CHAPTER 4 NUMERICAL MODELING

4.1 Overview

To study the seismic response of the aggregate pier foundation system, a robust computer code is to be used. The computer code has to have the ability to incorporate key factors and phenomena that influence the behavior of unimproved and improved ground. The flexibility of the computer code for alternative soil models has to be taken into account. Finally, the reported success of the use of the code and acceptance by geotechnical engineering profession also become a factor.

The computer code FLAC – Fast Lagrangian Analysis of Continua (FLAC, 2000) was selected because it meets the criteria mentioned above. It was first developed by Peter Cundall in 1986 (FLAC, 2000). The code is an explicit two-dimensional finite difference program that performs a Lagrangian analysis. Explicit means it uses a time stepping procedure to solve the problem without forming the stiffness matrix. The Lagrangian formulation enables the grid to move and deform with the material it represents since the incremental displacements are added to the coordinates.

The version that was used for this research is version 4.0 developed by Itasca Consulting Group, Inc. This version has new enhancements with particularly in graphics interface facilities. Graphical tools were developed in the GIIC (Graphical Interface for Itasca Codes) however the command line input is still available. Another importance is that FLAC (2000) has an embedded programming language called FISH (short for FLACish) that allows users to define new functions and variables. This version allows analysis to be performed for plane strain, plane stress, and axisymmetric problems. For this research, two-dimensional plane strain was assumed.

Figure 4.1 shows the nomenclature using in FLAC (2000) and will be often times referred to in this research.

4.1.1 Sign Conventions and Units

FLAC (2000) uses particular sign conventions and must be kept in mind especially in interpreting or reviewing the results. Figure 4.2 shows the sign convention for shear stress. For shear stress acting on a surface with outward normal in the positive x- and/or y- directions, a positive shear stress points to the positive coordinates of the second subscript. Conversely, for shear stress acting on a surface with outward normal in the negative x- and/or y- directions, a positive shear stress points to the negative coordinates of the second subscript.

For normal stresses, positive stresses indicate tension and negative stresses indicate compression. Conversely, for pore pressures, compressive fluid pore pressure is indicated by positive pore pressure and negative pore pressure indicates tension. For gravitational force, an element will move downward with positive gravity and upward with negative gravity.

Any consistent metric and English units can be used in FLAC. FLAC introduces some uncommon units as follows:

$$1\ slug = 1\,\frac{lb - \sec^2}{ft} = 14.59\ kg \qquad (4.1)$$

$$snail = \frac{lb - \sec^2}{in} \qquad (4.2)$$

$$FLAC\ \ permeability = \frac{k}{\gamma_w} \qquad (4.3)$$

where:

k = permeability (length/time)

γ_w = unit weight of water (force/length^3)

4.1.2 Grid Generation

The geometry of the problems in FLAC is defined by grids and gridpoints. FLAC organizes the grids and gridpoints in a row-and-column fashion. They are organized in horizontal direction, expressed in terms of “i” and in vertical direction, expressed in terms of “j”. The numbering of the rows and columns are started in the lower-left corner of the model. The rows go to the right while the columns continue upward.

One thing to keep in mind is that even though FLAC uses quadrilateral elements, it subdivides the quadrilateral elements into two constant strain triangular elements, which have constant strain, hence constant stress.

4.1.3 Boundary Conditions

Boundary conditions are variables that are prescribed to the boundary of the model. Of particular interest is that linear variation over specified range can be applied in FLAC through variables gradients, for example stress gradients.

Stress gradients can be applied by using the following equation:

$$S = S^{(s)} + \frac{x - x^{(s)}}{x^{(e)} - x^{(s)}} v_x + \frac{y - y^{(s)}}{y^{(e)} - y^{(s)}} v_y \qquad (4.4)$$

where:

