CHAPTER 6 ANALYSIS OF THE "STANDARD CASE" BREAKWATER

6.1 BREAKWATER MODELING

The "standard case" breakwater is 9.144 m (30 ft) long and has a radius of 1.524 m (5 ft). The shorter and longer mooring lines have stiffness values of 197,970 plf (pounds per linear foot) and 131,980 plf, respectively. For this breakwater, which is shown in Figure 5.1, it was decided that the breakwater should be modeled as a rigid cylinder made up of a number of panels that would give at least 0.5% accuracy on all results. With this in mind, several breakwaters constructed of 50 to 5000 panels were created. Given the short length of the cylinder, it was found that a breakwater composed of 120 panels resulted in a "worst case" of 0.21% discrepancy from the results (added mass, damping, and forces) obtained from the 5000 panel breakwater. Hence, a 120 panel cylinder is used as the "standard case" discretion of the breakwater surface.

6.2 FREE VIBRATION IN WATER

In Chapter 4, the natural frequencies and mode shapes of the "standard case" breakwater were presented. As established in Chapter 5, added mass and damping coefficients augment the equations of motion describing the structure's movement. Although the damping terms are not factors in calculating free vibration natural frequencies and mode shapes, the added mass terms (relatively large) greatly reduce the natural frequencies of the breakwater when submerged in water.

The eigenvalue problem which provided the dry (in air) natural frequencies ω_{dry} and mode shapes for the breakwater can be written as

$$\left|[K] - \omega_{dry}^2 [M]\right| = 0 \tag{6.1}$$

where [K] and [M] are the stiffness and mass matrices, respectively, and are of the form [6x6]. Now, with the addition of the added mass matrix $[M_A]$, the eigenvalue problem becomes

$$\left|[K] - \omega_{wet}^2 ([M] + [M_A])\right| = 0 \tag{6.2}$$

which yields the wet natural frequencies ω_{wet} and the associated mode shapes. In order to solve (6.2) for the wet natural frequencies, $[M_A]$ must be obtained, but it is dependent on this frequency. Therefore an iteration method is required. First, an initial guess for the first natural frequency is chosen. Secondly, the added mass matrix is obtained for that frequency and the first natural frequency is found by solving (6.2). The percent difference between the initial guessed frequency and the new value is found. This new value is then used as the new natural frequency and the added mass matrix is found again. Then (6.2) is solved and a new percent difference is found. The procedure is repeated until the percent difference is less than some specified value (2% was chosen). The final frequency value satisfying the 2% difference requirement is the first natural frequency of the cylinder in water. The procedure is then repeated for the other 5 natural frequencies. Table 6.1 compares the six dry natural frequencies to the six wet natural frequencies.

Table 6.1 – Comparison of Dry and Wet Natural Frequencies

Natural Frequency Number	Dry Natural Frequency (rad/s)	Wet Natural Frequency (rad/s)	Percent Difference
1	17.3	3.9	77.4 %
2	36.4	11.4	68.7 %
3	66.0	11.7	82.3 %
4	71.9	17.1	76.2 %
5	72.2	21.2	70.6 %
6	87.5	55.6	36.4 %

It is clear that most of the natural frequencies decrease sharply due to the presence of water. However, although the same coupling is present in the modes for the structure in water, these modes correspond to different natural frequencies. For instance, for the third dry natural frequency, the corresponding mode shape is heave alone. However, heave alone is the mode shape corresponding to the second wet natural frequency. This can be seen most clearly by comparing the two modal matrices

$$[u_{dry}] = \begin{bmatrix} 0 & 0.99 & 0 & 0 & 0 & -0.52 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0.74 & 0 & 0 & 0.65 & 0 & 0 \\ 0 & 0.14 & 0 & 0 & 0 & 0.86 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -0.68 & 0 & 0 & 0.76 & 0 & 0 \end{bmatrix} \tag{6.3}$$

and

$$[u_{wet}] = \begin{bmatrix} 0 & 0 & 0.99 & 0 & -0.38 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0.99 & 0 & 0 & 0 & 0 & 0.01 \\ 0 & 0 & 0.02 & 0 & 0.93 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0.01 & 0 & 0 & 0 & 0 & -0.99 \end{bmatrix} \tag{6.4}$$

where the six rows of each matrix represent surge, heave, sway, pitch, yaw, and roll, respectively. The first mode shape has remained sway combined with roll but the relative values have changed dramatically. The resulting first mode shape for the cylinder in water is almost entirely sway. Likewise, the sixth wet mode shape is mostly roll, and the third mode shape surge. The heave and yaw modes are the same as in air but have changed order. Therefore, no new figures are needed for these 5 modes. The fifth mode is combined surge and pitch and is very similar to the sixth mode shape for the cylinder in air. Hence, no new figure will be given here either (see Figure 4.9).

