CHAPTER 3

OPTIMIZED HARMONIC STEPPED-WAVEFORM TECHNIQUE (OHSW)

The objective of the proposed optimized harmonic stepped-waveform technique is to reduce, as much as possible, the harmonic distortion in the load voltage, working with a reduced switching frequency.

The quarter-wave symmetric waveform concept used in this thesis is proposed by Stefanovic et al. [3]. The paper also presents the total harmonic distortion minimization, which is very difficult to solve the switching angles because nonlinear equations tend to be much more complicated with the higher number of voltage levels. In [11], the authors present the multilevel waveform, which has very high THD with low modulation index. With the proposed technique, those two problems will be overcome.

Basically, the concept of the proposed technique is to combine the idea of the selective harmonic eliminated PWM presented by Patel et al [5] with the quarter-wave symmetric idea concept presented by Stefanovic et al [3]. The concept of the harmonic reduction is to eliminate the specific harmonics, which are the lowest orders.

In this Chapter, the method used to solve the appropriated switching angles of the Optimized Harmonic Stepped-Waveform (OHSW) technique will be presented. Equal step waveform with optimized switching angles will be focused in this thesis. The presented technique is to synthesize waveform by the multilevel inverter using cascaded-inverter with separated dc sources (SDCSs), which is presented in Chapter 2.

3.1 Introduction

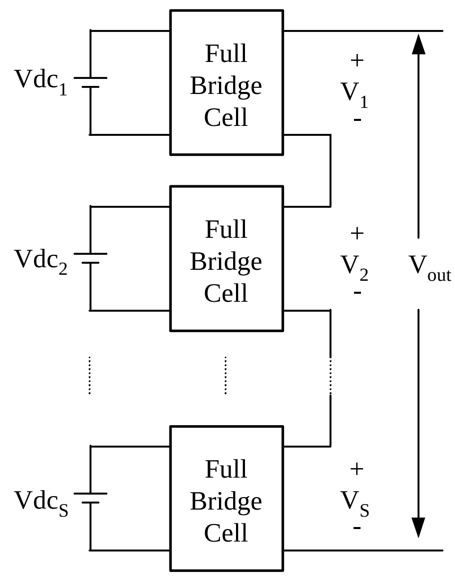

Figure 3.1 Schematic diagram of an s H-bridge series-connected multilevel inverter (single phase).

As shown in Fig. 3.1, s H-bridge cells are connected in series. An output voltage waveform can be generated by summation of the output voltage of each cell, i.e.

$$V_{out} = V_1 + V_2 + \ldots + V_S \qquad (3.1)$$

Fig. 3.2 illustrates a generalized waveform of s H-bridge inverters in series-connection. As discussed in Chapter 2, 2s+1 output levels can be synthesized with s H-bridge inverters and s separate dc sources (SDCSs). From the voltage waveform in Fig. 3.2, it consists of 4s switching angles, α_1, α_2, …, $\alpha_{(4s-1)}$, and α_{4s}, in each cycle. The voltage of the first level equals V_1; the voltage of the second level equals V_2 and so on. These voltage levels are supplied by SDCSs, whose amplitudes may be different. By considering the waveform in Fig. 3.2, there are three possible optimization techniques to reduce the voltage THD: 1) step heights are optimized with equally spaced steps; 2) step spaces are optimized with the steps of equal height; and 3) optimizing both heights and spaces.

Based on the circuit complexities and control possibilities, this thesis will focus on the second method, which uses equal voltage amplitude and optimizes the switching angles. To achieve these optimized angles, the numerical calculation will be applied and will be presented later.

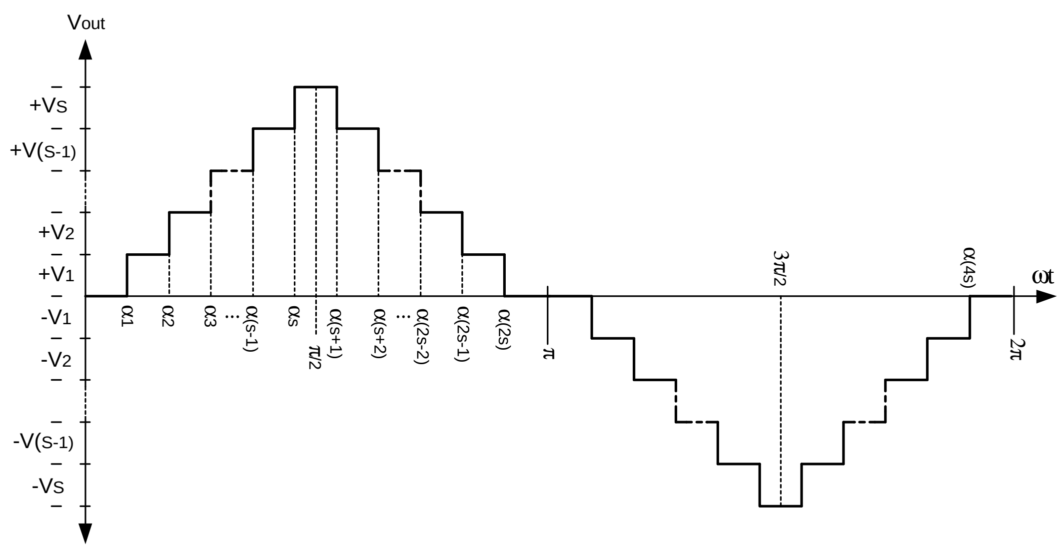

Figure 3.2 Output voltage waveform of the s H-bridge cell series-connected multilevel inverter.

