3.0 FINITE ELEMENT MODEL

In Chapter 2, the development of the analytical model established the need to quantify the effect of the thermal exchange with the dome in terms of a single parameter, T_d. In this chapter, a finite element model is described which offers detailed insight into the interaction of the sensor with the dome having a temperature gradient. This model is shown to be useful in determining the dome representative temperature.

3.1 The Finite Element Method

The finite element method is used to approximate functions describing physical relations in a physical domain. The development and advancement of computers has promoted the use of this method by allowing faster, more accurate, and more complete solutions than reasonably achievable with analytical methods. Difficult and cumbersome engineering effort is replaced with computer runtime. Analysis of physical processes is simplified by solving the physical equations over small elements with uniform properties and simple geometries. These elements are then connected in a mesh that represents the entire model. The elemental analysis allows for analyzing complex geometries and composite models more easily.

In thermal applications, finite element models provide information about temperature distributions in a geometry subject to given initial and boundary conditions. These may be specified temperatures or specified heat fluxes. Engineers can accurately simulate actual operating conditions of their design even before constructing a prototype. The method also provides information which may not be achievable from other methods. It is difficult to attain detailed information using experimental methods because the amount of experimental error increases as more precision is sought. For example, increasing the number of temperature sensors in an experiment may increase the amount of heat dissipated by the devices.

The development of a finite element model for a geometry begins with determining the physics governing the distribution of the unknown quantity, temperature. For example, in the domain of the geometry, the temperature distribution is governed by conduction, which may be expressed by Fourier's Law in Equation 3.1

$$q'' = -k\nabla T \ , \tag{3.1}$$

where ∇T = temperature gradient (K/m)
and q” = heat flux (W/m^2).

The physical description of the temperature gradient is converted from a continuum to a discretized domain of specific temperatures. The entire geometry is divided into small pieces, called elements. The elements hold information about the physical properties of their corresponding materials. Each element is bounded by nodal points, which are the discrete points for which a solution is sought. The differential equations describing the physics are replaced by difference quotients that apply to nodal points in the geometry [Reddy, 1984]. In linear conduction, the nodal temperature for the $n+1^{st}$ node affected by a heat flux q” from an adjacent node n at a distance Δx is found by

$$T_n = T_{n+1} + \frac{q''}{k} \Delta x \ . \tag{3.2}$$

Boundary conditions are included on the appropriate nodes in the finite element model. In the linear conduction example, the boundary nodes would either have a specified temperature T_o or a specified heat flux q”. The heat flux may be a function of the corresponding node temperature, as in the case of a surface exposed to convection $q_{conv} = h(T_o - T_\infty)$. The boundary condition for the node at x = 0 in three cases are

$$T(x=0) = T_o \, , \tag{3.3}$$

$$\frac{dT}{dx}(x=0) = \frac{q''}{k} \, , \tag{3.4}$$

and

$$\frac{dT}{dx}(x=0) = \frac{h(T_o - T_{air})}{k} \ . \tag{3.5}$$

In order to accurately build a system model, the geometry and material properties of each component must be established. This is an academic matter for designers since the real object does not yet exist. However, when trying to describe the physics of an existing object the model information that is not designated by the manufacturer or otherwise available must be assumed. If possible, these assumptions should be checked using experimental results.

Once the geometry is described, the remaining effort is in writing a program to solve a system of algebraic equations. Several commercial codes have been developed as tools for engineers and designers to simulate objects and approximate solutions to problems without personally developing the code to do so. Commercial codes may be oriented toward structural, fluid, or thermal applications. In all cases, geometry, material properties, and boundary and initial conditions are entered by the user. Other parameters, such as boundary condition relaxation parameters, may be entered by the user. Typically, these involve a tradeoff between solution accuracy and speed of calculation.

3.2 ALGOR Software

ALGOR is a commercial code with several analysis programs, one of which is a thermal analyzer. The program may be used to predict temperatures and heat fluxes resulting from subjecting a model to specified environments.

A model is created by drawing the surfaces with a graphical user interface. In a complex model with multiple materials, the user must be careful to designate each surface into a group, with each group representing a specific material. The material properties are later added into the model file, assigning the properties of each "group". The surfaces are then meshed to create block elements of the finite element model.

