Chapter 1: Research Objectives and Literature Review

1.1 Introduction

Since humans have developed machinery such as compressors, engines, and presses, which produce unwanted vibrations, they have been trying to control these vibrations. Approximately 80 years ago the methods used for vibration control were similar to the following:

"*The support consisted of layers of the following materials: 1/8 in. sheet lead, 4 in. pitch pine, 2 in. nonpareil cork, a No. 24 galvanised iron sheet, 2 in. nonpareil cork, 7/8 in. soft pine board, 4 in. soft pine beams, 1 in. piano-felt pads resting on concrete...*"
-Alec B. Eason (Eason, 1923)

This quotation is from the description of a vibration mounting developed for an air-circulating fan. It is apparent that a great deal of trial and error was involved in the development of this vibration isolator. Today vibration isolators can be procured "off the shelf", come in a variety of more "sophisticated" materials, and a lot of the guesswork has been eliminated from their design. A typical vibration isolator is made of a helical spring, rubber, or air springs (Macinante, 1984).

This research studies a novel type of vibration isolator utilizing the post-buckled stiffness of elastic struts. The advantage of using this post-buckled state is that ideally it can support more static load with a relatively small static deflection as compared to the typical isolators, but also exhibits a low axial stiffness when dynamic excitation is introduced.

Three models consisting of elastic, post-buckled or pre-bent struts supporting a mass will be studied. All analysis will be performed using the software *Mathematica* (Mathematica, 2003). The first model is the most basic, consisting of two buckled struts supporting a symmetric rigid bar that loads the struts just above their Euler buckling load. The boundary conditions of each strut are considered fixed at each end, and the bar is assumed to be rigid. The base of each strut is then subjected to a forced, vertical, harmonic displacement excitation. The second model is similar to the first except that the bar is asymmetric in shape; hence one strut is supporting more load than the other. The third model studied is different in that the supports are actually pairs of struts which are pre-bent rather than buckled, with a bonded filler located between the struts. See Figures 1.1, 1.2, and 1.3 for a physical description of each model.