In general, the modulation index of SPWM is the ratio of the modulating signal amplitude to the carrier signal amplitude. For the specified multilevel case, the modulation index is defined as follows:

$$M = \frac{V_{out}}{sV_{dc}} \tag{3.2}$$

where

V_{out} is the amplitude of the output voltage at the fundamental frequency.

s is the number of dc sources or H-bridge cells per phase.

and V_{dc} is the amplitude of dc sources.

3.2 Optimized Harmonic Stepped Waveform

3.2.1 Quarter-Wave Symmetric Multilevel Waveform

The optimized harmonic stepped waveform is assumed to be the quarter-wave symmetric. The relationship among the switching angles of the waveform shown in Fig. 3.2 can be found as follows:

In the second quarter;

$$a_{s+1} = p - a_s$$

$$\vdots$$

$$a_{2s-1} = p - a_2$$

$$a_{2s} = p - a_1$$

In the third quarter,

$$a_{2s+1} = p + a_1$$

$$\vdots$$

$$a_{3s-1} = p + a_{s-1}$$

$$a_{3s} = p + a_s$$

In the forth quarter,

$$a_{3s+1} = 2p - a_s$$

$$\vdots$$

$$a_{4s-1} = 2p - a_2$$

$$a_{4s} = 2p - a_1 \tag{3.3}$$

The first half cycle of the proposed quarter-wave symmetric waveform is depicted in Fig. 3.3. The output voltage level is zero from $\omega t = 0$ to $\omega t = \alpha_1$. At $\omega t = \alpha_1$, the output voltage level is changed from zero to $+V_1$, and from $+V_1$ to $+(V_1+V_2)$ at $\omega t = \alpha_2$. The process will be repeated until $\omega t = \pi/2$, and the output voltage level becomes $+V_1 +V_2+\ldots+V_{(S-1)}+V_S$. Then, in the second quarter, the level of output voltage will be decreased to $+V_1 +V_2+\ldots+V_{(S-1)}$ at $\omega t = \pi-\alpha_S$. The process will be repeated until $\omega t = \pi-\alpha_1$ and output voltage becomes zero again. In the second half of the waveform, the process will be repeated all of previous steps except the amplitude of the dc sources change from positive to negative. The next period will then repeat the same cycle.

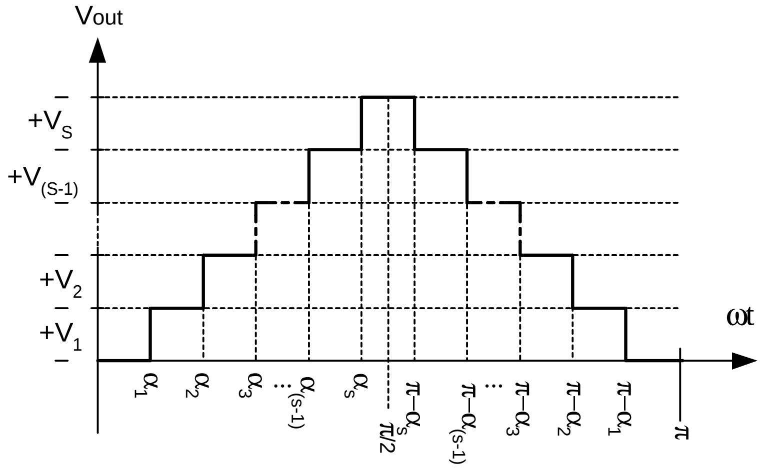

Figure 3.3 the quarter-wave symmetric s H-bridge cell multilevel waveform.

3.2.2 Fourier Series of the Proposed Waveform

Because of odd quarter-wave symmetric characteristic, which is illustrated in Fig. 3.4, the Fourier series coefficient are given by

$$a_n = \frac{4}{p}\int_0^{\frac{p}{2}} f(wt)\sin(nwt)d(wt) \qquad \text{,for odd n} \qquad (3.4)$$

$$a_n = 0 \qquad \text{,for even n} \qquad (3.5)$$

and

$$b_n = 0 \qquad \text{,for all n} \qquad (3.6)$$

where

$$f(wt) = V_{out}(wt)$$

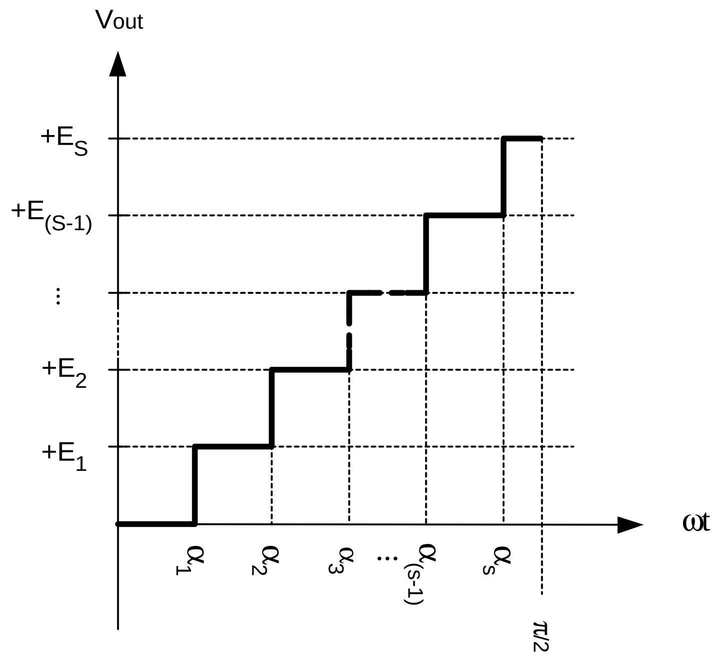

Figure 3.4 The first quarter of the quarter-wave symmetric waveform.

