CHAPTER 2

PHOTOVOLTAIC FUEL

2.1 Introduction

Along the lines of other generating sources of electricity, photovoltaic systems also require a fuel in order to generate electricity. As inferred from the name, photovoltaic, which is a combination of the Greek word “ Photo ” which means light and “ Voltaic ” which means electric, the fuel needed for this generating system is sunlight. In a nutshell, photovoltaic means light to electricity. Thus a designer of photovoltaic systems must be able to ascertain the amount of sunlight that exists per given area at different locations and times throughout the year. This chapter presents the empirical formulas used in estimating solar insolation with emphasis on direct and diffuse radiation which are the components of global radiation. Before presenting these formulas, some basic knowledge of solar geometry is required. The generalized model is developed after Rahman [15].

2.2 Solar Geometry

Figure 2.1 below shows the sun-earth relative positions at various times of the year. The earth rotates about its axis every twenty-four hours and completes one revolution around the sun every 365 days. While this is happening, the sun also rotates on its axis approximately once every earth month. The earth’s axis of rotation is called the polar axis and it is inclined at an angle of 23.5^{o} from the elliptic axis, which is normal to the elliptic plane. The elliptic plane is the plane of orbit of the earth around the sun. The inclination of the earth from the straight-up position in reference to the plane of orbit makes the northern hemisphere tilt toward the sun in summer and away from the sun in winter, thereby producing the seasonal variations on earth. While the earth rotates around the sun, it passes through four distinct points during every revolution as follows;

1. Winter Solstice: It occurs approximately December 21 of every year. At this time, the north pole is inclined 23.5^{o} away from the sun and all points on the earth’s surface north of the arctic circle, 66.5^{o} N are in complete darkness and all points south of the Antarctic circle 66.5^{o} S receive continuous sunlight.

