

The vehicle model is also subjected to symmetry verification by subjecting it to various steering inputs and longitudinal inputs. Table 2.2 shows the list of cases performed to verify the validity of the vehicle model.

**Table 2.2 Vehicle Model Validation Test Cases**

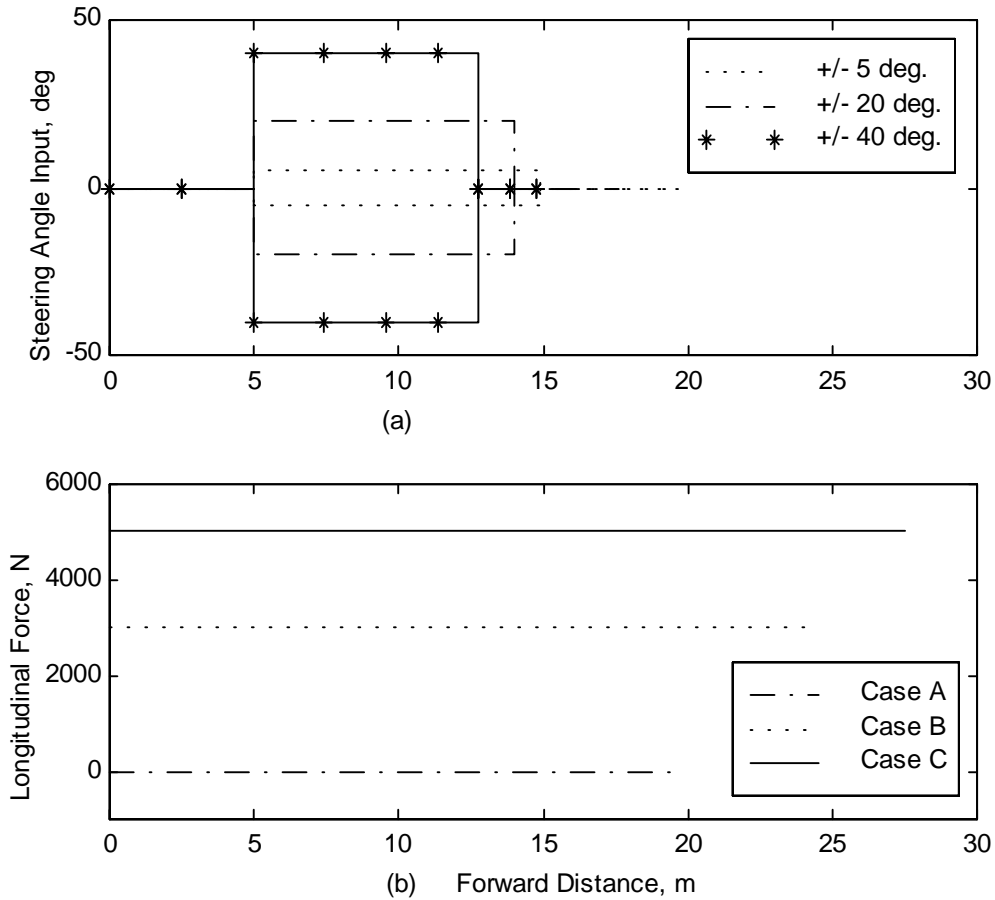
Case	Steering Angle Function (deg)	Longitudinal Force (N)
A	Step (5,20,40)	0
B	Step (5,20,40)	3000
C	Step (5,20,40)	5000
D	Saw-tooth (5,20,40)	0
E	Saw-tooth (5,20,40)	3000
F	Saw-tooth (5,20,40)	5000
G	4Hz sine wave (5,20,40)	0
H	4Hz sine wave (5,20,40)	3000
I	4Hz sine wave (5,20,40)	5000

These cases demonstrate whether there is symmetry between left and right turns. The vehicle model is subjected to a series of steering angle inputs consisted of three types of steering angle inputs, a step function, a saw-tooth function and a sinusoidal function with amplitudes of 5, 20, and 40 degrees, as indicated in Table 2.2. The vehicle model is also subjected to a series of longitudinal force inputs, again to demonstrate vehicle model symmetry. It must be noted that these tests were done open-loop, there is no feedback or optimization applied to these tests.

### **2.9.1 Vehicle Steering Symmetry Validation Using a Step Input**

For the first series, cases A, B, and C, the vehicle model is subjected to a step steering angle input of varying amplitudes with an initial velocity of 10 m/s. These amplitudes are held constant for the same amount of time. Initially the longitudinal force input is zero and then increased to 3000N and 5000N.

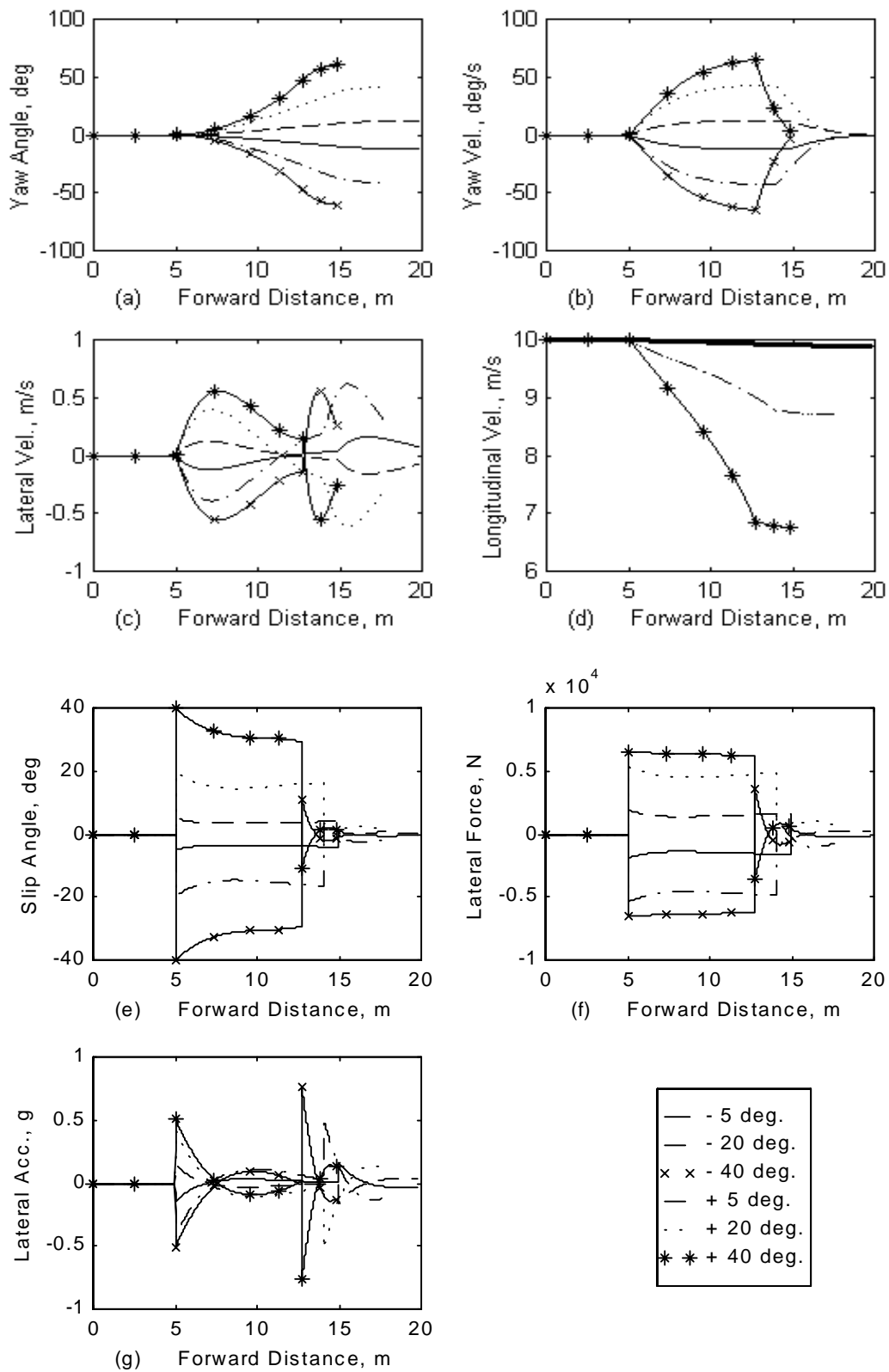
Figure 2.18 shows the input into the vehicle model, versus the distance traveled.



