4. Platooning on the AHS

4.1 Introduction

The introduction of advanced technologies into our transportation system allows planners and designers the ability to incorporate new operational concepts into our highway network. One of these concepts is that of platooning. Platooning consists of creating platoons of “linked” vehicles which would travel along the AHS acting as one unit. These vehicles would follow one another with very small headways – vehicle spacing as little as a couple of meters – and be “linked” through headway control mechanisms, such as radar-based or magnetic-based systems. The first vehicle in the platoon, the leader, continuously provides the other vehicles, the followers, with information on the AHS conditions, and what maneuvers, if any, the platoon is going to execute [10]. To properly utilize this new concept of platooning its operational characteristics must be explored.

4.2 Platoon Stability

When traffic conditions on today’s highways reach congested levels a phenomenon known as “stop and go traffic” occurs. In these conditions vehicles will come to a stop, then begin to accelerate only to have to stop again a few seconds later. A version of this also happens without traffic coming to a complete stop. Vehicles traveling along will have to slow down due to some traffic condition. After a little time the speed will increase only to slow down again a few seconds later. This causes a ripple effect which propagates back through the traffic stream.

This phenomenon is due to a driver’s reaction to the distance between his or her vehicle and the vehicle in front relative to his or her perceived safe following distance. As traffic begins to speed back up, the distance between vehicles becomes larger than the safe following distance, and thus the driver responds by increasing speed so as to close the gap. In doing so the distance
between vehicles becomes less than the safe following distance (especially if the leading vehicle applies his or her brakes), thus causing the driver to have to brake to again get back to a safe following distance. This will usually cause the next vehicle back to get too close and have to break. During this braking the gap will again widen to at least the safe following distance, thus the driver begins accelerating to maintain that distance. This acceleration, coupled with the braking of the following vehicle, opens that gap to more than the safe following distance causing that vehicle to accelerate to close the gap.

It can be easily seen how this can easily propagate back through the traffic stream, causing instability. With the introduction of automation to vehicles, the possibility of this adjustment happening several times a second (as opposed to once every couple of seconds) is introduced. These rapid adjustments must be avoided as the resulting ride would be far from comfortable. It could ultimately lead to drivers purposely using the conventional highway instead of the AHS, thus defeating the purpose of the system.

Thus the goal is to establish a control law for acceleration and deceleration which will create stability in the traffic system and therefore avoid the above-mentioned condition. In Section 3.3 the safe headway was found in terms of the stopping distance of the leading and following vehicles and the average vehicle length (Equation (3.11)). By assuming that the lead vehicle comes to an instantaneous stop \( d_{i-1} = 0 \) and letting the jerk, \( j \), equal infinity, and then substituting in Equation (3.27) with \( t_s = 0 \), Equation (3.11) becomes the familiar uniform deceleration model:

\[
 s_i = v_i t_r + L \tag{4.1}
\]

The “safe following model” of maintaining one car length for every 10 mph of speed originates from this special case.

To obtain a control law we want to incorporate time into Equation (4.1), thereby entering into traffic stream dynamics. The new form of Equation (4.1) is

\[
x_{i-1}(t-T) - x_i(t-T) = v_i(t) t_r + L_i \tag{4.2}
\]
where $T$ is the lag, or reaction, time. In the case of automated travel $T = t_r$, the sensor reaction time. By differentiating Equation (4.2) with respect to time and solving for acceleration one gets the linear car-following control law:

$$\dot{v}(t) = \left(t_r^{-1}\right) \times \left[v_{i-1}(t-T) - v_i(t-T)\right] \tag{4.3}$$

In this case the acceleration of the $i^{th}$ vehicle is determined by the speed of vehicle $i$ relative to vehicle $i-1$. The time lag $T$ signifies the time between the stimulus, relative velocity, and the response, vehicle acceleration. Therefore the acceleration is being controlled by the relative velocity times a sensitivity factor, $t_r^{-1}$.

Yet the linear car-following control law lacks the one property needed – the distance between vehicles. The sensitivity factor can thus be changed to include vehicle headway so as to become

$$\alpha \frac{[V_i(t-T)]^m}{[x_{i-1}(t-T) - x_i(t-T)]^n} \tag{4.4}$$

When this new sensitivity factor is incorporated into the linear car-following control law the non-linear control law is obtained:

$$\dot{v}(t) = \alpha \frac{[V_i(t-T)]^m}{[x_{i-1}(t-T) - x_i(t-T)]^n} \left[v_{i-1}(t-T) - v_i(t-T)\right] \tag{4.5}$$

Where $\alpha$ is a proportionality constant. This control law allows the vehicle’s acceleration to be dependent on the vehicle’s absolute speed and headway, along with relative speed. Lee and Drew [17] suggest that platoon stability can be reached for values of $m = 0.7$ and $n = 2.7$ for a six vehicle platoon under the following initial conditions: headway of 28 feet, speeds of 200 mph, a maximum acceleration/deceleration of 0.3g, and a lag time of 0.08 seconds. In Figure 4-1, the velocity-time, space-time, and headway-time graphs for this model are shown for the first two seconds of the operation. Notice that all the graphs are linear, representing a stable system. At the end of the acceleration period an average headway of 40.7 feet and speed of 300 mph were obtained. To execute this maneuver, an STS of just under 7000 feet (about 1.3 miles) is required. By using a control law like that of Equation (4.5) the STS operation can be successfully completed without creating disturbances in the system. This allows a smooth and safe transition from the higher speed UCS to the lower speed RCS and back.
Velocity, Space, Headway – Time Relationships

