Appendix D: Dynamic Program for Two Struts – Asymmetric Bar

Mathematica Code

Input and initial guesses for dynamic moment (gmd), shear force (gqd) and axial force (gpd):

```
(*Forced small vibrations about buckled equilibrium; (2) clamped struts supporting an asymmetric rigid
body; y1=x1e,y2=y1e,y3=theta1_e, y4=m1e, y5=x1d, y6=y1d, y7=theta1_d, y8=m1d,
y9=p1d,y10=q1d,y11=x2e,y12=y2e,y13=theta2_e,y14=m2e, y15=x2d, y16=y2d, y17=theta2_d, y18=m2d,
y19=p2d, y20=q2d;  given po, uo1, uo2, frequencies, damping coefficients c1 and c2, b1 and b2 where
b1>b2 and b1+b2=length of rigid body, the height of the rigid body varies from 2h-a to 2h+a where h is
half the average height of the body, and moments m1 and m2 , q1, q2, p1 and p2 are from
ClampedStrutEquil4PI^2asymmetric, this program will run a do loop so that several values of ω can be
evaluated*)

Clear[pi,uo1,po1,m1,c1,ω1,q1,gp1d,gq1d,gm1d,uo2,po2,m2,c2,ω2,q2,gp2d,gq2d,gm2d]
pi = N[π];
i=(-1)^0.5;
uo1=0.01;
uo2=0.01;
po=40;
p1=38.581;
p2=45.419;
m1=1.83269;
m2=-3.75116;
q1=-0.065956;
q2=0.065956;
(* use these values for the symmetric case, and a=0 and io = 6.9333333333 also
     p1=40;
  p2=40;
  m1=2.03772;
  m2=-2.03772;
  q1=-0.00000060591;
  q2=0.00000060591;*)
c1=1;
c2=1;
r=1;
 g=0;
b=0.5;
a=0.02;
h=.1;
b1=b*(1+.5*a/h);
b2=b*(1-.5*a/h);
(*io=(2*r*po/(3*h^2))*((h^4)+((h^2)*((b1+b2)/2)^2)+(.5*(h^2)*a^2)-((a^4)/48)-
((a^2)/12*((b1+b2)/2)^2));*)
io=6.916435556;
gp1d=0.360000+3.220000i;
gp2d=0.1490003+2.16820*i;
gq1d=0.818000+2.010058*i;
gq2d=0.67300-1.016575*i;
gm1d=0.2441000-3.031971*i;
gm2d=0.627420+1.010529482*i;
```

