

## **2.1 Implementation of the Cook-Felderman technique**

In this chapter a in-depth investigation on the Cook-Felderman technique will be made, including some experimental test cases done in the supersonic wind tunnel at Virginia Tech. The purpose of these experiments was to obtain some real heat transfer data for verification of the data reduction method. The basic analytical equation for the determination of the flux from the surface temperature, is given by *equation 2.1* [6]:

$$q(t_n) = \frac{2\sqrt{krC_p}}{\sqrt{p}} \sum_{j=1}^n \frac{T_j - T_{j-1}}{\sqrt{t_n - t_j} + \sqrt{t_n - t_{j-1}}} \quad (2.1)$$

## **2.2 Theoretical studies**

To evaluate this method here, it was used in a test problem for which the exact answer is known.

The problem was created as follows: Using the solution to the Heat Equation for a constant surface heat flux on the surface of a semi-infinite solid [11], a time dependent temperature profile was created:

$$T(x,t) - T_i = \frac{2q\sqrt{at/p}}{k} e^{\left(\frac{-x^2}{4at}\right)} - \frac{qx}{k} \left(1 - \operatorname{erf}\left(\frac{x}{2\sqrt{at}}\right)\right) \quad (2.2)$$

On the surface ( $x = 0$ ) this equation reduces to:

$$T(t) - T_i = \frac{2q\sqrt{at/p}}{k} \quad (2.3)$$

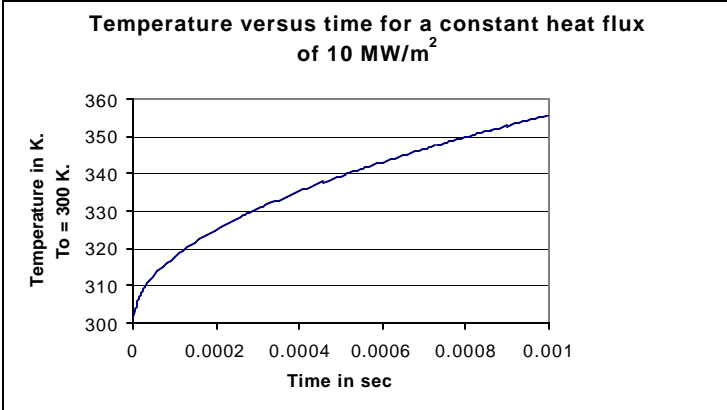
Where:

$$T_0 = 300 \text{ K}$$

$$\alpha = \frac{k}{rC_p}$$

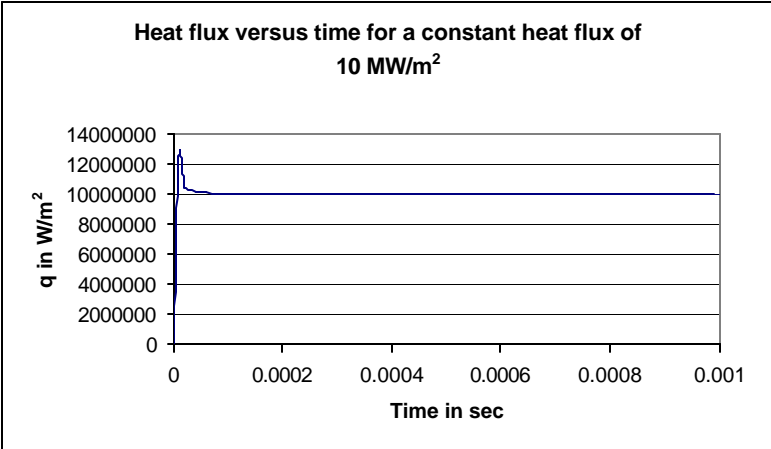
$$\tau = \text{Time}$$

By choosing an appropriate  $q = 10 \text{ MW/m}^2$ , the resulting time dependent surface temperature can be seen below.



