to Figure 2.1(a), which shows a surface described in 3-D space and Figure 2.1(b) which describes the same surface in the *u*, *v* parametric plane.

A surface is composed of an infinite number of points. However, it is impractical to

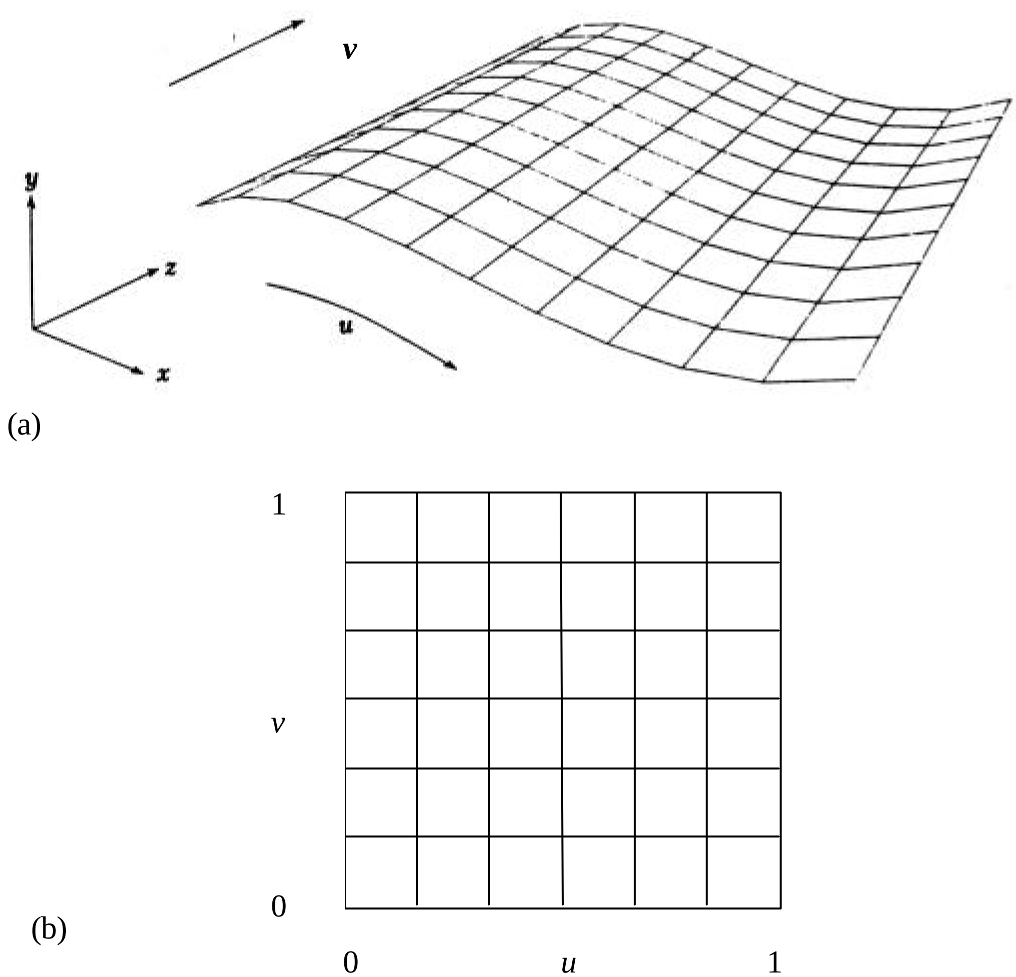

Figure 2.1: Description of a surface in 3-D space and in the parametric plane. (a) The cartesian system of coordinates x, y, and z completely describes the surface. (b) Points on the surface are mapped onto the *u*, *v* plane based on the parametric description of the surface.

describe a surface in terms of an infinite number of points. Therefore, position data is often sampled along evenly spaced curves (along one parametric direction) with a number of points being sampled along each individual curve. For convenience, let the number of points sampled in the two parametric directions, be the same; that is, the number of points along a parametric curve is the same as the number of curves in the other parametric direction. The motivation behind keeping the number of points in both directions equal is to enable analysis of the “error plot” in both directions with the same sampling rate.