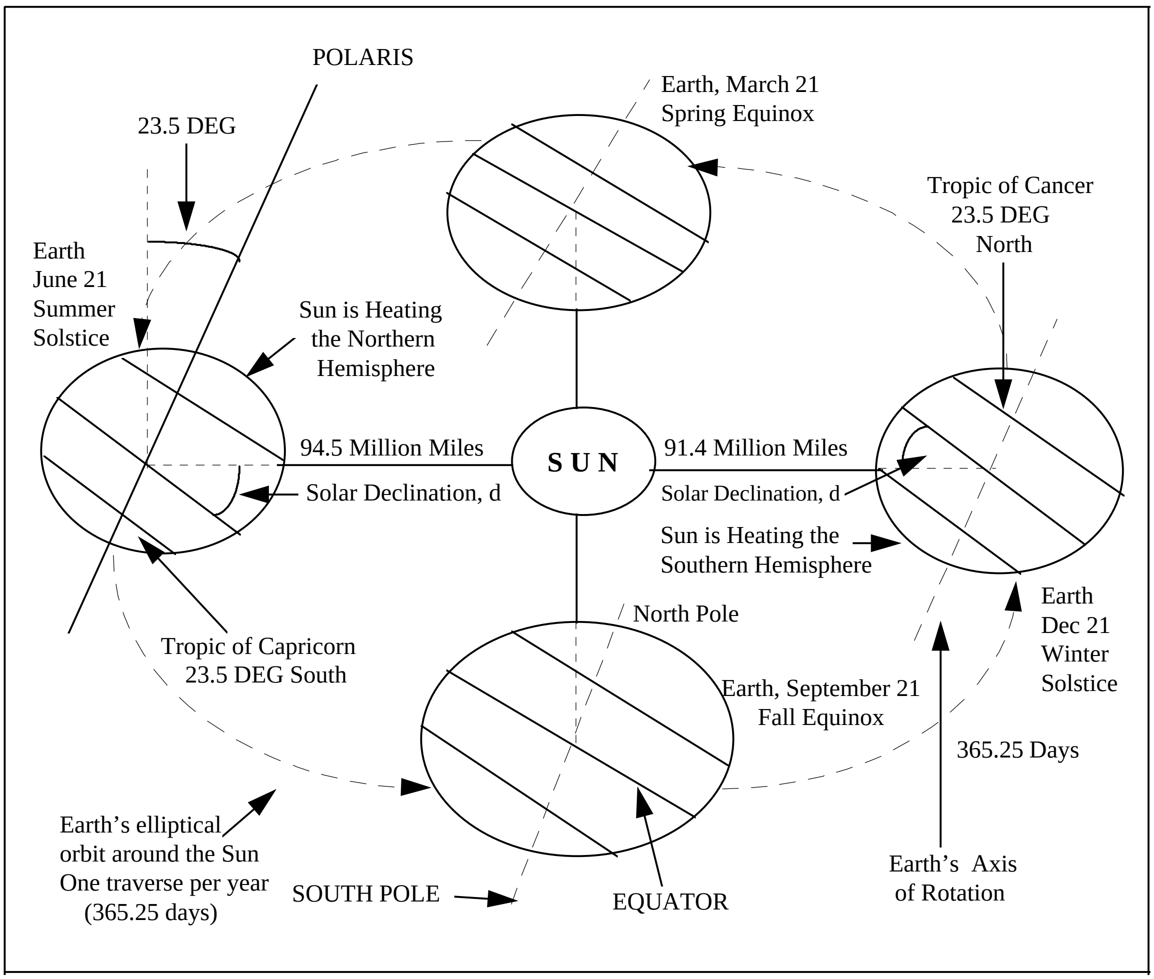

Figure 2.1: The Earth in its Yearly Traverse Around the Sun [15]

2. Summer Solstice: It occurs approximately June 21 of every year. At summer solstice, the earth experiences conditions opposite to winter solstice described above.

3. Spring Equinox: This is also called vernal equinox and it occurs approximately March 21 of every year. At this time, all points on the earth's surface experience 12 hours of daylight and 12 hours of darkness.

4. Fall Equinox: This is also called autumnal equinox and it occurs approximately September 21 of every year. All points on the earth's surface experience the same conditions described in spring equinox above.

2.2.1 Solar Constant (SC_0)

The solar constant is the total energy per unit time that is received on a unit area perpendicular to the direction of propagation, at an average sun-earth distance, outside the atmosphere. The SI unit for the solar constant is $1353 W / m^2$ or $1.353 KW / m^2$.

2.2.2 Earth-Sun Distance

The amount of solar radiation available outside the earth's atmosphere varies slightly according to the earth-sun distance. These variations have been estimated to be as high as 7 %. Daily variations have also been observed because of the elliptical orbit of the earth's rotation around the sun. As a result the solar constant is just an average value. The variations in total daily solar radiation can be computed using the following empirical formula by Lunde [11];

$$SC_d = SC_0[1.0 + 0.033 Cos\theta] \text{...} 2.1$$

where SC_d = Solar daily constant

SC_0 = Solar constant

θ = 360N/365

N = Julian date (January 1 = 1)

2.2.3 Declination Angle (DA_d)

The declination angle is the angle between the maximum elevation point of the sun's daily path across the sky and the same point of that path at either equinox as observed from a fixed point on the earth's surface. This angle varies between +23.45 degrees to -23.45 degrees in the northern hemisphere. These values are obtained at summer solstice, June 21 and winter solstice, December 21. The following empirical formula by Duffie and Beckman [3], can be used to estimate the declination angle.

$$DA_d = 23.45 Sin[360(N + 284) / 365] \text{...} 2.2$$

where DA_d = Declination angle
N = Julian date (January 1 = 1)
The quantity within the brackets is in degrees.

2.2.4 Hour Angle (HA)

The hour angle is the angular displacement of the sun east or west of the local meridian due to the spinning of the earth on its axis at the rate of 15 degrees per hour. The hour angle is negative during A.M hours and positive during P.M hours. The reference point for the hour angle is solar noon and its value is zero at this time. The hour angle is related to the time parameter, H, by the following equation [15];

$$H = HA + \frac{\pi}{2} \qquad \text{...2.3}$$

2.2.5 Solar Noon

Solar noon is the local time of day when the sun is at its highest altitude for that day for any given location on earth. That is, solar noon corresponds to the time when the sun crosses the observers meridian.