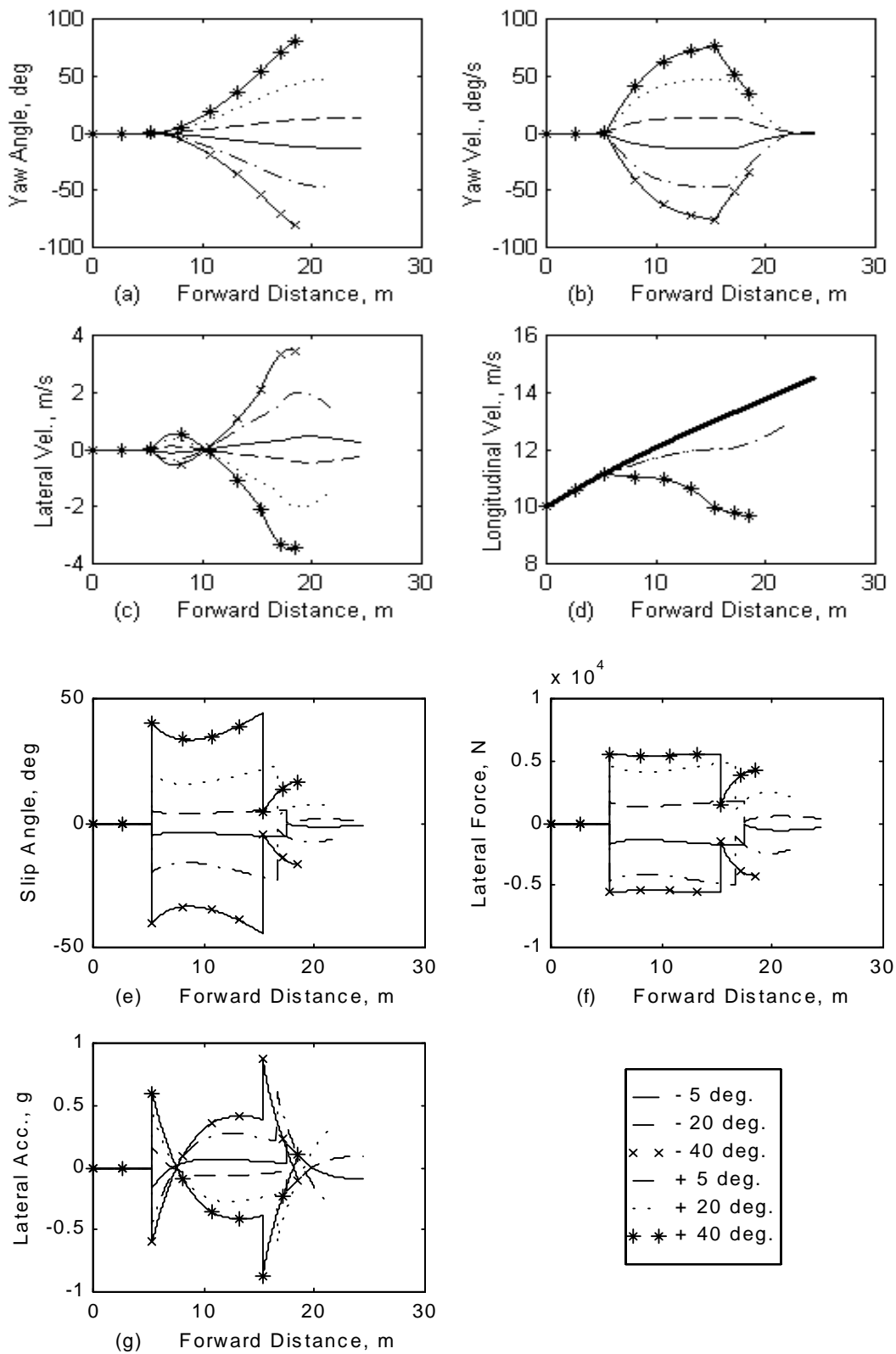
**Figure 2.18 System Inputs with Step Steering Angle for Cases A, B, and C**

As Figure 2.18.a shows the vehicle covers more distance longitudinally when subjected to smaller amplitude steering angle. This makes sense since the time for the step steering is held constant for all steering amplitudes. The profiles of Figure 2.18.b show that the vehicle covers more distance when subjected to larger longitudinal force.