Figure 4-1
Lu [25] also developed another control law, the Multiple-mode Vehicle Headway Control Model (MVHC). This model, which is based on the manual driving process, incorporates vehicle dynamics and current roadway conditions into control algorithms which apply to certain modes of vehicle operation. Under this system a vehicle will operate under one of five modes: cruise control, following control, rapid deceleration, slow deceleration, or emergency braking. The mode which a vehicle operates under depends upon its spacing and relative velocity between itself and the vehicle in front of it. Of the most interest is how the MVHC system brings a platoon of vehicles to a stop through the use of one of the last three modes.

To classify which mode a vehicle will be operating under, Lu first defined a safe distance function, $SD_i$, as follows:

$$SD_i = \begin{cases} 
    b_1 \frac{V_i^2}{\text{DMAX}_i} - b_2 \frac{V_{i-1}^2}{B_e} & V_i \geq V_{i-1} \\
    b_3 V_i & V_i < V_{i-1} 
\end{cases}$$  \hspace{1cm} (4.6)$$

where

$V_j = \text{velocity of the } j^{th} \text{ vehicle}$

$\text{DMAX}_j = \text{maximum deceleration of the } j^{th} \text{ vehicle}$

$B_e = \text{emergency braking rate}$

$b_1, b_2, b_3 = \text{constants}$

During a situation where a vehicle in a platoon must decelerate, either to slow down or come to a complete stop, one of the following modes will be activated according to the following set of classification equations:

- **rapid deceleration mode**

  $$a_2SD_i < \Delta_{i-1,i} < a_1SD_i$$ \hspace{1cm} (4.7)

  $$A_{i-1} \leq CD$$ \hspace{1cm} (4.8)

- **slow deceleration mode**

  $$a_2SD_i < \Delta_{i-1,i} \leq a_1SD_i$$ \hspace{1cm} (4.9)
\[
\frac{\Delta_{i-1,i}}{V_i} \geq \beta \frac{V_i}{DMAX_i}
\] (4.10)

- emergency braking mode

\[
\Delta_{i-1,i} < a_2SD_i
\] (4.11)

\[
\Delta V_{i-1,i} < CRV
\] (4.12)

where

\(\Delta_{i-1,i}\) = spacing between the \(i-1\)th vehicle and the \(i\)th vehicle

\(A_{i-1}\) = acceleration of the \(i-1\)th vehicle

\(V_i\) = velocity of the \(i\)th vehicle

\(DMAX_i\) = maximum deceleration of the \(i\)th vehicle

\(\Delta V_{i-1,i}\) = relative velocity of the \(i-1\)th vehicle and the \(i\)th vehicle

\(CRV\) = critical relative velocity

\(CD\) = critical deceleration

\(\beta, a_1, a_2\) = constants

One will notice that the classification is based on the rate of deceleration of the lead vehicle, the relative velocities of the lead and following vehicles, and the separation between the two vehicles. In this way an appropriate deceleration mode can be quickly and easily selected. Once assigned a mode of operation, the vehicle will then follow the associated control laws, which Lu defined as follows:

- rapid deceleration mode

\[
Ard_i = Krd_iKrd_i \frac{V_{i-1}-V_i}{ST+PRT}
\] (4.13)

- slow deceleration mode

\[
Asd_i = Ksd_i \frac{V_i}{c_1\Delta_{i-1,i} + c_2} + KSD
\] (4.14)

- emergency braking mode

\[
Ab_i = \min \left\{ DMAX_i, Kb \frac{V_{i-1}^2-V_i^2}{c_3\Delta_{i-1,i} + c_4} \right\}
\] (4.15)

where
\(\text{Ard}_i\) = acceleration of the \(i^{th}\) vehicle in rapid deceleration mode
\(\text{Asd}_i\) = acceleration of the \(i^{th}\) vehicle in slow deceleration mode
\(\text{Ab}_i\) = acceleration of the \(i^{th}\) vehicle in emergency braking mode
\(\text{Krd}_i\) = velocity related gain in rapid deceleration mode of \(i^{th}\) vehicle
\(\text{Krd}_h\) = headway related gain in rapid deceleration mode of \(i^{th}\) vehicle
\(ST\) = sensor time
\(\text{PTRT}\) = power train response time
\(\text{Ksd}_i\) = slow deceleration mode control gain of \(i^{th}\) vehicle
\(K_b, K_{SD}, c_1, c_2, c_3, c_4 = \) constants

Using these deceleration modes the vehicle would be able to come to a safe stop while using the most gradual deceleration possible. Therefore, in the platoon environment the entire platoon, when faced with an incident, would be able to safely come to a complete stop basically in reaction to the lead vehicle’s behavior. Since the constants and other undefined values can be defined for each individual vehicle, this control law will work for both passenger vehicle and truck operations, as well as platoons of these vehicles which have varied characteristics, such as maximum deceleration.