x and y = the coordinate of a gridpoint in the range

$x^{(s)}$, $y^{(s)}$ = the coordinate of the starting gridpoint

$x^{(e)}$, $y^{(e)}$ = the coordinate of the ending gridpoint

v_x, v_y = stress gradient in x- and y-direction, respectively

$S^{(s)}$ = the starting value of stress

S = the value of stress of interest

For example, a command line:

```
apply sxy=1500 var 200, -300 from 6,1 to 11,6
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means the value of the applied shear stress (sxy) is equal to 1500 at the starting gridpoint (6,1) and is equal to 1400 at the ending gridpoint (11,6). For gridpoints in between, a linear interpolation is applied, for example, the stress will be 1700 at gridpoint (11,1) and 1200 at gridpoint (6,6).

4.1.4 Initial Conditions

Initial conditions are initial variables that are prescribed to the model before any construction is started. The best-suited initial condition will be represented by field measurement. If no field measurement is available, efforts should be performed to imitate the condition at the site.

Stress gradients, as explained in the previous section, may also be applied for the initial conditions.

4.1.5 Constitutive Model

FLAC (2000) has ten constitutive models for soil embedded in the code. In addition to that, users are allowed to develop their own models. The ten models are divided into three groups:

1. Null model group: null model (to represent material that is removed from the model)
2. Elastic model group: isotropic model, transversely isotropic model
3. Elasto-plastic model group: Drucker-Prager model, Mohr-Coulomb model, ubiquitous-joint model, strain-hardening/softening model, bilinear strain-hardening/softening ubiquitous-joint model, double-yield model, modified cam-clay model

The characteristics of the material being modeled and the intended application of the material will determine which constitutive models will be used. It was decided that for this research to model the soil as an elasto-plastic material with a Mohr-Coulomb failure criterion. The Mohr-Coulomb criterion models only dilation at failure and not the densification during cyclic loading at stress below failure. Therefore, the Mohr-Coulomb model was modified in order to model the changes of permanent volumetric strains in drained cyclic loading or pore pressures in undrained cyclic loading. The changes in volumetric strains or pore pressures were modeled using the Finn model.

4.1.6 Silent Boundary

In dynamic analyses, the application of boundaries to the model may cause the applied propagating waves to reflect back into the model. Using a larger model may minimize this problem but as a consequence, a large computational time becomes a problem. An alternative is to use silent (quiet or viscous or absorbing) boundary to overcome the problem. Silent boundary was suggested by Lysmer and Kuhlemeyer (1969). Silent boundary operates in the time domain and was based on the use of independent dashpots in the normal and shear directions applied at the model boundaries. FLAC (2000) states that a silent boundary is effective at absorbing the propagating waves for waves arriving at angles of incidence larger than 30°.

