

Simulation of Batch Thickening Phenomenon for Young Sediments

by

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BATCH THICKENING SIMULATION

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Abstract

The present study consists of the development of a MATLAB version of computer program (FORTRAN) developed by Papanicolaou (1992) to solve the governing small strain consolidation equation of second order non-linear transit partial differential equation of parabolic type. This program is modified to integrate the settling and consolidation processes together in order to provide continuous results from start to end of the process in a single run of MATLAB program. The study also proposes a method to calculate the batch curve by considering the variation of solids concentration in the suspension region. Instead of the graphical approach available in the literature, the program uses numerical approach (Newton-Raphson method) to calculate the solids concentration in suspension region at the interface of suspension and sedimentation regions. This method uses the empirical relationship between solids flux and solids concentration. The study also proposes a method to calculate the solids concentration, throughout the settling column, using the concept of characteristic. The present work also simulates the large strain consolidation model (Gutierrez, 2003). The results of present work closely match with the results of small strain model (Diplas & Papanicolaou, 1997) available in literature.

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List of Symbols:

a	Parameter represents the permeability of solids
a'	Dimensionless Parameter
A_0	Gravity number
b	Permeability Index
b_0	Experimental parameter
b_1	Experimental parameter
b_2	Experimental parameter
C	Solids concentration
C_0	Initial concentration
C_c	Solids concentration in suspension
C_{co}	Compression Index
C_d	Solids concentration in sediment
C_{max}	Maximum concentration on the top of sediment layer
d_i	Diameter of the particle
D	Diameter of the column
e	Void ratio
e_0	Void ratio corresponding to reference stress
e'	Slope of the curve of void ratio versus effective stress

g	Gravitational constant
h	Height of the water column
H_0	Initial height
K	Permeability
K_0	Null stress permeability
N	Dimensionless parameter
P_T	Total pressure
P	Overpressure
P_n	Normalized overpressure
P_{sn}	Normalized effective stress
P_s	Vertical effective stress
$P_{s\max}$	Maximum vertical effective stress
S	Solids flux
S_c	Solids flux in suspension
S_d	Solids flux in sediment
t	Time
t_0	Reference time
u	Solids settling velocity
v	Velocity of characteristic
w	Sedimentation rate
x	Height

Greek symbols:

α	Pore compressibility factor
β	Compressibility coefficient
δ	Compressibility coefficient

ε_s	Solidosity
ε_{s0}	Maximum concentration on the top of sediment layer
λ	Parameter defined in term of compression index
η	Velocity of upward moving concentration
μ	Liquid viscosity
ϕ	Solids concentration in suspension
ϕ_0	Null stress porosity
ϕ_{s0}	Initial solids concentration in column
τ	Normalized time
ρ_s	Mass density of solids
ρ_w	Mass density of water
σ'_v	Effective vertical stress
σ'_{v0}	Reference vertical stress
$\sigma'_{v\max}$	Maximum vertical effective stress

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Chapter 1: Introduction

Section 1.1: Batch thickening phenomenon

Batch thickening is the process of separation of solids and liquid under the influence of only the gravitational force. This method of separation is very useful in the field of environmental engineering as it can be used to separate solids waste from the mixture of pollutants in secondary waste water settling tanks. It can also be used in mineral processing, chemical engineering and soil consolidation in geotechnical engineering.

In 1916, Coe & Clevenger identified the four regions in the settling column during the batch thickening test. 1) Supernatant liquid region. 2) Constant concentration region in suspension. 3) Transit region (varying concentration) in suspension. 4) Consolidation (Sediment) region. During the batch thickening, two kinds of processes take place. 1) Settling of the solids from the suspension region. 2) Consolidation of already settled solids at the bottom of the settling column. Thus, during the batch thickening process two kinds of interfaces can be seen. 1) Interface between supernatant liquid (clear liquid) and suspension zones, and 2) the interface between suspension and sediment zones. The variation of the height of supernatant-suspension interface with time is termed as batch curve. The variation of suspension-sediment interface with time is termed as L-curve. The batch curve can be further divided into three regions (Papanicolaou, 1992) as shown in Figure 1.1.1. The first region (AB) is termed as the constant settling region, where the particles settling velocity in the suspension region remains constant. In this region, particles are bond together in a flock structure, which is strong enough to avoid differential settling and, at the same time, is too tenuous to avoid crushing (Papanicolaou, 1992). The second region (BC) is termed as the transition region. In this region, particle's settling velocity is influenced by the velocity of upward moving liquid coming out from the sedimentation region as a result of consolidation. Due to the upward moving liquid from consolidating region, settling velocity of particles in suspension decreases. Thus, the rate of settlement of particles in suspension

decreases. The third region (CD) is the consolidation region, which occurs after the settling of all the solids from the suspension region.

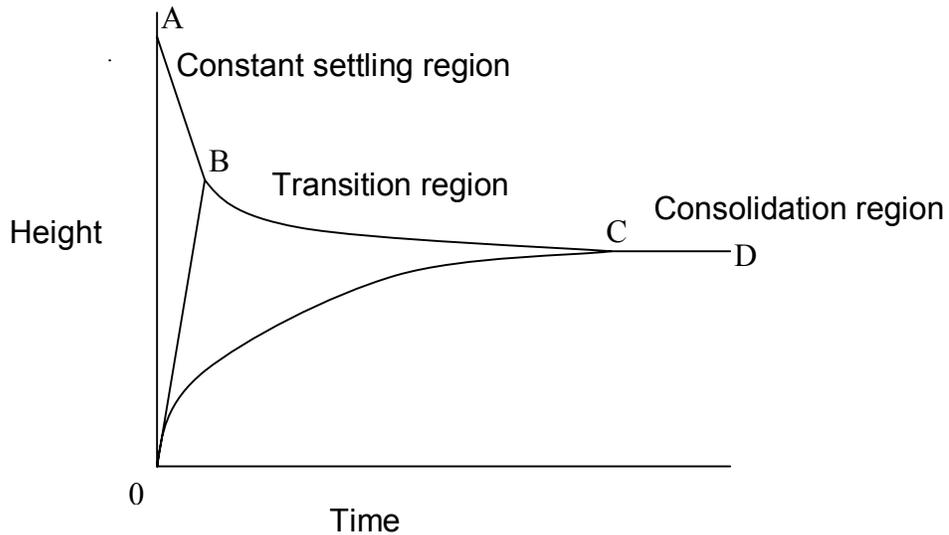


Figure 1.1.1 – Behavior of batch curve during batch thickening process

The primary factors which affect the process of batch thickening are: 1) initial height of the suspension column, 2) initial concentration of the solids in suspension region, and 3) liquid viscosity (Gaudin et al, 1959; Jivaheri, 1971; Bhargava and Rajgopal, 1990). The wall effect of the column may be an important factor in the settling of solids from suspension region but it can be neglected if $D > 100d_i$ (Cullum, 1988), where, D is the diameter of settling column, and d_i is the diameter of solids particles.

Section 1.2: Objectives of present study

The objectives of the present work are:

1. Simulation of small strain batch thickening model, given by Papanicolaou (1992), in MATLAB.
2. Integration of solids settling process (occurs in suspension region) to solids consolidation process (occurs in consolidation region). This integration will help in obtaining the continuous results from start to end of batch thickening process in single run of MATLAB program.
3. Development of a numerical method to calculate the solids concentration in the suspension-sediment interface.
4. Development of numerical method to calculate the batch curve.
5. Calculation of concentration of solids in settling column.
6. Simulation of large strain batch thickening model, given by Gutierrez (2003), in MATLAB.

Section 1.3: Thesis outlined

Chapter 2 begins with the literature review on small strain batch thickening phenomenon with main focus on the pioneer work of Kynch (1952). Later in this chapter, literature review on large strain consolidation model is presented. Chapter 3 begins with the description of small strain consolidation model presented by Papanicolaou (1992). Following the discussion, a numerical method to calculate the solids concentration in suspension region is proposed. Later in this chapter, a numerical method to calculate the batch curve is proposed. Also, a method to calculate the solids concentration, throughout the settling column is presented. Finally, subroutines, algorithms, and flow charts of MATLAB programs of small strain and large strain batch thickening phenomenon are presented. Chapter 4 begins with the presentation of input data required for the simulation of finite strain and large strain batch thickening

processes. Following that, results of small strain and large strain simulations are demonstrated. Finally, comparisons are shown between experimental and present simulation results, and between small strain and large strain simulations result. Chapter 5 summarizes the significance of work done in present research and the future course for this project to take.

Chapter 2: Literature Review

Section 2.1: Small strain batch thickening model

The foundation of the research work in the field of sedimentation and consolidation phenomenon was put forth by Kynch (1952). Kynch's theory was based on two explicit assumptions. First, one dimensional continuity and second, the settling rate of solids, u , at any point in the suspension column is a function of solids concentration, ϕ , at that point, i.e. $u = u(\phi)$. Kynch (1952) proposed the theory of kinematics for suspension region. He proposed the continuity equation of solids flux in suspension region as:

$$\frac{\partial \phi}{\partial t} + \frac{\partial(\phi u)}{\partial x} = 0 \quad 0 < x < L, t > 0 \quad (2.1.1)$$

Where, ϕ is the volume fraction of solids as a function of height (x) and time (t) and u is local velocity of solids as a function of local concentration solids. At time $t = 0$, the concentration of solids can be given as:

$$\phi(x,0) = \begin{cases} 0 & x = L \\ \phi_0 & 0 < x < L \\ \phi_{\max} & x = 0 \end{cases} \quad (2.1.2)$$

In 1983, Fitch interpreted Kynch's research work on sedimentation phenomenon in three theorems as:

1. Velocity of discontinuity of concentration in a sedimentation column can be given by,

$$v = -\frac{\Delta S}{\Delta \phi} \quad (2.1.3)$$

Where, S is the solids flux and ϕ is the solids concentration defined above and below the stable discontinuity of concentration. This theorem is valid for any kind of suspension medium, i.e. it is valid whether the assumption $u = u(\phi)$ is valid or not.

2. With the assumption $u = u(\phi)$ hold true, the locus of an upward moving concentration during sedimentation is a straight line and termed as characteristic. The velocity, v , of this characteristic is given by,

$$v = -\frac{dS}{d\phi} \quad (2.1.4)$$

3. The concentration, ϕ , of any characteristic (OB) generated from the origin during the sedimentation phenomenon can be given by,

$$\phi = \frac{\phi_0 H_0}{(u + v)t} \quad (2.1.5)$$

Where, t is the time taken by the characteristic to reach the supernatant-suspension liquid interface, v is the velocity of characteristic, u is the settling velocity of the solids of concentration ϕ , and ϕ_0 and H_0 are the initial concentration and initial height respectively (Figure 2.1.1).

In his model, Kynch ignored the particulate behavior of solids in suspension and assumed that solids are homogeneously distributed in the suspension region. He also ignored the diffusion effect and the inertial (acceleration and deceleration) behavior of solid particles. In 1982, Dixon considered the inertial behavior of solid particles in sedimentation phenomenon. According to Dixon, due to inertial effect, an apparent discontinuity between initial solids concentration and maximum solids concentration appears. He gave the expression for the dimensionless parameter N as:

$$N = \frac{g(\rho_s - \rho_w)}{\rho_s + \rho_w \left(\frac{\phi_i}{1 - \phi_i} \right) u_{s0}^2} \quad (2.1.6)$$

Where, g is gravitational constant, ρ_s and ρ_w are the density of solids and liquid respectively. If $N > 0.1$, then inertial effect and particulate effect can be neglected. Also, in suspension region, diffusion effect is negligible if the size of particles or aggregates is greater than 5-20 μm .

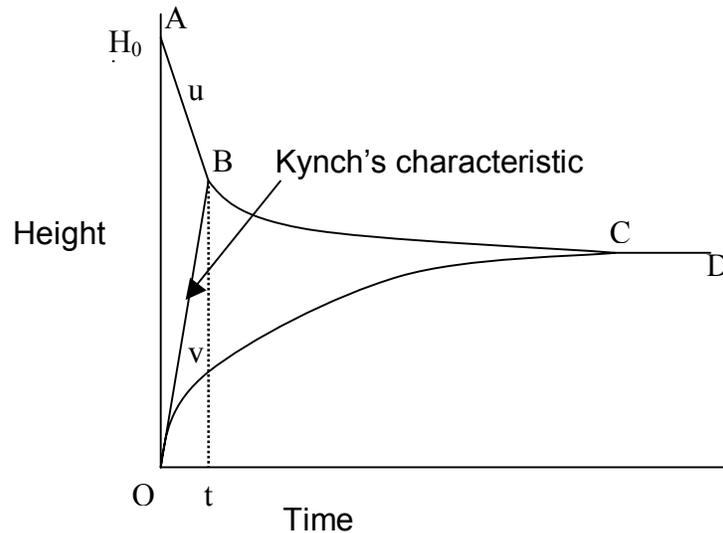


Figure 2.1.1 - Kynch characteristic

Kynch's sedimentation model is also valid for the flow through porous media formed by the solid particles, which is a basic principle of the filtration process (Burger and Wendland, 2001). Kynch neglected the rising of consolidating layer from the bottom of settling column. Thus, Kynch's assumption of $u = u(\phi)$ and theorem 3 (Fitch, 1983) are not valid for the consolidating region present at the bottom of the column. In other words we can say that the theorem 3 is not valid for the characteristic generated from the top of the consolidating layer (Fitch, 1983). In the consolidating region, u is a function of

local solids concentration and the solids stress gradient (Michael & Bolger, 1962; Fitch, 1966, 1975, 1979; Shirato, 1970). Also, the Kynch's theory $u = u(\phi)$ is not valid for the flocculated suspension as the dynamic effect, particularly the effective solids stress, plays an important role in this zone (Scott, 1968).

Tiller (1981) considered the rising of consolidating layer from the bottom of the column. He used the concept of mass conservation. According to Tiller, at any instant in time, the integrated amount of solids has crossed the sediment-suspension interface plus the amount of solid remaining in suspension, is equal to the amount of solids originally present in the suspension. He derived equation 2.1.8 to give the concentration of characteristic generated from the top of the consolidation layer, but Tiller paper did not use the Kynch theorems. Referring to Figure 2.1.2:

$$\int_0^{L_1} \frac{d\varepsilon_{s1}}{dt_1} dx + \varepsilon_{s1} \frac{dL_1}{dt_1} = \phi_{s2} \left(-\frac{dH_2}{dt_2} + \frac{dL_1}{dt_1} \right) \quad (2.1.7)$$

and

$$\phi_{s2} = \frac{\phi_{s0} H_0}{H_{12} - L_1} \exp \left(\int_0^{t_1} \left(\frac{\frac{dH_{12}}{dt_2} - \frac{dL_1}{dt_1}}{H_{12} - L_1} \right) dt_1 \right) \quad (2.1.8)$$

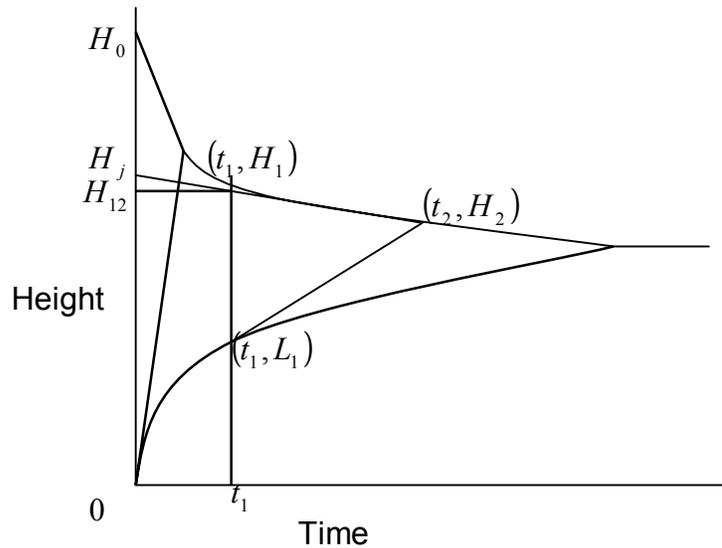


Figure 2.1.2 - Graphical method to calculate solids concentration (Tiller, 1981)

Where, t_1 is the time of generation of second characteristic, ϕ_{s2} is the concentration associated with the characteristic generated at time t_1 , H_1 is the height of batch curve at time t_1 , H_2 is the height where second characteristic, generated at time t_1 , hits the batch curve (supernatant- suspension interface), L_1 is the height of consolidation layer at time t_1 and ε_{s1} is the concentration, varying with height of consolidation layer, in consolidation layer at time t_1 . H_{12} is the ordinate corresponding to the time t_1 on the tangent passing from the point (t_2, H_2) .

Tiller (1981) generalized Kynch's third theorem for the characteristic generated from the top of the sediment layer. Referring to the Figure 2.1.3, the concentration of the characteristic, generated from the top of sediment layer, can be given as:

$$C_d = C_0 \left(\frac{H_0 - H_j}{H_i - H_j} \right) \quad (2.1.9)$$

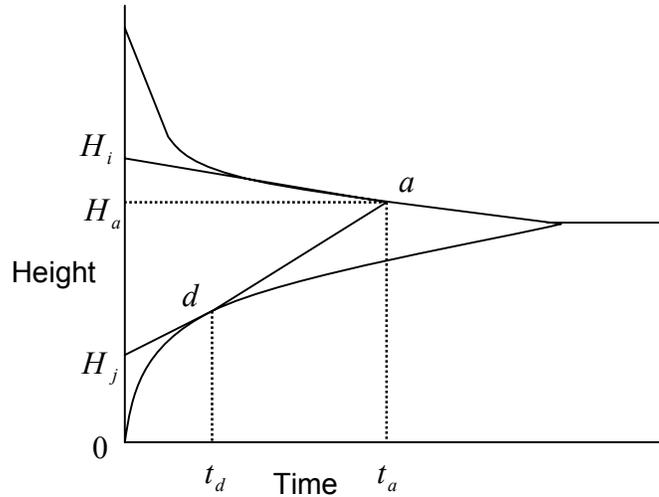


Figure 2.1.3 - Graphical method for concentration of characteristic (Tiller, 1981)

Where, C_0 is the initial concentration, C_d is the concentration of characteristic generated from the top of the sediment layer 'd', and H_i is the point of intersection of tangent generated at points 'a' with the ordinate (Height axis) and H_j is the point of intersection of tangent generated at point 'd' with the ordinate axis (Figure 2.1.3). Thus, Tiller's procedure requires Figure 2.1.3 and equation 2.1.9 to calculate the concentration of characteristic generated from the top of sediment layer. Moreover, Tiller's procedure does not give the relationship between the solids concentration and solids settling rate from one laboratory test.

After the Tiller's generalization of Kynch characteristic, Fitch (1983) proposed the characteristics of the concentration discontinuity. According to Fitch, if any part of the chord of concentration discontinuity goes above the Kynch's plot, solids flux (solids velocity multiply by solids concentration) versus solids concentration, then that discontinuity would be unstable and immediately gives rise to a different concentration

distribution. A concentrations discontinuity (AB) whose chord lies everywhere below the Kynch's plot would propagate stably. This implies that the locus of constant concentration must propagate either from the origin of the height versus time plot, or tangentially from the locus of sediment-suspension interface (Figure 2.1.4).

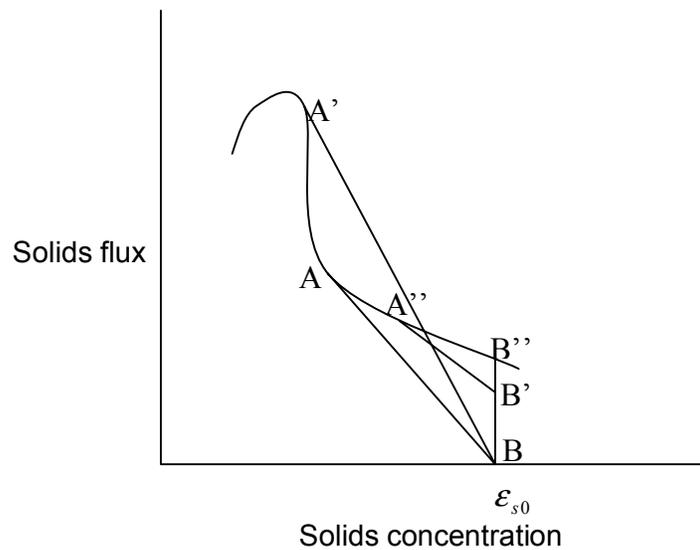


Figure 2.1.4 - Stability of concentration discontinuity (Fitch, 1983)

According to Fitch (1983), u tends to be small as ϕ tends to ϕ_{\max} and decreases suddenly to zero. This implies that the solids in sediment at ϕ_{\max} have a settling velocity of zero. “At the top of sediment layer the solids must have the concentration at which solids can exhibit comprehensive yield value. They can not have lower concentration as it would not support the comprehensive solid stress. Also, they can not have higher concentration as there are no solids above them not totally supported by hydraulic force” (Fitch, 1983). Chu (2002) tried to measure the spatiotemporal distribution of solidosity (solid volume ratio) for clay slurry using a computerized axial topography scanner (CATSCAN). According to Chu, it is difficult to find the exact value of null stress solidosity (ϕ_{\max} or ε_{s0}), but he proposed that, ideally, a sudden jump of concentration of

solid exists at sediment-suspension interface. Thus, ε_{s0} is equal to the solidosity at which the slope of ε_s versus height (z) curve has maximum slope. If the solids are compressible then the subsidence velocity just below the compression discontinuity will not be zero, except at the initial time at which the sediment zone depth is equal to zero. Liquid squeezes out from the sediment zone due to the consolidation of the solids and the velocity of squeezed liquid increases as the consolidation increases due to sedimentation of more solids on top of the sediment layer. As a result, the corresponding solids flux also increases. Thus, according to Kynch's first theorem, velocity of the characteristic can be given as:

$$v = \frac{S_c - S_d}{\phi_d - \phi_c} \quad (2.1.10)$$

Where, ϕ_d is the solids concentration at the top of the sediment zone and it is constant. ϕ_c and S_c are the solids concentration and solids flux in the suspension region and vary very slowly. S_d is solids flux in the sediment zone and it is a dominating factor in the calculation of the velocity of characteristic. Thus, velocity of the characteristic decreases with time.

Fitch (1983) concluded that the rise of sediment zone is usually not visible. Font (1988) proposed a method to estimate the variation of sediment layer with time when it is not visible. The velocity of rising sediment can be deduced by rearranging equation 2.1.7 applied to the sediment surface.

$$\frac{dL_1}{dt_1} = \frac{\phi_{s2} \left(\frac{dH_2}{dt_2} \right) - \int_0^{L_1} \frac{d\varepsilon_s}{dt_1} dx}{\phi_{s2} - \varepsilon_{s1}} = \frac{\phi_{s2} U_{s2} - \varepsilon_{s1} U_{s1}}{\phi_{s2} - \varepsilon_{s1}} \quad (2.1.11)$$

The Tiller (1981), Fitch (1983), and Font (1988) studies give the methods to calculate only the solids concentration in the suspension region. These methods require the prior knowledge of height variation of supernatant-suspension interface, i.e. batch curve and suspension-sediment interface, i.e. L-curve with time. Papanicolaou (1992) developed an integrated model of suspension and sediment components which did not require the prior knowledge of L-curve. This model required the slurry flux curve and gave the variation of the height of supernatant-suspension and suspension-sediment interfaces with time. In the integrated model, a numerical scheme was developed to solve the second order non-linear transit partial differential equation of parabolic type for the sediment zone equation. This numerical scheme comprised the Petrov-Galerkin method to construct the weak form of the governing equation, the Crank-Nicolson method to solve the time derivative part of the governing equation, and the Newton-Raphson method to solve the nonlinear part of the governing equation. In his model, Papanicolaou used Kynch's (1952) concept of mass balance in the suspension region to account for the propagation of discontinuities throughout the region. Papanicolaou used the graphical procedure of Fitch's (1983) concept in order to calculate the settling amount of solids with the help of a slurry flux curve. In his model, he coupled the components of suspension and sediment layers by taking into account the effects of added amounts of settled solids on top of the sediment layer and the effect of upward moving liquid on the settling rate of solids in the suspension region.

In 2000, Burger analyzed the settling process in a different way. Burger (2000b) treated suspension as a mixture of two superimposed continuous media of solids and fluid. He proposed the BV solution technique to solve the obtained sedimentation equation of second order partial differential equation of mixed hyperbola-parabolic type by considering it as a convection-diffusion equation.

In addition to Kynch's (1952), Tiller's (1981), Fitch's (1983) and Font's (1988) one dimensional theories of sedimentation with constant cross section of column, Anestis & Schneider (1983) proposed that the Kynch's theory $u_s = f(\phi_s)$ can also be applicable to the varying cross section of the column. He assumed that gravitational force and coriolis force are negligible compared to the centrifugal force. In this case, the characteristic and the isoconcentration curve no longer coincide and solutions are complicated compared to the batch settling solutions. In further development, Gustavsson & Ooppelstrup (2001) proposed a two-dimensional mathematical model by considering the forces and the flow of solids and fluid in two dimensions.

The above mentioned theories did not account for the effects of channels present in the sediment region. According to Perez, Font, and Raster (1998), a suspension region can be found in the form of solids movement in channels in the sediment region. But the expressions for effective pressure and the permeability, which are given for the suspension region, do not hold true for this due to interparticles friction and change in permeability. They also developed the MATLAB computer program, using the finite difference method, based on the sediment region model given by Michaels and Bolger (1962).

Section 2.2: Large Strain consolidation model

In 1958, Gibson developed a model for sedimentation-consolidation which provided the base for development of a large strain consolidation model. In his model, Gibson used Terzaghi's (1943) small strain consolidation equation with the assumptions of constant sedimentation rate of solids and constant sedimentation properties like compressibility, permeability and density. In 1992, Wangen (1992) and Audet & Flower (1992) modified the Gibson linear theory into non-dimensionalized, non-linear and large strain form. Wangen used dimensionless parameters in the formulation of consolidation equation.

These normalized parameters are Gravity number A_0 and normalized time τ and given by,

$$A_0 = \frac{K_0(\rho_s - \rho_w)}{\mu w} \quad (2.2.1)$$

and

$$\tau = \frac{t}{t_0} \quad (2.2.2)$$

Where, K_0 is permeability at the surface of sediment region, ρ_s is density of the solids, ρ_w is density of water, μ is viscosity of water, w is the sedimentation rate, and t_0 is reference time (time taken by settling solids with the settling rate of w to achieve maximum vertical effective stress) used to normalize the time and given by,

$$t_0 = \frac{1}{\alpha(\rho_s - \rho_w)gw} \quad (2.2.3)$$

Where, α is a pore compressibility parameter.

The equation of sedimentation-consolidation given by Wengen to obtain the normalized overpressure in sediment zone is:

$$\tau \frac{\partial P_n}{\partial \tau} + P_n - x \frac{\partial P_n}{\partial x} + A_0 \frac{1}{e'} \frac{\partial}{\partial x} \left[\frac{K(e)}{1+e} \left(\frac{\partial P_n}{\partial x} \right) \right] = 1 \quad (2.2.4)$$

Where, e is void ratio (ratio of volume of voids and the volume of solids), e' is slope of the curve of void ratio versus effective stress P_s , x is a normalized Langrangion coordinate ($0 \leq x \leq 1$). The equation 2.2.4 can be solved by using the following boundary

conditions and constitutive equations to obtain the normalized overpressure in sediment zone:

1. At the top of sediment layer normalized overpressure is zero.

$$P_n(\tau, x = 1) = 0 \quad (2.2.5)$$

2. At the bottom of the settling column velocity of solids and liquid is zero. Thus, no-slip condition holds true for both solids and liquid and the gradient of normalized overpressure at the bottom of the settling column can be defined as:

$$\frac{\partial P_n}{\partial x}(\tau, x = 0) = 0 \quad (2.2.6)$$

Equation 2.2.4 has three variables P_s , e' and K . The constitutive equations used to solve the Wangen sediment-consolidation equation are:

$$e' = \alpha \frac{\phi_0}{(1 - \phi_0^2)} \exp(-\alpha P_s) \quad (2.2.7)$$

and

$$K = K_0 \exp[a'(\phi - \phi_0)] \quad (2.2.8)$$

Where, ϕ is porosity, ϕ_0 is porosity at zero effective stress, K_0 is the surface permeability and a' is a dimensionless empirical parameter.