Lu ran a simulation model comparing the above control law against manual driver control, and it was found that not only did the MVHC system respond to situations better than did manual drivers, but the system was able to respond with a smooth and consistent response, thereby providing a safer and more efficient operation than possible with human interaction.

### 4.3 Interplatoon Spacing

Back in section 3.3, AHS Operations, a safe-following distance was found while considering operations along the AHS Magway. These operations consisted solely of longitudinal maneuvers – accelerating and decelerating. It was mentioned that along certain UCS there would be interchanges to load and unload vehicles from the system. These cause merging and weaving which adversely affect capacity. It was then that the concept of platooning was
introduced to provide the necessary gaps in the traffic flow without seriously compromising the system’s capacity potential.

The safe-following distance, $s$, that was found will essentially become our intraplatoon spacing, or the spacing between vehicles in the same platoon. By examining Figure 4-2, a potential platooning situation can be seen. In Figure 4-2(a) three platoons are shown with a platoon of $n$ vehicles between two platoons of $N$ vehicles. This would represent normal operations. Figure 4-2(b) shows what the platoons should look like if they were forced to come to a stop.

As mentioned in Section 4.1, platoons consist of two types of vehicles – leaders and followers. Leaders make all of the decisions for the platoon, including interaction with the rest of the Maglev AHS network. These decisions are passed along to the rest of the platoon, the followers. Followers only are concerned with responding to the information from the leader, whether it be reporting the vehicle’s condition or executing an emergency stop. Since the leader must interact with vehicles other than those in the platoon, sensor response time could be much greater. Therefore another headway, the interplatoon spacing, is needed as shown in Figure 4-2(a).

This spacing between platoons, $s_p$, is found in much the same way as the intraplatoon spacing, $s$, and thus is

$$s_p = vT + s$$

(4.16)

where $T$ is the reaction time. By examining Figure 4-2(a) for the first two platoons, the AHS density $k$ can be found to be

$$k = \frac{5280(N + n)}{(N - 1)s + (n - 1)s + 2s_p}$$

(4.17)

If a system of only free agents was considered, i.e. a system without platooning ($N = n = 1$), then $k = 5280/s_p$ as would be expected. Also from traditional flow theory, the traffic volume $q$ is equal to the density, $k$, times speed, $V$, in miles per hour. This gives

$$q = \frac{5280(N + n)V}{(N + n)s + 2.93VT}$$

(4.18)
Guideway Traffic Stream Dynamics

Figure 4-2
when substituting Equation (4.16) into Equation (4.17) before multiplying. Figure 4-3(a) shows a volume-speed curve for \( s = 20 \text{ ft}, T = 0.15 \text{ sec} \) and \( n = 1 \) and Figure 4-3(b) for \( s = 60 \text{ ft} \). These two values of \( s \) represent the headways that could exist on the car guideway and the truck guideway, respectively.

### 4.4 Guideway Volume-Speed Relationships

The expression for density and volume, Equations (4.17) and (4.18) can be expressed in terms of the parameters \( a \) and \( j \) from Section 3.3:

\[
k = \frac{5280(N + n)}{(N + n)s + 2(d_i - d_{i-1})}
\]

and

\[
q = \frac{5280(N + n)V_0}{(N + n)s + 2(d_i - d_{i-1})}
\]

based on the safe platoon headway \( s_p = d_i - d_{i-1} + s \). Substituting for \( d_i \) and \( d_{i-1} \) gives

\[
q = \frac{5280(N + n)V_0}{(N + n)s + 2\left[1.47V_0(a/j) - \left(\frac{1}{6}\right)a(a/j)^2\right]}
\]

Equation (4.21) is plotted in Figure 4-4 for parameters \( n = 1, s = 20, a = 32.2, \) and \( a/j = 0.5 \) for platoon sizes of \( N = 2 \) to \( N = 12 \). It is believed that the curves in Figure 4-4 are more realistic than those of Figure 4-3.

In Section 3.2 the radius of curvature of the guideway and the forces related to high speed travel through these curves were discussed. It was shown that at high speeds the centrifugal force which the driver is subjected to can become quite high. Conditions would closely match those experienced by some pilots during the World War II era. This requires some investigation into the effects of these forces on the driver and the comfort of the system.
Car Guideway Speed-Volume Relationship

Figure 4-3(a)

Truck Guideway Speed-Volume Relationship

Figure 4-3(b)
Guideway Speed-Volume Relationship

Figure 4-4
Under conditions of high centrifugal forces over long periods of time lead to a condition known as G-LOC, or loss of consciousness due to high G forces. While this requires a person to be subjected to accelerations in excess of five G’s (five times the acceleration of gravity) for periods of at least three seconds [23], which will never be experienced on any type of roadway or guideway, the effects on a person prior to reaching G-LOC are within the reach of today’s technology. As a person experiences increased accelerations over a prolonged period of time, several phases of reaction are experienced. The first condition would be visual symptoms, ranging from tunnel vision to gray-out. Beyond that a person will experience blackout, as the eyes are no longer receiving adequate, or any, blood. It is beyond this point where a person will experience G-LOC, brought on by the deprivation of blood to the brain.