Given that water wave frequencies are usually in the range of 0 to 6 rad/s, it is advantageous to change the diameter of the mooring lines such that the first natural frequency of the breakwater in water is in this range. By trial and error, it was found that by dividing the stiffness of the lines by 58, a first wet natural frequency of 1.25 rad/s is obtained. All six wet natural frequencies in rad/s become

$$[\omega_{wet}] = [1.25 \quad 1.64 \quad 2.32 \quad 3.19 \quad 3.83 \quad 17.94]^T \tag{6.5}$$

The added masses that caused the changes in natural frequencies will be displayed graphically in the next section and will not be listed here. The new stiffness values for the shorter and longer mooring lines become 3413 plf and 2276 plf, respectively.

6.3 FORCED VIBRATION OF THE BREAKWATER

It was discussed in the introduction of Chapter 5 how the structure would respond linearly to forces that act on the breakwater as the waves pass over it. In addition, it was noted that the added mass and damping coefficients are calculated for the structure's motion in still water. However, since the structure vibrates at the forcing frequency (frequency of the waves), it can be said that these terms are dependent on the frequency of the waves. Nevertheless, these terms are independent of the direction of the waves and are a function of the frequency only. The forces acting on the structure are determined for waves passing over a rigid structure and are dependent on both the angle of wave propagation and the frequency of the waves. Three directions of wave propagation are considered to include incident angles β of 0°, 15°, and 30°. The added mass and damping coefficients will be considered first, followed by hydrodynamic forces, and finally the structure's effectiveness for all three cases.

6.4 HYDRODYNAMIC COEFFICIENTS: ADDED MASS AND DAMPING

The added mass matrix was found for numerous wave frequencies and the elements are plotted vs. kR where k is the wave number and R is the radius of the cylinder. A(1,1) is element (1,1) of the [6x6] added mass matrix [M_A]. Figure 6.1 is a plot of a nondimensionalized A(1,1) where ρ is the density of ocean water and V is the volume of the breakwater. Likewise nondimensional A(2,2), A(3,3), A(4,4), and A(5,5) are plotted vs. kR as Figures 6.2 – 6.5. A(6,6) is always approximately zero due to the ability of the breakwater to undergo roll without much resistance from the water. Also, it should be noted that the added mass terms are "added moments" for the rotational degrees of freedom 4, 5, and 6 and these elements are divided by an additional "R" to nondimensionalize the quantities A(4,4) and A(5,5). Likewise the diagonal elements C(1,1) – C(5,5) of the damping matrix [C] are plotted as nondimensional quantities vs. kR as Figures 6.6 – 6.10 where ω is the frequency of the waves. Once again the roll mode damping C(6,6) is always approximately zero.

All of the nondimensional added mass curves seem to peak at some value of kR and then decrease, only to rise and level off. It is interesting that the maximum added mass for the first four degrees of freedom occurs at around kR=0.25, whereas the maximum added mass for the fifth degree of freedom is found at about kR=0.50.

The nondimensional damping curves are all somewhat "bell shaped". They all peak somewhere between kR=0.4 and kR=0.75. The rotational damping curves are more spread out over the independent variable kR than the translational ones.

6.5 HYDRODYNAMIC FORCES

The forces {F} making up the {6x1} matrix corresponding to the six degrees of freedom are plotted vs. kR for numerous frequencies for all three cases of β = 0°, 15°, and 30° shown in Figures 6.11, 6.12, and 6.13, respectively. Here, g is the acceleration of gravity, A is the amplitude of the waves (1 for all cases), and L is the length of the structure (9.144 m). For normal waves, only vertical (heave) and horizontal (sway) forces are developed as expected for a fixed structure. For oblique waves, all forces and moments are plotted vs. kR except the roll moment for the same reason there were negligible added mass and damping coefficients for this mode.

For normal waves, the two nondimensional hydrodynamic force curves (heave, F2 and sway, F3) are very similar in shape and both peak at a kR value of about 0.4. For these two force curves, the maximum value decreases and shifts slightly to the left (lower corresponding kR) as β increases to 15° and 30°. On the other hand, the nondimensional moments pitch (F4) and yaw (F5) along with the surge force (F1) all tend to increase as the incident angle increases. This is really quite logical, for the larger the incident angle, the more involved pitch, yaw, and surge become.