For all *n*, from equations (3.4) to (3.6), the Fourier series is given as

$$f(wt) = \sum_{n=1}^{\infty} a_n \sin(nwt) \tag{3.7}$$

From equation (3.4), let $a = wt$

Hence,

$$a_n = \frac{4}{p} \int_0^{\frac{p}{2}} f(a) \sin(na) d(a) \tag{3.8}$$

From equation (3.8) and Fig. 3.4,

$$a_n = \frac{4}{p}\left[\int_{a_1}^{a_2} E_1 \sin(na) da + \int_{a_2}^{a_3} E_2 \sin(na) da + \ldots + \int_{a_S}^{\frac{p}{2}} E_S \sin(na) da\right]$$

$$= \frac{4}{np}\left[- E_1 \cos(na)\Big|_{a_1}^{a_2} - E_2 \cos(na)\Big|_{a_2}^{a_3} - \ldots - E_S \cos(na)\Big|_{a_S}^{p/2}\right]$$

$$= \frac{4}{np}\left[- E_1 \cos(na_2) + E_1 \cos(na_1) - E_2 \cos(na_3) + E_2 \cos(na_2) - \ldots\right.$$

$$\left. - E_S \cos(n\frac{p}{2}) + E_S \cos(na_S)\right]$$

$$= \frac{4}{np}\left[E_1 \cos(na_1) + (E_2 - E_1)\cos(na_2) + \ldots + (E_S - E_{(S-1)})\cos(na_S)\right] \tag{3.9}$$

From Fig. 3.3 and 3.4, the following relationship can be found.

$$V_1 = E_1$$

$$V_2 = E_2 - E_1$$

$$\vdots$$

$$V_S = E_S - E_{(S-1)} \tag{3.10}$$

Substitute equation (3.10) into equation (3.9), we get

$$a_n = \frac{4}{np}\left[V_1 \cos(na_1) + V_2 \cos(na_2) + \ldots + V_S \cos(na_S)\right] \tag{3.11}$$

Suppose the steps of equal heights, as discussed in 3.1, let

$$V_1 = V_2 = \ldots = V_S = E \tag{3.12}$$

Therefore, for any s and odd *n*, a_n is given by

$$a_n = \frac{4E}{np}\left[\cos(na_1) + \cos(na_2) + \ldots + \cos(na_S)\right]$$

or

$$a_n = \frac{4E}{np}\sum_{k=1}^{S}\cos(na_k) \tag{3.13}$$

Finally, the Fourier series of the quarter-wave symmetric s H-bridge cell multilevel waveform is written as follows:

$$v_{out}(wt) = \sum_{n=1}^{\infty}\left[\frac{4E}{np}\sum_{k=1}^{S}\cos(na_k)\right]\sin(nwt) \tag{3.14}$$

where

a_k is the switching angles, which must satisfy the following condition

$$a_1, a_2, \ldots, a_S < \frac{p}{2}$$

s is the number of H-bridge cells.

n is odd harmonic order.

and E is the amplitude of dc voltages.

3.2.3 Optimized Harmonic Switching Angles

In this section, the method to solve the optimized harmonic switching angles will be explained. From equation (3.13) and (3.14), the harmonic components in the waveform can be describes as follows:

1) the amplitude of dc component equals zero

2) the amplitude of the fundamental component, $n = 1$, and odd harmonic component are given by

$$h_1 = \frac{4E}{p}\sum_{k=1}^{S}\cos(a_k) \quad \text{and} \quad h_n = \frac{4E}{np}\sum_{k=1}^{S}\cos(na_k) \tag{3.15}$$

where

s is the number of full-bridge cells.

E is the amplitude of dc voltage.

a_k are the switching angles.

n is the odd harmonic order.

3) the amplitude of all even harmonics equals zero

Thus, only the odd harmonics in the quarter-wave symmetric multilevel waveform need to be eliminated. The switching angles of the waveform will be adjusted to get the lowest output voltage THD.

Form the waveform shown in Fig. 3.2, s switching angles, namely a_1, a_2, …, and a_S, need to be known. Mathematically, s equations obtained from (3.15) are set up. Unfortunately, these equations are nonlinear as well as transcendental in nature, which suggests a possibility of multiple solutions. There are no general methods that can be applied to solve such equations. The practical method of solving these equations is a trial and error process. A numerical technique is the best approach in solving the equations. Usually, the Newton-Raphson method is used to solve such nonlinear equation systems.

3.2.3.1 The Newton-Raphson Method

The Newton-Raphson method for computing the root of an equation is a successive-approximation procedure, which is suitable for implementation in a computer program.

Generally, the system of nonlinear equation in s variables can be represented as

$$f_1(a_1, a_2, \ldots, a_S) = k_1$$

$$f_2(a_1, a_2, \ldots, a_S) = k_2$$

$$\vdots$$

$$f_S(a_1, a_2, \ldots, a_S) = k_S \tag{3.16}$$

Then, (3.16) can be written in vector notation as

$$F(a) = K \tag{3.17}$$

where

$F = [f_1, f_2, \ldots, f_S]^T$

$a = [a_1, a_2, \ldots, a_S]^T$

$K = [k_1, k_2, \ldots, k_S]^T$

and F, a, K are $s \times 1$ matrices

Nonlinear equation system can be solved by using a linearization technique, which the nonlinear equations are linearized about an approximate solution. As shown in Fig. 3.5, an equation in a single independent variable $y = f(a)$ one starts with an estimate of a near the root of $f(a) = k$. One then computes the intersection of the tangent line to the graph at this estimate with the a axis and uses the intersection as the abscissa of the new estimate. The process is then repeated until a desirable solution is achieved.