In order to compare corresponding points on the original and the new surface, the vector difference between the corresponding points on the original and new surface is used. The collection of all the vector differences is referred to as the “error plot.” Figure 2.2 shows an example of the “error plot.” The actual dimensions of the surface are mapped onto the parametric plane and this figure does not indicate the actual dimensions of the error surface.

As can be seen in Figure 2.2, the shape of the error plot helps in identifying areas on the new surface with significant position error. The elevation of the crests indicates the magnitude of the discrepancy. However, the parametric plot of error, per se has little value to the designer. To best understand this fact, consider the case where position data is sampled along 100 isoparametric curves on both the surfaces. This means that the designer has 10,000 values of error at different points. Some of these might be actually within acceptable tolerance limits. Typically, the designer is interested in say, the 10 or 20 points corresponding to maximum value of error. Therefore, a scheme for reducing this data is needed.

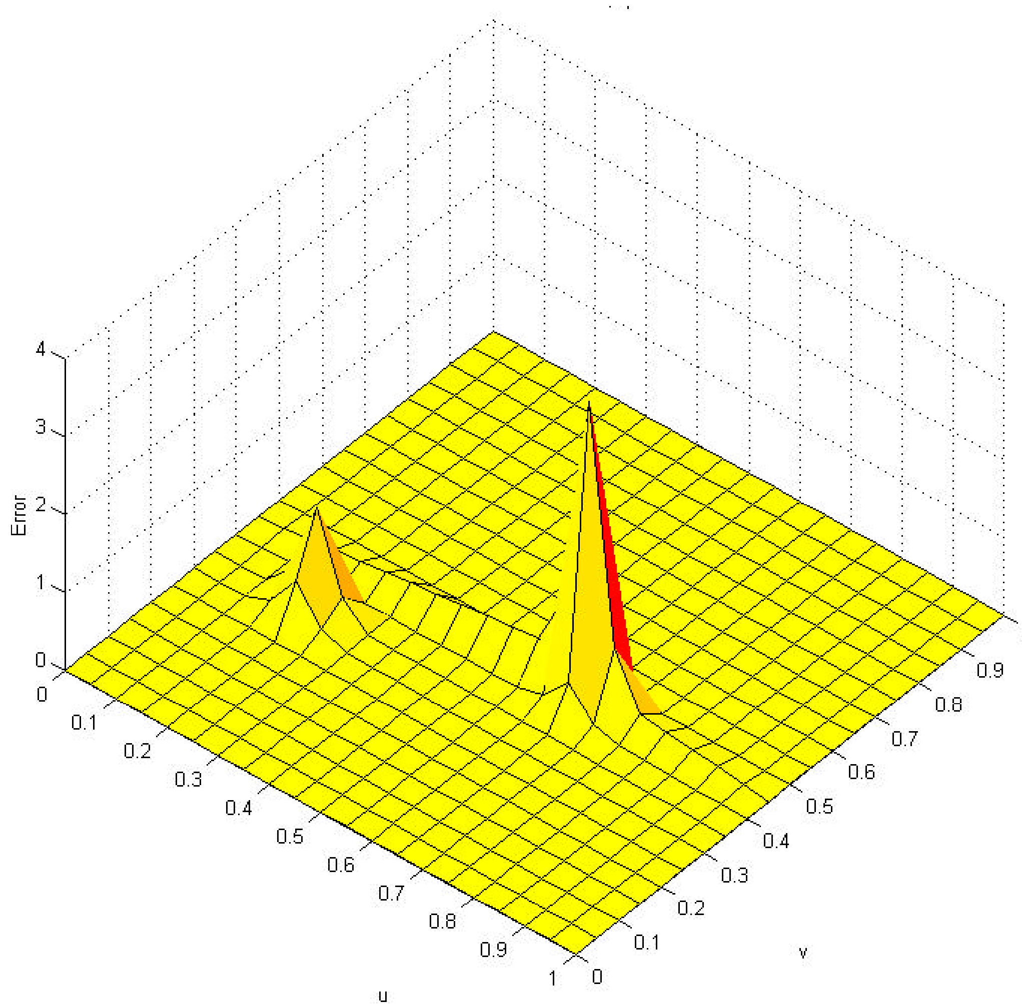

Figure 2.2: Error surface shown in u, v parametric plane. Note that the boundary of the patch is mapped onto the u, v plane, and that this figure does not indicate the actual dimensions of the patch.