The effective vertical stress P_s can be given in terms of normalized effective stress as:

$$P_s = P_{sn} \alpha \quad (2.2.9)$$

The normalized effective vertical stress P_{sn} can be defined in terms of normalized overpressure as:

$$P_{sn} = \tau(1 - x - P_n) \quad 0 \leq P_s \leq 1 \quad (2.2.10)$$

In 1997 Fox and Berles proposed a dimensionless piecewise-linear (piecewise iterative) finite difference model for the one dimensional large strain consolidation. The solutions of the model are independent of the initial height of the consolidating layer and the absolute value of the hydraulic conductivity. In the model, a correction factor is applied on the Terzaghi's theory in order to consider the effect of vertical strain and maximum excess pore pressure on the compressive layer. But in this approach, the results might be deviated by the cumulative effect of small incremental errors. Moreover, the models mentioned above did not integrate the processes of sedimentation and consolidation.

In 2003 Gutierrez proposed the model which integrated sedimentation and consolidation processes. He proposed compaction and permeability models with the following constitutive equations, in order to fit in Wangen model:

$$e = e_0 - \lambda \ln(P_{sn}) \quad (2.2.11)$$

$$K = K_o \exp[b(e - e_0)] \quad (2.2.12)$$

Where, e is void ratio, e_0 is void ratio corresponding to reference stress $P_{s \max}$, λ is a parameter defined in term of compression index C_{co} and equal to $0.434 C_{co}$, P_{sn} is normalized effective stress and defined as $P_{sn} = \frac{P_s}{P_{s \max}}$, and b is permeability index.

Moreover, in order to fit in Wangen's model, Gutierrez defined new normalized overpressure as:

$$P_n = \frac{P}{P_{s \max}} \quad (2.2.13)$$

Where, P is overpressure and defined as:

$$P = P_T - \rho_w gh \quad (2.2.14)$$

Where, P_T is total pore pressure and h is height of the water column.

And new reference time (time taken by settling solids with the settling rate of w to achieve maximum vertical effective stress, $P_{s \max}$) as:

$$t_0 = \frac{P_{s \max}}{(\rho_s - \rho_w)gw} \quad (2.2.15)$$

Chapter 3: Batch Thickening – Computer Simulation

Section 3.1: MATLAB programming for batch thickening process

The present study consists of the development of a MATLAB version of FORTRAN computer code, developed by Papanicolaou (1992), to solve the governing finite strain consolidation equation of second order non-linear transit partial differential equation of parabolic type. Papanicolaou used the basic concepts of static force balance, Darcy-Shirato law, volume balance equation, and continuity equation for liquid phase, to formulate the consolidation phenomenon. He formulated a partial differential equation called the diffusive-convective equation given as:

$$\frac{d\varepsilon_s}{dP_s} \frac{\partial P_s}{\partial t} + \frac{K\varepsilon_s}{\mu} \frac{\partial^2 P_s}{\partial x^2} + \frac{1}{\mu} \frac{d[K\varepsilon_s]}{dP_s} \left(\frac{\partial P_s}{\partial x} \right)^2 + \frac{g\Delta\rho}{\mu} \frac{d[K\varepsilon_s^2]}{dP_s} \frac{\partial P_s}{\partial x} = 0 \quad (3.1.1)$$

Where, P_s is vertical effective stress, K is permeability, ε_s is solidity, μ is liquid viscosity, x is height, t is time, g is gravitational constant, $\Delta\rho = \rho_s - \rho$ with ρ_s and ρ are the solids and liquid mass density respectively. The equation 3.1.1 contained three unknowns P_s , K and ε_s . Papanicolaou used constitutive equations (valid for P_s value up to 5-10 atm) for K and ε_s , in term of P_s , given by Tiller and Leu (1981) as:

$$\varepsilon_s = \varepsilon_{s0} (1 + aP_s)^\beta \quad (3.1.2)$$

and

$$K = K_0 (1 + aP_s)^{-\delta} \quad (3.1.3)$$

Where, β , δ and a are empirical parameters represent the degree of compactibility of solids. The parameters β and δ are known as compressibility coefficients and their values varies in the range of 0.0-0.05 and 0.0-0.25 respectively. In 1986, Tsai obtained empirical relationship for β and δ (based on experiments with 29 different materials) as:

$$\beta = 3.08(0.65 - \varepsilon_{s0})^{4.0} \quad (3.1.4)$$

and

$$\delta = 5.19(0.65 - \varepsilon_{s0})^{2.68} \quad (3.1.5)$$

Using constitutive equations 3.1.2 and 3.1.6 in equation 3.1.1, Papanicolaou formulated the governing consolidation equation with one unknown, P_s , as :

$$\frac{\partial P_s}{\partial t} - \frac{K_o(1 + aP_s)^{1-\delta}}{a\mu\beta} \frac{\partial^2 P_s}{\partial x^2} - \left(\frac{\partial P_s}{\partial x} \right) \frac{g\Delta\rho\varepsilon_o K_o(2\beta - \delta)(1 + aP_s)^{(\beta-\delta)}}{\mu\beta} - \frac{K_o(\beta - \delta)(1 + aP_s)^{-\delta}}{\mu\beta} \left(\frac{\partial P_s}{\partial x} \right)^2 = 0 \quad (3.1.6)$$

The equation 3.1.6 can be solved using two boundary conditions and one initial condition. The two boundary conditions are:

1. At the top of sediment layer effective pressure is zero.

$$P_s = 0 \quad (3.1.7)$$

2. At the bottom of the settling column velocity of solids and liquid is zero. Thus, no-slip condition holds true for both solids and liquid and the gradient of effective pressure at the bottom of the settling column can be defined (Papanicolaou, 1992, Eq. 5.40) as:

$$\left(\frac{\partial P}{\partial x}\right)_{x=0} = g(\rho_s - \rho_w)\epsilon_{s0}(1 + aP_s)^\beta \quad (3.1.8)$$

In second boundary condition effective pressure is unknown thus Newton-Raphson method is used to obtain the effective pressure by using the known value of effective pressure at the start of the time step (initial condition).

$$P_s = f(x) \quad (3.1.9)$$

The solution of the governing equation used the Petrov-Galerkin method to construct the weak form of the governing equation, the Crank-Nicolson method to solve the time derivative part of the governing equation, and the Newton-Raphson method to solve the nonlinear part of the governing equation (Papanicolaou, 1992).

Moreover, Papanicolaou used the following equation to calculate the settling amount of solids from the suspension region

$$\Delta x = u \Delta t \quad (3.1.10)$$

Where, u is the solids settling velocity and given by the empirical relationship $u = f(\phi)$ (Kynch, 1952), and Δt is the time step. But the Papanicolaou simulation does not give continuous results of finite strain batch thickening process in one run of the FORTRAN program. The present work integrates settling and consolidation processes in order to get the continuous results from start to end of finite strain batch thickening process in one run of MATLAB program.

Section 3.2: A Numerical method to calculate the solid concentration at the suspension-sediment interface

The present work also proposes a numerical method to calculate the solids concentration in suspension region at suspension-sediment interface. This method is simpler in comparison to the graphical approach mentioned in the literature. Referring to Figure 3.2.1,

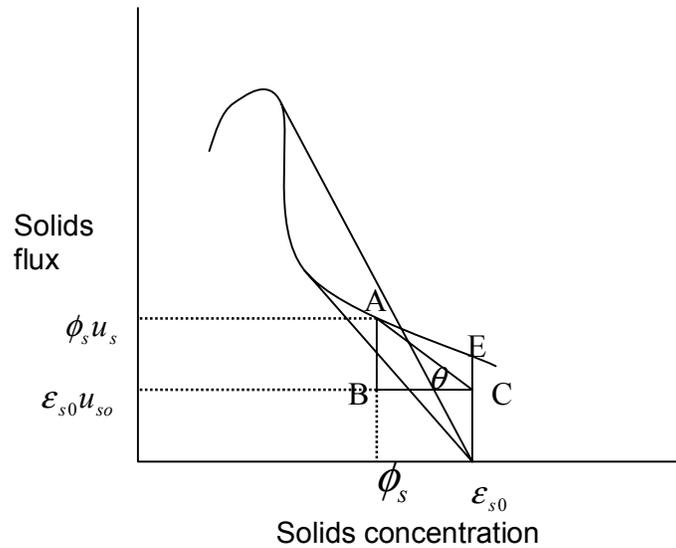


Figure 3.2.1 - Plot between solids flux and solids concentration

The tangent at the point A ($\tan \theta$) in the flux curve can be defined as:

$$\tan \theta = \frac{AB}{BC} = \frac{d(u_s \phi_s)}{d\phi_s} = \frac{u_s \phi_s - \epsilon_{s0} u_{s0}}{\epsilon_{s0} - \phi_s} \quad (3.2.1)$$

Papanicolaou (1992) obtained the relationship (Eq. 3.2.2) between solids settling velocity and solids concentration by the application of curve fitting over the experimental data given by Tiller and Chu (1981).

$$u_s = b_0 + b_1\phi_s + b_2\phi_s^2 \quad (3.2.2)$$

Where, b_0 , b_1 , and b_2 are coefficients obtained from the least square method in curve fitting of experimental data. Using the relationship 3.2.2 in equation 3.2.1 and after solving mathematically, the following cubic equation in term of solids concentration formulated.

$$4b_2\phi_s^3 + (3b_1 - 3b_2\varepsilon_{s0})\phi_s^2 + (2b_0 - 2b_1\varepsilon_{s0})\phi_s - \varepsilon_{s0}(b_0 + u_{s0}) = 0 \quad (3.2.3)$$

The equation 3.2.3 can be solved by the Newton-Raphson iteration method to obtain solids concentration in suspension region.

Section 3.3: A Numerical method to calculate the batch curve

The present work also proposes a numerical method to obtain the batch curve by considering the variation of solid concentration in the suspension region. The height of suspension-supernatant liquid interface can be calculated by applying the mass conservation principle. For applying the mass balance equation, the concentration of solids from the bottom of the cylinder to the top of the suspension layer should be known. At any instant in time, the concentration of solids in the consolidated region can be easily calculated by using the concepts of consolidation. At the same instant in time, the concentration of solids at any particular height in the suspension region is equal to the concentration of characteristic reaches at that height. Thus, it is important to know the number of characteristics present in the suspension region at any instant in time and

the existing period of characteristic (time taken by the characteristic to reach the supernatant liquid phase from the point of its generation). Referring to Figure 3.3.1, the mass balance equation at time t_1 can be given as:

$$\phi_{s0}H_0 = \int_0^{L_{t1}} \epsilon_s dx + \int_{L_{t1}}^{H_1} \phi_s dx \quad (3.2.4)$$

Where, ϕ_{s0} is initial concentration, H_0 is initial height, and ϵ_s and ϕ_s are concentration in consolidation and suspension region respectively. Thus, the volume of solids per unit cross sectional area existing in the suspension region at the time of generation of a characteristic is given by $\int_{L_{t1}}^{H_1} \phi_s dx$. This volume of solids crosses the characteristic by the time it reaches the supernatant liquid phase (Diplas & Papanicolaou, 1997).

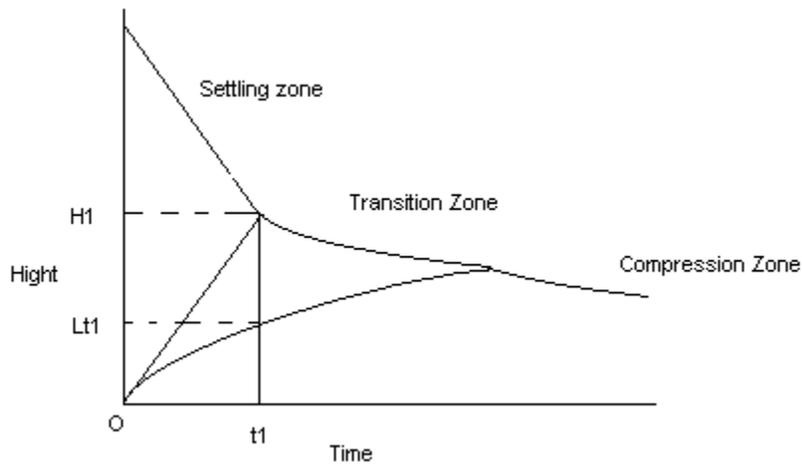


Figure 3.3.1 - Mass conservation in batch thickening curve

Referring to Figure 3.3.2, the volume of solids per unit cross sectional area that moves with the characteristic during its 'existing period', $t_2 - t_1$, is given by,

$$\phi_{s1}(v_{s1} + u_{s1})(t_{n1} - t_1) + \phi_{s1}(v_{s1} + u_{s1})(t_{n2} - t_{n1}) + \dots + \phi_{s1}(v_{s1} + u_{s1})(t_2 - t_{nn})$$

or,

$$\phi_{s1}(v_{s1} + u_{s1})(t_2 - t_1)$$

Where, v_{s1} and u_{s1} are the velocity of the characteristic and the solids settling velocity respectively at time t_1 . Thus, the 'existing period' of the characteristic is given by,

$$t_2 - t_1 = \frac{\phi_{s0} H_0 - \int_0^{L_{t1}} \epsilon_s dx}{\phi_{s1}(v_{s1} + u_{s1})} \quad (3.2.5)$$

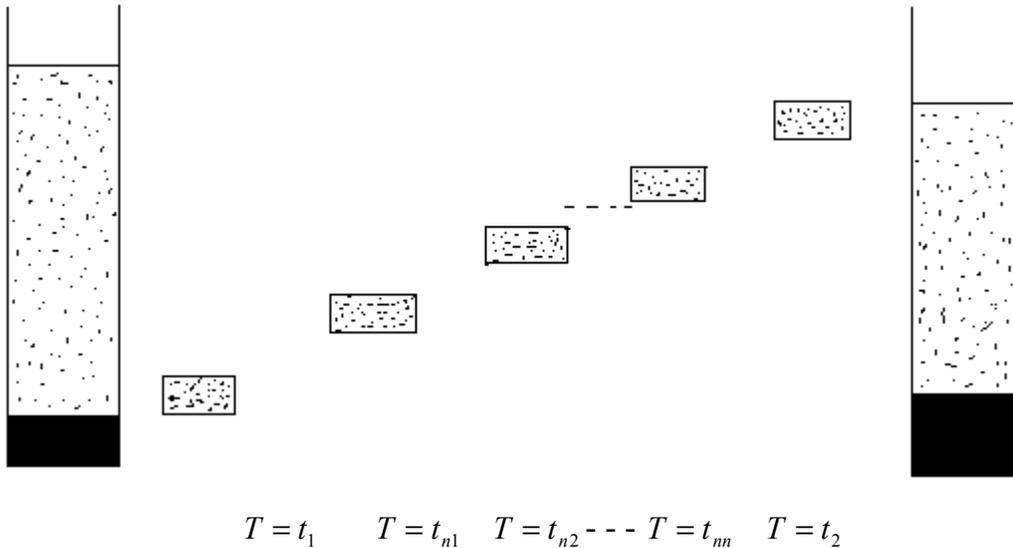


Figure 3.3.2 - Movement of solids with the characteristic

Now, referring to Figure 3.3.3, Let at any time t_a , three characteristics exist in the suspension region. The concentration of first and second characteristics influences the height DC. The concentration of second and third characteristics influences the height CB. Height ED is influenced by the initial concentration and concentration of first characteristic, while BA is the height of consolidation region. Now applying the mass balance equation:

$$\phi_{s0}H_0 = \int_0^{AB} \epsilon_s dx + (BC)(\phi_{s2} + \phi_{s3})/2 + (CD)(\phi_{s1} + \phi_{s2})/2 + (H - AB - BC - CD)(\phi_{s0} + \phi_{s1})/2 \quad (3.2.6)$$

Thus,

$$H = \frac{2\phi_{s0}H_0 - 2 \int_0^{AB} \epsilon_s dx - (BC)(\phi_{s2} + \phi_{s3}) - (CD)(\phi_{s1} + \phi_{s2}) + (AB + BC + CD)(\phi_{s0} + \phi_{s1})}{\phi_{s0} + \phi_{s1}} \quad (3.2.7)$$

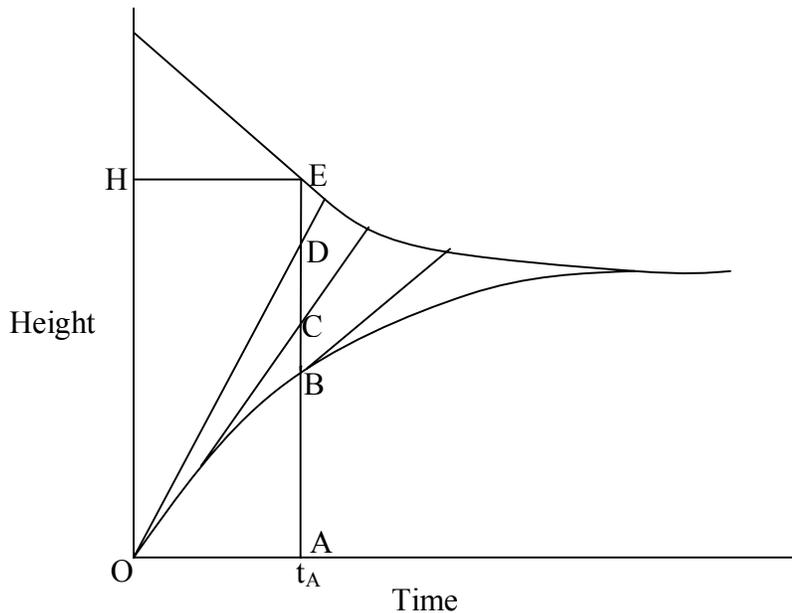


Figure 3.3.3 – Batch curve calculation using the concept of mass conservation

Section 3.4: Calculation of solids concentration in settling column:

At any instant in time, the settling column can be divided in three regions.

1) Supernatant liquid region. 2) Suspension region. 3) Consolidation region. Referring to figure 3.3.3, the solids concentration in supernatant region (H_0H_A) is considered to be zero as negligible amounts of solids are present in this region. Solids concentration in suspension region (AG) is varying and the value of solids concentration in suspension region at any particular height and at any time instant is given by the concentration of a characteristic present at that height at that particular time instant. The procedure to obtain the concentration of a characteristic is mentioned in section 3.2. For small time step, the difference between the heights of two consecutive characteristics present in suspension region is small. Thus, it can be assumed that the height weight average of concentrations of two consecutive characteristics is the solids concentration between the heights of those two characteristics. For example, the solids concentration in height AE is the height weight average of concentrations of characteristics OA and MB as shown in Fig. 3.3.4 and which can be calculated as:

$$\tan \theta = (C_E - C_A) / (H_A - H_E) \quad (3.2.8)$$

and

$$C_P = C_A + \tan \theta (H_A - H_P) \quad (3.2.9)$$

Where, C_A , C_E and C_P are the concentrations at heights H_A , H_E and H_P in the settling column respectively.

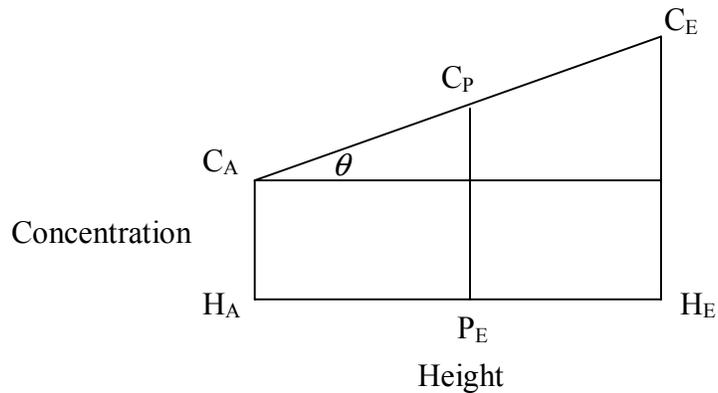


Fig 3.3.4. Calculation of solids concentration in suspension region

Similarly, the solids concentration for the heights EF and FG can be obtained.

In consolidation region, GK, the solids concentration can be calculated by Eq. 3.1.2. Moreover, The present work also simulates the large strain consolidation model proposed by Gutierrez (2003).

Section 3.5: Subroutines of computer code

A) Large strain model subroutines:

- 1) **large_strain:** This is the main program for the simulation of large strain consolidation model. It contains the following subroutines (2-7).
- 2) **consol_Is:** It evaluates the effective stress and solids concentration in the consolidated layer, depth of consolidated layer and the velocity of upward moving liquid from consolidated layer.
- 3) **efunc:** It evaluates the void ratio in consolidated layer.
- 4) **dfunc:** It evaluates the compressibility in consolidated layer.
- 5) **pfunc:** It evaluates the permeability in the consolidated layer.
- 6) **thomas:** It gives the solution of the trigonal system of equations.

7) **sediment_Is**: It evaluates the height of batch curve and L-curve, velocity of characteristics and their time to reach the supernatant-suspension interface (batch curve).

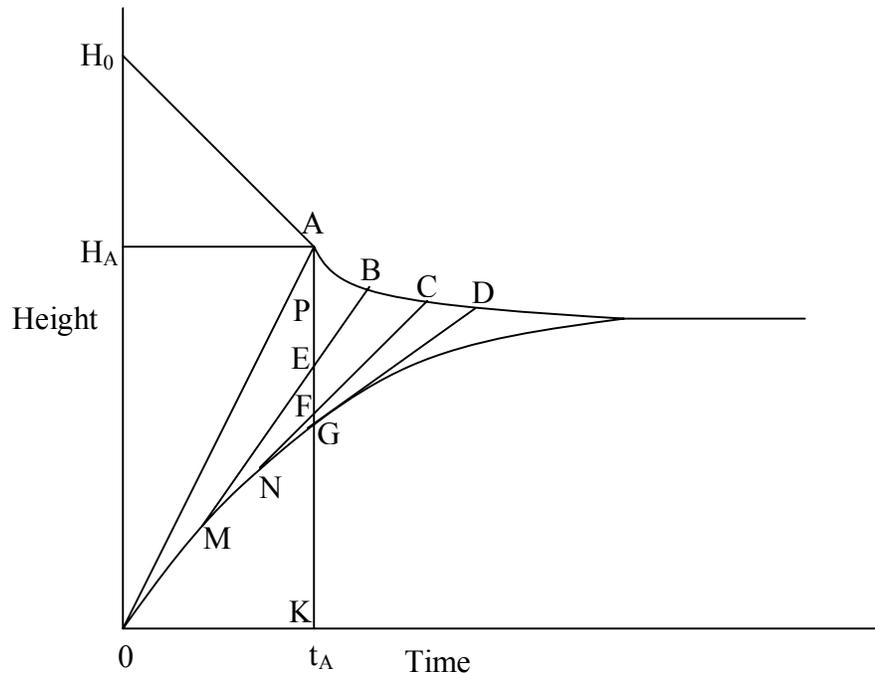


Figure 3.3.5 – Solids concentration in settling column using the concept of characteristics

B) Small strain model subroutines:

- 1) **finite_strain**: This is the main program for the simulation of finite strain model. It contains the following subroutines (2-7).
- 2) **consol_fs**: It evaluates the effective stress and the solids concentration in the consolidated layer, the depth of consolidated layer and the velocity of upward moving liquid from consolidated layer.

- 3) **connectivity**: It defines the element connections in the finite element mesh.
- 4) **oned_gauss**: It defines the gauss points for the element in the finite element mesh.
- 5) **coefmatrix**: It evaluates the stiffness matrix, mass matrix and source vector for the element in finite element mesh.
- 6) **solver**: It gives the solution for unsymmetrical system of equations.
- 7) **sediment_fs**: It evaluates the height of batch-curve and L-curve, velocity of characteristics and their time to reach the supernatant- suspension interface.

C) Subroutines for large and small strain models:

- 1) **newraps**: It evaluates the concentration of a characteristic.

Section 3.6: Algorithm and Flowchart of simulations

A) Algorithm for small strain computer program:

Step 1: Input the data corresponding to the characteristics of column, solids, fluid, and mixture of solids and fluid. For example column height, specific gravity of solids and fluid, initial concentration of the mixture, and relation between solids settling velocity and corresponding local concentration etc. are some initial data (Table 4.1.1)

Step 2: Calculate the height of initial settling solids, with very small initial time step, using initial the settling velocity of the solids corresponding to initial solids concentration. Calculate the height of batch curve (height of supernatant-suspension interface) corresponding the initial settling height.

Step 3: Calculate the properties of first and second characteristics (isoconcentration lines). For example, concentration associated with characteristics using 'newraps' subroutine, velocity of characteristics, time of generation of characteristics and time taken by characteristics to reach the supernatant-suspension interface.

Step 4: Calculate the decrease in height of settling layer using the subroutine 'consol_fs'. Solve the Non linear partial differential equation of parabolic type (Eq. 3.1.6) for the effective pressure in consolidation region using the subroutines 'consol_fs', 'connectivity', 'oned_gauss', 'coefmatrix' and 'solver'.

Step 5: Calculate the void ratio, compressibility and permeability in consolidation region.

Step 6: Using the subroutine 'sediment_fs', calculate the height of new settling solids and thus, height of L-curve (sediment-suspension interface). Calculate the properties of new characteristic, as in step 3, generating from the top of this sediment-suspension interface. Also, calculate the height of batch curve using mass conservation principle.

Step 7: Repeat step 4, 5 and 6 until the difference between batch curve and L-curve is grater than 5% of the initial height.

