CHAPTER 6

FINAL SUMMARY

6.1 Introduction

This chapter presents a summary of the scaling techniques developed in this thesis. The first section is intended to be a simple, direct working guideline for those who wish to scale model and prototype PM d.c motors. A summary is also offered on scaling dissimilar motors in robotic manipulators. The chapter ends with a list of conclusions along with suggestions for future research. These final remarks should assist those wishing to apply dimensional analysis to electro-mechanical devices.

6.2 Summary: Techniques for Scaling a PM d.c. Motor

Scaling a PM d.c. motor is achieved by satisfying the five Pi terms below. Use of these terms assumes that the candidate motors are not controlled, and testing will be conducted by measuring position step responses. Furthermore, the terms presented here assume that the motors are geared and will be used in linear displacement applications.

$$\Pi_1 = \left[\left(\frac{T_C}{T_{gs}}\right)\left(\frac{D_1}{K_D}\right)\right]_{\text{model}} = \left[\left(\frac{T_C}{T_{gs}}\right)\left(\frac{D_1}{K_D}\right)\right]_{\text{prototype}}$$

$$\Pi_2 = \left[\frac{N_T T_{L_{pre-geared}}}{T_{gs}}\right]_{\text{model}} = \left[\frac{N_T T_{L_{pre-geared}}}{T_{gs}}\right]_{\text{prototype}}$$

$$\Pi_3 = \left[\frac{x}{N_T r \omega_{NL} \tau_m}\right]_{\text{model}} = \left[\frac{x}{N_T r \omega_{NL} \tau_m}\right]_{\text{prototype}}$$

$$\Pi_4 = \left[\frac{t}{\tau_m}\right]_{\text{model}} = \left[\frac{t}{\tau_m}\right]_{\text{prototype}}$$

$$\Pi_5 = \left[\frac{T_s}{T_{gs}}\right]_{\text{model}} = \left[\frac{T_s}{T_{gs}}\right]_{\text{prototype}}$$

where

$$T_{gs} = \frac{K_t V_{ref}}{R}$$ [Stall torque for the reference voltage]

$$\tau_m = \frac{J_1 R}{K_t K_b}$$ [Mechanical time constant]

$$\omega_{NL} = \frac{V_{ref}}{K_b} \qquad \text{[No load speed for the reference voltage]}$$

$$K_D = \frac{K_t K_b}{R} \qquad \text{[Damping constant]}$$

and

T_C	[Coulomb friction torque]
T_s	[Static friction torque (stiction)]
T_L	[Torque load]
D_1	[Viscous friction constant]
J_1	[Motor's moment of inertia]
K_b	[Back emf constant]
K_t	[Torque constant]
x	[Scaled displacement]
N_T	[Total gear ratio]
r	[Equivalent radius]

(Note that the Pi terms are derived for motors that have an electrical time constant at least 10 times faster than the mechanical time constant.)

Evaluation of the Pi factors proceeds in a definite order with the Π_4 term solved first. This Pi group seeks to match transient responses of motors for a specified time of run. As Ipsen (1960) demonstrated, matching the ratio of the electrical and mechanical time constants is a more precise method of achieving similarity among a family of motors and may be more appropriate for velocity applications. However, the fourth Pi term as derived in this study, is better suited for matching position responses. This is because the time term in Π_4's numerator can be used to approximate the run time needed for the prototype motor to reach a specified position setpoint if its mechanical time constant is significantly different than the model's.

The third Pi term is satisfied next and can be manipulated in several ways. Most commonly, the model's reference displacement, reference voltage, gear ratio, radius, and time constant have been previously established by equations similar to (5.2) and (5.3), or:

$$\theta(t) = \left(\frac{V}{K_b} - \left(\frac{R}{K_t K_b} \right) T_C \right) \left(t - \tau_m + \tau_m e^{-\left(\frac{t}{\tau_m}\right)} \right) + \theta_0$$

where

$$x_{geared} = N_T r \theta(t)$$

With the model's design parameters known and the prototype's desired displacement and desired reference voltage established, the prototype's gear ratio is computed from:

$$N_{T_p} = N_{T_m}\left[\frac{K_{b_p}}{K_{b_m}}\right]\left[\frac{x_p}{x_m}\right]\left[\frac{V_{ref_m}}{V_{ref_p}}\right]\left[\frac{r_m}{r_p}\right]\left[\frac{\tau_{m_m}}{\tau_{m_p}}\right]$$