If one was to reconsider the use of today’s highway curvatures with magway speeds of 300 mph, the acceleration that would be experienced would be around 3.5 G’s. If this level of acceleration continued for about five seconds, it is possible that a passenger could experience the first signs of G-LOC, impaired vision. As stated previously, this is also outside the range of what anyone would consider comfortable travel, especially when compared to today’s driving conditions. Recall that when the speeds through a curved section was reduced to 200 mph, the G-factor was lowered into the range of 1.5 G’s. This value is well outside of the range of concern for experiencing any of the side effects related to G-LOC. Note that this was using a radius of curvature consistent with today’s highways, which normally have a design speed in the range of 80 mph. Obviously, increasing the radius of curvature will have the effect of reducing the centrifugal acceleration even more, and thus further from the danger of experiencing any of the symptoms associated with high G’s.

4.5 Moving Queues

As stated previously, platooning plays a significant role in AHS operations due primarily to the small headways which can be obtained. While determining what constitutes a platoon can be fairly arbitrary, it will be assumed that any two vehicles with a spacing too small to permit
another vehicle to enter are platooned. Following Drew’s “moving queues” development this platooning model is based on performing a Bernoulli test on each headway resulting in the individual platoon sizes forming a geometric distribution,

\[ P_n = p^{n-1}(1 - p) \]  \hspace{1cm} (4.22)

where \( p = P(x<s_p) \) and \( 1-p = P(x>s_p) \).

The average platoon size is

\[ N = \sum_{n=1}^{\infty} nP_n = (1 - p)^{-1} \]  \hspace{1cm} (4.23)

Taking the distribution of space headways, \( f(x) \), to be an Erlang distribution

\[ P(x > s_p) = \int_{s_p}^{\infty} \frac{(ka)^a}{(a-1)!} e^{-aks} x^{a-1} e^{-aks} dx = e^{-aks_p} \sum_{i=0}^{a-1} \frac{(aks_p)^i}{i!} \]  \hspace{1cm} (4.24)

The values of \( N \) for \( a = 1, 2, 3, \) and 4 are found to be

\[ N_1 = e^{ks_p} \]  \hspace{1cm} (4.25)

\[ N_2 = \frac{e^{2ks_p}}{1 + 2ks_p} \]  \hspace{1cm} (4.26)

\[ N_3 = \frac{e^{3ks_p}}{1 + 3ks_p + 4.5(ks_p)^2} \]  \hspace{1cm} (4.27)

\[ N_4 = \frac{e^{4ks_p}}{1 + 4ks_p + 8(ks_p)^2 + 10.67(ks_p)^3} \]  \hspace{1cm} (4.28)

Equations (4.25) and (4.26) are plotted in Figure 4-5. They show the average platoon size is dependent on the traffic density and interplatoon spacing. One can easily see how as the interplatoon spacing increases, so does the platoon size, and vice versa. This allows for some control over the spacing, thereby allowing platoon size management to ultimately control the availability of gaps in the traffic stream. The importance of gaps in the guideway traffic stream will be seen in the next section.
Platoon Size for Various Parameter Values

Figure 4-5
4.6 Merging and Weaving

When considering a system with vehicles under manual control, merging and weaving depend heavily on the size of gaps in the traffic flow and the probability that a driver will deem the gap large enough to safely move his or her vehicle into it. Therefore a critical gap is defined – a gap size which is greater than 50 percent of those accepted yet less than 50 percent of those rejected. The theory is that the critical gap can then be used as the guideline for gap acceptance. If a gap is larger than the critical gap, it is accepted, and if it is smaller it is rejected. This is supposed to accurately estimate the number of gaps accepted and rejected in the real system.

Once automated control is introduced the idea of gap acceptance becomes gap management. Since driver perception of the ideal gap size is replaced by the ability to actually measure the gap size, the critical gap takes on a different meaning. Now it becomes the test for whether or not a gap is large enough to merge into. That is, the critical gap is the smallest gap which can be accepted. Automation allows for the utilization of every available acceptable gap, which could not be assumed before. Along with the ability to measure and analyze the gaps by the minor stream, automated control also presents the opportunity to actually create acceptable gaps in the major stream for the minor stream.

Besides the critical gap, the ideal gap is also of interest. The ideal gap is the gap needed to allow for an ideal merge – one in which the entering vehicle enters the main traffic stream without causing the traffic stream to have to adjust its speed. Under automated control the ability exists to measure the available gaps and to determine if it is an ideal gap. Since any gap larger than the ideal gap will allow for an ideal merge, the ideal gap becomes the critical gap under automated control.