The boundary conditions of the model are specified on the surfaces. Internal nodes are not subject to convective or radiative heat transfer, but may be designated as heat sources. For purposes of visualization, each surface is typically designated using a different color. Each "color" represents a thermal exchange condition. A convective exchange is characterized by a convective sink temperature and a convective heat transfer coefficient h, where

$$Q_{conv} = h(T_{surf} - T_{\sin k}) . \tag{3.6}$$

A radiative exchange is characterized by a radiative heat sink temperature and a radiative exchange factor F where

$$Q_{rad} = F\sigma(T_{surf}^4 - T_{rad}^4) . \tag{3.7}$$

In ALGOR, any surface in a model may be exposed to convective heat transfer with a sink, radiative heat transfer with a sink, or both with the same or different sink temperatures.

3.3 Dome Glass Model

A model of the outside filter dome was created to describe the magnitude of the temperature gradient that may exist across it under various conditions. A model was created in

ALGOR that contained the geometrical and physical characteristics of the dome. The glass properties are:

ρ = density = 2500 kg/m^3,

c_p = specific heat = 750 J/kg K,

and k = thermal conductivity = 1.38 W/mK.

The geometry of the hemispherical dome is:

OR = Outside Radius = 0.0254 m

and IR = Inside Radius = 0.0234 m.

In the model, the volume of the dome is divided into 768 elements. The division consists of two elements through the thickness of the dome. There are 16 rings of elements from the tip of the dome to the rim. The rings are then divided in the azimuthal direction, with each element spanning 15-deg. The dome mesh is shown in Figure 3.1.

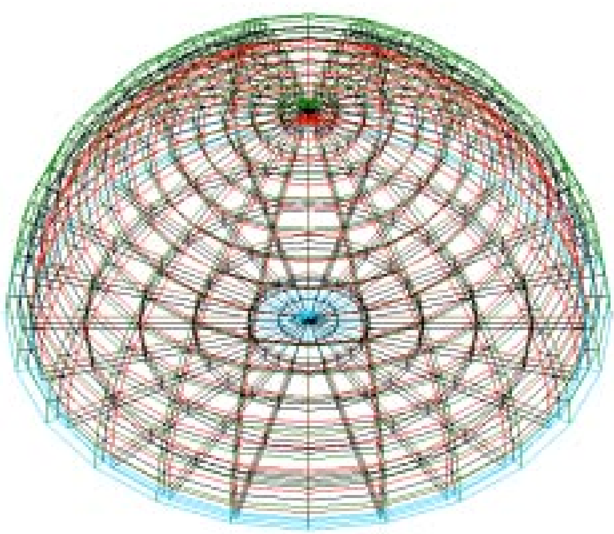

Figure 3.1 Three-dimensional view of glass outer dome mesh

The dome is exposed to conditions similar to nighttime conditions to simulate the environment that creates the zero offset in field observations. The base of the dome is maintained constant at 0°C to simulate thermal contact with the instrument body at a typical terrestrial nighttime temperature.

The inside surface of the dome is isolated from convective heat transfer and exchanges radiation only with the top of the floor inside the dome of the instrument, which is also assumed to be a constant 0°C. The F-value for this exchange is set to 0.95, the assumed emissivity of the dome surface. Though the inside of the dome exchanges radiation with itself as well as with the floor, the temperature of the dome is nearly 0°C, the same as the floor, so the F-value is again set to 0.95. It is not reduced so that it includes radiation exchange of the dome with itself as well as the floor.

The outside surface of the dome is exposed to convection with nighttime air at 0°C. The value of the convection coefficient of h is varied to correspond to different wind conditions. The outside of the dome is also exposed to radiation with the nighttime sky at 220 K. This is an established value for the equivalent blackbody radiative temperature of a clear nighttime sky [Kato]. The outside surface of the dome exchanges radiation only with the sky, so for this exchange, the F-value in the model is set to 0.95, the effective emissivity of the dome surface.

The results of this simulation are included in Section 5.1.

3.4 Mounted Dome Model

The model of the outside dome is modified in order to explore cases that simulate actual experimental conditions. An experimental trial consists of removing the dome from equilibrium in an environment and monitoring it's thermal state as it approaches equilibrium with a new environment. A more complete description of the experiment itself is given in Chapter 4. The steel rim that secures the glass dome to the instrument is added to the existing geometry. The steel properties are:

ρ = density = 7850 kg/m^3,

c_p = specific heat = 444 J/kg K,

and k = thermal conductivity = 42 W/mK.

The geometry is entered from measurements taken of the actual dome. These are shown in Figure 3.2.