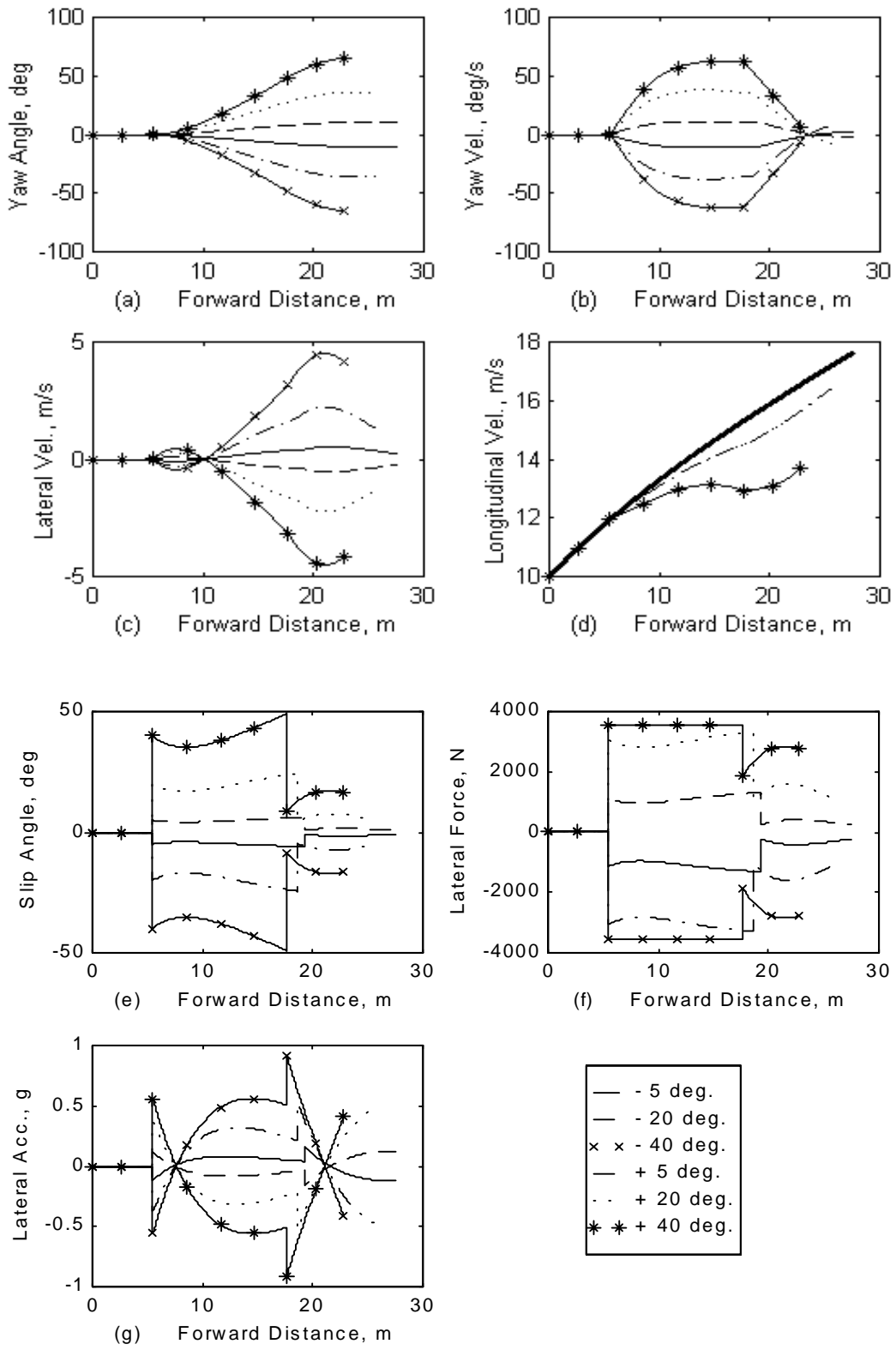
Figures 2.19, 2.20, and 2.21 show the response of the vehicle model in terms of yaw angle, yaw velocity, lateral velocity, longitudinal velocity, slip angle, lateral force, and lateral acceleration as a function of forward distance traveled.



**Figure 2.19 Results for Case A**



**Figure 2.20 Results for Case B**



**Figure 2.21 Results for Case C**

The yaw angle of the vehicle increases with increasing steering angle amplitudes as shown in Figures 2.19.a, 2.20.a, and 2.21.a. The yaw velocity also increases with increasing steering angle input amplitudes as shown in Figures 2.19.b, 2.20.b, and 2.21.b, because the execution time for turning is kept constant for all steering angle inputs. The same trend is seen with the lateral velocity, as shown in Figure 2.19.c, 2.20.c, and 2.21.c.

The longitudinal velocity, however, is unique. Figures 2.19.d, 2.20.d, and 2.21.d shows the longitudinal velocity actually decreases as the vehicle is subjected to higher steering angles. This is due to the fact that the longitudinal velocity highly depends on the longitudinal force, for this particular case the longitudinal force is zero.

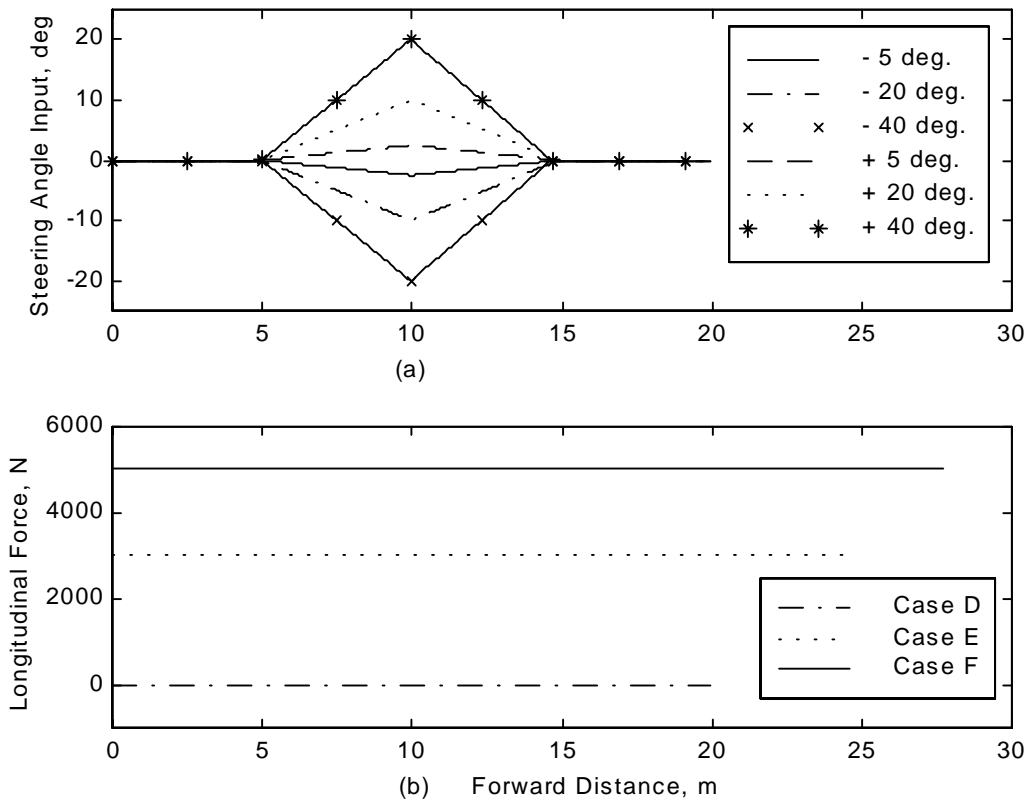
Figures 2.19.e, 2.20.e, and 2.21.e show the slip angle increasing with the steering angle. The lateral force and the lateral acceleration show the same trend which can be seen in Figures 2.19.f, 2.20.f, 2.21.f, 2.19.g, 2.18.g, and 2.21.g, respectively.