6.6 STRUCTURAL RESPONSE: RESPONSE AMPLITUDE OPERATOR (RAO)

Solving (5.1) for a given wave frequency (and structural response frequency) results in solutions for $\{\xi\}$, a {6x1} matrix defining the structure's amplitudes in each of the six degrees of freedom for the given wave frequency. In general, ξ_j is a complex number, but to simplify things ξ_j will be used from here out only as a single value, which is the magnitude of the complex number. (RAO)j for j=1 to 6 is the Response Amplitude Operator for each of the j degrees of freedom. It is equal to ξ_j/A where A is the amplitude of the incident wave. Since A is always one for our cases, we will simply plot

ξ_j/A vs. kR, keeping in mind that A is always one. Figures 6.14, 6.15, and 6.16 show this relationship for β equal to 0°, 15°, and 30°, respectively. The structural response is combined heave and sway for all normal waves. Therefore, only these two variables are plotted. Likewise, for oblique waves, translational motion is extremely dominant and only these values are plotted.

For normal waves, it is clear that sway is the dominant mode for a kR value less than about 0.4. The sway response peaks at around kR=0.3 and the heave response peaks at a kR value of nearly 0.4. Although the heave response barely changes as the incident angle is increased, the sway response perpetually decreases. Although the relative amplitude of the surge response is small for most values of kR, it is interesting that the curve peaks at a kR value slightly greater than 1.0. Here surge is clearly the dominant response.

6.7 INFLUENCE OF STRUCTURE ON WATER WAVES

The influence that the structure has on the waves passing over it will now be considered. The transmission coefficient T_c is defined as the ratio of the amplitude of the waves behind the structure to the incident wave amplitude. For convenience, the amplitude of the waves on the other side of the structure will be defined as the average value of the wave amplitude from the breakwater to 50 m downstream. It will be calculated from the center of the breakwater and in a direction normal to the cylinder's axis regardless of the direction of the incident wave.

The values of T_c as a function of kR are shown in Figure 6.17. It is evident that the structure is most effective for normal waves at kR approximately equal to 0.65, which corresponds to a wave frequency of 2.05 rad/s. For β equal to 15° the structure is most effective for kR equal to about 0.11 (wave frequency 1.85 rad/s). For β equal to 30°, the structure is most effective at the natural frequency of the cylinder in water (wave frequency 1.25 rad/s) and a corresponding kR of around 0.3. Figure 6.17 shows clearly that such a breakwater can be effective for a large bandwidth when β equals 0° or 15°.

6.8 FREE SURFACE ELEVATION AMPLITUDE

The transmission coefficient of Section 6.4 gives some idea of the overall effectiveness of the breakwater. However, since it is defined only for the center of the structure, its value may be misleading. Therefore, four specific cases will be considered to include the three most effective wave frequencies (1.25 rad/s, 1.85 rad/s, and 2.05 rad/s) discussed in the previous section and a typical wave frequency of $\pi/2$. For β equal to 0°, the free surface elevation amplitudes for the four aforementioned frequencies are shown in Figures 6.18-6.21, for β equal to 15° in Figures 6.22-6.25, and for β equal to 30° in Figures 6.26-6.29. It should be stated here that the free surface elevation amplitudes are not "snapshots" taken at some time t. On the contrary, the term free surface elevation amplitude refers to the maximum amplitude of a wave at any time t for a given location (X,Y). To make things more clear, consider the case of waves passing over the same region without a structure present. In this case (with our unit amplitude waves) the free surface elevation amplitude would simply be 1.0 for all (X,Y).

The breakwater is 9.144 m long and is located at X=0 and extends from Y=-4.572 m to 4.572 m. The Y-axis extends beyond this range in order to manifest the role of end effects on breakwater effectiveness.

For β equal to 0°, the effectiveness of the structure is apparent in all four plots. However, by comparison, it is clear that the most effective case is for ω=2.05 rad/s as indicated by the low transmission coefficient. On the other hand, careful inspection of Figure 6.20 reveals that at the ends of the structure, the structure's effectiveness diminishes more sharply for this wave frequency compared to ω=1.25 rad/s (Figure 6.18) and $\omega=\pi/2$ rad/s (Figure 6.21). Figures 6.18 and 6.21 show a broader band of effectiveness in the Y direction. However, since the effectiveness of the breakwater is considered behind the center of the structure (Y=0), it can be concluded that for normal waves, the transmission coefficient well represents the structure's effectiveness.

