

Chapter 4

SIMULATION OF GROUND-WATER FLOW

Modeling Objectives

A conceptual model of the study area is applied to a numerical ground-water flow model to: 1) quantify the hydrologic parameters and hydrologic budget for the Rio Cobre and Rio Minho-Milk river basins under steady-state conditions, 2) simulate the distribution of hydraulic head in the basins, 3) determine the impact of the SCFZ on ground-water flow, and 4) determine the overall water budget. A quasi three-dimensional ground-water flow approach is used to estimate hydrologic parameters and a hydrologic budget for the Rio Cobre and Rio Minho-Milk river basins. The Rio Cobre and Rio Minho-Milk river basins are represented in a ground-water flow model subdivided into a grid system that incorporates the White Limestone and the alluvium aquifers, rivers, the South Coast Fault Zone, estimated recharge, and discharge along the saltwater-freshwater interface.

Ground-water Flow in the Rio Cobre and Rio Minho-Milk River Basins

The Rio Cobre and Rio Minho-Milk river basins together represent a single hydrogeologic basin because no sub-surface flow occurs across the periphery of the basin. The hydrogeologic boundaries of the basin include 1) a topographic divide north of the impermeable plutonic and recrystallized rocks, which is represented as a general-head boundary, 2) the freshwater-saltwater interface to the south, which is represented as a no-flow boundary discharging to a constant head at the ocean surface, and 3) the eastern and

western boundaries, which are represented by no-flow conditions as they are believed to be topographic highs that coincide with ground-water divides. Most of the ground-water flow in the basin occurs within the White Limestone aquifer. Ground-water-flow in the White Limestone aquifer of the Rio Cobre and Rio Minho-Milk river basins is complex due to the unmapped pathways of solution-enlarged fractures. The fractures act as highly permeable zones for ground-water movement and the dissemination of flow through the White Limestone aquifer.

Recharge to the Rio Cobre and Rio Minho-Milk river basins occurs mainly in the form of infiltration from precipitation. Recharge to the White Limestone aquifer from rainfall occurs rapidly. The White Limestone has relatively low coefficients of storage from recharge at higher elevations where there is crystalline limestone and exposed basement rocks. Oxygen ($\delta^{18}\text{O}$) and hydrogen (δD) isotope data confirm recharge originates exclusively in the cooler, elevated upper regions of the Rio Minho-Milk River basin high above the Clarendon and Vere Plains (Howard et al., 1993). The degree of oxygen depletion observed in the $\delta^{18}\text{O}$ values (-2.5‰) of ground-water in the Rio Minho-Milk River basin is consistent with recharge occurring at elevations above 750 m (asl) (Mullings, 1993). The upper Rio Cobre Basin is considered to be a closed inland sub-basin consisting of a recharge area where there is infiltration of precipitation and a reservoir area, where there is rapid inflow and storage of meteoric water in the basin (White, 1980) (Figure 4.1).

