CHAPTER 4

SELECTIVE HARMONIC ELIMINATED PULSE WIDTH MODULATION (SHE PWM)

The Selective Harmonic Eliminated PWM (SHE PWM) technique is currently applied in conventional three-level inverter circuits. The concept of the SHE PWM technique will be presented in this chapter. It needed to be compared to the optimized harmonic stepped-waveform technique in several aspects. Mainly, the harmonic components and the harmonic characteristics will be focused.

4.1 Introduction

The SHE PWM technique is currently used to synthesize an output waveform of both a half-bridge and a full-bridge inverter. In this thesis, a three-level SHE PWM generated by a full-bridge inverter is considered. A full-bridge or H-bridge voltage source inverter, which comprises four switches and one dc source, is depicted in Fig. 4.1.

Three states of an output waveform such as positive, negative, and zero, can be obtained. Fig 4.2 shows a generalized three-level SHE PWM waveform, which is synthesized by using the inverter circuit shown in Fig 4.1. The output waveform is chopped N times per quarter. Each switch is, therefore, switched N times per cycle to generate such a waveform.

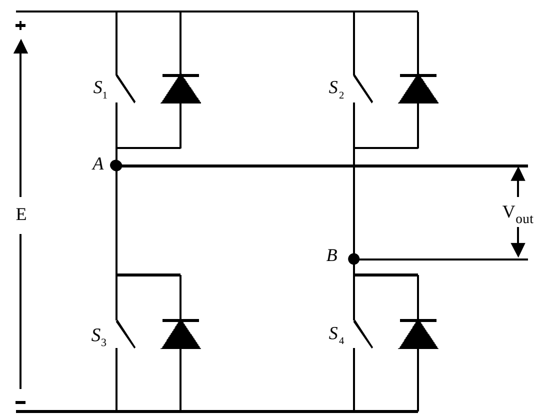

Figure 4.1 A full-bridge voltage source inverter.

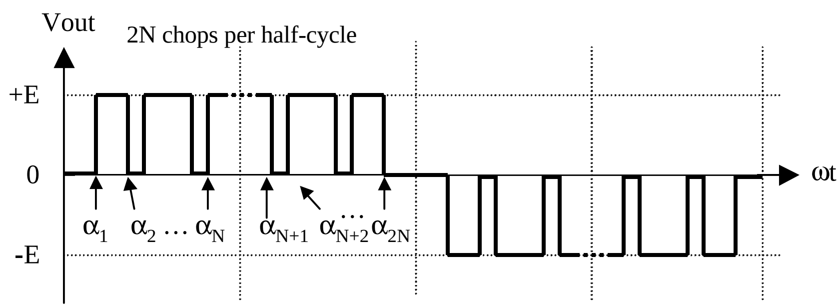

Figure 4.2 Generalized three-level SHE PWM waveform.

4.2 The SHE PWM Waveform

The H-bridge inverter as discussed above has three states of operation. Consider the generalized three-level SHE PWM waveform shown in Fig. 4.2. Let N be the number of switching angles per quarter-cycle. The output waveform is assumed to be odd quarter-wave symmetry, whose amplitude equals E.

4.2.2 The Fourier Series of the SHE PWM Waveform

Because of odd quarter-wave symmetry, the dc component and the even harmonics are equal to zero. Patel and Hoft [5] presented the Fourier series of the three-level SHE PWM as follows:

$$v_{out}(wt) = \sum_{n=1}^{\infty} a_n \sin(nwt) \tag{4.1}$$

where

$$a_n = \frac{4E}{np} \sum_{k=1}^{N} (-1)^{k+1} \cos(na_k) \qquad \text{, for odd n} \tag{4.2}$$

N is the number of the switching angles per quarter.

a_k is the switching angles, which must satisfy the following condition:

$$a_1 < a_2 < \ldots < a_N < \frac{p}{2} \tag{4.3}$$

E is the amplitude of the dc source.

and n is the harmonic order.

