

**INVESTIGATION INTO SNAP LOADING OF CABLES
USED IN MOORED BREAKWATERS**

by

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(ABSTRACT)

A two-dimensional, nonlinear dynamic analysis is conducted on a moored breakwater configuration to investigate snap loads in mooring lines. Breakwaters are structures used to attenuate or eliminate waves and protect shorelines, harbors, and other natural and man-made marine structures from wave damage. The breakwater in this investigation is modeled both as a point mass and as a rigid body. Both models are subjected to free undamped motions and forced undamped wave motion. Energy is dissipated through the use of a coefficient of restitution applied when a mooring line becomes taut (i.e., reaches its natural length). The mooring line is modeled as an inextensible cable with no axial or bending resistance when slack. Snap loading arises when a mooring line transitions suddenly from a slack condition to a taut condition. The analysis was conducted on a breakwater configured upside down and hanging by two mooring lines. The length of the mooring lines, coefficient of restitution, size and shape of the breakwater, initial position of the breakwater, amplitude of wave forcing, ratio of vertical to horizontal forcing, and frequency of forcing were all varied in the analysis. The results show that the rotations of the rigid body and the wave forcing have a significant role in the analysis, indicating that a rigid-body model for a moored breakwater under wave forcing is the more accurate model.

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Nomenclature

(Definitions of Symbols and Terms)

A,a:	horizontal dimension of breakwater, from center to edge
a_{mn} :	entry in velocity matrix
B,b:	vertical dimension of breakwater, from center to edge
b_{mn} :	entry in impact condition matrix
C:	center of mass of breakwater
c_n :	various constants of integration
CV_x :	distance from C to V in x direction
CV_y :	distance from C to V in y direction
CW_x :	distance from C to W in x direction
CW_y :	distance from C to W in y direction
e:	coefficient of restitution
f_{cr} :	critical force
f_o :	amplitude of harmonic forcing function
F_x, f_x :	harmonic force applied in x direction
F_y, f_y :	harmonic force applied in y direction
g:	gravitational acceleration
g_1 :	circular arc defined by right cable being taut
g_2 :	circular arc defined by left cable being taut
H,h:	height of region U
I_c :	mass moment of inertia about its center
J:	left connection of cable to breakwater
JV_x :	distance from J to V in x direction
JV_y :	distance from J to V in y direction
K:	right connection of cable to breakwater

KE:	kinetic energy
KW_x :	distance from K to W in x direction
KW_y :	distance from K to W in y direction
L:	Lagrangian
m:	mass of breakwater
PE:	potential energy
Q_q :	generalized force for coordinate q
q:	coordinate (x, y, θ , α , or β)
R/r:	length of cable (radius of g_1 or g_2 circular arc)
S,s:	spacing between centerline of the symmetric configuration and a support
T,t:	time
T_x, t_x :	phase for harmonic horizontal force component F_x
T_y, t_y :	phase for harmonic vertical force component F_y
U:	valley-shaped region defined by circular arcs g_1 and g_2
V:	left support of cable to breakwater
v_n :	normal velocity
v_t :	tangential velocity
v_x :	velocity in x direction
v_y :	velocity in y direction
W:	right support of cable to breakwater
X,x:	horizontal direction
\dot{x} :	velocity in x direction
\ddot{x} :	acceleration in x direction
Y,y:	vertical direction
\dot{y} :	velocity in y direction
\ddot{y} :	acceleration in y direction
α :	angle from vertical to taut mooring line

β :	angle from vertical to line between connection points
γ :	incoming and rebound angle
δ :	virtual displacement
δW :	virtual work
θ :	angle of rotation of breakwater measured about its centroid
$\dot{\theta}$:	angular velocity of breakwater measured about its centroid
$\ddot{\theta}$:	angular acceleration of breakwater measured about its centroid
$\lambda, \lambda_1, \lambda_2$:	Lagrange multipliers
v :	ratio of amplitude of f_y to amplitude of f_x
ϕ :	angle measured from horizontal to tangent line at point on g_1 circular arc
ψ :	angle measured from horizontal to tangent line at point on g_2 circular arc
Ω :	frequency of harmonic forces

Note:

- upper case letters represent dimensional parameters
- lower case letters represent nondimensional parameters
- subscript (i) indicates a variable at time of impact or new initial condition after impact
- subscript (m) integer indicates column position in matrix
- subscript (n) integer indicates row position in matrix
- subscript (x) indicates a variable in the horizontal direction
- subscript (y) indicates a variable in the vertical direction
- subscript (θ) indicates a rotational variable
- superscript (-) indicates a variable just before impact
- superscript (+) indicates a variable just after impact
- dot (\bullet) indicates a derivative
- subscript (o) indicates an initial condition
- hat (\wedge) indicates impulsive action