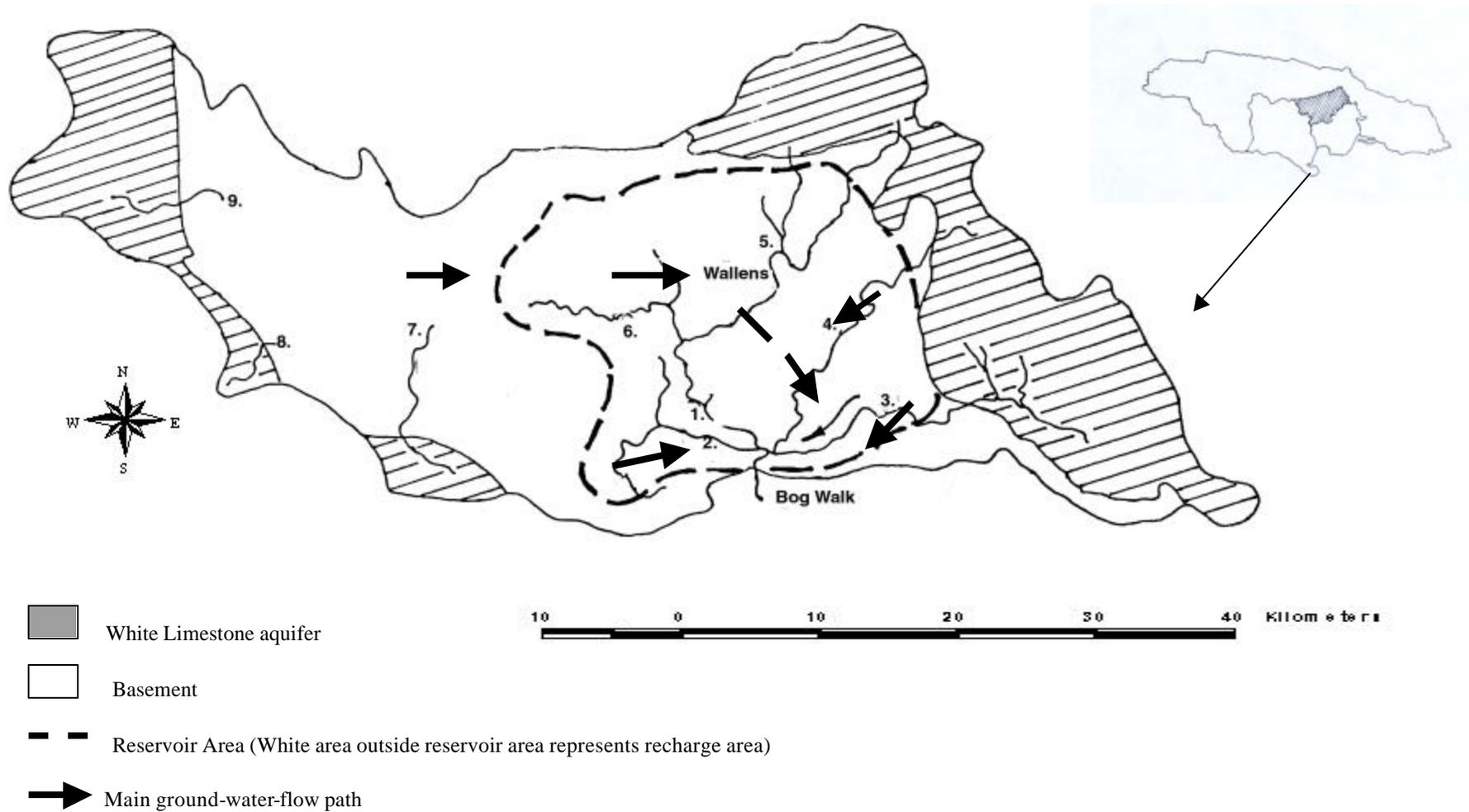


Figure 4.1 Division of the upper Rio Cobre basin into a recharge area and reservoir area (White, 1980): 1) Rio Cobre, 2) Thomas River, 3) Rio Pedro, 4) Rio D'Oro, 5) – Rio Magno, (6) Black River, 7) Murmuring River, 8) Savannah River, and 9) South River.

Conceptual Model of Ground-water-Flow

For the ground-water flow simulations, the Rio Cobre and Rio Minho-Milk river basins were conceptualized using two model layers (Figure 4.2). The upper model layer represents the unconfined portion of the White Limestone aquifer and the alluvial aquifer. Hydrologic stresses represented in the model are net recharge and discharge. Recharge to the upper model layer was estimated by subtracting the total estimated evapotranspiration and surface runoff from total precipitation. Local variations in recharge occur due to lithology, slope, and geology. Additional recharge to the aquifer can occur along streams and rivers, subsurface inflow from neighboring basins, and leakage from ponds, lakes, and reservoirs. The lower model layer represents the confined part of the White Limestone aquifer and overlies the crystalline bedrock. Most ground-water flow occurs in this layer as subsurface flow and originates from leakage of the overlying unconfined aquifer from upland areas at the north end of the study region. The confining unit (red marine clay) that separates the two model layers is not simulated because it is not areally extensive. Discharge from the study area occurs primarily as submarine discharge to the Caribbean Sea at the south edge of the study area. Additional discharge can occur as baseflow to streams and rivers, although this amount is small in comparison to submarine discharge.

Modeling Approach

A quasi three-dimensional model grid is used to represent three-dimensional flow through the two aquifers in which flow is basically assumed to be horizontal. Ground-water flow in the alluvial (unconfined) and White Limestone (confined) aquifers of the

