CHAPTER 3: Tube with Flat Length-to-Width Ratio of 2:1

3.1 Introduction

A tube with a flat length-to-width ratio of 2:1 was modeled. Parameters used in this study include:

Flat Length: $L_f = 3.0$m

Flat Width: $W_f = 1.5$m

Half Length: $a = 0.5L_f = 1.5$m

Half Width: $b = 0.5W_f = 0.75$m

Poisson's Ratio: $\nu = 0.45$

Thickness: t = 3mm

Young's Modulus: $E = 7.0346 \times 10^9$ Pa

Specific Weight: $\gamma_{slurry} = 1.5\gamma_{water}$

One-quarter of the tube was modeled. Two different soil stiffnesses were studied; one had a distributed spring stiffness, K_d, of 4.20×10^6 N/m^3, and the other had a distributed spring stiffness, K_d, of 8.40×10^4 N/m^3.

In this chapter, the influence of hydrostatic pressure on the geometry of the tube is investigated, including X-, Y-, and Z-translations. Also studied is the contact region between the ground and the tube, and the mid-surface stresses along the X- and Y-axes.

3.2 Distributed Spring Stiffness, K_d, of 4.20×10^6 N/m^3

3.2.1   Geometry

A quarter model with a tensionless distributed spring stiffness of $K_d = 4.20 \times 10^6$ N/m^3 was considered. The maximum hydrostatic pressure applied to this model at ground level (Z = 0) was 11,621 Pa. The pressure head was 0.79m from the reference ground level, so that at Z = 0.79m, the hydrostatic pressure was zero.

The model was first studied at an intermediate hydrostatic pressure. In this case, the hydrostatic pressure at ground level was 7355 Pa and the pressure head was 0.5m. Figure 3.2.1.1 shows the deflected shape of one-quarter of the tube at this pressure. The figure shows some wrinkling that occurs in the model. The top surface of the tube deflected upward 0.247m at the center from its original height of 0.0429m. The bottom surface deflected downward 1.79mm at the center from its original height of 0.0m.

This intermediate hydrostatic pressure of 7355 Pa was also applied to a tube with the same parameters as previously discussed except with a thickness, t, of 2.5mm. In this case, the tube deflected upward 0.270m at its center and downward 1.77mm. This indicates that reasonable changes in the tube's thickness do not have a significant impact on the tube's deflected shape.

The model with t = 3mm was analyzed at the applied hydrostatic pressure of 11,621 Pa. Figure 3.2.1.2 shows a three-dimensional plot of one-quarter of the tube. In this figure, the wrinkling is much more noticeable than in Figure 3.2.1.1. Cross-sectional plots of the deflected shape along the X- and Y-axes are shown in Figures 3.2.1.3a and 3.2.1.3b, respectively. At the center, the tube deflected upward 0.342m from its initial height of 0.0429m. The tube deflected downward at its center 2.84mm. The coordinates of the quarter tube's outer edge, in meters, at B_x, at B_y, and at the corner are (1.47, 0.0, 0.162), (0.0, 0.702, 0.143), and (1.48, 0.739, 0.171), respectively.

3.2.2 Contact Region

The area of contact between the tube and its elastic foundation is studied for the two hydrostatic pressure heads of 0.79m and 0.5m. Because the tube rests on a foundation modeled using tensionless springs, the springs are only active where there is downward deflection, i.e., where they are in compression. Therefore, to show the locations where the springs are active approximates the region of contact.

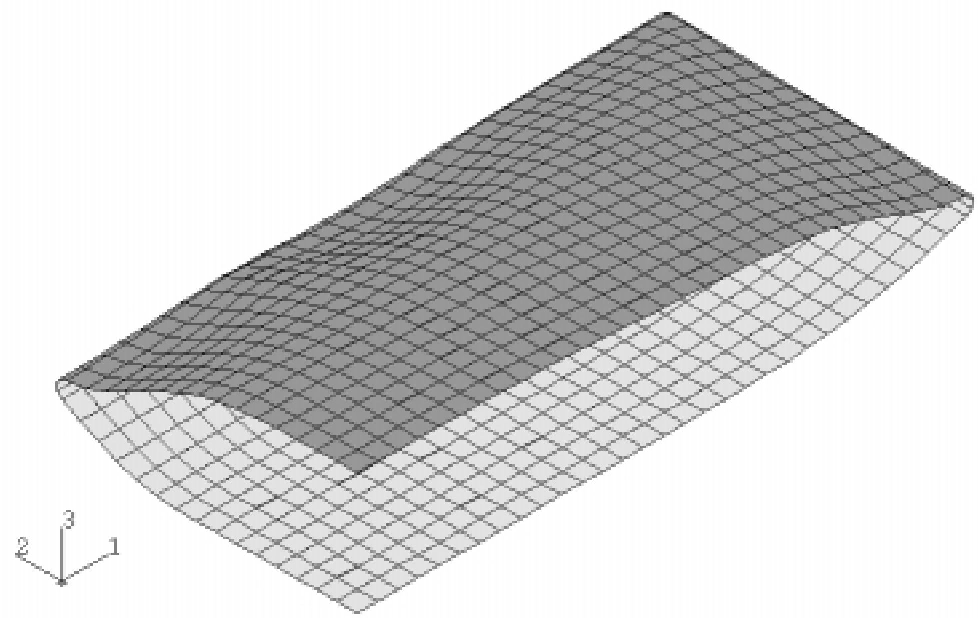


Figure 3.2.1.1: Pressure Head of 0.5m for K_d = 4.20 x 10^6 N/m^3 and L_f:W_f of 2:1