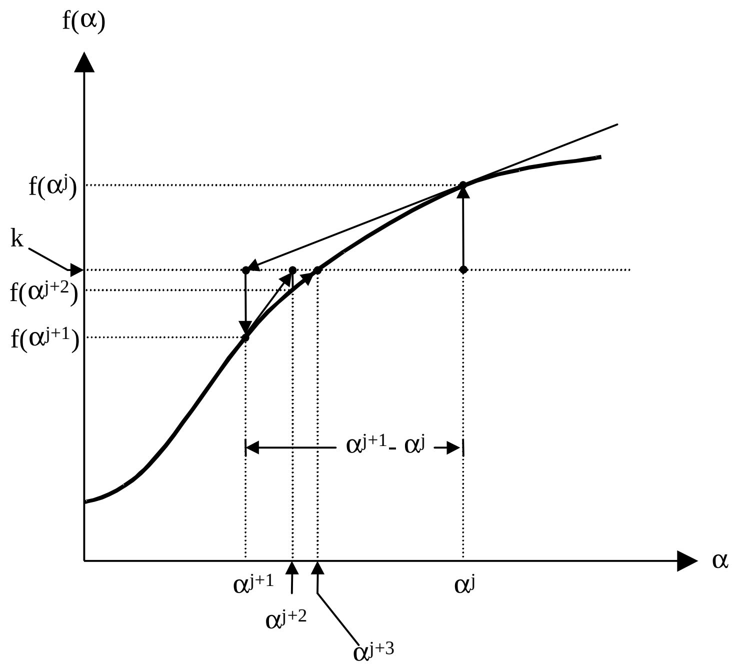

Figure 3.5 Linearization technique.

In a multivariable nonlinear system, a set of independent variables is formed in matrix format and the statement of the algorithm of Newton's method can be shown as follows:

1) Guess a set of initial values for α with $j = 0$

$$\text{Suppose } a^j = [a_1{}^j, a_2{}^j, \ldots, a_S{}^j]^T \tag{3.18}$$

2) Calculate the value of

$$F(a^j) = F^j \tag{3.19}$$

3) Linearize equation (3.17) about a^j

$$F^j + \left[\frac{\partial f}{\partial a}\right]^j da^j = K \tag{3.20}$$

where

$$\left[\frac{\partial f}{\partial a}\right]^{j} = \begin{bmatrix} \frac{\partial f_1}{\partial a_1} & \frac{\partial f_1}{\partial a_2} & \cdots & \frac{\partial f_1}{\partial a_S} \\ \frac{\partial f_2}{\partial a_1} & \frac{\partial f_2}{\partial a_2} & \cdots & \frac{\partial f_2}{\partial a_S} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_S}{\partial a_1} & \frac{\partial f_S}{\partial a_2} & \cdots & \frac{\partial f_S}{\partial a_S} \end{bmatrix}$$

and $$da^{j} = \left[da_1^{\,j} \quad da_2^{\,j} \quad \cdots \quad da_S^{\,j}\right]^T$$

4) Solve da^{j} from (3.20) by

$$da^{j} = INV\left[\frac{\partial f}{\partial a}\right]^{j} (K - F^{j}) \tag{3.21}$$

where $INV\left[\frac{\partial f}{\partial a}\right]^{j}$ is the inverse matrix of $\left[\frac{\partial f}{\partial a}\right]^{j}$

5) As updated the initial values,

$$a^{j+1} = a^{j} + da^{j} \tag{3.22}$$

6) Repeat the process, equations (3.19) to (3.21), until da^{j} is satisfied to the desired degree of accuracy.

3.2.3.2 Apply Newton's Method to Solve the Optimized Harmonic Switching Angles

From 3.2.3, the amplitude of odd harmonic component in quarter-wave symmetric multilevel waveform is written again here.

$$h_n = \frac{4E}{np}\sum_{k=1}^{S}\cos(na_k) \quad \text{, for } n = 3, 5, 7, \ldots \tag{3.23}$$

A set of nonlinear equation corresponding to (3.23) can be given as follows:

- The nonlinear equation of the fundamental component:

$$\frac{4E}{p}\left[\cos(a_1) + \cos(a_2) + \ldots + \cos(a_S)\right] = h_1 \qquad (3.24a)$$

- The nonlinear equations of the odd harmonic component:

$$\frac{4E}{3p}\left[\cos(3a_1) + \cos(3a_2) + \ldots + \cos(3a_S)\right] = h_3 \qquad (3.24b)$$

$$\frac{4E}{5p}\left[\cos(5a_1) + \cos(5a_2) + \ldots + \cos(5a_S)\right] = h_5 \qquad (3.24c)$$

$$\vdots$$

$$\frac{4E}{np}\left[\cos(na_1) + \cos(na_2) + \ldots + \cos(na_S)\right] = h_n \qquad (3.24n)$$

For an output voltage waveform of s full-bridge cell circuit, *s* switching angles, namely α_1, α_2,…, and α_S, need to be known. It is implied that s equations are required to solve such switching angles. Importantly, the expected solutions must be less than $p/2$.

