CHAPTER 1

INTRODUCTION

1.1 DISPLACEMENT BASED OPTIMIZATION AS AN ALTERNATIVE FORMULATION TO STRUCTURAL OPTIMIZATION PROBLEMS

Modern structural optimization problems can usually be formulated in several alternative, but equivalent, ways leading to identical solutions. The chosen formulation may affect significantly the solution process, and is characterized by the independent variables and analysis equations used, as well as the form of constraints. A comprehensive study for different available schemes to structural optimization problems is given by U. Kirsch and G.I.N. Rozvany (1994).

Displacement Based Optimization (DBO) methodology (McKeown 1977; S. Missoum et al, 1998) is a relatively new formulation to structural optimization problems. It uses displacements of structural response as design variables to design structures for optimum performance. The approach can be best described by comparing it to Classical Nested Approach (CNA) and Simultaneous Analysis and Design (SAND) (Haftka, 1985).

The most dominant formulation is the so-called Classical Nested Approach (CNA), where structural analysis equations are excluded from the mathematical programming formulation of design problem. Considering minimum weight structural design subjected to constrained static structural responses of displacements and stresses, the CNA approach can be formulated as:

$$\begin{aligned} &\text{Min} \quad && w(\mathbf{x}) \\ &\text{s.t.} \quad && g_j(\mathbf{x}) \le 0, \quad j = 1,2,\cdots,m \\ & && \text{bounds on } \mathbf{x} \end{aligned} \tag{1.1}$$

where w is structural weight. The design variable vector **x** is usually composed of cross sectional characteristic parameters of structural finite elements, such as cross sectional areas of trusses. Finally, the g_j's represent m structural displacements or stresses constraints. These displacement and stresses responses are generally computed by displacement based finite element analysis whenever searching in **x** design space needs to evaluate exact values of the displacements and stresses on new trial design. Analysis equations are as follows:

$$\mathbf{K}(\mathbf{x})\mathbf{u}=\mathbf{p} \tag{1.2}$$

where **K**(**x**) is stiffness matrix of structure, **u** is unknown displacement vector and **p** external loading vector. Once the displacements are evaluated, stresses are easily computed from the displacements **u**.

Simultaneous Analysis and Design (SAND) approach introduces additional inclusion of the displacement **u** as the design variables. The finite element equilibrium equations **Ku**=**p** are included explicitly as equality constraints of the optimization model. During optimization process, those constraints **Ku**=**p** are gradually satisfied as the search process approaches an optimum solution. This is quite different from the CNA approach that solves **Ku**=**p** exactly in each optimization iteration. The optimization model of SAND can be written as:

$$\begin{aligned} \text{Min } \; & w(\mathbf{x}) \\ \text{s.t. } \; & g_j(\mathbf{x},\mathbf{u}) \leq 0, \quad j = 1,2,\cdots,n \\ & \mathbf{K}(\mathbf{x})\mathbf{u} = \mathbf{p} \\ & \text{bounds on } \mathbf{x},\mathbf{u} \end{aligned} \tag{1.3}$$

where g_j's are constraints such as bounded element stresses.

The new Displacement Based Optimization (DBO) approach (McKeown 1977; S. Missoum et al, 1998) decomposes structural optimization problem into two levels as does the CNA approach. However, the inner and outer problems are reversed, and the displacements that are in the nested inner problem in CNA, i.e., in **K**(**x**)**u**=**p**, are moved to the outer level, and the traditional design variables **x** that are in the outer level in CNA are moved to the inner problem. The outer level of DBO, therefore, deals with the model:

$$\begin{aligned} \text{Min } \; & w(\mathbf{x}(\mathbf{u})) \\ \text{s.t. } \; & g_j(\mathbf{u}) \leq 0, \quad j = 1,2,\cdots,n \\ & \text{bounds on } \mathbf{u} \end{aligned} \tag{1.4}$$

where g_j's are usually constraints from the bounds on the element stresses. In this formulation, **x**(**u**) represents the dependence of **x** on **u**. After optimizer produces a new displacement field **u** in a particular outer level iteration, structural traditional design parameters **x** are recovered by solving a set of alternative representative equations of equilibrium relations while minimizing structure weight. For truss structures, the cross sectional areas of the elements **x** can be obtained from known displacement **u** by solving an inner level Linear Program (LP) problem:

$$
\begin{aligned}
\text{Min} \quad & w(\mathbf{x}) \\
\text{s.t.} \quad & \mathbf{T}(\mathbf{u})\mathbf{x} = \mathbf{p} \\
& \text{bounds on } \mathbf{x}
\end{aligned}
\tag{1.5}
$$

where **T** is LP constraints coefficients matrix. Equality constraints $\mathbf{T}(\mathbf{u})\mathbf{x}=\mathbf{p}$ is an alternative representation of $\mathbf{K}(\mathbf{x})\mathbf{u}=\mathbf{p}$. Here to provide a better understanding, a relationship diagram of design spaces of CNA, SAND and DBO approaches is shown in figure 1.1.