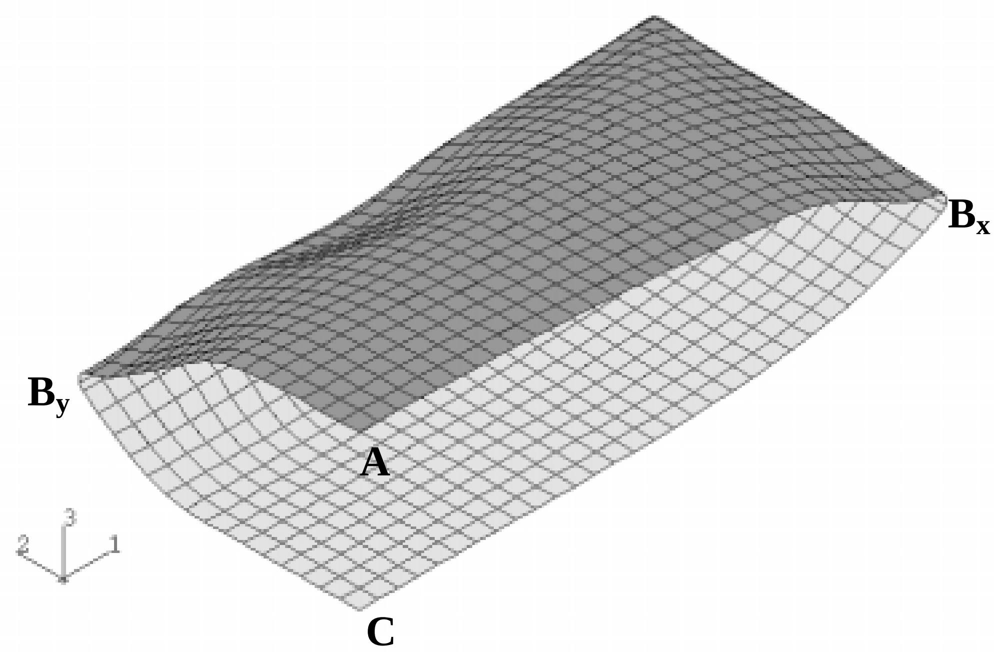
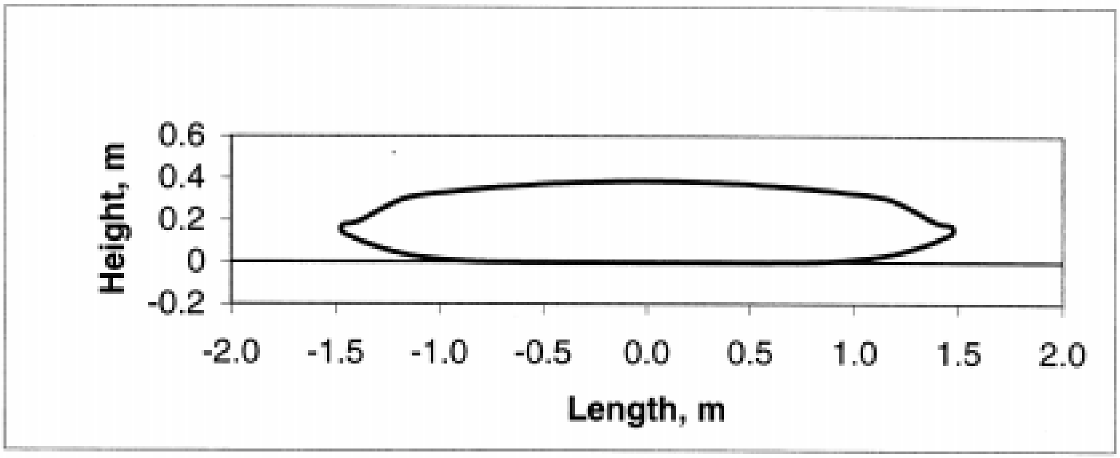
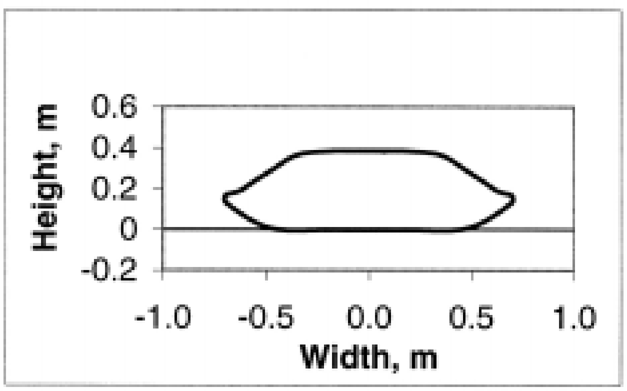


Figure 3.2.1.2: Pressure Head of 0.79m for K_d = 4.20 x 10^6 N/m^3 and L_f:W_f of 2:1



a: Along X-Axis.



b: Along Y-Axis.

Figure 3.2.1.3: Cross Sections of the Tube at a Pressure Head of 0.79m for $K_d = 4.20 \times 10^6$ N/m^3 and L_f:W_f of 2:1

Figure 3.2.2.1 shows the approximate contact regions for hydrostatic pressure heads of 0.79m and 0.5m for one-quarter of the tube. The circles denote the locations at which the springs are active for both pressure heads. The open squares denote where springs are active for the 0.5m pressure head case but not active for the 0.79m pressure head case. The plot shows that as the pressure head increases from 0.5m to 0.79m, the contact region decreases.

The contact region for the entire tube is shown in Figure 3.2.2.2 for a hydrostatic pressure head of 0.79m. The circles denote locations of active springs, i.e., springs which are in compression. As the figure shows, all of the points on the edge move toward the center, and the sides curve inward between the corners.

