5. Network Analysis

5.1 Introduction

With the continued growth of this country as it enters the next century comes the inevitable increase in the number of vehicles trying to use the already overtaxed transportation network. Since it is evident that our current practices of traffic management and the construction of additional lanes of highway can not hope to relieve the congestion being caused, another solution is needed. And AHS is that solution.

Assume by the middle of the next century an entire AHS Maglev network has been built as depicted in Figure 3-2. As the Interstate Highway System has over the last 20 years, growth patterns of both existing cities and new population centers will be centered around the AHS and its interchanges. With these changes will come new traffic patterns based around the guideways and the guideway-freeway interchanges. It is therefore of interest to see how the traffic will respond to the new transportation network.

Traditionally an analysis of a highway network is completed using what is known as the Urban Transportation Planning Process (UTPP). This process consists of four steps – trip generation, trip distribution, modal split, and traffic assignment. Trip generation is concerned with estimating the number of trip productions and trip attractions each node, or zone, produces. This depends heavily on the socio-economic characteristics of the population utilizing the network, and is therefore outside the scope of this work. Thus trip generation will not be considered and trips will be loaded onto the network at interchanges using arbitrary volumes. This should not be a problem as the goal is to get a feel for how the new network will behave rather than to model an actual system. As for modal split, this is actually taken care of by the network itself. Recall through Figure 3-2 that separate guideway and freeway lanes are provided for both cars and trucks and buses. By having the car traffic separated from the truck and bus traffic, modal split is actually performed between the respective roadways.
This leaves trip distribution and traffic assignment. Trip distribution is used to find out where the trips loaded at each production node are destined. This can then be used to estimate the number of trips on a link if performed along a single route (i.e. allowing no route choice). Traffic assignment is used to decide which route a trip will be made on. Using one of two available methods – user equilibrium and system optimization – the link volumes over an entire network can be found. In the case of an AHS parallel to the existing Interstate Highway System, the results of trip distribution and traffic assignment are of interest.

5.2 Trip Distribution

Here trip distribution will be used to determine what the link volumes between interchanges on the AHS guideway would be. Assume that interchanges along the AHS are equally spaced and \(2Q_r\) vehicle trips per hour are generated at each interchange – \(Q_r\) in each direction. Recall that \(Q_r\) is the ramp capacity of an interchange, found in Section 4.4. The ramp traffic then proceeds to travel downstream and exits at its destination interchange. The number of trips that exit at each interchange is inversely proportional to the number of interchanges between the origin and destination interchanges (see Figure 5-1). This can be mathematically expressed as

\[
Q_r = q_1 + q_2 + q_3 + \ldots + q_n = \sum_{k=1}^{n} q_k
\]

(5.1)

where

\[
q_1 = \frac{q_1}{k_x}
\]

(5.2)

and \(x\) is a measure of the importance of proximity to the traveler and is a function of the trip purpose. Substituting Equation (5.2) into Equation (5.1) and solving for \(q_1:\)

\[
q_1 = \frac{Q_r}{\sum_{k=1}^{n} k^{-x}}
\]

(5.3)

The link volume, \(q\), between adjacent interchanges can be expressed as
Trip Distribution Analysis

Figure 5-1
\[ q = \sum_{k=1}^{n} k q_k \]  

(5.4)

Then by substituting Equation (5.2) into Equation (5.4)

\[ q = q_1 \sum_{k=1}^{n} k^{1-x} \]  

(5.5)

Substituting Equation (5.3) into Equation (5.5) finally results in

\[ q = Q_r \frac{\sum_{k=1}^{n} k^{1-x}}{\sum_{k=1}^{n} k^{-x}} \]  

(5.6)

In Figure 5-2, \( q/Q_r \) is plotted against \( n \) for various values of \( x \).