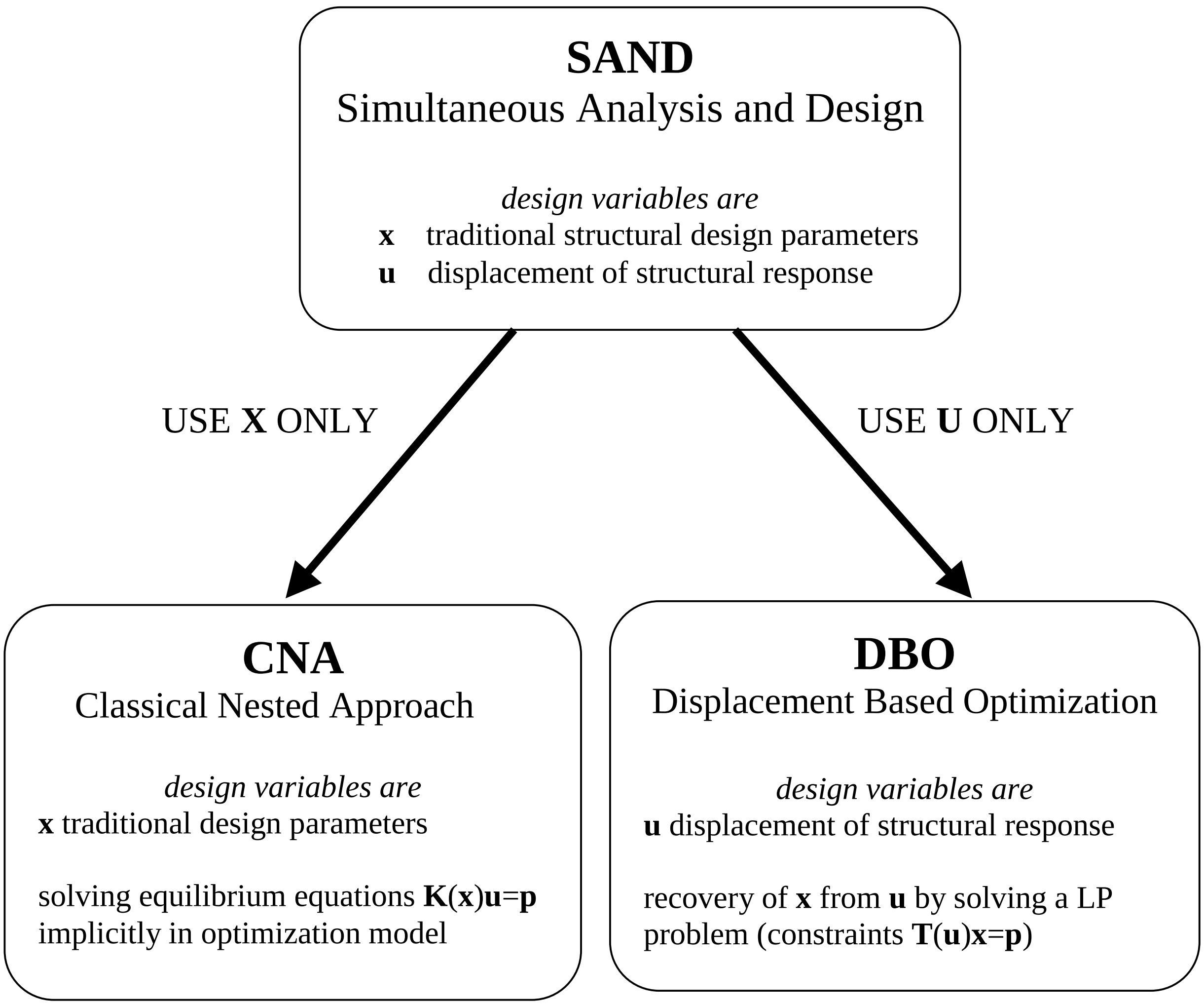

Fig. 1.1 Design space relationship among SAND, CNA and DBO

Based on the three approaches presented above, two major advantages could be addressed. First, compared to the SAND approach, the DBO approach uncouples the traditional structural design variables **x** from the state variables **u**. Thus, instead of searching in design space of both **x** and **u**, the DBO reduces the design space so as to improve efficiency of searching. Furthermore, the DBO avoids computational difficulty that is commonly assumed due to different characteristics between **x** and **u**. Second, compared to the CNA approach, the DBO eliminates the need to perform costly nonlinear structural analysis in each iteration of search process for nonlinear structural optimization problems, such as truss sizing with geometrical or material nonlinearity. As a result, the DBO will save computational cost dramatically.