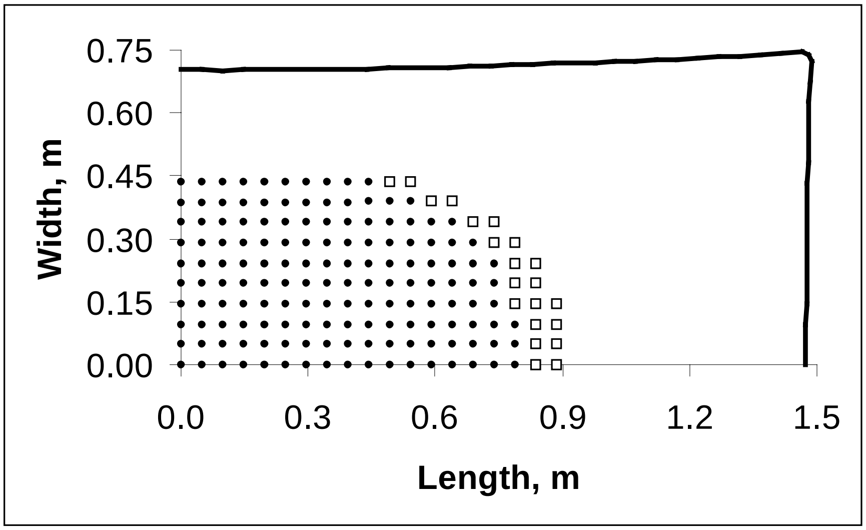


Figure 3.2.2.1: Quarter Contact Region at Pressure Heads of 0.5m and 0.79m for K_d = 4.20 x 10^6 N/m^3 and L_f:W_f of 2:1; circle: 0.5m and 0.79m, square: 0.5m

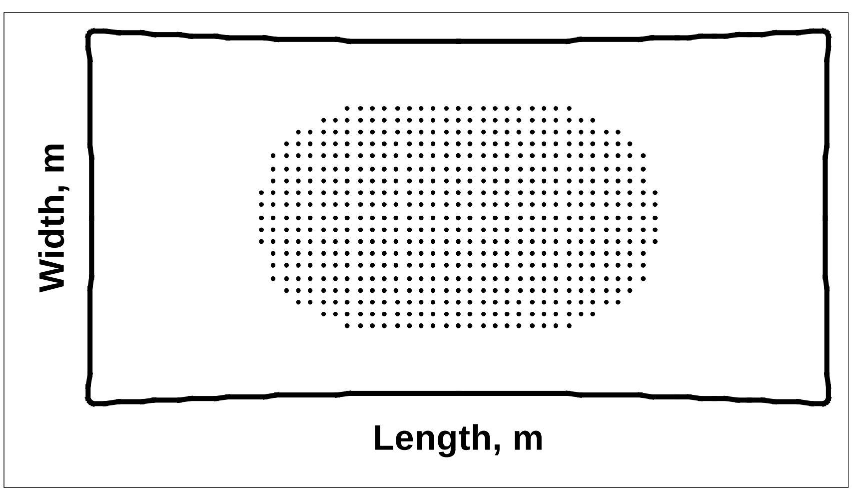


Figure 3.2.2.2: Whole Contact Region at a Pressure Head of 0.79m for K_d = 4.20 x 10^6 N/m^3 and L_f:W_f of 2:1

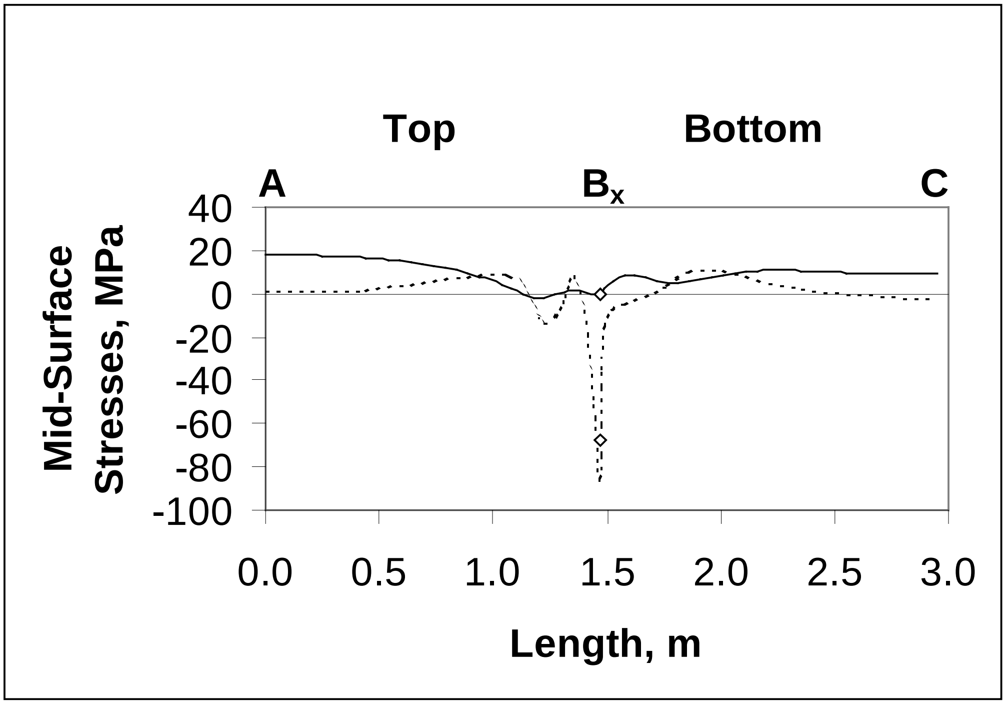
3.2.3 Mid-Surface Stresses

The mid-surface stresses are studied along the X- and Y-axes as shown in Figures 3.2.3.1a and 3.2.3.1b, respectively. In these figures, points A, B_x, B_y, and C correspond to those points shown in Figure 3.2.1.2. Point A refers to the top center of the tube, B_x to the edge of the tube along the X-axis, B_y to the edge of the tube along the Y-axis, and C to the bottom center of the tube. Therefore, the portion of the stress plot between A and B corresponds to the top of the tube, and the portion between B and C corresponds to the bottom of the tube. In Figure 3.2.3.1, the edge of the tube is denoted by an open diamond.