From nonlinear equations system (3.24), equation (3.24) is the nonlinear equation of the fundamental component of an output voltage waveform, whereas (3.24b), (3.24c), and (3.24n) are the nonlinear equation of the 3rd , the 5th, and the nth odd harmonic component of the output voltage, respectively. On the right hand side of these equations, they are the amplitudes of the components, which can be controlled. Basically, the lowest odd harmonic components should be eliminated from a single-phase system. In a three-phase system, because of 120 electrical degree phase shift, the lowest non-triplen harmonic components need to be removed from phase voltage. All even harmonics are not existed because of the symmetric characteristic of the waveform.

To control the amplitude of the fundamental component, (3.24a) will be considered. As introduced in 3.1, the modulation index for the multilevel waveform is given as

$$M = \frac{h_1}{sE} \tag{3.25}$$

where

h_1 is the amplitude of the fundamental component.

s is the number of dc sources or H-bridge cells.

and E is the voltages of the dc sources.

From (3.24a) and (3.25), the following equation can be obtained.

$$\cos(a_1) + \cos(a_2) + \ldots + \cos(a_S) = \frac{sMp}{4} \tag{3.26}$$

From equation (3.26), varying the modulation index value can control the amplitude of the fundamental component.

The other s-1 nonlinear equations, which are the undesirable harmonic components, can be eliminated. Usually, they are set to be zero. Therefore, in a single-phase multilevel inverter, the lowest s-1 odd harmonics can be removed, and in a three-phase multilevel inverter, the lowest s-1 non-triplen harmonics can be eliminated.

Table 3.1 shows the number of harmonic contents which can be eliminated as function of the number of dc sources, s, in both single-phase and three-phase system.

Table 3.1 The number of eliminated harmonic components of OHSW technique.

	Single-phase system	Three-phase system
The number of dc sources	*s*	*s* (per phase)
The number of output voltage levels	2*s*+1 phase voltage levels	4*s*+1 line to line voltage levels
The harmonics in the output phase voltage, which need to be eliminated	The lowest *s*-1 odd harmonics	The lowest *s*-1 non-triplen odd harmonics

Example 3.1

To explain how to apply Newton-Raphson method to the presented waveform, the output waveform of a nine-level inverter using cascaded-inverter with separated dc sources (SDCSs) shown in Fig. 3.6 will be used as an example. In this example, single-phase system is assumed, and modulation index of output voltage, M, is 0.85.

From the waveform shown in Fig. 3.6, four unknowns, α_1, α_2, α_3, and α_4, need to be known. Because of a single-phase system, the lowest three odd harmonics, i.e., the 3rd, 5th, and 7th, should be eliminated.

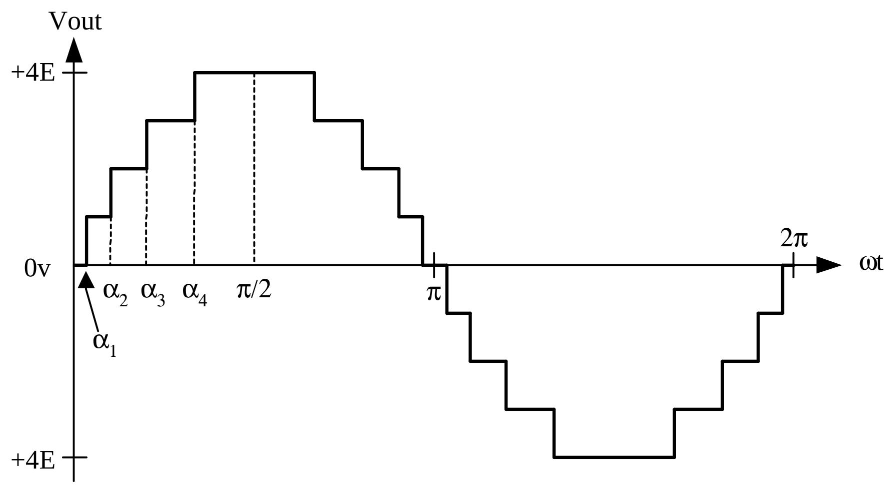

Figure 3.6 Output voltage waveform of a nine-level inverter.

To control the fundamental amplitude and eliminate the harmonics, four nonlinear equations can be set up as follows:

$$\cos(a_1) + \cos(a_2) + \cos(a_3) + \cos(a_4) = 0.85\frac{3p}{4} \tag{3.27a}$$

$$\cos(3a_1) + \cos(3a_2) + \cos(3a_3) + \cos(3a_4) = 0 \tag{3.27b}$$

$$\cos(5a_1) + \cos(5a_2) + \cos(5a_3) + \cos(5a_4) = 0 \tag{3.27c}$$

$$\cos(7a_1) + \cos(7a_2) + \cos(7a_3) + \cos(7a_4) = 0 \tag{3.27d}$$

To solve the switching angles of nonlinear equations (3.27a) to (3.27d), the Newton's method explained in 3.2.3.1 is applied, and the following matrices are implemented:

1) The switching angle matrix,

$$a^j = [a_1^j, a_2^j, a_3^j, a_4^j]^T \tag{3.28a}$$

2) The nonlinear system matrix,

$$F^j = \begin{bmatrix} \cos(a_1{}^j) + \cos(a_2{}^j) + \cos(a_3{}^j) + \cos(a_4{}^j) \\ \cos(3a_1{}^j) + \cos(3a_2{}^j) + \cos(3a_3{}^j) + \cos(3a_4{}^j) \\ \cos(5a_1{}^j) + \cos(5a_2{}^j) + \cos(5a_3{}^j) + \cos(5a_4{}^j) \\ \cos(7a_1{}^j) + \cos(7a_2{}^j) + \cos(7a_3{}^j) + \cos(7a_4{}^j) \end{bmatrix} \tag{3.28b}$$

and

$$\left[\frac{\partial f}{\partial a}\right]^j = \begin{bmatrix} -\sin(a_1{}^j) & -\sin(a_2{}^j) & -\sin(a_3{}^j) & -\sin(a_4{}^j) \\ -3\sin(3a_1{}^j) & -3\sin(3a_2{}^j) & -3\sin(3a_3{}^j) & -3\sin(3a_4{}^j) \\ -5\sin(5a_1{}^j) & -5\sin(5a_2{}^j) & -5\sin(5a_3{}^j) & -5\sin(5a_4{}^j) \\ -7\sin(7a_1{}^j) & -7\sin(7a_2{}^j) & -7\sin(7a_3{}^j) & -7\sin(7a_4{}^j) \end{bmatrix} \tag{3.28c}$$

3) The corresponding harmonic amplitude matrix

$$T = \begin{bmatrix} \frac{(0.85)(3)p}{4} & 0 & 0 & 0 \end{bmatrix}^T \tag{3.28d}$$

Then, equations (3.27a) to (3.27d) can be rewritten in the following matrix format:

$$F(a) = T \tag{3.29}$$

By using matrices (3.28a) to (3.28d) and the Newton's method, the statement of algorithm, which is suitable for implement in a computer program, is shown as follows:

1) Guess a set of initial values for α with $j = 0$

$$\text{Assume} \quad a^0 = [a_1{}^0, a_2{}^0, a_3{}^0, a_4{}^0]^T \tag{3.30}$$

2) Calculate the value of

$$F(a^0) = F^0 \tag{3.31}$$

3) Linearize equation (3.29) about a^0

$$F^0 + \left[\frac{\partial f}{\partial a}\right]^0 da^0 = T \tag{3.32}$$

and $$da^0 = \begin{bmatrix} da_1{}^0 & da_2{}^0 & da_3{}^0 & da_4{}^0 \end{bmatrix}^T$$

4) Solve da^0 from equation (3.32), i.e.

$$da^0 = INV\left[\frac{\partial f}{\partial a}\right]^0 (T - F^0) \tag{3.33}$$

where $INV\left[\frac{\partial f}{\partial a}\right]^0$ is the inverse matrix of $\left[\frac{\partial f}{\partial a}\right]^0$

5) As updated the initial values,

$$a^{j+1} = a^j + da^j \tag{3.34}$$

6) Repeat the process, equations (3.31) to (3.34), until da^j is satisfied to the desired degree of accuracy, and the solutions must satisfy the following condition:

$$a_1, a_2, a_3, a_4 < \frac{p}{2} \tag{3.35}$$

To implement this algorithm in a computer, MATLAB programming is used. After running the program, each iteration result is tabulated in table 3.2 In this example, the process is repeated until all elements in df^j matrix are less than 0.0001 The final solutions, in degree, are:

$$a1 = 5.2538$$

$$a2 = 28.1201$$

$$a3 = 46.3876$$

$$a4 = 84.0986 \tag{3.36}$$

Because these switching angles satisfy the condition (3.35), they are the correct solutions. By using these switching angles in the waveform, the 3rd, 5th, and 7th harmonics will be eliminated with the modulation index equal to 0.85. As the results, the first harmonics content, which shows in the waveform, is the 9th harmonic.

Table 3.2 Content of matrix a^j, F^j, and df^j from MATLAB program by using the Newton's method

Iteration index	a^j (degree)	F^j	df^j
j = 0	5	2.8756	-2.0271
	20	0.4659	5.8345
	40	0.559	6.5274
	80	-0.7129	5.8326
j = 1	2.9729	2.6594	1.9027
	25.8345	0.2265	2.5526
	46.5274	0.0837	-0.1464
	85.8326	0.2735	-1.7376
j = 2	4.8757	2.6689	0.3852
	28.3871	-0.0087	-0.2674
	46.381	-0.0006	0.007
	84.095	0.036	0.0047
j = 3	5.2608	2.6703	-0.007
	28.1197	0	0.0005
	46.388	-0.0003	-0.0004
	84.0996	-0.0004	-0.0011
j = 4	5.2538	2.6704	0.0000
	28.1202	0	-0.0001
	46.3876	0	0.0000
	84.0985	0	0.0000
j = 5	5.2538	2.6704	0.0000
	28.1201	0	0.0000
	46.3876	0	0.0000
	84.0986	0	0.0000

3.2.4 The Total Harmonic Distortion Calculation

As introduced in the first chapter, the total harmonics distortion (THD) is mathematically given by

$$THD = \frac{\sqrt{\sum_{n=2}^{\infty} H_{(n)}{}^2}}{H_1} \tag{3.37}$$

where

H_1 is the amplitudes of the fundamental component, whose frequency is w_0

and $H_{(n)}$ is the amplitudes of the nth harmonics at frequency nω_0

From (3.15), the amplitude of the fundamental and harmonic components of the quarter-wave symmetric multilevel waveform can be express as:

$$h_n = \frac{4E}{np} \sum_{k=1}^{S} \cos(n a_k) \tag{3.38}$$

From (3.37), let $H_{(n)} = h_n$ and $H_1 = h_1$

$$THD = \frac{\sqrt{\sum_{n=2}^{\infty} {h_n}^2}}{h_1} \tag{3.39}$$

Substitute equation (3.38) to (3.39),

$$THD = \frac{\sqrt{\sum_{n=2}^{\infty} \left(\frac{1}{n} \sum_{k=1}^{S} \cos(n a_k) \right)^2}}{\sum_{k=1}^{S} \cos(a_k)} \tag{3.40}$$

By using (3.40), therefore, output voltage THD of the presented waveform can be calculated. Theoretically, to get exact THD, infinite harmonics need to be calculated. However, it is not possible in practice. Therefore, certain number of harmonics will be given. It relies on how precise THD is needed. Usually, *n* = 63 is reasonably excepted.