The formulation of silent boundary can be written as:

$$t_n = -\rho C_p v_n \qquad (4.5)$$

$$t_s = -\rho C_s v_s \qquad (4.6)$$

where:

t_n = the normal stress at the model boundary

t_s = the shear stress at the model boundary

ρ = mass density

C_p = the p-wave velocity

C_s = the s-wave velocity

v_n = the normal component of the velocity at the model boundary

v_s = the shear component of the velocity at the model boundary

The values of the p-wave velocity (C_p) and s-wave velocity (C_s) can be determined using the following equations:

$$C_p = \sqrt{\frac{K + \frac{4G}{3}}{\rho}} \qquad (4.7)$$

$$C_s = \sqrt{\frac{G}{\rho}} \qquad (4.8)$$

where:

K = bulk modulus

G = shear modulus

Silent boundary is best suited for dynamic source applied within a grid. It should not be used along the side boundaries of a model when the dynamic source is applied at the top or bottom boundaries because the propagating wave will leak out of the side boundaries. For this case, free-field boundary should be used.

4.1.7 Free Field Boundary

The purpose of using free field boundary is similar to that of silent boundary in that it is used so that the outward waves propagating from inside the model can be properly absorbed by the side boundaries. The details of the use of free field boundary can be seen in a finite difference code NESSI that was developed by Cundall, et al. (1980).

Free-field boundary supplies similar conditions to that of an infinite model. A one-dimensional column width is created adjacent to the side boundaries of the model and are connected each other by viscous dashpots. The unbalanced forces generated in the free field grid, both in x- and y-directions, are applied to the side boundaries and can be determined using the following equations, which applies to the left side boundary:

$$F_x = -\left[\rho C_p \left(v_x^m - v_x^{ff}\right) - \sigma_{xx}^{ff}\right]\Delta S_y \qquad (4.9)$$

$$F_y = -\left[\rho C_s \left(v_y^m - v_y^{ff}\right) - \sigma_{xy}^{ff}\right]\Delta S_y \qquad (4.10)$$

where:

v_x^m = x-velocity of gridpoint in main grid at left side boundary

v_y^m = y-velocity of gridpoint in main grid at left side boundary

v_x^{ff} = x-velocity of gridpoint in left free field

v_y^{ff} = y-velocity of gridpoint in left free field

σ_{xx}^{ff} = mean free-field horizontal stress at gridpoint

σ_{xy}^{ff} = mean free-field shear stress at gridpoint

ΔS_y = mean vertical zone size at boundary gridpoint

ρ, C_p, C_s = the same as have been explained before

Similar equations can be derived for right side boundary.

4.1.8 Damping

Damping should be applied to a system subjected to dynamic loading, otherwise the system will oscillate indefinitely. FLAC (2000) has three options of damping embedded in the code, namely Rayleigh damping, local damping, and artificial viscosity. This section discusses Rayleigh damping because it is best suited for this research. Local damping is embodied in the FLAC static solution and may be used for dynamic analysis but it becomes unreliable for complex input time history. Artificial viscosity is best used for waves with a sharp front.