Alternatively, Π_3 could be used to predict the prototype's voltage requirement given a specified gear ratio, radius, time constant, and displacement. V_{ref} calculated from Π_3 is then used to compute the prototype's stall torque, or:

$$T_{gs_{prototype}} = \frac{V_{ref} K_t}{R}$$

The stall torque is used to solve the friction-based Π_1 and Π_5 terms, and from these dimensionless ratios, the candidate prototype motors' friction characteristics are evaluated. As was discovered in this thesis, matching static, Coulomb, and viscous friction terms using off-the-shelf motors is usually not an option, and similarity is rarely achieved without a controller.

The last term to consider is the external load Pi factor, Π_2. Like the friction torque terms, it evaluates the ratio of the desired torque load to the stall torque. For the application in this study, low gearing makes this term insignificant. In the case where torque loading is significant and it is not eliminated by gearing, then this term should be solved for immediately after the gear ratios have been fixed by Π_3. If the prototype motor cannot accommodate the desired load torque, then either lower gearing should be considered, or the entire Pi matching process should be repeated with another candidate motor that produces more torque.

Following the selection of the model and prototype motors, the velocity and position responses should be evaluated and confirmed through the dimensional and nondimensional closed form solutions developed in section 4.3 of Chapter 4. If all five Pi terms are satisfied, then experimental step responses will show that the model and prototype motors due in fact scale.

The prototype motor selection process outlined here is intended to serve as supplement to the traditional motor sizing methods which depend heavily on dimensioned speed versus torque curves supplied by motor manufacturers. It is felt by the author that for some applications the Pi method presented here may be more expedient. This is especially true for low velocity applications where friction effects must be closely scrutinized. The tools presented here offer a means to select prototype motors that avoid this regime. Furthermore, the traditional speed versus torque curves are usually drawn with transients set to zero. The Pi terms and the nondimensional equations developed here explicitly evaluate candidate prototype motors by their transient responses. Although the manufacturer's data sheets and speed vs. torque curves are adequate for selecting a *model* motor, they are less than ideal for selecting a similar *prototype* motor. The approach presented here is more rigorous, systematic, and ultimately more desirable for low velocity applications.

6.3 Review of Controlling Dissimilar Systems

Scaling a model and prototype PR manipulator with dissimilar motors is first considered possible if the selection of the prototype motor and gearing produces a reference voltage that is significantly greater than the prototype's break-out voltage. Secondly, the run time for the two systems must be sufficiently longer than the motors' mechanical time constants if the model and prototype constants are dissimilar. Finally, dissimilar friction effects must be canceled through a controller that utilizes an inverse model of the motors' friction components.

In this thesis, the development of a nondimensional trajectory planner was the first step taken to achieve similarity between model and prototype PR manipulators that had dissimilar motors. By controlling the run time of the two systems, and by controlling the shape of their trajectory paths, it was possible to scale the systems' controlled displacement. Equation (3.25) presents the final inverse form of the trajectory planner. Appendices E and F present Matlab and C code implementation of this algorithm. The sections in these programs that are related to the trajectory planner are titled as "Set Point Programmer." To fully understand this trajectory planner, it is recommended that the code in these Appendices be reviewed in some detail.

Controlling dissimilar motors was further assisted by combining the trajectory planner in a controller that was based on the computed torque algorithm. This open-loop control technique canceled the model and prototype motors' dynamics so that the reference produced by the trajectory planner resulted in the desired scaled motion. By using this controller, similarity between systems was achieved if motor friction was accurately modeled and estimated. However, experimentation revealed that accurately estimating the prototype's friction levels could be an insurmountable challenge.