2.3 Solar Radiation

Solar radiation is comprised of heat, visible light and ultraviolet radiation. The relative amount of these components in sunlight depends on the air mass and atmospheric conditions. Total solar radiation is composed of short-wave solar radiation and long-wave terrestrial radiation. The short-wave solar radiation is composed of the direct normal component of sunlight, the diffused component of skylight and the component reflected from natural surfaces. The amount of contribution from these three components varies according to time-of-day, and sky and ground conditions. When these three components are measured together on a horizontal surface, the resulting quantity is called global radiation. The long-wave terrestrial radiation has two components, namely, the incoming atmospheric component and the outgoing terrestrial component. Global radiation is measured using a pyranometer. Direct or beam radiation is the radiation coming from the sun and a small portion of the sky around the sun at normal incidence, when the collector is perpendicular to the ray of the sun. The pyrheliometer is the instrument commonly used to measure the beam component of insolation. Diffuse component of global radiation comes from the sky and surroundings, but not directly from the sun. Reflected component of global radiation comes from the surfaces that surround the solar array [15].

2.4 Location Factor

Assuming the earth is a perfect sphere, the spherical coordinate system can be used to locate a given point on the earth's surface, both in time and space. In the spherical coordinate system, the location factor (LF) is given by;

$$LF = Sin(P)Sin(H)Cos(DA_d) + Cos(P)Sin(DA_d) \quad \text{...} 2.4$$

where P = Position on earth given by $P = \frac{\pi}{2} - $ Latitude

H = Hour of day ($0 \le H \le 2\pi$)

DA_d = Declination angle

For detail derivation of equation 2.4, see reference number [2].

2.5 Atmospheric Transmittance (AT)

Before solar radiation reaches the earth's surface, it is subjected to (i) scattering by air molecules, water vapor and dust, and (ii) absorption by ozone, water molecules and carbondioxide. The effects of scattering and absorption on solar radiation varies with time as atmospheric condition and air mass change. Some attempts to model this phenomenon are quite complex, however, a simpler model developed by Hottel gives a reasonably good estimate. His model takes into account, the zenith angle, the altitude of a standard atmosphere and four climatic conditions.

Table 2.1 Location Parameters [2]

CLIMATE TYPE	R_0	R_1	R_2
Tropical	0.95	0.98	1.02
Mid-Latitude Summer	0.97	0.99	1.02
Subarctic Summer	0.99	0.99	1.01
Mid-Latitude Winter	1.03	1.01	1.00

The four climatic conditions are given in table 2.1 above. Under standard clear sky conditions of 23 kilometers visibility, the atmospheric transmittance (AT) is given by [15] ;

$$AT = B_0 + B_1 \exp(-B_2/LF) \qquad \text{...} 2.5$$

where LF = Location factor

$$B_0 = R_0[0.4237 - 0.00821(6.0 - A)^2]$$

$$B_1 = R_1[0.5055 - 0.00595(6.5 - A)^2]$$

$$B_2 = R_2[0.2711 - 0.01858(2.5 - A)^2]$$

A = Altitude in kilometers (maximum altitude = 2.5 kilometers)
Thus for a given location factor, the atmospheric transmittance under standard clear sky conditions can be determined for any zenith angle and altitude up to 2.5 kilometers.

2.6 Calculating Solar Energy and Power on a Flat Collector

So far presented the variables needed to determine how much power can be delivered at a place on the earth's surface have been presented. In this section, the formulas used in calculating solar energy and solar power available at any location on the earth's surface will be presented.