Rayleigh damping is frequency-dependent and has two components: mass-proportional and stiffness-proportional. FLAC (2000) enables the users to define the first component only or the second component only or the sum of both components. At lower frequency of the system, the first one is more dominant, while at high frequency the latter is more dominant.

Figure 4.3 shows that Rayleigh damping has a reasonably flat region that spans a 3:1 frequency range. The idea in FLAC is to adjust the center frequency (f_{min}) of Rayleigh damping so it lies within the flat region, i.e. the range of predominant frequencies in the system. The center frequency (f_{min}) is the frequency at which the mass-proportional and stiffness-proportional damping each contributes one-half of the total damping. For many cases, the predominant frequencies are related to the natural mode of oscillation of a system.

$$f = \frac{C}{\lambda} \qquad (4.11)$$

where:

f = the fundamental frequency related to the natural mode of oscillation of a system

C = the velocity of the wave related to the natural mode of oscillation of a system

= C_s or C_p, [equation (4.7) or (4.8)] depending on the problem analyzed

λ = the longest wavelength related to the natural mode of oscillation of a system

Kuhlemeyer and Lysmer (1973) showed that the wavelength (λ) determines the accuracy for wave propagation problems. They found that the value of λ relates to the element length in the direction of propagation (l) by a factor of one-tenth to one-eight with one-tenth factor suggested by FLAC (2000):

$$l \leq \frac{\lambda}{10} \qquad (4.12)$$

4.2 Earthquake Motions

This research requires dynamic analysis. Hence, earthquake time history is required to be applied in FLAC analysis. The time history may be applied as external dynamic loading at the model boundary or as internal dynamic loading at internal gridpoints. The application of the dynamic loading can be performed in four different ways:

1. as an acceleration time history,
2. a velocity time history,
3. a stress (or pressure) time history, or
4. a force time history

FLAC suggests that the first and the second options be applied for cases with rigid base and that the third and the fourth options be applied for cases with flexible base.

A velocity time history can be transformed into a stress time history by using the following equations:

$$\sigma_n = 2(\rho C_p)v_n \qquad (4.13)$$

$$\sigma_s = 2(\rho C_s)v_s \qquad (4.14)$$

where:

σ_n = applied normal stress

σ_s = applied shear stress

Note that there is factor 2 in both equations to take into account the input energy that is absorbed due to the use of silent boundary. Hence, the applied stress must be doubled.

FLAC (2000) suggests that the input time history be filtered and baseline corrected before it is applied to the grid. The following sections discuss the two processes.

4.2.1 Filtering

The input time history, as explained previously, may have high peak velocity and short rise-time (time at which the peak velocity occurs). To meet the requirement suggested by Kuhlemeyer and Lysmer (1973) in equation (4.12), FLAC will need a very fine grid generation. This problem can be overcome by filtering the input time history and by removing the high frequency components. This will allow a coarser grid generation to be used in FLAC without significantly affecting the results.