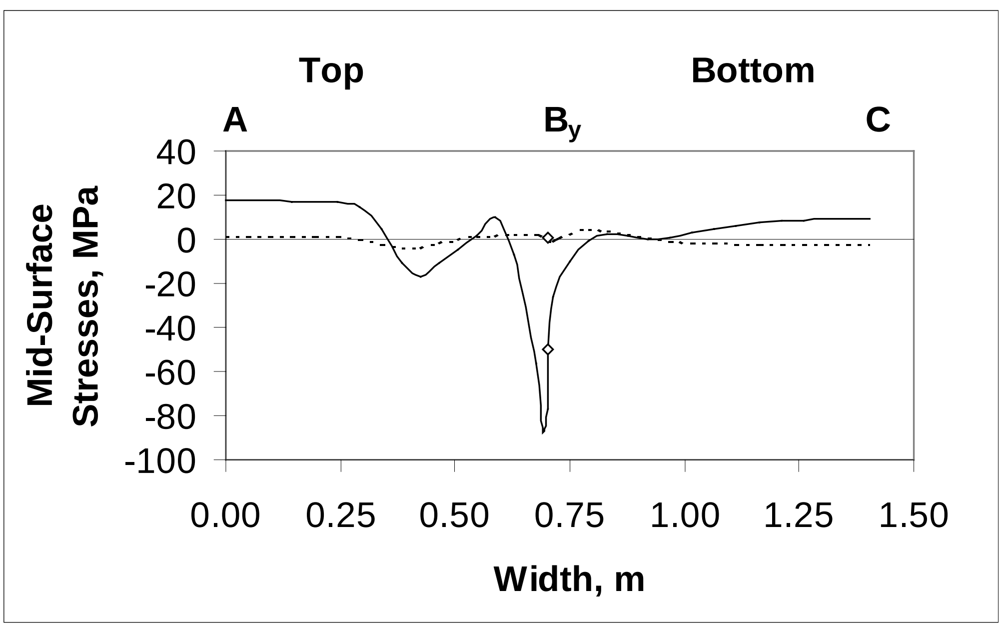
In Figure 3.2.3.1, the solid lines denote σ_{11}, and the dashed lines denote σ_{22}. For stresses along the X-axis, σ_{11} is the mid-surface in-plane stress, and σ_{22} is the stress which is normal to σ_{11} and parallel to the Y-axis. For stresses along the Y-axis, σ_{22} is the mid-surface in-plane stress, and σ_{11} is the stress which is normal to σ_{22} and parallel to the X-axis. The in-plane shear stresses along the X- and Y-axes are zero due to symmetry.

As Figure 3.2.3.1a shows, σ_{22} is almost zero near the center of the tube and is negative (compression) at the edge of the tube. Near the edge, on the top, the tube changes from being in tension to compression, back to tension, and into compression again. This indicates there is wrinkling in the structure, as is also shown in Figure 3.2.1.2.

In Figure 3.2.3.1a, σ_{11} is positive (tension) and relatively constant near the center for both the top and bottom surfaces, and goes to zero at the edge. Looking closely at σ_{11} near the edge, σ_{11} is zero at the top and positive (tension) on the bottom. This can be explained by the uplift that occurs on the bottom of the tube.



a: Along X-Axis.



b: Along Y-Axis

Figure 3.2.3.1: Mid-Surface Stresses at a Pressure Head of 0.79m for $K_d = 4.20 \times 10^6$ N/m^3 and $L_f:W_f$ of 2:1; σ_{11} solid, σ_{22} dashed

Figure 3.2.3.1b shows the mid-surface stresses along the Y-axis when X=0. Stress σ_{11} is basically constant and in tension near the center of the tube. On the top, σ_{11} changes from being in tension to compression, to tension again, and then back to compression at the edge. This indicates that there is wrinkling in the tube. Stress σ_{22} is almost zero on the top, on the bottom, and at the edge of the tube.

3.2.4 Pressure vs. Height

Figure 3.2.4.1 shows a plot of pressure vs. height for a distributed spring stiffness $K_d = 4.20 \times 10^6$ N/m^3. Several different hydrostatic pressure values were applied to the model; the hydrostatic pressure heads, measured from the reference ground level Z = 0, ranged from 0.0m to 0.79m. The values of the hydrostatic pressure at Z = 0 were plotted against the corresponding tube heights, measured from Z = 0, in order to obtain the plot shown. The pressure vs. height curve shows that the tube initially softens and then stiffens as the pressure increases.

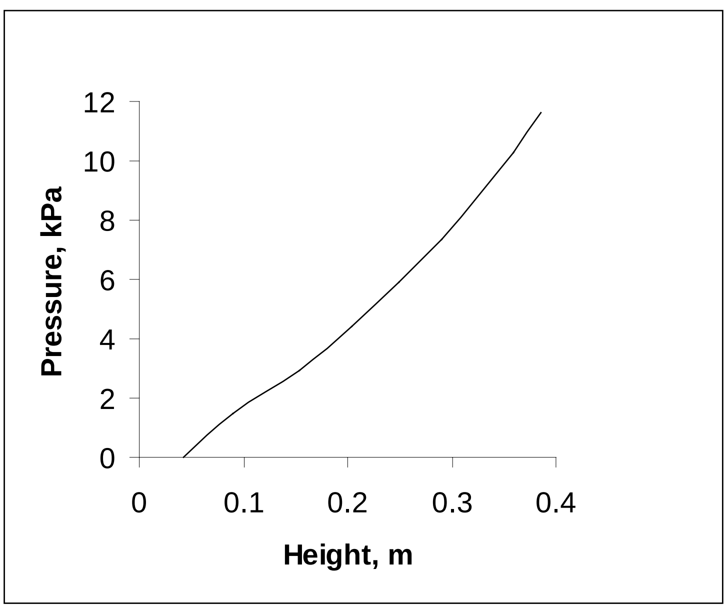


Figure 3.2.4.1: Pressure vs. Height for $K_d = 4.20 \times 10^6$ N/m^3 and L_f:W_f of 2:1