The THD calculation for example 3.1 will be presented in the following example.

Example 3.2

Replacing ∞ with 63 for (3.40), the equation becomes

$$THD = \frac{\sqrt{\sum_{n=2}^{63}\left[\frac{1}{n}\left(\cos(na_1)+\cos(na_2)+\cos(na_3)+\cos(na_4)\right)\right]^2}}{\left[\cos(a_1)+\cos(a_2)+\cos(a_3)+\cos(a_4)\right]} \tag{3.41}$$

By substituting the switching angle values from example 3.1 into equation (3.41), the output voltage THD of nine-level cascaded inverter waveform, which is calculated up to the 63rd harmonic, is 12.73%.

Verify the results from example 3.1 and 3.2 with PSPICE

The calculated results from example 3.1 and 3.2 will be verified by PSPICE. The nine-level inverter using four 100V dc sources is simulated. The operating frequency is 400 Hz and the modulation index is 0.85.

After simulated, the output voltage waveform of the nine-level inverter, its spectrum, and the output file of PSPICE are shown in Fig. 3.7, 3.8, and 3.9, respectively. As a consequence, the 3rd, 5th, and 7th is eliminated from the waveform as shown in Fig. 3.8. The fundamental component of the waveform is 340V, which is equal to 85% of 400V, or the modulation index is 0.85. The most significant harmonic, which is existed in the waveform, is the 9th and has the amplitude of 24.4V, which is 7.2% of the fundamental component.

From Fig. 3.9, the output voltage THD is shown as 12.73%, which is equal to the calculated result.

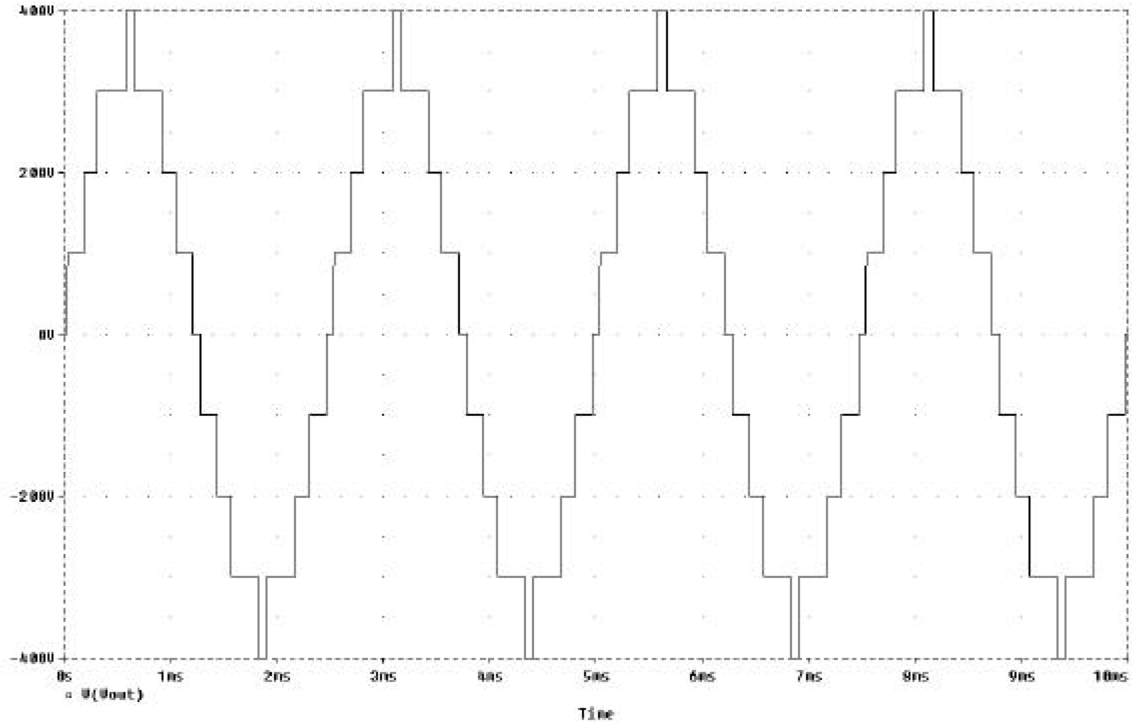

Figure 3.7 The output voltage waveform of a nine-level inverter.