B) Algorithm for large strain computer program:

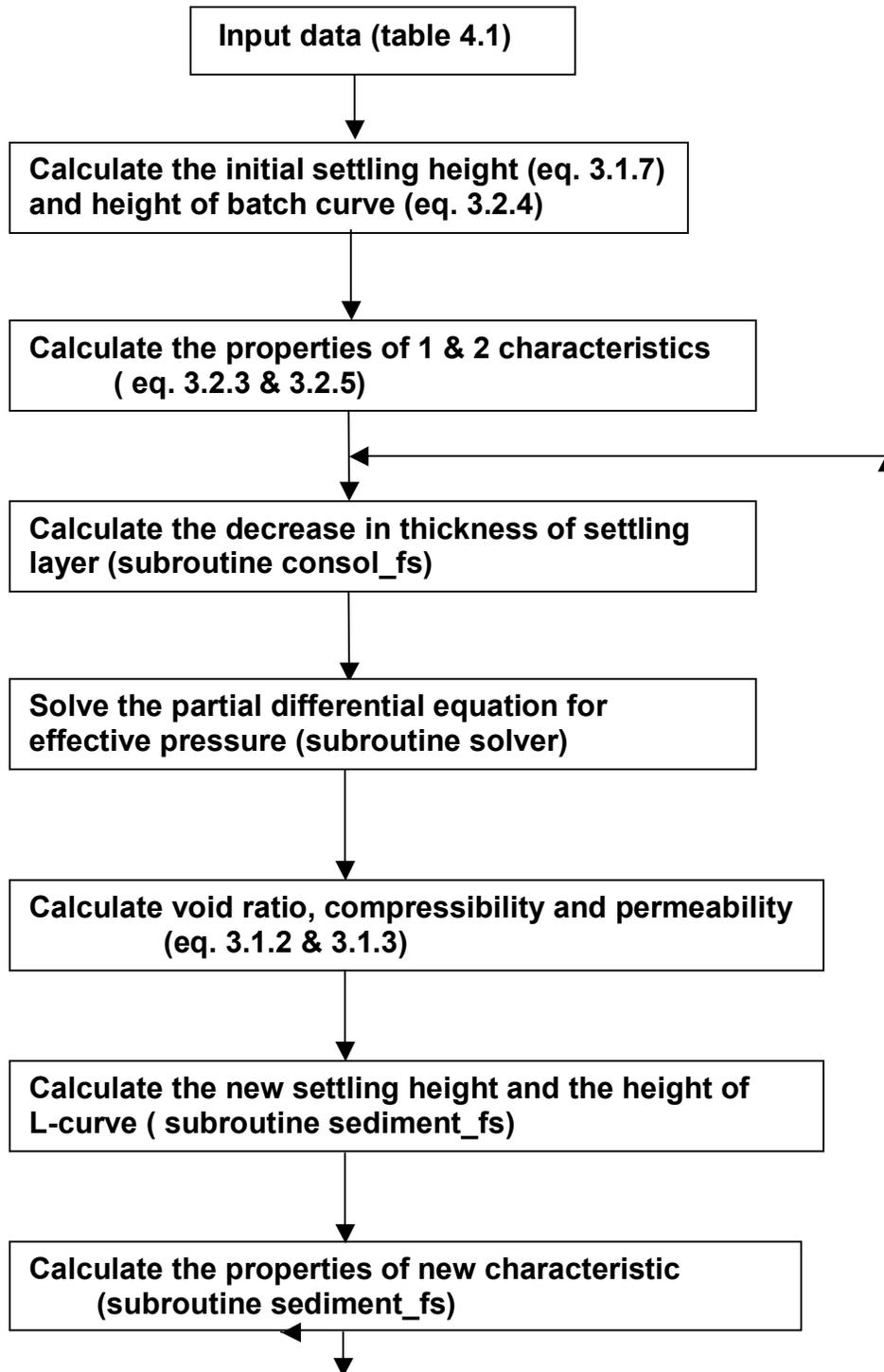
Step 1: Follow steps 1, 2 and 3 of algorithm for finite strain computer program.

Step 2: Calculate the effective pressure in the consolidation region by solving the trigonal system of equations using the subroutine thomas, and calculate the decrease in height of settling layer using the subroutine consol_ls.

Step 3: Calculate the void ratio, compressibility and permeability using the subroutine efunc, dfunc and pfunc respectively.

Step 4: Follow steps 6 and 7 of the algorithm for large strain computer program with subroutine sediment_ls corresponding to the subroutine sediment_fs of finite strain model.

C) Flow charts for computer programs: Following flowcharts, Figure 3.5.1 and Figure 3.5.2, are based on the algorithm of the computer program for small strain model and large strain model for batch thickening respectively.



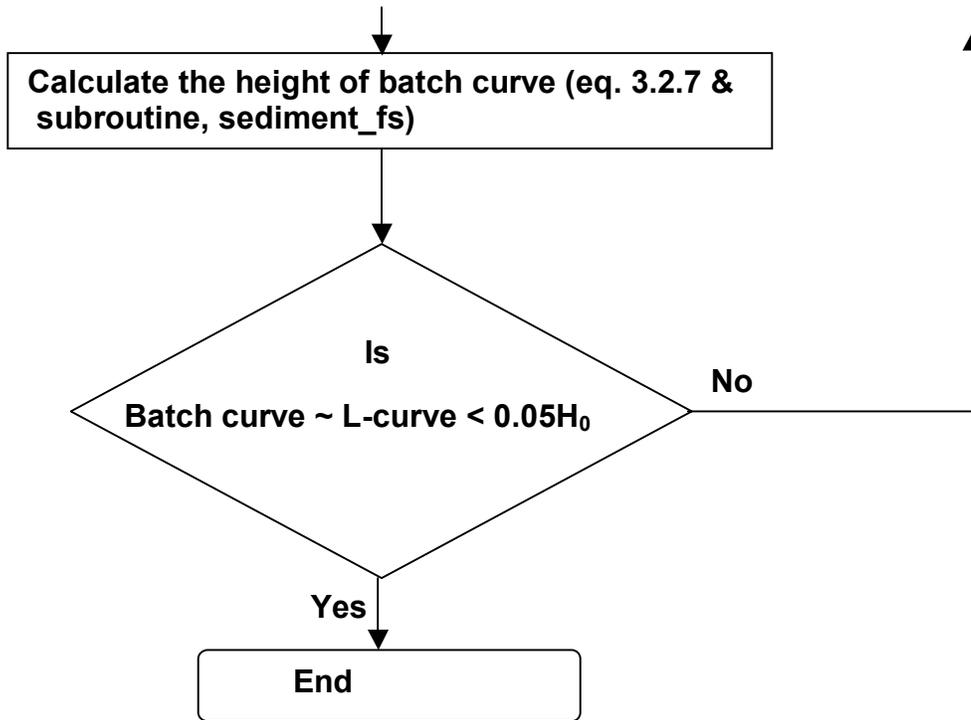
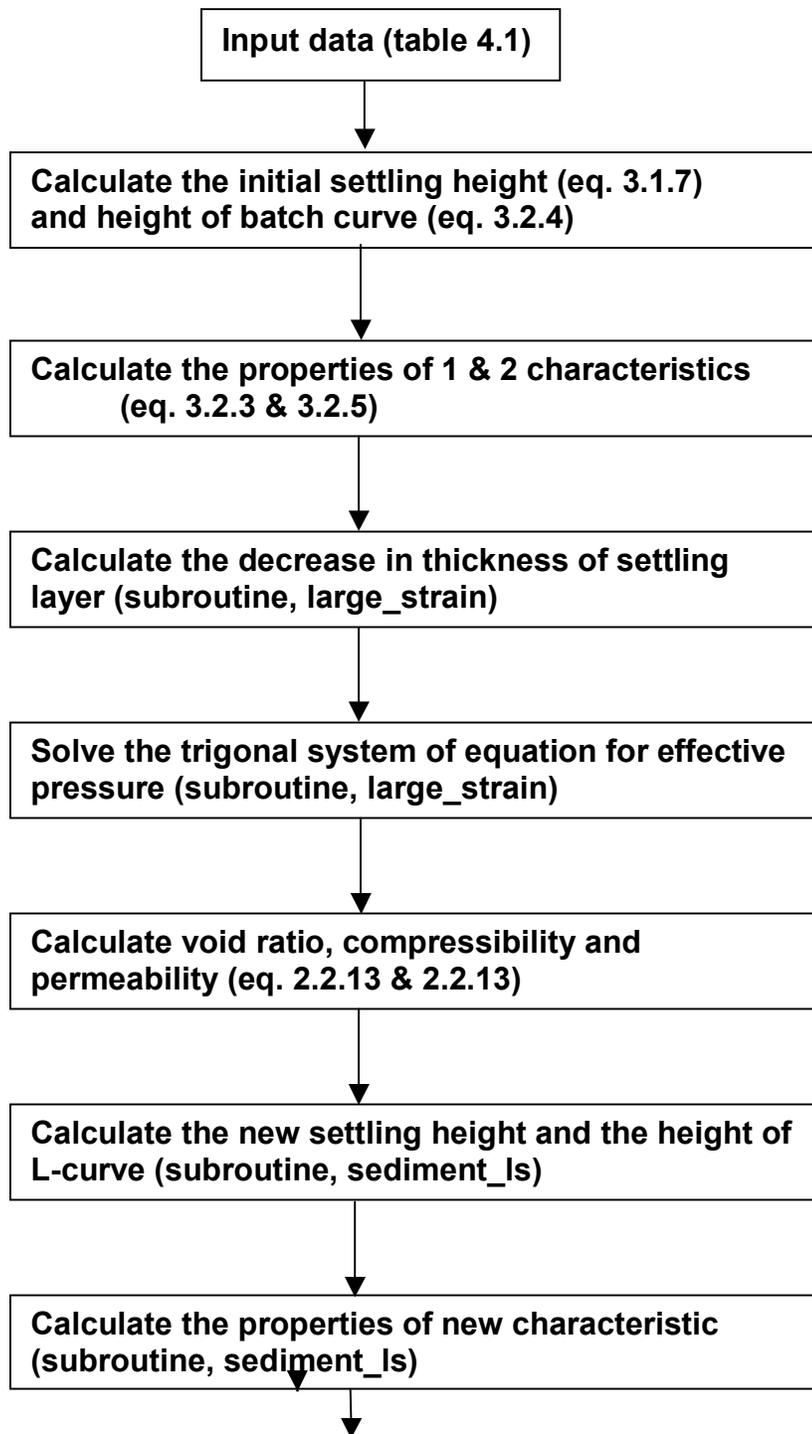


Figure 3.6.1 - Flow chart of small strain computer program



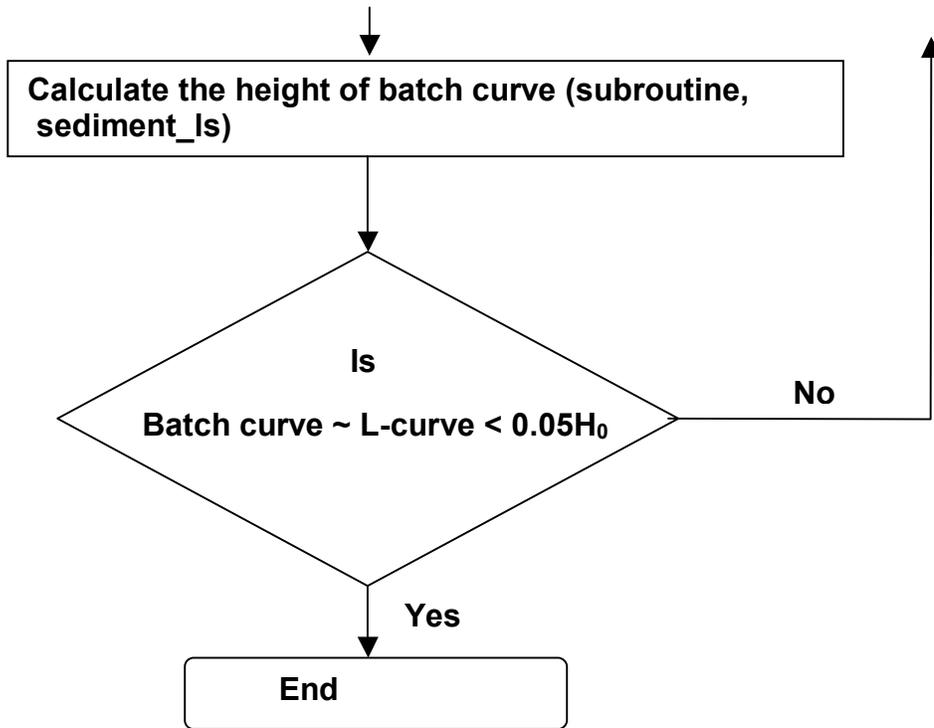


Figure 3.6.2 - Flow chart of large strain computer program

CHAPTER 4: Inputs and Outputs

Section 4.1: Input data

The present simulation has been run for the data provided by Tiller (1981) and Diplas & Papanicolaou (1997) for the attapulgate slurry (Table 4.1.1). The adjustment in null stress solidosity (ε_{s0}) has been made according to empirical relationship between solids velocity and its local concentration.

Table 4.1.1- Input data for small strain and large strain simulations

<i>Parameter</i>	<i>Symbol</i>	<i>Value</i>
Initial suspension height	H_0	0.4 (m)
Density of solids	ρ_s	2,300 (Kg/ m ³)
Density of liquid	ρ_w	1,000 (Kg/ m ³)
Initial solid volume ratio in suspension region	ϕ_{s0}	0.03
Initial solid settling velocity	u_{s0}	0.00003 (m/s)
Initial null stress volume fraction of solids	ε_{s0}	0.075
Initial intrinsic permeability	K_0	10^{-14} (m/s)
Dynamic viscosity	μ	0.001 (kg/m-s)
Relation between solid settling velocity (u_s) and solid volume ratio (ϕ_s)		$u_s = b_0 + b_1\phi_s + b_2\phi_s^2$

Curve fitting coefficient	b_0	0.000119711
Curve fitting coefficient	b_1	-0.00428381
Curve fitting coefficient	b_2	0.0418934
Empirical parameter	a	0.003
Empirical parameter	β	0.0361
Empirical parameter	δ	1.234
Parameter	α	0.00653
Maximum time for large strain Simulation	TMAX	20000

Section 4.2: Outputs of simulations

The results obtained by small strain model and large strain model are shown in Figures 4.2.1 - 4.2.9 and Figures 4.2.10 – 4.2.13 respectively. These results contain:

- a) Variation of batch curve and L-curve with time.
- b) Variation of effective pressure in consolidation region with height at various time steps.
- c) Variation of porosity in consolidation region with height at various time steps.
- d) Variation of permeability in consolidation region with height at various time steps.
- e) Variation of solids volume ratio in consolidation region with height at various time steps.
- f) Variation of solids volume ratio in column at various time steps.
- g) Variation of solids settling velocity with time.
- h) Variation of solids flux with solids concentration.

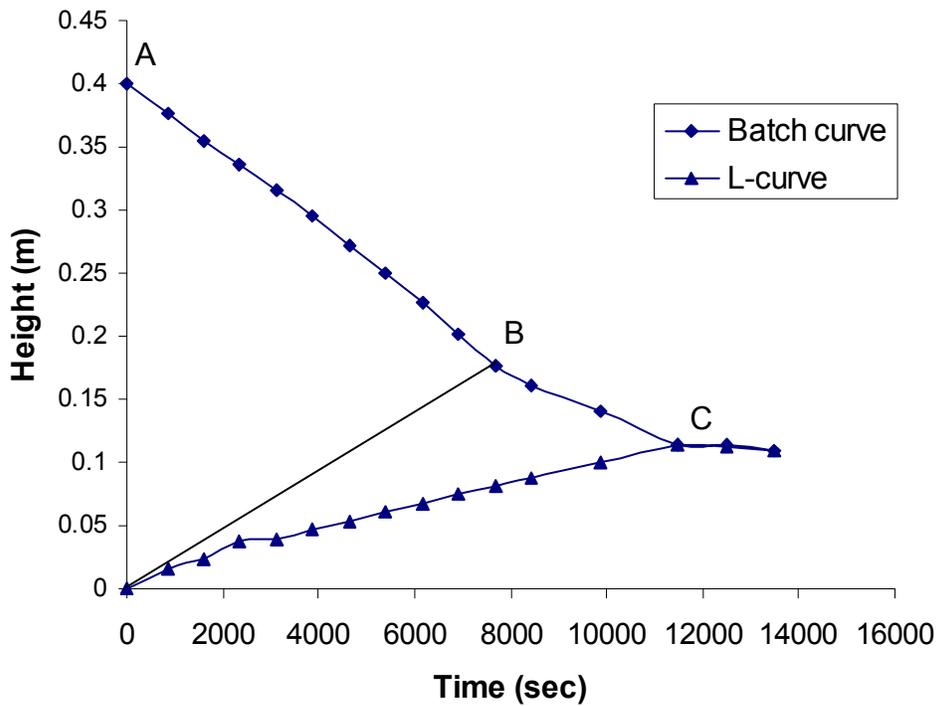


Figure 4.2.1 - Variation of batch curve and L-curve with time in small strain batch thickening simulation

Figure 4.2.1 shows the variation of batch curve and L-curve with time. The first characteristic, OB, generated from the bottom of settling column reaches the supernatant-suspension interface after 8000 seconds (approximately) and at the height 0.1700 m (approximately). In this Figure AB represents constant settling region and BC represents transition region.

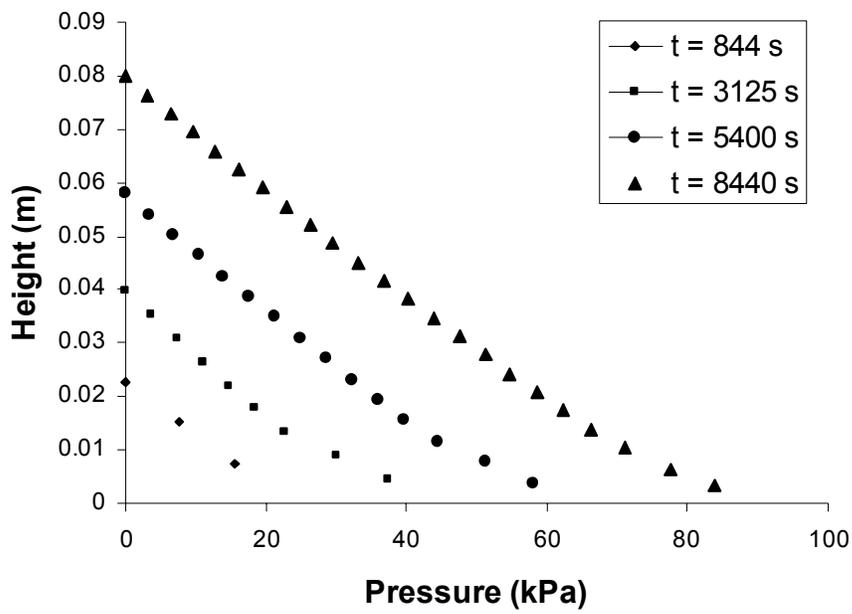


Figure 4.2.2 - Variation of effective pressure in consolidation region with height in small strain batch thickening simulation

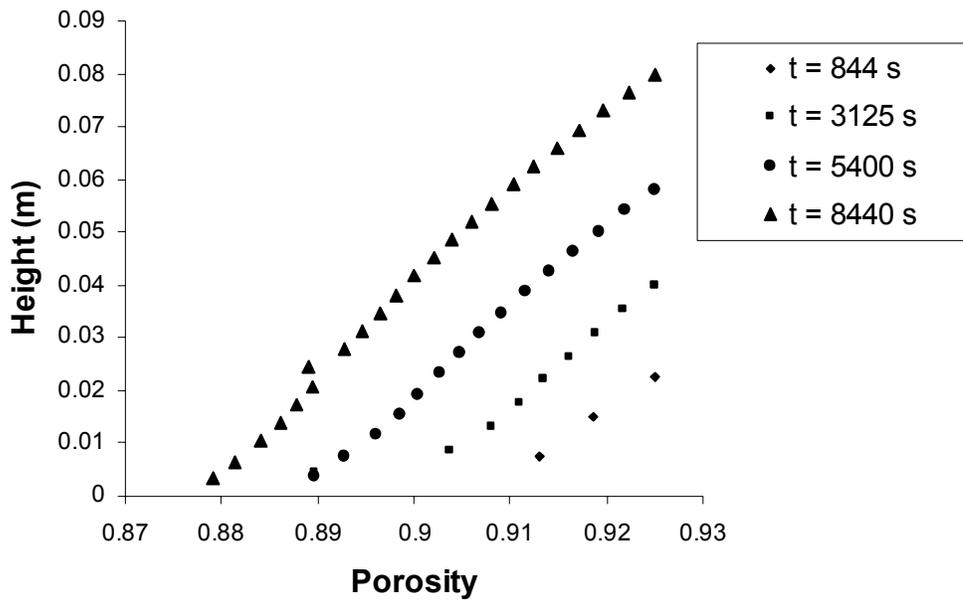


Figure 4.2.3 - Variation of porosity in consolidation region with height in small strain batch thickening simulation

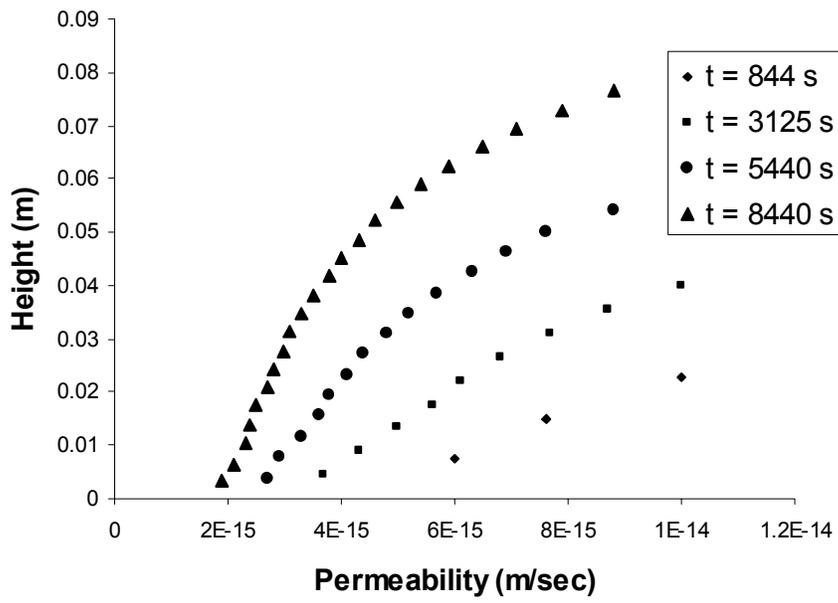


Figure 4.2.4 - Variation of permeability in consolidation region with height in small strain batch thickening simulation

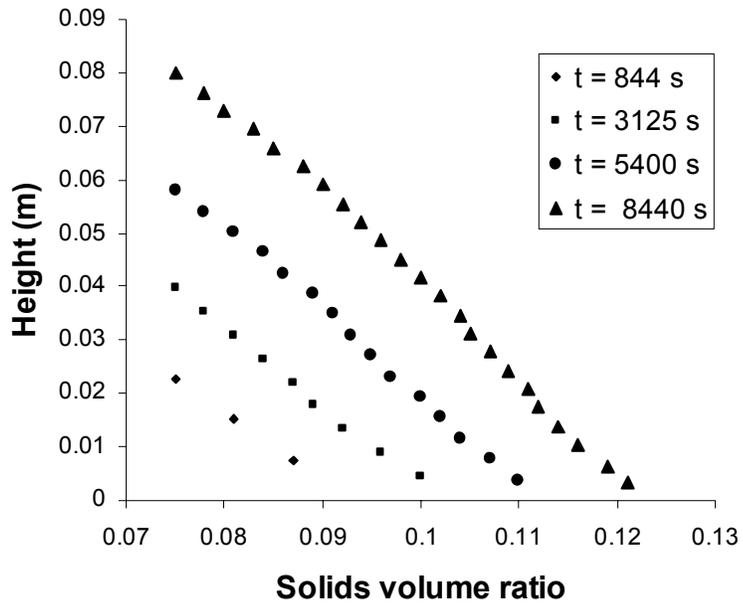


Figure 4.2.5 - Variation of solids volume ratio in consolidation region with height in small strain batch thickening simulation

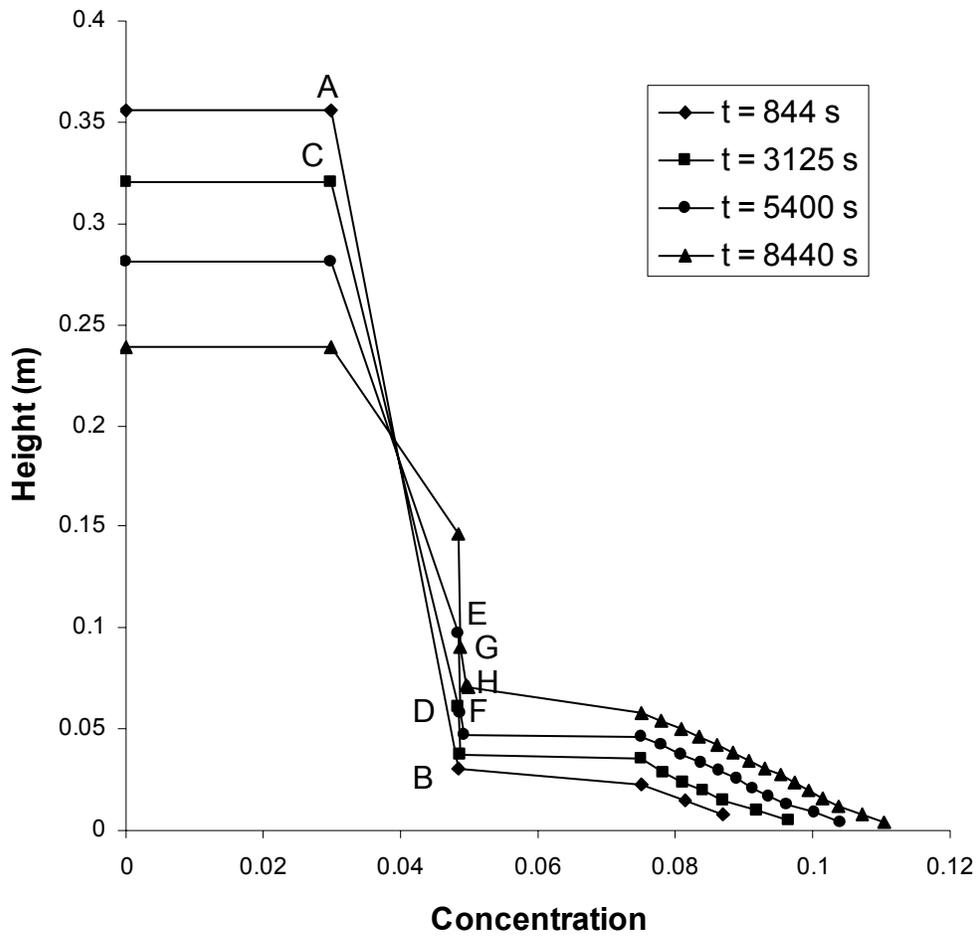


Figure 4.2.6 - Variation of solids concentration in column height in small strain batch thickening simulation

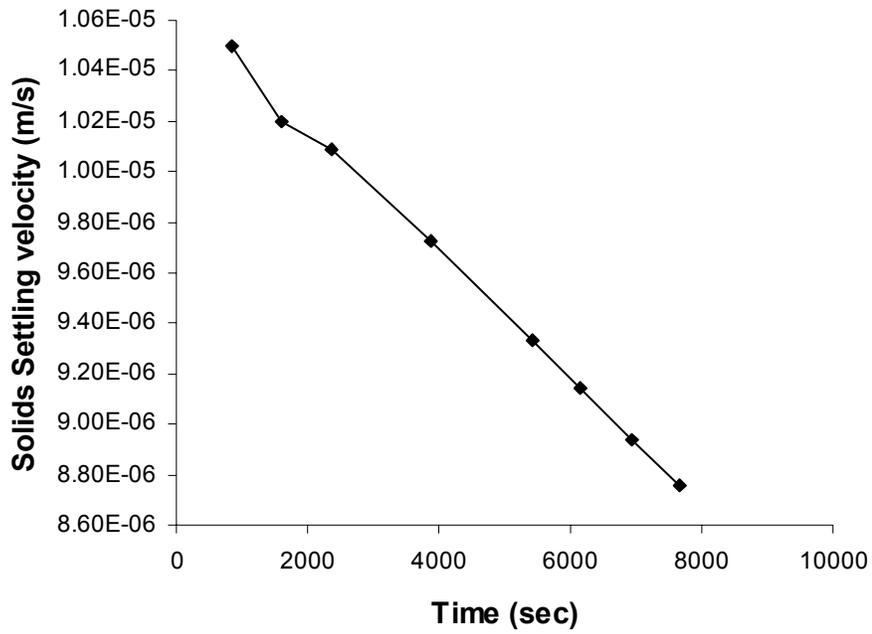


Figure 4.2.7 - Variation of solids settling velocity at sediment-suspension interface with time in small strain batch thickening simulation