The third Pi term was revised to more accurately predict the prototype motor's maximum voltage level for a desired displacement when controlled by the trajectory planner/computed torque algorithm. This new dimensionless grouping considered the time of run as part of its time term.

$$\left[\frac{x}{N_T r \omega_{NL}\left(t_f - \tau_m\right)}\right]_m = \left[\frac{x}{N_T r \omega_{NL}\left(t_f - \tau_m\right)}\right]_p$$

Rearranging this equation produces ratios of the model and prototype motor parameters.

$$V_{ref_p} = V_{ref_m}\left[\frac{K_{b_p}}{K_{b_m}}\right]\left[\frac{x_p}{x_m}\right]\left[\frac{N_{T_m}}{N_{T_p}}\right]\left[\frac{r_m}{r_p}\right]\left[\frac{t_f - \tau_{m_m}}{t_f - \tau_{m_p}}\right]$$

In this form, it is clear that the time terms cancel if the final run times are set equal in both systems and are significantly greater than the mechanical time constants. From this simple equation, the prototype's reference voltage can be quickly calculated and used to determine a stall torque. This term is then used in the first and fifth Pi terms, and friction effects are predicted as previously discussed. Thus, model and prototype PR manipulator systems can achieve similarity with dissimilar motors if the above conditions are met.

6.4 Conclusions and Recommendations for Future Work

6.4.1 Conclusions

The following is a list of findings and conclusions that were discovered over the course of this research. It should serve as a guide to investigators attempting to apply dimensional analysis to electro-mechanical devices.

- PM d.c. motors are scaleable if Π_1-Π_5 are satisfied.
- Dimensionless variables formed with a stall torque and friction terms can predict a motor performance at low velocities.
- Model and prototype motors should have reference voltages that are nearly equal and significantly higher than the friction threshold levels in order to avoid performance problems due to friction.
- Precisely matching friction characteristics among motors is often impossible, and to achieve similarity among motors, friction must be eliminated through an inverse model in open-loop control, or friction compensation with integral control in a closed-loop system. (However, the latter method is reported to lead to the stick-slip phenomenon.)
- Friction in well crafted motors is easily identified through simple off-line techniques. However, friction in cheaply made gearmotors with plastic nonbearing gears is extremely nonlinear at low velocities and difficult to estimate.
- Dissimilar model and prototype mechanical time constants are of little consequence if the time of run is equal and significantly long for both systems.
- The effects of the electrical time constant in PM d.c. motors is not perceptible in position responses where stiction is significant. Thus, it can be successfully ignored in system modeling.
- Manipulator link dynamics are eliminated through low gearing and low mass materials, and because of this, it can be ignored in the controllers' inverse model.
- Canceling all dynamics in a PR manipulator through the above techniques produces a gain of unity in the forward path. This results in a position response that is equal to the desired trajectory reference.
- Open-loop control is viable in robotic systems as long as friction is canceled and disturbances are not anticipated.
- Finally, a cubic polynomial based trajectory planner is easily nondimensionalized for a PR manipulator's kinematics.

6.4.2 Recommendations

The following are a few suggestions where more work with dimensional analysis might be considered with respect to servo systems and control.

- Pursue work that develops scaling laws for continuous duty applications where heating parameters are significant.
- Nondimensionalize backlash and compliance in joints and gearboxes.
- Apply the techniques developed here to scale micro brushless d.c. PM motors.
- Determine for what classes of motors are the Π_1-Π_5 scaling laws (or some variation thereof) applicable.

Investigating the question of what is the smallest motor class that these Pi terms can successfully scale is an obvious research topic that deserves attention. Scaling brushless PM motors is also an attractive research area because of the ever growing popularity of using these motors in high precision servo positioning systems. Finally, applying dimensional analysis to other micro devices, such as those used in modern medicine, promises to offer exciting research opportunities and future work.

A concluding remark regards dimensional analysis as it is applied to closed-loop control. As pointed out by Armstrong and Amin (1996), many in industry continue to tune servo systems by hand, and fail to adequately compensate for friction with PD and PID control. Dimensional analysis can offer assistance to these shortcomings, and lead to decisions in the *design* and *selection* phase that can minimize the effects of friction in systems that are especially vulnerable to the stick-slip problem.