3.3 Distributed Spring Stiffness, K_d, of 8.40 x 10^4 N/m^3

3.3.1 Geometry

A quarter model was also studied with a distributed spring stiffness of K_d = 8.40 x 10^4 N/m^3, 50 times smaller than in Section 3.2. The maximum hydrostatic pressure applied to the tube in this case was 10,150 Pa at the reference ground level of Z = 0. The hydrostatic pressure head which corresponds to this pressure is 0.69m, such that at Z = 0.69m the pressure applied is zero.

Figure 3.3.1.1 shows a three-dimensional plot of the model. The top surface of the tube deflected upward 0.270m from its original height of 0.0429m. The bottom surface deflected downward 0.104m at the center from its initial height of 0.0m. The figure shows that wrinkling has occurred in the model.

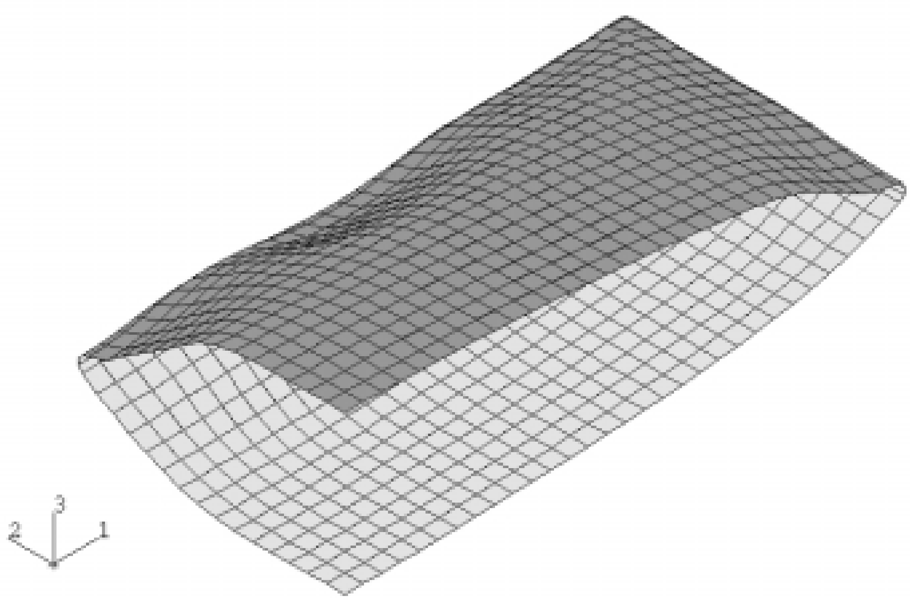
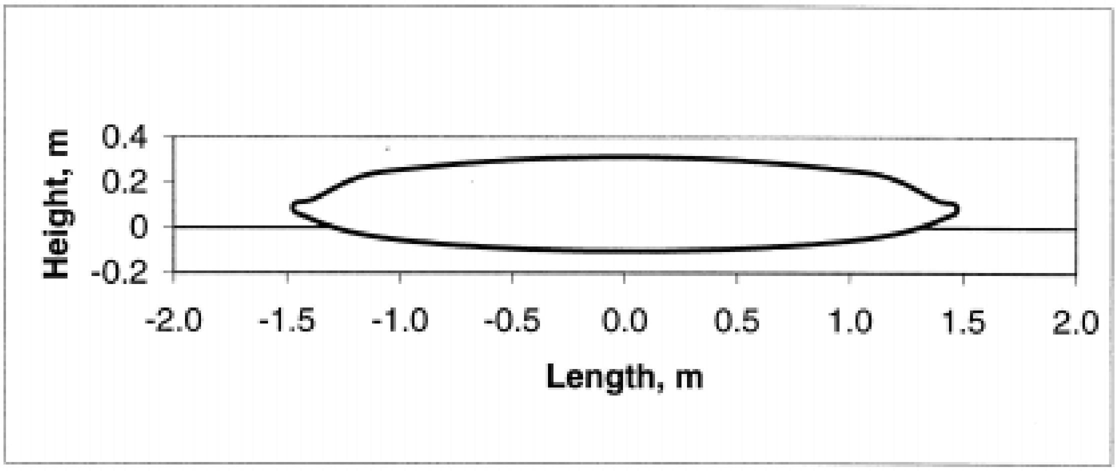
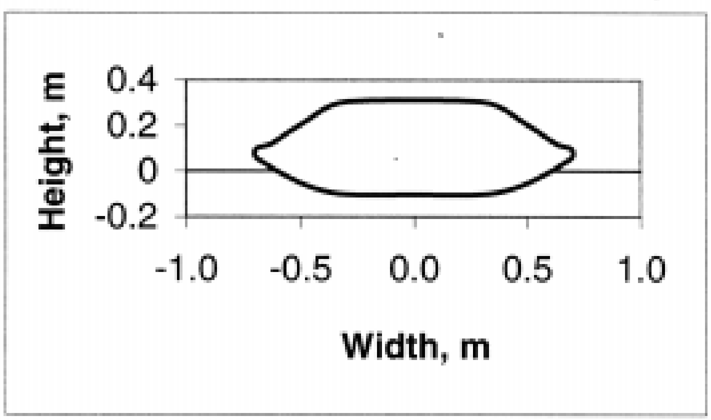