For this research, the filtering process was performed using a computer code Bandpass (Olgun, 2001). This program was used to remove low and high frequencies of an earthquake time history.

The following procedures were used in the computer code:

1. The program plots the unfiltered time history of acceleration, velocity and displacement and the corresponding spectral content.
2. Raw acceleration history is bandpass filtered using an Infinite Impulse Response (IIR) filtering method in accordance with the specified corner frequencies. Butterworth window was used for the filter
3. Low frequencies are removed to clear the motions that have periods longer than the duration of the earthquake shaking, such long period motions cause non-zero velocities and permanent sway at the end of velocity and displacement respectively.

The filtering procedure can be accomplished by using a Fourier series, known as Fourier transform technique. This procedure represents an input time history as the sum of a series of sinusoids of different amplitude, frequency, and phase. Figure 4.4 illustrates the procedure schematically. The procedure can be written in mathematical terms:

$$x(t) = c_0 + \sum_{n=1}^{\infty} c_n \sin(\omega_n t + \phi_n) \qquad (4.15)$$

where:

x(t) = a periodic function

c_n = the amplitude of the *n*th sinusoids

ϕ_n = the phase angle of the *n*th sinusoids

A plot of c_n versus frequency (ω_n) is known as a Fourier amplitude spectrum and a plot of ϕ_n versus frequency (ω_n) is known as a Fourier phase spectrum. In this research, the Fourier amplitude spectrum was used to express the frequency content of a motion.

A technique known as Fast Fourier Transform (FFT) was used in this research. The FFT technique was developed by Cooley and Tukey (1965) by using an algorithm for the case where $N=2^j$, where N is the number of points in the digitized time history and j is the calculation stage. The algorithm is repeated for each *j*th stage. The time required to complete the transform is proportional to $N\log_2 N$.

4.2.2 Baseline Correction

Baseline correction was performed to force both the residual velocity and displacement to be zero. The purpose of this procedure is to prevent FLAC from exhibiting continuing velocity or residual displacements after the motion has finished.

The process is shown on Figure 4.5 and can be explained as follows:

a. The velocity time history can be obtained by integrating the acceleration time history. The integration process may result in non-zero velocity (Figure 4.5a).
b. The integration of the velocity time history will result in a displacement time history, which may not be zero (Figure 4.5b).

c. A low frequency velocity wave can be selected so that both the final velocity and displacement become zero. This wave may be a polynomial or periodic function with free parameters (Figure 4.5c). These free parameters can be adjusted to get the desired results as shown on Figure 4.5d.

If baseline correction is not performed, the uncorrected acceleration time history will result in a linear error in velocity and a quadratic error in displacement (Kramer, 1996).