The ideal gap $T_i$ can be broken into three components [18]:

$$T_i = T_r + T_i + T_f$$  \hfill (4.29)
where $T_r$ is the safe time headway between the ramp and lead vehicles, $T_l$ is the time lost accelerating during merging, and $T_f$ is the safe time headway between the lag and ramp vehicles.

If $\tau$ is the reaction time then

$$T_r = \frac{L_l}{v_g} + \tau \quad (4.30)$$

and

$$T_f = \frac{L_r}{v_g} + \tau \quad (4.31)$$

where $L_l$ and $L_r$ are the lengths of the lag and ramp vehicles respectively and $v_g$ is the guideway speed. Also

$$T_l = T_2 - T_1 \quad (4.32)$$

where $T_2$ is the time needed for the merging ramp vehicle to accelerate from the ramp speed $v_r$ to the guideway speed $v_g$ and $T_1$ is the time to cover that same distance $x$ but at a constant speed $v_g$.

Using the non-uniform acceleration model

$$v = \frac{\alpha}{\beta} - \left( \frac{\alpha}{\beta} - v_0 \right) e^{-\beta t} \quad (4.33)$$

the distance traveled during the merging maneuver, $x$, is found to be

$$x = \frac{\alpha}{\beta} T_2 - \frac{\alpha}{\beta^2} \left( 1 - e^{-\beta \tau_2} \right) + \frac{v_r}{\beta} \left( 1 - e^{-\beta \tau_2} \right) \quad (4.34)$$

where $\alpha$ is the acceleration at the beginning of the maneuver and $\alpha / \beta = v_{\text{max}}$, thus $\beta = \alpha / v_{\text{max}}$. Also from the non-uniform acceleration model (Equation (4.33)) $T_2$ is found to be

$$T_2 = -\frac{1}{\beta} \ln \left( \frac{\alpha - \beta v_g}{\alpha - \beta v_r} \right) \quad (4.35)$$

It is interesting to quickly examine the case where the guideway speed $v_g$ is the maximum vehicle speed $v_{\text{max}}$. One will find that $T_2$ becomes infinity. This is due to the asymptotic behavior of the non-uniform acceleration model. As time increases the exponential term of the model approaches zero, thus making the second term of Equation (4.33) become small. Yet the exponential never actually becomes zero, thus preventing the term from ever totally dropping out. Therefore the velocity $v$ never actually reaches $\alpha / \beta$, the maximum velocity $v_{\text{max}}$. But it does approach $v_{\text{max}}$ and $t$ goes to infinity.
Continuing, substituting for \( x \) in

\[
T_i = \frac{x}{v_g}
\]  

(4.36)

gives

\[
T_i = \frac{\alpha}{v_g \beta} - T_2 - \frac{\alpha}{v_g \beta} \left( 1 - e^{-\beta \tau} \right) + \frac{v_r - \alpha}{v_g \beta} \left( 1 - e^{-\beta \tau} \right)
\]  

(4.37)

which when substituted in Equation (4.29) with \( T_2 \) gives

\[
T_i = \frac{v_g - v_r}{\beta v_g} + \frac{\alpha - \beta v_g}{\beta^2 v_g} \ln \left( \frac{\alpha - \beta v_g}{\alpha - \beta v_r} \right)
\]  

(4.38)

By substituting Equations (4.30), (4.31), and (4.32) into Equation (4.33), the ideal gap size is found to be

\[
T_i = \frac{L_i + L_i}{v_g} + 2\tau + \frac{v_g - v_r}{\beta v_g} + \frac{\alpha - \beta v_g}{\beta^2 v_g} \ln \left( \frac{\alpha - \beta v_g}{\alpha - \beta v_r} \right)
\]  

(4.39)

If under AHS merging \( v_g = v_r \) and \( L = L_r = L_i \), one obtains

\[
T_i = 2 \left( \frac{L}{v} + \tau \right)
\]  

(4.40)

where \( \tau \) is the sensor reaction time. The ideal gap size in Equation (4.40) is plotted for various values of \( \tau \) and \( v \) with \( L = 20 \) ft in Figure 4-6(a) and \( L = 60 \) ft in Figure 4-6(b).

Since it is the ideal gap which allows the main traffic stream to continue without having to adjust its speed, ultimately it would be in the system’s best interest to have every entering vehicle be able to conduct an ideal merge into an ideal, or better, gap. Thus the vehicle merging system needs to be able to adjust the merging vehicle’s speed and acceleration to allow it to make the ideal merge. This is done through control laws. In developing a vehicle merging control law, Li [24] suggests breaking the merging operation into two parts, or stages. The first stage, the speed adjustment stage, is where the merging vehicle adjusts its speed from its speed on the entrance ramp to the speed of the gap into which it will enter. The vehicle will also
Ideal Gap for Controlled Merge

Figure 4-6
position itself relative to the gap as to allow for smooth entry into the gap during the next stage. The second stage, the lane merging stage, is where the vehicle actually enters the selected gap. It is also at the beginning of this stage that the vehicle first becomes part of the main traffic stream. Ultimately the merging vehicle will be traveling at the same speed as the main traffic flow, thereby allowing for a smooth and unobtrusive merge into the main stream traffic. Therefore the control law Li developed ultimately places the vehicle in the position to smoothly merge into traffic, while having it traveling at the speed of the traffic with no residual acceleration to have to correct.