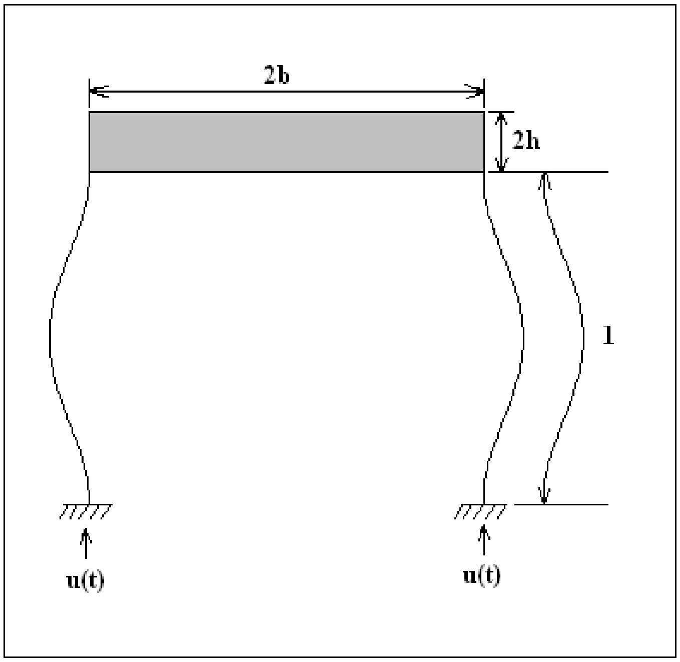

Figure 1.1 Two Struts Supporting Rigid Bar

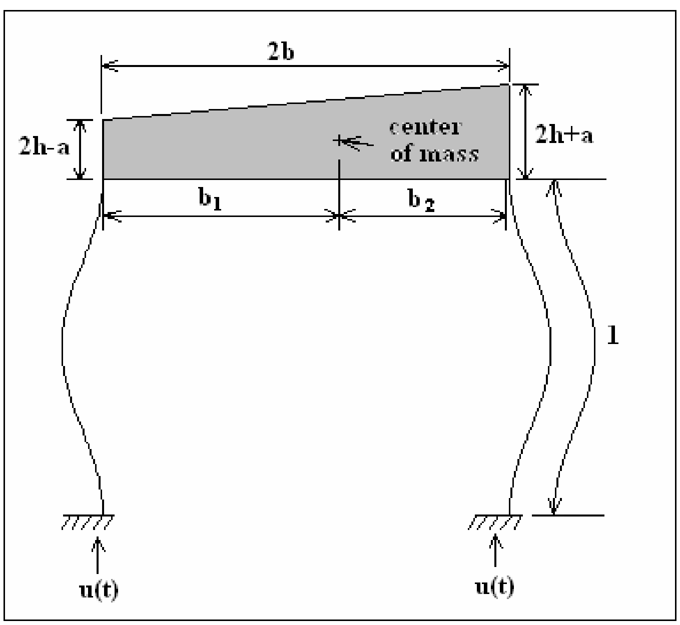

Figure 1.2 Two Struts Supporting Asymmetric Rigid Bar

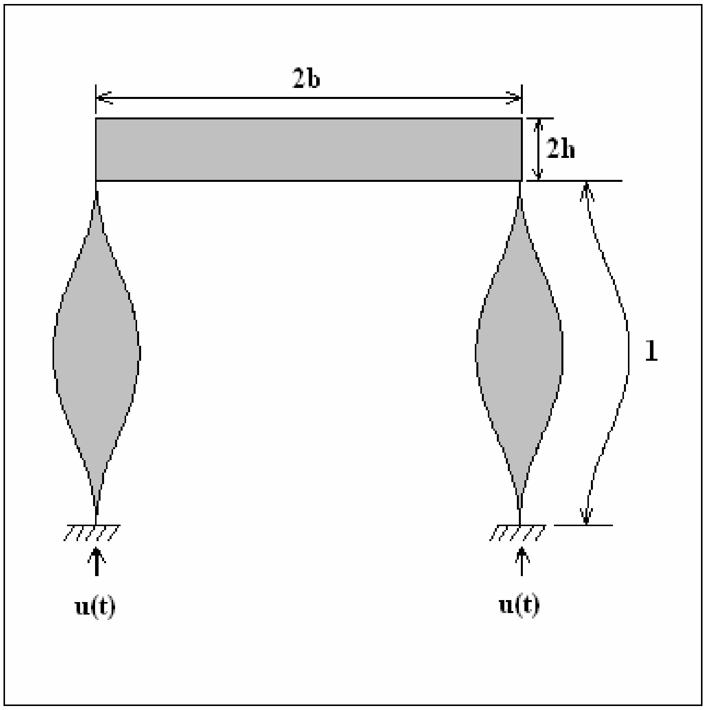

Figure 1.3 Two Pairs of Struts with Intermediate Bonded Filler, Supporting Rigid Bar

To determine the effectiveness of these models, the displacement transmissibility will be determined for a wide range of frequencies at which the model is being forced. Transmissibility is defined here as the ratio of the amplitude of the vertical displacement of the center of mass of the rigid bar to the amplitude of the harmonically forced base. For an isolator to be effective, the transmissibility ratio must be well below unity.

1.2 Literature Review

1.2.1 Describing Vibration

Vibration can be defined as "the cyclical change in the position of an object as it moves alternately to one side and the other of some reference or datum position" (Macinante, 1984). Vibration of rigid bodies can be rectilinear (or translational), rotational, or a combination of the two. Rectilinear vibration refers to a point whose path of vibration is a straight line, and rotational vibration refers to a rigid body whose vibration is angular about some reference line. Additionally, vibration of flexible bodies can be described by flexural or other elastic vibrations such as longitudinal, tension and compression, and torsional or twisting.

