

DESIGN OF USER-WEIGHT-BASED EXERCISE MACHINES

by

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(Abstract)

This thesis describes the process of designing exercise machines that raise the weight of the user as the primary source of resistance.

Most strength training machines use weight stacks or springs as the source of resistance. While such machines are highly evolved and provide an excellent workout, they typically have a number of disadvantages including high cost, and large size and weight. A user weight-based exercise design will reduce the cost, size, and weight of the machine.

The design process considers some important issues. Parallelogram linkages are implemented to provide non-rotary motion without the disadvantage of linear bearings. The user input is located with respect to the user providing correct relative motion for the exercise. The design also considers proper resistance curves during the design process.

Specific examples are given for each step of the design process. These examples include the evolution of ideas and the creation and use of kinematic and automatic tools.

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Chapter 1, Introduction

This thesis uses the engineering design process as described by Ertas and Jones (1996). This procedure is applied to design a home exercise machine. This machine uses a person's own weight as the source of resistance. The design process is used to take this idea from concept to prototype.

The design process can be used to devise a system, component or process to meet desired needs. Another definition of the design process is "The process of applying the various techniques and scientific principles for the purpose of defining a device, a process or a system in sufficient detail to permit its realization. Design may be simple or enormously complex, easy or difficult, mathematical or non-mathematical; it may involve a trivial problem or one of great importance." (Norton, 1992). There are many detailed methods for the engineering design process. They all have some common elements, such as brain-storming and some type of analysis. All design processes must include iterative decision making. Designs are never perfect the first time and will always improve with iteration.

1.1 The Engineering Design Process

The engineering design process as defined by Ertas and Jones (1996) will now be described. This process is generalized for large projects done by large organizations or companies. For small organizations or student projects, some of the steps can be simplified or ignored. Figure 1-1 (Ertas and Jones, 1996) shows a schematic block diagram of this process. It is important to first know about synthesis and analysis in the design process. This is not a discrete step, but is continuously ongoing. To synthesize means to combine parts into a complex whole. To analyze means to separate the whole into elements. They are interrelated during the design process. This usually happens right after an initial statement of need. A quick sketch of an idea to solve a problem is a

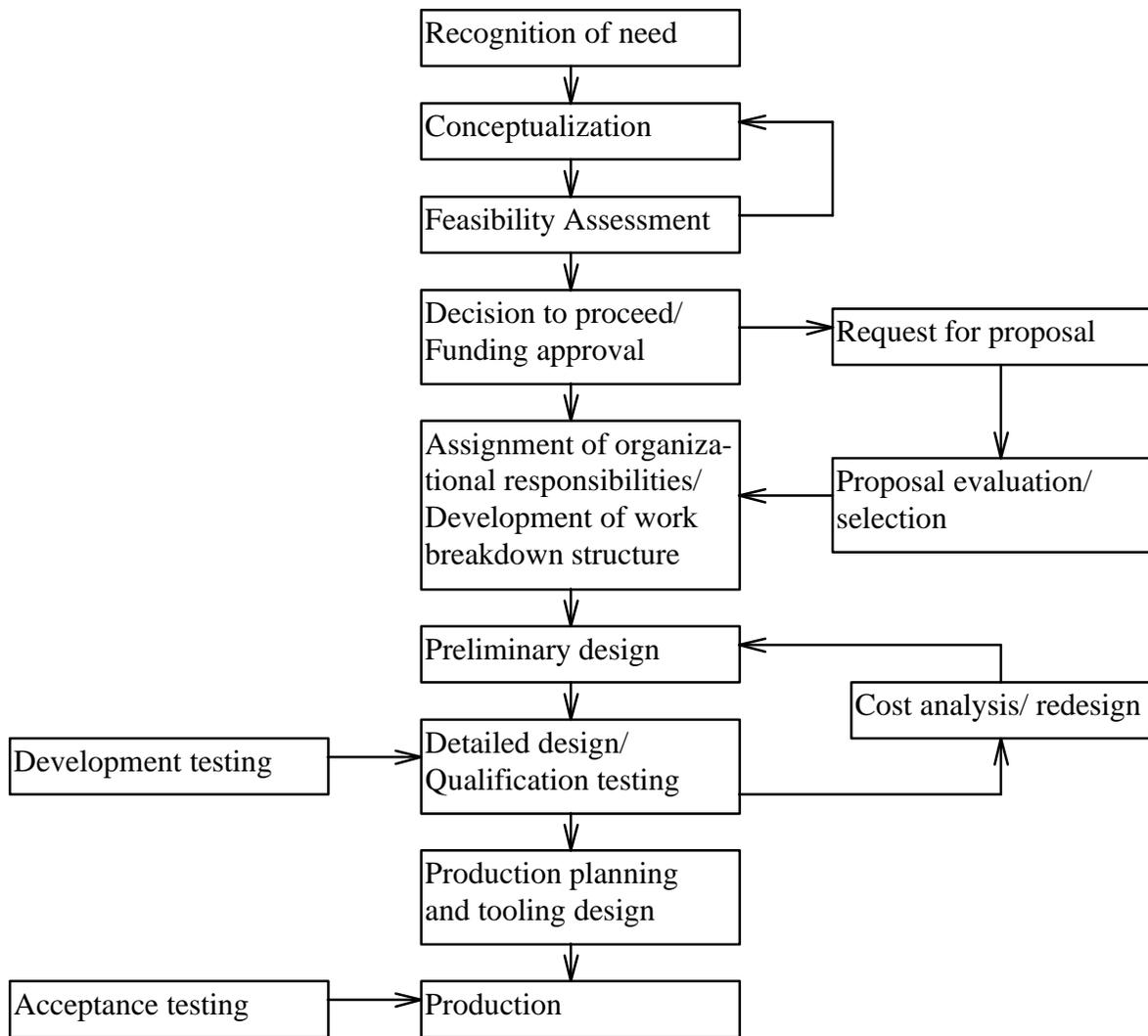


Figure 1-1, Steps in the Engineering Design Process

Adapted from (Ertas and Jones, 1996)

crude example of synthesis. This step would be followed directly by problems with the idea or better ideas based on components of the design.

Before a system can be analyzed, it must at least be conceptualized. Therefore, synthesis must occur first. An initial concept to solve the problem must be determined. Once a concept is approved, schematics and layouts are created to visually depict the concept to other groups of people. Eventually, this will lead to detailed design and detailed drawings of each component along with assembly drawings.

1.1.1 Recognition of need

The first step in the engineering design process is the recognition of need. When a need is given, it is usually brief and lacking detail. The design engineer must structure the problem statement. There are different categories for each of the five different types of needs. The first type of need is a formal request, also called a request for proposals. The second type is an informal request, this is a suggestion or an implication from a potential customer. The third type is when a need is felt to exist. The fourth type of need is an assignment from a supervisor. Finally, the fifth type is the need for a product for which a market could be developed. This is a marketing strategy to create a fad to sell an idea. In this case the need is created.

1.1.2 Conceptualization and Creativity

Once a need is recognized, concepts immediately follow. A design engineer will start to crudely synthesize and analyze the problem. This step is fun and frustrating and potentially the most satisfying. Few facts exist about the phenomenon of creativity. Some believe it can be taught, and others believe that it is inherited.

Although the process of conceptualization can not be well defined, it does seem to have a general trend. There is frequently an idea generation or brainstorming stage where people try to think of as many ideas as possible without judgment. Analysis will determine if the idea is unacceptable, but it is best to start with many ideas as possible. This is followed by a period of frustration when the ideas run out. At this point, the problem should be set aside to allow a period of incubation. During this time, the subconscious mind will still work. Ideally, this will cause a new idea to pop into the conscious mind, which will often seem to be an obvious solution (Norton, 1992).

When the concept is analyzed, unforeseen difficulties are often encountered. Therefore iteration or restart is necessary to refine the solution. The conceptualization stage is part of the synthesis and analysis process.

1.1.3 Feasibility Assessment

This step is used to determine the feasibility of a concept. This is to ensure that the design phase is entered with a concept that is feasible or achievable technically and with regard to cost. This is a type of analysis but not an in-depth analysis of individual components or of every concern. This is a preliminary analysis to determine if the idea qualifies to be further pursued and analyzed in greater depth. If the concept is not feasible, the process must return to conceptualization. For small projects, this may be part of the conceptualization stage.

1.1.4 Establishing Design Requirements

This step follows the feasibility assessment and will provide detailed task specifications for the design. I feel that this step should follow the recognition of need since this will also help determine the best concept. The list of requirements or specifications are important for communicating ideas to other engineering groups. It helps to save time and cost by minimizing wasted pursuits. Two types of specifications could be used. Performance specifications state what the design must do. Design specifications define how the design must do it. This step constrains the problem so that it can be solved and can be shown to have been solved. Care should be taken to not make the requirements too specific and, therefore, limit the designer's freedom.

1.1.5 Organizational Work / Breakdown Structure

This step is needed to maintain management accountability and prevent responsibilities from diffusing into other groups. This structure is a family tree to subdivide efforts. It also relates tasks of each group to each other. Obviously, for small project with only a few people, this step is not as important.

1.1.6 Preliminary Design

This stage is the bridge between design concept and detailed design. If there is more than one acceptable concept, an evaluation will be conducted. The cost will now

become more realistic and schematics, diagrams and layouts will be used. Also at this stage, some computation and analysis will be done. A component-level literature search can also be conducted. Vendor equipment will be evaluated and experts are consulted.

This is still a preliminary design, so not all requirements can be specified correctly on the first try. Iteration is still taking place on this step as well as the entire design process. Therefore feedback is required. Automated tools for detailed analysis are often powerful aids in this stage of design. These tools include commercial software packages or software code written by an engineer for specific analysis needs. Some examples are kinematic and dynamic software or finite-element analysis software.

1.1.7 Detailed Design

The detailed design stage is an extension of the preliminary design stage. There is still some synthesis and analysis occurring. Each part is evaluated to see if it meets the overall requirements. Specifications are given for each component, this is usually done in detail drawings.

The detail drawings define each component and how they are all assembled into subassemblies and main assemblies. These drawings specify the following elements for each component:

- a. Operating parameters
- b. Operating and non-operating environmental stimuli
- c. Test requirements
- d. External dimensions and tolerances
- e. Maintenance and testability provisions
- f. Materials requirements
- g. Reliability requirements
- h. External surface treatment
- i. Design life
- j. Packing requirements
- k. External marking
- l. Special requirements (lubrication, etc. ...)

Detail drawings for internal manufacturing are related by the “next assembly” on the drawing to inform that the part belongs to that assembly. That assembly drawing will have a Bill Of Materials (BOM) to provide information on which parts are needed to make the assembly.

There are other diagrams that are also useful in detailed design. Some examples are diagram drawings for piping and wiring. Installation drawings are also used for assembly or installation outside of the company.

The cost is checked during this step of the design process.

1.1.8 Production Process Planning and Tooling Design

At this stage, engineers need to determine the appropriate sequence of process operations for production. These operations and their sequence are determined by their geometry, dimensions, tolerances, materials, surface finish, etc. This step can be traversed methodically in the following order:

- a. Interpret the design drawing
- b. Select the material
- c. Select the production process
- d. Select the machines
- e. Determine the sequence of operations
- f. Select jigs, fixtures, tooling, and reference datum
- g. Establish tool cutting parameters like speed, feed, and depth
- h. Select inspection gages
- i. Calculate process time
- j. Process documentation and NC data

Producibility analysis may also be used at this stage. This optimizes the design by minimizing the cost of manufacturing. After the manufacturing processes have been determined, production planning is done. Production planning is defined as laying out the production line flow and production control is scheduling work, providing materials, and supplies.

1.1.9 Production

Production is ready after the prototype has been tested and is qualified. During the production, quality must still be tested. This can be done at various stages during the manufacturing processes. Additionally, quality assurance testing should be done after the product is complete.

1.2 Other Design Considerations

There are still some other design considerations that deserve some discussion. The steps of the design process have been laid out, but there are still other parts of the design process that can incorporate more than one of these steps. Some steps can overlap to save time. This is called concurrent engineering. One example is design for manufacturability. This is when the design engineer considers the manufacturing processes needed to produce a design. Some design changes can save time during the manufacturing processes. Another consideration is human factors data for a user friendly product.

1.2.1 Design for Manufacture and Assembly

Typically, between 70 and 80% of the production cost is a result of the design of a product (Ertas and Jones, 1996). The decisions made early in the design process have a greater effect on cost than later decisions for manufacturing improvements. As a result, design must be done such that manufacturing processes are minimized and optimized. The design and manufacturing of a product can not be separated. Therefore, it is important for the designer to understand process requirements and preferred manufacturing methods.

The following list is a good set of guidelines to follow:

- a. Simplify the design.
- b. Eliminate operations that require skill.
- c. Minimize the number of parts.
- e. Use a modular design.
- f. Minimize part variations.
- g. Use multi-functional designs. Design a part that does many things.

- h. Design parts for multi-use. A database would work well for general parts.
- i. Design to simplify fabrication. This can be accomplished with cheap materials, not wasting time, and designing self-securing parts.
- j. Minimize use of fasteners since cost of fastening is more than cost of fastener.
- k. Minimize assembly directions.
- l. Maximize compliance by using generous tapers, chamfers, location points for fixtures, etc. ...
- m. Minimize handling by positioning parts in proper position and making them easy to find.
- o. Eliminate or simplify adjustments. Use stopping points and detents.
- p. Avoid flexible component such as wires and cables.
- q. Minimize testing.

1.2.2 Human Factors

Human factors information is important for ease of use, safety, efficiency, and also for maintenance and repair. Some examples of important considerations are color indicators, warning lights, and location of switches. Another factor to realize is the aesthetic appeal of the product. This can be more important for sales than functionality.

The design process is applied to designing a home exercise machine. The following chapters give specific examples of the important steps.

Chapter 2, Identification of need

The first step in the engineering design process is to identify the need for a product, device or service. There is an obvious general need for exercise and strength training equipment.

2.1 The Need for Exercise

The need for strength training is the first issue to consider. The surgeon general states that daily activity for 35 to 40 minutes will improve overall health and quality of life. Some examples of such aerobic activity include biking, walking, even working around the house or yard work (Burns, 1996)

The effects of strength training go beyond just increasing strength. Several weeks of strength training will reduce resting blood pressure, a 2 month program can reduce it by 4 mm Hg. It also causes fat reduction in three ways. First, strength training will burn calories during the work out. Secondly, the after burn effect will burn more calories for hours after a work out. The after burn effect is the increase in metabolic rate and a higher body temperature. Lastly, for each pound of muscle, 35 calories are consumed each day to support tissue. Strength training will also increase bone density and improve glucose metabolism. It also causes faster gastrointestinal transit, produces better blood lipid levels, reduces lower back pain and reduces arthritic discomfort (Wescott, 1996).

Lack of time seems to be the most popular excuse for people who do not strength train. In fact, 60% percent of Americans do not engage in regular exercise and 25% do not exercise at all. However, an awareness of physical fitness is growing. Strength training needs to be shown to be fun and attractive (Wescott, 1996).

2.2 Background Research in Exercise Technology

It is important to understand the trends and facts of exercise to know the needs in the home exercise market. There are many different opinions from muscle magazines and

the field or sports medicine on the best methods of strength training. A wide variety of different workout routines are readily available. These routines specify how weights should be lifted, and how many sets and repetitions should be done. One major issue that is evident today is the controversy regarding concentric and eccentric exercises.

2.2.1 Concentric Versus Eccentric Training

Concentric training is based on training muscles as they contract. This is commonly known as the “positive” exercise. Eccentric training is based on training muscles as they lengthen. This is commonly known as the “negative” exercise.

Some concrete facts state that eccentric training is more effective for body building (Phillips, 1996). The eccentric exercise will cause hypertrophy, the growth of muscle tissue. Eccentric training is believed to cause more muscle growth because it causes greater tension and more stimulus to the muscle fibers and greater biological adaptation. The specific medical reasons are not important in this document. Another important fact is that eccentric strength is about 40% greater than concentric strength (Dibble, 1989). It is important for best performance not to rest between repetitions or motions during a repetition (Duchaine, 1996). Even though the eccentric exercise is most important, the concentric exercise still has some value. Concentric exercises do help increase strength. They also cause fatigue of muscle fiber during a set to cause the fiber to lock. This prepares the muscle fiber for an optimal eccentric repetition at the point of fatigue. When the muscle fiber locks, the eccentric part of the repetition will cause subcellular damage to the muscle. Therefore the concentric exercise is also a critical primary stimulus for the eccentric exercise (Phillips, 1996). Though this may sound destructive, it is most effective for building muscle since the cells are rebuilt. To conclude, exercises should be done continuously throughout the set. The negative part of the exercise should take about three times longer than the positive part with controlled motion (Phillips, 1996), and the weight should be increased up to 40% in the negative part of the exercise if possible. Knowing this, it is clear that isokinetic machines, which only apply resistance for the concentric motion, are not effective for building muscle (Poliquin, 1995).

2.2.2 Strength Curves Versus Resistance Curves

The strength curve is defined to be the maximum force or torque that a person can exert as a function of the position through a particular exercise motion (Telle, 1996). An example of a strength curve could be found by looking at a biceps muscle. The maximum strength of the biceps can be measured for each position of the elbow rotation. The curve could be seen by plotting the force as a function of elbow rotation. An example of a strength curve is shown in Fig. 2-1.

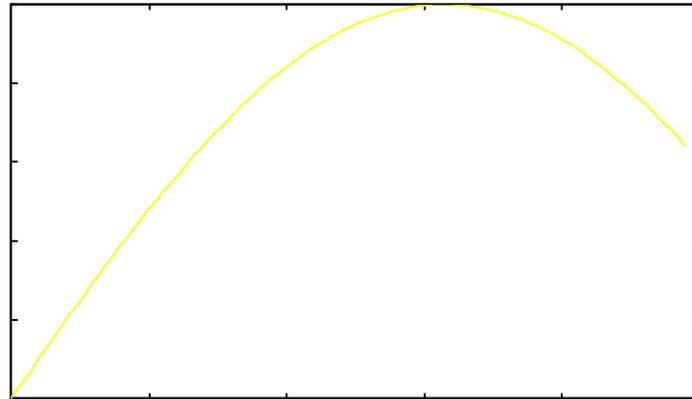


Figure 2-1, Example Strength Curve

Arthur Jones brought the variable resistance cam some attention in the fitness community. He worked with Nautilus to make strength training equipment that used a variable resistance cam to match strength curve. The resistance that the cam creates is called the resistance curve. Ideally, the resistance curve should exactly match the strength curve. This method should yield optimal results because it will train muscle fibers through the entire range of motion. It eliminates the “sticking point” that does not allow the exercise motion to continue. If the motion is not continued, the muscle that is used to complete the motion will not be trained. The variable resistance cam not only allows all the muscle fibers to be trained, but it also trains them at their maximum strength (Telle, 1996).