The first issue which must be explored is whether or not the merging vehicle, once it reaches the entrance ramp, actually has the ability to adjust to the position and speed of the gap which it is to merge into. This characteristic of the vehicle is referred to as its pursuit ability. To determine if the vehicle can merge with its associated gap, the pursuit ability index is found:

$$\psi = \frac{e}{e_{\text{max}}} = \frac{D_m - D_c}{D_{\text{max}} - D_c} = \frac{V_{c1} - V_m}{V_{\text{max}} - V_m}$$  \hspace{1cm} (4.41)

where

- $D_{\text{max}}$ = the maximum or minimum allowed distance of the merging vehicle
- $V_{\text{max}}$ = the maximum or minimum allowed speed of the merging vehicle
- $e_{\text{max}}$ = the maximum allowed distance or speed error of the merging vehicle
- $V_{c1}$ = the actual speed of the targeted gap
- $D_{c1}$ = the actual distance of the targeted gap
- $V_m$ = the actual speed of the merging vehicle
- $D_m$ = the actual distance of the merging vehicle
- $e$ = the actual distance or speed error of the merging vehicle

If $-1 < \psi < 1$, the vehicle has the ability to successfully pursue and eventually merge into the gap. Note that a negative pursuit ability index signifies that the merging vehicle is traveling faster than the gap into which it is to merge. If the calculated pursuit index is outside of this range, then the gap is rejected and a new gap is found.
Once a gap has been classified as pursuable, the next step is to guide the merging vehicle into place as to ideally intercept the gap. This is done by developing a control law which will adjust the position and velocity of the vehicle continuously so as to allow it to merge successfully. Li attempted to do this using a Linear Control Law. Under this control law the speed profile of the merging vehicle followed a linear path. While this control law worked fine under certain circumstances, if prior to beginning the maneuver the vehicle was not able to utilize a constant acceleration, then the Linear Control Law was not able to minimize the acceleration noise. That is, it was not able to optimize the acceleration changes so that they would be kept to a minimum. Therefore Li then developed an Optimal Control Law. Under this control law the speed profile was able to follow a circular path, meaning the acceleration was no longer constant. This allowed the law to minimize the acceleration noise realized during the maneuver. Yet there was a problem with the Linear Control Law which the Optimal Control Law could not fix. Both of these laws, except under certain special cases, would have the vehicle arrive in the appropriate position to merge with residual acceleration. That is, the vehicle would have to make a near instantaneous acceleration change at the end of the maneuver so that the vehicle could begin its merging maneuver with zero acceleration. This would create a large jerk at the end of the maneuver. Therefore the Parabolic Control Law was developed. This law is a hybrid of the Optimal Control Law, therefore the ability to minimize acceleration noise was maintained. But this control law also has the ability to select a speed profile which leaves the vehicle with zero acceleration at the end of the maneuver. The Parabolic Control Law is as follows [24]:

\[
D_{des} = \left( \frac{V_m}{V_g} + 2 \right) \frac{D_g}{3}
\]

\[
V_{des} = \left( \frac{3D_m}{D_g} - 2 \right) V_g
\]

\[
\dot{V}_{des} = \frac{2(V_g - V_m)}{T} - \frac{V_m - V_{c3}}{T} + \frac{V_{c3} V_m}{V_g} \dot{V}_g
\]

where

\[
D_{des} = \text{desired distance of the merging vehicle}
\]

\[
V_{des} = \text{desired speed of the merging vehicle}
\]
\( \dot{V}_{des} \) = desired acceleration of the merging vehicle
\( D_g \) = actual distance of the gap
\( V_g \) = actual speed of the gap
\( \dot{V}_g \) = actual acceleration of the gap
\( D_m \) = actual distance of the merging vehicle
\( V_m \) = actual speed of the merging vehicle
\( V_{c3} \) = critical speed of the merging vehicle
\( T \) = total time needed to successfully complete the maneuver

It would be through this control law that a vehicle would successfully complete the speed adjustment stage. At this point the merging vehicle would effectively become part of the main stream traffic flow and would laterally maneuver into the gap.

Of interest is to find the minor stream capacity, \( Q_r \), which can be accepted by the major stream. This capacity depends on the major stream volume, \( q \), and the ideal gap, \( T_c \) (Note: \( T_c \) will be used instead of \( T_i \) to avoid confusion later). To find the equation for \( Q_r \), take the reciprocal of the average delay to a minor stream vehicle which is ready to merge, \( d \):

\[
d = \bar{n} \times \bar{r}
\]

where \( \bar{n} \) is the average number of gaps rejected before accepting a gap and \( \bar{r} \) is the average length of a rejected gap.