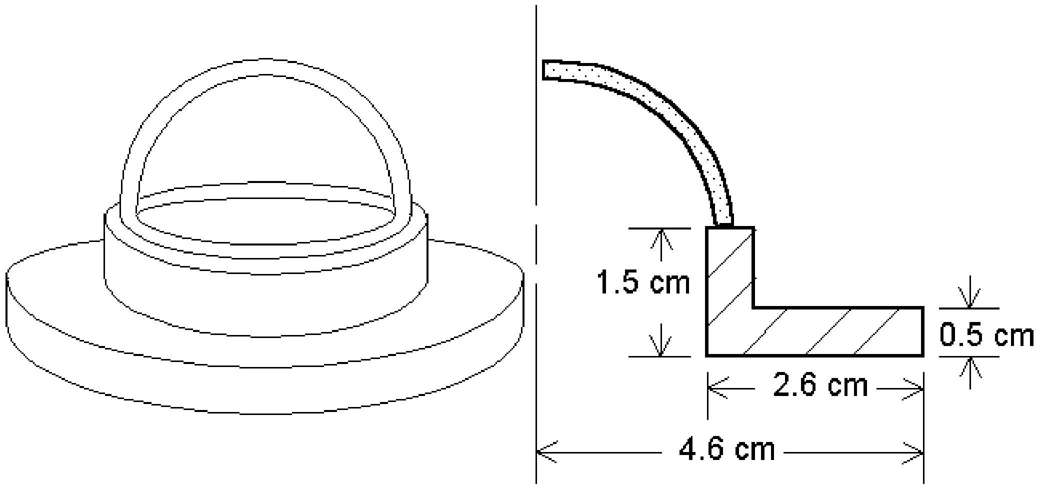

Figure 3.2 Dimensions of glass dome and steel rim

3.5 Model Boundary Conditions

The boundary conditions, specifically the ambient temperatures during experiments, vary between trials. Therefore, the boundary conditions of the model must be tailored for each experiment. The results of the mounted dome model are transient, or time varying. The temperature distribution on the dome is determined for several time steps. For this case, it is sufficient to define the boundary and initial conditions with initial temperatures for each node and sink temperatures for each surface that has convective or radiative exchanges. The initial temperature for all nodes is set to the steady-state temperature of the dome before the temperature surrounding the dome is changed. The environment of the dome is assumed to be at a constant temperature. The dome is assumed to exchange radiatively with the surrounding surfaces at the same temperature as the air with which the dome exchanges convectively. This "sink" temperature is the temperature which all thermistors appear to approach as they approach steady state conditions. The simulated transient thermal behavior during the cooling experiment is demonstrated in Figure 3.3.