4.2.3 Apply the Newton's method to solve SHE PWM switching angles

From (4.2), the nonlinear equation system of SHE PWM waveform can be written as follows:

$$\cos(a_1) - \cos(a_2) + \ldots \pm \cos(a_N) = \frac{p}{4} M \tag{4.4a}$$

$$\cos(3a_1) - \cos(3a_2) + \ldots \pm \cos(3a_N) = \frac{3p}{4E} h_3 \tag{4.4b}$$

$$\cos(5\alpha_1) - \cos(5\alpha_2) + \ldots \pm \cos(5\alpha_N) = \frac{5\pi}{4E} h_5 \qquad (4.4c)$$

$$\vdots$$

$$\cos(N\alpha_1) - \cos(N\alpha_2) + \ldots \pm \cos(N\alpha_N) = \frac{N\pi}{4E} h_n \qquad (4.4n)$$

where

M is the modulation index, and $M = \frac{h_1}{E}$.

From equations (4.4a) to (4.4b), the cosine terms of α_N are negative with even N and positive with odd N. To control the amplitude of the fundamental component, the modulation index in (4.4a), M, is given. According to the above nonlinear system, N-1 surplus harmonic can be eliminated from the output waveform by setting equations (4.4b) to (4.4n) to zero. Basically, the lowest odd harmonic components need to be eliminated from a single-phase system. In a three-phase system, the lowest non-triplen harmonic components are eliminated. Generally, all triplen-harmonics in line-to-line voltage will be eliminated by 120 electrical degree phase shift characteristic.

Example 4.1

A three-level five-switching angle SHE PWM waveform, which is shown in Fig. 4.3, will be used as an example to explain how to apply the Newton-Raphson method to solve the SHE PWM switching angles. In this example, a single-phase system is assumed, and modulation index of output voltage, M, is 0.85.

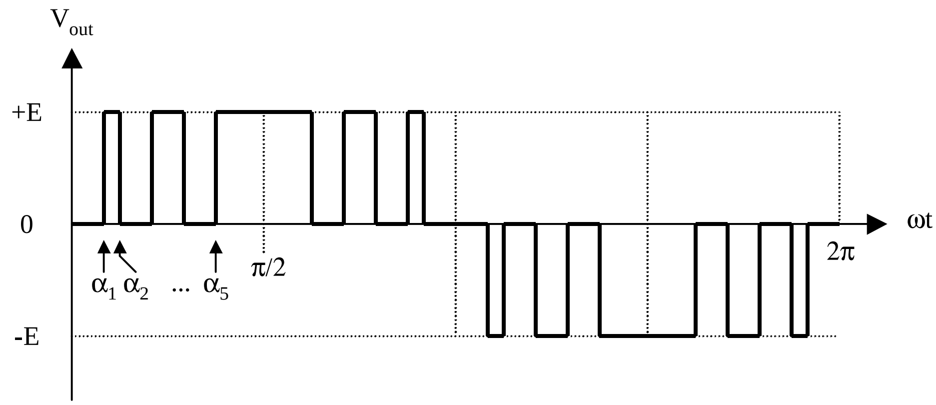

Figure 4.3 Output voltage waveform of a three-level five-angle SHE PWM.

From the output waveform shown in Fig. 4.3, five unknowns, α_1 to α_5, need to be solved. Because of a single-phase system, the lowest four odd harmonics, i.e., the 3rd, 5th, 7th, and 9th will be eliminated. From equations (4.4a) to (4.4n), five nonlinear equations will, therefore, be set up as follows:

$$\cos(\alpha_1) - \cos(\alpha_2) + \cos(\alpha_3) - \cos(\alpha_4) + \cos(\alpha_5) = 0.85\frac{\pi}{4} \quad (4.5a)$$

$$\cos(3\alpha_1) - \cos(3\alpha_2) + \cos(3\alpha_3) - \cos(3\alpha_4) + \cos(3\alpha_5) = 0 \quad (4.5b)$$