There is still some criticism about the variable resistance cam. The first problem is that fatigue happens most rapidly at the end of an exercise motion. Therefore, as repetitions increase, the resistance curve at the end of the cam rotation may exert forces greater than the strength curve. So strength curves change even in the course of a work

out. Some companies have designed variable resistance cams with less resistance at the end of rotation. Another criticism is that the cams match an average strength curve. However, strength curves among members of a population vary. Isokinetic machines would seem to solve the problem. These machines exert resistance that is directly related to the speed of the exercise motion. These machines use hydraulics or electronic controls to always match the force exerted by the user. However, most only allow concentric training. Therefore free weights are still advocated as the best solution (Telle, 1996). Free weights accommodate changes in the strength curve by allowing the user to accelerate the mass when stronger. Consequently, when the person is weaker, the kinetic energy stored in the mass will help the motion continue. This idea is based on energy storage as the mass acts like an energy buffer.

2.3 Home Exercise Market

The home exercise market is replete with various types of cardiovascular and strength training equipment. Advertisements continually show new ideas for cardiovascular workout or abdominal workout machines. There is also an ever-increasing range of strength training equipment. The attraction of home exercise equipment is the convenience of not going to a gym. There also may be a cost savings over a long period of time.

The concept of using a person's body weight for a cardiovascular work out is very common now. I feel that one reason that the idea is successful is because the exercise appears to be fun.

Strength training equipment offers a wide range of exercises. However, the low end machines use some basic exercises and modify them to create more exercises. Some strength training machines use weight plates as a source of resistance. SoloFlex, BowFlex and NordicTrack machines do not use plates for resistance. These non-plate loaded machines cost about \$1000 (NordicTrack Inc, 1996)

The home equipment market is potentially larger and more profitable.

2.3.1 Existing Machines and Trends

It is important to look specifically at the products available in the market. This will help save time in the design process by not repeating work that has already been done. This can also give ideas to improve and change existing designs.

Some electronic equipment has been designed by Universal Gym and LifeFitness. The goal of these machines is to provide an intelligent workout that optimizes the user's time. This is done by measuring the strength curve of the user and then matching it. The machines can also add more resistance for the eccentric motions and adjust for fatigue. The electronic machines also reduce boredom by creating an interactive experience. They also coach the user with feedback on fitness results. Universal Gym and LifeFitness designed a system that stores each user's information on a magnetic strip and can be used for future workouts. These machines use dc motors and a motor controller to create resistance (Dibble, 1989).

Nautilus is still using the variable resistance cam and improving on those machines with the 2ST series. These machines are quieter and smoother with needle bearings. Also the seat is easily adjusted to any position using a gas spring. The weight stack can be changed while seated and can change by increments of 1 pound (0.2248 N). Another line of machines makes use of four-bar linkages to generate a resistance curve. These linkages are loaded with plates (Barnett, 1996). Soper (1995), Tidwell (1995), Scardina (1996), and Bokelberg (1990) have documented techniques of designing these four-bar linkages.

Another type of machine that creates a resistance curve is produced by Strive. This is a most recent idea for generating strength curves. The concept is based on placing plates on a rotating plane. As they are rotated through the exercise motion, the imbalance forces will create a varying strength curve. This concept is shown in Fig. 2-2. As each plate is changed, the strength curve changes. This allows for an adjustable strength curve for each person. However, the use of this type of machine does seem confusing (Brown, 1996).

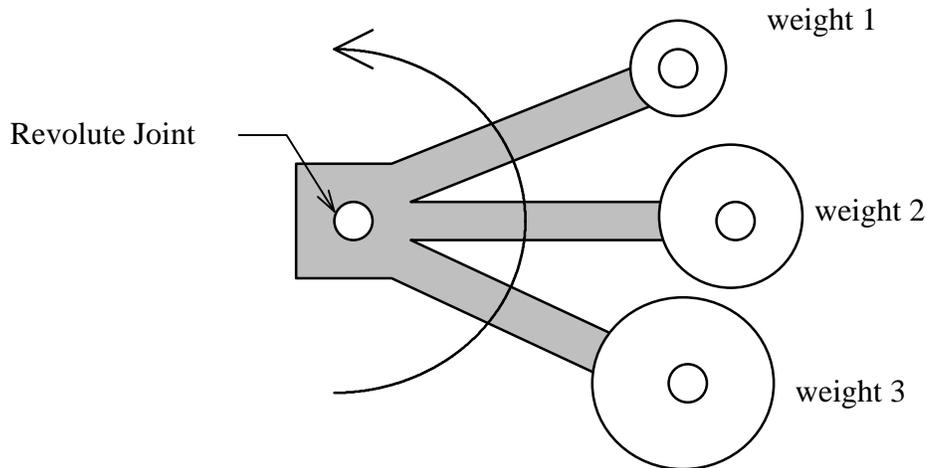


Figure 2-2, The Strive Concept

Cybox has developed an isokinetic electronic machine (Dibble, 1989). The velocity of the exercise motion is controlled electronically. Hydra Fitness is another isokinetic machine that uses hydraulic actuation to control the speed (Dibble, 1989). Again, these machines do not provide eccentric training.

All of these machines are high-end machines typically found in fitness clubs. There is also a large range of home exercise equipment for resistance training. A very popular example is the SoloFlex shown in Fig. 2-3.

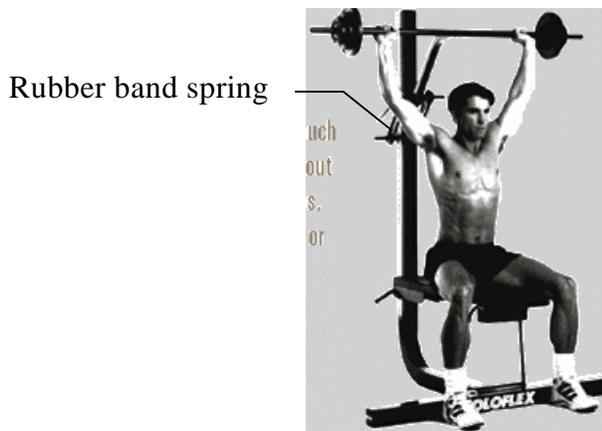


Figure 2-3, SoloFlex Machine
 (Adapted from SoloFlex, <http://www.soloflex.com>:81/)

This machine uses an elastic band as the source of resistance. However, the resistance curve provided by the elastic band is a linearly increasing force starting at zero. This

results in poor resistance curves for the user in most exercises. The BowFlex, shown in Fig. 2-4, is another example of a machine that uses spring energy.

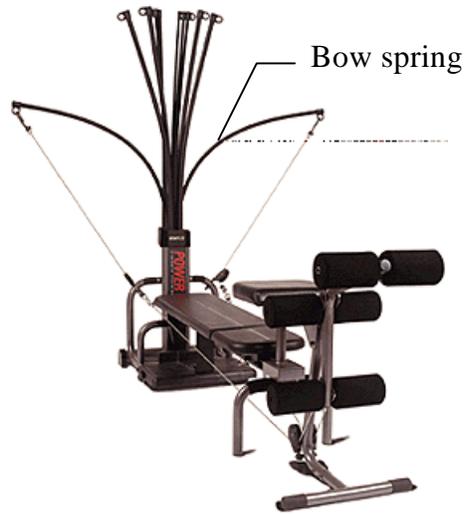


Figure 2-4, BowFlex Machine
(Adapted from BowFlex, <http://www.bowflex.com/>)

The NordicFlex Gold is an isokinetic machine for home use. These machines do not use weights, but have other sources of resistance. Another machine was created by NordicTrack called the NordicFlex UltraLift. This machine uses the person's weight as the primary source of resistance. It is based on a four-bar linkage with a roller under one of the links. The Ultralift concept is shown in Fig. 2-5. This machine was not found until after the final design of this thesis had been completed. CML Group Inc., the owner of NordicTrack, announced the release of the NordicTrack UltraLift on November 21, 1996 (PRNewswire, 1996). It is interesting to notice how similar it is to some of the initial four bar concepts that are documented in Chapter 3. This idea does eliminate the need to change cable lengths. It also allows the resistance to be adjusted without lifting the linkage.

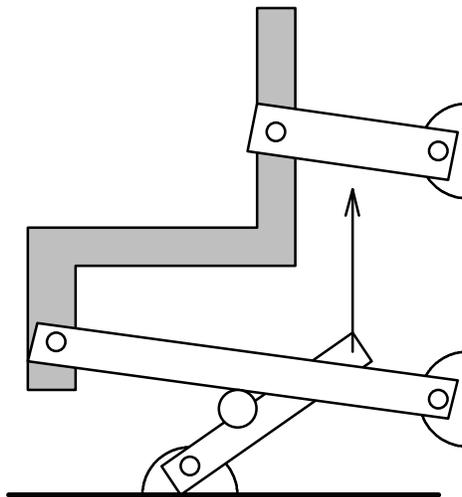


Figure 2-5, UltraLift Concept
(Adapted from NordicTrack, <http://www.nordictrack.com/>)

2.4 The Defined Need

Based on the topics discussed in this chapter, it is good to define a clear need. This need is not a specific request, but a sensing of a need. There is a need for strength training to improve overall health. Eccentric training is important for building muscle. The strength curve should also be considered. Home exercise machines are attractive to people who would rather save time, money and avoid the inconvenience of a gym. The

retail cost should be under \$1000 to compete with similar exercise equipment. The need for using a person's weight exists because it is fun and attractive. It also saves cost for materials and manufacturing.

Another way to define the need is to make a goal statement.

A home exercise machine that uses a person's own weight as the source of resistance. This resistance must be variable for people of different strengths. This machine needs to include eccentric exercises and the resistance of each exercise should be designed with the strength curve in mind. The machine should be competitively priced for less than \$1000 and offer a complete range of exercises.

Chapter 3, Conceptualization / Creativity

This chapter discusses the “conceptualization of solutions” stage in the engineering design process. This stage immediately follows the “identification of need”. This phase initiates the synthesis of the final solution.

3.1 Initial Concepts

The general concept developed from the need statement was to develop a mechanism to lift the person vertically. This mechanism needed to allow adjustments in the mechanical advantage without lifting the user’s weight. Additionally, since the motion of an exercise stroke is linear, the required input motion was linear. A linkage was conceived with only prismatic joints causing linear motion of each joint. This idea appears in Fig. 3-1. The platform link is used to support a platform for the user. Therefore, the output motion will raise the user. The adjustment link is used to vary the resistance by varying how high the user is raised. Since prismatic joints are expensive and difficult to implement, this linkage was modified by replacing prismatic joints with revolute joints.

3.2.1 Evolution of Prismatic Mechanisms to Revolute Mechanisms

While prismatic joints produce the desired motion in a conceptually simple way, these joints have several drawbacks in application. They are relatively expensive to produce, and they are prone to higher frictional forces and stick-slip motion. This led to a systematic attempt to replace all prismatic joints with revolute joints. A prismatic joint can be replaced by straight line linkages such as Watt’s four bar linkage. Each Watt’s linkage adds four more revolute joints. The Watt’s mechanism is shown in Fig. 3-2. The two rockers in this linkage are of equal length and the midpoint of the coupler forms an approximately linear coupler curve.

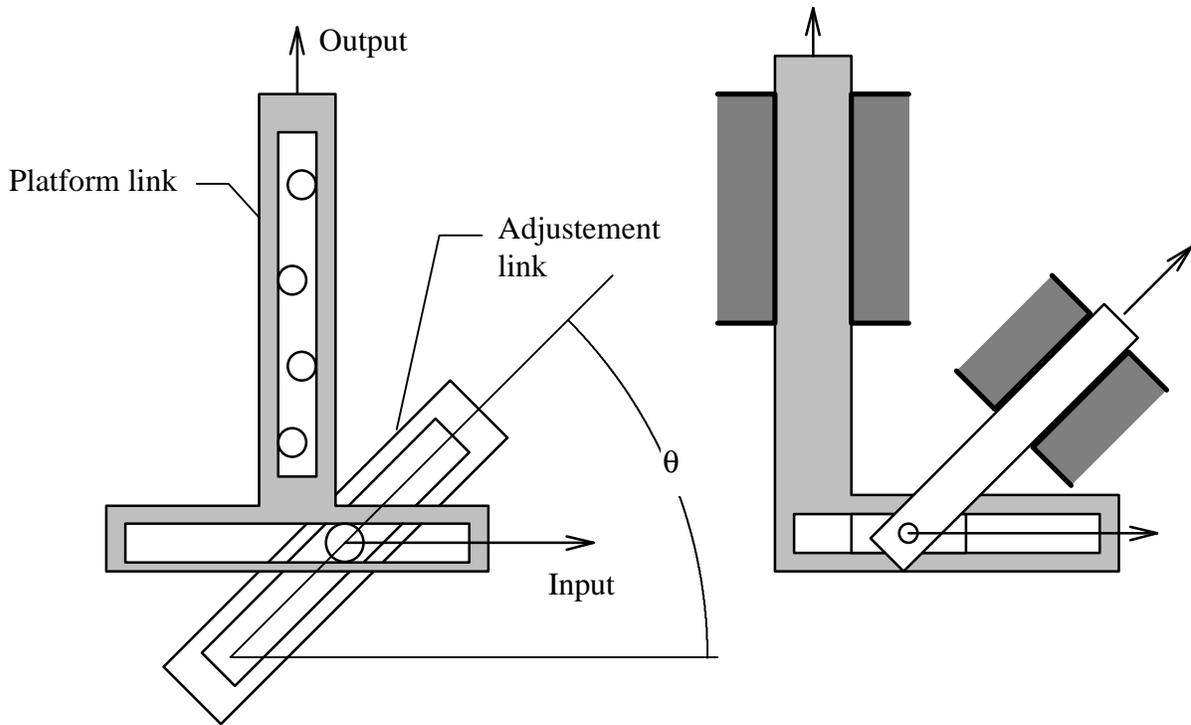


Figure 3-1, Fully Prismatic Linkage

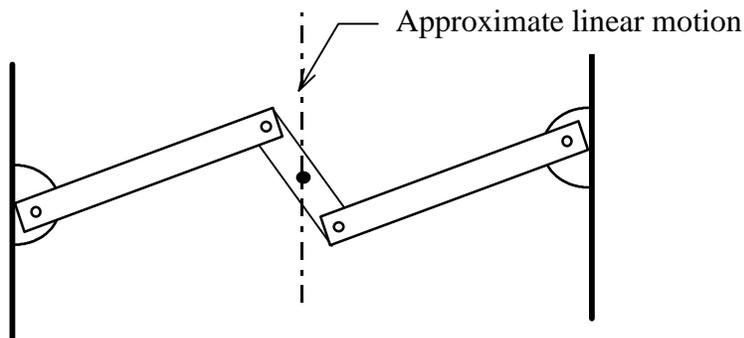


Figure 3-2, Watt's Straight Line Mechanism

Figure 3-1 shows a schematic of a fully prismatic linkage. This linkage has 1 revolute joint and 3 prismatic joints. There are a total of 4 links including the ground link. Gruebler's equation (Gruebler, 1917) for planar linkages shows that there is 1 degree of

freedom.

$$m = 3(n - 1) - 2 f_1 - f_2$$

$$m = 3(4 - 1) - 2(4) = 1$$

- m : Mobility or degrees of freedom
- n : Number of links $n \neq 0$ including ground
- f_1 : Number of one degree of freedom joints
- f_2 : Number of two degree of freedom joints

The vertical prismatic joint was replaced using two Watt's four bar linkages, as shown in Fig. 3-3. This linkage has 11 revolute joints, 2 prismatic joints and 10 total links including ground. This linkage does have a mobility of one.

$$m = 3(n - 1) - 2 f_1 - f_2$$

$$m = 3(10 - 1) - 2(13) = 1$$

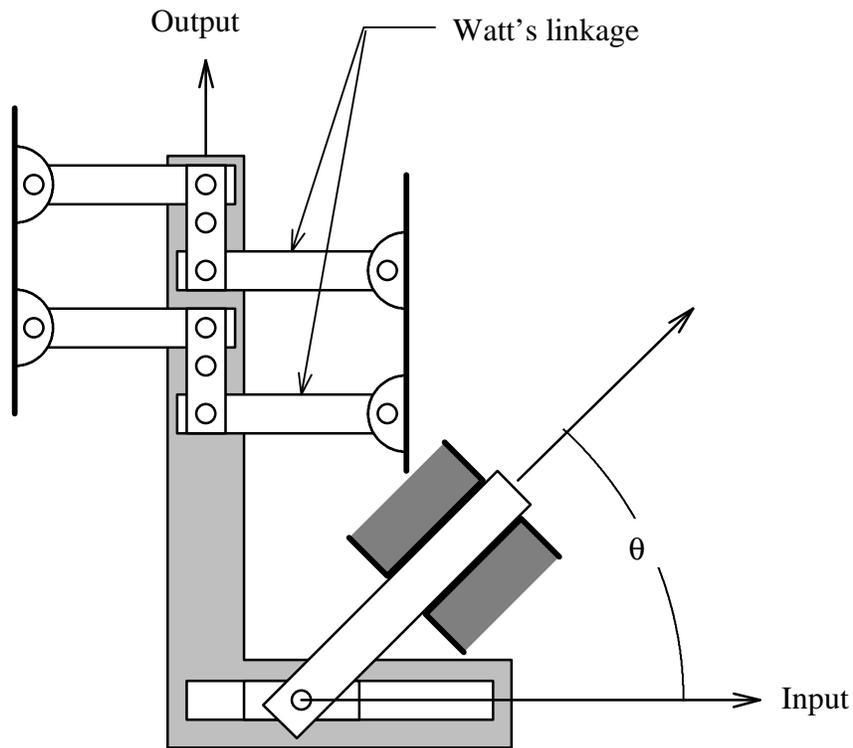


Figure 3-3, One Replace Prismatic

Next, the adjustable prismatic joint was replaced with a single Watt's linkage. One linkage was used since this does not require keeping another link in constant orientation. This schematic is shown in Fig. 3-4. This linkage contains 1 prismatic joint and 15

revolute joints. There are a total of 12 links including ground. The mobility of this linkage is also one.