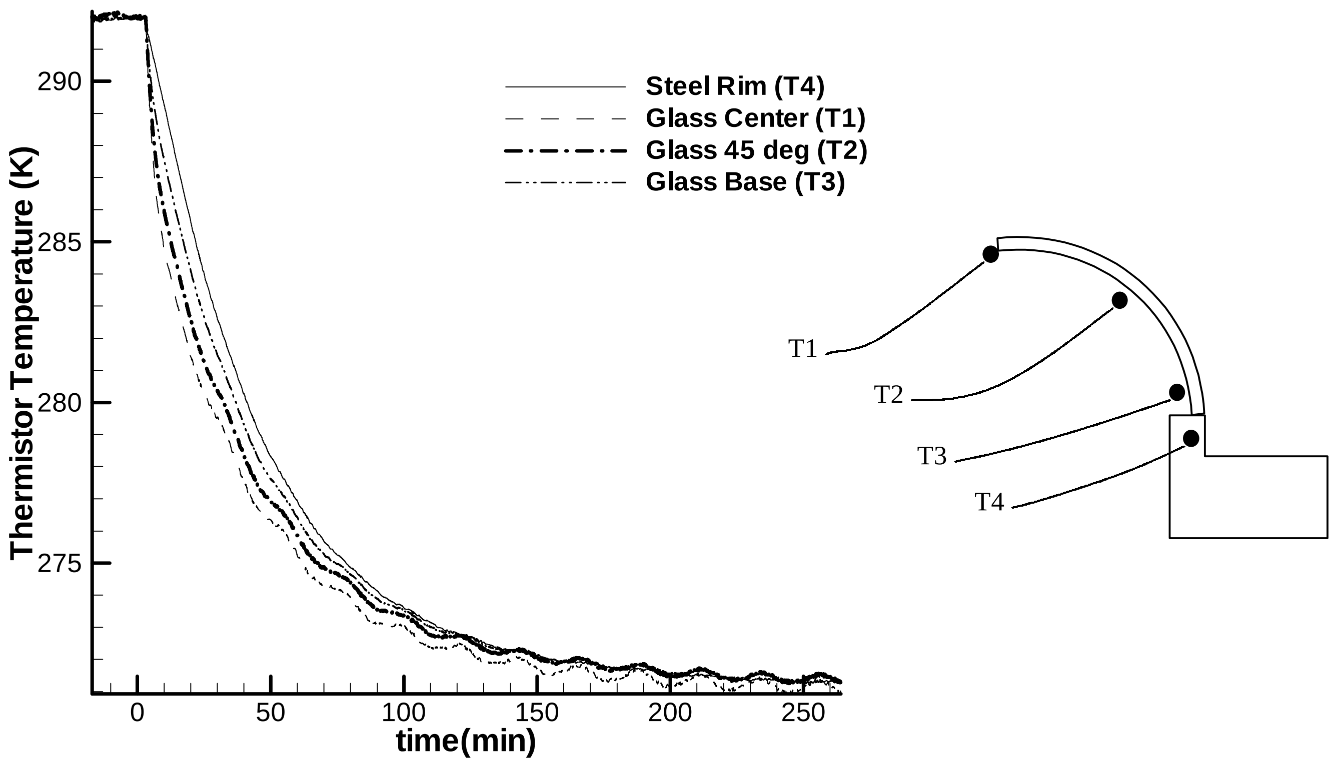

Figure 3.3 Representative data from cooling experiment

The convection coefficients cannot be determined exactly using analytical methods because the air currents present during experiment are poorly defined and unsteady. In order to determine an appropriate value to use for the convection coefficient, several sets of transient data from actual experiments were visually compared with model results. The convection coefficient was varied in the model to find the closest match to the data results. Results and this approach are given in Chapter 5.

The radiative environment within the refrigerator is not uniform. A freezer compartment sits in the corner of the refrigerator and remains several degrees cooler than the rest of the refrigerator. Though this does not affect the air temperature that is convectively cooling the dome, it does affect the temperature with which the exposed surface of the dome exchanges radiation. This means that the side of the dome exposed to the freezer compartment cools faster and reaches lower temperatures than the side facing away from the freezer compartment. The entire dome also experiences quicker cooling and lower temperatures than if the freezer compartment were absent because the exposed surfaces are conductively connected to the unexposed surfaces.

In order to accurately model the refrigerator environment, a freezer compartment is included in the numerical model as well. The surfaces directly exposed to the freezer compartment are determined using the geometry shown in Figure 3.4. The surfaces are then located in the numerical model and the radiation sink temperature is set to 5°C below the sink temperature for the other surfaces.

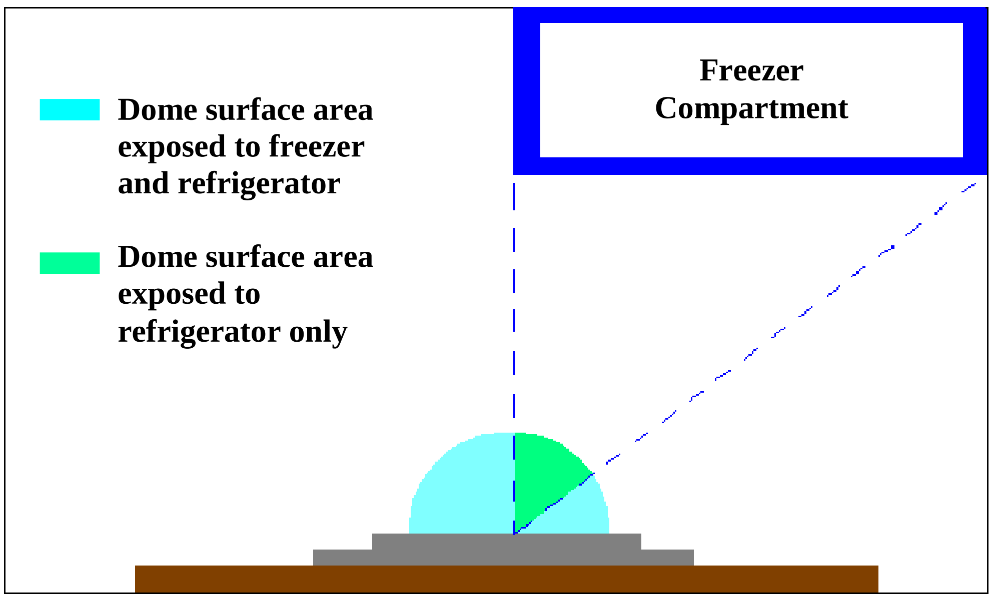

Figure 3.4 Refrigerator environmental conditions

The model calculates the transient temperature distributions on the dome resulting from exposure to the experimental conditions. The experimental measurement data is used to validate the model results. The experimental procedure is described in detail in Chapter 4.