It is proposed that, if the error data taken along isoparametric curves is fit with a linear least squares fit, a plot of the ordinate intercept of the fits would be a good measure of the error between the surfaces.

Other norms besides ordinate intercept values obtained from linear least squares fit were also considered. Given below is a list of norms that were considered:

- Maximum value of error along a curve;

- Mean value of error along a curve;
- Median values of error along a curve;
- One of the parameters of the linear least fit; and
- One of the parameters from a cubic fit or a combination thereof

All these norms were applied to very simple test cases and their ability to accurately model a wide variety of data tested. For instance the first norm or the maximum value norm treats data with one huge spike and data with constant difference equal to the magnitude of the spike, as the same. The mean value does not accurately model the presence of peaks or indendations in the data. The use of median as a norm seems to work well in cases where the data seems to ordered uniformly in terms of magnitude. The cubic fit seems to be the best candidate for the norm intuitively speaking, since the data is nothing but the difference between two cubic B-spline curves. However, depending on the specifics of each matching surface, the error data may vary in complexity. Consider the case, where the matching surface is offset to the original surface. In such a scenario, the use of a cubic model to fit the data actually introduces error and complicates the issue. The linear least squares fit, on the other hand attempts to smooth out variations in the data and models the indentations of the data with ease.

In particular, the slope and the maximum value of error along each isoparametric curve were found to be less effective measures compared to the ordinate intercept plot. The following paragraph will explain this hypothesis graphically, followed by a simple test case that validates it. Results of the correlation between the error values and the ordinate intercept values will be also presented to bolster the hypothesis.

Figure 2.3 shows the error plot for two surfaces. As mentioned earlier, position data is collected along a number of isoparametric curves. Consider all the points along one such a curve. Linear least squares is used to find the best-fit line for the error data along each curve. Figures 2.4 and 2.5 show the error data and the corresponding best fit lines along each isoparametric curve in the *u* and *v* parametric direction respectively. The ordinate intercepts of all such best-fit lines in a parametric direction are plotted against the parameter value to obtain the ordinate intercept plot that can be used as a measure of error. Figures 2.6, 2.7 and 2.8 show the ordinate intercept plots for both *u* and *v* direction, respectively. It can be easily seen that these last two plots replicate the shape of the error plot in either direction. Table 2.1 shows the statistical correlation between the maximum values of error between two surfaces and the ordinate intercept values obtained from the above plots.

The high correlation between the error and the ordinate intercepts indicates that the ordinate intercept values are a good measure of the error between two surfaces. The primary purpose of data reduction is to quickly locate areas of substantial error even though the maximum values of the error can also be predicted. The predicted value of maximum error and percentage error between the actual and predicted error are included to give the reader a better idea of the usefulness of the data reduction mechanism.

Table 2.1: Table showing the correlation between the ordinate intercept values and the maximum value of error in both u and v directions. All units in mm.

The magnitude of maximum error	0.111916
Results along u parametric direction	
Correlation coefficient between 21 maximum values of error and ordinate values	0.959777
The predicted value of maximum error	0.110256
Percentage error between actual and predicted value	1.48373
Results along v parametric direction	
Correlation coefficient between 21 maximum values of error and ordinate values	0.922946
The predicted value of maximum error	0.111535
Percentage error between actual and predicted value	0.340623

It was mentioned earlier that calculating vector difference of position data is just one of the ways of characterizing the difference between two surfaces. The difference between orientation of surface normals on the two surfaces is another method that has been implemented in this research. It uses the same data reduction scheme discussed earlier in this section and gives the reader valuable information about the difference in orientation of two surface at corresponding points in E^3.