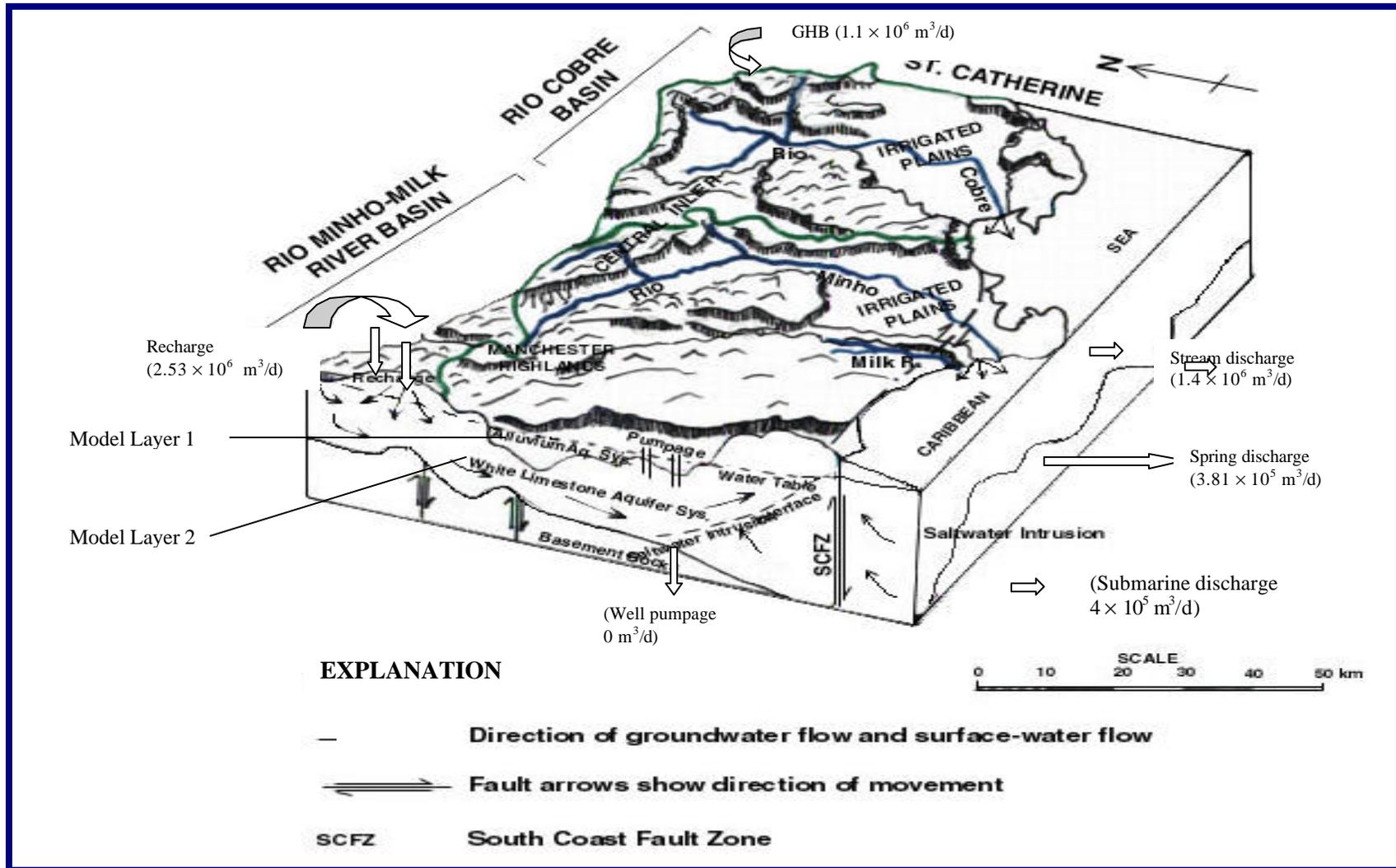


Figure 4.2 Conceptualization of the Rio Cobre and Minho-Milk river basins.

Rio Cobre and Rio Minho-Milk river basins was simulated using the U.S. Geological Survey modular three-dimensional finite-difference code commonly known as MODFLOW (McDonald and Harbaugh, 1988). Flow between layers is assumed to occur vertically and is a function of the difference in heads simulated in the two layers and a specified vertical conductance value (Anderson and Woessner, 1992).

General Assumptions and Model Limitations

Several general simplified assumptions are used in the simulation of ground-water flow, and the development and calibration of the model :

- The average conditions for 1998 are assumed. Pumpage occurring during this year is not considered to significantly affect the average conditions within the basins.
- Fractures and conduits through which water flows may be considered porous medium to which Darcy's Law is assumed applicable.
- Pumping does not significantly affect the level of hydraulic heads; therefore, pumping wells are not simulated.
- Net recharge from precipitation is not areally uniform because there is heterogeneity in the spatial distribution of hydraulic conductivity, variations in geology, total precipitation, slope, and vegetation.
- Karstification, aquifer heterogeneity, vertical anisotropy, and the presence of large voids and faults impact the spatial distribution of hydraulic conductivity. Consequently, a zonation approach is adopted whereby similar K values are assigned to specific regions on the basis of these factors.

- Flow in the White Limestone aquifer occurs in the uppermost 650 m, and vertical flow is assumed controlled by the intervening confining unit between the White Limestone and alluvial aquifer and is sufficiently lower than the aquifers (Neuman and Witherspoon, 1969). Vertical conductivity is controlled by vertical anisotropy within a single layer or by leakance across different aquifers.
- Evapotranspiration was not directly simulated in the model. Recharge rates were considered as “net recharge” implying that the effects of evapotranspiration were already accounted for in the net precipitation rates.
- Submarine discharge occurs from the aquifer along the coast where the aquifers are hydraulically connected to the sea. The equality of freshwater head and saline water head at the seabed must exist at the boundary between fresh ground-water and saline ground-water if: (1) the boundary is a sharp interface; (2) the saline ground-water is static (nonflowing); (3) the flow system is in a state of dynamic equilibrium; (4) and the only forces acting on the ground-water are head gradients arising from gravitational forces (Hubbert, 1940).

Selection of Computer Code

A ground-water flow model for the alluvium and White Limestone aquifers is based on MODFLOW, the USGS modular three-dimensional finite-difference computer code commonly used to simulate ground-water flow and determine hydrologic budgets in various physiographic regions (McDonald and Harbaugh, 1988). Many versions of this code have been created since 1988 and version 2.81.22 was used in this modeling study. The computer program uses finite-difference techniques and a block-centered

formulation to solve the ground-water flow equation for three-dimensional, steady state flow in the anisotropic, heterogeneous, porous media of the Rio Cobre and Rio Minho-Milk river basins. In order to use a finite-difference approximation, a grid is superimposed over the study area, and aquifer hydraulic parameters necessary to solve the flow equation are averaged over the area of cell or grid block and assigned at a node at the center of the block. Finite-differences compute the average head value for a cell at the node. In the block-centered formulation, the nodes for which water levels are simulated are located at the center of the grid cells. These cells are the smallest volumetric units over which the hydraulic properties are assumed constant.

Governing Flow Equation

A three-dimensional finite-difference ground-water flow model developed by McDonald and Harbaugh (1998) was used for the computer simulations. The mathematical model simulates flow indirectly by means of a governing equation thought to represent the physical processes that occur in the system. This model is based on the governing equation for anisotropic, heterogeneous steady flow expressed as follows:

$$\frac{\partial}{\partial x} \left(K_{xx} \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left(K_{yy} \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left(K_{zz} \frac{\partial h}{\partial z} \right) + W(x, y, z) = 0 \quad (1)$$

where

h is the hydraulic head in the aquifer (L);
 x, y, and z are the cartesian coordinates aligned with the principal components of the hydraulic conductivity tensor K_{xx} , K_{yy} , and K_{zz} ,

$W(x,y,z)$ is a volumetric flux per unit volume (1/T) and is referred to as a source or sink term.

K_{xx} , K_{yy} , and K_{zz} , are the hydraulic conductivities expressed in the principal directions of anisotropy [L/t].