When a vehicle attempts to enter the guideway, it might have to reject a certain number of gaps before an acceptable gap, one larger than the ideal gap, becomes available. The number of rejected gaps is \( n \), and \( \bar{n} \) is the average of these:

\[
\bar{n} = \sum_{n=0}^{\infty} nP_n
\]

where \( P_n \), the probability of rejecting \( n \) gaps before accepting a gap, is

\[
P_n = (1 - p)p^n \quad n = 1, 2, 3, \ldots
\]

where

\[
p = P(t < T_c) = \int_{0}^{T_c} f(t)dt
\]
and \( f(t) \) is the probability density function of gaps in the major stream. Substituting Equation (4.47) into Equation (4.46) gives

\[
\bar{n} = (1 - p) \sum_{n=0}^{\infty} np^n = \frac{p}{1 - p}
\]  

(4.49)

By substituting Equation (4.48) into Equation (4.49)

\[
\bar{n} = \frac{\int_0^{\infty} f(t)dt}{\int_0^{\infty} f(t)dt}
\]

(4.50)

The term \( \bar{r} \) in Equation (4.45) is the total time spent in rejected gaps divided by the total number of rejected gaps which can be represented as

\[
\bar{r} = \frac{\int_0^{T_c} tf(t)dt}{\int_0^{\infty} f(t)dt}
\]

(4.51)

By substituting Equations (4.50) and (4.51) into Equation (4.45):

\[
d = \frac{\int_0^{T_c} tf(t)dt}{\int_0^{\infty} f(t)dt}
\]

(4.52)

If \( f(t) \) conforms to an Erlang distribution, it can be shown using the approach taken by Drew that

\[
Q_r = \frac{q \sum_{i=0}^{a-1} \left(aqT_c\right)^i}{i!} - e^{aqT_c} - \sum_{i=0}^{a} \frac{(aqT_c)^i}{i!}
\]

(4.53)

For Erlang values of \( a = 1, 2, 3, \) and \( 4 \) the equations for \( Q_r \) become

\[
Q_{r,1} = \frac{q}{e^{aqT_c} - 1 - qT_c}
\]

(4.54)

\[
Q_{r,2} = \frac{q(1+2qT_c)}{e^{2aqT_c} - 1 - 2qT_c - 2(qT_c)^2}
\]

(4.55)

\[
Q_{r,3} = \frac{q(1+3qT_c + 4.5(qT_c)^2)}{e^{3aqT_c} - 1 - 3qT_c - 4.5(qT_c)^2 - 4.5(qT_c)^3}
\]

(4.56)
\[ Q_{r,4} = \frac{q \left( 1 + 4qT_c + 8(qT_c)^2 + 10.67(qT_c)^3 \right)}{e^{4qT_c} - 1 - 4qT_c - 8(qT_c)^2 - 10.67(qT_c)^3 - 10.67(qT_c)^4} \]  

(4.57)

Since the merging capacity \( Q_m \) is just the sum of the guideway volume, \( q \), and the ramp volume \( Q_r \), it becomes for \( a = 1 \)

\[ Q_{m,1} = \frac{q(e^{qT_c} - qT_c)}{e^{qT_c} - 1 - qT_c} \]  

(4.58)

Equations (4.54) and (4.55) are plotted in Figure 4-7(a) and Figure 4-7(b) for various values of \( T_c \).

While merging incorporates vehicles entering into a traffic stream from an entrance ramp, weaving is actually the crossing of two or more traffic streams traveling in the same direction. It can be thought of as more like merging into one stream and then diverging back into separate streams. When merging was considered, vehicles entering from the ramp (minor stream) were assumed to merge one at a time into the major stream. But in this case the weaving capacity \( Q_w \) will be developed allowing for platoons on both the major and minor traffic streams.

Consider the situation where there is a single lane of ramp traffic entering a single lane of guideway traffic, a portion of which will exit downstream. If the gap in the major stream is less than the critical gap, \( T_c \), then no ramp traffic will enter; if the gap is between \( T_c \) and \( T_c + T' \), one vehicle enters; if it is between \( T_c + T' \) and \( T_c + 2T' \), two vehicles enter; etc. The ability of the guideway lane to absorb ramp vehicles per unit time becomes:

\[ Q_r' = \sum_{i=0}^{\infty} (i + 1)P[T_c + iT' < t < T_c + (i + 1)T'] \]  

(4.59)

Take the distribution of the gaps on the guideway to conform to an Erlang distribution, and setting \( a = 1 \), results with

\[ Q_r' = \frac{q e^{-qT_c}}{1 - e^{-q\tau}} \]  

(4.60)

The weaving capacity \( Q_w \) is the sum of the guideway flow \( q \) and \( Q_r' \) which simplifies to
Guideway Ramp Capacity versus Guideway Volume

Figure 4-7
Now consider the case where $T_c = T'$. This would occur when weaving involves individual
vehicles instead of platoons but allowing multiple vehicles to enter one gap if it is of sufficient
size. Equations (4.60) and (4.61) become

$$Q_w = q \frac{1 - e^{-qT} + e^{-qT'}}{1 - e^{-qT'}} \quad (4.61)$$

$$Q_r = \frac{q e^{-qT_c}}{1 - e^{-qT_c}} \quad (4.62)$$

and

$$Q_w = \frac{q}{1 - e^{-qT_c}} \quad (4.63)$$

Figure 4-8 shows plots of Equations (4.62) and (4.63) for various values of $T_c$.