Similarly, the difference in radii of curvature at different points along corresponding isoparametric curves has also been found to be a valuable measure of characterizing the differences between two surfaces. This method also uses the data reduction scheme discussed above and is useful for validating the results obtained from the vector difference of position method.

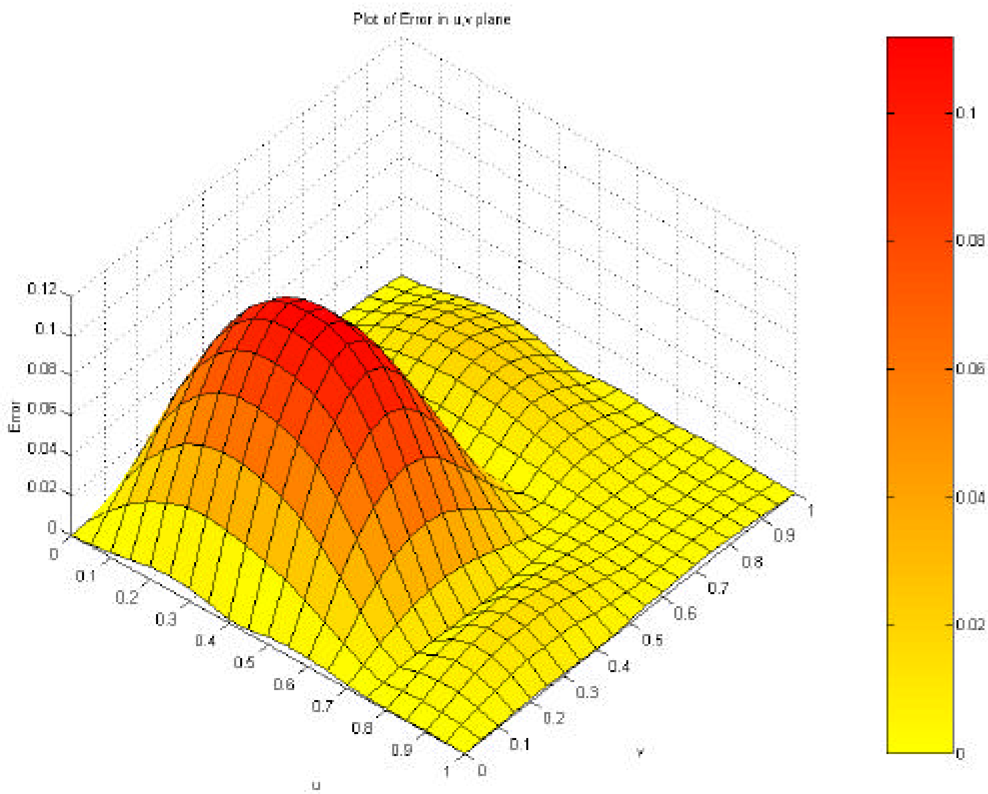

Figure 2.3: Parametric plot of error or "error surface." The proposed method of plotting ordinate intercepts will be demonstrated using this as an example.