2.6.1 Solar Power Available at a Location

The following convention will be used for the expressions given below, if a location is in the northern hemisphere, its Latitude (L) is positive, (L > 0)and if the location is in the southern hemisphere, its Latitude (L) is negative,(L < 0). The solar power available at a location is given by [15] ;

In the Northern Hemisphere:

$$P_N = (SC_d).(AT).[Cos(L)Cos(HA)Cos(DA_d) + Sin(L)Sin(DA_d)] \qquad \text{..........……......................} 2.6$$

In the Southern Hemisphere:

$$P_N = (SC_d).(AT).[Cos(L)Cos(HA)Cos(DA_d) - Sin(L)Sin(DA_d)] \qquad \text{.......................................} 2.7$$

2.6.2 Instantaneous Solar Energy Available at a Location

The instantaneous solar energy available at a location can only be computed between sunrise and sunset. As a result it is vital to know the actual sunrise and sunset times at a given location. A variable that can help determine the actual sunrise and sunset times at a location is either the hour angle at sunrise (HA_r) or the hour angle at sunset (HA_s). HA_r and HA_s are easily computed using the equations 2.6 or 2.7 above with the fact that P_N and P_S are both equal to zero either at sunrise or at sunset. Using equation 2.6 above with $P_N = 0$, the hour angle at sunset HA_s is given by [15] ;

$$Ha_s = Cos^{-1}[-\tan(DA_d)\tan(L)] \qquad \text{2.8}$$

The numerical values for both HA_r and HA_s are the same because sunrise and sunset times are symmetric about solar noon. Having computed HA_s, the instantaneous solar energy available at a location is given by [15];

$$H_D = \frac{(SC_d)(AT)}{p}\left[Cos(L)Sin(HA_s)Cos(DA_d) + (HA_s)Sin(L)Sin(DA_d)\right] \qquad \text{2.9}$$

HA_s is expressed in radians.

2.6.3 Average Daily Solar Energy Available at a Location

Since we know that day length is 24 hours, we can use the expression for H_D given in equation 2.9 above to determine the average daily solar energy available at a given location. The average daily solar energy available at a location is given by [2];

$$D_{AV} = I_0\left[Cos(L)Sin(HA_s)Cos(DA_d) + (HA_s)Sin(L)Sin(DA_d)\right] \qquad \text{2.10}$$

where

$$I_0 = \frac{[24.(SC_d).(AT)]}{p}$$

2.7 Solar Energy on a Tilted Plane

In order to increase energy collection especially at higher altitudes, the flat plate collector is tilted south towards the sun if the location is in the northern hemisphere. To understand the conditions under which maximum energy may be obtained from a flat collector, equation 2.6 in section 2.6.1 above will be used. From this equation, observe that, the energy collected will be maximum when HA = 0. This means the energy collected will be maximum at solar noon. Substituting HA = 0 into equation 2.6, the solar power available at solar noon is given by [15];

$$P_N = (SC_d)(AT)[Cos(L)Cos(DA_d) + Sin(L)Sin(DA_d)] \quad \text{......2.11}$$

Using trigonometric identities, equation 2.11 can be simplifies as:

$$P_N = (SC_d)(AT)[Cos(L - DA_d)] \quad \text{......2.12}$$

Equation 2.12 tell us that the power available at solar noon will be maximum when the latitude (L) is equal to the declination angle (DA_d). This observation is contradictory because as discussed in section 2.2.3 above, the declination angle varies between +23.45 and -23.45 degrees throughout the year. However, the latitude has a much wider range, from -90 degrees at the south pole to +90 degrees at the north pole. To make sense of this contradiction, a new variable T, (tilt angle in degrees) can be introduced into equation 2.12 above. The resulting equation is [15] ;

$$P_N = (SC_d)(AT)[Cos(L - T - DA_d)] \quad \text{......2.13}$$

From equation 2.13, it is observed that the best tilt to obtain maximum power at solar noon is given by [15] ;

$$\begin{aligned} \mathrm{L - T = DA_d} &\quad \text{for the northern hemisphere} \\ \mathrm{L + T = DA_d} &\quad \text{for the southern hemisphere} \end{aligned}$$

Also from equation 2.13 above, the following conclusions can be derived;

i. For a flat collector, the maximum power available at solar noon is given by [15] ;

$$P_{NH} = (SC_d)(AT)[Cos(L - DA_d)]$$

where T = 0

ii. For a tilt collector in the northern hemisphere ($\mathrm{L - T = DA_d}$), the maximum available power at solar noon is given by [15] ;

$$P_{NT} = (SC_d)(AT)[Cos(0)]$$

$$\Rightarrow P_{NT} = (SC_d)(AT)$$

Thus the maximum power available at solar noon on a surface with optimum tilt is independent of the location (L) and the day of the year.