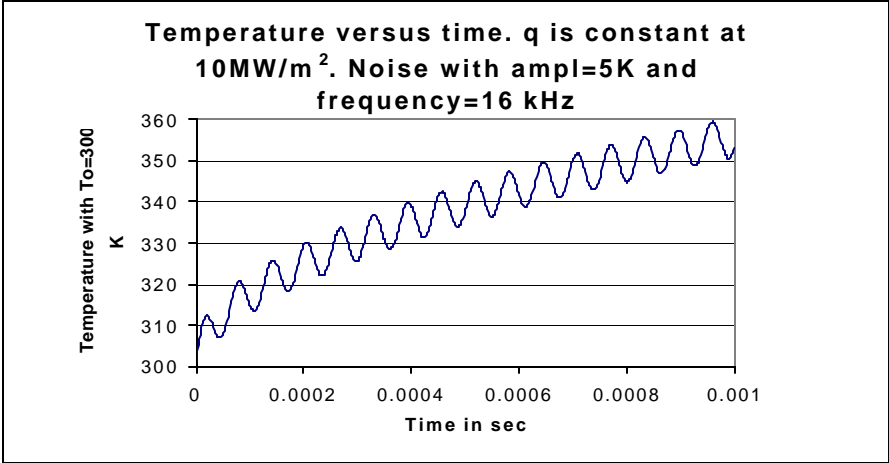
**Figure 2.1 Theoretical surface temperature curve calculated from a constant flux of 10 MW/m<sup>2</sup>.**

Suppose this is taken as the output of a thermocouple on the surface of the object through which the heat flux should be calculated. This can then be used as the input to the Cook-Felderman technique, and a comparison can be made to see if the “known” heat flux of 10 MW/m<sup>2</sup> can be retrieved. Utilizing this technique with a time step of 0.00001, the graph in *figure 2.2*, was created. Apart from the overshoot in the beginning, the flux is on the 10MW/m<sup>2</sup> mark.



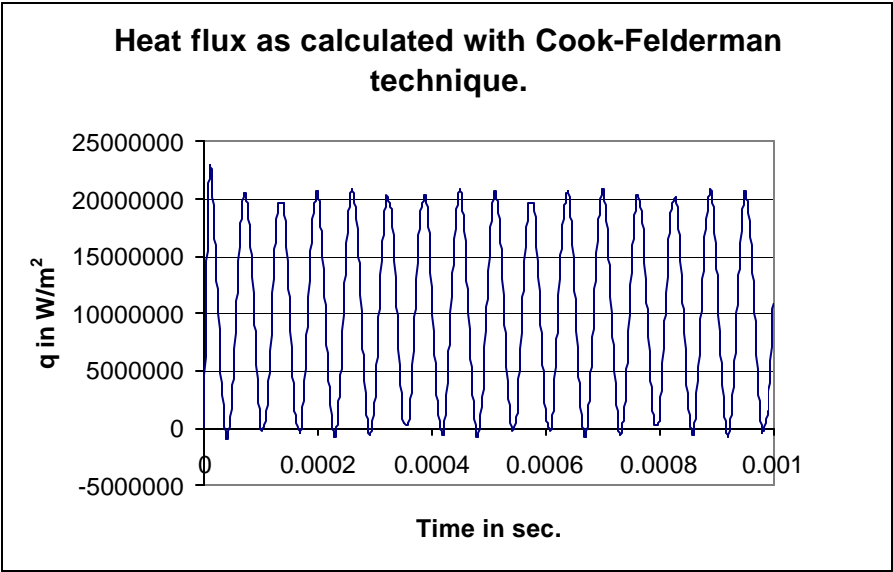
**Figure 2.2 The heat flux deduced from the temperature curve in figure 2.1, using Cook-Felderman.**

To look at the effect of noise on the temperature signal, a rather large sinusoidal noise wave was superimposed on the input temperature-time signal, as illustrated in *figure 2.3*. The amplitude in this case was set at 5 K and the frequency was 16 kHz.