In the case of simulations that do not include changes in hydraulic head with respect to time (steady-state), the right hand side of equation (1) goes to zero and estimates of storage are disregarded.

Application of Finite-Difference Equations

The partial differential equation representing ground-water flow (Eq.1) may be closely approximated by finite-difference equations, which are sets of algebraic expressions that are solved simultaneously in MODFLOW using the strongly-implicit procedure. The set of algebraic equations that result when approximating a ground-water flow model using the method of finite differences is normally solved using a combination of matrix and iterative solution techniques (Anderson and Woessner, 1992). The construction of the three-dimensional grid in which each model cell represents hydrologic properties that approximate the physiography of an area is fundamental to the solution of the algorithm involved in the strongly-implicit procedure. The solution of the finite difference scheme is achieved with the WHS solver in VISUAL MODFLOW. The WHS solver implements a conjugate-gradient algorithm, called the bi-conjugate gradient stabilized (Bi-STAB) acceleration routine, using Stone incomplete decomposition as the

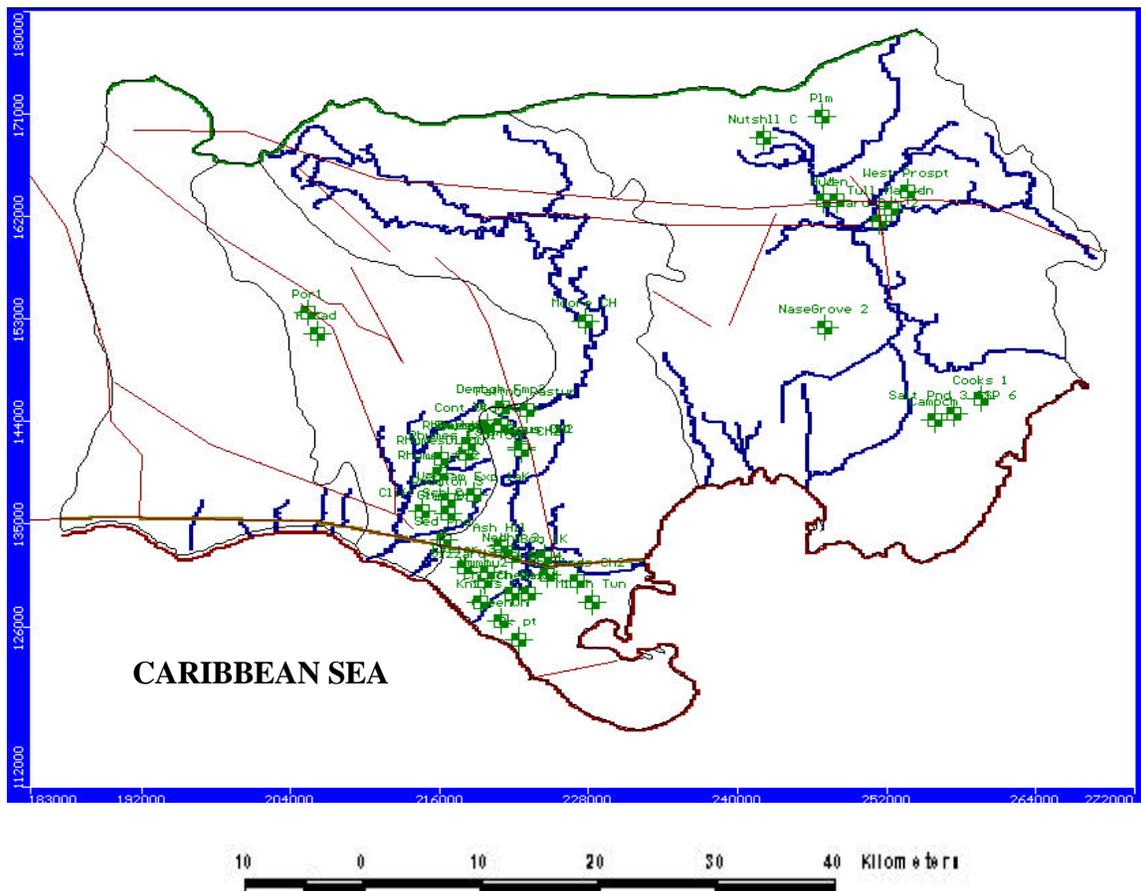
pre-conditioning method (Obretch, 1994). Pre-conditioning of the equations allows two factorization levels to be chosen. A factorization level of 1 results in fewer iterations, but requires more memory, whereas a factorization level of 0 requires less memory but performs more iterations. A factorization level of 0 was used for the simulation in this thesis.

MODEL DESIGN

Spatial Discretization of Model Grid

The geographic boundaries of the model grid were determined by using a map created in AutoCAD Drawing Exchange Format (DXF) covering the extent of the both the Rio Cobre and Rio Minho-Milk river basins. The DXF map was projected in a Geographic Information System (GIS) using metric coordinates (UTM) and then imported into MODFLOW. This map represents the areal extent of the physiographic region. A finite-difference grid superimposed (Figure 4.3) over a 2550 km² study area was designed and constructed based on the simplification of a conceptual model representing the physical properties of the ground-water system (Figure 4.2). The grid network has a constant spacing of 200 m by 160 m. A total of 375 rows and 450 columns and 337,500 cells are used to cover the study area. The White Limestone and the alluvium are simulated as two aquifers. Two model layers are used to represent the study region extending vertically from 400 meters above mean sea level to 250 meters below mean sea level (Figure 4.3). The two model layers slope with the land surface and thickness is highly varied as illustrated by the grid cross sections in elevation view and plan view (Figures 4.4 - 4.7).