It can be seen from Figure 5-2 that the guideway link volume \( q \) increases as the length of trip \( n \) increases. This is especially true for low values of \( x \). As said before, \( x \) is a measure of the importance of proximity to the driver, and is a function of trip purpose. For example, if the trip is for work, a person is just as willing to drive a long distance as a short one – i.e. the proximity to the workplace is not an issue. Meanwhile, for shopping a person is much more likely to go to a nearby location than to drive a long distance to shop. Therefore proximity is an issue when discussing a shopping trip. To represent these differences, each trip type is given values of \( x \) which best demonstrate its pattern. Historically, \( x < 1 \) has been used for work trips; \( 1 \leq x \leq 2 \) for school and shopping trips; and \( x > 2 \) for social and recreational trips. By examining the equation for \( q/Q_r \), obtained from Equation (5.6), the requirement for convergence can be seen to be \( x \geq 2 \).

Now look at the case where there is a minimum trip length of \( a \) interchanges on the guideway. Then

\[ Q_r = q_a + q_{a+1} + q_{a+2} + \ldots + q_n = \sum_{k=a}^{n} q_k \]  

(5.7)

where
Volume Ratio versus Length of Trip

Figure 5-2
\[ q_k = \frac{q_a}{(k - a + 1)^x} \]  

(5.8)

Then the link volume, \( q \), between interchanges can be expressed as

\[ q = \sum_{k=a}^{n} k q_k \]  

(5.10)

By substituting Equation (5.8) into Equation (5.10)

\[ q = q_a \sum_{k=a}^{n} \frac{k}{(k - a + 1)^x} \]  

(5.11)

Substituting Equation (5.9) into Equation (5.11) finally gives

\[ q = Q_r \frac{\sum_{k=a}^{n} k}{\sum_{k=a}^{n} (k - a + 1)^{-x}} \]  

(5.12)

In Figure 5-3, \( q/Q_r \) is plotted against \( n \) for various values of \( a \) using \( x = 2 \).

While trip distribution was able to find what the maximum link volumes would be depending on what the ramp capacity is, there is no way to determine whether the vehicle trips would use the guideway over the freeway, especially for short trips. Therefore traffic assignment will be used to determine how vehicle trips will distribute themselves over the guideway and freeway for a trip with the same origin and destination.

5.3 Traffic Assignment

While in the previous section it was assumed that the volume of traffic entering the guideway was equivalent to the ramp capacity, the actual volume which will access the guideway will depend on the volume of traffic entering the guideway-freeway system and how these vehicles assign themselves to either the freeway or the guideway to reach their ultimate destination. This assignment decision is based mostly on travel time which is dependent on the characteristics of the transportation network. This section will demonstrate the interactions and equilibrium
Relationship Between Volume Ratio and Length of Trip

Figure 5-3
between the guideway and the freeway for both cars and trucks, and will show the dynamic character of the system that causes adjustments to establish new equilibrium states based on controlled changes to land-use activity and/or to the guideway-freeway interface.

In relation to the proposed guideway-freeway system, traffic assignment becomes the splitting of the entrance ramp volumes at each interchange into the guideway entrance ramp volume, $Q_r^g$, and the freeway entrance ramp volume, $Q_r^f$, so that

$$Q_r^g + Q_r^f = Q_r$$  \hspace{1cm} (5.13)

This route choice is a classic equilibrium problem, since travelers’ route choice decisions are primarily a function of route travel times, which are determined by traffic flow. Traffic flow is itself a product of route choice decisions. This relationship between travel time and traffic volume, called the highway supply function, has many mathematical forms that have evolved over the years.

Two theoretical approaches to obtaining a mathematical expression of the highway supply function are: (1) continuous flow phenomenon such as in a one-dimensional compressible fluid, and (2) discrete flow using queuing theory as a foundation. The former gives rise to a relationship between speed and flow on a conventional roadway that is parabolic in nature, with an increasing reduction in speed as the volume approaches the roadway’s capacity. However, when the reciprocal of speed is taken, it has been found that the travel time found is significantly underestimated.