$$m = 3(n - 1) - 2f_1 - f_2$$

$$m = 3(12 - 1) - 2(16) = 1$$

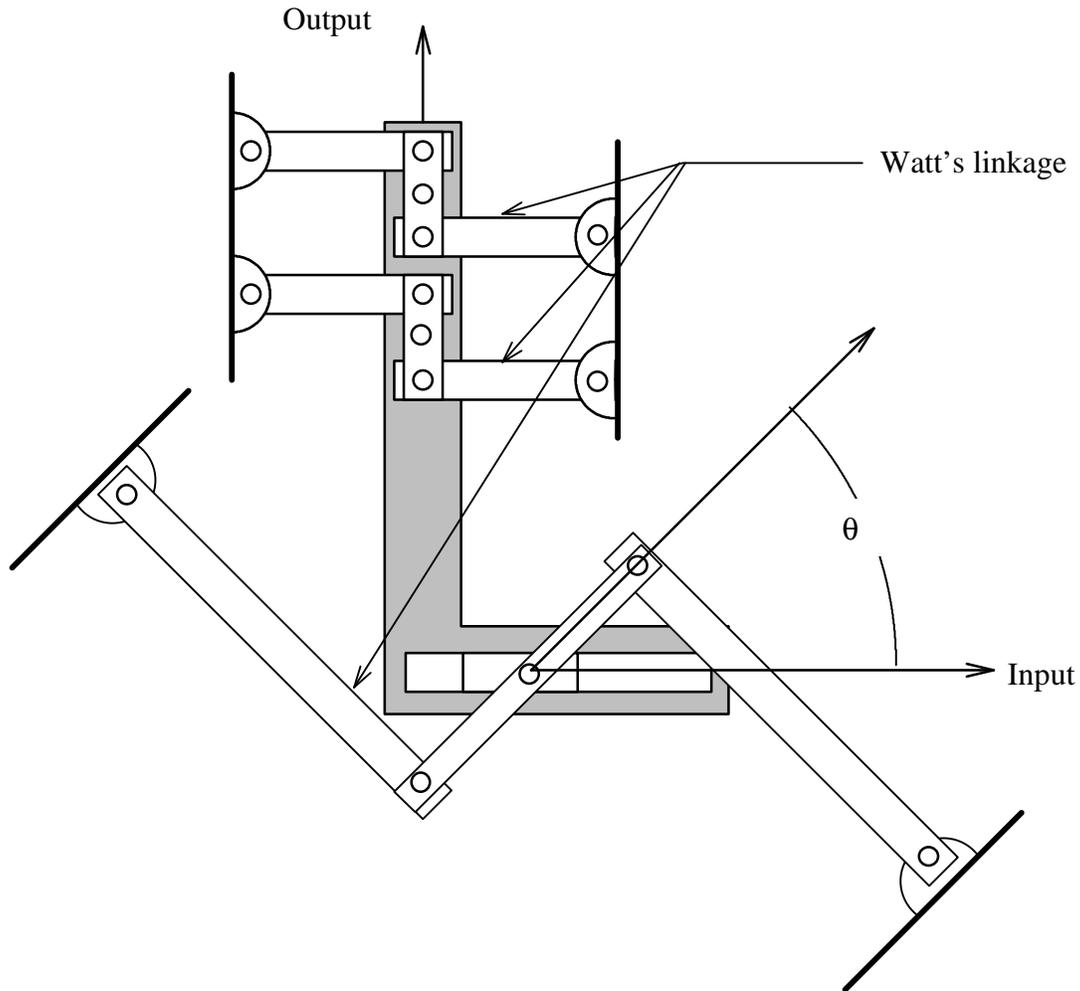


Figure 3-4, Two Replaced Primitives

The most complex straight-line-equivalent mechanism was created by replacing the last horizontal prismatic joint with another Watt's four bar. This is shown in Fig. 3-5. This linkage has 19 revolute joints and no prismatic joints. There are a total of 14 links. The mobility of this linkage is still one.

$$m = 3(n - 1) - 2f_1 - f_2$$

$$m = 3(14 - 1) - 2(19) = 1$$

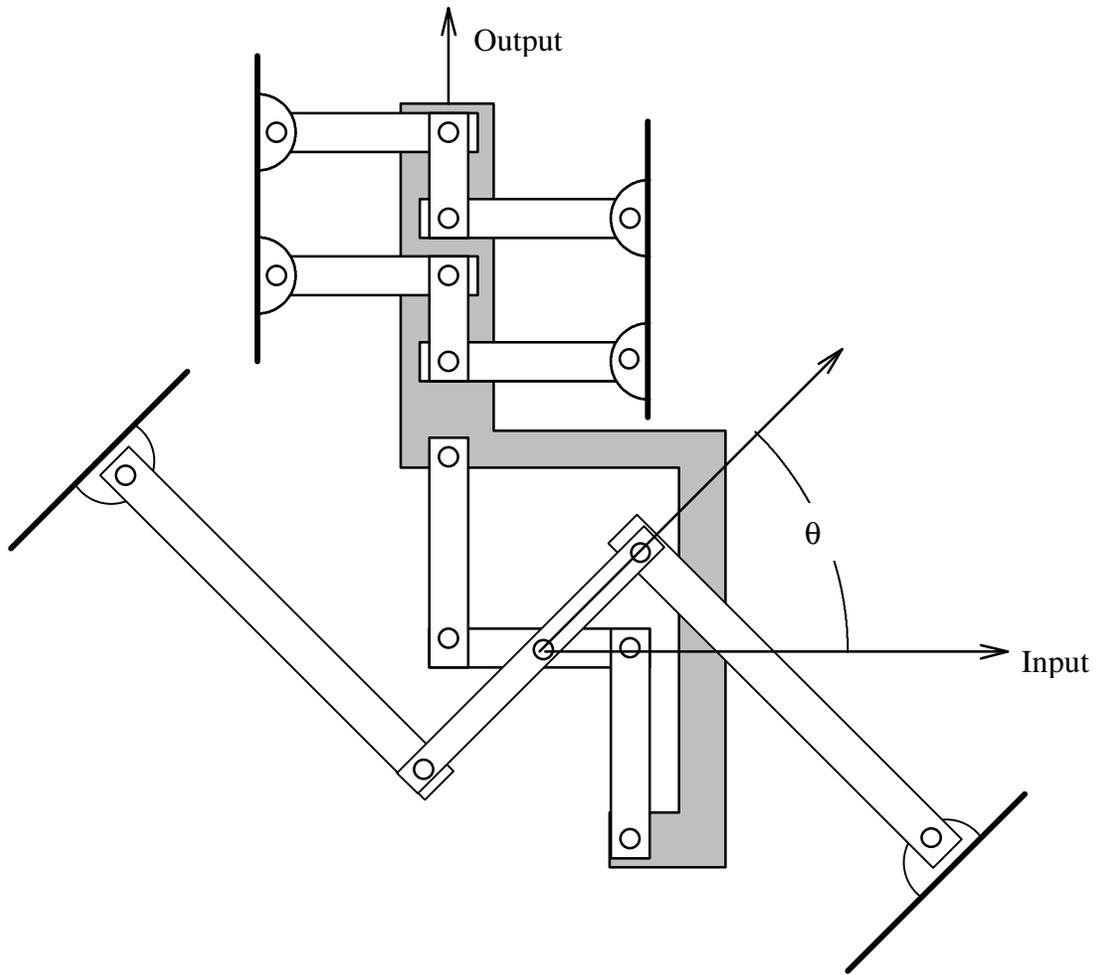


Figure 3-5, Fully Revolute Linkage

Each one of these linkages has one degree of freedom. Since, the platform link should maintain a constant orientation, a prismatic joint would most likely be a linear bearing. This would be more expensive to implement than a linkage. However, two Watt's linkages are complicated and add 10 revolute joints to the system. Other concepts of creating a vertical motion for the platform must be considered. The horizontal prismatic joint can still be created with a simple roller.

3.2 Refined Concepts

Concepts were generated to create linear, vertical motion without using a prismatic joint or two Watt's linkages. Linkages were considered first. One linkage idea used a scissors-type linkage as shown in Fig. 3-6.

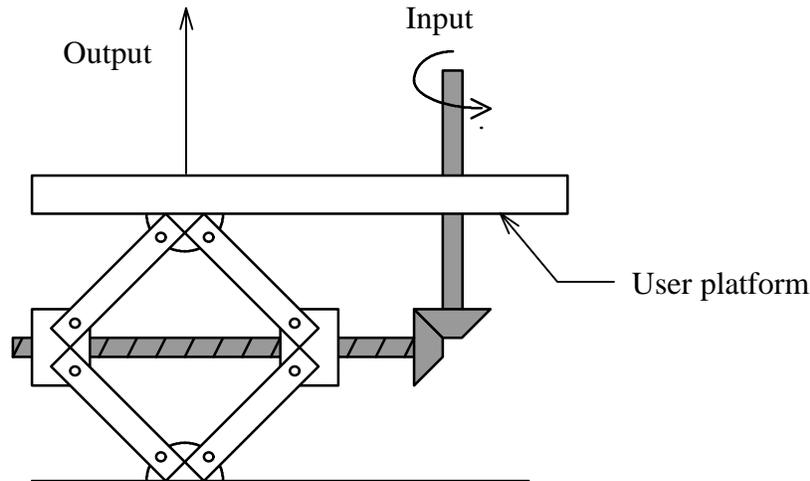


Figure 3-6, Scissors Linkage

This concept also gives a linear vertical lift motion. The linkage is based on rotating a bevel gear set to compress the scissors links and lift the platform. The concept is not complete since some issues are not resolved. The input shaft must remain vertically fixed relative to ground to maintain gear contact. Also, a method of adjusting the resistance is not determined.

The second linkage concept was a four bar with two parallel rockers. A parallel four bar would be a good choice to eliminate links and joints. The arced motion is not ideal for the feel of the user, since the motion is not vertically linear. The radius of the arc depends on the length of the rockers. This compromise was worth investigating. A schematic is shown in Fig. 3-7. In this concept, the four-bar linkage is used to elevate the person. The input is a cable with a roller under the bottom link. As the cable is pulled, the linkage is forced to move upwards.

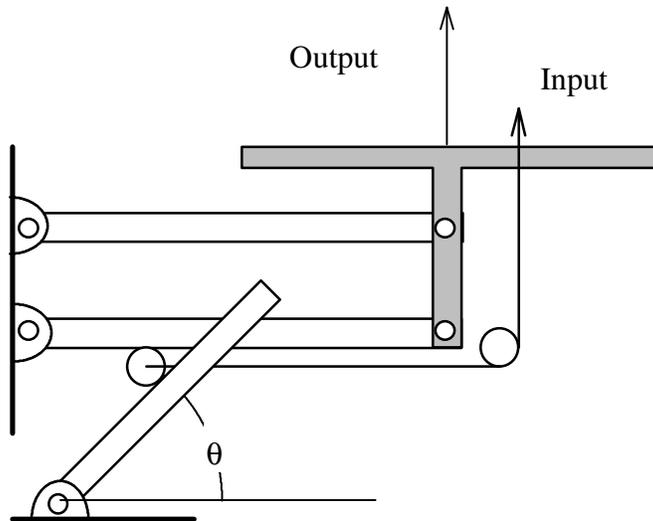


Figure 3-7, Parallel Linkage

Another version of this idea uses a cam under the linkage to replace the roller on an inclined surface. As the cam rotates from the cable tension, the change in radius lifts the linkage. This concept is shown in Fig. 3-8.

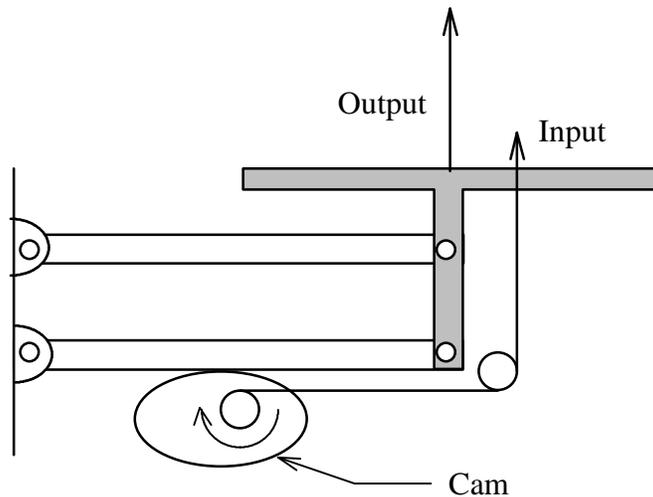


Figure 3-8, Parallel Linkage with a Cam

Another linear motion linkage was found from Mechanisms and Mechanical Devices Sourcebook by Chironis. This eight-bar linkage not only causes linear motion but also keeps the link in a constant orientation. This linkage is shown in Fig. 3-9.

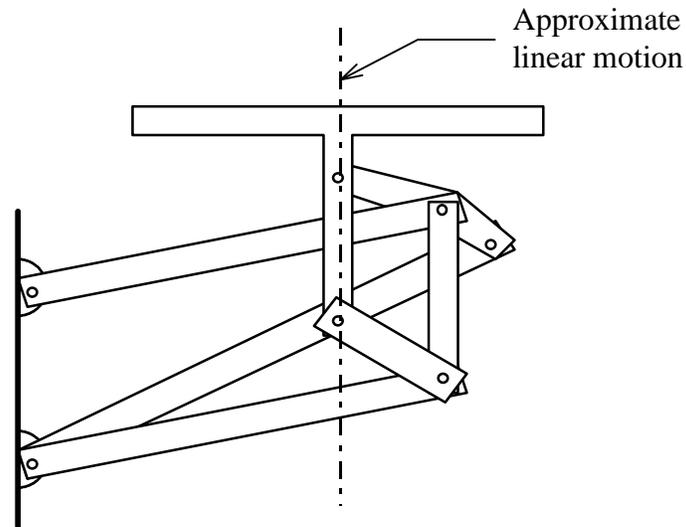


Figure 3-9, Constant orientation linear linkage

Another idea was also considered that was not based on a linkage mechanism. This idea was a hydraulically actuated machine. Resistance can be created by lifting a person's weight hydraulically. A simple schematic of a hydraulic mechanism is shown in Fig. 3-10. This mechanism has an input lever that compresses one cylinder. The fluid is

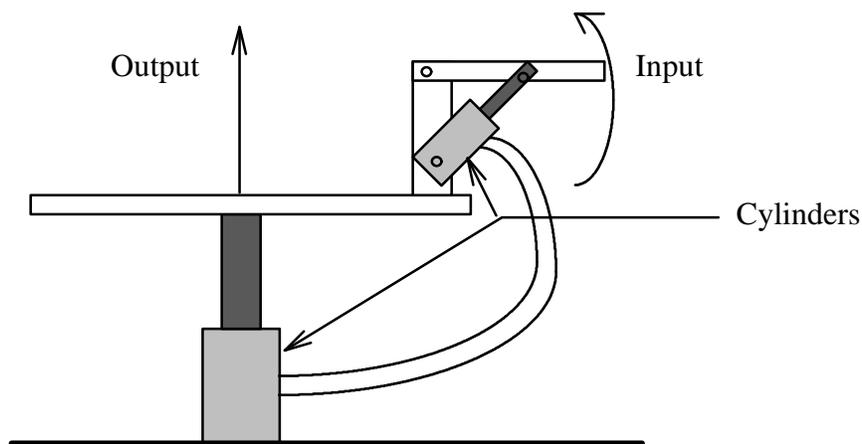


Figure 3-10, Hydraulic Mechanism

then forced into the cylinder below the support platform. As a result, the user is lifting his body weight.

The process of conceptualization led the first concepts toward a parallel linkage with a roller to create two revolute joints. Clearly, during this step of the engineering design process, there is synthesis and some analysis. Each time a problem is found or a new idea comes to mind, the design engineer iterates on the concept.

3.3 Feasibility Assessment

The feasibility assessment is the next step in the design process. A new chapter will not be devoted to this subject since this is a small-scale design project.

The hydraulic lift concept will likely be higher in cost and the required maintenance of hydraulic cylinders is a disadvantage. Another problem with hydraulic cylinders is the potential leaking of fluid. This type of may also have noticeable friction.

Therefore, the focus turned to linkage concepts. In general, radial plate cams are more expensive to manufacture than linkages. Since a roller on a ramp, as shown in Fig. 3-7, will accomplish the same task as a radial plate cam, as shown in Fig 3-8, the linkage with rollers were further considered. One advantage of a radial cam is that its surface can be shaped to control the mechanical advantage. This could be an advantage for designing a resistance curve. There is still another potential problem with a linkage that uses a roller. As the angle of the inclined surface that the roller follows is adjusted, the length of cable may change. The cable length may need to be adjusted for different adjustments in the resistance. Therefore, a new design consideration is that the roller must always start at the same position.

Chapter 4, Establishing Design Requirements

After going through some conceptualization and feasibility analysis, some design requirements become more apparent. Most of these requirements are known after the need is defined and some background research is done. However, after some conceptualization and feasibility analysis, more specifications are found. This topic will be discussed in this chapter even though it is a concurrent step in the engineering design process.

4.1 Human Factors

Human factors data can be directly applied to this project. This information can be found from the Human Factors Design Handbook (Woodson et al., 1992). The important data is included in Appendix A. The available data is for the ranges of the 5th, 50th and 95th percentile of the coed, adult population.

4.2 Design Requirements for the Home Exercise Machine

At this point of the engineering design process, a list can be made of specific design requirements. This list should be as exhaustive as possible. As the design process continues, more can be added to this list. Based on Chapter 2, human factors data, and design for manufacture issues, the following list was compile for the home exercise machine.

- a. The machine must use the person's weight as a source of resistance.
- b. The resistance must be variable for strengths of different people.
- c. The resistance should be reasonably constant.
- d. The machine must include concentric and eccentric resistance.
- e. The retail price should be between \$500 to \$1000.
- f. The machine should be light enough for two people to carry.
- g. The machine must fit through a door opening of 83 in (210.82 cm) by 35 in (88.9 cm).

- h. The operating height should be less than 7 ft (213.36 cm).
- i. The machine should include one major exercise for each muscle group, alterations of these exercises will add to list of exercises. These major exercises include a chest press or fly, a biceps curl, a row, a lateral pull-down, a triceps extension, a leg-press, a leg curl, a calf exercise, and a military press.
- j. There should be no more than 2 adjustments to operate the machine.
- k. The machine must be safe.
- l. The design should have as few parts to assemble as possible
- m. The manufacturing processes should be efficient.

4.3 Organizational Work /Breakdown Structure

The next step is to organize a work structure. This would be a very important step for a large scale project in a large organization. If there are different teams of engineers, each group must know their responsibilities. The method of management is very important here and project engineers are responsible for communicating among the different disciplines.