4.3 Dynamic Pore Pressure Generation

FLAC contains an embedded constitutive model that takes into account the pore pressure build up in soils during cyclic loading. The model is known as the Finn model. The model was developed by Martin, et al. (1975) and then modified by Byrne (1991).

The Finn model was proposed by Martin, et al. (1975) using the following equation for the development of volumetric strains during cyclic loading:

$$\Delta\varepsilon_{vd} = C_1\left(\gamma - C_2\varepsilon_{vd}\right) + \frac{C_3\varepsilon_{vd}^{\,2}}{\gamma + C_4\varepsilon_{vd}} \qquad (4.16)$$

where:

C_1, C_2, C_3, C_4 = constants

γ = shear strain

ε_{vd} = the volumetric strain

$\Delta\varepsilon_{vd}$ = the incremental volumetric strain

The value of the incremental volumetric strain ($\Delta\varepsilon_{vd}$) is related to pore pressure (Δu) in one load cycle by the following equation:

$$\Delta u = \overline{E}_r \Delta\varepsilon_{vd} \qquad (4.17)$$

where:

Δu = increase in residual excess pore water pressure for the cycle

$\overline{E}_r$ = tangent modulus of the one-dimensional unloading curve at a point corresponding to the initial vertical effective stress (σ_{v0}')

Martin, et al. (1975) proposed that the value of the tangent modulus ($\overline{E}_r$) could be determined as:

$$\overline{E}_r = \frac{(\sigma_v')^{1-m}}{mk_2(\sigma_{v0}')^{n-m}} \qquad (4.18)$$

where:

m, n, and k_2 = constants determined using at least three unloading curves in a consolidometer

σ_v' = the current vertical effective stress

σ_{v0}' = the initial vertical effective stress

Byrne (1991) used the data from tests performed by Martin, et al. (1975) and modified the Finn model and suggested the use of the following simpler equation:

$$\frac{\Delta\varepsilon_{vd}}{\gamma} = C_1 \exp\left(-C_2\left(\frac{\varepsilon_{vd}}{\gamma}\right)\right) \qquad (4.19)$$

The values of C_1 and C_2 are constants that can be estimated using the following empirical correlations:

$$C_1 = 8.7(N_1)_{60}^{-1.25} \qquad (4.20a) \text{ or}$$

$$C_1 = 7600(D_r)^{-2.5} \qquad (4.20b)$$

$$C_2 = \frac{0.4}{C_1} \qquad (4.21)$$

where:

$(N_1)_{60}$ = the corrected SPT blow count (refer to Section 3.1)

D_r = the relative density of soil

Byrne (1991) related the incremental volumetric strain ($\Delta\varepsilon_{vd}$) to pore pressure by using the following series of equation:

$$u_g = \sum \Delta u \qquad (4.22)$$

$$\Delta u = M\Delta\varepsilon_v^p \qquad (4.23)$$

$$M = K_m p_a \left(\frac{\sigma_v'}{p_a} \right)^m \qquad (4.24)$$

$$\Delta\varepsilon_v^p = 0.5\gamma C_1 \exp\left(-C_2 \left(\frac{\varepsilon_{vd}}{\gamma} \right) \right) \qquad (4.25)$$

where:

u_g = the pore water pressure generated per half cycle loading

Δu = change in pore water pressure per half cycle loading

M = the constrained modulus

σ_v' = effective vertical stress in half cycle loading

p_a = atmospheric pressure with the same unit as σ_v'

$\Delta\varepsilon_v^p$ = change in volumetric strain in half cycle loading