Figure 3.3.1.1: Pressure Head of 0.69m for K_d = 8.40 x 10^4 N/m^3 and L_f:W_f of 2:1

Figures 3.3.1.2a and 3.3.1.2b show cross sections of the deflected tube along the X- and Y-axes, respectively. The coordinates of the quarter tube's edge, in meters, at B_x, B_y, and its corner are (1.48, 0.0, 0.0907), (0.0, 0.702, 0.0711), and (1.48, 0.739, 0.0980), respectively.



a: Along X-Axis



b: Along Y-Axis

Figure 3.3.1.2: Cross Sections of the Tube at a Pressure Head of 0.69m for $K_d = 8.40 \times 10^4$ N/m^3 and L_f:W_f of 2:1

3.3.2 Contact Region

Figure 3.3.2.1 shows the region of contact for a quarter tube with hydrostatic pressure heads of 0.5m and 0.69m. In this plot, circles denote locations where the springs are active for a pressure head of 0.5m and a pressure head of 0.69m. Open squares denote the location of springs which are active for a pressure head of 0.5m but not for a pressure head of 0.69m. Open triangles signify locations at which a spring is active for a 0.69m pressure head but not for a 0.5m pressure head. This plot shows that the contact region decreases as the pressure head increases from 0.5m to 0.69m except where the triangle is located. At the location of the triangle, the tube changed from being in contact at a 0.0m pressure head, to not being in contact at a 0.5m pressure head, to being in contact again at a pressure head of 0.69m. When moving along the X-axis when Y = 0.568m, the tube changes from being in contact, to not being in contact, to being in contact again, to not being in contact again; this is due to the wrinkling in the model.

Figure 3.3.2.2 shows the contact region for the whole tube when a pressure head of 0.69m is applied to the model. As the figure shows, all of the points on the edge move toward the center, and the sides curve inward between the corners.

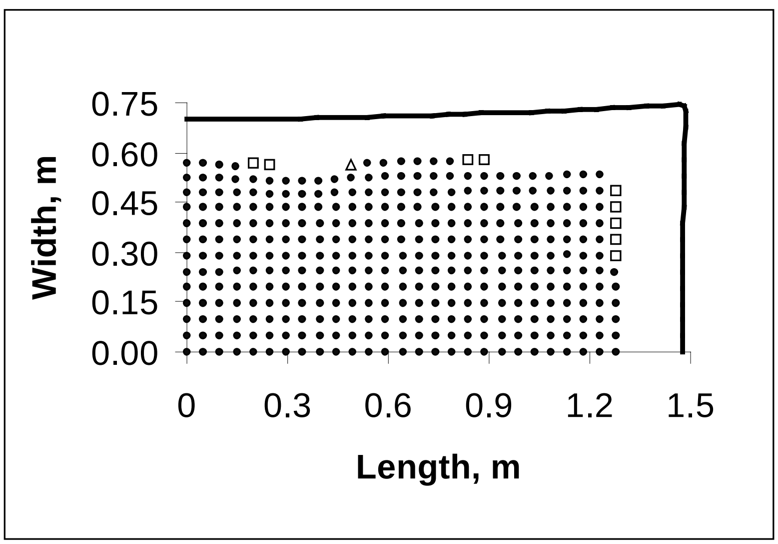


Figure 3.3.2.1: Quarter Contact Region at Pressure Heads of 0.5m and 0.69m for K_d = 8.40 x 10^4 N/m^3 and L_f:W_f of 2:1; circle: 0.5m and 0.69m, square: 0.5m, triangle: 0.69m

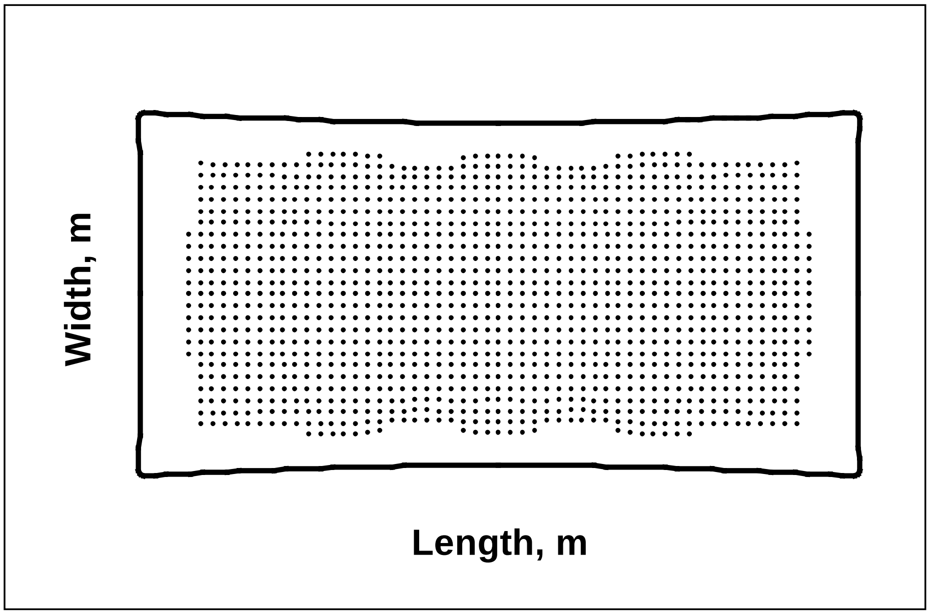
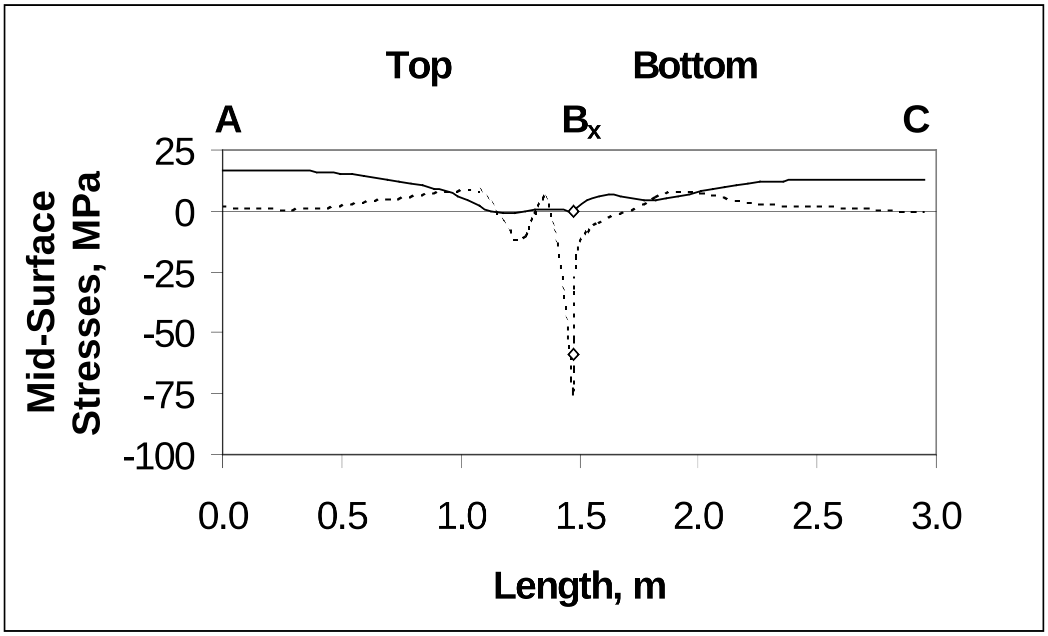