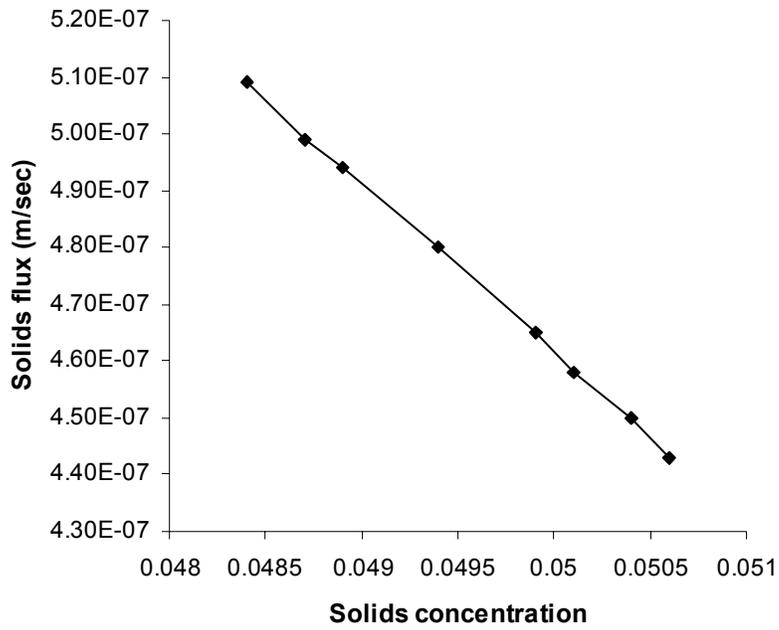


Figure 4.2.8 - Variation of solids flux with solids concentration at Sediment-suspension interface in small strain batch thickening simulation

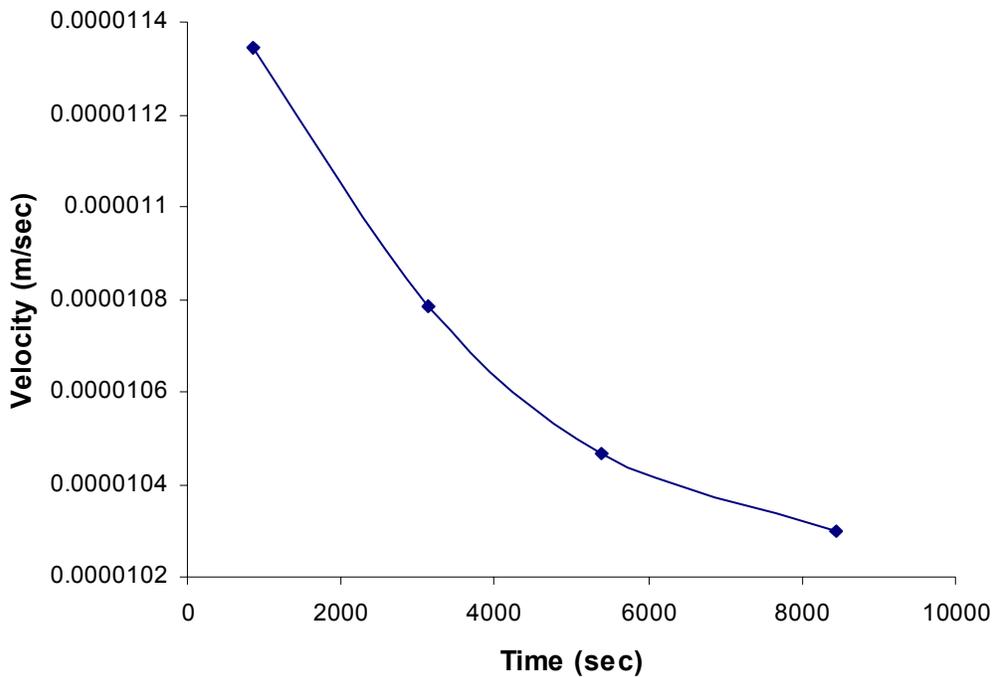


Figure 4.2.9 - Variation of solids settling velocity at 0.075 m from the bottom of settling column with time in small strain batch thickening simulation

Figure 4.2.9 shows variation of solids settling velocity with time at the height of 0.075m from the bottom of settling column. The variation of concentration of characteristics with time at this height is shown in Figure 4.2.6. The concentration at height 0.075m can be calculated by following the procedure mentioned in section 3.4 and velocity can be calculated by eq. 3.2.2.

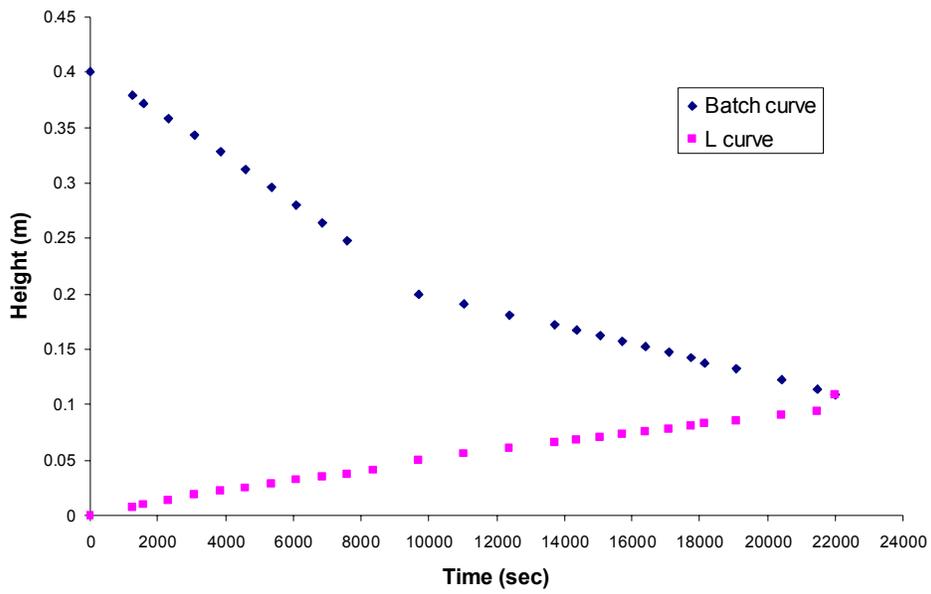


Figure 4.2.10 - Variation of batch curve and L-curve with time in large strain batch thickening simulation

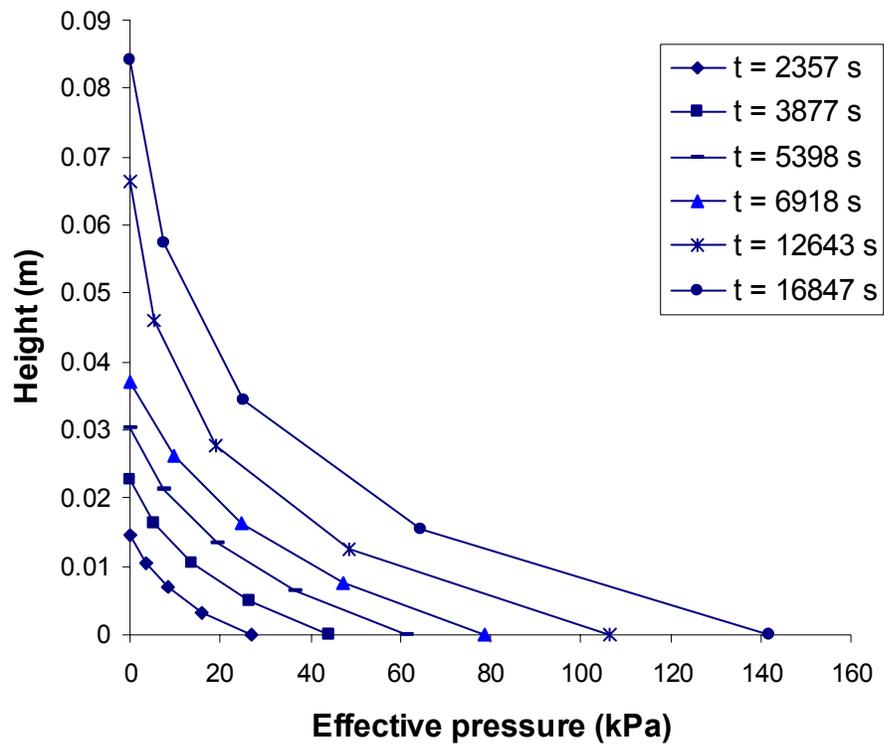


Figure 4.2.11 - Variation of effective pressure in consolidation region with height in large strain batch thickening simulation

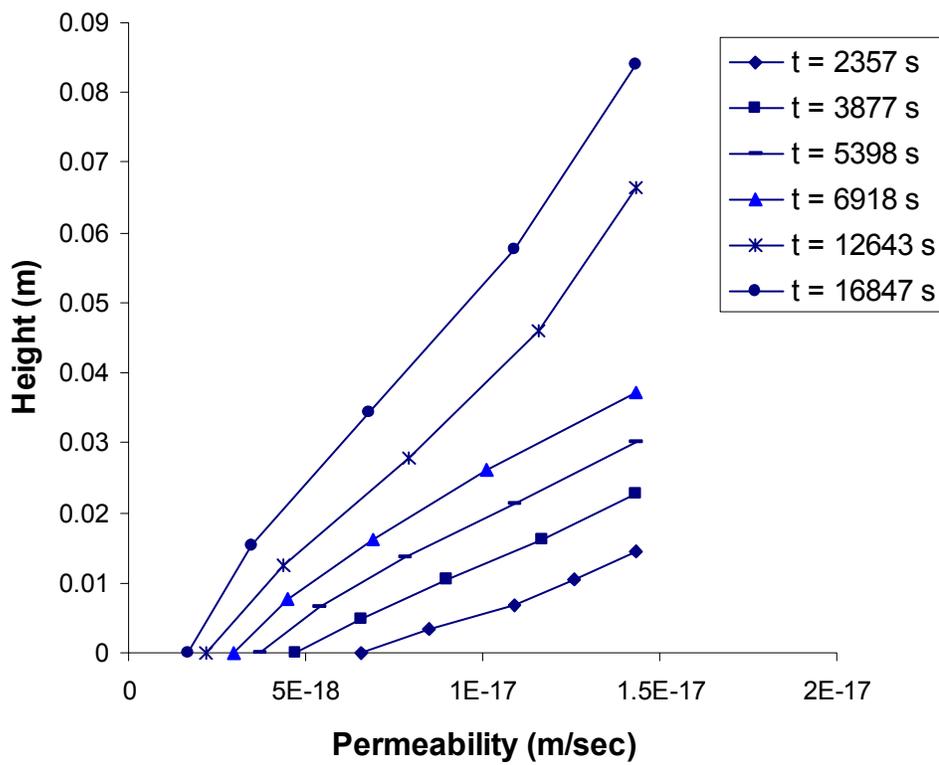


Figure 4.2.12 - Variation of permeability in consolidation region with height in large strain batch thickening simulation

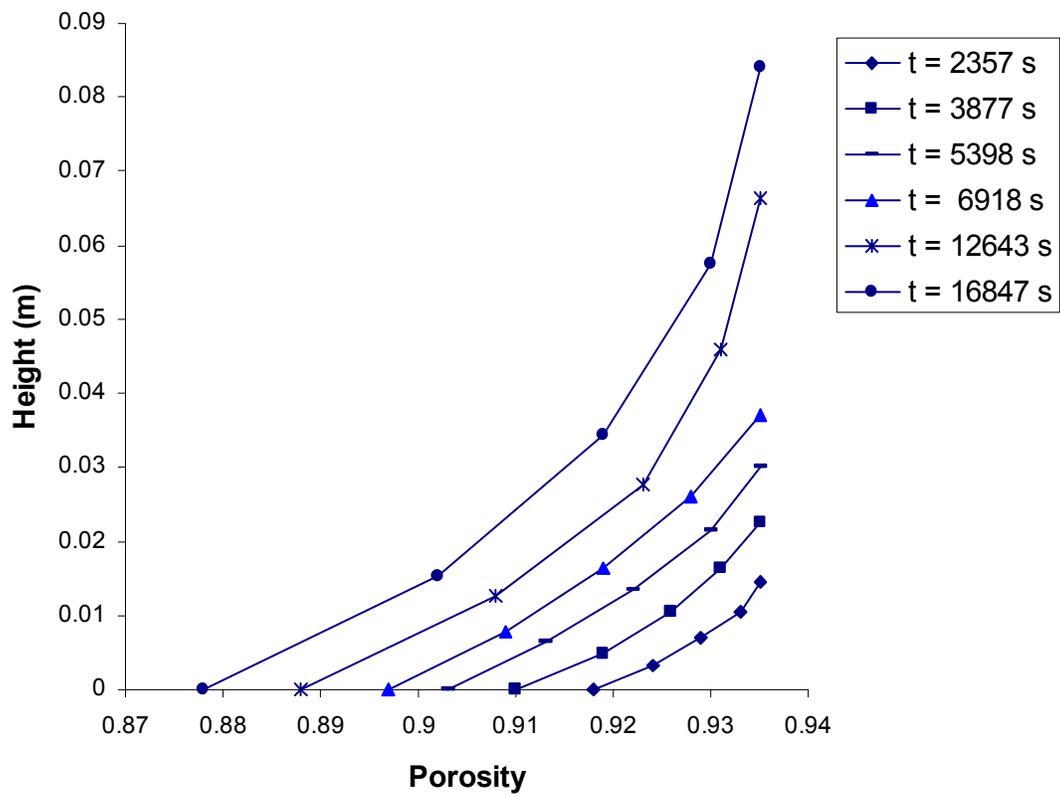


Figure 4.2.13 - Variation of porosity in consolidation region with height in large strain batch thickening simulation

Section 4.3 : Comparisions of results

The results of variation of batch curve and L-curve with time of present small strain simulation have been compared with the results of Diplas & Papanicolaou's (1997) small strain simulation (Figure 4.3.1), and the experimental data obtained by Tiller (1981) (Figure 4.3.2). Also, results of present simulations of small strain model (Papanicolaou, 1992) and large strain model (Gutierrez, 2003) have been compared (Figure 4.3.2).

Table 4.3.1 - Present small strain simulation's output data

SN	Time (sec)	Present simulation	
		Batch curve height (m)	L- curve height (m)
1	0	0.4000	0.0000
2	844	0.3761	0.0161
3	1604	0.3552	0.0241
4	2365	0.3363	0.0371
5	3125	0.3161	0.0393
6	3885	0.2942	0.0466
7	4645	0.2726	0.0538
8	5405	0.2498	0.0609
9	6166	0.2262	0.0677
10	6926	0.2020	0.0744
11	7686	0.1770	0.0809
12	8446	0.1615	0.0873
13	9866	0.1401	0.1002
14	11484	0.1148	0.1140
15	12484	0.1121	0.1121
16	13484	0.1094	0.1094
17	14484	0.1067	0.1067
18	15484	0.1042	0.1042
19	16484	0.1017	0.1017

Table 4.3.2 - Papanicolaou's small strain simulation's output data

SN	Time (sec)	Papanicolaou's simulation	
		Batch curve height (m)	L- curve height (m)
1	0	0.4000	0.0000
2	2000	0.3400	0.0400
3	4000	0.2750	0.0600
4	6000	0.2200	0.0900
5	7500	0.1800	0.0950
6	8500	0.1600	0.1020
7	9500	0.1450	0.1050
8	11000	0.1350	0.1200
9	12000	0.1200	0.1250
10	15000	0.1100	0.1100
11	16000	0.1075	0.1075

Table 4.3.3 - Tiller's small strain experimental data

SN	Time (sec)	Tiller experiment	
		Batch curve height (m)	L- curve height (m)
1	0	0.4000	0.0000
2	2000	0.3400	0.0375
3	4000	0.2800	0.0600
4	6000	0.2200	0.0800
5	7500	0.1800	0.0950
6	8500	0.1600	0.1020
7	9500	0.1450	0.1050
8	11000	0.1400	0.1200
9	12000	0.1225	0.1225
10	15000	0.1100	0.1100
11	16000	0.1075	0.1075

Table 4.3.4 - Present large strain simulation's output data

SN	Time (sec)	Large strain simulation	
		Batch curve height (m)	L-curve height (m)
1	1259	0.3792	0.0079
2	1582	0.3723	0.0101
3	2335	0.3580	0.0142
4	3088	0.3432	0.0181
5	3842	0.3279	0.0218
6	4595	0.3123	0.0252
7	5348	0.2964	0.0285
8	6106	0.2803	0.0317
9	6854	0.2640	0.0348
10	7608	0.2474	0.0378
11	9702	0.1991	0.0501
12	11043	0.1903	0.0566
13	12384	0.1812	0.0610
14	13725	0.1719	0.0661
15	14395	0.1671	0.0687
16	15066	0.1624	0.0712
17	15736	0.1575	0.0737
18	16407	0.1527	0.0761
19	17077	0.1478	0.0785
20	17748	0.1428	0.0809
21	18418	0.1378	0.0832
22	19089	0.1328	0.0856
23	20430	0.1227	0.0901
24	21502	0.1145	0.0937
25	22000	0.1096	0.1096

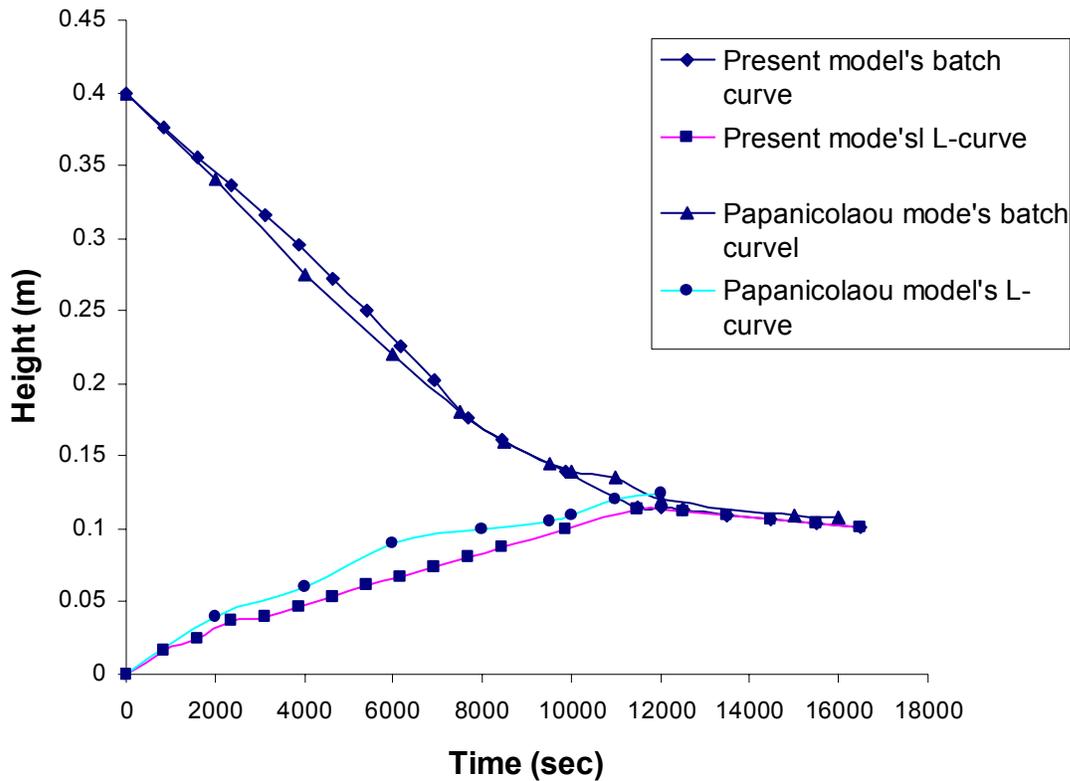


Figure 4.3.1 – Comparison between present small strain simulation and Papanicolaou's small strain simulation

Comparison between present simulation, Papanicolaou's simulation and Tiller's experiment: In present MATLAB simulation for batch thickening process, the batch curve and L-curve meet (approximately) at height of 0.1140 m from the bottom of the column and after 11,500 seconds (approximately) from the start of experiment. In Papanicolaou's FORTRAN simulation, the batch curve and L-curve meets (approximately) at the height of 0.1250 m from the bottom of settling column and after

11,800 seconds (approximately) from the start of the experiment. The difference in this final result of both methods is less than 10%. Figure 4.3.1 demonstrates the comparison between both simulations result with time. According to the Tiller (1981) experiment, the batch curve and L-curve meet (approximately) at the height of 0.1225 m (approximately) from the bottom of the settling column and after 12000 seconds (approximately) from the start of the experiment. Figure 4.3.2 demonstrates the comparison between the results of present MATLAB simulation and Tiller's experiment with time.

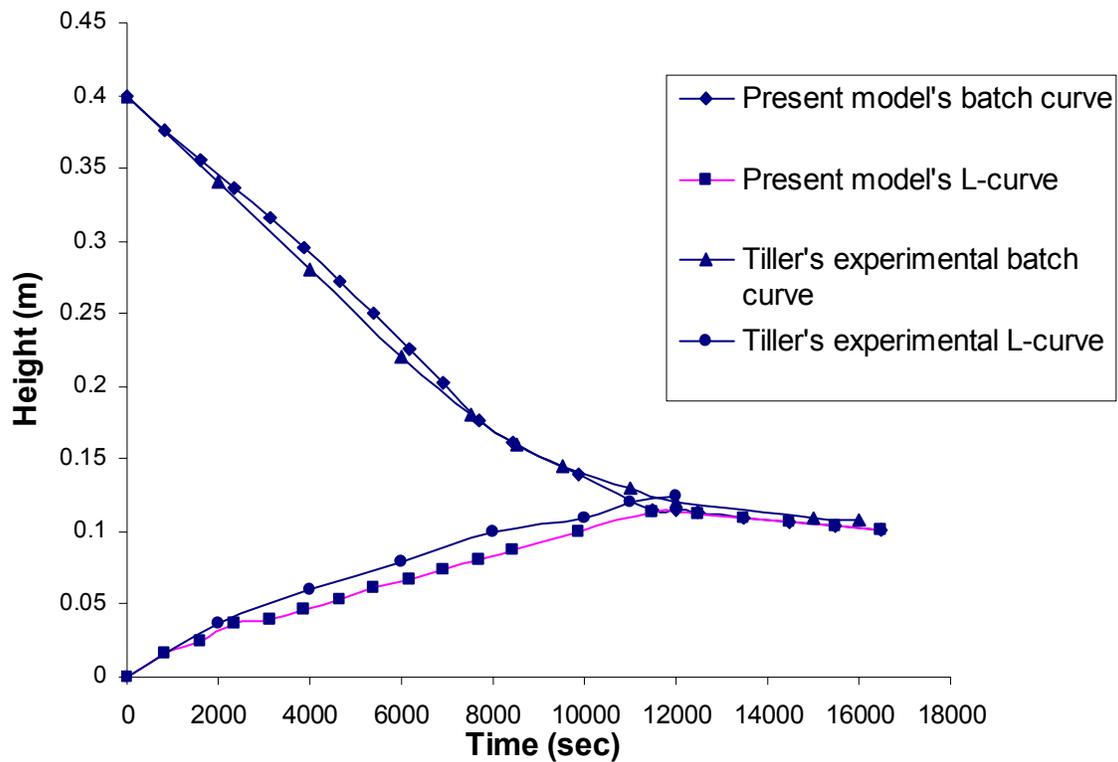


Figure 4.3.2 - Comparison between present small strain model and Tiller's small strain experiment

Comparison between finite strain and large strain models: The result of MATLAB simulations of small strain model (Papanicolaou, 1992) and large strain model (Gutierrez, 2003) has been compared in Figure 4.3.3. From the comparison, it can be concluded that in the large strain model, the rate of consolidation is higher than that of small strain consolidation. Thus, batch thickening process takes grater time to complete in large strain model compared to small strain model. In large strain model simulation, the batch curve and L-curve meets (approximately) at the height of 0.1096 m from the bottom of the settling column after 22,000 seconds (approximately) from the start of the experiment.

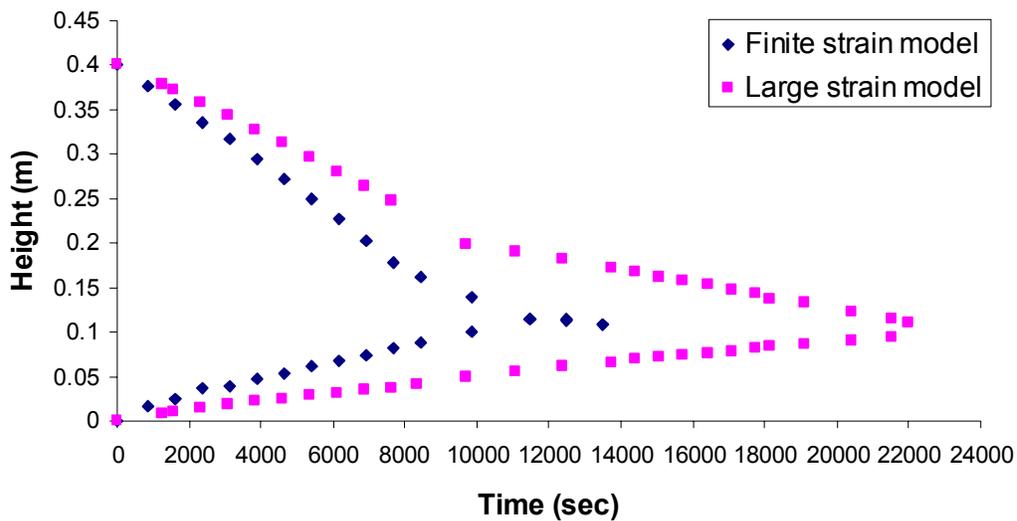


Figure 4.3.3 - Comparison between small strain and large strain models

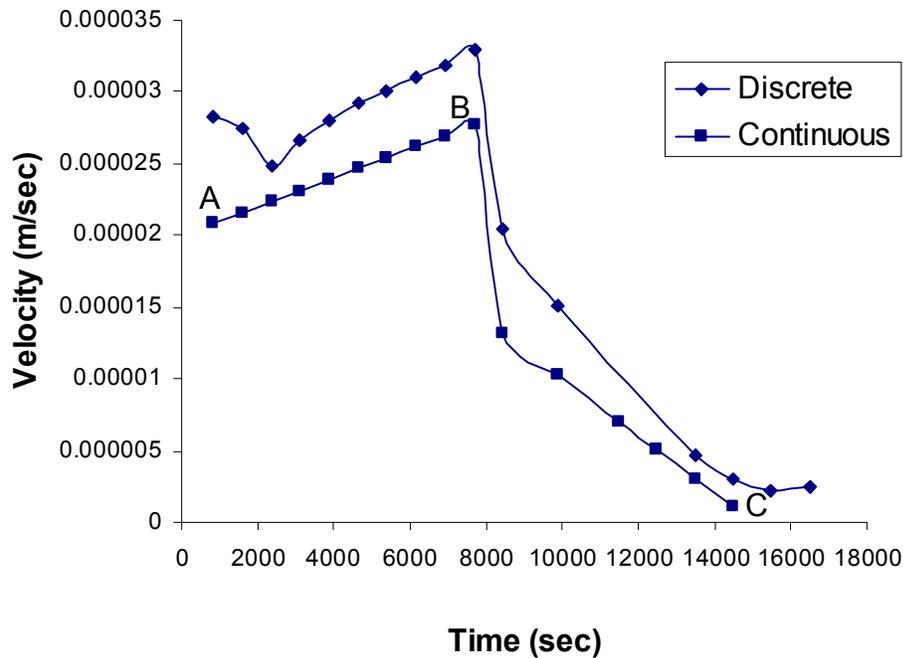


Figure 4.3.4 - Variation of velocity of suspension-sediment interface with time in small strain batch thickening simulation

Figure 4.3.4 shows the variation of velocity of supernatant-suspension interface with time. The discrete curve has been obtained with the help of table 4.3.1. The continuous curve has been obtained by curve fitting of batch curve shown in Figure 4.2.1. Two curves have been fitted between A & B and B & C. Later derivative of those two curves has produced the velocity curves AB and BC shown in Figure 4.3.4.