HYDROGEOLOGY AND SIMULATED WATER BUDGET OF THE RIO COBRE AND RIO MINHO-MILK RIVER BASINS, JAMAICA, WEST INDIES.



EXPLANATION

-  No-flow boundary
-  Stream channel
-  Fault
-  Location of observation well used in model calibration
-  Horizontal-flow-barrier
-  Constant head boundary
-  General head boundary

Figure 4.3 Finite difference grid and boundary conditions used in model analysis.

HYDROGEOLOGY AND SIMULATED WATER BUDGET OF THE RIO COBRE AND RIO MINHO-MILK RIVER BASINS, JAMAICA, WEST INDIES.

Row 246

Vertical Exaggeration x 100

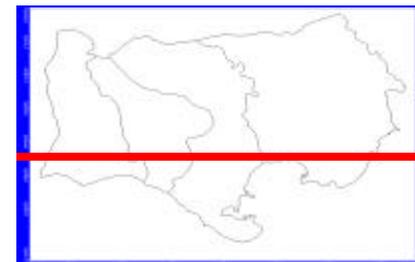
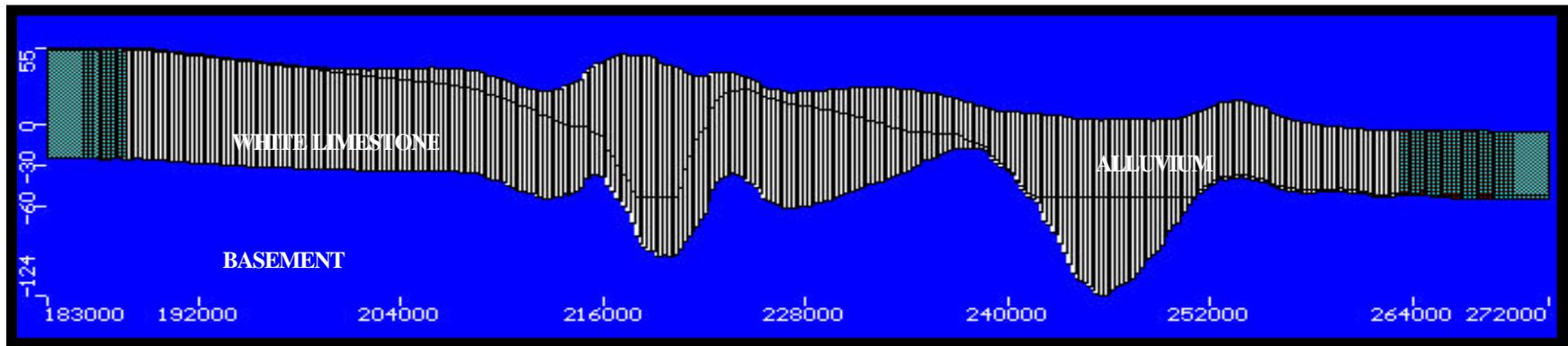
- Modeled cell
- Inactive cell
- Bottom of layer 1

Cheapside

Fort Clarence

W

E



Row 246

Figure 4.4 Section along row 246 of model grid (grid location shown in inset below).

HYDROGEOLOGY AND SIMULATED WATER BUDGET OF THE RIO COBRE AND RIO MINHO-MILK RIVER BASINS, JAMAICA, WI.

Row 136

Vertical Exaggeration x 100

- Modeled cell
- Inactive cell
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Shirehampton

Cavaliers

W

E

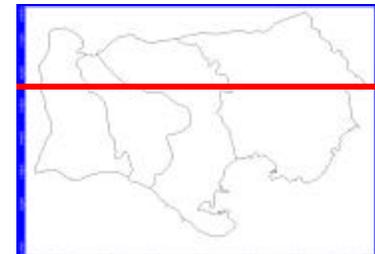
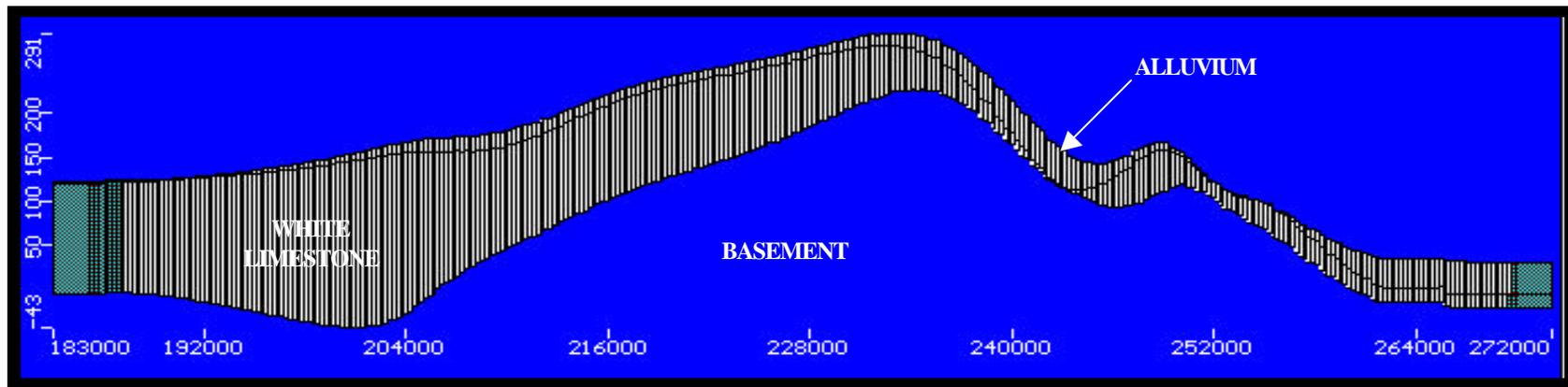


Figure 4.5 Section along column row 136 of model grid (grid location shown in inset below).