Blunden [22] has applied the second approach to the development of the following supply function:

$$T = T_f \left( \frac{1-(1-j)\rho}{1-\rho} \right)$$  \hspace{1cm} (5.14)

where $T$ is the travel time, $T_f$ is the free-flow travel time, $j$ is the level of service factor, and $\rho$ is the volume to capacity ratio, $q/Q$. In relation to the proposed system, $T$ and $T_f$ would be travel times between interchanges.
In applying the above model, consider the following scenario. Both the guideway and freeway lanes run parallel thus providing travelers with the choice of two trips. Assume that the minimum length of a guideway trip is $a = 3$. The section being analyzed will consist of four interchanges with eastbound traffic entering at interchange 0 destined for interchange 2 and westbound traffic entering at interchange 3 destined for interchange 1. A 50-50 directional split will be assumed, so that the number of vehicles traveling from interchange 0 to interchange 2 is equal to the number of vehicles traveling from interchange 3 to interchange 1. The choice for eastbound travelers is either to travel links 0-1 and 1-2 on the freeway or 0-1, 1-2, and 2-3 on the guideway and then 3-2 on the freeway. The choice for vehicles entering at interchange 3 and traveling west is similar, but reversed. Figure 5-4 sketches out the hypothetical network.

Of interest in the scenario is to find the link volumes using two theories of travel route choice: user equilibrium and system optimization. With user equilibrium a traveler selects his or her route by which route would have the shortest travel time. The affect on the other users of the system is not considered. The ultimate result is that as long as vehicles are using all available routes the travel times on all those routes would be equal. Under system optimization the traveler is assigned to the path which increases the total system travel the least. In this instance the vehicle would not necessarily be assigned to the route with the best travel time for itself as in user equilibrium. Also, the travel times on different routes do not have to be equal like in user equilibrium, even if vehicles are loaded onto all the available routes. The travel times which will be used in the example originate from the highway supply function shown in Equation (5.14), and are:

\[
T_c^f = T_f^f \left( \frac{Q_c^f - (1 - j_c^f)q_c^f}{Q_c^f - q_c^f} \right) \tag{5.15}
\]

\[
T_c^g = T_f^g \left( \frac{Q_c^g - (1 - j_c^g)q_c^g}{Q_c^g - q_c^g} \right) \tag{5.16}
\]

\[
T_t^f = T_f^f \left( \frac{Q_t^f - (1 - j_t^f)q_t^f}{Q_t^f - q_t^f} \right) \tag{5.17}
\]
Sample AHS Network

Figure 5-4
$$T^g_i = T^g_j \left( \frac{Q^g_i - (1 - j^g_i)q^g_i}{Q^g_i - q^g_i} \right)$$

(5.18)

where

- $T^j_i$ = travel time on route $j$ by vehicle type $i$
- $T^f_j$ = free-flow travel time on route $j$
- $Q^j_i$ = capacity of route $j$ for vehicle type $i$
- $q^j_i$ = volume of vehicle type $i$ on route $j$
- $j^j_i$ = level of service factor for route $j$ for vehicle type $i$

The indices $i$ and $j$ can be classified as follows: vehicle type $i$ can be either car ($c$) or truck ($t$), and route $j$ can be either freeway ($f$) or guideway ($g$).

The first point to notice about the system is the symmetry of the network (see Figure 5-4). Because loading is identical at both interchange 0 and interchange 3, the link volumes throughout the system will be symmetric. That is, the volume on freeway link 0-1, $q^f_{0-1}$, is equal to the volume on freeway link 3-2, $q^f_{3-2}$. Also note that the link volumes on the entire guideway in both directions are equal. Because of this and that the minimum trip on a guideway is three interchanges, the guideway links 0-1, 1-2, and 2-3 will be considered as one link, 0-3. Likewise in the other direction the guideway will be called link 3-0. By examining the symmetry of the network, the following relations can be found:

$$q^g_{0-3} = q^g_{3-0} = Q^g_r$$

(5.19)

$$q^f_{1-2} = q^f_{2-1} = Q^f_r$$

(5.20)

$$q^f_{0-1} = q^f_{3-2} = Q^f_r + Q^g_r$$

(5.21)

$$q^f_{1-0} = q^f_{2-3} = 0$$

(5.22)

Due to this symmetry, all of the following work will be done with traffic traveling in the eastbound direction, namely from interchange 0 to interchange 2. The reader should realize that all the results obtained hereafter apply in the westbound direction also.