Suppose that all the guideway vehicles exit off the guideway while all the ramp vehicles enter
onto the guideway and do not exit. This would amount to the switching of lanes for all vehicles
entering the weaving area. The weaving ratio would then be

$$R = \frac{q}{Q'} \quad (4.64)$$

Substituting Equation (4.62) in Equation (4.64),

$$R = \frac{1 - e^{-qT_c}}{e^{-qT_c}} = e^{qT_c} - 1 \quad (4.65)$$

It follows that

$$e^{qT_c} = R + 1 \quad (4.66)$$

and

$$q = \frac{1}{T_c} \ln(R + 1) \quad (4.67)$$

By substituting Equation (4.67) in Equation (4.64) and solving for $Q'_{r}$

$$Q'_r = \frac{\ln(R + 1)}{RT_c} \quad (4.68)$$

It follows that
Weaving Capacities Based on Multiple Entries

Figure 4-8
\[ Q_w = \frac{(R+1)}{R} \frac{1}{T_c} \ln(R+1) \]  

(4.69)

\[ Q' \text{ and } Q_w \text{ have been plotted against } R \text{ for various values of } T_c \text{ in Figure 4-9.} \]

It is envisioned that weaving areas would consist of two parallel guideway lanes as shown in Figure 4-10. The required length of the merging area \( M \) depends on the width of a guideway lane, \( w \), the lateral velocity, \( v_l \), and the vehicle’s longitudinal velocity, \( v \). This relationship becomes

\[ M = \frac{w v}{v_l} \]  

(4.70)

Vehicles will go through the gap acceptance procedure before actually entering the merge area. Ideally the velocities of both traffic streams would be equal. The vehicles would go through any gap analysis procedure while still on the ramp, possibly waiting in a queue. Therefore an analysis of the interface between the freeway system and the guideway system should be done.

### 4.7 Guideway-Freeway Interface

As suggested in the previous section, under high guideway volumes where acceptable gaps are not readily available, the potential exists for a queue to develop on the entrance facility for the guideway. The potential is actually greater when considering the exit facility from the guideway onto the freeway. The average delay to a ramp vehicle in position to merge, \( d \), derived in Section 4.6, has been interpreted as the average service time for the queue of ramp vehicles, \( Q_r \) [19]. Letting \( f(t) \) be the distribution of service times at the head of the queue, the entrance ramp merging system can be considered a classical queuing system. Kendall [20] has derived the steady state equations for Poisson arrivals and arbitrary service time distribution as

\[ L = \rho + \frac{\rho^2 + q_r^2 \sigma^2}{2(1 - \rho)} \quad \rho < 1 \]  

(4.71)

where \( \rho = q_r/Q_r \) (\( q_r \) is the entrance ramp demand) and \( \sigma^2 \) is the variance of service times. If \( f(t) \) is a gamma distribution, then
Weaving Capacities Based on Weaving Ratio

Figure 4-9
GUIDEWAY WEAVING AREA

Figure 4-10
\[ \sigma^2 = \frac{a}{(aQ_r)^2} \]  

Substituting Equation (4.72) in Equation (4.71)

\[ L = \rho + \frac{\rho^2 (1 + a^{-1})}{2(1 - \rho)} \]  

where \( L \) is the expected queue length on the ramp, or the ramp queue length. In Fig. 4-11(a), (b), (c) and (d), \( L \) is plotted against \( q_r \) and \( Q_r \) for \( a = 1, 2, 3 \) and 4.

Of interest is when the ramp demand, \( q_r \), exceeds the ramp capacity, \( Q_r \) (\( \rho > 1 \)). Deterministic queuing is used to analyze the system. Lee [21] has shown that for \( \rho > 1 \) \( L \) is given by

\[ L = \frac{(cq_r - Q_r)T_s}{2} \quad q_r < Q_r \]  

where \( c \) is the ratio of the overload demand to the normal demand, \( q_r \), and \( T_s \) is the duration of the overload. In the context of Equation (4.73)

\[ \rho = \frac{cq_r}{Q_r} \quad q_r < Q_r \]  

giving

\[ L = \frac{(\rho - 1)Q_r T_s}{2} \quad \rho < 1 \]  

Letting \( \rho = q_r / Q_r \) (\( q_r > Q_r \))

\[ L = \frac{(q_r - Q_r)T_s}{2} \]  

Equation (4.77) is plotted in Figure 4-11 in dotted lines.

While some queuing is bound to occur at some point in time, an event such as when \( q_r > Q_r \) can create serious repercussions through the freeway network much in the same way overloaded freeway ramps spill onto arterials now. Especially with an exit ramp queue backing on the guideway, serious reductions of the benefits of the AHS could ultimately lead to the questioning of the entire system. But proper planning for adequate queue storage areas, both for entrance and exit ramps, can allow for the efficient and safe operation of the AHS.
Ramp Queue Lengths

Figure 4-11