CHAPTER 5: Conclusion

The present work has simulated the small strain batch thickening model, given by Papanicolaou (1992), in MATLAB. The simulation used the Petrov-Gaterkin method (to construct the weak form of PDF), the Crank-Nicolson method (to solve the time derivative part of PDF), and the Newton-Raphson method (to solve the non linear part of PDF) to solve the second order non linear transit partial differential equation of parabolic type. The result obtained from this simulation, as shown in Figures 4.3.1 and 4.3.2, closely matches with the result obtained by Diplas & Papanicolaou (1997) and experimental result (Tiller, 1981) available in literature.

To obtain continuous results from simulation, the integration of solids settling process with solids consolidation process has been done. During the integration of two processes, it is assumed that all the settlement of solids from suspension region takes place at the end of time step of the consolidation process in each time step.

A numerical method has been developed to calculate the solids concentration in suspension region at the sediment-suspension interface. This method used the empirical relationship between solids flux and solids concentration to obtain algebraic equation of degree 3. This algebraic equation has been solved using the Newton-Raphson method to determine the solids concentration. This numerical method is simpler in comparison to the graphical approach available in literature to calculate solids concentration. The calculation of batch curve, at any time instant in a column, is based on the mass conservation. The proposed method has considered the presence of characteristics (line of constant concentration) in suspension region. This approach also gives the variation of solids concentration in suspension region.

The simulation of large strain batch thickening model, given by Gutierrez (2003),

has also included the proposed numerical methods to calculate the solids concentration and batch curve. The final height of settling solids resulted from finite strain simulation closely matches with that of large strain simulation (Figure 4.3.3). In future, the small strain and large strain models can be modified by accounting the effect of channeling phenomenon. Moreover, integration of small strain and large strain simulations can also be considered as a future course.

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Appendix:

A1: Algorithm to calculate batch curve

Used constants:

$$\phi_0 = 0.03$$

$$\varepsilon_{s0} = 0.075$$

Initialize variables:

$$npc = 1$$

Referring to Figure A2.1, the first characteristic (the line of constant concentration) started from a point 'A₁', i.e. the bottom of the cylinder at time $t = 0$

The number of characteristic, $char$, for the first characteristic:

$$char = 1 \quad (1)$$

The starting time, ts_start , for the first characteristic is

$$ts_start(char) = 0.0 \quad (2)$$

The total time, $tsum$, at the start of the first characteristic is

$$tsum = 0 \quad (3)$$

The height of falling material, $falx$, at $t = 0$:

$$falx = 0 \quad (4)$$

The total height of settled material, $totalx$, at $t = 0$:

$$totalx = 0 \quad (5)$$

The solid volume ratio, ϕ_s , i.e. the concentration for the first characteristic is given by the equation:

$$\frac{u_s \phi_s}{\epsilon_{s0} - \phi_s} = \frac{d(u_s \phi_s)}{d\phi_s} \quad (6)$$

Where, u_s is the solid settling velocity at $t = 0$ and given as:

$$u_s = b_0 + b_1 \phi_s + b_2 \phi_s^2 \quad (7)$$

The solid volume ratio, ϕ , for the first characteristic:

$$\phi(char) = \phi_s \quad (8)$$

The solid settling velocity, u_s , for the first characteristic:

$$u_s(char) = b_0 + b_1 \phi(char) + b_2 \phi(char)^2 \quad (9)$$

The velocity of propagation for the first characteristic is given as:

$$v = -\frac{d(u_s \phi_s)}{\phi_s} \quad (10)$$

and using the empirical relationship between solids settling velocity and solids concentration:

$$v(char) = abs(b_0 + 2b_1 phi(char) + 3b_2 phi(char)^2) \quad (11)$$

The time at which the characteristic reaches the supernatant-suspension interface, ts_reach , and for the first characteristic, i.e. A₅A₁ in Figure A2.1,

$$ts_reach(char) = \frac{H_0}{(v(char) + sus(char))} \quad (12)$$

For the first time step, ts_1 , it is assumed that the consolidation is not taking place. Thus, this time step should be very small, i.e. A₂A₁ in Figure A2.1,

$$ts_1 = ts_reach(char) / n \quad (13)$$

Where, n is any positive integer.

The total time at which a characteristic reaches to the supernatant-suspension interface is termed as $total_ts_reach$ and for first characteristic:

$$total_ts_reach(char) = ts_reach(char) + (char - 1)ts \quad (14)$$

For the first time step, it is assumed that only the settling of the particles occurs without any consolidation. Thus, referring to Figure A2.1 at point A₂,

$$char = char + 1 \quad (15)$$

Total time, $tsum$,

$$tsum = tsum + ts_1 \quad (16)$$

The height of the characteristic, $height$, is given as:

$$height(char - 1) = v(char - 1)(tsum - ts_start(char - 1)) \quad (17)$$

The height of falling material in the time step, ts , is given as:

$$falx(char) = height(char - 1) \quad (18)$$

The height of consolidated layer, x_2 , is given as:

$$x_2(char) = 0.0 \quad (19)$$

Thus total height of the settled layer, $totalx$, is given as:

$$totalx(char) = x_2(char - 1) + falx(char) \quad (20)$$

The effective height (the height over which the concentration of the characteristic is taken in to account in order to calculate the height of supernatant-suspension interface) of first characteristic at the time of consideration (after time step of ts) is given by,

$$h_reach(char - 1) = height(char - 1) - (totalx(char)) \quad (21)$$

Thus, supernatant-suspension height, $mudline_h2$, (like A_2C_2 in Figure A2.1) at point A_2 is given by:

$$mudline_h2 = \frac{(H_0\phi_0 - falx(char)\epsilon_{s0} - h_reach(char-1)\phi(char-1) + height(char)\phi_0)}{\phi_0} \quad (22)$$

Referring to Figure A2.1, the new characteristic is generated at the point B_2 . Since no consolidation has been occurred till this time thus, the characteristic generated at this point has the same concentration and coincided with the first characteristic.

$$\phi(char) = \phi(char-1) \quad (23)$$

The starting time for this characteristic is given by,

$$ts_start(char) = ts_start(char-1) + ts_1 \quad (24)$$

The new time step for the other characteristics can be larger than ts_1 thus, new time step can be given as:

$$ts = \frac{ts_reach(1) - ts_1}{N} \quad (25)$$

Where, N is a positive integer.

Now for the next time step, ts , consolidation occurs over the settled material of height 'totalx(char)'. Let $x_2(char)$ is the consolidated height (outcome of subroutine 'consol').

Then,

$$dx_2 = \frac{x_2(char)}{NEM} \quad (26)$$

Where, NEM is the number of element in the finite element method (FEM) analysis of the problem.

Define variable sum_1 as:

$$sum_1 = \sum_{NEM} \epsilon_s dx_2 \quad (27)$$

Where, ϵ_s is the solid concentration in the consolidated layer.

The suspension velocity, sus , of the solid just above the consolidation-suspension interface is given as:

$$sus(char) = abs(b_0 + b_1 phi(char) + b_2 phi(char)^2) \quad (28)$$

The velocity of characteristic if given as:

$$v(char) = abs(b_0 + 2b_1 phi(char) + 3b_2 phi(char)^2) \quad (29)$$

and the total time for this characteristic to reach the mudline-suspension interface is same as the total time of first characteristic.

The third characteristic starts from the point B_3 (Figure A2.1). At this point or any point afterwards like B_4 , B_5 , B_6 and so on.

Total time is

$$tsum = tsum + ts \quad (30)$$

Number of characteristics:

$$char = char + 1 \quad (31)$$

The height of falling material:

$$falx(char) = abs\left(\frac{sus(char - 1)phi(char - 1) - us(nnodes - 1)\epsilon_{s0}}{\epsilon_{s0} - phi(char - 1)}\right)ts + tddx_1 \quad (32)$$

Where, $us(nnodes - 1)$ is the consolidating velocity of the upper most layer of consolidation layer, $nnods$ is the number of nodes in which consolidating layer is divided for FEM analysis and $tddx_1$ is the reduced height of consolidating layer during the process of consolidation.

The total height of settled layer (like A_3B_3 in Figure A2.1)

$$totalx(char) = x_2(char - 1) + falx(char) \quad (33)$$

Modification is required in the parameter ' npc ' which tells about the number of characteristic, which has been reached to the mudline-suspension interface and that can be done as:

$$\begin{aligned} & \text{if } (abs(tsum - total_ts_reach(npc + 1) < 10) \\ & \quad npc = npc + 1 \end{aligned}$$

The height of all the originated characteristics (as C_3Z_3 in Figure A2.1) at this time of observation (i.e. at point B_3) is given by,

$$\begin{aligned} & \text{for } i = npc + 1 : char - 1 \\ & \quad height(i) = v(i) * (tsum - ts_start(i)) \end{aligned}$$

Define a new variable $height_2$ (like A_2B_2 , A_3C_3 , and A_4D_4 etc. in Figure A2.1), which is helpful in calculating the mudline-suspension height and it is given as:

$$\begin{aligned} & \text{if}(npc == 1) \\ & \quad height_2 = v(npc)(tsum - ts_start(npc)) + totalx(npc) \\ & \text{else} \\ & \quad height_2 = v(npc + 1)(tsum - ts_start(npc + 1)) + totalx(npc + 1) \end{aligned} \quad (34)$$

The points at which the height of mudline-suspension interface has to be calculated can be divided in two categories. First, it is the point at which any characteristic reaches the mudline-suspension interface (like A_5 in Figure A2.1) and second, it is the point that lies between the reaching points of two successive characteristics.

In the first case the supernatant-suspension interface height can be given as:

$$mudline_h2 = height2 \quad (35)$$

and new time step needs to define which can be given by,

$$ts = \frac{total_ts_reach(npc + 1) - total_ts_reach(npc)}{N} \quad (36)$$

In second case the supernatant-suspension interface height can be given by adopting the following steps.

$$1) \quad \text{height}(\text{char}) = 0.0 \quad (37)$$

2) Define new variable sum_2 and assign it to the initial value of zero

$$\text{sum}_2 = 0.0 \quad (38)$$

3) Calculate the effective height (as mentioned earlier) like B_3C_3 in Figure A2.1 for all the originated characteristics which are present at the point of observation.

for $i = \text{char} - 1 : 1 : 1 + \text{npc}$

$$\begin{aligned} h_char(i) &= \text{height}(i) - \text{height}(i+1) - (\text{totalx}(i+1) - \text{totalx}(i)) \\ \text{sum}_2 &= \text{sum}_2 + h_char(i)\text{phi}(i) \end{aligned} \quad (39)$$

4) Finally the height of supernatant-suspension interface can be given by,

if $\text{npc} = 1$

$$\text{mudline_h2} = \frac{(H_0\text{phi}_0 - \text{sum}_1 - \text{falx}(\text{char})\epsilon_{s0} - \text{sum}_2 + \text{height}_2\text{phi}_0)}{\text{phi}_0}$$

else

$$\text{mudline_h2} = \frac{(H_0\text{phi}_0 - \text{sum}_1 - \text{falx}(\text{char})\epsilon_{s0} - \text{sum}_2 + \text{height}_2\text{phi}_0)}{\text{phi}_0} \quad (40)$$

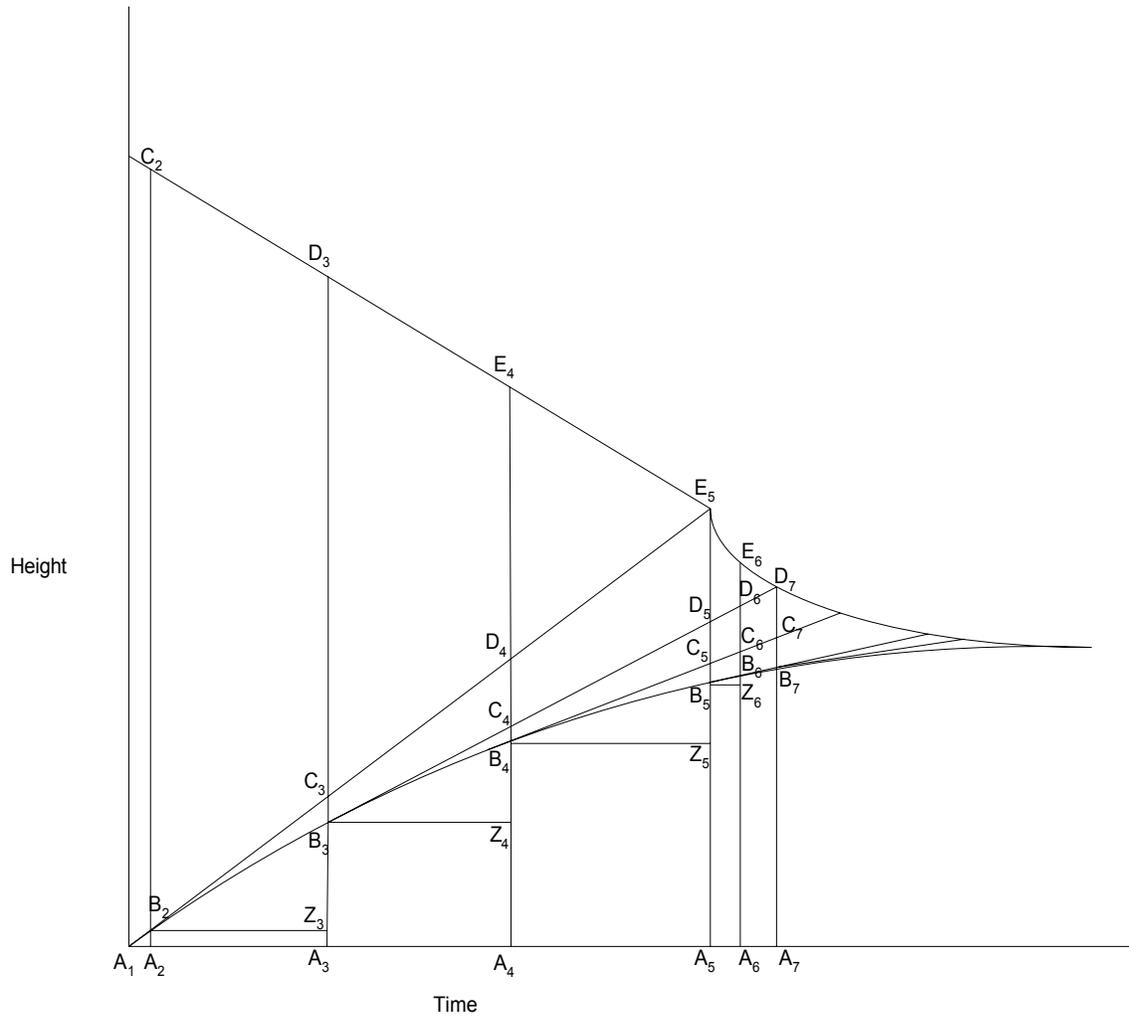


Figure A2.1 - Diagram used to calculate batch curve

Appendix A2: MATLAB code

Finite Strain simulation subroutines

- 1) **finite_strain**: This is the main program for the simulation of finite strain batch thickening process.

```
%*****
% Simulation of the batch thickening phenomenon for young sediments for finite
% strain model given by Papanicolaou (1992)
% This calculates:
% The height of batch curve and L-curve with time
% The solid concentration throughout the column at any time instant
% The effective pressure variation in consolidation region at any time instant
% The permeability in consolidation region
% The compressibility in consolidation region
% The velocity of characteristics present at any time instant in suspension region
%*****

function[] = finite_strain;

%*****
% Definition of variables used in the program
%*****

% A1,A2      = Parameters used in time approximation scheme
% ALPHA     = Parameters used in time approximation scheme
% BPS       = Bottom effective pressure
% CNVCOEF   = Convergence limit
% C0        = Initial concentration
% CONN      = Connectivity matrix
% DBPS      = Secondary variable at bottom
% EK        = Intrinsic permeability at effective pressure equal to zero
% DOFPELM   = Degree of freedom per element
% ELF1      = Force vector estimated at ant time T
% ELF2      = Force vector estimated at time T + TS
% EM        = Intrinsic permeability elsewhere except effective pressure equal
%            to zero
% EPS       = Algebraic value of effective pressure at each node
% GPS       = Effective pressure solution vector
% GPS0      = Effective pressue at time T
% GPS0AV    = Average effective pressure in sublayer at time T
% HBW       = Half bandwidth of global coefficient matrix, [GLK]
% INTPFUN   = Indicator of the type of interpolation function
% INTVL     = Interval among the NTS value
% ISPV      = Index value of specific primary variables
% ITER      = Number of iteration count
% ITLIM     = Max allowable iteration limit within each load step
% LOADSTEP  = Load step counter
% N         = Element number
% NELM      = Number of elements
% NELM1     = Temporary variable defined in program
% NEQ       = Number of equations in the model before boundary conditions
% NGP       = Number of gauss point used in evaluation of element coefficients,
%            [ELK],{ELF} and [ELM]
% NNODS     = Number of nodes in FE mesh
% NDOF      = Number of degree of freedom per node
% NODMESH   = Number of nodes in FE mesh
% NODN      = Global node number
% NODPELM   = Number of nodes per element
% NPRNT     = Indicator to print element matrices
```

```

% NSPV      = Number of specified primary variables
% NSSV      = Number of specified secondary variables
% NTS       = Indicator of the current time step
% PV        = Dirichlet type boundary condition
% PVARINI   = Primary variables initial value in vector form
% SOLVEC    = Column vector with the total solution up to current iteration with
%           each load step
% SV        = Neumann type boundary condition
% TIME      = Current time
% TMAX      = Max value of time that can be reach in the time loop
% TS        = Time step
% VSPV      = Value of specified primary variables

%*****
% Initialization of required matrices
%*****

% MXNOD     = input('enter the max number of nodes used in FE mesh');
MXNOD      = 3;

% MXEQ      = input('enter the max number of equation used to solve FE mesh');
MXEQ       = MXNOD;

% MXELM     = input('enter the max number of element used in FE mesh');
MXELM      = MXNOD-1;

% MXPDOF    = input('enter the max number of degree of freedom of primary variable
%                 used in FE mesh');
MXPDOF     = 1;

% MXSDOF    = input('enter the max number of degree of freedom of seconday variable
%                 used in FE mesh');
MXSDOF     = 1;

% Global coefficient matrix
GLK        = zeros(MXEQ,MXEQ);

% Element coefficient matrix at time T.
ELK        = zeros(MXEQ,MXEQ);

ELA        = zeros(MXEQ,MXEQ);

% Element coefficient matrix at time T+TS
ELKI       = zeros(MXEQ,MXEQ);

% Column vector of global forces before going into the subroutine Solver and after
% coming out of the subroutine .solver it contains the Increment of the solution of
% each iteration.
GLF        = zeros(MXEQ,1);

% Element force factor at time T
ELF        = zeros(MXEQ,1);

% Element force vector at time T + TS
ELFI       = zeros(MXEQ,1);

% Global mass matrix
GLM        = zeros(MXEQ,MXEQ);

```

```

% Element mass matrix
ELM      = zeros(MXEQ,MXEQ);

% Global coordinates of an element
GLX      = zeros(MXNOD,1);

% Local coordinates of an element
ELX      = zeros(MXEQ,1);

% The height of sublayers in consolidation region at any time T + TS
DX2      = zeros(MXNOD,1);

% The decrement in height of sublayers in consolidation region
% at ant time T+TS
DDX2     = zeros(MXNOD,1);

% The height of sublayers in consolidation region at any time T
DX1      = zeros(MXNOD,1);

% The decrement in height of sublayers in consolidation region
% at ant time T
DDX1     = zeros(MXNOD,1);

% Effective pressure
SOLVEC   = zeros(MXEQ,1);
GPSO     = zeros(MXEQ,1);
GPSOAV   = zeros(MXEQ,1);

% Connectivity of elements in finite element mesh
CONN     = zeros(MXELEM,3);

% Primary and secondary variables index
ISPV     = zeros(MXPDOF,2);
ISSV     = zeros(MXSDOF,2);

% Values of primary and secondary variables
VSPV     = zeros(MXPDOF,1);
VSSV     = zeros(MXSDOF,1);

% Solid concentration in consolidation region
ES       = zeros(MXEQ,1);

% Consolidation rate in consolidation region
MV       = zeros(MXNOD,1);

% Average linear concentration in consolidation sublayer at any time T
C2AV     = zeros(MXEQ,1);

% Average linear concentration in consolidation sublayer at time T+TS
C1AV     = zeros(MXEQ,1);

% Velocity of upward moving liquid coming out from sublayers of consolidation
% region
UL2      = zeros(MXNOD,1);

% Velocity of settling down solids in sublayer of consolidation region
US2      = zeros(MXNOD,1);

% Weight of solids in sublayer of consolidation region
WS       = zeros(MXNOD,1);

```

```

% Permeability in sublayer of consolidation region
K2      = zeros(MXNOD,1);

% Effective pressure in element (sublayer) of consolidation region
EPS     = zeros(MXEQ,1);

%*****
% Input data
%*****

% NDOF      = input('enter the degree of freedom of nodes');
% NODPELM  = input('enter the number of nodes per element in a FE mesh');
% NELM     = input('enter the number of elements in FE mesh');
% INTPFUN  = input('enter the interpolation function indicator');
% CNVCOEF  = input('enter the convergence coefficient');
% ALPHA    = input('enter the value of alpha for time approximation');
% TS       = input('enter the time step');
% TMAX     = input('enter the max value of time that can be reach in a time loop');
% NSPV     = input('enter the number of specified primary variables');
% NSSV     = input('enter the number of specified secondary variables');
% NGP      = input('enter the number of gauss point for the problem');
% PS0(i)   = input('enter the value of effective pressure at T');
% C0(i)    = input('enter the value of concentration at time T');
% X0       = input('enter the height of bed at T');
% X1       = input('enter the thickness of solids at T');
% TDDX2    = input('assume the total decrease in thickness at time T+TS');
% FALLX    = input('enter the height of falling material at time T');
% ITLIM    = input('enter the iteration limit');
% MXEQ     = (MXELM+1)*3;

HBW      = 0.0;

% NODPELM  = input('enter the number of nodes per element in a FE mesh');
NODPELM  = 2;

% NDOF     = input('enter the degree of freedom of nodes');
NDOF     = 1;

% Half band width
HBW      = NODPELM*NDOF;

% Number of nodes calculation
% NELM     = input('enter the number of elements in FE mesh');
NELM     = MXELM;

NNODS    = NELM*(NODPELM-1)+1;

DOFPELM  = NODPELM*NDOF;

% Defined parameter
NELM1    = NELM+1;

% Number of equation required
NEQ      = NNODS*NDOF;

%*****
% Input parameters
%*****

% Values of empirical parameters

```

```

A          = 0.033;
PHIS0     = 0.03;
SUS0     = abs(0.0000946754 - 0.0026552*PHIS0 + 0.018926*PHIS0*PHIS0);
SUS0     = 0.00003;
C0        = 0.075;
B         = 0.361;
% B       = 3.08*(0.65-C0)^4;
D         = 1.234;
% D       = 5.19*(0.65-C0)^2.68;
EK        = 10^(-14);
RHOS     = 2300;
RHOL     = 1000;
DRHO     = 1300;
MU       = 0.001;
H0       = 0.40;

%*****First Kynch characteristic at T=0*****

% X0 = input('enter the height of bed at T0')
X0       = 0.0;

% Number of characteristic at initial time
CHAR     = 1

% Define a parameter
NPC      = 1;

% Total time
TSUM     = 0.0;

% Height of supernatant-suspension interface
MUDLINE_H2 = 0.4;

% Total height of consolidation layer
TOTALX(CHAR)= 0.0;
CON      = TOTALX(CHAR);

% Plot of batch curve and L-curve
subplot(4,2,1);
plot (TSUM,MUDLINE_H2,'*');
hold on;
plot (TSUM,CON,'o');
% axis([0 15000 0 0.5]);
axis('normal');

% Sediment deposited in first time step of Kynch
C0       = 0.036;

% Concentration of characteristic
PHIS(CHAR) = 0.03;
PHIS1     = PHIS(CHAR);

% Solid settling velocity
% SUS(CHAR) = abs(0.000119711- 0.00428381*PHIS(CHAR)+ 0.0418934*PHIS(CHAR)*...
%           PHIS(CHAR));
SUS(CHAR) = abs(0.0000946754 - 0.0026552*PHIS(CHAR) + 0.018926*PHIS(CHAR)*...
              PHIS(CHAR));

% Velocity of characteristic
V(CHAR)   = abs(0.0000946754 - 2*0.0026552*PHIS(CHAR) + 3*0.018926*PHIS(CHAR)^2);

% Starting time of the characteristic

```

```

TS_START (CHAR) = 0.0;

% Calculation of the time for characteristic to reach the supernatant-suspension
% interface
TS_REACH(CHAR) = H0*PHIS0/(PHIS(CHAR)*(V(CHAR) + SUS(CHAR)));

%*****Second Kynch characteristic*****

% Calculation of concentration of characteristic
PHIS_IP = PHIS1;
PHIS0 = PHIS1;
US2(NNODS-1) = 0.0;
C0 = 0.045;
[PHI] = newraps(US2,NNODS,PHIS_IP,C0);
PHIS2 = PHI;

% Define parameters
Z = 1;
N = 1;

while(PHIS2<=PHIS1 & Z<200 )
    % PHIS_IP = input('enter the value of solid concentration greater than phis1 and
    % less than c0');
    PHIS_IP = PHIS_IP+0.01;
    [PHI] = newraps(US2,NNODS,PHIS_IP,C0);
    PHIS2 = PHI;
    Z = Z+1;
end

% Solid concentration above the consolidation-suspension interface at initial time
if(Z>200 | PHIS2< PHIS1)
    PHIS(CHAR) = PHIS0;
else
    PHIS(CHAR) = PHIS2;
end

PHIS1 = PHIS(CHAR);

% Solis settling velocity
SUS(CHAR) = abs(0.0000946754 - 0.0026552*PHIS(CHAR) + 0.018926*PHIS(CHAR)*...
    PHIS(CHAR));

% Velocity of characteristic
V(CHAR) = abs(0.0000946754 - 2*0.0026552*PHIS(CHAR) + 3*0.018926*PHIS...
    (CHAR)^2);

% Starting time of the characteristic
TS_START (CHAR) = 0.0;

% Calculation of the time for characteristic to reach the supernatant-suspension
% interface
TS_REACH(CHAR) = H0*PHIS0/(PHIS(CHAR)*(V(CHAR) + SUS(CHAR)));

%*****Third Kynch characteristic *****

% Calculation of concentration of characteristic
PHIS_IP = PHIS1;
PHIS0 = PHIS1;