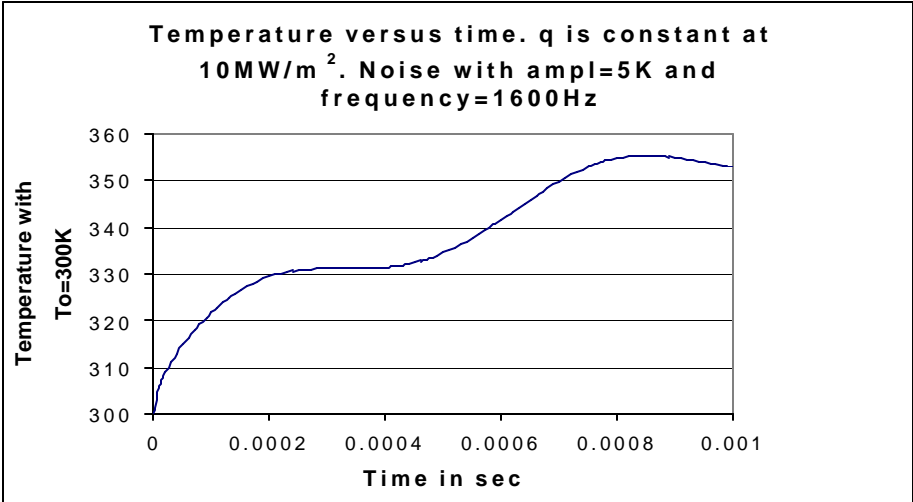
**Figure 2.3** Temperature curve calculated from a known flux of  $10 \text{ MW/m}^2$ , but this time with superimposed noise.

As can be seen in *figure 2.4*, this noise is amplified when converting the temperature data to heat flux by using Cook-Felderman.



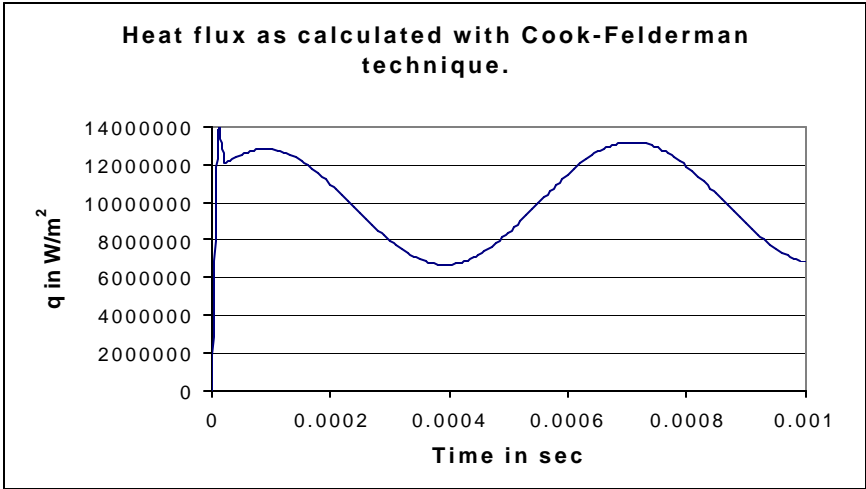
**Figure 2.4** Cook-Felderman predicted heat flux using noisy temperature input.

This may indicate a need to filter the measured surface temperature or the deduced heat flux, or both. If the frequency of the noise is decreased as in *figure 2.5*, the heat flux results are shown in *figure 2.6*.



**Figure 2.5 Temperature curve calculated from a known flux of 10 MW/m<sup>2</sup> , but this time with lower frequency noise superimposed.**

This clearly shows that the effect of a decrease in the frequency of the noise is very large.