For a project of this size, the organizational structure is not a major issue. However, a team of two undergraduate students assisted in the fabrication of prototypes and models. Communication was important as concepts were analyzed and prototypes were fabricated.

Chapter 5, Preliminary Design

After a reasonable design concept has been developed, it must be analyzed to determine more specific design issues. These can include motion, forces, and stresses. This step includes additional synthesis and more extensive analysis. It is an iterative process that will refine the design. This step may also reveal that a concept is too difficult to implement if, for example, the motion is unacceptable, or internal forces are too extreme. In this case, the process will have to return to the conceptualization stage.

The concept that has been selected for further development is a four-bar linkage with a roller and an inclined link to adjust the linkage movement. This is seen in Fig. 3-7. Some analysis was performed on this idea to predict the behavior. It is important to perform a kinematic, force and stress analysis on the linkage. This will determine the motion of the links, and the forces that the user must input. Internal forces are also important for stress and fatigue analysis to avoid failure. The velocity and acceleration of the linkage were not considered, since the motion is assumed to be slow. Only static force analysis has been considered.

5.1 Kinematic Analysis

For a preliminary analysis, a free-body diagram approach was simplified with simplifying assumptions was developed. The first assumption was that all of the user's weight is applied at the end of the parallel link in contact with the roller. This is possible if user's weight is applied at the center of the platform in Fig. 3-7. Another assumption was that the links were massless. The last assumption was that the roller is a point. The geometry determined the force vector directions. A free-body diagram of the lower link is shown in Fig. 5-1.

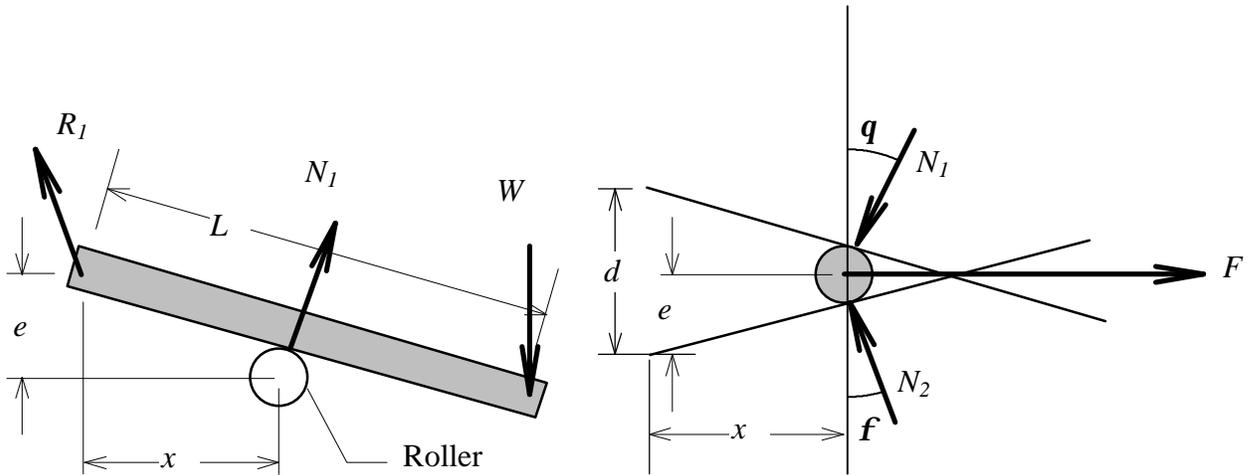


Figure 5-1, Free Body Diagrams of Initial Design

In this problem, the force W and the angle f are given and the user input force F is to be determined. Since the input force should be a function of stroke, the angle of the links is defined in terms of the roller position x . The angle f is a fixed angle set by the user. It determines the amount of resistance by varying how far the platform will lift.

$$\begin{aligned}
 (f) &= - \quad = \quad (f) \\
 q &= - \quad q = - \left\{ \frac{-}{-} \right\}
 \end{aligned}$$

Summing moments of the link about the fixed pivot allows the normal force N_1 on the roller to be found. This normal force can then be used to determine the input force F .

$$\begin{aligned}
 & \left[\begin{array}{c} +(-) \\ + \quad - \quad f \end{array} \right] - (q) = \\
 & = \frac{q}{\left[\begin{array}{c} + \quad - \quad f \end{array} \right]}
 \end{aligned}$$

Once this normal force is known, the forces can be summed in the vertical and horizontal directions for the roller to determine F .

$$\sum \frac{q}{f} = \frac{q + f}{f}$$

The equation for F can be expanded by using the solutions Δt_i and N_2 .

The user force, F, as a function of position x is desired. Matlab was used to solve for this force by using an M file. This is a Matlab specific macro file. The code for this type of file is shown in appendix B. This file is called ffunc_va.m. This file can be used in Matlab to generate plots and data. The results of running this file are shown below and the output plot is seen in Fig. 5-2.

```

» ffunc_va
Enter value of phi:20
phi =
    20
Enter stroke:24
stroke =
    24

```

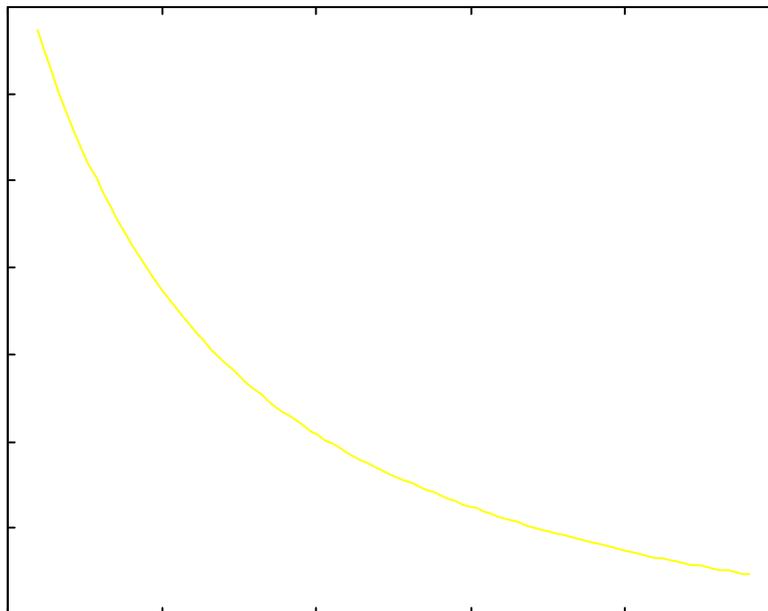
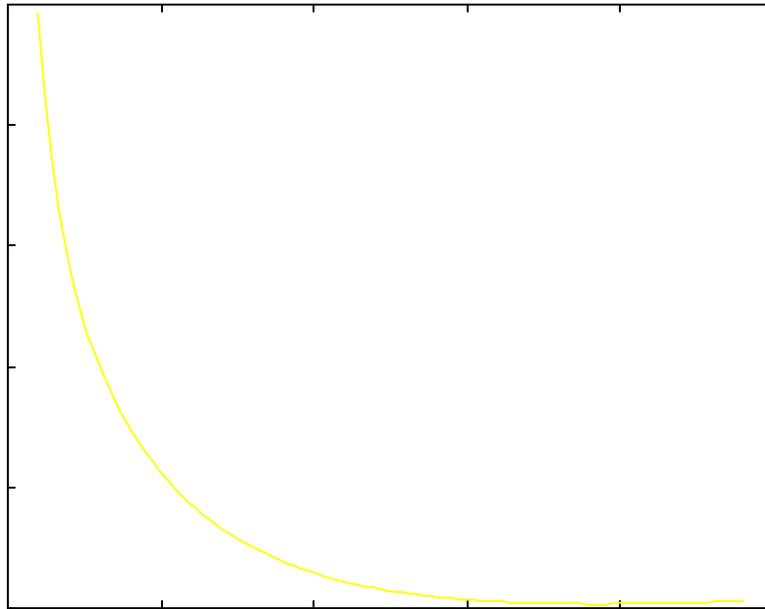


Figure 5-2, ffunc_var.m Output

This M file is an interactive tool, allowing the designer to input the fixed ramp angle and the stroke of the exercise. This allows the designer to focus more on the design parameters and less on tedious calculations. This design tool can be used to iterate through different link dimensions and ramp angles.

After iterating on these parameters, the force curves were still not very constant. Another variation of this linkage was used. In this concept, the roller is placed directly under the platform which is part of the coupler link. In addition, a gas spring was added to the ramp link. A schematic is shown in Fig. 5-3.

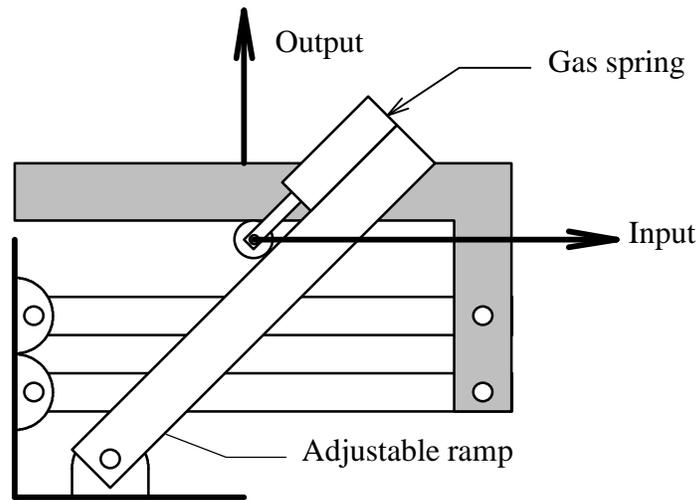


Figure 5-3, Second Roller Schematic

This configuration shows the adjustable ramp link. Similar to the first design, this iteration requires the user to pull on a cable and force the roller to move up the ramp. As a result, the platform moves upward.

The same basic kinematic analysis was done on this mechanism. The forces were determined by using static free-body diagrams. Figure 5-4 shows the free body diagram of the roller. P is the force applied by the gas spring at an angle θ . The angle θ is not necessarily equal to the angle α . The force of P linearly increases at a rate of k .

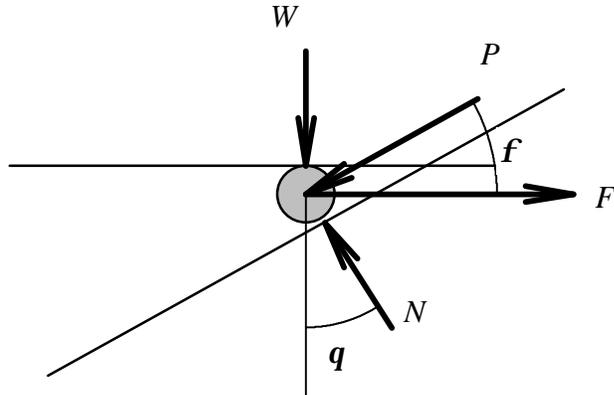


Figure 5-4, Free Body Diagram of Roller Forces

Once again, the assumption was made that the user's weight is applied at a single point and the links are massless. The roller is also assumed to be a point. If the weight is applied on the platform at a point location over the roller, then all of the weight is applied onto the roller. By summing forces in the horizontal and vertical direction, the user's input force (F) can be determined again.

$$\begin{aligned} \sum & \quad - \quad q + \quad - \quad f = \\ \sum & \quad - \quad - \quad f + \quad q = \\ = & \end{aligned}$$

The force P is the gas spring force. This force linearly increases by a rate Kf . This is a characteristic of the spring. Friction around the seal causes the spring force to increase or decrease depending on compression or expansion of the spring. Another M file was created to solve for the user input force (F). This is found in Appendix B. This file is titled `gas_con.m`. The output of this file is a mesh plot of the force as a function of stroke and ramp angle in Fig. 5-5.

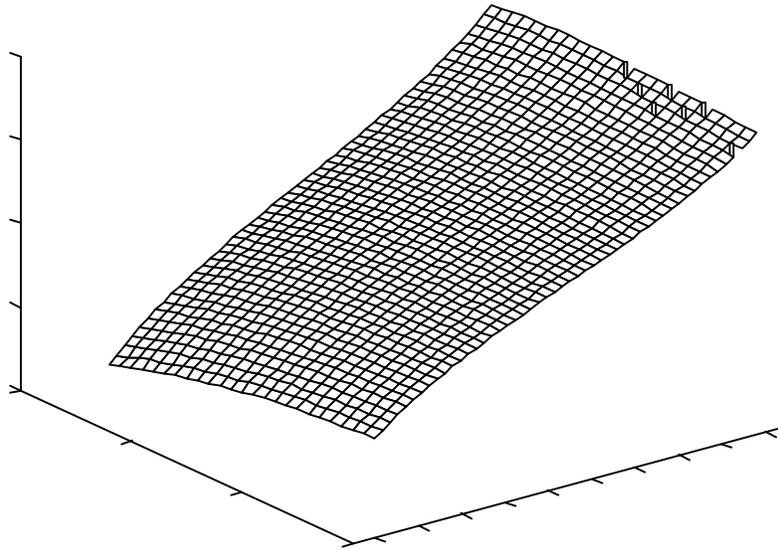


Figure 5-5, gas_con.m Output

The force as a function of stroke is much more constant for this mechanism for a wide range of ramp angles. Therefore, this mechanism was further analyzed with loop closure equations.

5.1.1 Loop-Closure Equations

The loop-closure method is a way to determine the exact displacement, velocity and accelerations of each link. When doing this, each link is represented by a vector in real and imaginary coordinates. The vectors for this machine are represented in Fig. 5-6. For consistency, the vector directions are all pointing up and to the right. The angles of each vector are all measured counter clockwise from the positive real axis.

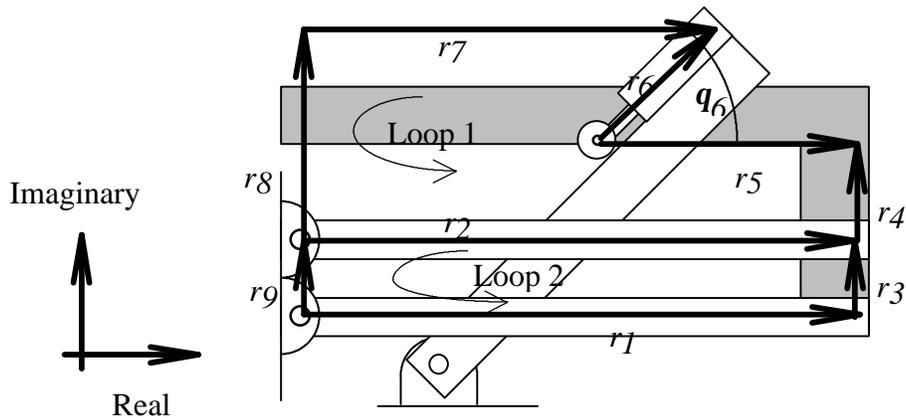


Figure 5-6, Loop Vectors

The vectors can be summed in a loop, for a closed loop the sum is zero. The orientation of \vec{r}_9 , \vec{r}_3 , \vec{r}_4 , and \vec{r}_8 are all assumed to be in the positive vertical direction. Since this assumption is made, loop 2 does not provide any additional information. However, the first loop, loop 1, is very important for position analysis and force generation. This loop can be expressed by the following equations:

$$\begin{aligned}
 &+ \quad - \quad + \quad - \quad - \quad = \\
 &^q \quad + \quad ^q \quad - \quad ^q \quad + \quad ^q \quad - \quad ^q \quad - \quad ^q \quad = \\
 &\quad \mathbf{q} = \frac{\mathbf{p}}{\quad} \quad \mathbf{q} = \quad \mathbf{q} = \quad \mathbf{q} = \frac{\mathbf{p}}{\quad} \\
 &\mathbf{q} \quad + \quad \mathbf{q} \quad + \quad - \quad + \quad \mathbf{q} \quad + \quad \mathbf{q} \quad - \quad - \quad = \\
 &\quad \mathbf{q} \quad - \quad + \quad \quad \mathbf{q} \quad - \quad = \\
 &\quad \mathbf{q} \quad + \quad + \quad \quad \mathbf{q} \quad - \quad =
 \end{aligned}$$

The displacement of interest is the vertical motion of the platform. This can be represented by $\mathbf{x} \sin(\mathbf{q})$. Therefore, the displacement of the platform can be expressed as:

$$\begin{aligned}
 \mathbf{q} &= - \quad \mathbf{q} \quad - \quad + \\
 &= \frac{\quad}{\mathbf{q}}
 \end{aligned}$$

5.1.2 Velocity Analysis

The time derivative of the closed-loop equation must be found to perform a virtual work analysis. Since complex numbers are used, the derivative is straight forward.

Derivatives of constant vectors are zero and left out of the equation.:

$$\begin{aligned} \dot{q} - \dot{q} + \dot{q} &= \\ - \dot{q} + \dot{q} &= \\ \dot{q} + \dot{q} &= \end{aligned}$$

These equations can be used to solve for \dot{q} . This can be done by dividing the imaginary equation by $-\frac{q}{q}$ and adding the two equations.

$$\dot{q} = \frac{\dot{q}}{\left[q + \frac{q}{q} \right]}$$

5.1.3 Virtual Work

The most important piece of kinematic information for this problem is the amount of force or resistance that the user will experience. This can be determined by the method of virtual work. This method applies the principle of energy conservation. The work done on the system by an external force must be equal to work done by system against another outside force. This method requires a differential of the displacement. The easiest way to obtain that is to differentiate with respect to time to obtain velocity. Therefore, the method can be thought of as power conservation. If the weight of the links are neglected, along with friction in the bearings, then the weight of the user and the input force are the only external forces. The weight of the user is applied vertically downward against the velocity of the platform. The input force is transmitted through a cable and is applied horizontally along vector \vec{r}_5 . The virtual work equation is as follows:

$$\begin{aligned}
 \vec{r}_1 \cdot \dot{\vec{r}}_1 + \vec{r}_2 \cdot \dot{\vec{r}}_2 &= \\
 -\dot{q} &= \\
 = -\frac{\dot{q}}{q} &= \\
 &= \frac{\dot{q}}{q}
 \end{aligned}$$

This equation shows that the ratio of forces are proportional to the ratio of velocities. The velocity of vector 5 (\dot{r}_5) is an input that can be arbitrarily assigned a value of 1. The weight of the user is also another arbitrary constant. The angular velocity of angle $\dot{\phi}$ must be determined. This can be found from a velocity analysis of the loop-closure equation.

All of these calculations were automated by a Matlab M file. This was also written to determine the forces as a function of stroke. The M file is in Appendix B and is titled old_ramp.m. The output of this M file is a plot of the displacement and input force as a function of stroke.