Row 136

HYDROGEOLOGY AND SIMULATED WATER BUDGET OF THE RIO COBRE AND RIO MINHO-MILK RIVER BASINS, JAMAICA, WEST INDIES.

Column 161

Vertical Exaggeration x 70

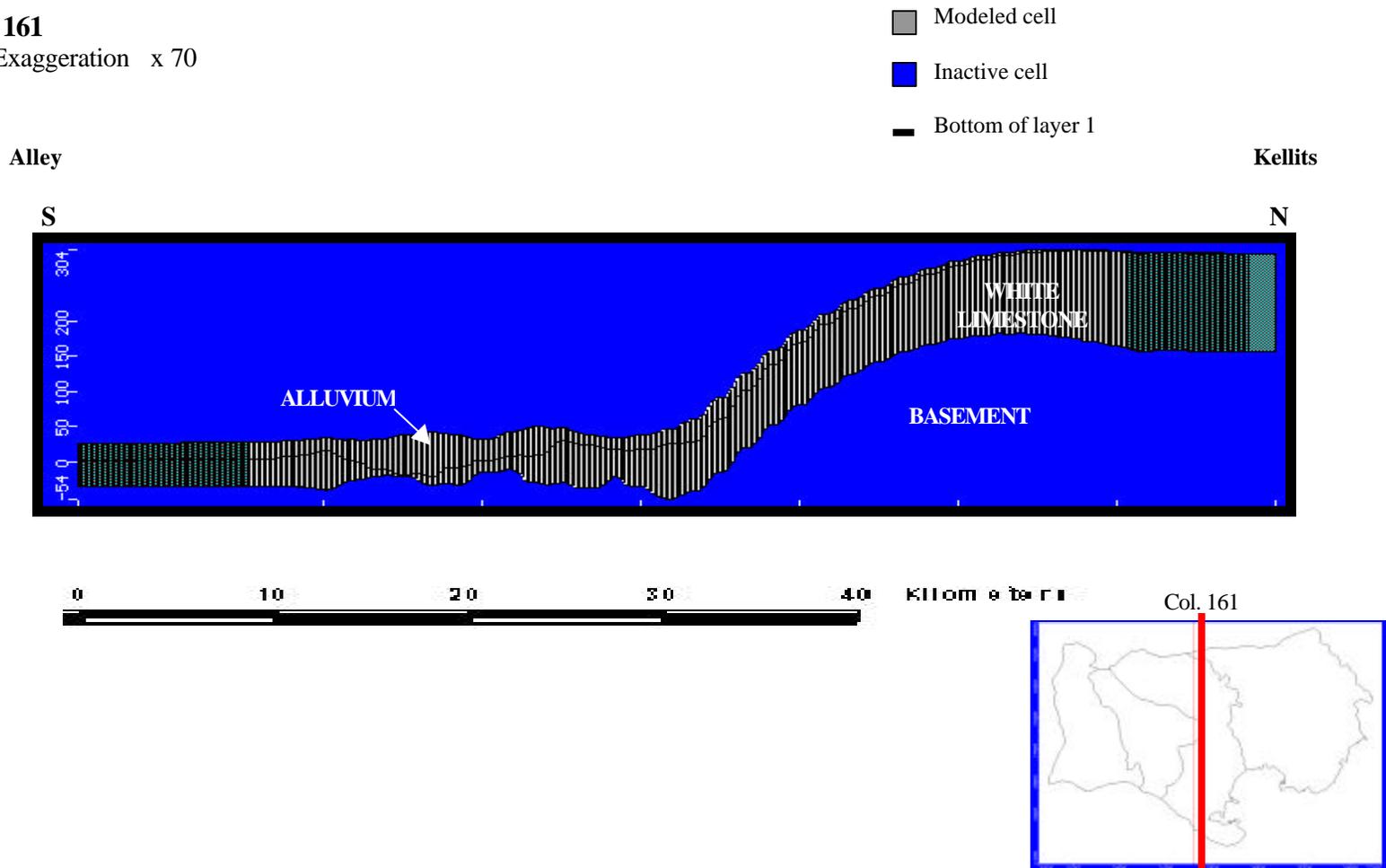


Figure 4.6 Section along column 161 of model grid (grid location shown in inset below).

HYDROGEOLOGY AND SIMULATED WATER BUDGET OF THE RIO COBRE AND RIO MINHO-MILK RIVER BASINS, JAMAICA, WEST INDIES.