When describing vibration, a waveform diagram is typically used. A waveform is a diagram or mathematical function that shows how the position of the vibrating point is changing with time. The most studied form of vibration is simple harmonic motion (sinusoidal or harmonic), in which the waveform is a sine or cosine curve. Mathematically, harmonic vibration of a single-degree-of-freedom system can be given by the expression

$$u(t) = u_0 \sin \omega t \tag{1.1}$$

where

$u(t)$ = the position of the point with respect to time t

u_o = the peak or maximum displacement (i.e. the amplitude) of the point from a datum line

ω = the circular frequency

t = time

Graphically, this waveform is represented in Figure 1.4. T is generally the notation given for the period. The period is defined as the length of time from a point on the waveform to the next point where the wave repeats itself, and the units are time (typically seconds). Its reciprocal is known as the cyclic frequency, f, and is measured in cycles per second, or hertz. The relations between ω, f, and T are given below:

$$T = \frac{2\pi}{\omega} \text{sec} \qquad f = \frac{\omega}{2\pi} \text{ cycles per second} \qquad (1.2, 1.3)$$

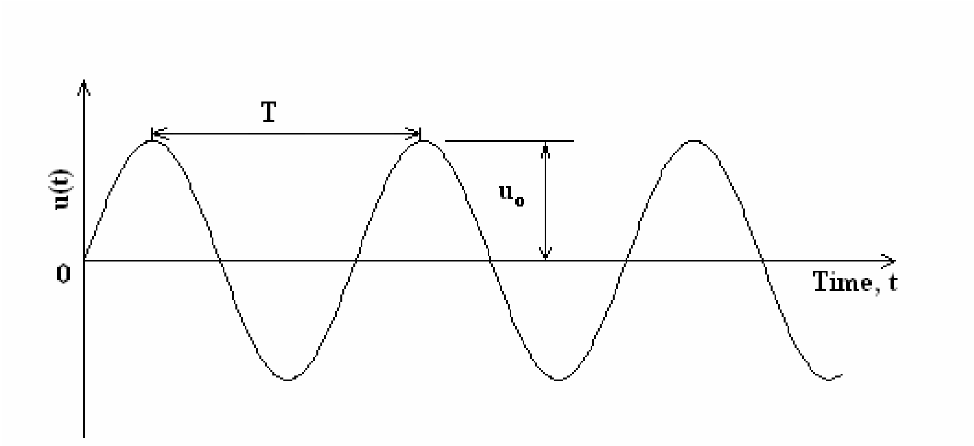

Figure 1.4 Waveform of Simple Harmonic Motion

Expressions describing the velocity and acceleration of the vibrating point can be determined by taking the first and second derivatives of Equation 1.1 with respect to time. Therefore

$$\frac{du}{dt} = u_0 \omega \cos \omega t \quad \text{(velocity)} \tag{1.4}$$

$$\frac{d^2u}{dt^2} = -u_0 \omega^2 \sin \omega t \quad \text{(acceleration)} \tag{1.5}$$

From studying these equations, one can see that the displacement value u(t) and the acceleration value are at a maximum when the velocity is equal to zero. This means the vibrating point momentarily comes to a stop at its maximum position from the reference datum, and then begins accelerating at its maximum rate.

Harmonic motion is a form of periodic motion. Periodic motion is defined as a vibration whose waveform is repetitive. Many types of engines, compressors, pumps, and other machinery that run continuously generate a form of periodic vibration. If a motion is periodic, its velocity and acceleration are also periodic. Vibration can also be random, but is much more difficult to analyze mathematically, which is why periodic (harmonic in particular) motion is used in this research for the purposes of studying vibration and its effects. Because many types of machinery produce periodic vibrations, studying harmonic motions can be very useful.

Another important parameter to discuss when describing vibration is damping. If a system is initially displaced a certain distance and then released, such as a pendulum, it will vibrate about a certain datum line for a finite amount of time before coming to rest. The amplitude of the motion decays, and the cause of this decay in motion, or dissipation of energy, is referred to as damping. It is present naturally, and if a system is not being forced to vibrate by an external source, its motion will eventually decay because of the intrinsic damping that is present. Damping can also be introduced into a system as a means of controlling the vibrations.

1.2.2 Vibration Isolation

1.2.2.1 Overview

Vibration isolation typically involves a system that includes a source of the vibration, a path along which the vibration is transmitted, and the receiving end of the vibration. For example, a large air-circulation fan mounted to the floor of a factory might be causing a disturbance to employees whose offices are located on the same floor. The fan would be the source, the floor would be the transmission path, and the employee would be the receiver.