Figure 3.3.2.2: Whole Contact Region at a Pressure Head of 0.69m for K_d = 8.40 x 10^4 N/m^3 and L_f:W_f of 2:1

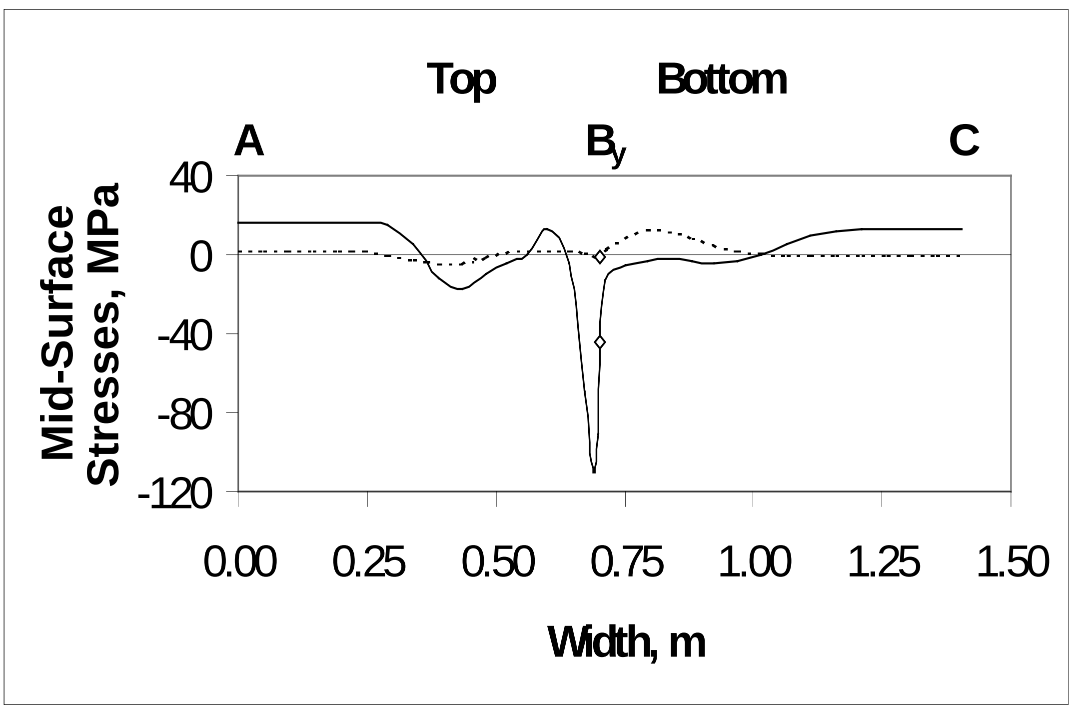
3.3.3 Mid-Surface Stresses

For K_d = 8.40 x 10^4 N/m^3, the mid-surface stresses are studied along the X- and Y-axes. Figures 3.3.3.1a and 3.3.3.1b show the mid-surface stresses along the X- and Y-axes, respectively.

The stresses in Figure 3.3.3.1a follow the same trend as those shown in Figure 3.2.3.1a. Stress σ_{11} is relatively constant and is positive (tension) near the center of the top surface and goes to zero near, and at, the edge. On the bottom, near the edge, the tube is in tension. This is where uplift occurred and the springs are inactive. Stress σ_{11} levels off to a constant tension value as it approaches the bottom center. Stress σ_{22} is almost zero near the center of both the top and bottom surfaces, but is negative (compression) at the edge. The top surface changes from being in tension to compression, to tension again, and back to compression at the edge. This is due to wrinkling.



a: Along X-Axis



b: Along Y-Axis

Figure 3.3.3.1: Mid-Surface Stresses at a Pressure Head of 0.69m for $K_d = 8.40 \times 10^4$ N/m^3 and L_f:W_f of 2:1; σ_{11} solid, σ_{22} dashed

Figure 3.3.3.1b shows the mid-surface stresses for the cross section when X = 0. The stresses shown in this figure follow the same trend as those shown in Figure 3.2.3.1b. Stress σ_{22} is almost zero near the center and edge of the tube. It is positive (tension) near the edge on the bottom where uplift has occurred. Stress σ_{11} is positive (tension) and constant near the center of both the top and bottom surfaces and is negative (compression) at the tube's edge. The top surface changes from being in tension to compression, to tension again, and back to compression at the edge due to wrinkling. Stress σ_{11} is almost zero on the bottom where uplift has occurred. Due to symmetry, the shear stresses along the X- and Y-axes are zero.