Column 257
Vertical Exaggeration x 70

- Modeled cell
- Inactive cell
- Bottom of layer 1

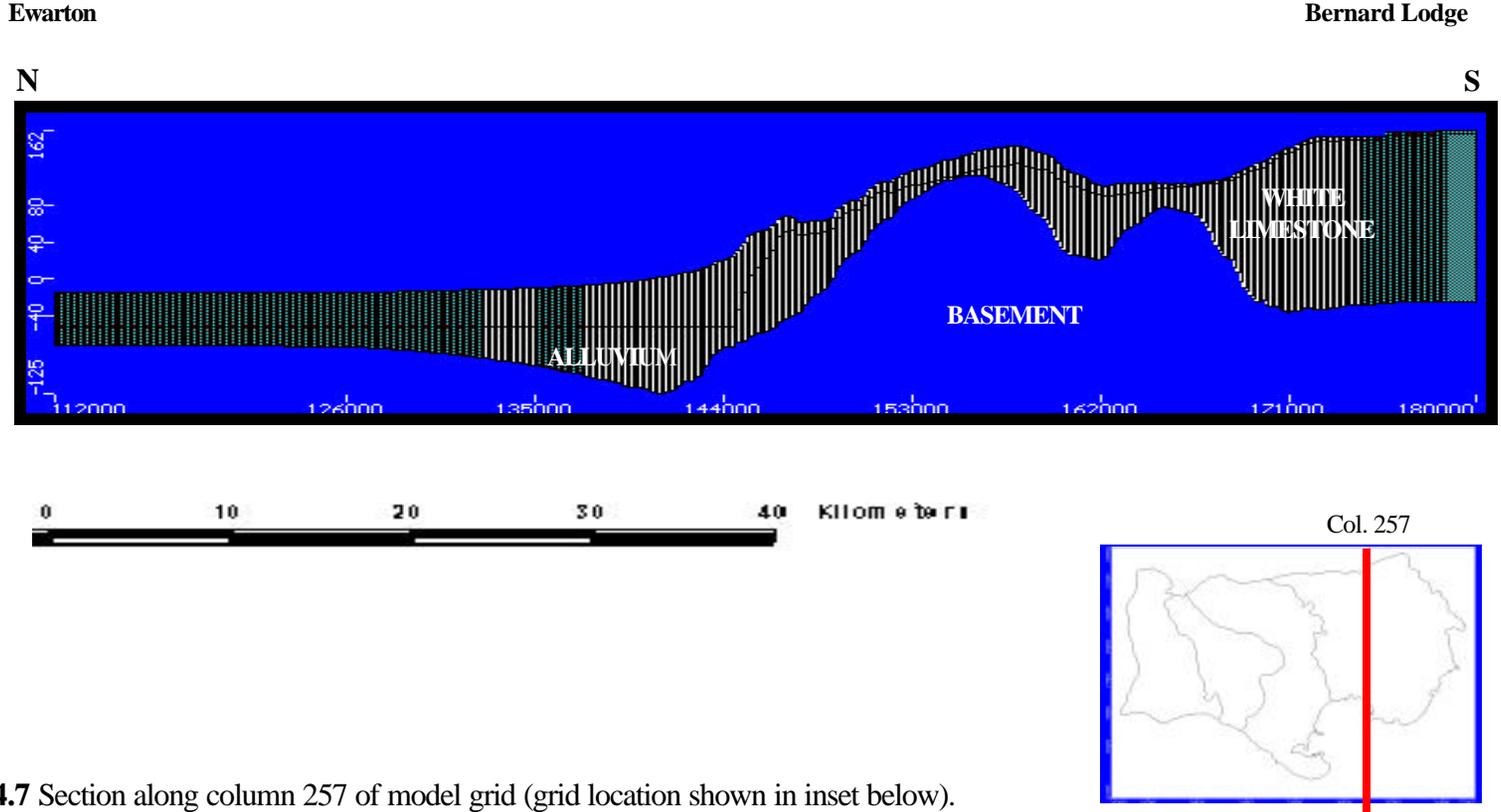


Figure 4.7 Section along column 257 of model grid (grid location shown in inset below).

Model Data Input

The finite-difference model was developed by incorporating geologic data and measured and inferred hydrologic data for the period 1998. Contour maps of the potentiometric surfaces of the two aquifers were developed and are based on the interpolation and extrapolation of heads from measured points (Figure 3.11-3.12). The maps were used to provide initial specified-head as reference elevations with which steady-state heads could be calculated. Input parameters to the model included horizontal and vertical aquifer hydraulic conductivity of the aquifers; hydraulic head for both aquifers, estimated recharge values to the topmost model layer, streambed conductance, and layer aquifer thickness. Each active grid cell was assigned values according to its location within the study area.

Boundary Conditions

Boundary conditions are specified for the upper surface, sides and bottom of the modeled area. The ground-water flow model area extends from eastern border of St. Catherine to within 5 kilometers of the western border of Manchester, and as much as 10 km from the Central Inlier (anticlinal axis) in the center of the island to the Caribbean Sea to the south (Figure 4.3). The water table surface is considered a specified-flux boundary where the flux is areal recharge. The lateral boundary of the Rio Cobre and Rio Minho-Milk river basins represents the periphery of active of model cells and is a no-flow boundary associated with topographic divides. These no-flow boundaries on the east and west are the result of the assumption that ground-water does not flow across rivers and streams, but discharges into them. The vertical surfaces that bound the model domain on the east and west are specified-flux (no-flow) boundaries. In addition, topographic divides are also ground-water divides.

The base of the White Limestone aquifer coincides with the basement volcanics where flow is negligible. Because no flow is assumed to the bottom of the White Limestone aquifer, it is a specified-flux (no-flow) boundary. A general-head boundary was used to simulate the northern edge of the study area because of the potential for upland subsurface inflow to the study area along this region that lies topographically below the topographic divide, the Central Inlier. General-head boundaries allow flow to occur into and from the model cell nearest to the boundary and is dependent on the conductance of the material at the boundary and on the difference between the specified-head at the boundary and the simulated head in the model cell. A constant-head boundary is used to represent the Caribbean Sea coast at the southern edge of the model area. Rivers are simulated as general-head boundaries where the river stage and riverbed conductance are specified. Three major rivers draining the area are Rio Cobre, Rio Minho, and the Milk River.