The term used when we speak of treating or correcting this condition of unwanted vibrations is *vibration isolation.* Vibration isolation can be employed at any of the three parts of the system previously described. Referring to this simple example, vibration isolation may be performed at the source by trying to balance the fan to reduce the vibrations or by mounting the fan on a seismic mounting or vibration isolator. Or, it may be preferred to treat the path of the vibration, which is the floor of the factory in our example. Because vibration becomes a problem when transmitted to a floor structure when the natural frequency of vibration of the floor structure is similar to the vibrating frequency of the source (resonance), designing the floor structure to avoid this is the best way to treat the problem. Another option is to install a dynamic absorber, which is designed to absorb the energy from the vibrating structure (see Den Hartog, 1956, Inman, 2001, and Macinante, 1984, for more discussion on absorbers). Finally, treatment at the receiver can also be considered. The desks, chairs, and computer equipment could possibly be mounted on isolators; however it may be simpler to move the offices to another floor. In this example, treatment of the receiver is probably the most unlikely candidate for an economically feasible option. But, in other cases, such as a piece of sensitive equipment on the receiving end, the most viable option may be to isolate the receiver.

1.2.2.2 Types of Isolators

The focus of this research is on vibration isolators located at the source or the receiver rather than absorbers. In general, isolators can be categorized into area isolators or unit isolators. Area isolators are typically comprised of similar materials as unit isolators, but they are in the form of a mat or pad on which the vibrating source is supported (or on which the receiving, sensitive equipment rests). Unit isolators are smaller, individually separate pieces that support and isolate the equipment. Many types of materials have been used as area or unit isolators, as suggested by the quotation at the beginning of this chapter, including "metal springs, rubber, cork, felt, airbags, squash balls, tennis balls, basketballs, glass fiber, knitted wire mesh, lead, asbestos, soil, sand, gravel, and even a pile of newspapers" (Macinante, 1984). Typical unit isolators used today include rubber isolators, air springs, helical springs, and a unit of some combination of metal spring, rubber, and/or pneumatic elements.

1.2.2.3 Single-Degree-of-Freedom System

When choosing an isolator, it is necessary to determine the static load carrying capacity of the isolator(s), the amount of static deflection to expect, the axial stiffness k of the isolator, the damping c present in the isolator, the mass m of the system being supported, the frequency ω of the vibration created by the source, and the natural frequencies ω_n of the isolators. Figure 1.5 shows these parameters in a simple one-degree-of-freedom system. One, or single, degree of freedom simply means that the motion is restricted to one direction only, or in other words, the displacement of the system can be described with one independent number. Actual vibration problems are usually much more complicated than this, involving several degrees of freedom needed to describe the displacement, but the single-degree-of-freedom system is typically used to illustrate the fundamentals.

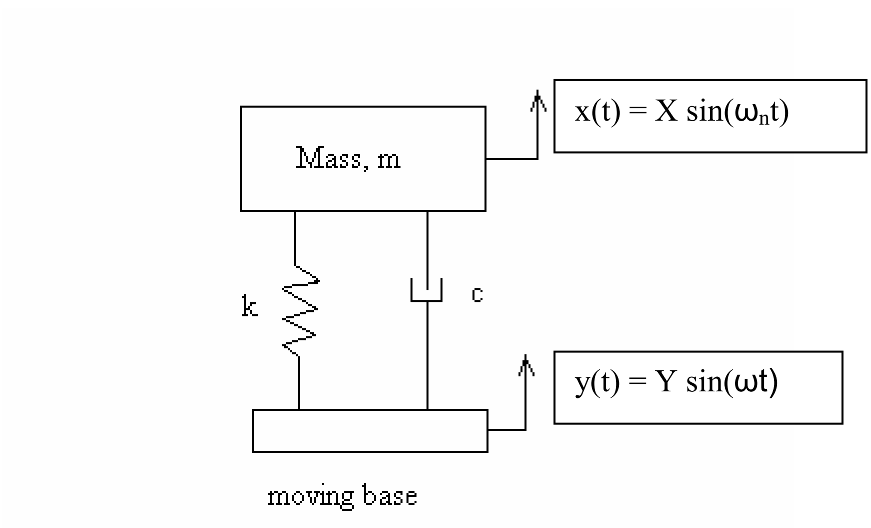

Figure 1.5 Single-Degree-of-Freedom System.