Despite the above two advantages, the DBO is far from mature since there are little earlier works. Thus, extensive numerical testing is needed to learn how the method behaves. That is the motivation of the works described in this thesis.

1.2 BACKGROUND TO TRUSS SIZING WITH MATERIAL NONLINEARITY

A recent work (S. Missoum et al, 1998) showed that the DBO approach was computationally efficient and robust in solving several standard linear elastic truss sizing problems. The same paper also envisioned advantages of DBO to tackle optimization problems with nonlinear structures. The work described hereafter explores feasibility to apply DBO for truss sizing problems with material nonlinearity. In other words, one dimensional stress-strain nonlinear material laws are incorporated in truss element for the truss design problems studied.

Time saving gain was one key element to provoke this DBO methodology study. If not considering computational cost, Classical Nested Approach (CNA) can be easily used to solve large classes of nonlinear structural design problems including the truss sizing with material nonlinearity. For example, recent release of VisualDOC 1.2 (VR&D, 1999) provided linking of optimization tools with commercial nonlinear structural analysis tools such as ABAQUS or DYNA3D. Using VisualDOC 1.2, users can perform finite difference to calculate gradient information required by the optimizers, though they are computationally expensive. For some nonlinear structural problems, sensitivity equations may be available to compute analytical gradient so that computation of the finite difference gradient is avoided. Nonetheless, a fully nonlinear structural analysis is still required for each iteration of optimization process. To avoid those time-consuming nonlinear structural analyses, the concept of Simultaneous Analysis and Design (SAND) was proposed (Haftka, 1985). Generally speaking, SAND may face numerical difficulties arising from coupling traditional structural design parameters with structural response displacement in one design space. Fortunately, these possible difficulties didn’t show up when a school of researchers tackled special types of problems such as the truss sizing with nonlinear material by method essentially in line with the SAND approach (though SAND concept for nonlinear structural design was proposed later). Next paragraph will briefly introduce their approaches up-to-date. Computational efficiency of DBO is compared to this type of SAND approach that is the most efficient algorithm available.

Last several decades witnessed evolution of development of efficient approaches for structural plasticity design problems. For simplicity, the truss sizing problem with material nonlinearity has been extensively used to test such methodologies. Limit Design approach (Haftka & Gurdal, 1992) can formulated minimum weight design of truss structures with Elastic-Perfectly-Plastic (EPP) material as a single Linear Programming (LP) problem. However, structural response displacement or truss member strain is outside of consideration in the Limit Design format. As a result, Limit Design cannot include displacement or strain constraints that may be important to design practical structures. Kaneko and Maier (1981) proposed an important class of minimum weight truss design problem that was a natural expansion from their pioneering work about truss analysis problem with elastoplastic material. For linear strain hardening material, that analysis problem represented the transformation from elastic to plastic region as complementarity constraints. In other words, those complementarity constraints mathematically express the perpendicularity of two sign-constrained vector and mechanically describe an inherent property of plasticity. As a result, the analysis problem becomes the so-called Linear Complementarity Problem (LCP). The design counterpart of this analysis problem under displacement constraints can be formulated as nonlinear programming problem (NLP) involving complementarity constraints. This formulation included equilibrium equations of nonlinear structure as constraints of NLP so that it is a special case within the category of SAND approach. Mathematically, such a design problem is referred to in the optimization literature as Mathematical Program with Equilibrium Constraints (MPEC). In their seminal work, Kaneko and Maier (1981) used an iterative branch-and-bound method to solve such a MPEC problem. Cinquini and Contro (1984) developed optimal criteria via Lagrange multiplier technique to solve the same problem. Both the authors illustrated some simple truss problems to verify their methodologies. Tin-Loi (1999) recently revisited such a MPEC problem by using a smoothing scheme to approach complementarity condition throughout the optimization process. This paper also showed several successfully solved truss problems. Despite the generality of such a SAND approach, for truss sizing with material nonlinearity, all past papers showed working truss examples only for Linear Strain Hardening material. Finally we emphasize that the above papers all worked with a set of design variables including cross sectional areas and nodal displacements.

1.3 SUMMARY OF THESIS

We investigated the truss sizing problem with material nonlinearity using the Displacement Based Optimization (DBO) approach. The holonomic (path-independent) elastoplastic laws were assumed so that linear elastic truss sizing in the DBO setting was naturally extended for our study. In chapter 2, we discussed problem formulations for truss sizing with material nonlinearity, and various important aspects, such as scaling of displacement, analytical gradient, material laws. Chapter 3 shows and studies results of several test problems, with comparing them to available results from literatures. Concluding remarks is given in Chapter 4. While many examples prove the DBO approach is suitable for holonomic elastoplastic truss sizing problems, we also address that some procedures, such as scaling of displacement or relaxed form used in the inner problem, should be improved.