```

```

US2(NNODS-1) = 0.0
C0          = 0.048
[PHI]       = newraps(US2,NNODS,PHIS_IP,C0);
PHIS2       = PHI;

% Define parameters
Z           = 1;
N           = 1;

while(PHIS2<=PHIS1 & Z<200 )
% PHIS_IP   = input('enter the value of solid concentration greater than phis1
%           and less than c0');
    PHIS_IP = PHIS_IP+0.01;
    [PHI]   = newraps(US2,NNODS,PHIS_IP,C0);
    PHIS2   = PHI;
    Z       = Z+1;
end

% Solid concentration above the consolidation-suspension interface at initial time
if(Z>200 | PHIS2< PHIS1)
    PHIS(CHAR) = PHIS0;
else
    PHIS(CHAR) = PHIS2;
end

PHIS1      = PHIS(CHAR);

% Solid settling velocity
SUS(CHAR) = abs(0.0000946754 - 0.0026552*PHIS(CHAR) + 0.018926*PHIS(CHAR)*...
    PHIS(CHAR));

% Velocity of characteristic
V(CHAR)   = abs(0.0000946754 - 2*0.0026552*PHIS(CHAR) + 3*0.018926*PHIS...
    (CHAR)^2);

% Starting time of the characteristic
TS_START (CHAR) = 0.0;

% Calculation of the time for characteristic to reach the supernatant-suspension
% interface
TS_REACH(CHAR) = H0*PHIS0/(PHIS(CHAR)*(V(CHAR) + SUS(CHAR)));

%*****FOURTH KYNCH CHARACTERSTICS*****

% Calculation of concentration of characteristic
PHIS_IP   = PHIS1;
US2(NNODS-1) = 0.0
C0          = 0.056
[PHI]       = newraps(US2,NNODS,PHIS_IP,C0);
PHIS2       = PHI;

% Define parameters
Z           = 1;
N           = 1;

while(PHIS2<= PHIS1 & Z<200 )
% PHIS_IP   = input('enter the value of solid concentration greater than phis1
% and less than c0');
    PHIS_IP = PHIS_IP+0.01;
    [PHI]   = newraps(US2,NNODS,PHIS_IP,C0);
    PHIS2   = PHI;
    Z       = Z+1;
end

```

```

end

% Solid concentration above the consolidation-suspension interface at initial time
if( Z>200 | PHIS2< PHIS1)
    PHIS(CHAR) = PHIS0;
else
    PHIS(CHAR) = PHIS2;
end

PHIS1      = PHIS(CHAR);

% Solid settling velocity
SUS(CHAR)  = abs(0.0000946754 - 0.0026552*PHIS(CHAR) + 0.018926*PHIS(CHAR)*...
    PHIS(CHAR));

% Velocity of characteristic
V(CHAR)    = abs(0.0000946754 - 2*0.0026552*PHIS(CHAR) + 3*0.018926*PHIS...
    (CHAR)^2);

% Starting time of the characteristic
TS_START (CHAR) = 0.0;

% Calculation of the time for characteristic to reach the supernatant-suspension
% interface
TS_REACH(CHAR) = H0*PHIS0/(PHIS(CHAR)*(V(CHAR) + SUS(CHAR)));

%*****Characteristic at the start of sediment layer*****

% Calculation of concentration of characteristic
PHIS1      = 0.047
PHIS_IP    = PHIS1;
US2(NNODS-1) = 0.0
C0         = 0.075
[PHI]      = newraps(US2,NNODS,PHIS_IP,C0);
PHIS2      = PHI;

% Define parameters
Z          = 1;
N          = 1;

while(PHIS2<=PHIS1 & Z<500 & PHIS2>C0 )
    % PHIS_IP = input('enter the value of solid concentration greater than phis1
    % and less than c0');
    PHIS_IP  = PHIS_IP + 0.0001;
    [PHI]    = newraps(US2,NNODS,PHIS_IP,C0);
    PHIS2    = PHI;
    Z       = Z+1;
end

% Solid concentration above the consolidation-suspension interface at initial time
if( Z>500 | PHIS2< PHIS1 | PHIS2>C0)
    PHIS(CHAR) = PHIS1;
else
    PHIS(CHAR) = PHIS2;
end

PHIS1      = PHIS(CHAR);

% Solid settling velocity
% SUS(CHAR)  = abs(0.000119711- 0.00428381*PHIS(CHAR)+ 0.0418934*PHIS(CHAR)*...

```

```

%          PHIS(Char));
SUS(Char) = abs(0.0000946754 - 0.0026552*PHIS(Char) + 0.018926*PHIS(Char)*...
PHIS(Char));

% Velocity of characteristic
V(Char)    = SUS(Char)*PHIS(Char)/(C0 - PHIS(Char));

% Starting time of the characteristic
TS_START (Char) = 0.0;

% Calculation of the time for characteristic to reach the supernatant-suspension
% interface
TS_REACH(Char) = H0*PHIS0/(PHIS(Char)*(V(Char) + SUS(Char)));

% Define time step
TS1         = TS_REACH(Char)/10;
SUS1        = SUS(Char);
TOTAL_TS_REACH(Char) = TS_REACH(Char) + (Char-1)*TS1;

%*****Second characteristic at T= TS*****

% Number of characteristics after time period TS
Char        = Char + 1;

% Starting time of the characteristic
TS_START(Char) = TS_START(Char-1)+TS1;
TSUM          = TSUM + TS1;

% PHIS_IP = input('enter the value of solid concentration greater than phis0
% and less than c0')
PHIS_IP = PHIS1;

% The height of the characteristic
HEIGHT(Char-1) = V(Char-1)*(TSUM - TS_START(Char-1));

% Height of the falling material at the start of the characteristic
% FALX(Char) = (PHIS(Char-1)*SUS(Char-1))/(C0 -PHIS(Char-1))*TS;
FALX(Char) = HEIGHT(Char-1);

% Height of consolidation layer
X2(Char)    = 0.0;
X1(Char)    = FALX(Char);

% Total height of consolidated plus falling material
TOTALX(Char) = X2(Char) + FALX(Char);
CON          = TOTALX(Char);

% Height of supernatant-suspension liquid interface
H_REACH(Char-1) = HEIGHT(Char-1) - TOTALX(Char);
MUDLINE_H2      = (H0*PHIS0 - FALX(Char)*C0 + HEIGHT(Char-1)*PHIS0)/PHIS0;

% The second characteristic coincide with the first characteristic as consolidation
% does not till the end of first time step, TS1, thus,
PHIS(Char)    = PHIS(Char-1);
V(Char)       = V(Char-1);
SUS(Char)     = SUS(Char-1);
TOTAL_TS_REACH(Char) = TOTAL_TS_REACH(Char-1);

% Define the new time step for the rest of characteristics
TS            = (TS_REACH(1)-TS1)/10;

```

```

% Plot of batch curve and L- curve
subplot(4,2,1);
plot (TSUM,MUDLINE_H2,'*');
hold on;
plot (TSUM,CON,'o');
xlabel('Time (sec)');
ylabel('Height (m)');
legend ('Batch curve', 'L-curve');
title('(a) Batch curve and L-curve vs Time');
axis('normal');

% Element length at initial stage
DX      = X1(CHAR)/NELM;
for I = 1 : NEQ
    GPS0(I) = 9.81*C0*DRHO*(X1(CHAR) -(I-1)*DX);
    C1(I)   = C0;
end

% Average initial concentration in sublayer at time T1
for I = 1 : NNODS-1
    C1AV(I) = (C1(I+1)+C1(I))/2;
end

% Number of initial element
NELM_IN = NELM;

% Number of initial nodes
NNODS_IN = NNODS;

% Decrement in thickness in sublayer of consolidated region
TDDX2A = [0.00095,0.00040,0.00015];
TDDX2  = TDDX2A(1);

% Define parameter
DTDDX2 = 0.0001;

% Define a parameter
Z0= 0.0;

% Condition for termination of program (When batch curve meets L-curve)
while (abs(MUDLINE_H2-TOTALX)> 0.02)
    Z0 = Z0+1;
    DIFFERENCE(1) = 10^10;
    X1(CHAR) = TOTALX(CHAR);
    while (abs(DIFFERENCE(1))>15*GPS0(1)/100);

        % Define parameters
        ALPHA = 0.00653;
        A1    = ALPHA*TS;
        A2    = (1.0-ALPHA)*TS;

        %*****
        % Consolidation process
        %*****

        [BPS,GLF,SOLVEC,C2AV,US2,TDDX1,X2,CONN,K,GLX] = consol_fs(CHAR,NELM,NNODS,...
            NNODS_IN,MXEQ,MXNOD,GPS0,X0,X1,FALX,X2,C1AV,TS,TSUM,TDDX2,A1,A2,C0);

        %*****

```

```

% Values effective pressure, permeability, volume fraction of solid, porosity,
% void ratio, compressibility
for I = 1:NEQ
    GLF(I) = SOLVEC(I);
    K(I)    = EK*(1+A*GLF(I))^( -D);

    % Permeability within consolidation region
    K2(I) = (K(I));

    % Volume fraction of solids within consolidation region
    ES(I) = C0*(1+A*GLF(I))^(B);

    % Porosity in consolidation region
    p(I) = 1 - ES(I);

    % Void ration in consolidation region
    e(I) = 1/ES(I) - 1;

    % Compressibility in consolidation region
    G(I) = -A*B/(C0*(1+A*GLF(I))^(1+B));
end

% Plots
if(Z0==1 | Z0==3 | Z0==5 | Z0==7)

    subplot(4,2,2);
    plot(GLF,GLX,'*');
    xlabel('Effective Pressure (kPa)');
    ylabel('Height (m)');
    title( '(b) Effective pressure vs Height in consolidation region');
    hold on;

    subplot(4,2,3);
    plot(p, GLX,'*');
    xlabel('Porosity ');
    ylabel('Height (m)');
    title( '(c) Porosity vs Height in consolidation region');
    hold on;

    subplot(4,2,4);
    plot(K2,GLX,'*');
    xlabel('Permeability (m/s)');
    ylabel('Height (m)');
    title( '(d) Permeability vs Height in consolidation region');
    hold on;

    subplot(4,2,5);
    plot(ES, GLX,'*');
    xlabel('Solid volume ratio');
    ylabel('Consolidation region Height (m)');
    title( '(e) Solid volume ratio vs Height in consolidation region');
    hold on;
end

for I = 1:NEQ
    DIFFERENCE(I) = abs(GLF(I))-abs(GPS0(I));
end
DIFFERENCE(1) = GPS0(1)-GLF(1);
TDDX2        = TDDX2 -DTDDX2;
if (TDDX2<0)
    TDDX2 = TDDX2+DTDDX2;
end

```

```

        DTDDX2 = DTDDX2/10;
        TDDX2 = TDDX2-DTDDX2;
    end
end % End of the while loop for TDDX2

%*****
% Calculation of height of batch curve
%*****

[PHIS,PHIS1,PHIS2,SUS,FALX,X2,MUDLINE_H2,TOTALX,TSUM,TS,CHAR,TS_START,V,...
TS_REACH,TOTAL_TS_REACH,NPC,N] = sediment_fs(Z0,GLX,N,CHAR,NPC,TS_START,...
TS_REACH,TOTAL_TS_REACH,NELM,X0,X1,X2,TOTALX,FALX,TS,TS1,TSUM,TDDX1,TDDX2,...
C1,ES,PHIS1,PHIS,US2,SUS,NNODS,C0,V);

subplot(4,2,1);
CON = TOTALX(CHAR);
plot (TSUM,MUDLINE_H2, '*');
hold on;
plot (TSUM,CON, 'o');
xlabel('Time (sec)');
ylabel('Height (m)');
% axis([0 15000 0 0.5]);
axis('normal');
legend ('Batch curve', 'L-curve');
title( '(a) Batch curve and L-curve vs Time');

% Combination of consolidation region and falling material region

% Pressure distribution in falling material region
PS = DRHO*9.81*C0*FALX(CHAR);

% Number of new elements
NELM = NELM_IN + NELM;

% Number of new nodes
NNODS = NNODS_IN+NNODS-1;
NEQ = NNODS*NDOF;

% Number of equation required to solve
MXEQ = NNODS;

% Maximum number of nodes
MXNOD = NNODS;
X1 = TOTALX;
for I = NNODS- NNODS_IN+1: NNODS
    DX1(I) = FALX(CHAR)/(NNODS_IN-1);
end

% Calculation of the effective pressure in combined region
for I = 1:NNODS-NNODS_IN
    PS(I) = DRHO*9.81*C0*FALX(CHAR);
    GPS0(I) = GLF(I)+ PS(I);
end
for I = NNODS- NNODS_IN+1: NNODS
    GPS0(I) = 9.81*C0*DRHO*(FALX(CHAR) -(I-(NNODS-NNODS_IN+1))*DX1(I));
end

% Calculation of the solid volume ratio in combine region
for I = 1:NNODS-NNODS_IN
    C1AV(I) = ES(I);
end

```

```

    for I = NNODS- NNODS_IN+1: NNODS-1
        ClAV(I) = C0;
    end

    % Define the new solid volume ratio in suspension region
    PHIS1 = PHIS2;
    % C0 = 0.075+ 1.1*10^(-6)* TSUM;
end% End of the condition for which batch curve meets L-curve

% Simulation for batch thickening phenomenon after batch curve meets L-curve

if(abs(MUDLINE_H2 - TOTALX(CHAR))<= 0.02)

    TOTALX(CHAR) = MUDLINE_H2;
    MUDLINE_H2 = TOTALX(CHAR);
    CON = TOTALX(CHAR);
    X2(CHAR) = TOTALX(CHAR);

    subplot(4,2,1);
    plot (TSUM,MUDLINE_H2, '*');
    hold on;
    plot (TSUM,CON, 'O');
    xlabel('Time (sec)');
    ylabel('Height (m)');

    for I = 1:5
        CHAR = CHAR+1;
        X1(CHAR) = X2(CHAR-1);
        FALX(CHAR) = 0.0;
        if(FALX(CHAR) == 0.0)
            for I = 1:NNODS-1
                DX1(I) = X2(CHAR-1)/(NNODS-1);
            end
        end
        TDDX2 = TDDX2 -DTDDX2;
        if (TDDX2<0)
            TDDX2 = TDDX2+DTDDX2;
            DTDDX2 = DTDDX2/10;
            TDDX2 = TDDX2-DTDDX2;
        end
        TS = 1000;
        TSUM = TSUM+TS;
        [BPS,GLF, SOLVEC,C2AV,US2,TDDX1,X2,CONN,K,GLX] = consol_fs(CHAR,NELM,...
        NNODS,NNODS_IN,MXEQ,MXNOD,GPS0,X0,X1,FALX,X2,ClAV,TS,TSUM,TDDX2,A1,A2,C0);
        DIFFERENCE(1) = GPS0(1)-GLF(1);
        TOTALX(CHAR) = X2(CHAR);
        CON = TOTALX(CHAR);

        subplot(4,2,1);
        MUDLINE_H2 = CON;
        plot (TSUM,MUDLINE_H2, '*');
        hold on;
        plot (TSUM,CON, 'O')
        xlabel('Time (sec)');
        ylabel('Height (m)');
    end
end
end

```

2) Subroutine 'consol_fs'


```

% Calculation of the total decrement in height that occurs at the beginning of the
% consolidation period , TS
%*****

% DX1(I) = The height of each sublayer at time
% MV(I) = The consolidation layer for each sublayer at time T
% GPS0(I)= The initial value of effective pressure at T
% DXX1(I)= The decrement that occurs due to consolidation at each sublayer due to
% consolidation at time T
% TDDX1 = The total decrement of all the layer at time T

TDDX1 = 0.0;
if (FALX(CHAR) == 0.0)
  for I = 1:NNODS-1
    DX1(I) = X2(CHAR-1)/(NNODS-1);
  end
end

if (X2 == 0.0)
  for I= 1:NNODS-1
    % LENGH OF EACH SUBLAYER AT T
    DX1(I) = FALX(CHAR)/(NNODS-1);
  end
else
  for I = 1:NNODS-NNODS_IN
    DX1(I) = X2(CHAR-1)/(NNODS-NNODS_IN);
  end
  for I = NNODS-NNODS_IN+1:NNODS-1
    DX1(I) = FALX(CHAR)/(NNODS_IN-1);
  end
end

for I = 1:NNODS-1
  % Average effective pressure in sublayer at time T
  GPS0AV(I) = ( GPS0(I+1)+GPS0(I))/2;

  % Consolidation rate of each layer at time T
  MV(I) = (A*B)/(1+A*GPS0AV(I));

  % Decrease in sublayer's height at time T
  DDX1(I) = DX1(I)*MV(I)*GPS0AV(I);

  % Total decrease in height at time T
  TDDX1 = TDDX1+DDX1(I);
end

for I = 1:NNODS-1
  R(I) = DDX1(I)/TDDX1;
  TDDX1*(1-C1AV(I))/(MV(I)*GPS0AV(I));
  TDDX1*(I)/(MV(I)*GPS0AV(I));
end

%*****End of step 1*****
%*****
% Step 2
%*****
% DDX2 = Decrease in height of sublayer at time T + TS
% TDDX2 = Total assumed decrement at time T + TS
% C1AV = Average initial concentration in sublayer at time T
% C2AV = Average concentration in sublayer at time T + TS

```

```

% TS      = Time step for consolidation period
% UL2(I) = Velocity of upward moving liquid coming out from sublayers of
%          consolidation region
% US2(I) = Velocity of settling down solids in sublayer of consolidation region
% K2(I)  = Permeability at each sublayer at time T + TS
% EWP2(I)= Excess water pressure at time T + TS
% TWP2   = Total excess water pressure at time T + TS
% WS2(I) = Weight of solids in sublayer at time T + TS
% TWS2   = Total weight of solid at time T + TS
% BPS2   = Bottom effective pressure at time T + TS
% DIFFERENCE = Difference between simulation output and theoretical value of
%             effective pressure

for I= 1:NNODS-1
    % Decrease in height of sublayer at time T + TS
    DDX2(I) = R(I)*TDDX2;
end

% Average concentration in sublayer at time T + TS
for I = 1:NNODS-2
    C2AV(I) = ((DX1(I))/(DX1(I)-DDX2(I)))*C1AV(I);
end
C2AV(NNODS-1) = C0;

% Permeability at each sublayer at time T + TS
for I=1:NNODS-1
    K2(I)   = EK*(C0/C2AV(I))^(B/D);
end

% Liquid and solid velocity in sublayer at time T + TS
UL2(1) = -0.5*(DDX1(1)/TS)*(1/(1-C2AV(1)));
US2(1) = 0.5*(DDX1(1)/TS)*(1/C2AV(1));

SUMM = 0.0;
for I = 2:NNODS-1
    SUMM = SUMM + R(I-1);
    UL2(I) = -0.5*(1/TS)*TDDX1*(R(I)+2*SUMM)*(1/(1-C2AV(I)));
    US2(I) = 0.5*(1/TS)*TDDX1*(R(I)+2*SUMM)*(1/C2AV(I));
end

% Define variables
AA = UL2(NNODS-1);
BB = US2(NNODS-1);

% Excess water pressure and solid weight at time T + TS
TEWP2 = 0.0;
TWS2 = 0.0;
for I= 1:NNODS-1
    EWP2(I) = -(((UL2(I)-US2(I))*MU*(1-C2AV(I)))/K2(I))*(DX1(I)-DDX1(I));
    TEWP2 = TEWP2+ EWP2(I);
    WS2(I) = (DRHO)*9.81*C2AV(I)*(DX1(I)-DDX1(I));
    TWS2 = TWS2 + WS2(I);
end

% Bottom effective pressure at time T + TS
BPS = TEWP2 + TWS2;

% The secondary variable at the bottom of the sediments
DBPS = - DRHO*9.81*C0*(1+A*BPS)^(B-D+1)*EK*(1/A*MU*R);

```

```

%*****End of step 2*****
%*****End of 1st method*****

%*****2nd method*****
% Use of finite element
%*****

% X2 = Final height of consolidation layer after decrement
% DX2 = Final height of consolidation sublayer after decrement

X2(CHAR) =X1(CHAR)- TDDX1 - TDDX2;

for I = 1: NELM1
    if ( I==1)
        DX2(I) = 0.0;
    else
        DX2(I) = X2(CHAR)/NELM;
        % DX2(I) = DX1(I)-DDX1(I)-DDX2(I);
    end
end

%*****
% Geometry Module: defined the connectivity of the elements in finite element mesh
%*****
[GLX,CONN] = connectivity(NDOF,NNODS,DX2,NODPELM);

%*****
% First iteration
%*****

% For the first iteration set the current solution vector, GPS, to the vector of
% the initial condition
for I = 1:NEQ
    SOLVEC(I) = GPS0(I);
end

% Initialize the iteration
ERR      = 0.0;
CNVCOEF = 0.0;
DIFFERENCE = 1;
ITER     = 0;
% ITLIM  = input(' enter the value of iteration limit');
ITLIM    = 100;
for ITER = 1:ITLIM
    ITER = ITER+1;
    for I = 1:NEQ
        GLF(I) = 0.0;
        for J = 1: NEQ
            GLK(I,J)= 0.0;
            GLM(I,J)= 0.0;
        end
    end
end

for N = 1:NELM
    K=0.0;
    for I = 1: NODPELM
        NODN = CONN(N,I);
        % Transfer the global data to the element data
    end
end

```

```

        ELX(I) = GLX(NODN);
        KI     = NODN-1;
        for J = 1:NDOF
            KI = KI+1;
            K  = K+1;
            % Element effective pressure
            EPS0(K) = GPS0(KI);
            EPS(K)  = SOLVEC(KI);
        end
    end
% NGP = input(' enter the number of gauss points used in the problem');
NGP = 2;
% INTPFUN = input('enter the indicator for interpolation function');
INTPFUN = 1;

% Calculation of the coefficient matrices GLX,GLK,GLM.
[ELK,ELF,ELM] = coefmatrx(NDOF,NODPELM,N,NODN,EPS,EPS0,INTPFUN,...
    NELM,NNODS,NGP,ELX,ELA,A1,A2,TS,TSUM,C0);

% N      = Element number
%{ELF}   = Element source vector,{F}
%{GLF}   = Global source vector
%{ELK}   = Element coefficient matrix,[K]
%{GLK}   = Global coefficient matrix
%{GLM}   = Global mass matrix
%{CONN}  = Node connectivity

for I = 1:NODPELM
    NR = (CONN(N,I)-1)*NDOF;
    for II = 1:NDOF
        NR = NR+1;
        M = (I-1)*NDOF+II;
        GLF(NR) = GLF(NR)+ELF(M);
        for J = 1:NODPELM
            NCL = (CONN(N,J)-1)*NDOF;
            for JJ = 1:NDOF
                Q = (J-1)*NDOF+JJ;
                NC = NCL+JJ-NR+HBW;
                if (NC>0)
                    GLK(NR,NC) = GLK(NR,NC)+ELK(M,Q);
                end
            end
        end
    end
end
end
end
end
end

%*****
% Imposition of essential and natural type of boundary condition
%*****

% MXEBC = Maximum number of essential boundary conditions
% MXNBC = Maximum number of natural boundary conditions
NBW = 2.0*HBW-1;
for I = 1:NSPV
    IE = (ISPV(I,1)-1)*NDOF + ISPV(I,2);
    for J = 1:NBW
        GLK(IE,J) = 0.0;
    end
    GLK(IE,HBW) = 1.0;
    GLF(IE)     = VSPV(I);
end

```

```

end

%*****
% After the first iteration set the value of primary variables to zero
if (ITER == 1)
    for I = 1:NSPV
        VSPV(I) = 0.0;
    end
end

%*****
% Solve the finite element equations
%*****

% {GLF} denotes the global source before going into solver and denotes the global
% solution vector when it comes out of solver subroutine

BW      = 2*HBW;
for I = 1:NEQ;
    GLK(I,BW) = GLF(I);
end
HBW     = NODPELM*NDOF;
ITERM   = HBW;
GLK     = solver(GLK,MXEQ,NEQ,ITERM);
for I = 1:NEQ
    GLF(I) = GLK(I,BW);
end

%*****
% Update the solution and check for the convergence
%*****

DIFF = 0.0;
SOLN = 0.0;
for I = 1:NEQ
    DIFF      = DIFF +GLF(I)*GLF(I);
    SOLVEC(I) = SOLVEC(I)+GLF(I);
    SOLN      = SOLN + SOLVEC(I)*SOLVEC(I);
end
ERR      = sqrt(DIFF/SOLN);
% CNVCOEF = input(' enter the value of convergence limit or coefficient');
CNVCOEF = 0.01;
end % end of while loop
if (ERR>CNVCOEF)
    if(ITER == ITLIM)
        fprintf(' The scheme did not converge and increase the iteration limit');
    end
end
end

```

3) Subroutine 'connectivity'

```

%*****
% Function connectivity evaluates the connection of the elements of finite
% element mesh
%*****

function [GLX,CONN] = connectivity(NDOF,NNODS,DX2,NODPELM);

if (NDOF==1)
    % Generate a mesh of the unit square with linear elements

```

```

    NELM = NNODS-1;

% Generate node coordinates
GLX = zeros(NNODS,1);
GLX(1) = DX2(2);
if(NODPELM==2)
    for J=1:NELM
        GLX(J+1) = GLX(J)+DX2(J+1);
    end
end

% Generate element connectivity
CONN = zeros(NELM, 2);
N = 0;
for J=1:NNODS-1
    N = N + 1;
    CONN(N,:) = [J,J+1];
end
end

```

4) Subroutine 'oned_gauss'

```

%*****
% Function oned_gauss define the gauss points
%*****

function [R,W] = oned_gauss(NGP);
% Usage: [R,W] = oned_gauss(NGP)

% Variables: NGP
% NGP = Number of Gauss points:
% R = Gauss points located between (-1, 1)
% W = Gauss weights corresponding to R

R = zeros(NGP,1);
W = zeros(NGP,1);

if NGP == 1
    R(1) = 0;
    W(1) = 2;
elseif NGP == 2
    R(1) = -1.0d0 / sqrt(3.0d0);
    R(2) = -R(1);
    W(1) = 1.0;
    W(2) = 1.0;
elseif NGP == 3
    R(1) = -sqrt(3.0d0/5.0d0);
    R(2) = 0.0;
    R(3) = -R(1);
    W(1) = 5.0d0 / 9.0d0;
    W(2) = 8.0d0 / 9.0d0;
    W(3) = W(1);
elseif NGP == 4
    R(1) = -sqrt((3.0d0+2.0*sqrt(6.0d0/5.0d0))/7.0d0);
    R(2) = -sqrt((3.0d0-2.0*sqrt(6.0d0/5.0d0))/7.0d0);
    R(3) = -R(2);
    R(4) = -R(1);
    W(1) = 0.5d0 - 1.0d0 / ( 6.0d0 * sqrt(6.0d0/5.0d0) );
    W(2) = 0.5d0 + 1.0d0 / ( 6.0d0 * sqrt(6.0d0/5.0d0) );
    W(3) = W(2);
    W(4) = W(1);