Equilibrium equations, boundary conditions and solution (solving for transmissibility) for a range of ω in "do" loop:

```
0>>caseoutdamp.txt
Do[{{ω1=m;
      endpt[p1d_?NumberQ,q1d_?NumberQ,m1d_?NumberQ,
p2d_?NumberQ,q2d_?NumberQ,m2d_?NumberQ]:=First[NDSolve[{y1'[t]==Cos[y3[t]],y2'[t]==Sin[y3[t]],y
3'[t]==y4[t],y4'[t]==-p1*Sin[y3[t]]+q1*Cos[y3[t]],y5'[t]==-
y7[t]*Sin[y3[t]],y6'[t]==y7[t]*Cos[y3[t]],y7'[t]==y8[t],y8'[t]==(y10[t]-p1*y7[t])*Cos[y3[t]]-
(y9[t]+q1*y7[t])*Sin[y3[t]],y9'[t]==(ω1^2-i*ω1*c1)*y5[t],y10'[t]==(ω1^2-i*ω1*c1)*y6[t],
            y11'[t]==Cos[y13[t]],y12'[t]==Sin[y13[t]],y13'[t]==(y14[t]),y14'[t]==-
p2*Sin[y13[t]]+q2*Cos[y13[t]],y15'[t]==-
y17[t]*Sin[y13[t]],y16'[t]==y17[t]*Cos[y13[t]],y17'[t]==(y18[t]),y18'[t]==(y20[t]-p2*y17[t])*Cos[y13[t]]-
(y19[t]+q2*y17[t])*Sin[y13[t]],y19'[t]==(ω1^2-i*ω1*c2)*y15[t],y20'[t]==(ω1^2-
i*ω1*c2)*y16[t],y1[0]==0,y2[0]==0,y3[0]==0,y4[0]==m1,y5[0]==uo1,y6[0]==0,y7[0]==0,y8[0]==m1d,y9[0]==p
1d,y10[0]==q1d,y11[0]==0,y12[0]==0,y13[0]==0,y14[0]==m2,y15[0]==uo2,y16[0]==0,y17[0]==0,y18[0]==m2d
,y19[0]==p2d,y20[0]==q2d},{y1,y2,y3,y4,y5,y6,y7,y8,y9,y10,y11,y12,y13,y14,y15,y16,y17,y18,y19,y20},{
t,0,1},MaxSteps→1000]];

      endpt[gp1d,gq1d,gm1d, gp2d, gq2d, gm2d];

f1[p1d_?NumberQ,q1d_?NumberQ,m1d_?NumberQ,p2d_?NumberQ,q2d_?NumberQ,m2d_?NumberQ]:=
y1[1]/.endpt[p1d,q1d,m1d,p2d,q2d,m2d];

f2[p1d_?NumberQ,q1d_?NumberQ,m1d_?NumberQ,p2d_?NumberQ,q2d_?NumberQ,m2d_?NumberQ]:=
y2[1]/.endpt[p1d,q1d,m1d,p2d,q2d,m2d];

f5[p1d_?NumberQ,q1d_?NumberQ,m1d_?NumberQ,p2d_?NumberQ,q2d_?NumberQ,m2d_?NumberQ]:=
y5[1]/.endpt[p1d,q1d,m1d,p2d,q2d,m2d];

f6[p1d_?NumberQ,q1d_?NumberQ,m1d_?NumberQ,p2d_?NumberQ,q2d_?NumberQ,m2d_?NumberQ]:=
y6[1]/.endpt[p1d,q1d,m1d,p2d,q2d,m2d];

f7[p1d_?NumberQ,q1d_?NumberQ,m1d_?NumberQ,p2d_?NumberQ,q2d_?NumberQ,m2d_?NumberQ]:=
y7[1]/.endpt[p1d,q1d,m1d,p2d,q2d,m2d];

f8[p1d_?NumberQ,q1d_?NumberQ,m1d_?NumberQ,p2d_?NumberQ,q2d_?NumberQ,m2d_?NumberQ]:=
y8[1]/.endpt[p1d,q1d,m1d,p2d,q2d,m2d];

f9[p1d_?NumberQ,q1d_?NumberQ,m1d_?NumberQ,p2d_?NumberQ,q2d_?NumberQ,m2d_?NumberQ]:=
y9[1]/.endpt[p1d,q1d,m1d,p2d,q2d,m2d];

f10[p1d_?NumberQ,q1d_?NumberQ,m1d_?NumberQ,p2d_?NumberQ,q2d_?NumberQ,m2d_?NumberQ]:
=y10[1]/.endpt[p1d,q1d,m1d,p2d,q2d,m2d];

f11[p1d_?NumberQ,q1d_?NumberQ,m1d_?NumberQ,p2d_?NumberQ,q2d_?NumberQ,m2d_?NumberQ]:
=y11[1]/.endpt[p1d,q1d,m1d,p2d,q2d,m2d];

f12[p1d_?NumberQ,q1d_?NumberQ,m1d_?NumberQ,p2d_?NumberQ,q2d_?NumberQ,m2d_?NumberQ]:
=y12[1]/.endpt[p1d,q1d,m1d,p2d,q2d,m2d];
```

```
f15[p1d_?NumberQ,q1d_?NumberQ,m1d_?NumberQ,p2d_?NumberQ,q2d_?NumberQ,m2d_?NumberQ]:
=y15[1]/.endpt[p1d,q1d,m1d,p2d,q2d,m2d];

