

**An Analytical Study of the Weak Radiating Cell as a Passive  
Low Frequency Noise Control Device**

by

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## **(ABSTRACT)**

At low frequency where the acoustic wavelength is greater than the size of the vibrating structure, the radiated acoustic power is directly related to the volume velocity of the structure. Thus, minimizing the volume velocity is an effective noise reduction approach for low frequency structurally radiated noise. This thesis analytically investigates a *passive* volume velocity noise control device for acoustic surface treatment of planar structures. The device is referred to as a weak radiating cell. This device consists of two mechanically coupled surfaces such that, when placed on a vibrating structure, the response of the two surfaces are nearly out-of-phase and of equal strength over a wide frequency range. The response of the two surfaces forms a local acoustic dipole, with minimum volume velocity, that results in noise reduction. Thus, the control of low frequency structurally radiated noise is achieved by covering the structure with an array of these weak radiating cells. Several numerical models are developed to investigate the weak radiating cell noise control mechanisms. The first model consists of a simply supported beam treated with an array of weak radiating cell. In this model, the dynamic interaction effects between the beam and the cells are included. Results from this model predict an overall sound power level reduction of 9.8 dB between 0-1600 Hz and 20 dB

between 0–251 Hz. In addition, this model is used to investigate techniques to improve the noise reduction capabilities of the device. A model of weak radiating cells applied to a simply supported plate is next developed as an extension of the beam model. The results from this model are compared to previous experimental data. Good agreement is observed between results, which validates the modeling technique. Lastly, a model of an infinite 2D plate treated with weak radiating cells is developed. The model does not consider any dynamic interaction effect between the structure and the cells. Only the acoustic behavior of the weak radiating cell is included in this model. In addition, both the structural and acoustic responses are obtained in closed form through a wavenumber transform approach. Each of these models and their results offer valuable information that results in a better understanding the weak radiating cell and its potential as a low frequency passive noise control device.

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