The measurement typically used in determining whether an isolator is effective or not is called transmissibility. As defined previously, it is the ratio of the amplitude of the vertical displacement of the supported system to the amplitude of the base. Transmissibility can also be in the form of a force ratio, but reducing the displacement is the goal of this research, so when transmissibility is discussed, it is actually displacement transmissibility. In Figure 1.5, the transmissibility of the system at a certain frequency could be determined by measuring the displacement amplitude of the supported mass, X, and dividing it by the displacement amplitude of the base, Y. If this number is equal to or greater than one, the isolator is ineffective, and actually making the situation worse.

1.2.2.4 Stiffness in Vertical Vibration Isolators

The system shown in Figure 1.5 gives an example of vertical vibration. Vertical vibration has the added component of the gravitational force that must be overcome. Effective isolators have a relatively low natural frequency compared to the forcing frequency of the source they are isolating. Low natural frequencies are desirable because they ensure a low transmissibility. To obtain this low natural frequency, the stiffness of the isolator must be low as well. Typically, the stiffness of vibration isolators such as springs is linear; i.e., the static displacement of the mass is directly proportional to the force or load on the isolator. If the stiffness is low, the static displacement can be quite large relative to the dynamic displacement. To avoid the large static displacement, the stiffness of the spring can be increased, hence increasing the natural frequency. Therefore, an isolator with a nonlinear stiffness might be helpful in avoiding this problem. The elastically buckled strut, as discussed next, is proposed as a solution to this problem.

1.2.3 Elastically Buckled Struts as Isolators

Slender columns, or struts, made of an elastic material will support a static load up to a critical load, at which point the strut will buckle after the load exceeds this critical load. This critical load is known as the Euler buckling load and its value depends upon the support conditions at each end of the strut. For a strut that is fixed at both ends (not free to rotate or to deflect transversely), the critical load is $P_{cr} = 4\pi^2EI/L^2$, and for a strut that is pinned at both ends (free to rotate) the critical load is $P_{cr} = \pi^2EI/L^2$. In each of these equations, E is the modulus of elasticity of the elastic material of the strut, I is the moment of inertia of the cross section of the strut about the axis of bending, and L is the length of the strut.

The static energy required to support the load up to the critical buckling load of the strut is zero. After the strut has buckled, its force versus displacement ratio is non-linear, i.e., the displacement is not proportional to the load. In this post-buckled state, the strut can still support a significant amount of load, but the lateral stiffness has been greatly reduced. In many engineering applications this is to be avoided and is considered a failure of the strut or column. But, in the case of a vibration isolator, this low stiffness is desirable because of its low natural frequency and hence low transmissibility. This is the ideal condition for an efficient isolator, and has been proposed by Winterflood, et al. (2002 a,b,c) as a new type of vertical vibration isolator.

1.3 Research Scope

1.3.1 Overview

The purpose of this research is to study a new type of vibration isolator, utilizing the post-buckled stiffness of elastic struts (columns). The advantage of the post-buckled state, as described previously, is that ideally it can support more static load with a relatively small static deflection than traditional vibration isolators such as springs or rubber mounts, but also exhibit a low axial stiffness when dynamic excitation is introduced. Previous work on this subject has been performed by Professor R. H. Plaut with graduate students Laurie Alloway and Jenny Sidbury, and by Professor L. N. Virgin and his students at Duke University, as can be seen in the following references: Alloway, 2003, Plaut et al. 2003, Plaut et al. 2004, Sidbury, 2003, and Virgin and Davis, 2003. Much of this work included study of a single strut in static equilibrium, buckled and initially curved struts under forced harmonic axial excitation with pinned/pinned end conditions, clamped/clamped end conditions, and single-frequency or two-frequency axial excitation. As a continuation of this research, the proposed work is outlined below.

1.3.2 Research Scope

Three new cases of buckled struts serving as vibration isolators will be studied as part of this research. The essential difference from the previous work will be that the system will be comprised of two buckled struts or two pairs of pre-bent struts supporting a horizontal rigid mass. The system will be subjected to an axial harmonic displacement excitation at the base as before, and external damping will be included. The three new cases to be studied are:

1. Two buckled struts supporting a symmetric rigid bar (Figure 1.1).
2. Two buckled struts supporting an asymmetric rigid bar (Figure 1.2).
3. Two pairs of pre-bent struts with an intermediate bonded filler, supporting a symmetric rigid bar (Figure 1.3).