```

```

elseif NGP == 5
    R(1) = -sqrt(5.0d0+4.0d0*sqrt(5.0d0/14.0d0)) / 3.0d0;
    R(2) = -sqrt(5.0d0-4.0d0*sqrt(5.0d0/14.0d0)) / 3.0d0;
    R(3) = 0.0d0;
    R(4) = -R(2);
    R(5) = -R(1);
    W(1) = 161.0d0/450.0d0-13.0d0/(180.d0*sqrt(5.0d0/14.0d0));
    W(2) = 161.0d0/450.0d0+13.0d0/(180.d0*sqrt(5.0d0/14.0d0));
    W(3) = 128.0d0/225.0d0;
    W(4) = W(2);
    W(5) = W(1);
elseif NGP == 6
    R(1) = -0.2386191861;
    R(2) = -0.6612093865;
    R(3) = -0.9324695142;
    R(4) = - R(1);
    R(5) = - R(2);
    R(6) = - R(3);
    W(1) = .4679139346;
    W(2) = .3607615730;
    W(3) = .1713244924;
    W(4) = W(1);
    W(5) = W(2);
    W(6) = W(3);
else
    error('Quadrature rule not supported');
    keyboard;
end

```

5) Subroutine 'coefmatrx'

```

%*****
% Function coefmatrx evaluation of coefficient matrices
%*****

function [ELK,ELF,ELM] = coefmatrx(NDOF,NODPELM,N,NODN,EPS,EPS0,INTPFUN,NELM,...
NNODS,NGP,ELX,ELA,A1,A2,TS,TSUM,C0);

% X    = Global coordinates GLX(I)
% XI   = Local normalized coordinate ELX(I)
% L    = Element length
% {SF} = Shape function or element interpolation function
% {EDSF}= First derivative of shape function
% J     = Jacobian of transformation
% [GAUSPT] = 4x1 Matrix of Gauss points
% [GAUSWT] = 4x1 Matrix Gauss weight
% [ELK] = Element coefficient matrix, [K]
% [ELM] = Element mass matrix
% {ELF} = Element source vector, {F}
% EPS   = Algebraic values of effective pressure at each node

% Constants
A      = 0.033;
PHISO = 0.03;
US0    = 0.00003;
B      = 0.361;           % 3.08*(0.65-C0)^4;
D      = 1.234;           % 5.19*(0.65-C0)^2.68;
EK     = 10^(-14);        % 1.869*10^(-11.0);
RHOS   = 2300;
RHOL   = 1000;

```

```

DRHO = 1300;
MU = 0.001;
H0 = 0.4;

% Calculation for Gauss points
[GAUSPT, GAUSWT] = ONED_GAUSS(NGP);
NN = NODPELM*NDOF;

% Length of an element
L = ELX(NODPELM)-ELX(1);

% Initialization of all array
for I = 1:NN
    ELF1(I) = 0.0; % Force vector estimated at time T
    ELF(I) = 0.0; % Force vector estimated at time T+TS
    for J = 1:NN
        ELK1(I,J) = 0.0; % K matrix at time T
        ELK(I,J) = 0.0; % K matrix at time T+TS
        ELA(I,J) = 0.0; % Extra term in K tangent at T+TS
        ELM(I,J) = 0.0;
    end
end

%***** Calculation of force vector*****

if (N == 1)
    for I = 1:NODPELM
        if(I == 1)
            ELF1(I) = 9.81*EK*(1/A)*DRHO*C0*((1+A*EPS0(1)))^(B-D+1)/(MU*B);
            ELF(I) = 9.81*EK*(1/A)*DRHO*C0*((1+A*EPS(1)))^(B-D+1)/(MU*B);
        else
            ELF1(I) = 0.0;
            ELF(I) = 0.0;
        end
    end
elseif(N == NELM)
    for I = 1 : NODPELM
        if(I < NODPELM)
            ELF1(I) = 0.0;
            ELF(I) = 0.0;
        else
            ELF1(I) = (EK/(A*MU*B))*(EPS0(NODPELM)*(1/L)+EPS0(NODPELM-1)*(-1/L));
            ELF(I) = (EK/(A*MU*B))*(EPS(NODPELM)*(1/L)+EPS(NODPELM-1)*(-1/L));
        end
    end
elseif(N > 1 & N < NELM)
    for I = 1 : NODPELM
        ELF1(I) = 0.0;
        ELF(I) = 0.0;
    end
end

% Apply loop on Gauss point
for NODN = 1 : NGP
    XI = GAUSPT(NODN);

    % Evaluation of interpolation function and their derivatives at Gauss point

    % Lagrange interpolation functions used for linear and quadratic approximation
    % of second equation

```

```

if (INTPFUN == 1)
    % LINEAR INTERPOLATION FUNCTIONS(NODPELM=2)
    SF(1) = 0.5*(1.0-XI);
    SF(2) = 0.5*(1.0+XI);
    EDSF(1) = -0.5;
    EDSF(2) = 0.5;
elseif( INTPFUN>1)
    % QUADRATIC INTERPOATION FUNCTION (NODPELM=3)
    SF(1) = -0.5*XI*(1.0-XI);
    SF(2) = 1.0 - XI*XI;
    SF(3) = 0.5*XI*(1.0+XI);
    EDSF(1) = -0.5*(1.0-2.0*XI);
    EDSF(2) = -2.0*XI;
    EDSF(3) = 0.5*(1.0+2.0*XI);
end

% Compute derivative of interpolation function w.r.t. X
J = 0.5*L;
for I = 1:NODPELM
    GDSF(I) = EDSF(I)/J;
end
%*****

CONST = J*GAUSWT(NODN);
X = ELX(1)+0.5*L*(1.0+XI);
PS1 = 0.0;
DPSDX1 = 0.0;
PS2 = 0.0;
DPSDX2 = 0.0;

for I = 1:NODPELM
    % For the time T
    PS1 = EPS0(I)*SF(I)+PS1;
    DPSDX1 = EPS0(I)*GDSF(I) +DPSDX1;
    % For the time T+TS
    PS2 = EPS(I)*SF(I)+PS2;
    DPSDX2 = EPS(I)*GDSF(I) + DPSDX2;
end

% Calculation of [K] matrices
for I = 1:NN
    for J = 1:NN % NN = NODPELM*DOF.

        % Calculation of [K] matrix at time T
        ELK1(I,J) = ELK1(I,J)+ CONST*(GDSF(I)*GDSF(J)*EK*(1/(A*MU*B))*...
            (1+PS1*A)^(1-D)- SF(I)*GDSF(J)*((9.81*DRHO*C0*EK*...
            (2*B-D))/(MU*B))*(1+A*PS1)^(B-D)+SF(I)*GDSF(J)*...
            (DPSDX1)*EK*(1-B)*(1/(MU*B))*(1+A*PS1)^(-D));

        % Calculation of [K] matrix at time T+TS
        ELK(I,J) = ELK(I,J)+ CONST*(GDSF(I)*GDSF(J)*EK*(1/(A*MU*B))*...
            (1+PS2*A)^(1-D)- SF(I)*GDSF(J)*((9.81*DRHO*C0*EK*...
            (2*B-D))/(MU*B))*(1+A*PS2)^(B-D)+ SF(I)*GDSF(J)*...
            (DPSDX2)*EK*(1-B)*(1/(MU*B))*(1+A*PS2)^(-D));

        % Calculation of mass matrix
        ELM(I,J) = ELM(I,J)+ SF(I)*SF(J)*CONST;

        % Extra term of K tangent at time T+TS
        ELA(I,J) = ELA(I,J)+(A1*(EK/(MU*B))*GDSF(I)*SF(J)*(1-D)*DPSDX2*...
            (1+PS2*A)^(-D)- A1*SF(I)*SF(J)*(B-D)*DPSDX2*(2*B-D)*...
            ((9.81*DRHO*C0*EK)/(MU*B/A))*(1+A*PS2)^(B-D-1)+ A1*...

```



```

end
end
end
end

```

6) subroutine 'solver'

```

%*****
% Function solver gives the solution of unsymmetrical system of equations for
% effective pressure
%*****

function [GLK] = solver(GLK,MXEQ,NEQ,ITERM);

CERO = 1.D-8;
%CERO = 0.2;
PARE = CERO^2;
NBND = 2*ITERM;
NBM = NBND-1;

% Begin elimination of the lower left
GLK;
for I=1:NEQ
    if(abs(GLK(I,ITERM))<CERO)
        if(abs(GLK(I,ITERM))<PARE)
            fprintf('Computation stopped due to zero in the main diagonal')
        end
    else
        JLAST = min(I+ITERM-1,NEQ);
        M = ITERM+1;
        for J = I:JLAST
            M = M-1;
            if(abs(GLK(J,M))>=PARE)
                C = GLK(J,M);
                for K = M:NBND
                    GLK(J,K) = GLK(J,K)/C;
                end
                if(I==NEQ)
                    M= ITERM-1;
                    for IT =2:NEQ
                        for JT= 1:M
                            if (NEQ+1-IT+JT<=NEQ)
                                GLK(NEQ+1-IT,NBND)= GLK(NEQ+1-IT,NBND)-...
                                    GLK(NEQ+1-IT+JT,NBND)*GLK(NEQ+1-IT,ITERM+JT);
                            end
                        end
                    end
                end
            end
        end
    else
        M = 0.0;
        JFIRST = I+1;
        if(JLAST<I)
            M= ITERM-1;
            for IT =2:NEQ
                for JT= 1:M
                    if (NEQ+1-IT+JT<=NEQ)
                        GLK(NEQ+1-IT,NBND)= GLK(NEQ+1-IT,NBND)-...
                            GLK(NEQ+1-IT+JT,NBND)*GLK(NEQ+1-IT,ITERM+JT);
                    end
                end
            end
        end
    end
end
end

```

```

end
else
for JT = JFIRST:JLAST
M = M+1;
if( abs(GLK(JT,ITERM-M))<PARE)
M= ITERM-1;
for IT =2:NEQ
for JTT= 1:M
if (NEQ+1-IT+JTT<=NEQ)
GLK(NEQ+1-IT,NBND)= GLK(NEQ+1-...
IT,NBND)-GLK(NEQ+1-IT+JTT,...
NBND)*GLK(NEQ+1-IT,ITERM+JTT);
end
end
end
else
for K = ITERM:NBM
GLK(JT,K-M) = GLK(JT-M,K)-GLK(JT,K-M);
end
GLK(JT,NBND) = GLK(JT-M,NBND)-GLK(JT,NBND);
if(I>NEQ-ITERM+1)
M= ITERM-1;
for IT =2:NEQ
for JTT= 1:M
if (NEQ+1-IT+JTT<=NEQ)
GLK(NEQ+1-IT,NBND)= GLK(NEQ+1-...
IT,NBND)-GLK(NEQ+1-IT+JTT,...
NBND)*GLK(NEQ+1-IT,ITERM+JTT);
end
end
end
else
for K =1:M
GLK(JT,NBND-K) = -GLK(JT,NBND-K);
end
end
end
end
end
M= ITERM-1;
for IT =2:NEQ
for JT= 1:M
if (NEQ+1-IT+JT<=NEQ)
GLK(NEQ+1-IT,NBND)= GLK(NEQ+1-IT,NBND)-GLK(NEQ+1-...
IT+JT,NBND)*GLK(NEQ+1-IT,ITERM+JT);
end
end
end
end
M = ITERM;
end
end
end

```

7) Subroutine 'sediment_fs'

```

%*****
% Function sediment_fs calculates the height of batch curve with the consideration
% of characteristics present in the suspension region
%*****

```

```

function [PHIS,PHIS1,PHIS2,SUS,FALX,X2,MUDLINE_H2,TOTALX,TSUM,TS,CHAR,TS_START,...
V,TS_REACH,TOTAL_TS_REACH,NPC,N,W] = sediment_fs(Z0,GLX,N,CHAR,NPC,TS_START,...
TS_REACH,TOTAL_TS_REACH,NELM,X0,X1,X2,TOTALX,FALX,TS,TS1,TSUM,TDDX1,TDDX2,C1,...
ES,PHIS1,PHIS,US2,SUS,NNODS,C0,V);

% V          = Velocity of characteristic
% MUDLINE_H2 = Mudline interface height
% PHIS2      = Solid concentration in the suspension region at T2 = T1 + TS
% SUS        = Solid settling velocity at T2 = T1 + TS
% FALX       = The height of the falling solids on the top of the bed during a
% time period TS
% TOTALX     = Total height of the solids at T2 = T1 + TS
% TS_START(CHAR) = Starting time of a characteristic

% Empirical parameters
H0 = 0.4;
PHIS0 = 0.03;
SUS0 = 0.00003;

%*****Computation for the point of generation of next characteristic*****

DX2(CHAR) = X2(CHAR)/NELM;

% Calculation of the concentration of characteristic for given characteristic number

% Define a parameter
Z =1;

if ( CHAR <3)
    US2(NNODS-1) = 0.0;
end
if(CHAR ==2)
    PHIS(CHAR) = PHIS(CHAR-1);
    PHIS2      = PHIS(CHAR);
else
    PHIS_IP = PHIS1;
    [PHI]   = newraps(US2,NNODS,PHIS_IP,C0);
    PHIS2   = PHI;
    while(PHIS2<=PHIS1 & Z<500 & PHIS2>C0)
        % PHIS_IP = input('enter the value of solid concentration greater than phis1
        % and less than c0');
        PHIS_IP = PHIS_IP+0.0001;
        [PHI]   = newraps(US2,NNODS,PHIS_IP,C0);
        PHIS2   = PHI;
        Z       = Z+1;
    end
end

if( Z>=500 | PHIS2<PHIS1 | PHIS2>C0)
    PHIS(CHAR) = PHIS(CHAR-1);
else
    PHIS(CHAR) = PHIS2;
end

% Velocity of the solid settling velocity
SUS(CHAR) = abs(0.0000946754- 0.0026552*PHIS(CHAR)+ 0.018926*PHIS(CHAR)...
*PHIS(CHAR));
FLUX(CHAR) = 0.0000946754*PHIS(CHAR)- 0.0026552*PHIS(CHAR)^2 + 0.018926...
*PHIS(CHAR)^3;

% Plot of solid settling velocity with time

```

```

if(Z0==1 | Z0==2 | Z0==3 | Z0==5 | Z0==7 | Z0==8 | Z0==9 | Z0==10)
    U_S = SUS (CHAR);
    subplot(4,2,7);
    plot(TSUM, U_S, '*');
    xlabel('Time (sec)');
    ylabel('Settling velocity (m/sec)');
    title( '(g) Settling velocity vs Time');
    hold on;
end

% Plot of flux with concentration
if(Z0==1 | Z0==2 | Z0==3 | Z0==5 | Z0==7 | Z0==8 | Z0==9 | Z0==10)
    FLUX_S = FLUX(CHAR);
    C_S = PHIS(CHAR);
    subplot(4,2,8);
    plot(C_S, FLUX_S, '*');
    xlabel('Concentration');
    ylabel('Flux (m/sec)');
    title( '(h) Flux vs Concentration');
    hold on;
end

% Velocity of the characteristic
V(CHAR) = abs((SUS(CHAR)*PHIS(CHAR) - C0* US2(NNODS-1))/(PHIS(CHAR) - C0));

% Calculation of amount of solids in consolidation region
SUM1 = 0.0;
for I = 1: NNODS-1
    SUM1 = SUM1 + ES(I)*DX2(CHAR);
end

% Condition on the second and last characteristics generated in batch thickening
% phenomenon
if(CHAR==2)
    TS_REACH(CHAR) = TS_REACH(CHAR-1)-TS1;
    TOTAL_TS_REACH(CHAR) = TOTAL_TS_REACH(CHAR-1);
else
    if( PHIS0*H0 >= SUM1)
        TS_REACH(CHAR) = (PHIS0*H0 - SUM1)/(PHIS(CHAR)*(V(CHAR) + SUS(CHAR)));
    else
        TS_REACH(CHAR) = (0.00001)/(PHIS(CHAR)*(V(CHAR)+ SUS(CHAR)));
    end

    % Calculation of total time taken by a characteristic to reach mudline interface
    % (batch curve)
    TOTAL_TS_REACH(CHAR) = TS_REACH(CHAR) + TS_START(CHAR);
end

%*****
%*****Computation for the batch curve *****

% Total time at the point of generation of next characteristic
TSUM = TSUM + TS;

% Number of characteristics
CHAR = CHAR+1;

% Time of starting of the characteristic
TS_START(CHAR)= TS_START(CHAR-1)+TS;

```

```

% Height of falling material at the rising of characteristic
FALX(CHAR) = SUS(CHAR-1)*TS;

% Total height of consolidation region at the point of generation of characteristic
TOTALX(CHAR) = FALX(CHAR) + X2(CHAR-1);

% Determination of the number of characteristic reached the batch curve
if(abs(TSUM -TOTAL_TS_REACH(NPC+1))<10)
    NPC = NPC+1;
end

if(abs(TSUM - TOTAL_TS_REACH(NPC))<10)
    MUDLINE_H2 = V(NPC)*(TSUM - TS_START(NPC)) + TOTALX(NPC);
    TS
        = (TOTAL_TS_REACH(NPC+1) - TOTAL_TS_REACH(NPC))/3;
    N
        = 0.0;
else
    for I = NPC+1:CHAR-1
        % Height of characteristic contribution region
        HEIGHT(I) = V(I)*(TSUM -TS_START(I));
    end
    HEIGHT(CHAR) = 0.0;

% Define parameters
SUM2 = 0.0;
SUM3 = GLX(NNODS);

% plot of concentration throughout the height of column
if(Z0==1| Z0==3| Z0==5| Z0==7)
    subplot(4,2,6);
    plot (ES,GLX,'*');
    title('(f) solid concentration in column');
    hold on;
end

if(NPC==1)
    HEIGHT2 = V(NPC)*(TSUM - TS_START(NPC)) + TOTALX(NPC);
    for I = CHAR-1:-1:1+NPC
        H_CHAR(I) = HEIGHT(I)-HEIGHT(I+1) -(TOTALX(I+1) - TOTALX(I));
        SUM2 = SUM2 + H_CHAR(I)*PHIS(I);
    end

% Height of batch curve
MUDLINE_H2 = (PHIS0*H0 - SUM1 - FALX(CHAR)*C0 - SUM2 + HEIGHT2*PHIS0)/PHIS0;

for I = CHAR-1:-1:1+NPC
    if (H_CHAR(I)<0)
        H_CHAR(I)=0.0;
    end
    SUM3 = SUM3 + H_CHAR(I);
    V_S = PHIS(I);

% Rest part of plot of concentration throughout the height of column
if(Z0==1| Z0==3| Z0==5| Z0==7)
    subplot(4,2,6);
    plot (V_S,SUM3,'*');
    xlabel('Concentration');
    ylabel('Height (m)');
    hold on;
end
end
end

```

```

% Rest part of plot of concentration throughout the height of column
if(Z0==1| Z0==3| Z0==5| Z0==7)
    subplot(4,2,6);
    plot (PHIS0,MUDLINE_H2,'*');
    hold on;
    plot (0, MUDLINE_H2,'*');
    xlabel('Concentration');
    ylabel('Height (m)');
    axis('normal');
    hold on;
end
else
HEIGHT2 = V(NPC+1)*(TSUM - TS_START(NPC+1)) + TOTALX(NPC+1);
for I = CHAR-1:-1:1+NPC
    H_CHAR(I) = HEIGHT(I)-HEIGHT(I+1) -(TOTALX(I+1) - TOTALX(I));
    SUM2 = SUM2 + H_CHAR(I)*PHIS(I);
end

% Height of batch curve
MUDLINE_H2 = (PHIS0*H0 - SUM1 - FALX(CHAR)*C0 - SUM2 + HEIGHT2*PHIS...
(NPC))/PHIS(NPC);
V_S_I = PHIS(NPC);
for I = CHAR-1:-1:1+NPC
    if (H_CHAR(I)<0)
        H_CHAR(I)=0.0;
    end
    SUM3 = SUM3 + H_CHAR(I);
    V_S = PHIS(I);

% Rest part of plot of concentration throughout the height of column
if(Z0==1| Z0==3| Z0==5| Z0==7)
    subplot(4,2,6);
    plot (V_S,SUM3,'*');
    xlabel('Concentration');
    ylabel('Height (m)');
    hold on;
end
end
%HEIGHT3 = MUDLINE_H2 - SUM3;

% Rest part of plot of concentration throughout the height of column
if(Z0==1| Z0==3| Z0==5| Z0==7)
    subplot(4,2,6);
    plot (V_S_I,MUDLINE_H2,'*');
    hold on;
    plot (0, MUDLINE_H2,'*');
    %pause
    xlabel('Concentration');
    ylabel('Height (m)');
    axis('normal');
    hold on;
end
end
end
end

N = N +1;
PHIS1 = PHIS2;

```

8) Subroutine 'newraps'

```

%*****
% Function newwraps evaluates the concentration of the characteristic
%*****

function [PHIS] = newwraps(US2,NNODS,PHIS_IP,C0);

% Empirical parameters for flux curve
B0 = 0.0000946754;
B1 = 0.0026552;
B2 = 0.018926;

% Equation need to solve for solid volume ratio, PHIS
A = 4*B2*PHIS_IP^3 + (3*B1-3*B2*C0)*PHIS_IP^2+ (2*B0 - 2*B1*C0)*PHIS_IP-C0*(B0+...
    US2(NNODS-1));
% Derivative of A with respect to PHIS
B = 12*B2*PHIS_IP^2 + 2*(3*B1-3*B2*C0)*PHIS_IP+(2*B0 - 2*B1*C0);

while(B ==0)
    % PHIS_IP = input('enter the value of solid concentration greater than phis0...
    % and less than c0')
    A = 4*B2*PHIS_IP^3 + (3*B1-3*B2*C0)*PHIS_IP^2+ (2*B0 - 2*B1*C0)*...
        PHIS_IP-C0*(B0+US2(NNODS-1));
    B = 12*B2*PHIS_IP^2 + 2*(3*B1-3*B2*C0)*PHIS_IP+(2*B0 - 2*B1*C0);
end
if (A/B>0)
    PHIS2 = PHIS_IP - A/B;
    if (abs( PHIS_IP-PHIS2)<= 0.01)
        PHIS = PHIS2;
    else
        PHIS_IP = PHIS2;
        PHIS = newwraps(US2,NNODS,PHIS_IP,C0);
    end
else (A/B<0)
    PHIS2 = PHIS_IP + abs(A/B);
    if (abs( PHIS2-PHIS_IP)<= 0.01)
        PHIS = PHIS2;
    else
        PHIS_IP = PHIS2;
        PHIS = newwraps(US2,NNODS,PHIS_IP,C0);
    end
end
end
A = 4*B2*PHIS^3 + (3*B1-3*B2*C0)*PHIS^2+ (2*B0 - 2*B1*C0)*PHIS-C0*(B0+...
    US2(NNODS-1));

```

9) Large strain main program 'large_strain' based on consolidation model of Gutierrez (2003)

```

%*****
% Simulation of the batch thickening phenomenon for young sediments for large
% strain model given by Gutierrez (2003)
% This calculates:
% The height of batch curve and L-curve with time
% The solid concentration throughout the column at any time instant
% The effective pressure variation in consolidation region at any time instant
% The permeability in consolidation region
% The compressibility in consolidation region
% The velocity of characteristics present at any time instant in suspension region
%*****

```

```

function[]= large_strain;

%*****
% Definition of variables used in the program
%*****

% C0      = Initial concentration
% NNODS   = Number of nodes in FE mesh
% NODMESH = Number of nodes in FE mesh
% NODN    = Global node number
% NODPELM = Number of nodes per element

%*****
% Initialization of required matrices
%*****

% MXNOD   = input('enter the max number of nodes used in FE mesh');
MXNOD    = 3;

% MXEQ    = input('enter the max number of equation used to solve FE mesh');
MXEQ     = MXNOD;

% MXELM   = input('enter the max number of element used in FE mesh');
MXELM    = MXNOD-1;

% The height of sublayers in consolidation region at any time T + TS
DX2      = zeros(MXNOD,1);

% The decrement in height of sublayers in consolidation region
% at ant time T + TS
DDX2     = zeros(MXNOD,1);

% The height of sublayers in consolidation region at any time T
DX1      = zeros(MXNOD,1);

% The decrement in height of sublayers in consolidation region
% at ant time T
DDX1     = zeros(MXNOD,1);

% Effective pressure
sigv     = zeros(MXEQ);
SOLVEC  = zeros(MXEQ,1);
GPSO    = zeros(MXEQ,1);
GPSOAV  = zeros(MXEQ,1);

% Consolidation rate in consolidation region
MV       = zeros(MXNOD,1);

% Average linear concentration in consolidation sublayer at any time T
C2AV    = zeros(MXEQ,1);

% Average linear concentration in consolidation sublayer at time T+TS
C1AV    = zeros(MXEQ,1);

% Velocity of upward moving liquid coming out from sublayers of consolidation
% region
UL2     = zeros(MXNOD,1);

% Velocity of settling down solids in sublayer of consolidation region

```

```

US2      = zeros(MXNOD,1);

%*****
% Input data
%*****

% TS      = input('enter the time step');
% TMAX    = input('enter the max value of time that can be reach in a time
%          loop');
% X1      = input('enter the thickness of solids at T');
% TDDX2   = input('assume the total decrease in thickness at time T + TS');
% FALX    = input(' enter the height of falling material at time T');
% MxEQ    = (MXELM+1)*3;

% NODPELM = input( 'enter the number of nodes per element in a FE mesh');
NODPELM  = 2;

% NDOF    = input('enter the degree of freedom of nodes');
NDOF     = 1;

% Number of nodes calculation
% NELM    = input('enter the number of elements in FE mesh');
NELM     = MXELM;

NNODS    = NELM*(NODPELM-1)+1;

DOFPELM  = NODPELM*NDOF;

NELM1    = NELM+1;

% Number of equation required
NEQ      = NNODS*NDOF;

%*****
% Input parameters
%*****

% Values of empirical parameters
PHIS0    = 0.03;
SUS0     = abs(0.0000946754 - 0.0026552*PHIS0 + 0.018926*PHIS0*PHIS0);
SUS0     = 0.00003;
C0       = 0.075;
EK       = 10^(-14);      %1.869*10^(-11.0);
RHOS     = 2300;
RHOL     = 1000;
DRHO     = 1300;
MU       = 0.001;
H0       = 0.40;

%*****First Kynch characteristic at T=0*****

% X0 = input('enter the height of bed at T0')
X0      = 0.0;

% Number of characteristic at initial time
CHAR    = 1;

% Define a parameter
NPC     = 1;

% Total time

```

```

TSUM = 0.0;

% Height of supernatant-suspension interface
MUDLINE_H2 = 0.4;

% Total height of consolidation layer
TOTALX(Char)= 0.0;
CON = TOTALX(Char);

% Plot of batch curve and L-curve
% subplot(2,2,1);
plot (TSUM,MUDLINE_H2,'*');
hold on;
plot (TSUM,CON,'o');
% axis([0 15000 0 0.5]);
axis('normal');

% Sediment deposited in first time step of Kynch
C0 = 0.036;

% Concentration of characteristic
PHIS(Char) = 0.03;
PHIS1 = PHIS(Char);

% Solid settling velocity
% SUS(Char) = abs(0.000119711- 0.00428381*PHIS(Char)+ 0.0418934*PHIS(Char)...
% *PHIS(Char));
SUS(Char) = abs(0.0000946754 - 0.0026552*PHIS(Char) + 0.018926*PHIS(Char)...
*PHIS(Char));

% Velocity of characteristic
V(Char) = abs(0.0000946754 - 2*0.0026552*PHIS(Char) + 3*0.018926*PHIS...
(Char)^2);

% Starting time of the characteristic
TS_START (Char) = 0.0;

% Calculation of the time for characteristic to reach the supernatant-suspension
% interface
TS_REACH(Char) = H0*PHIS0/(PHIS(Char)*(V(Char) + SUS(Char)));

%*****Second Kynch characteristic*****

% Calculation of concentration of characteristic
PHIS_IP = PHIS1;
PHIS0 = PHIS1;
US2(NNODS-1) = 0.0;
C0 = 0.045;
[PHI] = newraps(US2,NNODS,PHIS_IP,C0);
PHIS2 = PHI;

% Define parameters
Z = 1;
N = 1;

while(PHIS2<=PHIS1 & Z<200 )
% PHIS_IP = input('enter the value of solid concentration greater than phis1
% and less than c0');
PHIS_IP = PHIS_IP+0.01;
[PHI] = newraps(US2,NNODS,PHIS_IP,C0);