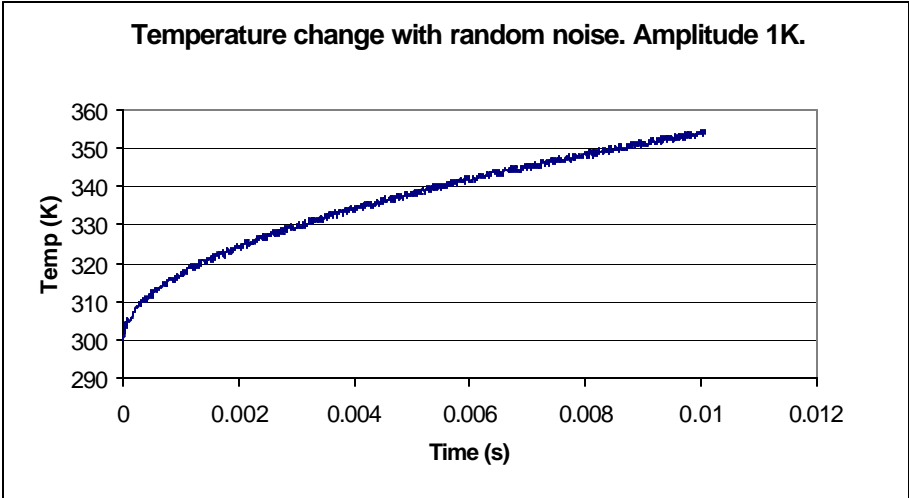


**Figure 2.6 Predicted heat flux using Cook-Felderman with the temperature in figure 2.5 as input.**

As seen in *figure 2.6* the noise generated is much less than that generated when the frequency and amplitude of the noise on the temperature signal, were higher. This means that the noise on

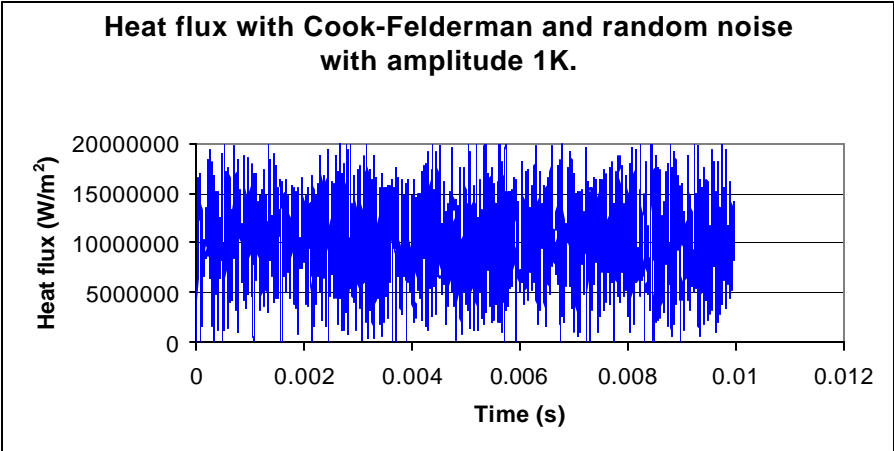
the temperature signal should be kept as smooth as possible to ensure a good result in the heat flux estimation.

Repeating this same evaluation with evenly distributed, random noise the following results were obtained. The results are very similar to those obtained with the superimposed sinusoidal noise.



**Figure 2.7** Created temperature signal with superimposed random noise.

As seen in the above figure the noise amplitude is much smaller than the previous theoretical analysis in *figure 2.3*. This was done because the amplitude of 5K is somewhat over conservative, but was used to show the basic effect of the noise on the eventual heat flux signal. In the next figure, *figure 2.8*, the heat flux was calculated with the Cook-Felderman method.