The strut is modeled as an elastica, and the boundary conditions are clamped/clamped for all cases. First the equilibrium geometry is determined by solving the governing differential equations using a shooting method. The equations of motion are then written about this equilibrium state, and these linearized differential equations for small motions are also solved by a shooting method. Because the purpose of the struts is to reduce unwanted vibrations, determining the transmissibility of the system is the main goal of this research. Transmissibility versus frequency plots will be generated for all cases, and parameters will be changed to determine how each parameter affects this relationship. Vibration shapes will also be developed for certain frequencies so that the physical behavior of the system can be studied.

1.3.2.1 Two Buckled Struts Supporting a Rigid Bar

Figure 1.1 shows the configuration of the system in nondimensional terms. The length of each strut is equal to 1, the width of the rigid bar is equal to 2b, and the height of the rigid bar is equal to 2h. The bases of the struts are subjected to vertical displacement u(t), which is a simple harmonic with frequency ω. Small steady-state motions of the system about its equilibrium configuration are considered. A transmissibility versus frequency plot is generated for a range of frequencies from ω=0.1 to 200, and vibration shapes at certain frequencies are generated and studied as well. Chapter 2 will discuss the model, nondimensional quantities, and the equilibrium and dynamic analysis in more detail.

1.3.2.2 Two Buckled Struts Supporting an Asymmetric Rigid Bar

Because the case in Figure 1.1 represents a situation that may not easily be achieved in practice, it was decided to study the effects of an asymmetrical rigid bar. By varying the value of “a” (see Figure 1.2), the center of gravity of the rigid bar shifts (changing the value of b_1) from the geometric center of the two struts. This also induces a rotation of the rigid bar, which will be plotted against the same range of forcing frequencies (ω=0.1 to 200) along with the transmissibility plots. Several values of “a” will be studied to determine how this shift in center of gravity affects the system’s transmissibility and rotation of the rigid bar. Vibration shapes will also be generated for certain values of ω.

The shift in the center of gravity away from the center of the bar creates a tilt or static rotation of the bar if the stiffness parameter of each strut is equal. Realizing that this tilt would be undesirable when supporting a piece of vibrating machinery, the stiffness of each strut is adjusted so that the bar is horizontal in the equilibrium state. This creates a new set of equilibrium values that are then applied to the dynamic program, and transmissibility plots are again generated and compared to the case of the symmetric bar. A detailed discussion of the model and analytical procedures is found in Chapter 2.

1.3.2.3 Two Pairs of Pre-bent Struts with an Intermediate Bonded Filler, Supporting a Rigid Bar

Two pairs of pre-bent struts with an intermediate bonded filler will also be studied (see Figure 1.3*)*. The concept of using pairs of buckled struts for vibration isolation was proposed by Winterflood et al. (2002 a,b,c). A similar device using initially-bent steel members with a filler has been considered for application with regard to seismic excitation of frames by Charney and Ibrahim (2004). The bonded filler material will have a stiffness parameter, k, assumed to act in the horizontal direction, and a damping parameter, c, but will be assumed to provide no axial resistance to the system. The rigid bar will be symmetric, as in the first case. Transmissibility versus frequency plots will be generated as in the first two cases. Because the struts are pre-bent rather than buckled, an initial shape must be given to the struts. The amplitude of the initial shape will be varied to see how it affects the transmissibility. Other parameters will also be varied, such as the initial bar weight, p_o , the filler stiffness, k, and the filler damping, c. Finally, a system will be analyzed using values of an actual elastic material and section properties that may be used in a physical model of an isolator. A nondimensional value of c will be varied to determine transmissibility plots similar to that of a typical single-degree-of-freedom system with damping corresponding to a damping ratio commonly found in physical structures. Details on the model and results can be found in Chapter 3.

1.3.3 Objectives

The objective of this research is to determine if the buckled and pre-bent pairs of struts provide an effective vibration isolator, and to compare the results among the three cases studied. All variables used in the analysis are nondimensionalized so that the results are not dependent on a certain value for the modulus of elasticity of the elastic material, or the section property of moment of inertia for a certain cross section, or the length of the strut. In this way, results are valid for any elastic material, regardless of the type of elastic material or section properties.