# Optimal Vehicle Path Generator
# Using Optimization Methods

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This research explores the idea of developing an optimal path generator that can be used in conjunction with a feedback steering controller to automate track testing experiment. This study specifically concentrates on applying optimization concepts to generate paths that meet two separate objective functions; minimum time and maximum tire forces.

A three-degree-of freedom vehicle model is used to approximate the handling dynamics of the vehicle. Inputs into the vehicle model are steering angle and longitudinal force at the tire. These two variables approximate two requirements that are essential in operating a vehicle. The Third order Runge-Kutta integration routine is used to integrate vehicle dynamics equations of motion. The Optimization Toolbox of Matlab is used to evaluate the optimization algorithm. The vehicle is constrained with a series of conditions, includes, a travel within the boundaries of the track, traction force limitations at the tire, vehicle speed, and steering.

The simulation results show that the optimization applied to vehicle dynamics can be useful in designing an automated track testing system. The optimal path generator can be used to develop meaningful test paths on existing test tracks. This study can be used to generate an accelerated tire wear test path, perform parametric study of suspension geometry design using vehicle dynamics handling test data, and to increase repeatability in generating track testing results.

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