First the theory of user equilibrium will be explored. As stated before, the concept is to balance the travel times on the two routes so that they will be equal. Therefore the travel time along the
guideway link 0-3 plus the freeway link 3-2 must be equal to the travel time along the freeway link 0-1 plus the freeway link 1-2. Mathematically, it would be shown as

\[ T_{0-3}^g + T_{3-2}^f = T_{0-1}^f + T_{1-2}^f \]  \hspace{1cm} (5.23)

Note from Equation (5.21) that \( T_{3-2}^f = T_{0-1}^f \) so that we end up with

\[ T_{0-3}^g = T_{1-2}^f \]  \hspace{1cm} (5.24)

for user equilibrium to exist. To model this system vehicles were entered into the network singly, each one checking what the existing travel times were at that time with the traffic loading that was present from those who entered the system beforehand. For example, if a vehicle prepares to enter the system and finds that the guideway route has the shorter travel time, that vehicle is added to the volume on that route. The next vehicle is then prepared to be loaded. When the travel times are checked, this time the freeway route may have the shorter travel time as the vehicle just loaded has increased the guideway volume and therefore the guideway travel time.

This model was run for both car and truck systems using the following values for the freeway and guideway characteristics: for cars \( T_{c,f}^f = 6.0 \) min., \( Q_{c,f}^f = 2400 \) veh/hr, \( j_{c,f}^f = 0.25 \), \( T_{c,g}^g = 3.6 \) min., \( Q_{c,g}^g = 30000 \) veh/hr, and \( j_{c,g}^g = 0.10 \); for trucks \( T_{t,f}^f = 6.0 \) min., \( Q_{t,f}^f = 1200 \) veh/hr, \( j_{t,f}^f = 0.50 \), \( T_{t,g}^g = 5.4 \) min., \( Q_{t,g}^g = 12000 \) veh/hr, and \( j_{t,g}^g = 0.20 \). The results of the model were outputted as plots of the guideway volume \( Q_r^g \) and the freeway volume \( Q_r^f \) versus the total volume \( Q_r \) for cars in Figure 5-5 and trucks in Figure 5-6. By setting the guideway travel time to free flow travel time (the freeway travel time when there are no vehicles loaded) and solving for the guideway volume it was found that

\[ Q_r^g = \frac{Q^f \left( \frac{T_{f}^f}{T_{g}^g} - 1 \right)}{\left( \frac{T_{f}^f}{T_{g}^g} - 1 + j^g \right)} \]  \hspace{1cm} (5.25)

before any vehicles begin to choose the freeway as a route. As a note, the model was set up so that if a vehicle was to encounter identical travel times on the two routes, the vehicle would select the route with the shorter distance, namely the freeway route. Using the above
Traffic Assignment of Cars Under User Equilibrium Conditions

Figure 5-5
Traffic Assignment of Trucks Under User Equilibrium Conditions

Figure 5-6
parameters, it was found that this required guideway loading was $Q_r^g = 26087$ veh/hr for cars and $Q_r^s = 4286$ veh/hr for trucks.

Once the loading on the guideway reaches that point, the trips begin to be distributed among both routes as can be seen in Figures 5-5 and 5-6. Also by examining the graphs one will notice that once loading has begun on both the routes, the pattern appears to follow a quadratic form. It would be of interest to find the function for which this loading follows to help gain an understanding of the relationships between the guideway and freeway ramp volumes, $Q_r^g$ and $Q_r^f$, and the total ramp volume, $Q_r$. Three forms of equations were fitted:

$$q_j = a + bq$$  \hspace{1cm} (5.26)

$$q_j = a + bq + cq^2$$  \hspace{1cm} (5.27)

$$q_j = a + bq^2$$  \hspace{1cm} (5.28)

where $j$ indicates the route type. These equations were fit using the least squares method with the results being shown in Appendix A in Figures A-1 through A-6 for cars and Figures A-7 through A-12 for trucks. The regression curves are plotted as a dashed line while the actual data curve is shown as a solid line. Upon examining the curves one will recognize that the fitted curves created by Equation (5.27), in Figures A-2, A-5, A-8, and A-11, best estimate the curves created by the model. Statistical analysis of the regression curves confirmed that the form of Equation (5.27) best explained the relationship between the guideway and freeway volumes and the total volumes. The four equations for this network are