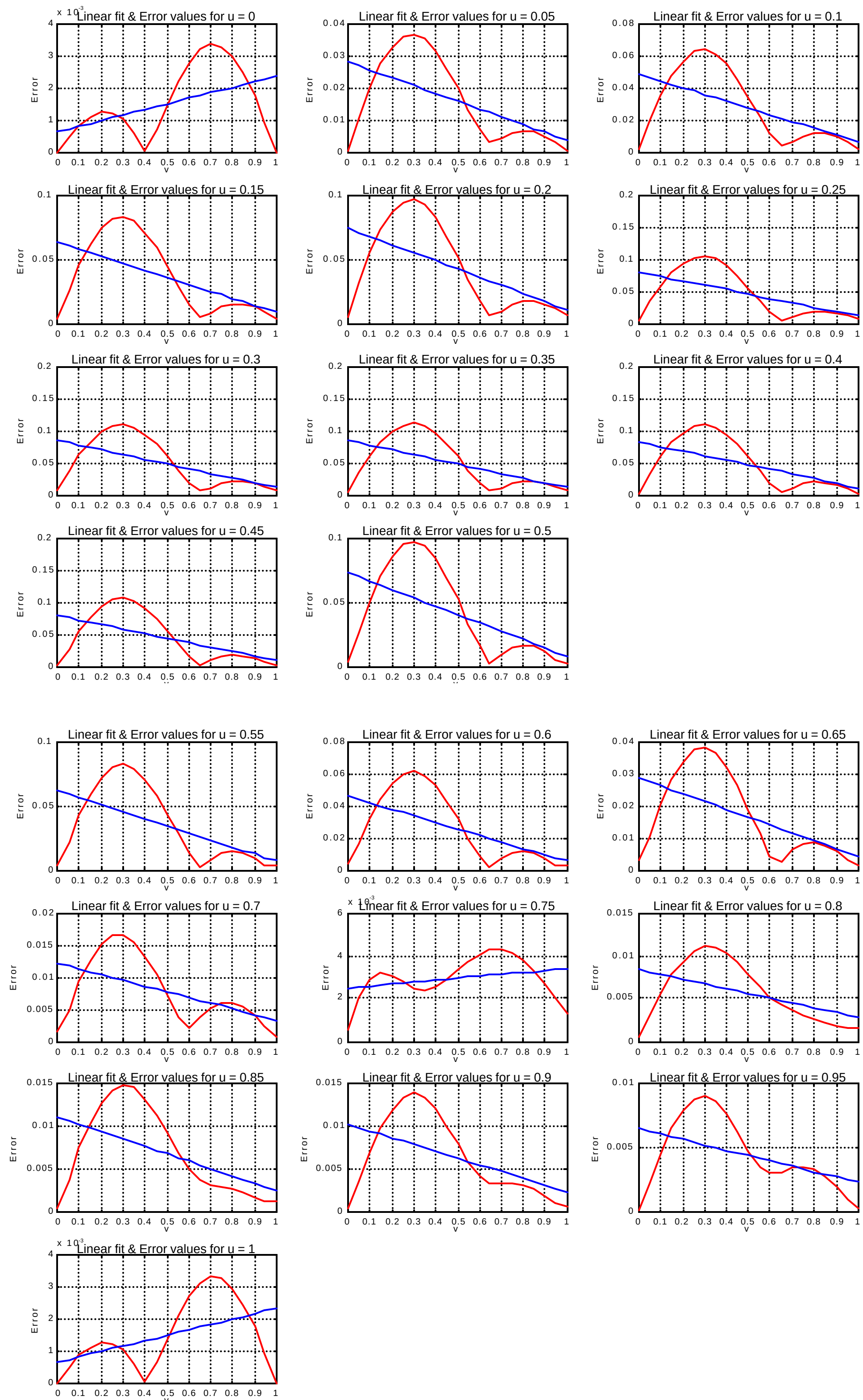

Figure 2.4: Error data and linear least squares fit along isoparametric curves in the u direction. The ordinate intercept values obtained here prove to be a good measure of the error between the two surfaces in that direction.