Figures 2.19, 2.20, and 2.21 show that the symmetry between left and right turns exists in the vehicle model. To further validate the symmetry of the vehicle model, the next section discusses the same vehicle model subjected to the saw-tooth steering angle input.

### **2.9.2 Vehicle Steering Symmetry Validation: Saw-Tooth Input**

For the second series, cases D, E, and F, the vehicle model is subjected to a saw-tooth steering angle input of varying amplitudes with an initial velocity of 10 m/s. These amplitudes are help constant for the same amount of time. Initially the longitudinal force input is zero and then increased to 3000N and 5000N.

Figure 2.22 shows the system input into the vehicle model.



**Figure 2.22 System Inputs with Saw Tooth Steering Angle for Cases D, E, and F**

As Figure 2.22.a shows the vehicle covers more distance longitudinally when subjected to smaller amplitude steering angle. This makes sense since the time for the step steering is held constant for all steering amplitudes. The profiles of Figure 2.22.b show that the vehicle covers more distance when subjected to larger longitudinal force.

Figures 2.23, 2.24, and 2.25 show the response of the vehicle model in terms of yaw angle, yaw velocity, lateral velocity, longitudinal velocity, slip angle, lateral force, and lateral acceleration as a function of forward distance traveled.

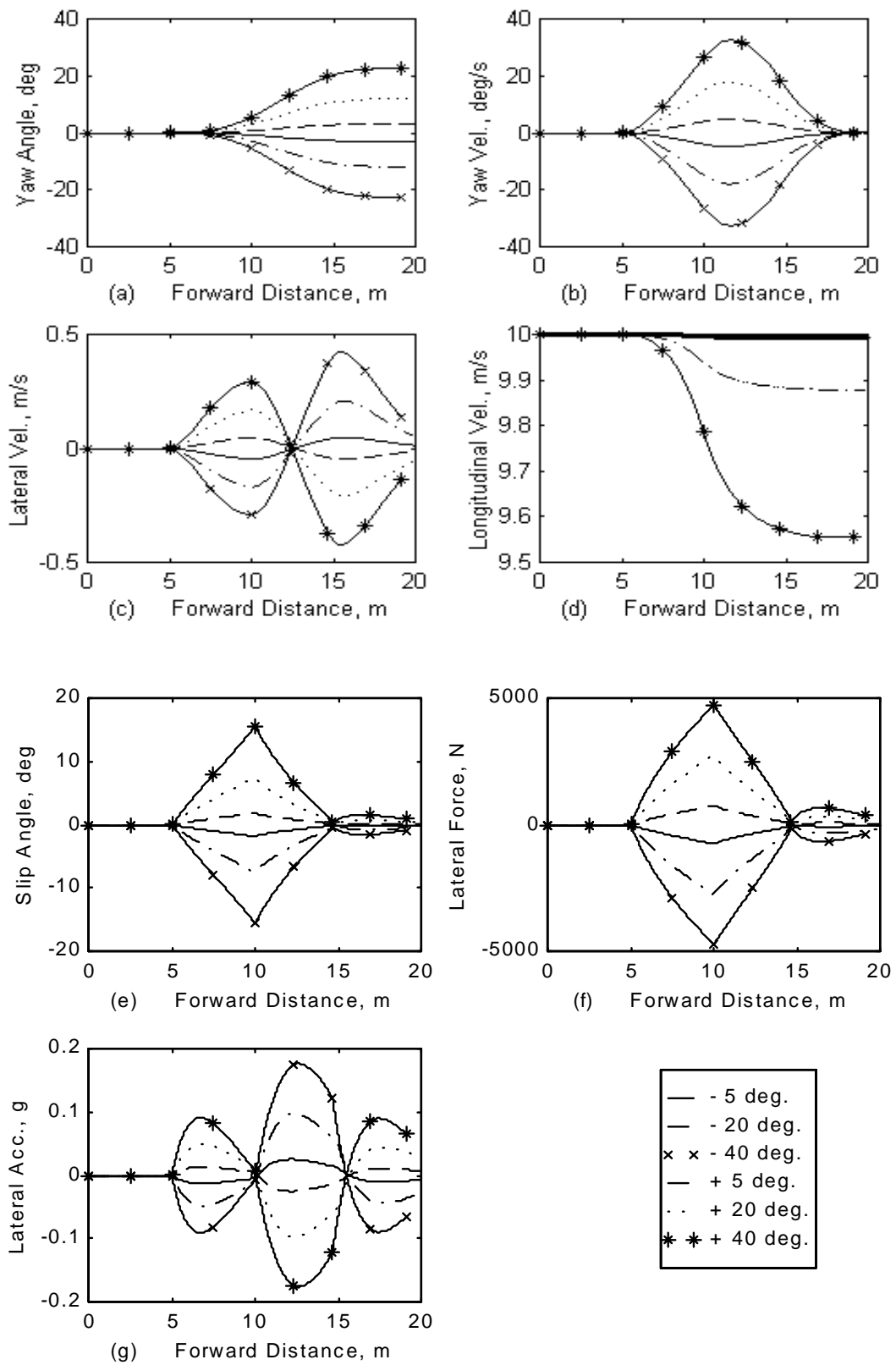


Figure 2.23 Results for Case D



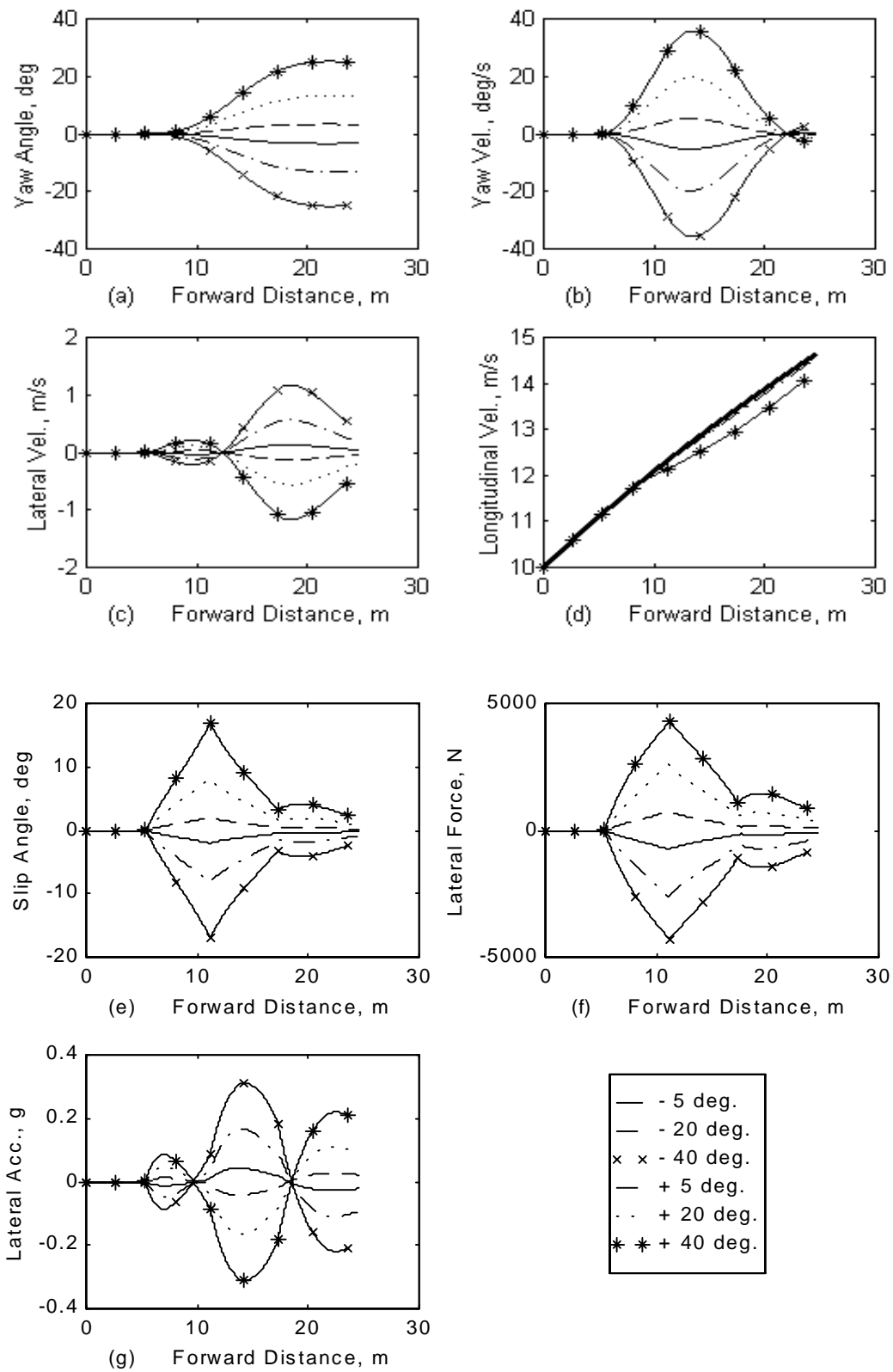
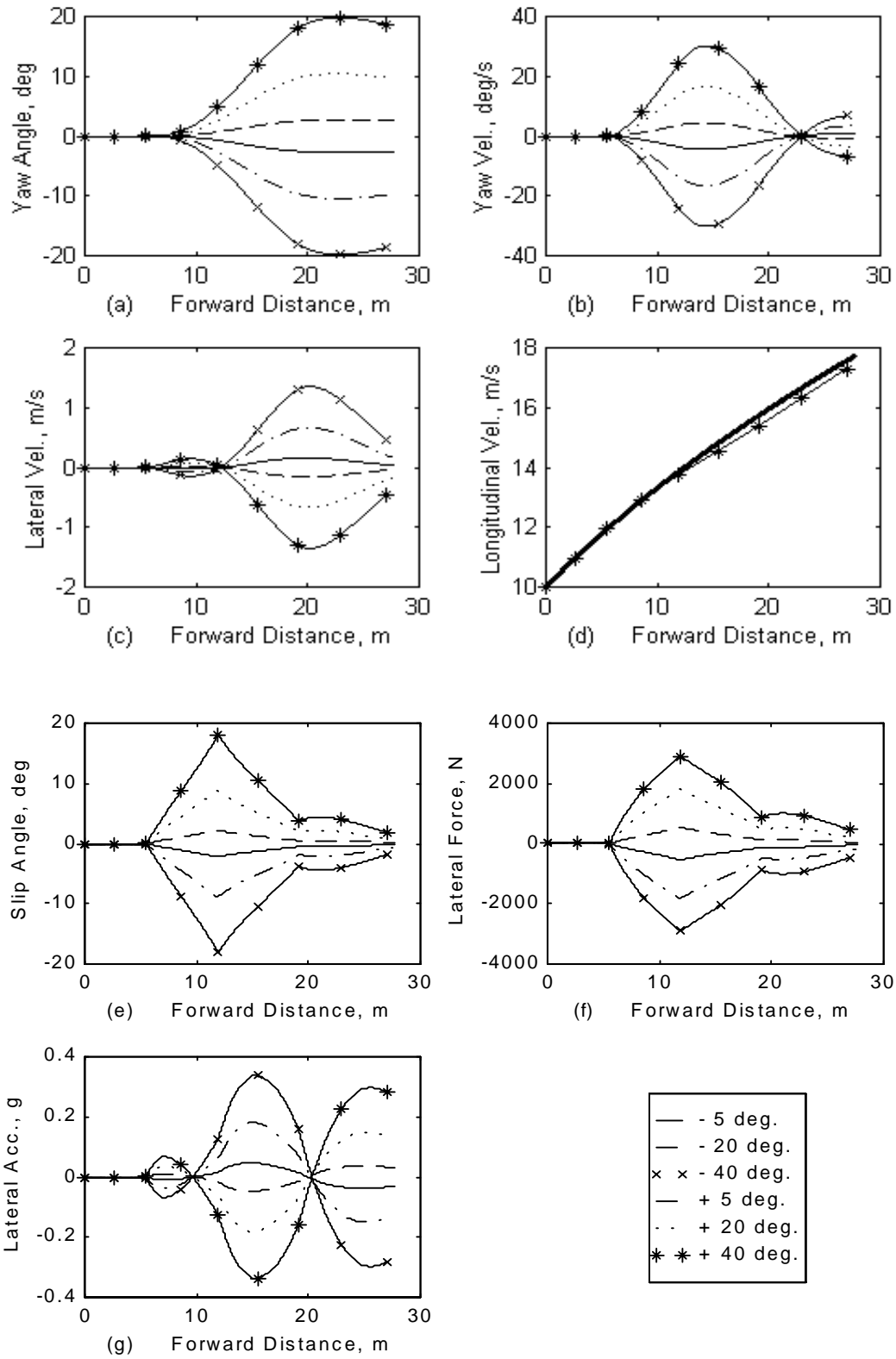