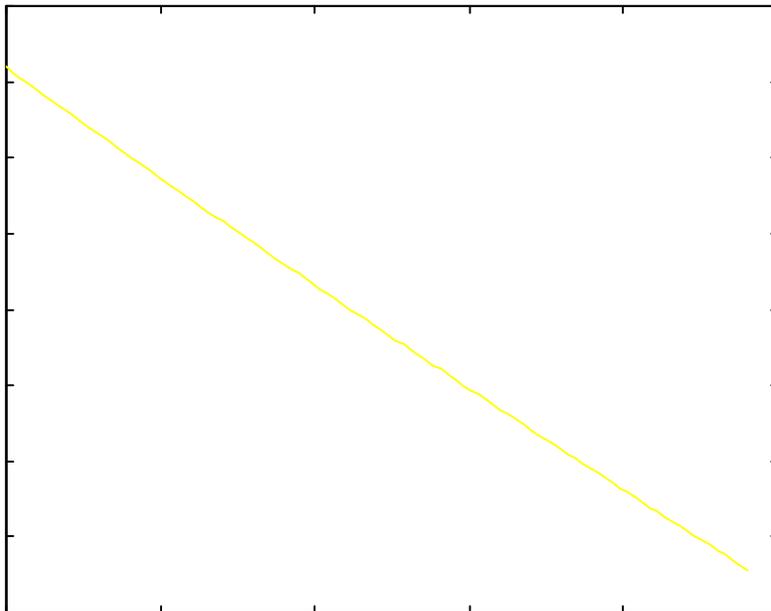
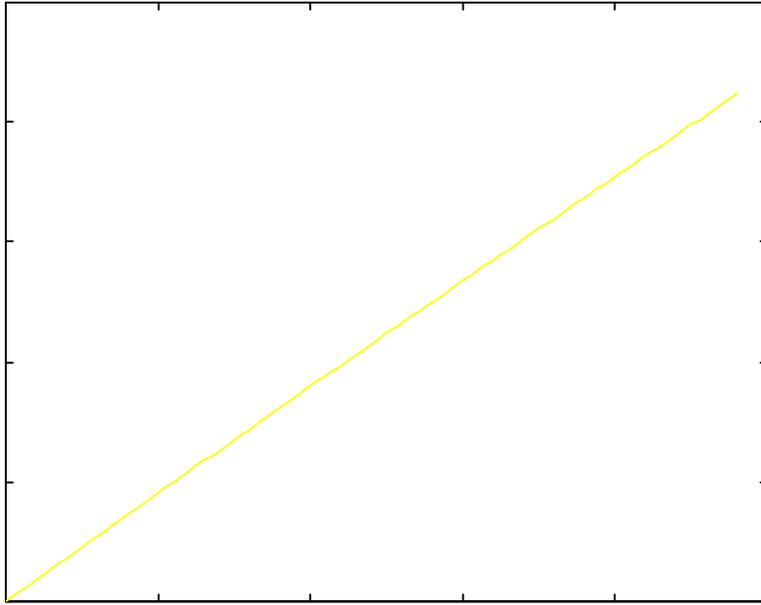


Figure 5-7, old_ramp.m Output

This file is interactive allowing the designer to change the ramp angle for variable resistance. This is a powerful tool to iterate loop closure equations and virtual work

equations. The dimensions for the links (vectors) were set in the M file. They were determined based on a footprint of 5 ft (152.4 cm) by 2.5 ft (76.2 cm).

After using this M file, the results seemed acceptable. The generated forces were relatively constant. At this point, physical models become important.

5.2 Model Making

A model of this concept was needed to test the physical behavior. This will determine if the mathematical model is correct and provide ideas for improvements. A crude prototype was built.

Upon completion of the prototype some problems were discovered. This is important for iterating on the design. The prototype was fabricated with two planes of linkages. Each plane contained rollers that were attached to one shaft. The bar that the rollers were connected to had to remain perpendicular to the cable. This only happened with some help from other people as the machine was used. A carriage of some sort would need to be designed to keep a constant orientation. Another problem was the machine had many parts. Since this design was built with identical linkages in two planes, the machine seemed to complicated, heavy and expensive to fabricate. This did not agree with the specification of minimizing parts. The revolute joints also caused trouble. The links connecting to the joints were not stable. This was due to the fact that the link was angle stock. There was only one plane of material to restrict the link from twisting around the pin. The rollers did not stay in a fixed position while the machine was idle. They could move freely since no forces were being applied to the cable.

All of these problems directed the design back to conceptualization to fix the problems in this design. Then the preliminary design work can be done on an improved concept.

The next chapter is devoted to the preliminary design of a second, improved concept.

Chapter 6, Second Preliminary Design

As discussed in Chapter 5, the first four-bar linkage with a roller under the platform presented problems that warranted a new design. Additional brainstorming with Dr. Reinholtz and two undergraduate students involved in fabrication, focused on overcoming these problems and on solving the problem with the minimum number of parts. Kinematic, force and stress analysis were performed in greater depth for this new design.

6.1 New Ideas

Additional brainstorming resulted in the simple lever shown in Fig. 6.4. This gave birth to a new approach in actuating the parallel four-bar linkage.

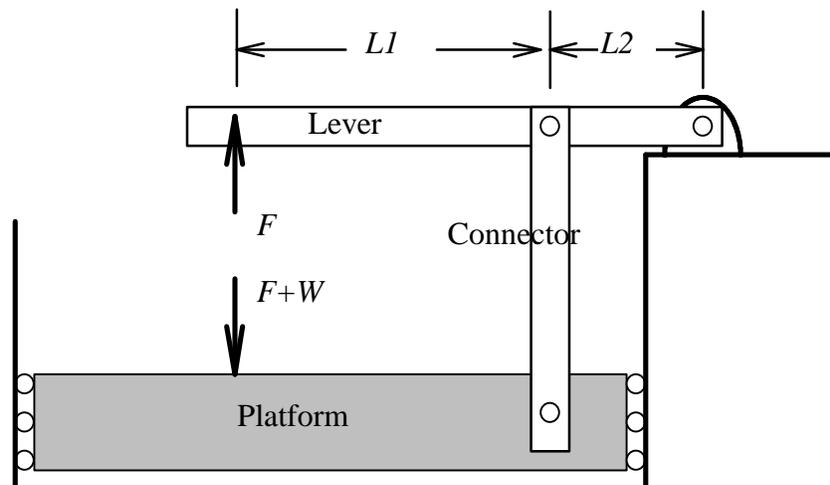


Figure 6-1, Lever Concept

The force (F) is applied by the user pushing up on the leverage bar. The equal and opposite force is applied down on the platform. The weight of the person is also included in the downward force. The motion of the platform as a result of the input force may seem counter-intuitive. However, a simple summation of forces on the lever and platform proves this to be a worthy idea. Figure 6-2 displays the free-body diagrams.

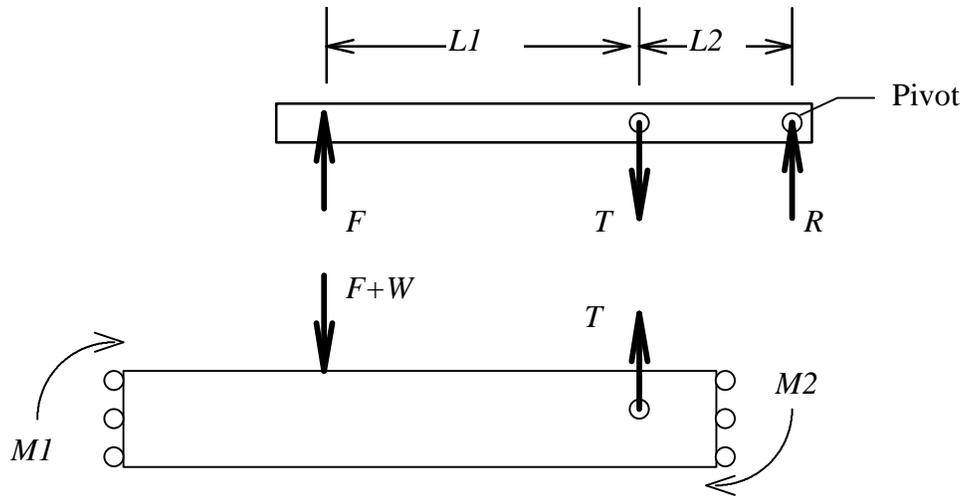


Figure 6-2, Free Body Diagrams of the Lever Concept

Summing moments of the lever bar and solving for T gives the following expression.

$$\sum \text{moments}_{\text{pivot}}: -F(L1 + L2) + T(L2) = 0$$

$$T = F + F \frac{L1}{L2}$$

The vertical forces of the platform can now be found by summing forces in the vertical direction.

$$\sum \text{forces}_{\text{vertical}}: -(W + F) + (F + F \frac{L1}{L2}) = 0$$

$$F = W \frac{L2}{L1}$$

As $L1$ approaches zero, the input force becomes infinitely large. Also, $L2$ approaches zero, the force becomes zero. In order for the input force to be equivalent to the weight, $L1$ would be equal to $L2$.

This lever idea can be adopted to the previous concept that was shown in Fig. 5-3 to remove the roller and the cable. A schematic of this new hybrid concept is shown in Fig. 6-3.

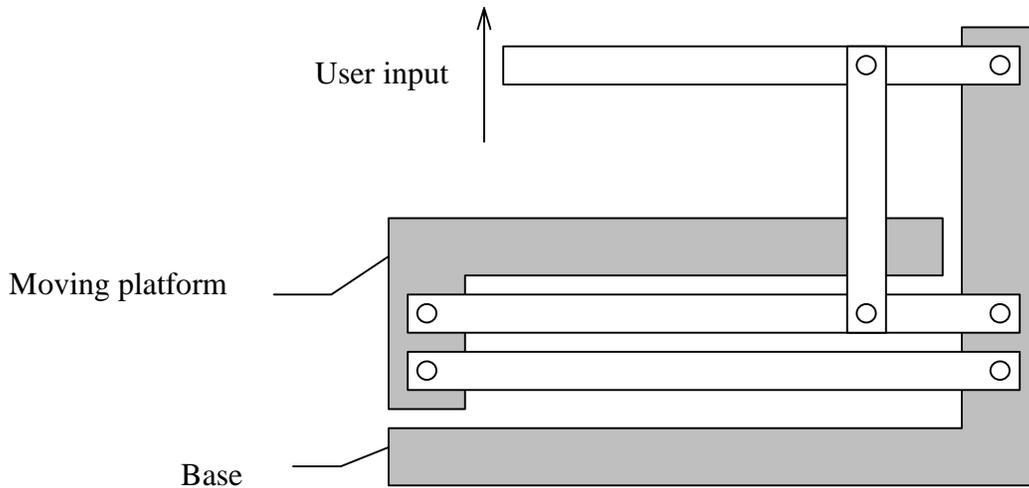


Figure 6-3, Four-Bar Lever Concept

In this concept, the input lever bar moves relative to ground rather than the platform. Such relative motion feels awkward to the user because the motion of the platform relative to the ground changes as the mechanical advantage is changed. In the next iteration, the input leverage link was attached to the platform link as shown in Fig. 6-4. A mobility analysis shows that this 6-bar linkage has one degree of freedom.

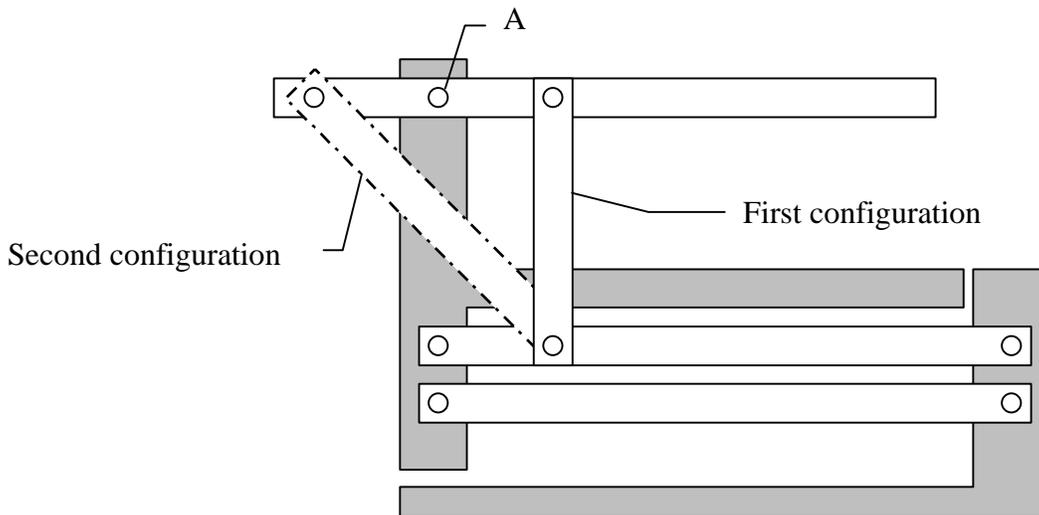


Figure 6-4, Modified Four-Bar Lever Concept

6.1.1 Physical Models

A cardboard model of the concept shown in Fig. 6-4 was created. It revealed that the platform will move up if the bar is pulled down. This behavior did not seem intuitive and shows the importance of physical models. Another realization was the difference between attaching the connector link to the right or to the left of the revolute joint labeled A in Fig. 6-4. If the link is connected to the left of joint A, the lever bar must be pushed up to lift the seat. If the connecting bar is attached to the right of the pivot, as shown in Fig. 6-4, the lever bar must be pulled down to lift the seat. Therefore, this mechanism allows for pushing and pulling exercises. In both configurations, the connecting link is in compression. It was noted that the bottom of the connecting link could be placed to the left of the revolute joint labeled B in Fig. 6-5. This gives the same capabilities and places the connecting link in tension. Tension in this member is desirable to avoid buckling which allows a lighter link to be used. This type of linkage is shown in Fig. 6-5.

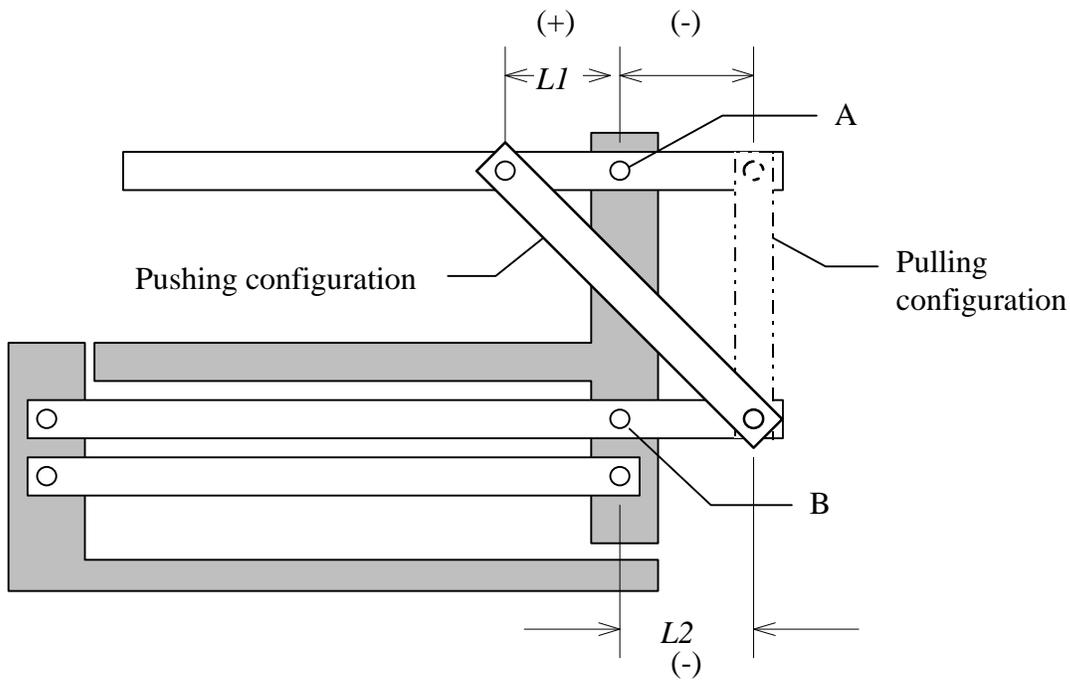


Figure 6-5, Tension Configuration

The model was also used in analyzing the effects of changing the dimensions $L1$ and $L2$. These parameters are used to adjust the mechanical advantage of the mechanism.

If the absolute value of L_I is increased in either the positive or negative direction as seen in Fig. 6-5, the platform displacement increases. This indicates that the input force increases.

6.2 Kinematic Analysis

Displacement and velocity analysis are needed to further study this concept. Velocity is subsequently used to perform a virtual work analysis when solving for the input force. Position analysis begins with a vector loop-closure equation for the mechanism. The vectors representing each link are shown in Fig. 6-6.

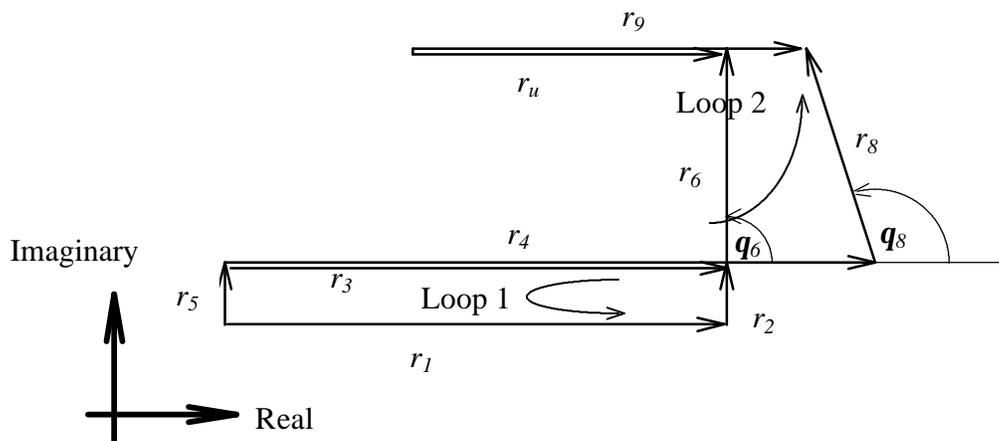


Figure 6-6, Vector Loops For Second Machine

The lengths of vectors \vec{r}_3 and \vec{r}_1 are assumed to be equal and vectors \vec{r}_5 and \vec{r}_2 are assumed to maintain a vertical orientation. The upper loop in the mechanism can be represented by the following closure equation.

$$\begin{aligned}
 \vec{r}_3 + \vec{r}_5 - \vec{r}_2 + \vec{r}_8 - \vec{r}_6 - \vec{r}_4 &= 0 \\
 r_3 \cos(q_3) + r_5 - r_2 \cos(q_2) + r_8 \cos(q_8) - r_6 - r_4 &= 0 \\
 r_3 - r_2 + r_8 \cos(q_8) - r_6 - r_4 &= 0 \\
 (r_3) - (r_2) + (r_8) \cos(q_8) - (r_6) - (r_4) &= 0 \\
 (r_3) - (r_2) + (r_8) \cos(q_8) - (r_6) - (r_4) &= 0
 \end{aligned}$$

The displacement of the platform q , as a function of the user input angle q , is the desired output. This is determined by the stroke of the exercise. In order to solve for q , the vector angle q must be eliminated. This can be done by solving the real equation for

(q) and the imaginary equation for (q) and squaring and adding both equations to eliminate q . This step yields the following equation:

$$= \left[\begin{matrix} + & + & - & - \\ - & q & + & \end{matrix} \right] q + \left[\begin{matrix} - & q \end{matrix} \right] q$$

Using the tangent-half-angle identity, an explicit equation q can be derived. The identities for sine and cosine as a function of the half-angle tangent are as follows:

$$\begin{aligned} q &= \frac{\sin t}{1 + \cos t} \\ q &= \frac{1 - \cos t}{1 + \cos t} \\ &= \left(\frac{q}{1 + q} \right) \end{aligned}$$

Using these identities in the loop closure equation results in a second degree equation in q . This can be solved using the quadratic equation and then t is solved by using the identity of t .