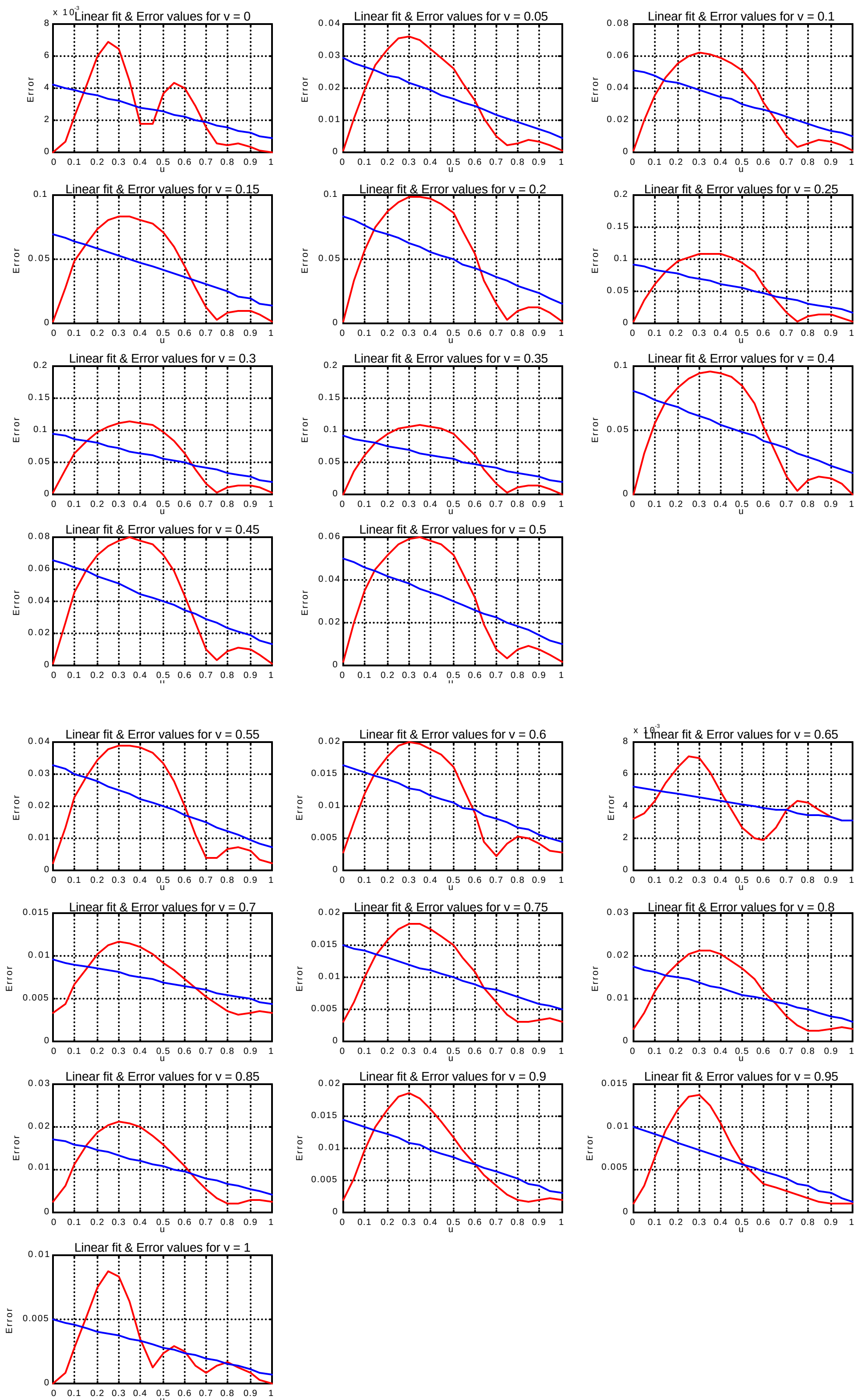

Figure 2.5: Error data and linear least squares fit along isoparametric curves in the v direction. The ordinate intercept values obtained here prove to be a good measure of the error between the two surfaces in that direction.