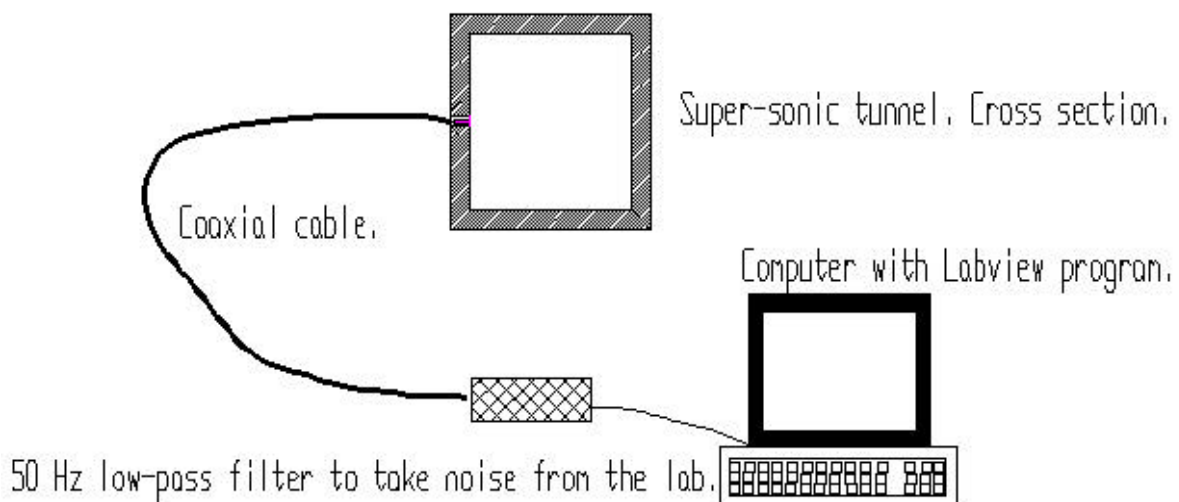


**Figure 2.8** The flux for the temperature in figure 2.7 using Cook-Felderman.

The amount of noise on the signal overwhelms the flux signal completely. In the next section a test on some experimental data will be performed. This will show the true nature of the Cook-Felderman technique.

### 2.3 Experimental set-up

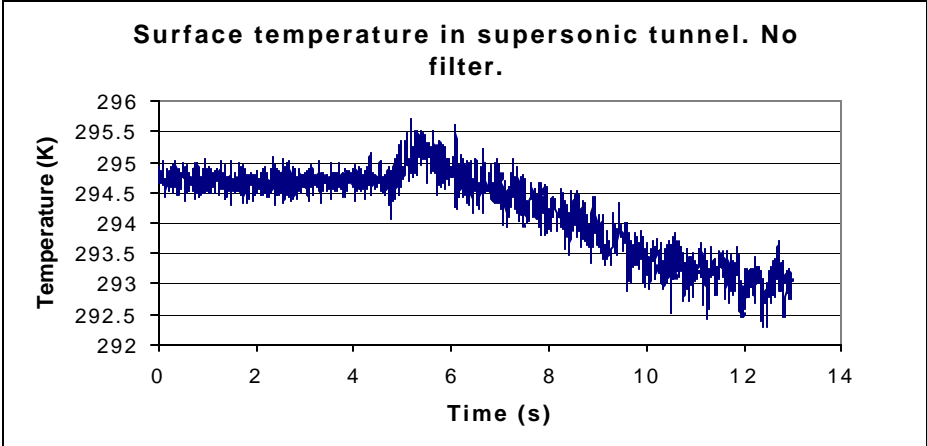
The layout of the experimental setup is shown in *figure 2.9*. A type-K thermocouple in an existing skin friction gage was installed in the side-wall of the supersonic tunnel. See Appendix C. The wall of the tunnel is constructed of an aluminum alloy. This material's thermo-physical properties are much different than the properties of the skin-friction gage, which is constructed mainly of copper. The heat flux calculated at the end of the data reduction will thus not be the true flux through the tunnel wall, since the flux through the wall and that through the gage will be different due to the differences in the physical properties. However, the aim of this experiment was to evaluate the Cook-Felderman technique and to see if reasonable results could be obtained with real data. From the tunnel, a coaxial cable transported the voltage, through a 50Hz low-pass filter (see Appendix B) to the computer. We made use of a Lab-View program to convert the measured voltage to the equivalent temperatures.