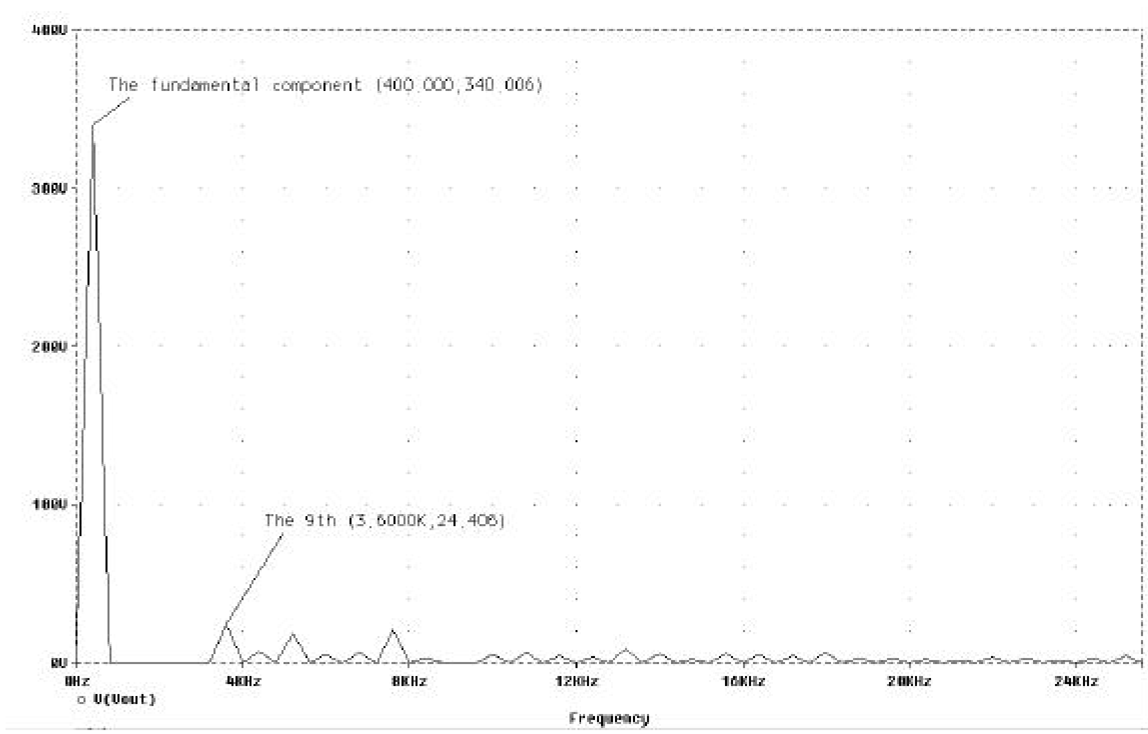

Figure 3.8 The frequency spectrum of the waveform in Fig 3.7.

```
**** 07/30/99 10:07:25 ********* NT Evaluation PSpice(July 1997)************
* C:\My Documents\Thesis\multi_5_7_11_test.sch
****     FOURIER ANALYSIS                 TEMPERATURE =   27.000 DEG C
****************************************************************************
FOURIER COMPONENTS OF TRANSIENT RESPONSE V(Vout)
DC COMPONENT =   0.000000E+00

HARMONIC   FREQUENCY    FOURIER    NORMALIZED    PHASE        NORMALIZED
   NO         (HZ)      COMPONENT    COMPONENT    (DEG)       PHASE (DEG)
    1      4.000E+02    3.400E+02    1.000E+00   -7.220E-02    0.000E+00
    2      8.000E+02    6.614E-03    1.945E-05    5.549E+01    5.557E+01
    3      1.200E+03    4.775E-03    1.404E-05    1.669E+02    1.670E+02
    4      1.600E+03    4.367E-03    1.284E-05    5.371E+01    5.378E+01
    5      2.000E+03    6.180E-03    1.818E-05   -1.771E+02   -1.770E+02
    6      2.400E+03    1.709E-03    5.025E-06    4.317E+01    4.324E+01
    7      2.800E+03    5.712E-03    1.680E-05   -1.739E+02   -1.738E+02
    8      3.200E+03    9.181E-03    2.700E-05   -9.673E+01   -9.666E+01
    9      3.600E+03    2.441E+01    7.178E-02   -6.529E-01   -5.807E-01
   10      4.000E+03    4.339E-03    1.276E-05    7.435E+01    7.442E+01
   11      4.400E+03    7.027E+00    2.067E-02    1.792E+02    1.793E+02
   12      4.800E+03    6.900E-03    2.029E-05    9.255E+01    9.262E+01
   13      5.200E+03    1.845E+01    5.427E-02   -9.380E-01   -8.658E-01
    ⋮
   60      2.400E+04    5.398E-03    1.587E-05   -1.553E+02   -1.552E+02
   61      2.440E+04    3.137E+00    9.227E-03   -4.412E+00   -4.340E+00
   62      2.480E+04    3.548E-03    1.043E-05   -1.611E+02   -1.610E+02
   63      2.520E+04    4.624E+00    1.360E-02   -4.546E+00   -4.474E+00

                TOTAL HARMONIC DISTORTION =   1.273219E+01 PERCENT
```

Figure 3.9 The output file of PSPICE.

3.3 Conclusions

To optimized the output multilevel voltage THD, the optimized harmonic stepped-waveform technique is presented. By applying this technique, the s-1 lowest odd harmonics can be eliminated from a single-phase output voltage. In three-phase system, the triplen harmonics in line voltage do not exist; therefore, the s-1 non-triplen harmonics need to be eliminated from a phase voltage.

By using the Newton-Raphson method, the switching angles can be easily solved. This method is very useful and effective technique, which is easy to apply in a computer program. In this thesis, MATLAB program is a main mathematical tool to do the job. An m-file used to solve the switching angles is shown in Appendix B.

PSPICE is used to verify the MATLAB solutions. As an example, the simulation results of the 9-level output voltage are very consistent with the calculated results.

In chapter 5, a set of switching angles of the OHSW technique, which will be considered from five levels up to fifteen levels, will be solved by MATLAB program using the Newton-Raphson method. The MATLAB results will be then verified by PSPICE to support the theory.

In the following chapter, the Selective Harmonic Eliminated Pulse Width Modulation (SHE PWM) technique will be discussed. The purpose to present this technique is because it will be used to compare with the OHSW technique in chapter 5 and 6. The comparison will be presented in several aspects.