$$q_c^f = -28968.7 + 1.70048q - 2.261 \times 10^{-5} q^2$$  \hspace{1cm} (5.29)

$$q_c^g = 28968.7 - 0.700479q + 2.261 \times 10^{-5} q^2$$  \hspace{1cm} (5.30)

$$q_f^f = -169.780 + 0.0123851q + 0.674 \times 10^{-5} q^2$$  \hspace{1cm} (5.31)

$$q_f^g = 169.780 + 0.987615q - 0.674 \times 10^{-5} cq^2$$  \hspace{1cm} (5.32)

where Equations (5.29) and (5.30) are for car volumes on the freeway and guideway, respectively, and Equations (5.31) and (5.32) are for truck volumes on the freeway and guideway, respectively. It is interesting to note how once loading begins on both routes, the freeway appears to get more loading relative to its capacity than does the guideway. But as
loading continues to increase the volumes closer to capacity, the guideway again takes most of the traffic loading.

While as stated above the higher order models shown in Equations (5.29) through (5.32) are statistically more significant, they don’t provide a practical relationship for use. It is therefore useful to have the linear relationships available for use as they are also much more appealing conceptually. The linear form for cars is converted to the form

$$q_c' = (q_c - 25700) \times 0.378$$ \hspace{1cm} (5.33)

and is plotted in Figure 5-7. The linear form for trucks is converted to the form

$$q_t' = (q_t - 4800) \times 0.122$$ \hspace{1cm} (5.34)

and is plotted in Figure 5-8. The equations suggest that traffic begins to divert from the two guideways when traffic volumes on the guideways reach 25700 cars per hour and 4800 trucks per hour.

While user equilibrium is an interesting case to examine – in fact necessary to examine since it is in exactly this manner that today’s roadways are loaded – the introduction of automation to the transportation system provides the opportunity to actually put system optimal operations into affect. System optimal operations actually attempt to achieve the lowest possible travel time for the transportation system as a whole. The problem with implementing this in the past has been the requirement that some drivers actually access the route with the longer travel time, something that is hard, if not impossible, to get done. With the automated vehicles it is possible to do just that and ultimately achieve a system optimal operation. When modeling the user equilibrium scenario the decision process upon entering the system was straight-forward – the vehicle was assigned the route with the lowest travel time. With system optimal operations the decision process becomes more complicated. Each vehicle must analyze the system and decide which route would increase the total travel time by the least amount if it was to join it. To develop the functions needed to allow a comparison to be made, start with the minimization function:
Freeway Traffic Volume as a Function of the Total Traffic Volume For Cars Under User Equilibrium Conditions

Figure 5-7
Freeway Traffic Volume as a Function of the Total Traffic Volume For Trucks Under User Equilibrium Conditions

Figure 5-8
\[
\min T = \sum_{i=1}^{n} t_i q_i
\]  \hspace{1cm} (5.35)

where \(i\) is the route and \(n\) is the number of routes in the network, in this case two. The highway supply functions from Equations (5.15) and (5.16) are inputted for \(t\), thus giving

\[
T = T_f \left( \frac{Q_f - (1 - j_f) q_f}{Q_f - q_f} \right) q_f + T_g \left( \frac{Q_g - (1 - j_g) q_g}{Q_g - q_g} \right) q_g
\]  \hspace{1cm} (5.36)

Taking the partial derivatives of Equation (5.36) in terms of \(q_f\) and \(q_g\) and simplifying produces

\[
\frac{\partial T}{\partial q_f} = T_f \left( \frac{Q_f - 2(1 - j_f) q_f}{Q_f - q_f} + q_f \left( \frac{Q_f - (1 - j_f) q_f}{(Q_f - q_f)^2} \right) \right)
\]  \hspace{1cm} (5.37)

\[
\frac{\partial T}{\partial q_g} = T_g \left( \frac{Q_g - 2(1 - j_g) q_g}{Q_g - q_g} + q_g \left( \frac{Q_g - (1 - j_g) q_g}{(Q_g - q_g)^2} \right) \right)
\]  \hspace{1cm} (5.38)

These two equations become the decision functions for the system optimal loading. Equations (5.37) and (5.38) determine the change in \(T\) in terms of \(q_f\) and \(q_g\). The route with the smallest change is the one which will receive the next vehicle entering the system.