3.3.4 Pressure vs. Height

A pressure vs. height curve for $K_d = 8.40 \times 10^4$ N/m^3 is shown in Figure 3.3.4.1. It contains heights measured from the reference ground level Z = 0 for hydrostatic pressure heads ranging from 0.0m to 0.69m. Again, the curve shows that the tube is initially soft and becomes more stiff as the pressure increases.

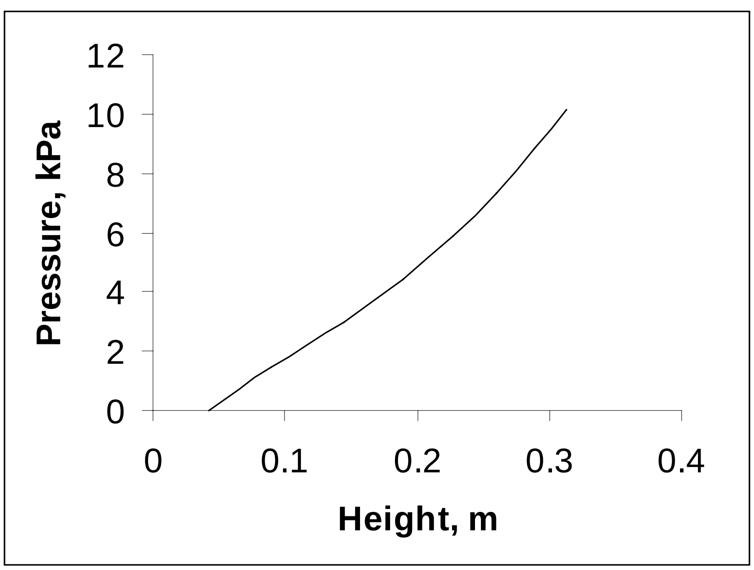


Figure 3.3.4.1: Pressure vs. Height for $K_d = 8.40 \times 10^4$ N/m^3 and L_f:W_f of 2:1

3.4 Discussion

The results obtained for the two distributed spring stiffnesses of $K_d = 4.20 \times 10^6$ N/m^3 and $K_d = 8.40 \times 10^4$ N/m^3 were compared for a length-to-width ratio of 2:1. As expected, the comparison showed that the contact region increases as the ground stiffness decreases (with one exception). The stresses along the X- and Y- axes follow the same pattern for both distributed spring stiffnesses, with the magnitudes at the edges varying by a small amount. The previous pressure vs. height plots for $K_d = 4.20 \times 10^6$ N/m^3 and $K_d = 8.40 \times 10^4$ N/m^3 are plotted together in Figure 3.4.1. The line representing the pressure vs. height plot for $K_d = 4.20 \times 10^6$ N/m^3 is dashed, and the line representing the plot for $K_d = 8.40 \times 10^4$ N/m^3 is solid. At a given pressure, the tube height, measured from Z = 0, for a lower distributed spring stiffness is less than that for a higher distributed spring stiffness.

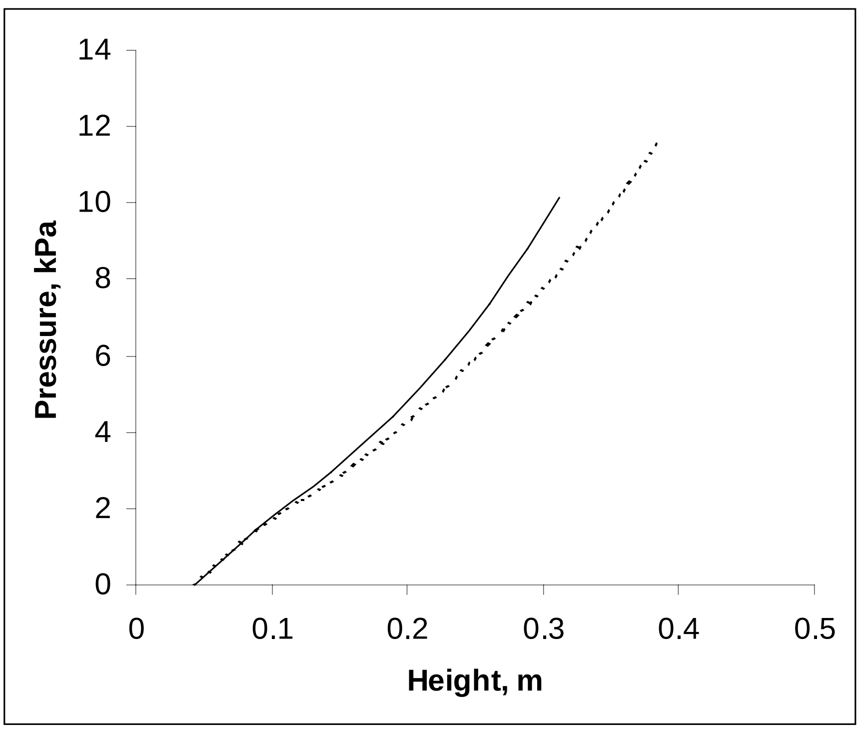


Figure 3.4.1: Pressure vs. Height for $K_d = 4.20 \times 10^6$ N/m^3 (dashed) and $K_d = 8.40 \times 10^4$ N/m^3 (solid) for L_f:W_f of 2:1