f16[p1d_?NumberQ,q1d_?NumberQ,m1d_?NumberQ,p2d_?NumberQ,q2d_?NumberQ,m2d_?NumberQ]:
=y16[1]/.endpt[p1d,q1d,m1d,p2d,q2d,m2d];

f17[p1d_?NumberQ,q1d_?NumberQ,m1d_?NumberQ,p2d_?NumberQ,q2d_?NumberQ,m2d_?NumberQ]:
=y17[1]/.endpt[p1d,q1d,m1d,p2d,q2d,m2d];

f18[p1d_?NumberQ,q1d_?NumberQ,m1d_?NumberQ,p2d_?NumberQ,q2d_?NumberQ,m2d_?NumberQ]:
=y18[1]/.endpt[p1d,q1d,m1d,p2d,q2d,m2d];

f19[p1d_?NumberQ,q1d_?NumberQ,m1d_?NumberQ,p2d_?NumberQ,q2d_?NumberQ,m2d_?NumberQ]:
=y19[1]/.endpt[p1d,q1d,m1d,p2d,q2d,m2d];

f20[p1d_?NumberQ,q1d_?NumberQ,m1d_?NumberQ,p2d_?NumberQ,q2d_?NumberQ,m2d_?NumberQ]:
=y20[1]/.endpt[p1d,q1d,m1d,p2d,q2d,m2d];

        Clear[p1d,q1d,m1d,p2d,q2d,m2d];
        rts=FindRoot[{(f15[p1d,q1d,m1d,p2d,q2d,m2d]-
f5[p1d,q1d,m1d,p2d,q2d,m2d])*(f11[p1d,q1d,m1d,p2d,q2d,m2d]-
f1[p1d,q1d,m1d,p2d,q2d,m2d])==(f16[p1d,q1d,m1d,p2d,q2d,m2d]-
f6[p1d,q1d,m1d,p2d,q2d,m2d])*(b1+b2+f2[p1d,q1d,m1d,p2d,q2d,m2d]-
f12[p1d,q1d,m1d,p2d,q2d,m2d]),f7[p1d,q1d,m1d,p2d,q2d,m2d]==(f15[p1d,q1d,m1d,p2d,q2d,m2d]-
f5[p1d,q1d,m1d,p2d,q2d,m2d])/(b1+b2),f17[p1d,q1d,m1d,p2d,q2d,m2d]==f7[p1d,q1d,m1d,p2d,q2d,m2d],
            -
ω1^2*2*r*po*(b2*f5[p1d,q1d,m1d,p2d,q2d,m2d]+b1*f15[p1d,q1d,m1d,p2d,q2d,m2d]+h*f6[p1d,q1d,m1d,
p2d,q2d,m2d]-
h*f16[p1d,q1d,m1d,p2d,q2d,m2d])==(b1+b2)*(f9[p1d,q1d,m1d,p2d,q2d,m2d]+f19[p1d,q1d,m1d,p2d,q2d,
m2d]),-
ω1^2*2*r*po*(b2*f6[p1d,q1d,m1d,p2d,q2d,m2d]+b1*f16[p1d,q1d,m1d,p2d,q2d,m2d]+h*f15[p1d,q1d,m1
d,p2d,q2d,m2d]-
h*f5[p1d,q1d,m1d,p2d,q2d,m2d])==(b1+b2)*(f10[p1d,q1d,m1d,p2d,q2d,m2d]+f20[p1d,q1d,m1d,p2d,q2d,
m2d]),io*ω1^2(f5[p1d,q1d,m1d,p2d,q2d,m2d]-
f15[p1d,q1d,m1d,p2d,q2d,m2d])/(b1+b2+f2[p1d,q1d,m1d,p2d,q2d,m2d]-
f12[p1d,q1d,m1d,p2d,q2d,m2d])==f19[p1d,q1d,m1d,p2d,q2d,m2d]*b2-f9[p1d,q1d,m1d,p2d,q2d,m2d]*b1-
f20[p1d,q1d,m1d,p2d,q2d,m2d]*h-f10[p1d,q1d,m1d,p2d,q2d,m2d]*h+(f2[p1d,q1d,m1d,p2d,q2d,m2d]-
f12[p1d,q1d,m1d,p2d,q2d,m2d])*(f19[p1d,q1d,m1d,p2d,q2d,m2d]*b2-f9[p1d,q1d,m1d,p2d,q2d,m2d]*b1-
f20[p1d,q1d,m1d,p2d,q2d,m2d]*h-
f10[p1d,q1d,m1d,p2d,q2d,m2d]*h)/(b1+b2)+(f6[p1d,q1d,m1d,p2d,q2d,m2d]-
f16[p1d,q1d,m1d,p2d,q2d,m2d])*(p2*b2-p1*b1-q2*h-
q1*h)/(b1+b2)+(f9[p1d,q1d,m1d,p2d,q2d,m2d]*h+f19[p1d,q1d,m1d,p2d,q2d,m2d]*h+f20[p1d,q1d,m1d,p2
d,q2d,m2d]*b2-f10[p1d,q1d,m1d,p2d,q2d,m2d]*b1)*(f11[p1d,q1d,m1d,p2d,q2d,m2d]-
f1[p1d,q1d,m1d,p2d,q2d,m2d])/(b1+b2)+(2*po*h+q2*b2-q1*b1)*(f15[p1d,q1d,m1d,p2d,q2d,m2d]-
f5[p1d,q1d,m1d,p2d,q2d,m2d])/(b1+b2)-f8[p1d,q1d,m1d,p2d,q2d,m2d]-
f18[p1d,q1d,m1d,p2d,q2d,m2d]},{p1d,gp1d},{q1d,gq1d},{m1d,gm1d},{p2d,gp2d},{q2d,gq2d},{m2d,gm2
d},AccuracyGoal→3,MaxIterations→300000];

        p1d=p1d/.rts;
        q1d=q1d/.rts;
        m1d=m1d/.rts;
        p2d=p2d/.rts;
        q2d=q2d/.rts;
        m2d=m2d/.rts;
        ω1,
```