Figure 2.24 Results for Case E



**Figure 2.25 Results for Case F**

The yaw angle of the vehicle increases with increasing steering angle amplitudes as shown in Figures 2.23.a, 2.24.a, and 2.25.a. The yaw velocity also increases with increasing steering angle input amplitudes as shown in Figures 2.23.b, 2.24.b, and 2.25.b, because the execution time for turning is kept constant for all steering angle inputs. The same trend is seen with the lateral velocity, as shown in Figure 2.23.c, 2.24.c, and 2.25.c.

The longitudinal velocity, however, is unique. Figures 2.23.d, 2.24.d, and 2.25.d shows the longitudinal velocity actually decreases as the vehicle is subjected to higher steering angles. This is due to the fact that the longitudinal velocity highly depends on the longitudinal force, for this particular case the longitudinal force is zero.

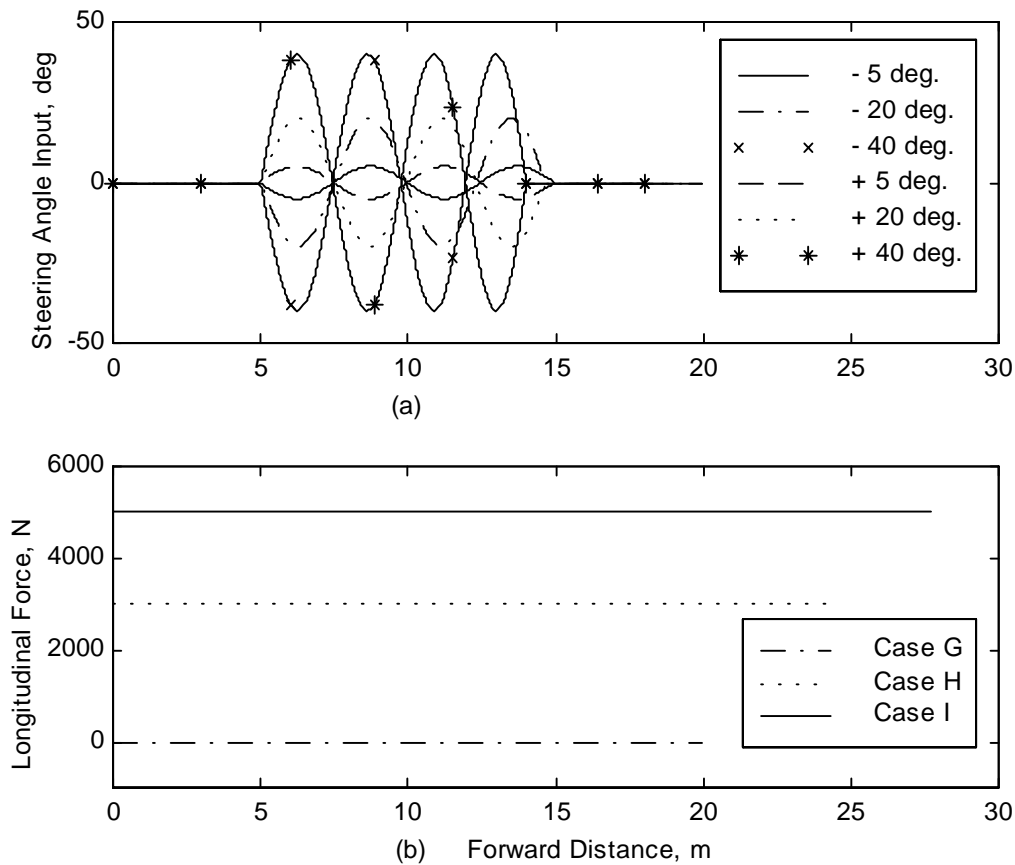
Figures 2.23.e, 2.24.e, and 2.25.e show the slip angle increasing with the steering angle. The lateral force and the lateral acceleration show the same trend which can be seen in Figures 2.23.f, 2.24.f, 2.25.f, 2.23.g, 2.24.g, and 2.25.g, respectively.

Figures 2.23, 2.24, and 2.25 show that the symmetry between left and right turns exists in the vehicle model. To further validate the symmetry of the vehicle model, the next section discusses the same vehicle model subjected to the sinusoidal steering angle input.

### **2.9.3 Vehicle Steering Symmetry Validation: Sine Wave Input**

For Cases G, H, and I, the vehicle model is subjected to a 4 Hz sine wave steering angle input of varying amplitudes with an initial velocity of 10 m/s. These amplitudes are kept constant for the same amount of time. Initially the longitudinal force input is zero and then increased to 3000N and 5000N.