This method of solving for the displacements was automated using a Matlab M file. The file, titled psh_pull.m, is listed in Appendix B. This file is generalized for all types of link closures to allow pushing and pulling exercises.

6.2.1 Velocity Analysis

The velocity of the links is needed as input to the virtual work method of force analysis. The loop-closure equation can be differentiated with respect to time and the real and imaginary components can be extracted. The following equations show this step:

$$\begin{aligned} \dot{q}^q + \dot{q}^q - \dot{q}^q &= \\ \dot{q}^q + \dot{q}^q - \dot{q}^q &= \\ \dot{q}^q + \dot{q}^q - \dot{q}^q &= \end{aligned}$$

The angular velocity of vector q is needed for virtual work analysis. Solving the real and imaginary equations above for \dot{q} gives:

$$\dot{\mathbf{q}} = \begin{pmatrix} - \\ - \end{pmatrix} \begin{bmatrix} \dot{q} & q & -\dot{q} & q & q \\ q & q & - & q & \end{bmatrix}$$

$$\mathbf{q} = \begin{bmatrix} \dot{q} & q & + & \dot{q} & q \\ \dot{q} & q & + & \dot{q} & q \end{bmatrix}$$

This calculation was also automated in the M file titled psh_pull.m in Appendix B.

6.3 Force Analysis

Forces were analyzed to determine the user input force and the internal forces of each link. These user input force was determined by two methods. The first method was the virtual work method and the second was a matrix method.

6.3.1 Virtual Work

The method of virtual work is used to solve for the input force. For the linkage configuration that causes a pulling exercise, shown in Fig. 6-7, the virtual work equation is:

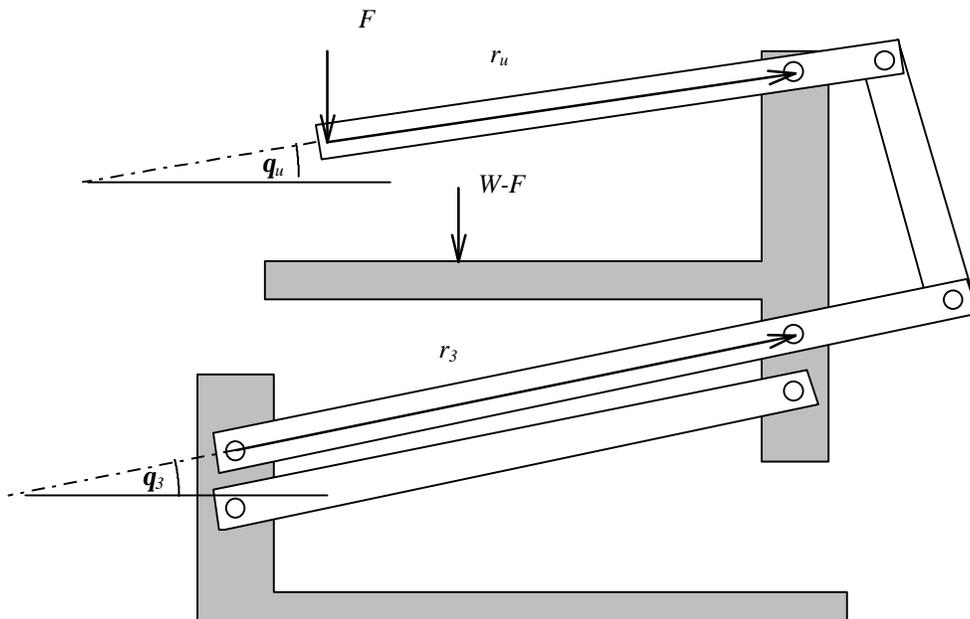


Figure 6-7, Pulling Configuration for Virtual Work

$$\vec{r}_1 \cdot \dot{\vec{r}}_1 + \vec{r}_2 \cdot \dot{\vec{r}}_2 + \vec{r}_3 \cdot \dot{\vec{r}}_3 =$$

$$= -\vec{r}_1 \cdot \dot{\vec{r}}_1$$

After expanding the dot products, the input force F_I can be found.

$$= -\frac{\dot{q}}{q} \frac{q}{q}$$

As expected, the input force is a function of the velocity ratios of the input and output links.

6.3.2 Matrix Method

A force analysis was performed to determine the internal forces for each link. This analysis also determines the input force needed for any position of the mechanism. This analysis will neglect force associated with the dynamic forces for each link. This approach is reasonable, because the machine is intended to be used in a near static mode. In other words, if dynamic forces become significant, the machine is being misused. A matrix method was used to solve for internal forces. A free-body diagram, such as the one shown in Fig. 6-6 was created for each link. A summation of forces and moments yielded three equations for each link. In total, a set of fifteen linear equations was found for the system. F_{ij} is the force link i exerts on link j . This force is shown as components in the x and y direction. Figures 6-8 through 6-12 show free-body diagrams for each link of the machine.

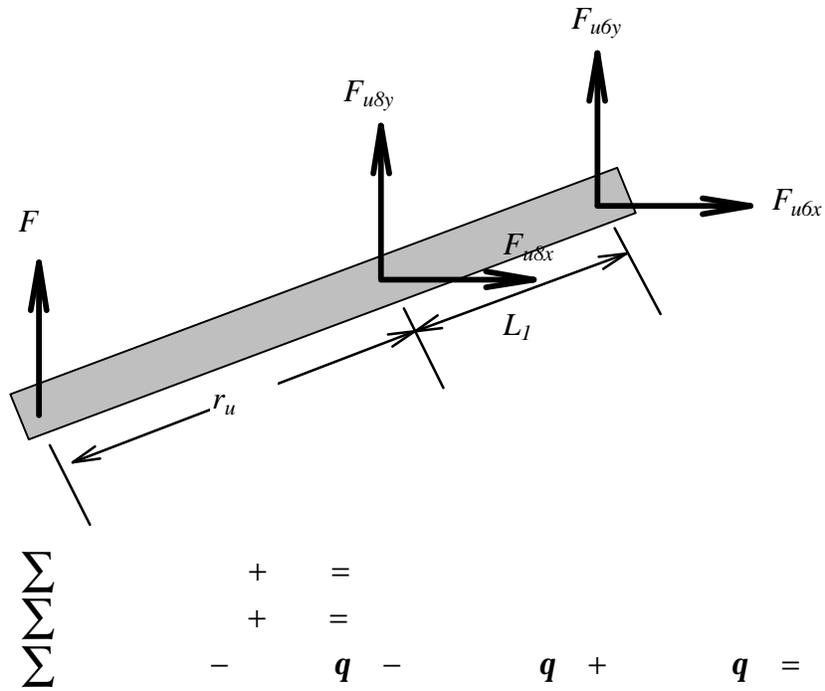


Figure 6-8, Free-Body Diagram of Input Link

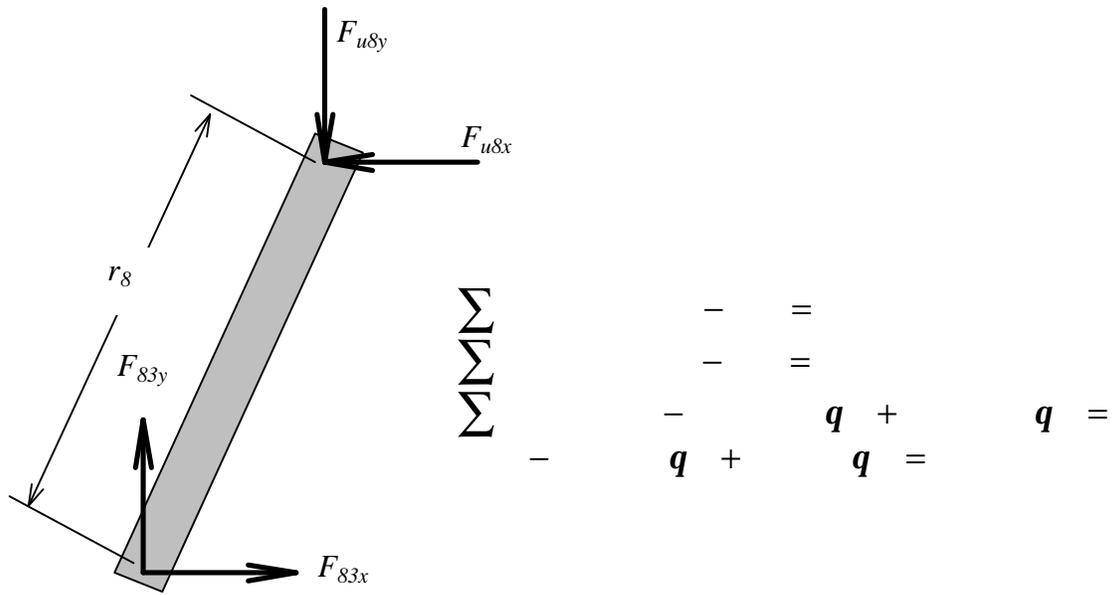


Figure 6-9, Free-Body Diagram of Connecting Link

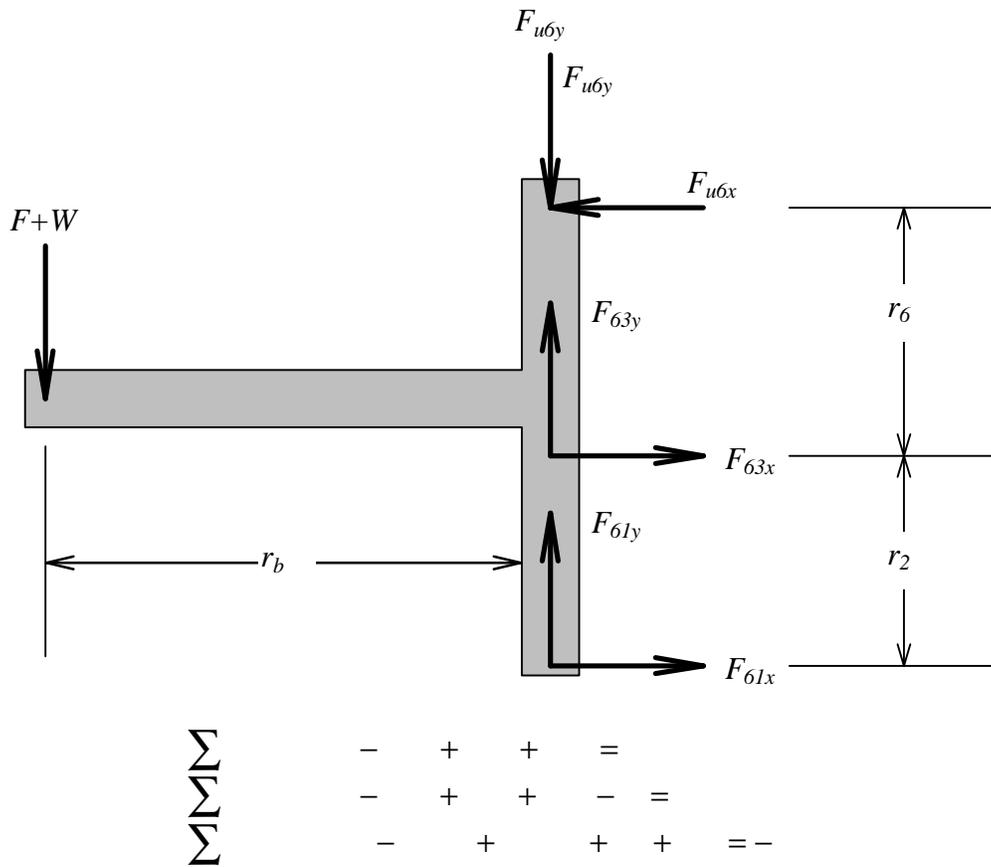


Figure 6-10, Free-Body Diagram of the Platform

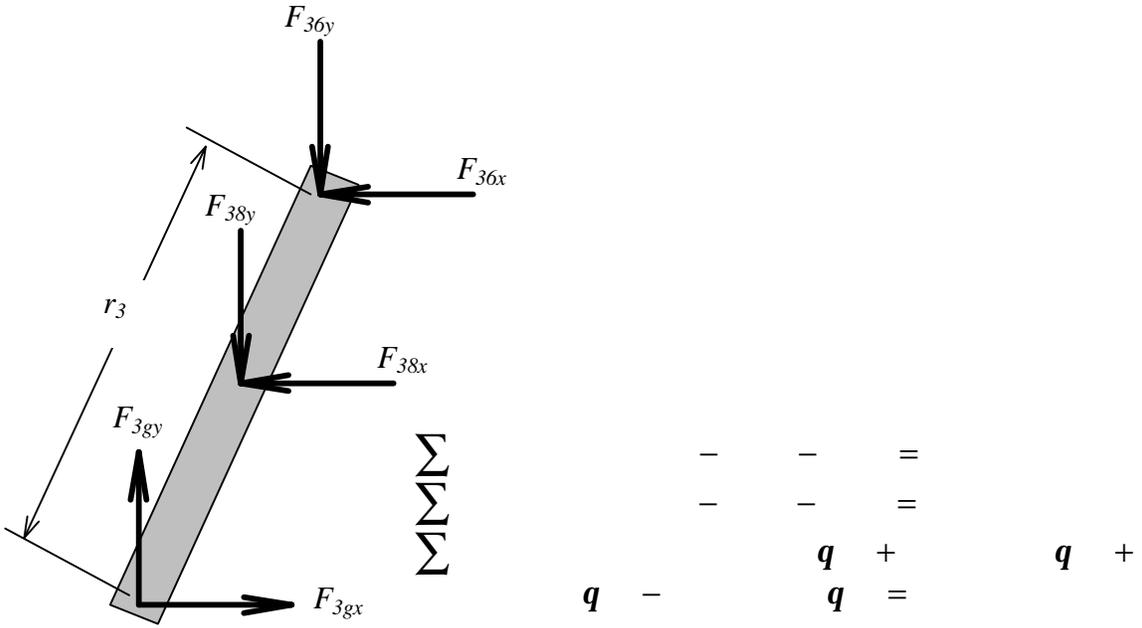


Figure 6-11, Free-Body Diagram of Upper Parallel Link

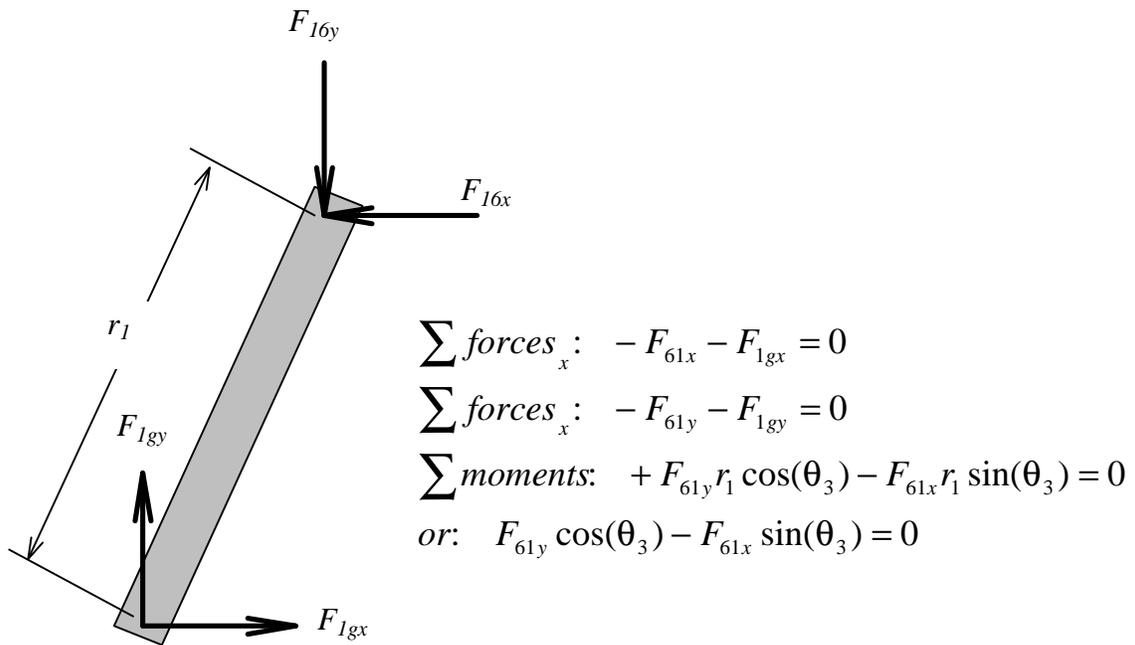


Figure 6-12, Free-Body Diagram of Lower Parallel Link

This linear system is solved using the Matlab file found in appendix B (pin_forc.m.) This file is generalized for all pushing and pulling closures.

6.4 Stress Analysis

The preceding force analysis can be used to find all internal forces in the linkage for a given configuration, input handle position and user weight. The rest of the internal forces provide information to determine the internal stresses for each link.

For two-force members loaded in compression, buckling was considered. If more than two forces were applied to a link, the bending moments were calculated.