$$\cos(5\alpha_1) - \cos(5\alpha_2) + \cos(5\alpha_3) - \cos(5\alpha_4) + \cos(5\alpha_5) = 0 \quad (4.5c)$$

$$\cos(7\alpha_1) - \cos(7\alpha_2) + \cos(7\alpha_3) - \cos(7\alpha_4) + \cos(7\alpha_5) = 0 \quad (4.5d)$$

$$\cos(9\alpha_1) - \cos(9\alpha_2) + \cos(9\alpha_3) - \cos(9\alpha_4) + \cos(9\alpha_5) = 0 \quad (4.5e)$$

To solve these switching angles, the Newton-Raphson method in 3.2.3.1 is applied, and the following matrices are implemented:

1) The switching angle matrix,

$$a^{j} = [a_1^{\,j}, a_2^{\,j}, a_3^{\,j}, a_4^{\,j}, a_5^{\,j}]^T \tag{4.6a}$$

2) The nonlinear system matrix,

$$F^{j} = \begin{bmatrix} \cos(a_1^{\,j}) - \cos(a_2^{\,j}) + \cos(a_3^{\,j}) - \cos(a_4^{\,j}) + \cos(a_5^{\,j}) \\ \cos(3a_1^{\,j}) - \cos(3a_2^{\,j}) + \cos(3a_3^{\,j}) - \cos(3a_4^{\,j}) + \cos(3a_5^{\,j}) \\ \cos(5a_1^{\,j}) - \cos(5a_2^{\,j}) + \cos(5a_3^{\,j}) - \cos(5a_4^{\,j}) + \cos(5a_5^{\,j}) \\ \cos(7a_1^{\,j}) - \cos(7a_2^{\,j}) + \cos(7a_3^{\,j}) - \cos(7a_4^{\,j}) + \cos(7a_5^{\,j}) \\ \cos(9a_1^{\,j}) - \cos(9a_2^{\,j}) + \cos(9a_3^{\,j}) - \cos(9a_4^{\,j}) + \cos(9a_5^{\,j}) \end{bmatrix} \tag{4.6b}$$

and

$$\left[\frac{\partial f}{\partial a}\right]^{j} = \begin{bmatrix} -\sin(a_1^{\,j}) & +\sin(a_2^{\,j}) & -\sin(a_3^{\,j}) & +\sin(a_4^{\,j}) & -\sin(a_5^{\,j}) \\ -3\sin(3a_1^{\,j}) & +3\sin(3a_2^{\,j}) & -3\sin(3a_3^{\,j}) & +3\sin(3a_4^{\,j}) & -3\sin(3a_5^{\,j}) \\ -5\sin(5a_1^{\,j}) & +5\sin(5a_2^{\,j}) & -5\sin(5a_3^{\,j}) & +5\sin(5a_4^{\,j}) & -5\sin(5a_5^{\,j}) \\ -7\sin(7a_1^{\,j}) & +7\sin(7a_2^{\,j}) & -7\sin(7a_3^{\,j}) & +7\sin(7a_4^{\,j}) & -7\sin(7a_5^{\,j}) \\ -9\sin(9a_1^{\,j}) & +9\sin(9a_2^{\,j}) & -9\sin(9a_3^{\,j}) & +9\sin(9a_4^{\,j}) & -9\sin(9a_5^{\,j}) \end{bmatrix} \tag{4.6c}$$

3) The corresponding harmonic amplitude matrix,

$$T = \begin{bmatrix} \frac{(0.85)p}{4} & 0 & 0 & 0 & 0 \end{bmatrix}^T \tag{4.6d}$$

Thus, equation (4.6a) to (4.6d) can be rewritten in the following matrix format:

$$F(a) = T \tag{4.7}$$

By using matrices (4.6a) to (4.6d) and the Newton-Raphson method, the statement of algorithm is shown as follows:

1) Guess a set of initial values for a^{j} with $j = 0$

Assume $$a^{0} = [a_1^{\,0}, a_2^{\,0}, a_3^{\,0}, a_4^{\,0}, a_5^{\,0}]^T \tag{4.8}$$

2) Calculate the value of

$$F(a^{0}) = F^{0} \tag{4.9}$$

3) Linearize equation (4.7) about a^{0}

$$F^0 + \left[\frac{\partial f}{\partial a}\right]^0 da^0 = T \tag{4.10}$$

and
$$da^0 = \left[da_1^0 \quad da_2^0 \quad da_3^0 \quad da_4^0 \quad da_5^0\right]^T \tag{4.11}$$

4) Solve da^0 from equation (4.10), i.e.

$$da^0 = INV\left[\frac{\partial f}{\partial a}\right]^0 (T - F^0) \tag{4.12}$$

where $INV\left[\frac{\partial f}{\partial a}\right]^0$ is the inverse matrix of $\left[\frac{\partial f}{\partial a}\right]^0$

5) As updated the initial values,

$$a^{j+1} = a^j + da^j \tag{4.13}$$

6) Repeat the process for equations (4.9) to (4.13), until da^j is satisfied to the desired degree of accuracy, and the solutions must satisfy the condition:

$$a_1 < a_2 < a_3 < a_4 < a_5 < \frac{p}{2} \tag{4.14}$$

By using MATLAB program, the results of iteration are tabulated in Table 4.1. From Table 4.1, the final solutions, which are switching angles in degree, are

$$a_1 = 22.5835$$

$$a_2 = 33.6015$$

$$a_3 = 46.6433$$

$$a_4 = 68.4980$$

$$a_5 = 75.0978 \tag{4.15}$$

Table 4.1 The results from MATLAB program.

Iteration index	a^{j} (degree)	F^{j}	df^{j}
j = 0	20.0000 30.0000 50.0000 70.0000 80.0000	0.5481 0.0000 0.1316 0.7879 0.0000	3.0612 4.0541 -3.3863 -0.6677 -3.9607
j = 1	23.0612 34.0541 46.6137 69.3323 76.0393	0.6668 0.0172 -0.0772 -0.0044 -0.0567	-0.4761 -0.4500 0.0431 -0.7903 -0.9083
j = 2	22.5851 33.6040 46.6568 68.5420 75.1310	0.6676 -0.0002 -0.0011 0.0043 -0.0052	-0.0016 -0.0024 -0.0134 -0.0440 -0.0332
j = 3	22.5835 33.6016 46.6434 68.4980 75.0978	0.6676 0.0000 0.0000 0.0000 0.0000	-0.00002 -0.00005 -0.00006 -0.00001 0.00001
j = 4	22.5835 33.6015 46.6433 68.4980 75.0978	0.6676 0.0000 0.0000 0.0000 0.0000	-0.0939e-9 -0.1345e-9 -0.0010e-9 0.1000e-9 0.0518e-9
j = 5	22.5835 33.6015 46.6433 68.4980 75.0978		

By applying these switching angles, the 3rd, 5th, 7th, and 9th harmonics will be eliminated from the output voltage waveform. The output voltage THD, which is also calculated by MATLAB, is 65.15%. The results will be simulated and verified by PSPICE, which will be shown in following section.

4.2.4 Verify the results with PSPICE

A three-level five-switching angle SHE PWM output waveform is synthesized by an H-bridge voltage source inverter. The calculated switching angle in example 4.1 are employed in the simulation. The H-bridge inverter is supplied by a 400V dc source and operates at 400Hz. The given modulation index is 0.85, and the expected fundamental component of the output voltage is 340V. After running PSPICE, the output voltage waveform and its frequency spectrum are shown in Fig. 4.4 and 4.5, respectively.

From the spectrum in Fig. 4.5, the 3rd, 5th, 7th, and 9th harmonic are eliminated from the output voltage waveform as expected. The amplitude of the fundamental is 340V, which is 85 percent of the dc source value. The first surplus harmonic is the 11th, whose amplitude is approximately 155V or 46 percent of the fundament component. The voltage THD, which is equal to the THD from MATLAB program, is shown in the output file of PSPICE in Fig. 4.6.