TR1=((((Re[f5[p1d,q1d,m1d,p2d,q2d,m2d]])^2+(Im[f5[p1d,q1d,m1d,p2d,q2d,m2d]])^2))^0.5)/uo1;

TR2=((((Re[f15[p1d,q1d,m1d,p2d,q2d,m2d]])^2+(Im[f15[p1d,q1d,m1d,p2d,q2d,m2d]])^2))^0.5)/uo2;
TR3=((uo1*TR1)+(uo2*TR2))/(uo1+uo2),
ϕ=Abs[(Re[f15[p1d,q1d,m1d,p2d,q2d,m2d]]-Re[f5[p1d,q1d,m1d,p2d,q2d,m2d]])]/(2*b),
gp1d=p1d*.98;
gp2d=p2d*.98;
gq1d=q1d*.98;
gq2d=q2d*.98;
gm1d=m1d*.98;

gm2d=m2d*.98;}>>>caseoutdamp.txt}Print[PaddedForm[TableForm[{{ω1,TR3,p1d,p2d,q1d,q2d,m1d,m2d}}],{5,6}]],{m,.1,.4,.1}]

ω	TR3	p1d	p2d	q1d
0.100000	1.152300	0.004622 + 0.001227 i	-0.016423 + 0.001134 i	0.001677 - 0.000066 i
0.200000	0.718740	-0.053765 - 0.004067 i	0.182660 + 0.014285 i	-0.019508 - 0.000217 i
0.300000	0.477220	-0.017842 + 0.002345 i	0.055908 + 0.002330 i	-0.006329 + 0.000557 i
0.400000	0.580210	-0.015749 + 0.003618 i	0.044576 + 0.002572 i	-0.005448 + 0.000665 i