The stress analysis was automated by an M file called stress.m. This file is also located in appendix B. This file determines the shear and moment forces in each member. Another file called dimen.m, also found in appendix B, is used to determine the actual dimensions needed to support the internal force loading. The final results from these programs are the dimensions needed for the members of this linkage assuming a safety factor of two.

In this chapter, analytical techniques were used to determine approximate component dimensions needed. Chapter 7 is devoted to the discussion of the automated tools used to perform these analytical techniques. The results of using these tools are discussed in chapter 7.

Chapter 7, Analytical Tools

Computer models and simulations are good examples of automated analytical tools that can aid in making engineering decisions. Code can be written in a variety of computer languages to perform repetitive analysis tasks. Graphics are helpful for and visual modeling. Matlab was chosen for this research since programming is simple flexible. Matlab also offers graphical functions to easily make plots. The focus of this chapter is to present the application of the Matlab tools referred to in Chapter 6.

7.1 M File psh_pull.m

The M file called psh_pull.m is used to analyze the displacement, velocity, and input forces of the linkage. This M file was written to include every configuration of the linkage. A total of four configurations are possible, two pulling and two pushing.

The configuration is determined by user inputs. The positive and negative values of L_1 and L_2 determine the configuration. These two inputs are shown in Fig. 6-5. If the signs of L_1 and L_2 are the same, the closure is a pulling exercise and if L_1 and L_2 have opposite signs, the closure is a pushing exercise. This is important to know for displacement and virtual work equations since the direction of motion of the input bar is needed.

The psh_pull.m file first performs a crude animation of the linkage through the exercise motion. This is done by plotting the vectors for each linkage position. Then the plots of displacement and force input are created.

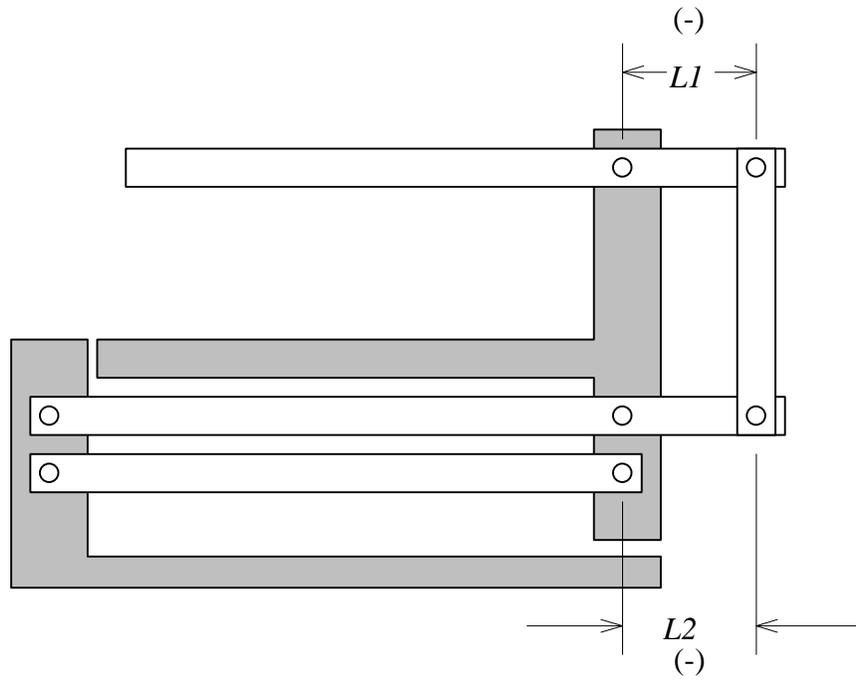


Figure 7-1, Case 1 Pulling Exercise

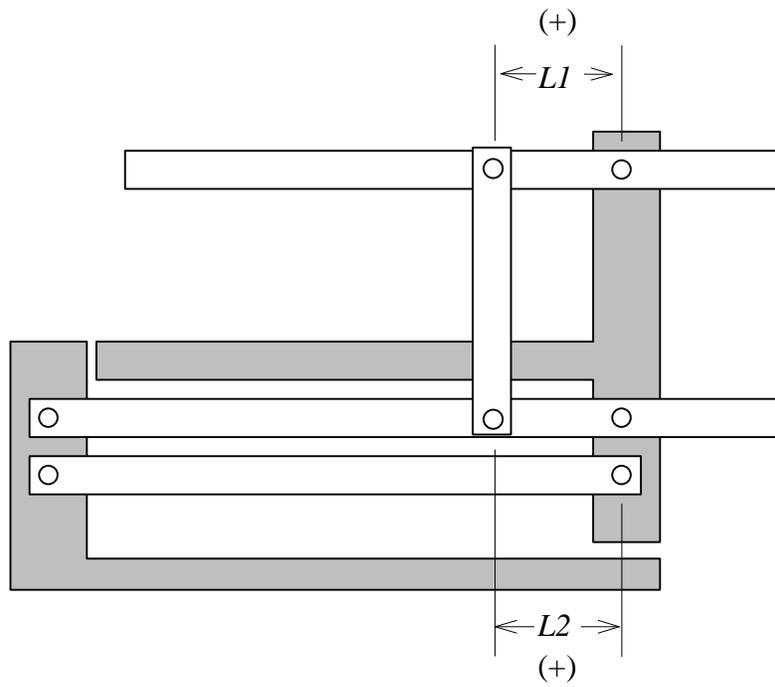


Figure 7-2, Case 2 Pulling Exercise

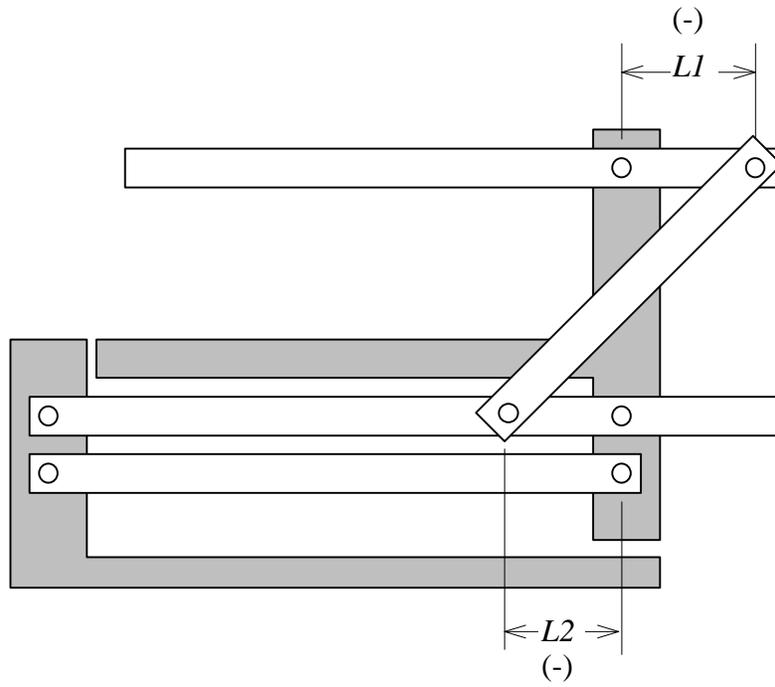


Figure 7-3, Case 3 Pushing Exercise

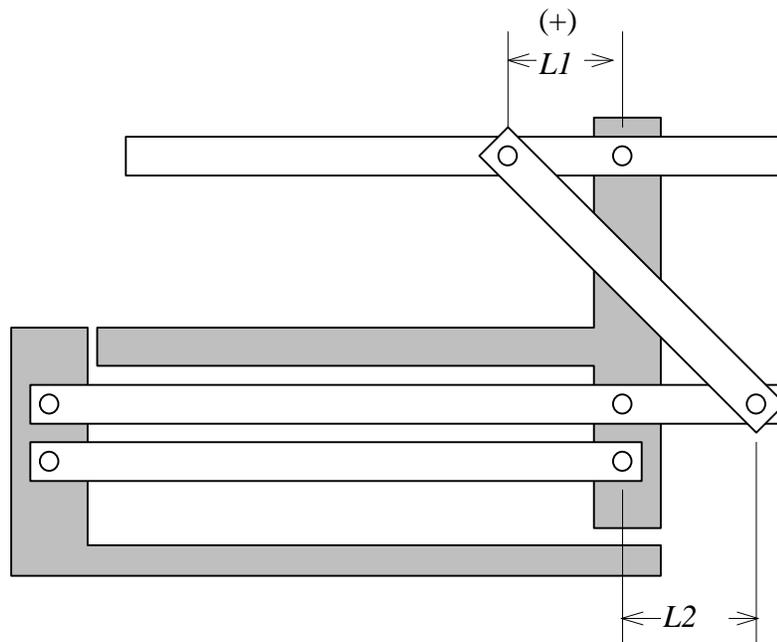


Figure 7-4, Case 4 Pushing Exercise

7.2 M File pin_forc.m

After the displacements and velocities are determined for the linkage, internal forces acting on the pins can be determined. This was done using the matrix method, as described in Section 6.3 of Chapter 6. The M file called pin_forc.m fills in the matrix elements for each matrix. Matlab contains functions to easily manipulate matrices

The output of this file is the internal tension or compression forces of the connector link (r_8) and the lower parallel link (r_7), since they are two-force members. The forces are positive for tension and negative for compression. User input force is also plotted. This has already been determined and serves as a good check to compare the force curves from the virtual work method.

7.3 M File stress.m

The third file that was created was called stress.m. This file determines the internal shear and moments as a function of position in the member. These internal forces are determined by using the pin forces calculated from pin_forc.m. Shear and moment diagrams are determined for each position of the linkage. The maximum shear and maximum moment are stored for each of the links at each linkage position. The output is a plot of the shear and moment diagram for each member that has internal bending (r_8 and r_7). These output plots represent the shear and moment diagrams corresponding to the position of maximum shear and maximum moments. The maximum shear in a link does not necessarily occur at the same linkage position as the maximum bending moments. Therefore, the shear and moment diagrams for one link may not correspond to the same position. The maximum stress is of interest since the parts of the mechanism should be designed for maximum loading.

7.4 M File dimen.m

This file is also interactive and determines the necessary dimensions of the link cross sections. This file was written with the assumption that rectangular tubing is used to create each link. The user must input the outside dimensions of the cross section and the

material being used. The length of the links have already been assigned a value from previous calculations. This file determines the wall thickness of the tubing based on the stress calculations. Figure 7-5 shows the dimensions of an arbitrary cross section of rectangular tubing.

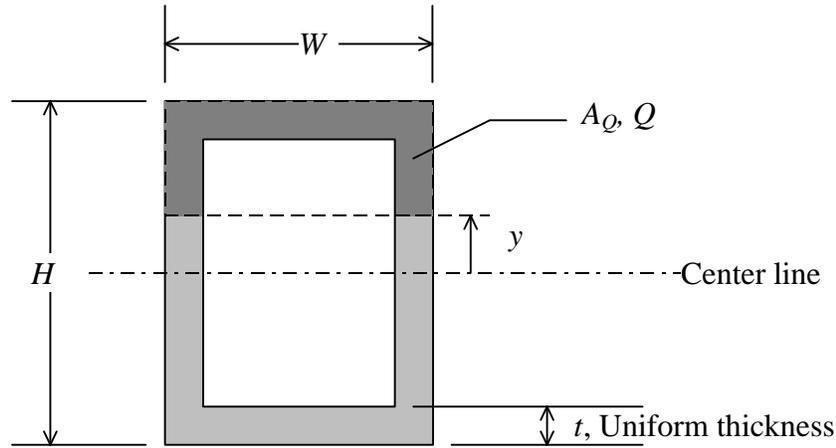


Figure 7-5, Cross Section of Rectangular Tubing

The file determines the needed wall thickness of the link based on the transverse shear stresses.

$$\text{Transverse shear stress: } \tau = \frac{VQ}{It}$$

Q is the first area moment of the section above the point of interest in the cross section. Q is a function of area A_Q and centroid (\bar{y}) of the section above the point of interest located at y . The point of maximum shear stress is at the center of the cross section. Q was simplified for this point as follows:

$$Q = \frac{W}{2} \left(\frac{H}{2} - y \right) \left(\frac{H}{4} - y \right) + \frac{W}{2} \left(\frac{H}{2} + y \right) \left(\frac{H}{4} + y \right)$$

Using this equation, the shear stress was determined as a function of H and W .

$$\tau = \frac{V}{It} \left[\frac{W}{2} \left(\frac{H}{2} - y \right) \left(\frac{H}{4} - y \right) + \frac{W}{2} \left(\frac{H}{2} + y \right) \left(\frac{H}{4} + y \right) \right]$$

substituting for $I(W, H, t)$,

$$\tau = \frac{V}{t} \frac{\left[\frac{W}{2} \left(\frac{H}{2} - y \right) \left(\frac{H}{4} - y \right) + \frac{W}{2} \left(\frac{H}{2} + y \right) \left(\frac{H}{4} + y \right) \right]}{\left[\frac{W}{12} \left(H^3 - 4Hy^2 + 6y^3 \right) + \frac{W}{12} \left(H^3 - 4Hy^2 + 6y^3 \right) \right]}$$

The ultimate shear strength of the material is assigned to τ . The wall thickness (t) can be solved numerically using Newton's method. Assuming a value of 1/8 in (0.3175 cm) and solving the shear stress and the bending stress, shows that the bending stress dominates. The bending stress is about 7 times larger than the maximum shear stress. The m file always provides the largest needed thickness based on the highest stress, which in this case is due to bending. The links are long enough to design for bending stress only.

The needed area moment of inertia due to bending stresses is determined for each link based on the bending stress equation.

$$\text{Bending stress: } \sigma = \frac{M}{I} = \frac{P \cdot L}{I}$$

Only the maximum internal moment and shear are used for these calculations. This maximum is found for any part of the link for all positions of the linkage. The yield strength of the selected materials is assigned to σ . Then the area (A) and area moment of inertia (I) are determined. Geometry of the cross section is then used to determine the thickness needed to meet the area and area moment of inertia requirements. The maximum thickness found from shear and normal stress calculations are used.

For two-force links that are in compression, the buckling equation is considered along with the normal stress. The maximum safe link length is given by:

$$L = \frac{\pi}{2} \sqrt{\frac{EI}{P}}$$

The compression force (P) is the largest compression force determined for all positions of the linkage. The area moment of inertia (I) is calculated from this equation.

$$I = \frac{P L^2}{\pi^2 E}$$

The thickness of the tubing wall is also calculated from this area moment of inertia. The largest thickness from the normal stress calculation and the buckling equation are used.

7.5 Results of Automated Tools

These M files were created to be used as design tools. They allow the user to pick dimensions of the mechanism and see the results. An M file called design.m was written to call all the M files in sequential order. This global program saves effort for the Matlab user. An example of the interaction with this code, using a weight of 150 lbs (667.2 N), and the output follow. Figures 7-6 through 7-16 are the output figures.

```
» design
Max stroke can be ruser.
ruser =
    30 (76.2 cm)
Enter stroke of exercise: 20 (50.8 cm)
Enter the connecting distance on input link: 2 (5.08 cm)
Enter the connecting distance on the 4 bar link: -10 (-25.4 cm)
This will be a pushing exercise.
```