**Figure 2.9** Experimental set-up for tests conducted in the supersonic windtunnel.

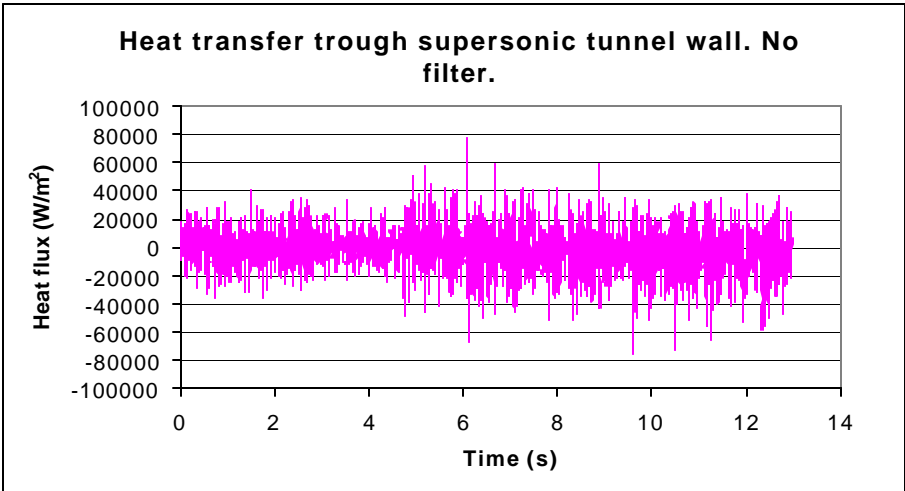
The temperature profile on the surface of the supersonic tunnel wall was measured, and the profile of the temperature change can be seen in *figure 2.10*. In this case, no filter were used,

because filters might, in certain applications, filter out some important data. A filter was used at a later stage, because the noise overwhelmed the data reduction scheme.



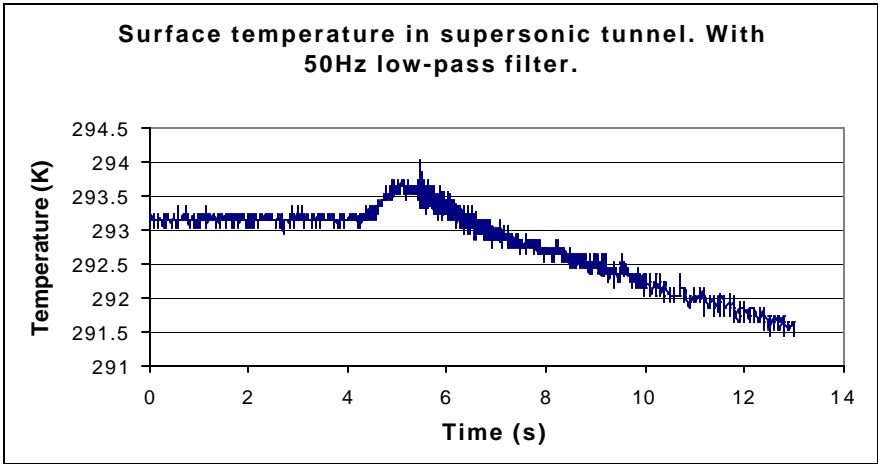
**Figure 2.10** Temperature measured on the supersonic wind tunnel wall.

The substantial, rapid variations in the temperature signal, of up to 1°C, shown in *figure 2.8*, are amplified a great deal when converting to heat flux. See *figure 2.11*. The time step here is the same as the sampling rate – 0.01s. These variations in temperature, and in flux (shown in *figure 2.10* and *2.11*) are commonly referred to as signal noise. This is clearly a very noisy temperature signal. This noisy signal contributes to the fact that the calculated flux as seen in the next figure, (*figure 2.11*) is almost unusable.



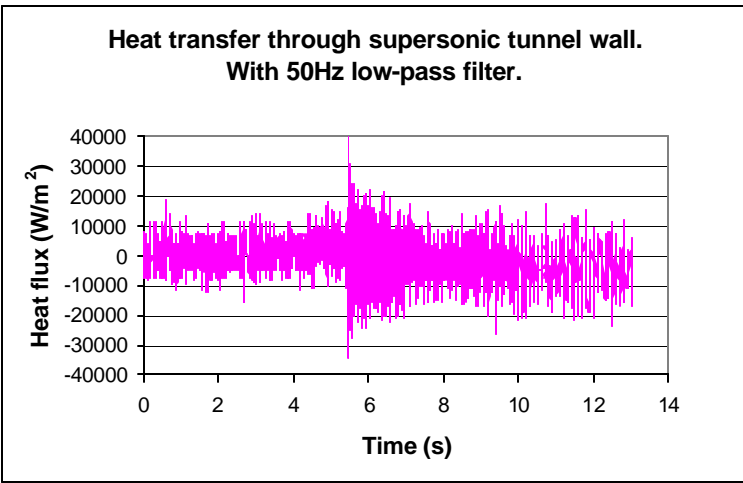
**Figure 2.11** Predicted heat flux from figure 2.10 calculated with Cook-Felderman.