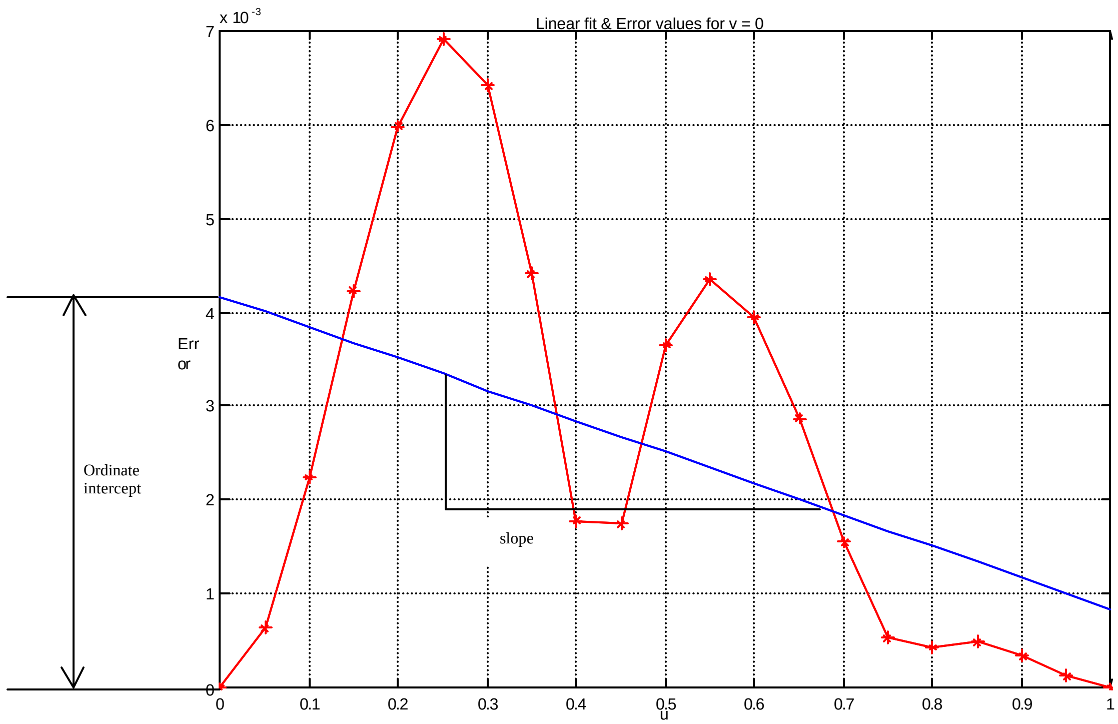

Figure 2.6: Plot showing linear least squares fit to the error data and the corresponding ordinate intercept and slope values.

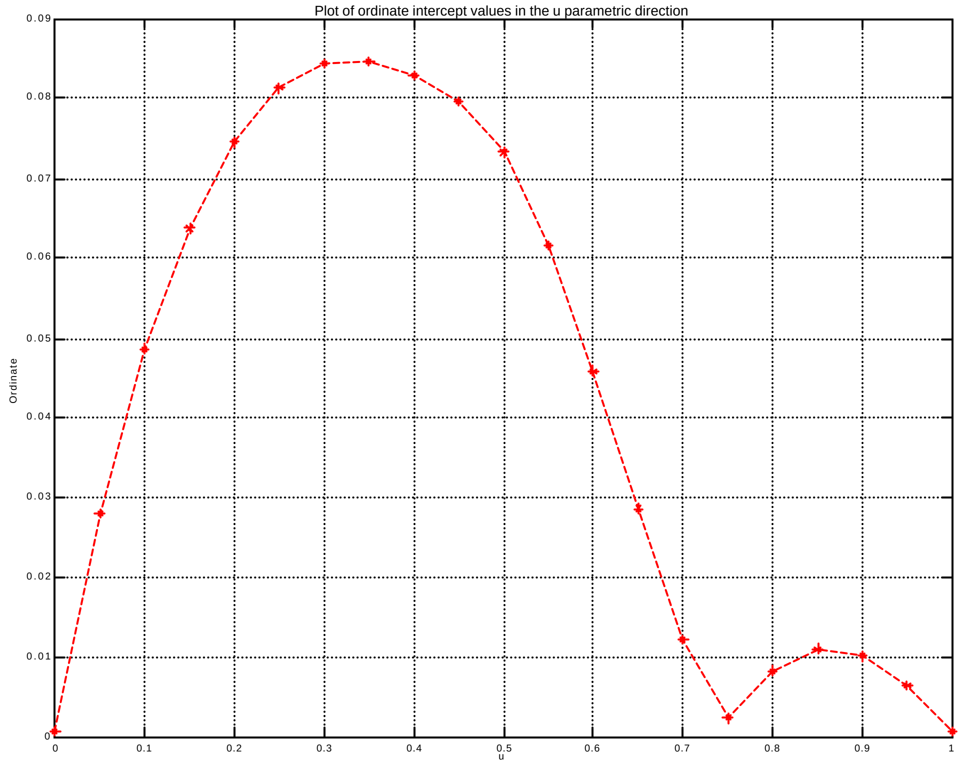

Figure 2.7: Plot of the ordinate intercept value taken from least squares fit along isoparametric curves in u direction. It can be seen that the shape of the error plot along u direction is replicated in the plot above.