Byrne (1991) suggested to use the values of K_m = 1600 and m = 0.5 for the above equations. The equations proposed by Byrne (1991) are more preferable because the constants C_1 and C_2 can be estimated using Standard Penetration Test (SPT) tests.

The Finn model incorporates both equations (4.16) and (4.19) into the Mohr-Coulomb model.

One of the most important processes in determination of pore pressure in FLAC would be the selection of the bulk modulus of water (K_w). This process will determine the length of time to take the system into equilibrium. FLAC (2000) suggested that for a coupled problem, the true diffusivity (c) is controlled by the stiffness ratio (R_k), which is determined as:

$$c = k\left(K + \frac{4}{3}G\right)\frac{1}{1+1/R_k} \qquad (4.26)$$

$$R_k = \frac{K_w/n}{K + 4G/3} \qquad (4.27)$$

where:

k = the mobility coefficient

K = the bulk modulus of soil matrix

G = the shear modulus of soil matrix

K_w = the bulk modulus of water = 2 * 10^9 Pa = 4.2 * 10^7 psf

n = the porosity of soil matrix

In equation (4.26), the mobility coefficient (k) is defined as the coefficient of the pore pressure term in Darcy's law.

$$k = \frac{k_H}{g\rho_w} \qquad (4.28)$$

where:

k_H = the hydraulic conductivity used in Darcy's law

g = the gravitational force

ρ_w = the density of water

It can be seen from equation (4.26) that if the value of R_k is very large, the system is coupled with a diffusivity governed by the soil matrix. In FLAC (2000) the modeling can be performed with uncoupled flow-mechanical approach, i.e. with the flow calculation in flow-only mode (**SET flow on, SET mech off**), and then in mechanical-only mode (**SET flow off, SET mech on**), to bring the model into equilibrium. For the latter, K_w is set to zero, while for the former, K_w should be adjusted to K_w^a by using the following equation:

$$K_w^a = \frac{n}{\frac{n}{K_w} + \frac{1}{K + 4G/3}} \qquad (4.29)$$

For a coupled flow-mechanical problem (**SET flow on, SET mech on**), the value of K_w should be adjusted so that the value of R_k is ≤ 20 by using equation (4.27).

The next chapter will explain the use of numerical modeling presented here to analyze an aggregate pier foundation system.

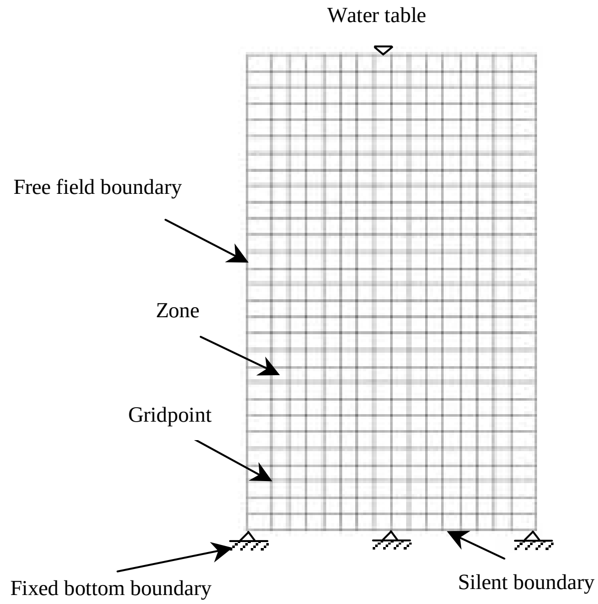


Figure 4.1 Nomenclature used in FLAC

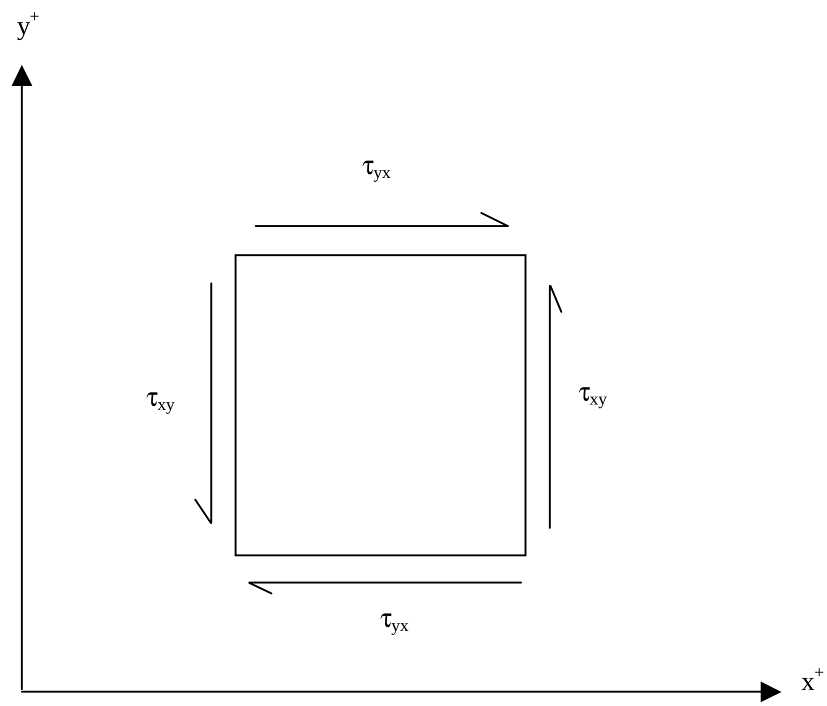


Figure 4.2 Sign convention for positive shear stresses (after FLAC, 2000)

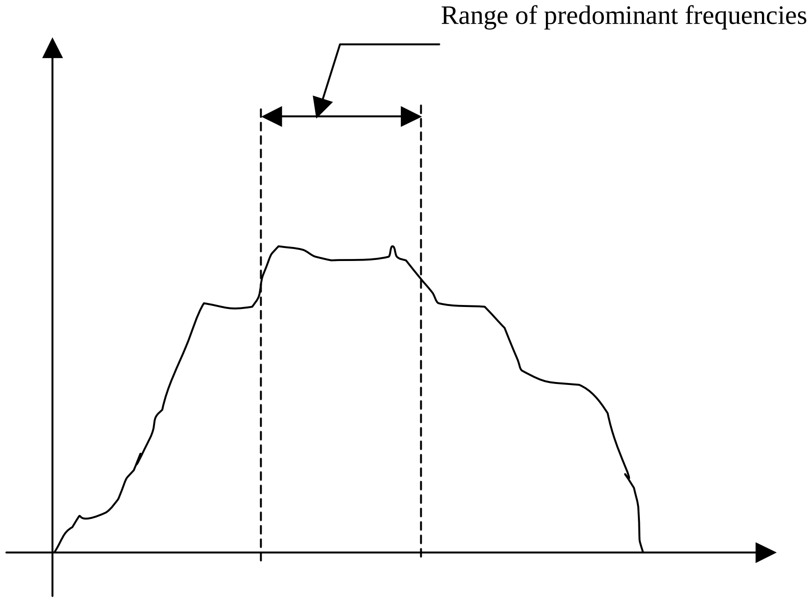


Figure 4.3 Plot of velocity spectrum versus frequency (after FLAC, 2000)

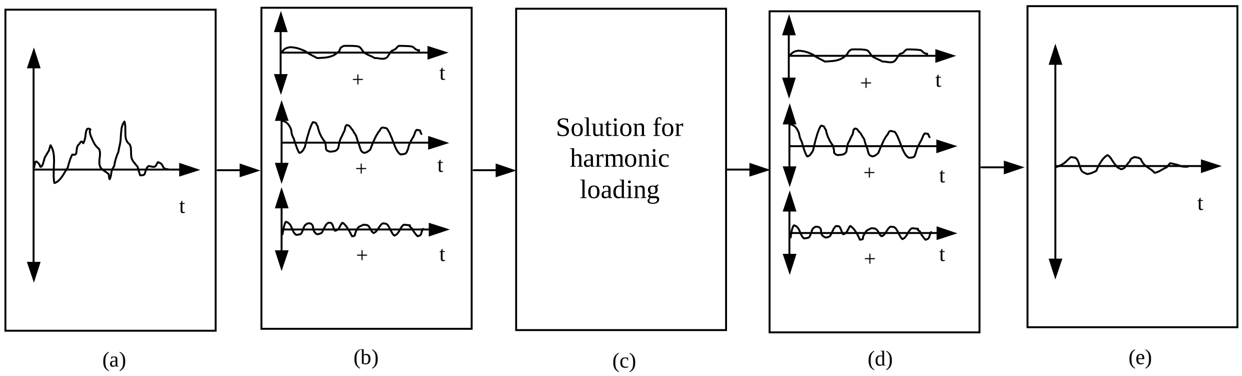
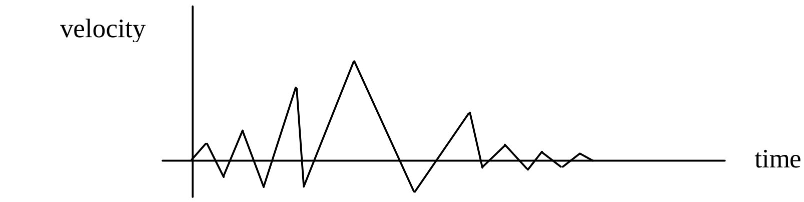
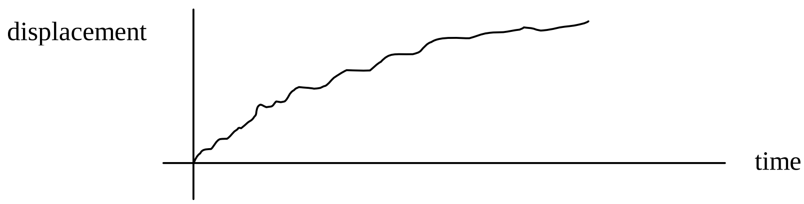


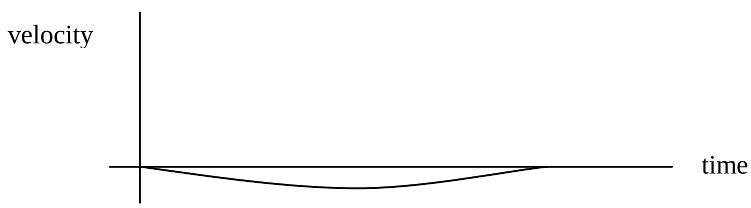
Figure 4.4 Process of Fourier transform: (a) Time history of loading, (b) Representation of time history of loading as sum of series of harmonic loads, (c) Calculation of response for each harmonic load, (d) Representation of response as sum of series of harmonic responses, (e) Summation of harmonic responses to produce time history of response (after Kramer, 1996)



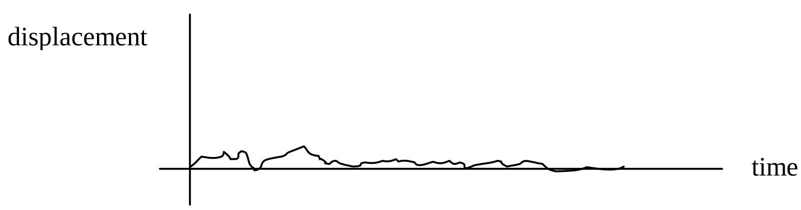
(a) Velocity time history



(b) Displacement time history



(c) Low frequency velocity wave



(d) Resultant displacement time history

Figure 4.5 The baseline correction process (after FLAC, 2000)