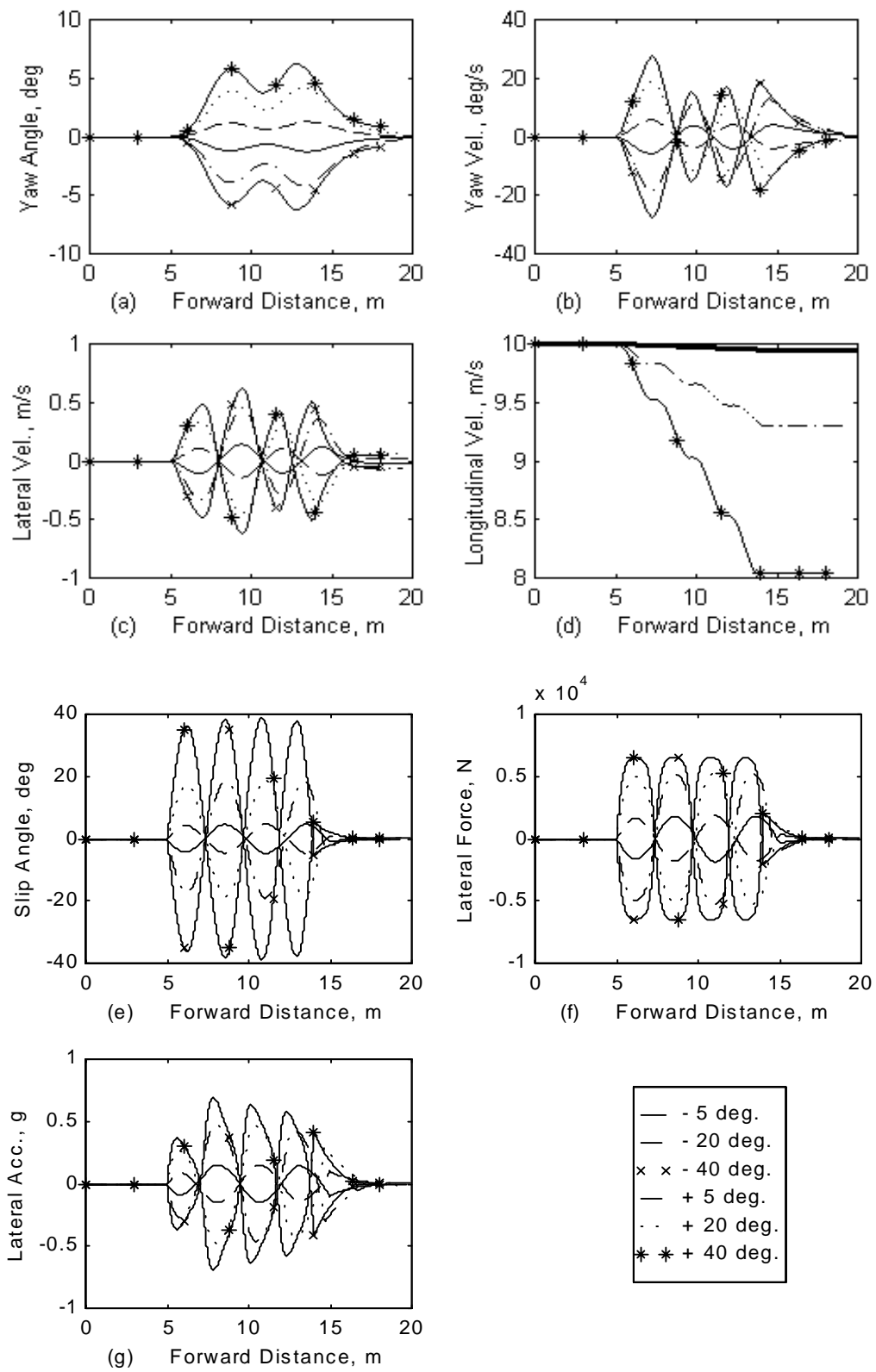
Figure 2.26 shows the system input into the vehicle model.



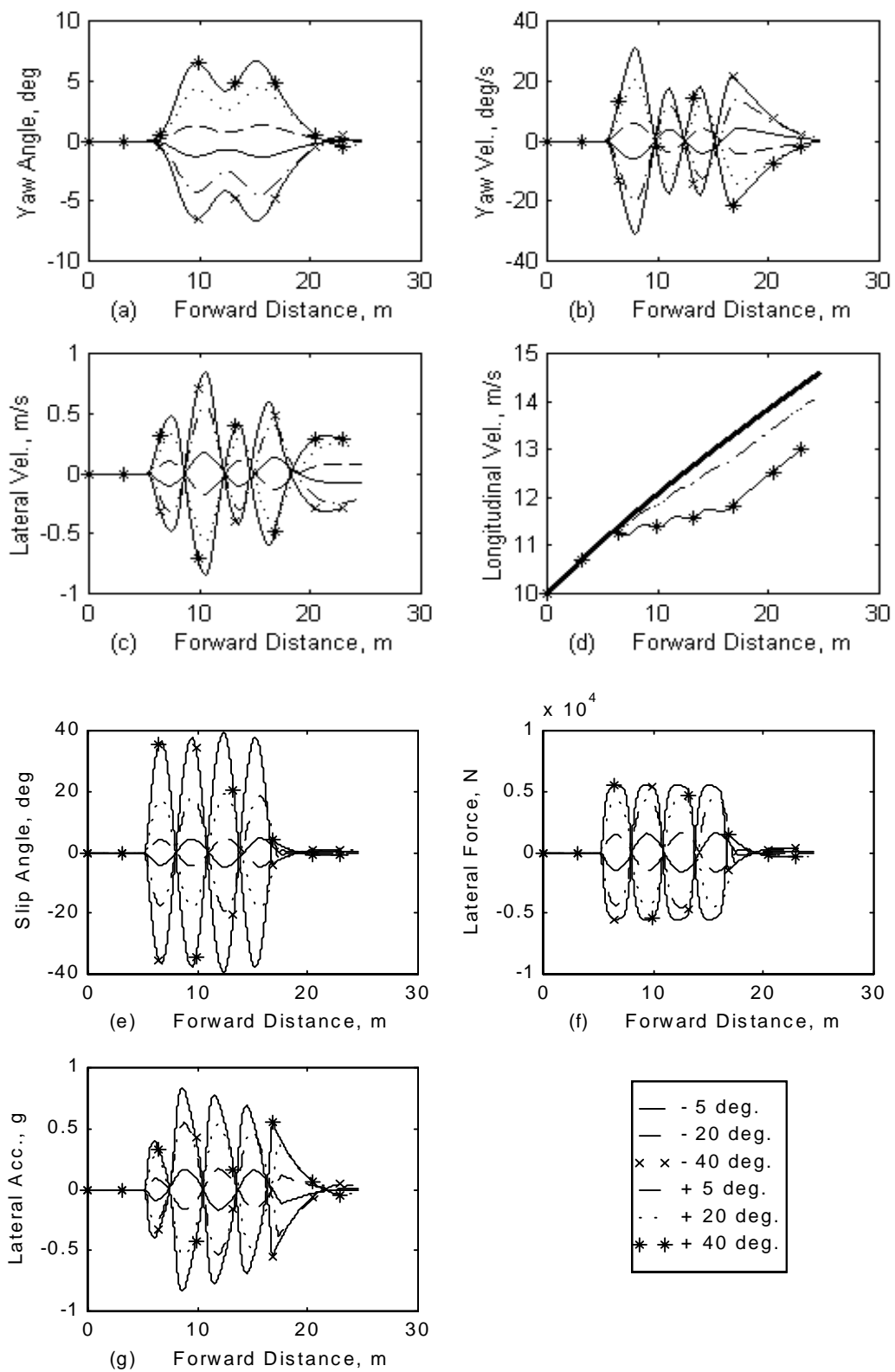
**Figure 2.24 System Inputs with Sinusoidal Steering Angle for Cases G,H, and I**