Improvement to this situation was achieved by using a 50 Hz low-pass filter. It was built by the author and installed in the coaxial cable just before the temperature signal entered the computer. (See *figure 2.9.*) For further detail on this low-pass filter, see Appendix B. The reason a 50 Hz filter was used was because most of the electrical noise in the laboratory seemed to be in the band 50 Hz and above. The resulting temperature signal can be seen in *figure 2.12*.



**Figure 2.12** Temperature signal after filtering was done.

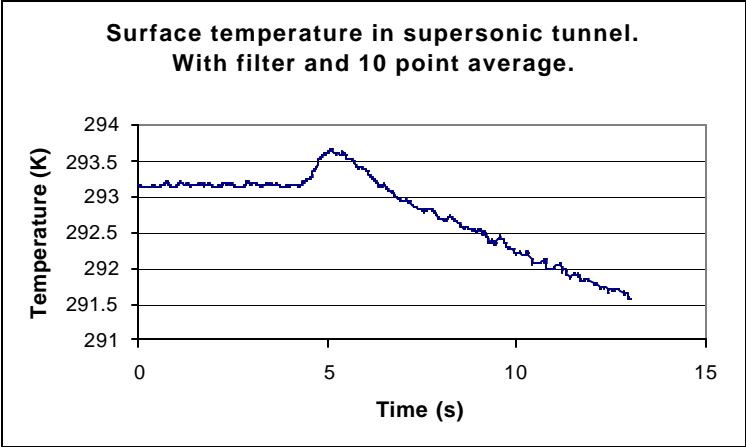
The corresponding deduced heat flux, in *figure 2.13*, is less noisy than that shown in *figure 2.11*, but still the noise is too much to be able to make any reasonable conclusions.



**Figure 2.13** Predicted heat flux from the temperature in figure 2.12 calculated with the Cook-Felderman technique.

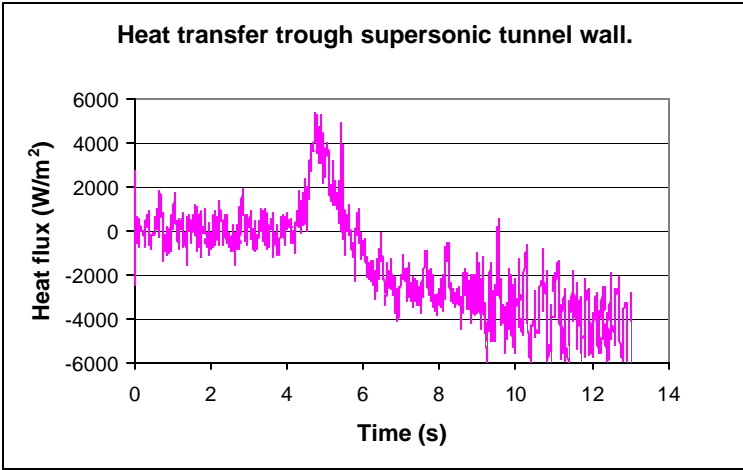


To make the data even more sensible a 10 point averaging scheme was used to smooth the measured temperature profile. This means that the average value of the temperature over every 0.1s was used to create the temperature profile as shown in *figure 2.14*.



**Figure 2.14 Average of the temperature profile in figure 2.12.**

If this data is used to calculate the flux through the wall it looks much better. See *figure 2.15*.



**Figure 2.15 Predicted heat flux from the temperature in figure 2.14 calculated with the Cook-Felderman technique.**

The deduced heat flux profile as seen in the above graph is much more sensible, in the sense that the noise is reduced a great deal. However, this continuous smoothing of each profile can cause problems, because some important effects might be lost in the smoothing process. But, if one is

intent on using the Cook-Felderman scheme, the data will have to be smoothed to obtain a reasonable answer. To capture more refined detail in the heat flux trend, another scheme will have to be incorporated that wouldn't need as much smoothing of the experimental data.