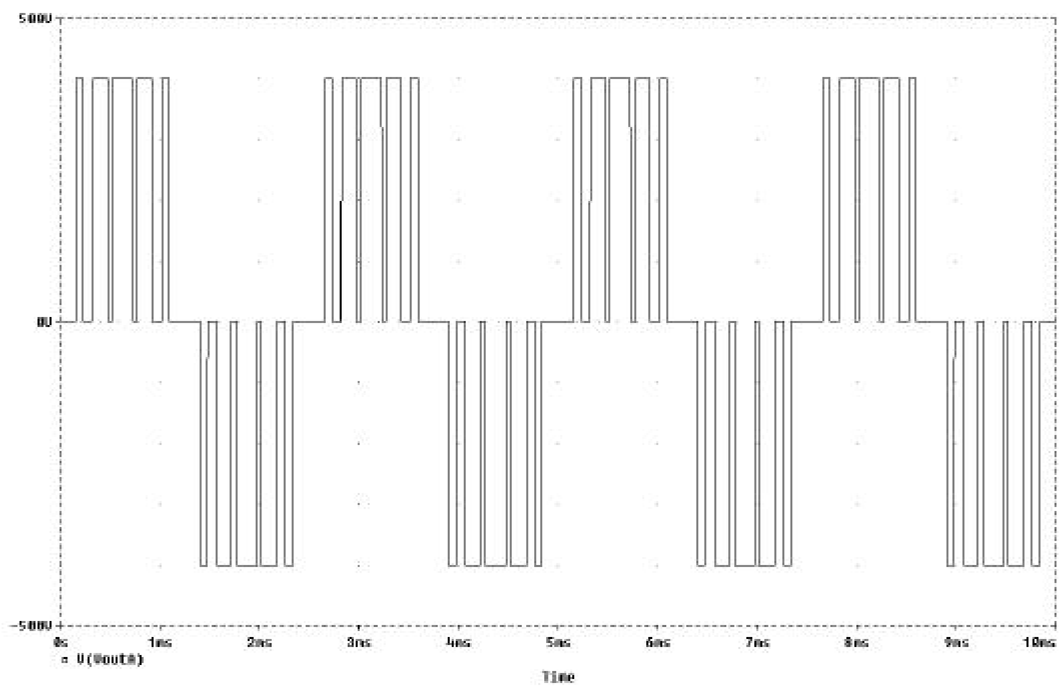

Figure 4.4 The output voltage waveform.