As with user equilibrium, the loads on both the guideway and freeway routes are plotted against the total traffic volume, in Figure 5-9 for cars and Figure 5-10 for trucks. The values used in the user equilibrium example are also used here. As expected, the traffic initially enters onto the guideway and continues to do so until the system travel time change is equal for both routes. To find this point Equation (5.37) was set equal to six (the value of Equation (5.38) with \(q_f = 0\)). Upon solving the resulting quadratic traffic volumes of \(q_g = 19165\) veh/hr for cars and \(q_g = 23779\) veh/hr for trucks were found to satisfy the conditions. Again the volume must reach large values before any loading will even begin on the freeway. Once loading does begin on both routes simultaneously, the load curves again resemble a parabolic curve. Due to the similarities between the nature of the curves found under user equilibrium and system optimization the same basic forms, Equations (5.26), (5.27), and (5.28), were used to fit these curves. Again the least squares method of regression was applied with the results being shown in Figures A-13 through A-18 for cars and Figures A-19 through A-24 for trucks. Upon examining the curves, and
Traffic Assignment of Cars Under System Optimal Conditions

Figure 5-9
Traffic Assignment of Trucks Under System Optimal Conditions

Figure 5-10
through statistical analysis, the regression curves portrayed in Figures A-14, A-17, A-20, and A-23 did the best job of explaining the relationship between the freeway and guideway volumes, \( q_f \) and \( q_g \), and the total traffic volume, \( q \). The resulting equations are

\[
q_c^f = -8010.65 + 0.572217q - 0.801 \times 10^{-5}q^2 \tag{5.39}
\]

\[
q_c^g = 8010.65 + 0.427783q + 0.801 \times 10^{-5}q^2 \tag{5.40}
\]

\[
q_t^f = -125.907 + 0.0423544q + 0.459 \times 10^{-5}q^2 \tag{5.41}
\]

\[
q_t^g = 125.907 + 0.957646q + 0.459 \times 10^{-5}q^2 \tag{5.42}
\]

where Equations (5.39) and (5.40) are for car volumes on the freeway and guideway, respectively, and Equations (5.41) and (5.42) are for truck volumes on the freeway and guideway, respectively. One will notice that again the freeway takes on most of its loading in the beginning and begins to level off at the end, while the guideway continuously increases the growth in traffic volume.

Again, while the higher order models are more statistically significant, the linear relationships have more conceptual appeal and are more practical for application and use. The linear relationship for cars is converted to the form

\[
q_c^f = (q_c - 18700) \times 0.203 \tag{5.43}
\]

and is plotted in Figure 5-11. The linear form for trucks is converted to the form

\[
q_t^f = (q_t - 2800) \times 0.099 \tag{5.44}
\]

and is plotted in Figure 5-12. The equations suggest that traffic begins to diverge from the guideway to the freeway when traffic volumes on the guideway reach 18700 cars per hour and 2800 trucks per hour under system optimal conditions.

Applying the two different theories of traffic assignment to the network brought one clear issue to the forefront – the volume needed to utilize both routes. The ability to obtain such an accurate curve when compared to the actual loading curve created by the models is a bit surprising but not unfounded. The model is ultimately based on a few equations which when compared with one another decided which route a given vehicle would be loaded onto. Because of this there is a strong relationship between the decision functions and the final output curve.
Freeway Traffic Volume as a Function of the Total Traffic Volume For Cars Under System Optimal Conditions

Figure 5-11
Freeway Traffic Volume as a Function of the Total Traffic Volume
For Trucks Under System Optimal Conditions

Figure 5-12
Since the curve was ultimately created by equations, it is reasonable to expect that there exists a function which would match its behavior extremely closely.