```

```

        PHIS2 = PHI;
        Z     = Z+1;
    end

% Solid concentration above the consolidation-suspension interface at initial time
if(Z>200 | PHIS2< PHIS1)
    PHIS(CHAR) = PHIS0;
else
    PHIS(CHAR) = PHIS2;
end

PHIS1 = PHIS(CHAR);

% Solis settling velocity
SUS(CHAR) = abs(0.0000946754 - 0.0026552*PHIS(CHAR) + 0.018926*PHIS(CHAR)...
    *PHIS(CHAR))

% Velocity of characteristic
V(CHAR) = abs(0.0000946754 - 2*0.0026552*PHIS(CHAR) + 3*0.018926*PHIS...
    (CHAR)^2)

% Starting time of the characteristic
TS_START (CHAR) = 0.0;

% Calculation of the time for characteristic to reach the supernatant-suspension
% interface
TS_REACH(CHAR) = H0*PHIS0/(PHIS(CHAR)*(V(CHAR) + SUS(CHAR)));

%*****Third Kynch characteristic *****

% Calculation of concentration of characteristic
PHIS_IP = PHIS1;
PHIS0 = PHIS1;
US2(NNODS-1) = 0.0
C0 = 0.048
[PHI] = newraps(US2,NNODS,PHIS_IP,C0);
PHIS2 = PHI;

% Define parameters
Z = 1;
N = 1;

while(PHIS2<=PHIS1 & Z<200 )
    % PHIS_IP = input('enter the value of solid concentration greater than phis1
    % and less than c0');
    PHIS_IP = PHIS_IP+0.01;
    [PHI] = newraps(US2,NNODS,PHIS_IP,C0);
    PHIS2 = PHI;
    Z = Z+1
end

% Solid concentration above the consolidation-suspension interface at initial time
if(Z>200 | PHIS2< PHIS1)
    PHIS(CHAR) = PHIS0;
else
    PHIS(CHAR) = PHIS2;
end

PHIS1 = PHIS(CHAR);

% Solid settling velocity

```

```

SUS(Char) = abs(0.0000946754 - 0.0026552*PHIS(Char) + 0.018926*PHIS(Char)...
            *PHIS(Char));

% Velocity of characteristic
V(Char) = abs(0.0000946754 - 2*0.0026552*PHIS(Char) + 3*0.018926*PHIS...
            (Char)^2);

% Starting time of the characteristic
TS_START (Char) = 0.0;

% Calculation of the time for characteristic to reach the supernatant-suspension
% interface
TS_REACH(Char) = H0*PHIS0/(PHIS(Char)*(V(Char) + SUS(Char)));

%*****FOURTH KYNCH CHARACTERISTICS*****

% Calculation of concentration of characteristic
PHIS_IP = PHIS1;
US2(NNODS-1) = 0.0
C0 = 0.056
[PHI] = newraps(US2,NNODS,PHIS_IP,C0);
PHIS2 = PHI;

% Define parameters
Z = 1;
N = 1;

while(PHIS2<= PHIS1 & Z<200 )
    % PHIS_IP = input('enter the value of solid concentration greater than phis1
    % and less than c0');
    PHIS_IP = PHIS_IP+0.01;
    [PHI] = newraps(US2,NNODS,PHIS_IP,C0);
    PHIS2 = PHI;
    Z = Z+1;
end

% Solid concentration above the consolidation-suspension interface at initial time
if( Z>200 | PHIS2< PHIS1)
    PHIS(Char) = PHIS0;
else
    PHIS(Char) = PHIS2;
end

PHIS1 = PHIS(Char);

% Solid settling velocity
SUS(Char) = abs(0.0000946754 - 0.0026552*PHIS(Char) + 0.018926*PHIS(Char)...
            *PHIS(Char));

% Velocity of characteristic
V(Char) = abs(0.0000946754 - 2*0.0026552*PHIS(Char) + 3*0.018926*PHIS...
            (Char)^2);

% Starting time of the characteristic
TS_START (Char) = 0.0;

% Calculation of the time for characteristic to reach the supernatant-suspension
% interface
TS_REACH(Char) = H0*PHIS0/(PHIS(Char)*(V(Char) + SUS(Char)));

```

```

%*****Characteristic at the start of sediment layer*****

% Calculation of concentration of characteristic
PHIS1      = 0.047
PHIS_IP    = PHIS1;
US2(NNODS-1) = 0.0
C0         = 0.075
[PHI]      = newraps(US2,NNODS,PHIS_IP,C0);
PHIS2      = PHI;

% Define parameters
Z          = 1;
N          = 1;

while(PHIS2<=PHIS1 & Z<500 & PHIS2>C0 )
    % PHIS_IP = input('enter the value of solid concentration greater than phis1
    % and less than c0');
    PHIS_IP = PHIS_IP + 0.0001;
    [PHI]   = newraps(US2,NNODS,PHIS_IP,C0);
    PHIS2   = PHI;
    Z      = Z+1;
end

% Solid concentration above the consolidation-suspension interface at initial time
if( Z>500 | PHIS2< PHIS1 | PHIS2>C0)
    PHIS(CHAR) = PHIS1;
else
    PHIS(CHAR) = PHIS2;
end

PHIS1      = PHIS(CHAR);

% Solid settling velocity
% SUS(CHAR) = abs(0.000119711- 0.00428381*PHIS(CHAR)+ 0.0418934*PHIS(CHAR)...
%           *PHIS(CHAR));
SUS(CHAR) = abs(0.0000946754 - 0.0026552*PHIS(CHAR) + 0.018926*PHIS(CHAR)...
               *PHIS(CHAR));

% Velocity of characteristic
V(CHAR)    = SUS(CHAR)*PHIS(CHAR)/(C0 - PHIS(CHAR));

% Starting time of the characteristic
TS_START (CHAR) = 0.0;

% Calculation of the time for characteristic to reach the supernatant-suspension
% interface
TS_REACH(CHAR) = H0*PHIS0/(PHIS(CHAR)*(V(CHAR) + SUS(CHAR)));

% DEFINE THE TIME STEP
TS1          = TS_REACH(CHAR)/10;
SUS1         = SUS(CHAR);
TOTAL_TS_REACH(CHAR) = TS_REACH(CHAR) + (CHAR-1)*TS1;

%*****SECOND CHARACTERISTIC AT T = TS *****

% DEFINE THE QUANTITIES AT THE POINT OF GENERATION OF SECOND CHARACTERISTIC AFTER
% TIME PERIOD TS
CHAR        = CHAR +1;

% TIME OF STARTING OF THE GIVEN CHARACTERISTIC

```

```

TS_START(Char)= TS_START(Char-1)+TS1;
Tsum      = Tsum + TS1;

% PHIS_IP = input('enter the value of solid concentration greater than phis0 and
% less than c0')
PHIS_IP = PHIS1;

% THE HEIGHT OF CHARACTERISTIC
HEIGHT(Char-1) = V(Char-1)*(Tsum - TS_START(Char-1));

% HEIGHT OF FALLING MATERIAL AT THE RISING OF GIVEN NUMBER OF CHARACTERISTIC
FALX(Char)    = HEIGHT(Char-1);
X2(Char)     = 0.0;
X1(Char)     = FALX(Char);
TOTALX(Char) = X2(Char) + FALX(Char);
CON          = TOTALX(Char);
H_REACH(Char-1) = HEIGHT(Char-1) - TOTALX(Char);
MUDLINE_H2   = (H0*PHIS0 - FALX(Char)*C0 + HEIGHT(Char-1)*PHIS0)/PHIS0;

% THE SECOND CHARECTRISTIC COINCIDE WITH THE FIRST ONE BECAUSE CONSOLIDATION DOES
% NOT START TILL THE END OF TS1 THUS,
PHIS(Char) = PHIS(Char-1);
V(Char)    = V(Char-1);
SUS(Char)  = SUS(Char-1);
TOTAL_TS_REACH(Char) = TOTAL_TS_REACH(Char-1);

% DEFINE THE NEW TIME STEP FOR REST OF CHARACTERISTIS
TS = (TS_REACH(1)-TS1)/1000;

subplot(2,2,1);
plot (Tsum,MUDLINE_H2,'*');
hold on;
plot (Tsum,CON,'o');
xlabel('Time (sec)');
ylabel('Height (m)');
% axis([0 15000 0 0.5]);
legend ('Batch curve', 'L-curve');
title('(a) Batch curve and L-curve vs Time');
axis('normal');

% element length at initial stage
DX = X1(Char)/NELM;
for I =1:NEQ
    GPS0(I) = 9.81*C0*DRHO*(X1(Char) -(I-1)*DX);
    C1(I)    = C0;
end

% AVERAGE INITIAL CONCENTRATION IN SUBLAYER AT TIME T1
for I = 1:NNODS-1
    C1AV(I) = (C1(I+1)+C1(I))/2;
end

NELM_IN = NELM;
NNODS_IN = NNODS;

%*****LARGE STRAIN PARAMETERS*****

% Define parameter
LS = 1;

% Maximum time in which process finish
TMAX = 20000;

```

```

% Height of the falling material
FALX_L = FALX(CHAR);

% Maximum sedimentation rate
W0 = FALX_L/TS1*C0;

% Sedimentation rate
W = W0;

% Maximum effective pressure
PSMAX = TMAX*(RHOS-RHOL)*9.81*W0;

% Total height of sedimentation region at any time (Settling + Consolidated)
X1_L = TOTALX(CHAR);

% Number of equations
N = NEQ;

% Coordinate of element node in one dimension
X = zeros(N);

% Overpressure in consolidation region
U = zeros(N);

% Initialization of arrays
AA = zeros(N);
BB = zeros(N);
CC = zeros(N);
DD = zeros(N);
Z = zeros(N);
PHI = zeros(N);
PS = zeros(N);
PS0 = zeros(N);
PSD = zeros(N);
PSU = zeros(N);
PE = zeros(N);
RK = zeros(N);
DEPTH = zeros(N);
UU = zeros(N);
SIGV = zeros(N);

%*****

while(abs(MUDLINE_H2-TOTALX)> 0.005)
% Normalized sublayers height in consolidation region
DX = 1./(NEQ-1);

%*****
% Effective pressure and concentration in consolidation region
%*****

[ SIGV,DEPTH,UZ,ES,X,U,AA,BB,CC,DD,Z,PHI,PS,PSO,PSD,PSU,PE,RK,UU,LS] = ...
consol_ls(PSMAX,TMAX,W,NEQ,DX,TSUM,TS,X,U,AA,BB,CC,DD,LS,Z,PHI,PS,PSO,...
PSD,PSU,PE,RK,DEPTH,UU,SIGV);

% Depth of consolidation region
DEPTH_F = DEPTH(NEQ);

% Effective Pressure in consolidation region

```

```

GLF      = SIGV;
GPS0     = SIGV;

% Decrement in height due to consolidation in time step TS
DELTA_X1 = X1_L - DEPTH_F;
TDDX1    = DELTA_X1;
US2      = UZ;
X2(CHAR) = DEPTH_F;

%*****
% Calculation of height of batch curve
%*****

[PHIS,PHIS1,PHIS2,SUS,FALX,X2,MUDLINE_H2,TOTALX,TSUM,TS,CHAR,TS_START,V,...
TS_REACH,TOTAL_TS_REACH,NPC,N,W] = sediment_ls(N,CHAR,NPC,TS_START,...
TS_REACH,TOTAL_TS_REACH,NELM,X0,X1,X2,TOTALX,FALX,TS,TS1,TSUM,TDDX1,...
C1,ES,PHIS1,PHIS,US2,SUS,NNODS,C0,V);

X1_L = TOTALX(CHAR);

% Plots
subplot(2,2,1);
CON = TOTALX(CHAR);
plot (TSUM,MUDLINE_H2,'*');
hold on;
plot (TSUM,CON,'o');
xlabel('Time (sec)');
ylabel('Height (m)');
% axis([0 15000 0 0.5])
legend ('Batch curve', 'L-curve');
title( '(a) Batch curve and L-curve vs Time');
axis('normal');

% DEINE THE NEW SOLID VOLUME RATIO IN SUSENSION REGION FOR NEW TIME STEP
PHIS1 = PHIS2;
end % END OF THE batch curve and L- curve condition

```

10) Subroutine 'consol_ls'

```

%*****
% Function consol_ls calculates the effective pressure and solid concentration
% in consolidation region for large strain model given by Gutierrez (2003)
%*****

function[SIGV,DEPTH,UZ,ES,X,U,AA,BB,CC,DD,Z,PHI,PS,PSO,PSD,PSU,PE,RK,UU,LS]...
= consol_ls(PSMAX,TMAX,W,NEQ,DX,TSUM,TS,X,U,AA,BB,CC,DD,LS,Z,PHI,PS,PSO,...
PSD,PSU,PE,RK,DEPTH,UU,SIGV);

% Empirical parameters

```

```

GRAV = 9.81;
XKO = 10^(-14);
RHOS = 2300;
RHOW = 1000;
XMU = 0.001;
TO = TMAX
TM = TO;
L0 = XKO*(RHOS-RHOW)*GRAV/(XMU*W);

fprintf('Deposition rate, W (m/s) = %g \n', W);
fprintf('Surface permeability, KO (mD) = %g \n', XKO);
fprintf('Sediment density, RHOS (kg/m^3) = %g \n', RHOS);
fprintf('Fluid density, RHOW (kg/m^3) = %g \n', RHOW);
fprintf('Fluid viscosity, MU (Pa*s) = %g \n', XMU );
fprintf('Max eff vert stress, SVMAX (kPa) = %g \n', PSMAX);
fprintf('Ref. time, TO (s) = %g \n', TM );
fprintf('Gravity number, Lo ( ) = %g \n', L0);

N = NEQ;

for I=1:N
    X(I) = (I-1)*DX;
end

% NORMALIZED CURRENT TIME
T_N = TSUM/TMAX;

% NORMALIZED CURRENT TIME STEP
TS_N = TS/TMAX;

% CURRENT TIME
TT = TSUM;

for I=1:N
    UO(I) = U(I);
    if I>1
        UD(I) = U(I-1);
    else
        UD(I) = UO(I);
    end
    if I<N
        UU(i) = U(I+1);
    else
        UU(i) = UO(i);
    end
end

%
E = efunc(SIGV,NEQ);
EO = efunc(PSO,NEQ);
ED = efunc(PSD,NEQ);
EU = efunc(PSU,NEQ);

%
RKO = pfunc(PSO,NEQ);
RKD = pfunc(PSD,NEQ);
RKU = pfunc(PSU,NEQ);

for I=1:N
    RKO(I) = RKO(I)/(1+EO(I));
    RKD(I) = RKD(I)/(1+ED(I));
end

```

```

        RKU(I) = rku(I)/(1+EU(I));
    end

% Compressibility in consolidation region
    G = dfunc(PSO,NEQ);

% Permeability in consolidation region
    XK = pfunc(SIGV,NEQ);

    for I=1:N
        if I==N
            DUX(I) = (U(I)-U(I-1))/DX;
        else
            DUX(I) = (U(I+1)-U(I))/DX;
        end

        C(i) = L0*TS/(TS_N*DX*DX);

        AA(i) = G(i)*(TS_N+TS)-2*C(I)*(RKU(I) + 2*RKO(I) + RKD(I))/4.0;
        BB(i) = C(I)*(RKO(I) + RKD(I))/2.0;
        CC(i) = C(I)*(rko(I) + RKU(I))/2.0;
        DD(i) = G(I)*(U(I)*(TS_N+TS) + (X(I)*DUX(I) -U(I) + 1)*TS);
    end

% Application of Neumann boubadry conditions
    CC(1) = 0.0;
    AA(1) = -1.0;
    BB(1) = 1.0;
    DD(1) = 0.0;

% Application of Dirichlet boundary conditions
    CC(N) = 0.0;
    AA(N) = 1.0;
    BB(N) = 0.0;
    DD(N) = 0.0;

% Result included boundary conditions
    Z = thomas(AA,BB,CC,DD,N);

    for I=1:N
        U(I) = Z(I);
    end

    for I=N:-1:1
        PHI(I) = E(I)/(1+e(I));
        ES(I) = 1/(1+e(I));
        if I==N
            DUX(I) = -U(I-1)/DX;
        else
            DUX(I) = (U(I+1)-U(I))/DX;
        end

        if I==N
            Z(I) = 0.0;
        else
            Z(I) = Z(I+1) + DX/(1-PHI(I));
        end

        PE(I) = T_N*U(I);
        PS(I) = T_N*(1 -U(I)-X(I));
        PSO(I) = T_N*(1 -UO(I) - X(I));
        PSD(I) = T_N*(1 -ud(I) - X(I));
    end

```

```

PSU(I) = T_N*(1 -uu(I) - X(I));

UU(I) = T_N*U(I)*PSMAX;
SIGV(I) = PS(I)*PSMAX;
RK(I) = XK(I)*XKO;

UZ(I) = -L0*XK(I)*DUX(I)/(1+E(I));
if (I==N) & (UZ>0.5)
    UZ(I) = 0.5;
end
US = UZ(1);
end

for I=N:-1:1
    DEPTH(I) = W*TSUM*(Z(1)-Z(I));
end

if (LS==250 | LS ==500 | LS ==750 | LS ==1000 | LS ==1250 | LS==1500)
    hold on; subplot(2,2,2);
    plot(SIGV,DEPTH);
    xlabel('effective vertical stress (kPa)');
    ylabel('thickness (m)');
    title('(b) Effective Pressure vs Height of consolidation region');

    hold on; subplot(2,2,3);
    semilogx(RK, DEPTH);
    xlabel('permeability (m^2)');
    ylabel('thickness (m)');
    title('(c) Permeability vs Height of consolidation region');

    hold on; subplot(2,2,4);
    plot(PHI, DEPTH);
    xlabel('porosity');
    ylabel('thickness (m)');
    title('(d) Porosity vs Height of consolidation region');
    hold on
end
LS = LS+1;

```

11) Subroutine 'dfunc'

```

%*****
% Function dfunc calculates the compressibility in consolidation region
% for large strain model given by Gutierrez (2003)
%*****

function [G] = dfunc(SIGV,NEQ);

% Empirical parameters
A = 0.033;
B = 0.361;
C0 = 0.065;
for I = 1:NEQ
    % Compressibility calculation
    G(I) = -A*B/(C0*(1+A*SIGV(I))^(1+B));
end

```

12) Subroutine 'efunc'

```

%*****
% Function efunc calculates the void ratio in consolidation region
% for large strain model given by Gutierrez (2003)
%*****

```

```
function [E] = efunc(SIGV,NEQ);
```

```

% Empirical parameters
C0 = 0.065;
B = 0.361;
A = 0.033;
for I = 1:NEQ
    % Solid volume ratio
    ES(I) = C0*(1+A*SIGV(I))^(B);

    % CALCULATION OF POROSITY
    p(I) = 1 - ES(I);

    % VOID RATIO CALCULATION
    E(I) = 1/ES(I) - 1;
end

```

13) Subroutine 'pfunc'

```

%*****
% Function pfunc evaluates the permeability in the consolidation region
% of large strain model given by Gutierrez (2003)
%*****

```

```
function [K] = pfunc(SIGV,NEQ);
```

```

% Empirical equation
EK = 1.43*10^(-3);
D = 1.234;
A = 0.033;
for I = 1:NEQ
    K(I) = EK*(1+A*SIGV(I))^(D);
end

```

14) Subroutine 'thomas'

```

%*****
% Function thomas calculates the solution of tridigonal system of
% equations
%*****

```

```
function[X] = thomas(A,B,C,D,N);
```

```

% Solution of tridiagonal system of equations using thomas method
% A - main diagonal
% B - superdiagonal
% C - sub-diagonal
% D - R.H.S. vector
% X - solution vector
% N - order of matrix

```

```

G = zeros(N);
Q = zeros(N);
X = zeros(N);

```

```
Q(1) = A(1);
```

```

for I = 2:N
    Q(I) = A(I)- C(I)*B(I-1)/Q(I-1);
end

G(1) = D(1)/A(1);
for I = 2:N
    G(I) = (D(I)-C(I)*G(I-1))/Q(I);
end

X(N) = G(N);
for I = N-1:-1:1
    X(I) = G(I)-B(I)*X(I+1)/Q(I);
end

```

15) Subroutine 'sediment_ls'

```

%*****
% Function sediment_ls calculates the height of batch curve with the consideration
% of characteristics present in the suspension region
%*****

function [PHIS,PHIS1,PHIS2,SUS,FALX,X2,MUDLINE_H2,TOTALX,TSUM,TS,CHAR,TS_START,...
V,TS_REACH,TOTAL_TS_REACH,NPC,N,W] = sediment_ls(N,CHAR,NPC,TS_START,TS_REACH,...
TOTAL_TS_REACH,NELM,X0,X1,X2,TOTALX,FALX,TS,TS1,TSUM,TDDX1,C1,ES,PHIS1,PHIS,US2,...
SUS,NNODS,C0,V);

% V          = Velocity of characteristic
% MUDLINE_H2 = Mudline interface height
% PHIS2      = Solid concentration in the suspension region at T2 = T1 + TS
% SUS       = Solid settling velocity at T2 = T1 + TS
% FALX      = The height of the falling solids on the top of the bed during a time
%            period TS
% TOTALX    = Total height of the solids at T2 = T1 + TS
% TS_START(CHAR) = Starting time of a characteristic

% Empirical parameters
H0 = 0.4;
PHIS0 = 0.03;
SUS0 = 0.00003;

%*****Computation for the point of generation of next characteristic*****

DX2(CHAR) = X2(CHAR)/NELM;

% Calculation of the concentration of characteristic for given characteristic number
Z =1;
if ( CHAR <3)
    US2(NNODS-1) = 0.0;
end
if(CHAR ==2)
    PHIS(CHAR) = PHIS(CHAR-1);
    PHIS2      = PHIS(CHAR);
else
    PHIS_IP    = PHIS1;
    [PHI]      = NEWRAP12A(US2,NNODS,PHIS_IP,C0);
    PHIS2      = PHI;
    while(PHIS2<=PHIS1 & Z<500 & PHIS2>C0)
        % PHIS_IP = input('enter the value of solid concentration greater than phis1
        % and less than c0');
        PHIS_IP = PHIS_IP+0.0001;
    end
end

```

```

        [PHI]   = NEWRAP12A(US2,NNODS,PHIS_IP,C0);
        PHIS2  = PHI;
        Z      = Z+1;
    end
end

if( Z>=500 | PHIS2<PHIS1 | PHIS2>C0)
    PHIS(CHAR) = PHIS(CHAR-1);
else
    PHIS(CHAR) = PHIS2;
end

% Velocity of the solid settling velocity
SUS(CHAR) = abs(0.0000946754- 0.0026552*PHIS(CHAR)+ 0.018926*PHIS(CHAR)*...
    PHIS(CHAR));
FLUX(CHAR) = 0.0000946754*PHIS(CHAR)- 0.0026552*PHIS(CHAR)^2 + 0.018926*...
    PHIS(CHAR)^3;

% Velocity of the characteristic
V(CHAR) = abs((SUS(CHAR)*PHIS(CHAR) - C0* US2(NNODS-1))/(PHIS(CHAR) - C0));

% Calculation of amount of solids in consolidation region
SUM1 = 0.0;
for I = 1: NNODS-1
    SUM1 = SUM1 + ES(I)*DX2(CHAR);
end

% Condition on the second and last characteristics generated in batch thickening
% phenomenon
if(CHAR==2)
    TS_REACH(CHAR) = TS_REACH(CHAR-1)-TS1;
    TOTAL_TS_REACH(CHAR) = TOTAL_TS_REACH(CHAR-1);
else
    if( PHIS0*H0 >= SUM1)
        TS_REACH(CHAR) = (PHIS0*H0 - SUM1)/(PHIS(CHAR)*(V(CHAR) + SUS(CHAR)));
    else
        TS_REACH(CHAR) = (0.00001)/(PHIS(CHAR)*(V(CHAR)+ SUS(CHAR)));
    end

    % Calculation of total time taken by a characteristic to reach mudline interface
    % (batch curve)
    TOTAL_TS_REACH(CHAR) = TS_REACH(CHAR) + TS_START(CHAR);
end

%*****
%*****Computation for the batch curve *****
% Total time at the point of generation of next characteristic
TSUM = TSUM + TS;

% Number of characteristics
CHAR = CHAR+1;

% Time of starting of the characteristic
TS_START(CHAR)= TS_START(CHAR-1)+TS;

% Height of falling material at the rising of characteristic
FALX(CHAR) = SUS(CHAR-1)*TS;

%
```

```

W      = (FALX(Char)/TS)*PHIS(Char-1)

% Total height of consolidation region at the point of generation of characteristic
TOTALX(Char) = FALX(Char) + X2(Char-1);

% Determination of the number of characteristic reached the batch curve
if(abs(TSUM -TOTAL_TS_REACH(NPC+1))<10)
    NPC = NPC+1;
end

if(abs(TSUM - TOTAL_TS_REACH(NPC))<10)
    MUDLINE_H2 = V(NPC)*(TSUM - TS_START(NPC)) + TOTALX(NPC);
    TS        = (TOTAL_TS_REACH(NPC+1) - TOTAL_TS_REACH(NPC))/3;
    N          = 0.0;
else
    for I = NPC+1:Char-1
        % Height of characteristic contribution region
        HEIGHT(I) = V(I)*(TSUM -TS_START(I));
    end
    HEIGHT(Char) = 0.0;
    % Define parameter
    SUM2          = 0.0;
    if(NPC==1)
        HEIGHT2 = V(NPC)*(TSUM - TS_START(NPC)) + TOTALX(NPC);
        for I = Char-1:-1:1+NPC
            H_CHAR(I) = HEIGHT(I)-HEIGHT(I+1) -(TOTALX(I+1) - TOTALX(I));
            SUM2      = SUM2 + H_CHAR(I)*PHIS(I);
        end
        % Height of batch curve
        MUDLINE_H2    = (PHIS0*H0 - SUM1 - FALX(Char)*C0 - SUM2 + HEIGHT2*...
            PHIS0)/PHIS0;
    else
        HEIGHT2      = V(NPC+1)*(TSUM - TS_START(NPC+1)) + TOTALX(NPC+1);
        for I = Char-1:-1:1+NPC
            H_CHAR(I) = HEIGHT(I)-HEIGHT(I+1) -(TOTALX(I+1) - TOTALX(I));
            SUM2      = SUM2 + H_CHAR(I)*PHIS(I);
        end
        % Height of batch curve
        MUDLINE_H2    = (PHIS0*H0 - SUM1 - FALX(Char)*C0 - SUM2 + HEIGHT2*PHIS...
            (NPC))/PHIS(NPC);
    end
end

end

N = N +1;
PHIS1 = PHIS2;

```

VITA

The author was born on August 25, 1978 in India. He was enrolled in Indian Institute of Technology (I. I. T.), Roorkee in 1998 in the program of Civil Engineering and completed his Bachelor of Technology (B.Tech) in May 2002. He was admitted to Virginia Polytechnic Institute and State University in August 2002 as a Master of Science (MS) student in the program of Civil engineering. During the study, he was awarded with Graduate Teaching Assistantship (GTA) and Graduate Research Assistantship (GRA).

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