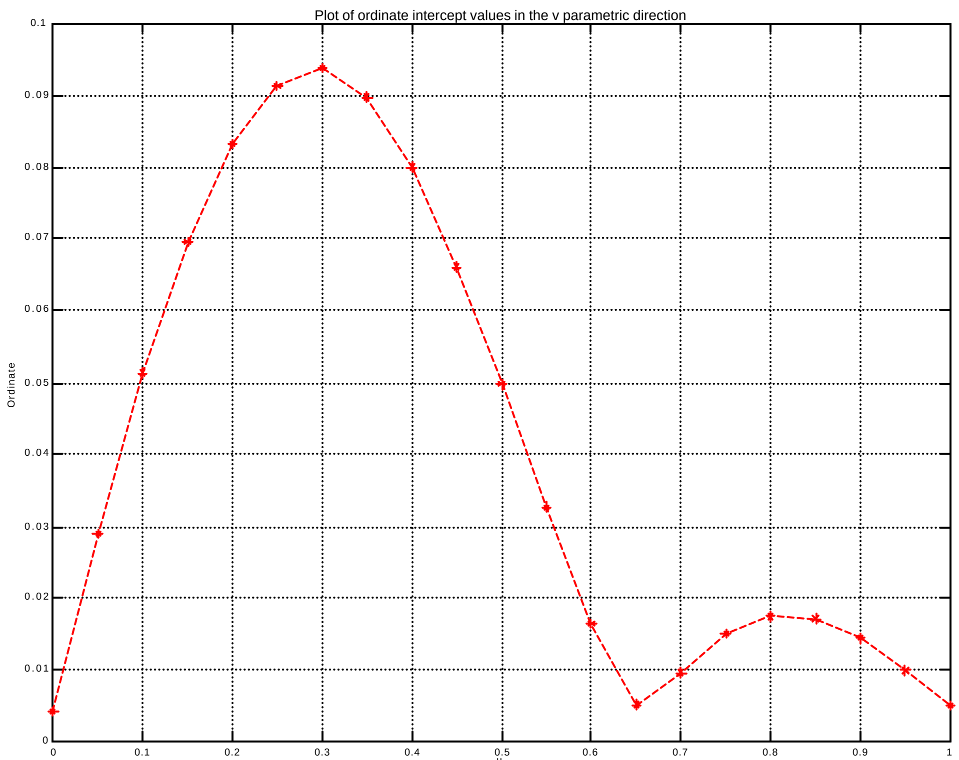

Figure 2.8: Plot of the ordinate intercept value taken from least squares fit along isoparametric curves in v direction. It can be seen that the shape of the error plot along v direction is replicated in the plot above.

2.4 Thesis Organization

The methods proposed in this research investigate the difference between two closely matched surfaces. Till now, much of the discussion has been focussed on the need for error measures. The reader has also had a brief overview of the important features of the proposed methodology. Given below is a brief outline of the topics discussed in this thesis:

1. Introduction
2. Problem Definition, Proposed Methods, Research Objectives, and Thesis Outline
3. Literature Review
4. Non-uniform B-splines
5. Curve and Surface Theory
6. Frequency Determination
7. Results
8. Conclusions

This material is presented with a view to allow readers at different levels to quickly locate areas of interest. The B-splines section gives a brief review of the mathematical representation from a geometric modeling perspective. The section on curve and surface theory familiarizes the reader with computing normals on a surface. The subsequent section on frequency determination discusses the use of harmonic analysis as an error measure. The implementation of these methods in Matlab is included in the appendix along with the data used for one of the test cases described in this research.