Simulation of Upland Subsurface Inflow

A general-head boundary (GHB) was assigned along the northern boundary of the simulated area (Figure 4.3). Head values along the boundary were estimated from interpolation of the depth to water based on known depths to water in a similar geological province (limestone) in the Rio Camuy, Puerto Rico (Torres-Gonzalez, 1996) under similar physiographic conditions. Head values assigned along this boundary are listed in Appendix II: Table I. Flow across the boundary (Q_b) was calculated as the product of the conductance (C_b) of the boundary and the difference between the head at the boundary h_b and the simulated head in the aquifer (h) in Eq. 2.

$$Q_b = C_b(h_b - h) \quad (2)$$

The conductance values of the General Head Boundary were determined from the product of the hydraulic conductivity of the head-dependent boundary cells (K_b) and the cross-sectional area (MW) of the boundary through which flow is simulated divided by the length (L) of the flow path:

$$C_b = \frac{K_b MW}{L} \quad (3)$$

where

- K_b is the conductivity of the general-head boundary cells
- M is the thickness of each layer
- W is the width of the each general-head boundary source cells
- L is the length of each general-head boundary source cells

The role of the variables M and L in this conductance equation is inverted for the computation of the streambed conductance as horizontal flow is simulated in this general-head condition, whereas vertical flow is simulated in the river aquifer interaction.

Simulation of Saltwater Interface

The saltwater-freshwater interface along the southern boundary of the Rio Cobre and Rio Minho-Milk river basins may be treated as a no-flow boundary where flow across a saltwater interface is considered negligible and diffusion is limited (LeGrand, 1976). Therefore in the bottom layer, a no-flow boundary is applied at the saltwater interface. The location of the interface can be approximated according to the Ghyben-Herzberg (1888; 1909) model in Figure 4.8:

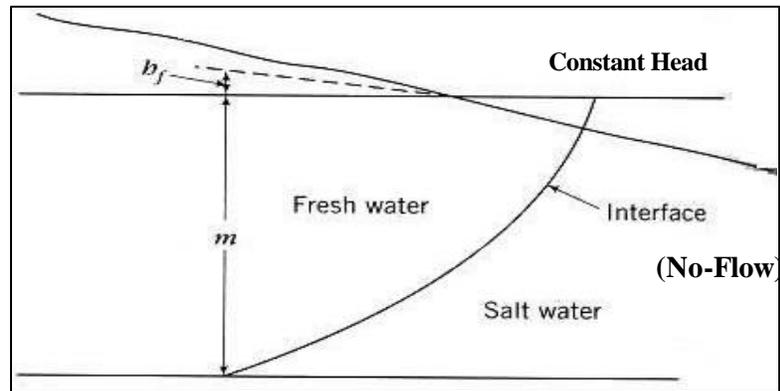


Figure 4.8 Idealized geometry to calculate the saltwater wedge (Domenico and Schwartz, 1990).

The contact zone between the two water bodies can be approximated using the Ghyben-Herzberg equation:

$$b = \frac{\rho_f}{\rho_s - \rho_f} H_f \quad (4)$$

where,

b = the depth of the interface below sea level

ρ_f = the density of freshwater density (1.000 kg/m³)

ρ_s = the saltwater density (1.025 kg/m³)

H_f = the elevation of the water table above saline water body elevation

$$b = 40H_f \quad (5)$$

The White Limestone aquifer is hydraulically connected to the ocean along its coastal margin. According to the Ghyben-Herzberg model, the interface between saline and fresh water ground-water in a coastal setting can be defined as the contact zone between

seawater intruding into rocks of a coastal area and the overlying fresh ground-water in a coastal aquifer flowing seaward (Yechieli, 2000). The distribution of freshwater head along this boundary is a function of saline water depth and the difference in their respective densities. The depth to the freshwater/saltwater interface in a coastal aquifer under steady-state conditions is approximately 40 times the height of the freshwater head (h_f) above sea level. In a coastal aquifer freshwater recharge mainly from lateral inflow forms a lens that floats on top of a base of higher density seawater.

Changes in recharge and/or induced conditions of pumpage and artificial recharge may disturb the hydrostatic equilibrium that results in the movement of the location of the interface and may also create a transition zone. The transition zone is a zone of mixing between salt and fresh water. Under equilibrium conditions (1) the interface that separates both fresh and salt water is fixed and (2) discharge occurs along a seepage face as shown in Figure 4.9.

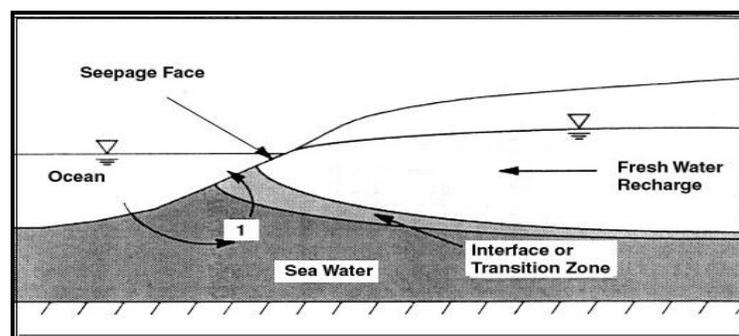


Figure 4.9 Occurrence of seawater in coastal aquifers

During pumping in a coastal aquifer, the freshwater head is drawn down and the saltwater-freshwater interface cones upward to establish a new hydrostatic equilibrium.