As Figure 2.26.a shows the vehicle covers more distance longitudinally when subjected to smaller amplitude steering angle. This makes sense since the time for the step steering is held constant for all steering amplitudes. The profiles of Figure 2.26.b show that the vehicle covers more distance when subjected to larger longitudinal force.

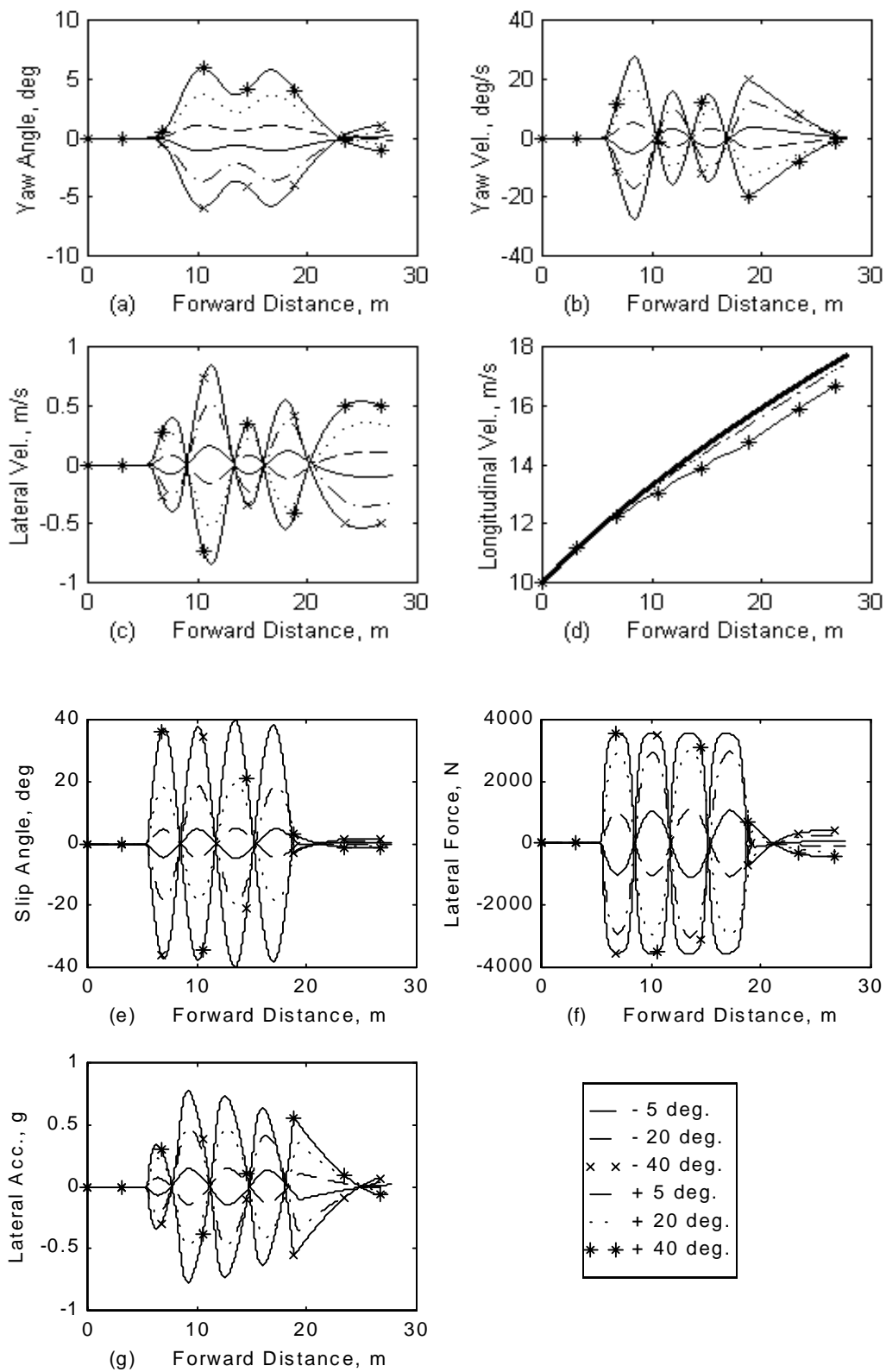
Figures 2.27, 2.28, and 2.29 show the response of the vehicle model in terms of yaw angle, yaw velocity, lateral velocity, longitudinal velocity, slip angle, lateral force, and lateral acceleration as a function of forward distance traveled.



**Figure 2.27 Results for Case G**



**Figure 2.28 Results for Case H**



**Figure 2.29 Results for Case I**

The yaw angle of the vehicle increases with increasing steering angle amplitudes as shown in Figures 2.27.a, 2.28.a, and 2.29.a. The yaw velocity also increases with increasing steering angle input amplitudes as shown in Figures 2.27.b, 2.28.b, and 2.29.b, because the execution time for turning is kept constant for all steering angle inputs. The same trend is seen with the lateral velocity, as shown in Figure 2.27.c, 2.28.c, and 2.29.c.

The longitudinal velocity, however, is unique. Figures 2.27.d, 2.28.d, and 2.29.d shows the longitudinal velocity actually decreases as the vehicle is subjected to higher steering angles. This is due to the fact that the longitudinal velocity highly depends on the longitudinal force, for this particular case the longitudinal force is zero.

Figures 2.27.e, 2.28.e, and 2.29.e show the slip angle increasing with the steering angle. The lateral force and the lateral acceleration show the same trend which can be seen in Figures 2.27.f, 2.28.f, 2.29.f, 2.27.g, 2.28.g, and 2.29.g, respectively.

Figures 2.27, 2.28, and 2.29 show that the symmetry between left and right turns exists in the vehicle model.