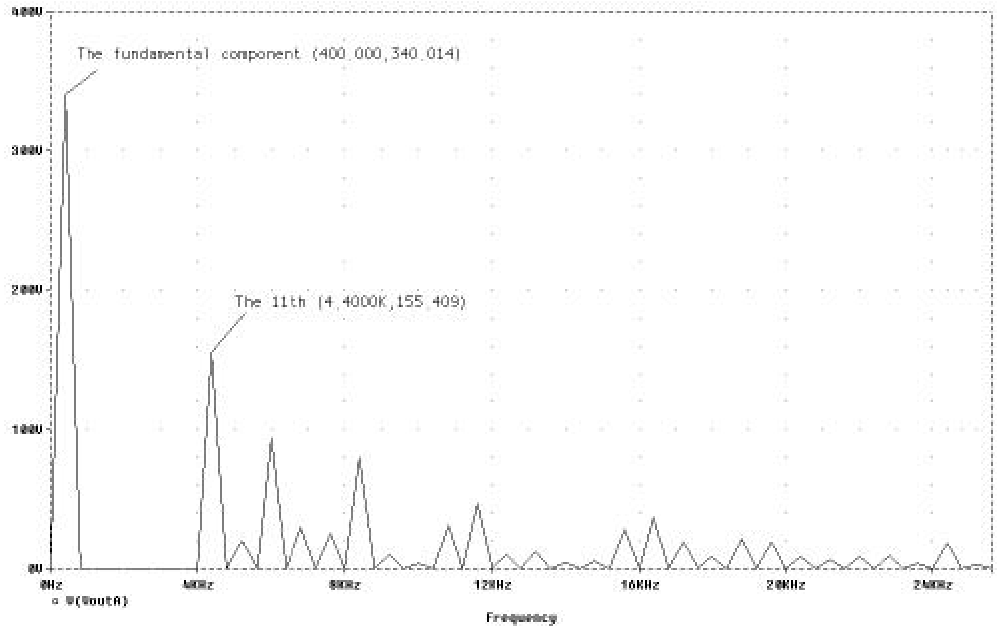

Figure 4.5 The frequency spectrum of the output waveform shown in Fig 4.4.

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**** 08/03/99 10:48:17 ********* NT Evaluation PSpice (July 1997)
* C:\My Documents\Thesis\she5angle_M85_sph.sch
****     FOURIER ANALYSIS                 TEMPERATURE =   27.000 DEG C
*****************************************************************************
FOURIER COMPONENTS OF TRANSIENT RESPONSE V(VoutA)

DC COMPONENT =   0.000000E+00

 HARMONIC   FREQUENCY    FOURIER    NORMALIZED    PHASE       NORMALIZED
    NO         (HZ)      COMPONENT   COMPONENT    (DEG)       PHASE (DEG)

     1      4.000E+02    3.400E+02   1.000E+00   -7.192E-02    0.000E+00
     2      8.000E+02    1.465E-02   4.310E-05    6.933E+01    6.940E+01
     3      1.200E+03    2.411E-02   7.091E-05   -1.660E+02   -1.659E+02
     4      1.600E+03    6.953E-03   2.045E-05   -8.747E+01   -8.740E+01
     5      2.000E+03    1.240E-02   3.648E-05   -4.176E+01   -4.169E+01
     6      2.400E+03    1.819E-02   5.350E-05   -1.292E+02   -1.291E+02
     7      2.800E+03    5.541E-02   1.630E-04    5.520E-01    6.240E-01
     8      3.200E+03    2.263E-02   6.656E-05    1.238E+02    1.239E+02
     9      3.600E+03    2.095E-02   6.160E-05    4.673E+01    4.680E+01
    10      4.000E+03    4.210E-03   1.238E-05    6.963E+01    6.970E+01
    11      4.400E+03    1.554E+02   4.571E-01    1.792E+02    1.793E+02
    12      4.800E+03    3.015E-02   8.868E-05    5.292E+01    5.299E+01
    13      5.200E+03    2.029E+01   5.967E-02   -9.411E-01   -8.692E-01
    14      5.600E+03    2.285E-02   6.722E-05   -3.565E+01   -3.558E+01
    15      6.000E+03    9.364E+01   2.754E-01   -1.095E+00   -1.023E+00
   ⋮
    62      2.480E+04    2.109E-02   6.202E-05   -1.122E+02   -1.122E+02
    63      2.520E+04    3.083E+00   9.068E-03   -4.545E+00   -4.473E+00

                TOTAL HARMONIC DISTORTION =   6.513426E+01 PERCENT
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Figure 4.6 The harmonic components and THD from PSPICE output file.

4.3 Conclusions

The selective harmonic eliminated PWM technique is introduced. The output waveform is shopped N times per quarter-cycle. The N-1 harmonics can be removed from the waveform and the fundamental amplitude can be controlled.

The simulated results agree with the calculated results very well.

As discussed in Chapter 3, the condition that the solutions of OHSW must be satisfied is $a_1, a_2, a_3, a_4 < \frac{p}{2}$. Compared to the SHE PWM condition, $a_1 < a_2 < a_3 < a_4 < a_5 < \frac{p}{2}$, it is easier to get the solutions because its condition has more degree of freedom. This is an advantage of OHSW over SHE PWM technique.