Oblique angles yield more interesting results. For β equal to 15°, Figure 6.23 (lowest transmission coefficient case) suggests a very effective structure over the entire Y region shown. However, it also shows that the waves in the upper portion (+Y values) of the downstream region are more sharply reduced. On the other hand, for ω=1.25 rad/s (Figure 6.22) the effectiveness is dramatically more pronounced for the lower portion of the waves in the downstream region. The role of end effects is shown in all four plots.

Finally, for β equal to 30°, the transmission coefficient may be somewhat misleading. It is not, however, for ω=1.25 rad/s which portrays a very effective structure. The lower portion of the structure is even more effective than the transmission coefficient suggests. Nevertheless, the other cases selected reveal some serious concerns. For a short breakwater (L=9.144m) used in largely oblique waves, the waves seem to pass around the structure and are actually amplified in some cases after some distance beyond the structure. Therefore, if used as a single breakwater, the structure should be close enough to the shore to prevent this effect. It should be kept in mind, however, that such structures are intended to be used in series with one another, which should minimize this effect. This topic is considered in Chapter 7.

One final point: when comparing Figures 6.22 and 6.23, it seems that the breakwater of Figure 6.22 is much more effective. However, the transmission coefficient is lower for the breakwater of Figure 6.23. This shows clearly the importance of free surface elevation plots. It should be recalled that T is calculated over the first 50 m after the structure. The amplitudes close to the structure are extremely low for Figure 6.23 but soon increase and balance out. The amplitudes for Figure 6.22 are more close to constant at the center of the structure over the 50 m span. Therefore the values near the structure for the breakwater of Figure 6.23 bring down the average T over the region and make the structure appear more effective. However, if the structure being protected is 100 m straight back from the breakwater and is subject to both cases, the larger waves will be felt as a result of the breakwater of Figure 6.23.

6.9 FREE SURFACE ELEVATION PLOTS

Figures 6.18-6.29 show the amplitude of the free surface for a given wave frequency. However, it is also interesting to show how the free surface looks at instants in time. These “snapshots” are cyclic for some period T, which is dependent on the wave frequency ω. For β equal to 0°, the free surface elevation plots for the four frequencies considered in the previous section (1.25 rad/s, 1.85 rad/s, 2.05 rad/s, and π/2 rad/s) are shown in Figures 6.30-6.33, for β equal to 15° in Figures 6.34-6.37, and for β equal to 30° in Figures 6.38-6.41. The figures all contain six parts (a)-(f) which are plots at a time t which is a fraction of the period T and include t equal to 0, 0.2T, 0.4T, 0.6T, 0.8T, and

T. Although the plot for t=T is redundant (same as the plot for t=0), it is useful in showing a complete cycle.

For β equal to 0°, the free surface elevation plots show the effectiveness of the structure. It is apparent that the size (amplitude) of the waves after the structure have similar values for any t considered. However, the superposition of the incident and reflected waves before the structure is shown by the varying wave amplitudes for different times (compare Figure 6.30 (a) to 6.30 (b)). Since our main concern is the amplitude of the waves downstream of the breakwater, we will focus on these waves. The free surface elevation amplitude plots of the previous section show that for normal waves the most effective case is when the wave frequency is 2.05 rad/s. By comparing Figures 6.30-6.33, this is more difficult to determine, because the structure is effective for all four cases and no color scheme is used to compare relative amplitudes. Nevertheless, some interesting points can be made. The end effects are very obvious. Figure 6.18, which suggests a large effective bandwidth in the Y direction, shows a sharply decreased wave amplitude near the cylinder's ends compared to the wave amplitude at Y=0. This is shown clearly in Figure 6.30 (a) and (b) as three-dimensional plots for time t=0 and t=0.2T. On the contrary, Figure 6.20 shows that for a wave frequency of 2.05 rad/s, the wave amplitudes near the ends of the structure are much larger than those near the center. This is bolstered by Figure 6.32 (a), which shows wave amplitudes that are larger at the cylinder's ends.

For β equal to 15°, some interesting points can be made as well. It was previously determined that at a wave frequency of 1.25 rad/s, the breakwater is much more effective at the bottom (-Y) than the top (+Y) of the structure. This is evident in Figure 6.34 (a) by comparing the wave heights at these locations at positions after the structure. The most effective case (based on the transmission coefficient) is when the wave frequency is 1.85 rad/s. However, this is primarily because the wave amplitudes at the center of the structure are temporarily reduced (for nearly 20 m) but then increase. This can be seen by examining the wave just after the structure in Figures 6.35 (a)-(f).

The main concern of the cases that consider an incident angle of 30° is the amplification of waves some distance past the structure. This is seen by examining the change in wave amplitude at the center of the breakwater (Y=0) as X increases (Figure 6.39 (e) as well as many others).