So far only the special case when the hour angle (HA) is zero has been discussed. In general, the average daily energy available on a tilt collector is given by [1];

$$P_T = [Cos(L - T)Cos(DA_d)Sin(HA_W) + (HA_W)Sin(L - T)Sin(DA_d)] \quad \text{......2.14}$$

where

$$HA_W = \min\left\{ \begin{matrix} Cos^{-1}[-\tan(DA_d)\tan(L)] \\ Cos^{-1}[-\tan(DA_d)\tan(L - T)] \end{matrix} \right\}$$

and I_0 is given by equation 2.10 above.

So far, it has been observed that tilting a collector increases the amount of energy that can be collected. The question of how large should we tilt the collector and when should this take place are also very important. It suffices to say that the answers to these questions depends on the location. For instance, in the northern hemisphere, the surface angle and position that faces the sun most directly throughout the year is directed due south and tilted from the horizontal at an angle equal to the location's latitude. An area facing south will obtain the optimum amount of direct-beam solar radiation if the tilt angle is equal to the location latitude. To maximize the amount of solar energy received on collectors located in the northern hemisphere during winter the months, it is recommended that the tilt angle should approximately equal the locations latitude plus 15 degrees and for the summer months, the tilt angle should approximately equal the locations latitude minus 15 degrees. For fall and spring months, the tilt angle should equal the location's latitude.

2.8 Adjustment of Clear Sky Model

All the values of solar insolation obtained using the equations presented above are valid only for clear sky conditions. We know that throughout the year not all days will be sunny. The days of a particular month can be classified according to insolation levels using the following criteria below [16];

1. A day will be classified as sunny if its insolation level is greater than or equal to one of the following conditions:
- If the month is approaching summer solstice, use the insolation level on the 10th day of the clear sky model.
- If the month is approaching winter solstice, use the insolation level on the 21st day of the clear sky model.
2. A mostly sunny day will be classified as a day whose insolation level greater than or equal to 90 % of the insolation level used in group 1 and less than group 1 insolation level.
3. A partly sunny day will be classified as a day whose insolation level is greater than or equal to 75 % of the insolation level used in group 1 and less than 90 % of the insolation level in group 1.
4. A partly cloudy day will be classified as a day whose insolation level is greater than or equal to 60 % of the insolation level used in group 1 and less than 75 % of the insolation level in group 1.
5. A cloudy day will be classified as a day whose insolation level is greater than or equal to 45 % of the insolation level used in group 1 and less than 60 % of the insolation level in group 1.
6. An overcast day will be classified as a day whose insolation level is greater than or equal to 30 % of the insolation level used in group 1 and less than 45 % of the insolation level in group 1.
7. A rain or snow day will be classified as a day whose insolation level is less than 30 % of the insolation level used in group 1.

After the range of insolation levels for classifying the days have been determined, the following generic rules with actual measured insolation are used to adjust the clear sky model. The generic rules are [16] ;

1. For a month approaching summer solstice (excluding June), multiply the insolation level of the clear sky model for the first 21 days by the mid-range multiplier according to day type. The remaining days of the month should be multiplied by the lower-range multiplier. For June, multiply all the days by the mid-range multipliers.
2. For a month approaching winter solstice (excluding December), multiply the insolation level of the clear sky model for the last 21 days by the mid-range multiplier according to day type. The remaining days of the month should be multiplied by the lower-range multiplier. For December, multiply all the days by the mid-range multipliers.The mid and lower range multipliers are given in table 2.2 below.

Table 2.2 Multipliers for Adjusting Clear Sky Model-Generated Estimates [15]

Day Type	**Mid-Range Multiplier**	**Lower-Range Multiplier**
Sunny	110.0 %	100 %
Mostly Sunny	95.0 %	90 %
Partly Sunny	82.5 %	75 %
Partly Cloudy	67.5 %	60 %
Cloudy	52.5 %	45 %
Overcast	37.5 %	30 %
Rain/Snow	20.0 %	20 %