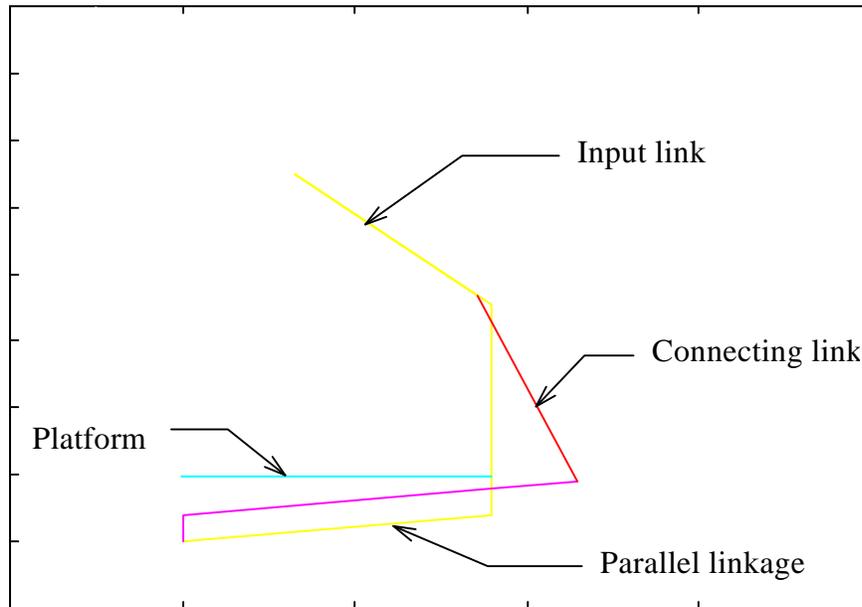


Figure 7-6, Final Position of Linkage

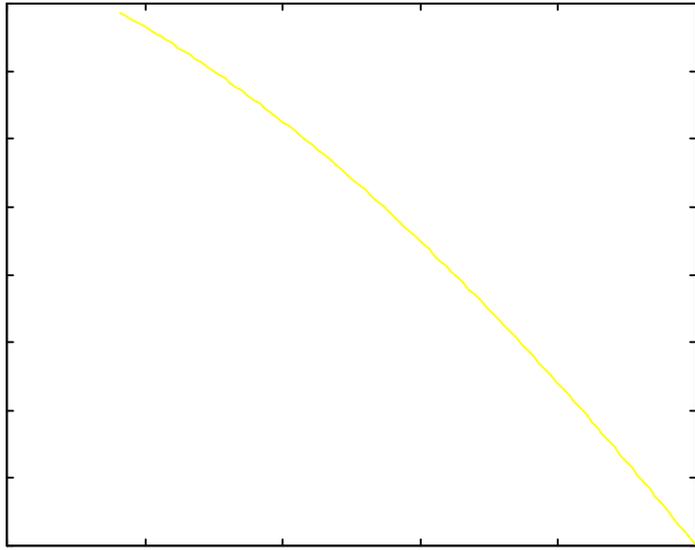


Figure 7-7, Height Versus q_u

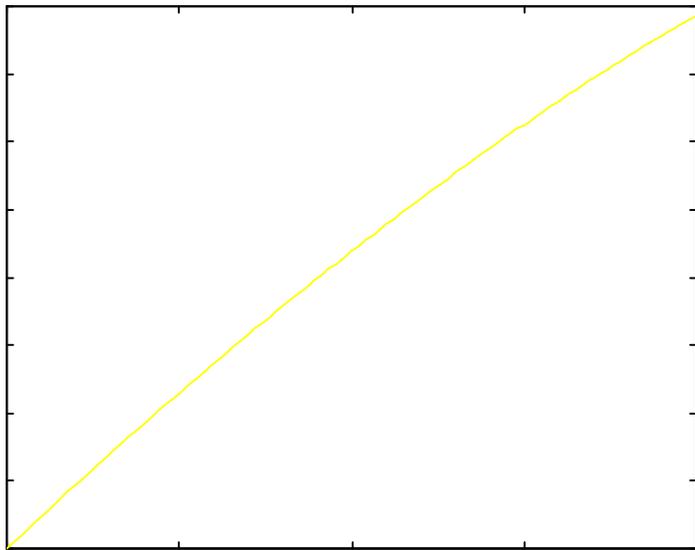


Figure 7-8, Height Versus Stroke

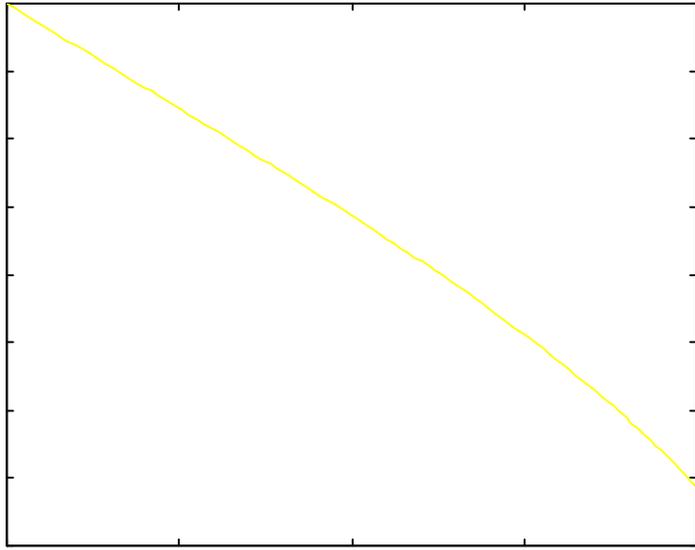


Figure 7-9, Force Versus Stroke for Virtual Work

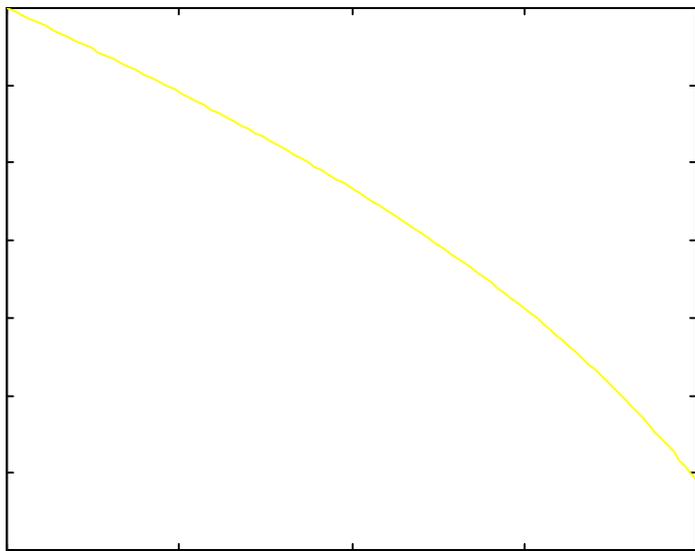


Figure 7-10, Force Versus Stroke for Matrix Method

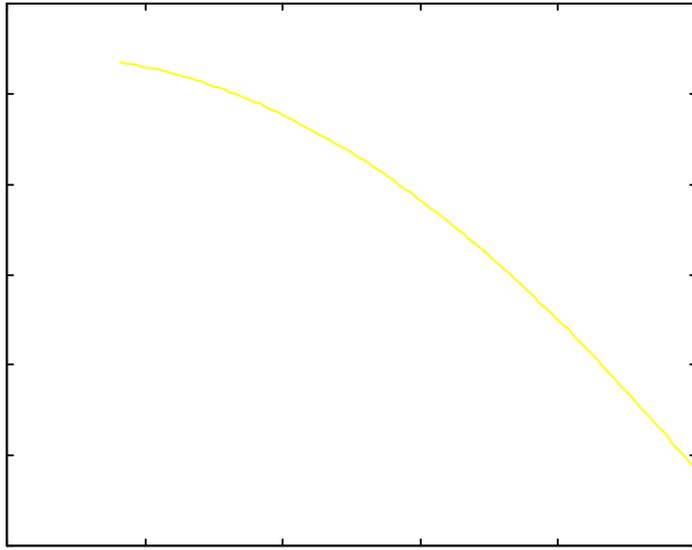


Figure 7-11, Connector Tension Versus θ_u

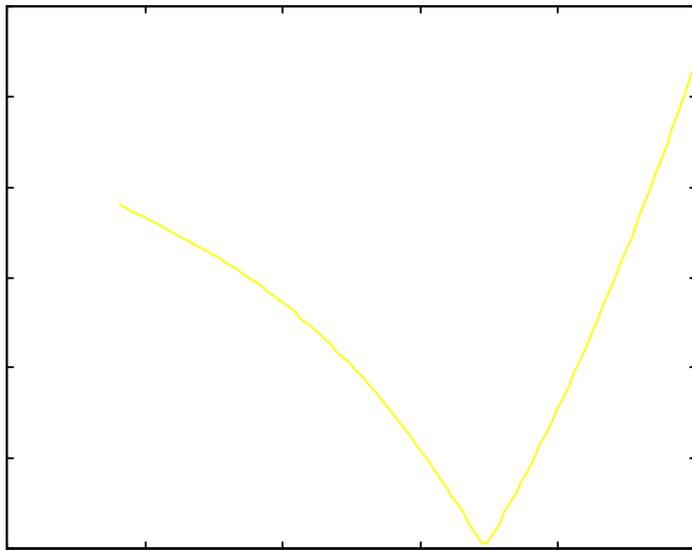


Figure 7-12, Tension of r Versus θ_u

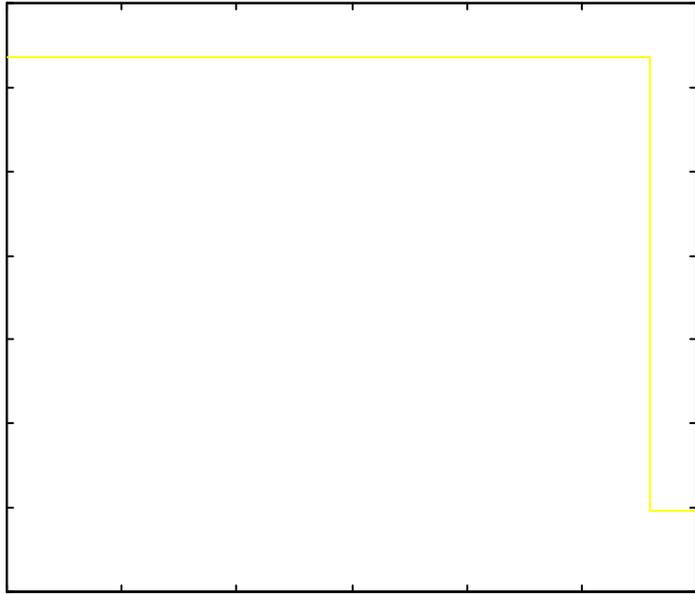


Figure 7-13, Shear Diagram of f_u

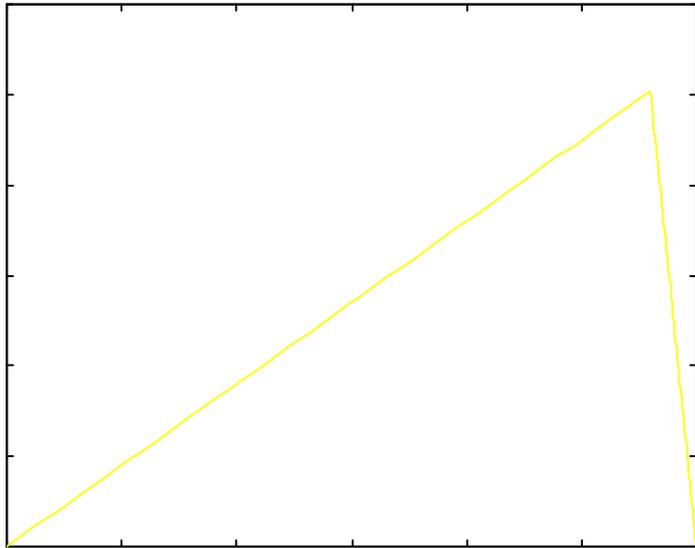


Figure 7-14, Moment Diagram of f_u

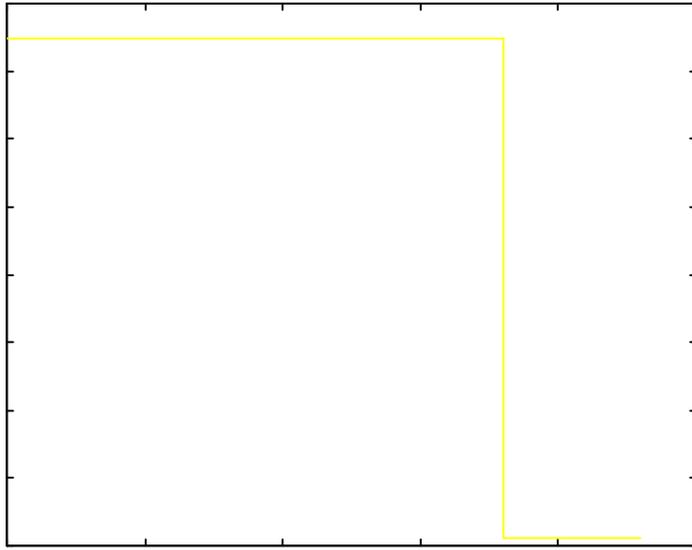


Figure 7-15, Shear Diagram of r_3

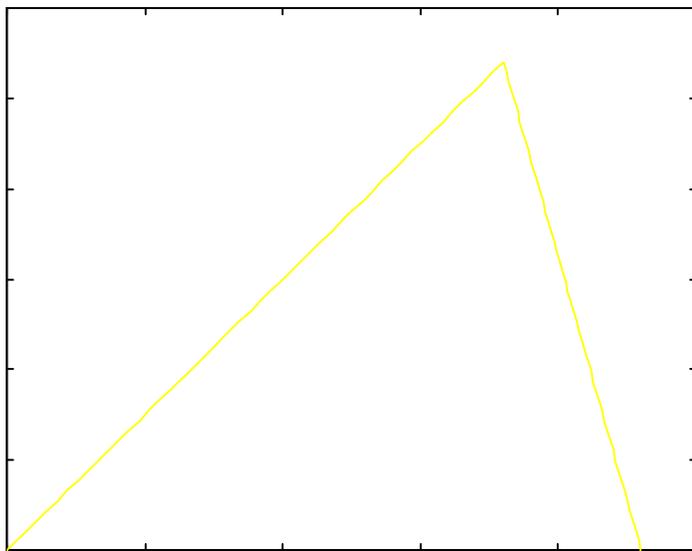


Figure 7-16, Moment Diagram of r_3

Enter the widest cross sectional dimension of tubing: 2 (5.08 cm)
Enter the smallest cross sectional dimension of tubing: 1.5 (3.81 cm)
Enter the material 1-steel, 2-aluminum: 1
The needed wall thickness for the input bar is:

$t_u =$
0.0065

The needed wall thickness for bar 8 is:

$t_8 =$
0.0024

The needed wall thickness for bar 3 is:

$t_3 =$
0.0367

The needed wall thickness for bar 1 is:

$t_1 =$
2.1431e-004

»

Figure 7-6 is one position in a generated animation. Figures 7-7 through 7-9 are plots using virtual work. Figure 7-9 is the input force curve calculated from virtual work. Figures 7-10 through 7-12 are plots of internal and external forces calculated from a matrix method. Figure 7-10 is also the input force curve. Figures 7-13 through 7-16 are shear and moment diagrams for links with more than two forces acting on them.

The results of this simulation shows that the input force curve, also called the resistance curve, varies from the initial force by about 30%. The resistance force curve that was plotted using the virtual work method is nearly identical to the force curve using the matrix method. The link dimensions used created a footprint of 4 ft (121.9 cm). The width of the footprint is determined by the base support. This is a detailed design issue. The smallest safety factor is found using the link with the largest needed wall thickness. Link r_3 required a wall thickness of 0.0367 in (0.0932 cm). Since the actual wall thickness was 1/8 in (0.3175 cm), the safety factor was 3.406 for a user wieght of 150 lbs (667.2 N). A safety factor is needed since there are unknown stress concentrations in the links where the revolte joints are attached. Overall, this analysis tool proves that a detailed design should be pursued to create a prototype.

7.6 Preliminary Design Results

The M file were used to perform a preliminary design. These programs require interactive user input. These input values include the stroke of the exercise, the value of parameters L_1 and L_2 , the material selection of steel or aluminum, and the tubing cross sectional dimensions. Other variables are assigned a value in the M files. These variables include the link lengths. The lengths were chosen to fit in a footprint of 83 in (210.82 cm) by 35 in (88.9 cm) as follows:

$W=150$ lb (667.23 N)
 $r_1=30$ inches (76.2 cm)
 $r_3=30$ inches (76.2 cm)
 $r_2=4$ inches (10.2 cm)
 $r_5=4$ inches (10.2 cm)
 $r_6=36$ inches (91.4 cm)
 $r_{user}=36$ inches (91.4 cm)

The angular speed of the input link was assigned a value of one since it is arbitrary ($\dot{q} = 1$). This angular speed will determine the velocity ratio from the velocity analysis. The values of L_1 and L_2 were incremented to determine the effects of these parameters on the resistance curve. The pushing closure was first analyzed, Table 7-1 shows the results of this analysis.

Table 7-1, User Force for Values of L_1 for Pushing Configuration

User Stroke	L_1	L_2	Forces
20 in (50.8 cm)	14 in (35.6 cm)	-10 in (-25.4 cm)	255 to 190 lbs (57.4 to 42.8 N)
20 in (50.8 cm)	12 in (30.5 cm)	-10 in (-25.4 cm)	216 to 160 lbs (48.6 to 36 N)
20 in (50.8 cm)	10 in (25.4 cm)	-10 in (-25.4 cm)	180 to 125 lbs (40.5 to 28.1 N)
20 in (50.8 cm)	8 in (20.3 cm)	-10 in (-25.4 cm)	145 to 95 lbs (32.6 to 21.4 N)
20 in (50.8 cm)	6 in (15.24 cm)	-10 in (-25.4 cm)	107 to 92 lbs (24.1 to 20.7 N)
20 in (50.8 cm)	4 in (10.2 cm)	-10 in (-25.4 cm)	72 to 48 lbs (16.2 to 10.8 N)
20 in (50.8 cm)	2 in (5.08 cm)	-10 in (-25.4 cm)	36 to 24 lbs (8.1 to 5.4 N)
20 in (50.8 cm)	1 in (2.54 cm)	-10 in (-25.4 cm)	18 to 12 lbs (4 to 2.7 N)

The pulling closure was also analyzed, Table 7-2 shows the results of this analysis. The range of forces increase for increasing values d_f for pushing closures. This makes sense, since the pushing closure reaches a toggle earlier in the stroke than does the pulling closure.

Table 7-2, User Force for Values of L_1 for Pulling Configuration

User Stroke	L_1	L_2	Forces
20 in (50.8 cm)	-10 in (-25.4 cm)	-10 in (-25.4 cm)	-170 to -170 lbs (-38.2 N)
20 in (50.8 cm)	-8 in (-20.3 cm)	-10 in (-25.4 cm)	-165 to -145 lbs (-37.1 to -32.6 N)
20 in (50.8 cm)	-6 in (-15.24 cm)	-10 in (-25.4 cm)	-135 to -108 lbs (-30.3 to -24.3 N)
20 in (50.8 cm)	-4 in (-10.2 cm)	-10 in (-25.4 cm)	-93 to -72 lbs (-20.9 to -16.2 N)
20 in (50.8 cm)	-2 in (-25.9 cm)	-10 in (-25.4 cm)	-47 to -36 lbs (-10.6 to -8.1 N)
20 in (50.8 cm)	-1 in (-2.54 cm)	-10 in (-25.4 cm)	-24 to -18 lbs (-5.4 to 4 N)

Changing the dimensions of each link would also change the results of these analyses. Certainly more advanced tools could be made to optimize the mechanism by iterating on more than one design variable.

To make the forces more constant, the link lengths of r_2 and r_1 would have to be increased. However, the trade off is that the footprint of the machine would increase along with the amount of material used and the resulting bending stresses. The results presented in Tables 6-1 and 6-2 were considered to be acceptable for the preliminary design.

The results of the this preliminary design satisfy the design criteria stated in Chapter 4. This design allows the user to vary the resistance. The resistance curve moves up for pulling exercise and moves down for pushing exercises. The shape of the resistance curve is opposite to the needed strength curve. However, the resistance does not vary more than 30% during the input motion. More consideration needs to be given to the strength curve in future research. The length of the links, and the position of attachment between links will effect the resistance curve. The design is compact enough and light enough to allow two people to carry it through a door. The pushing and pulling capability

allows for all major muscle groups to be trained. The motion is closely linear and only one adjustment is required without lifting the machine. The resistance varies from 0 to 170% of the user's body weight for this range of L_j . 170% of the user's weight is a large resistance that can be used for leg press exercises. Other considerations such as manufacturing issues are considered during detailed design.

Chapter 8, Detailed Design

After a preliminary design has been completed and accepted, detailed design must be completed. This stage of the design process ultimately results in detailed drawings of a tested design for manufacturing. This is inevitably an iterative process of design, testing and redesign. The first iteration of this step is demonstrated with a prototype design and initial test results.

8.1 Manufacturability Issues

One of the first steps in designing for manufacturability is material selection. This will impact how the parts are manufactured and assembled. For this prototype, rectangular steel tubing was selected. Tubing was chosen because of its availability and because it is lighter than solid steel while being strong enough. The outside dimensions were chosen from standard stock to be 2 by 1.5 in (5.08 by 3.81 cm) and the wall thickness was chosen to be 1/8 in (0.3175 cm). Based on the stress analysis, this produced a safety factor greater than two.

8.1.1 Revolute Joints

The stability of revolute joints was known to be a concern from the first prototype described in Chapter 5. To generate new designs for these joints, similar exercise equipment was inspected. Figure 8-1 shows the schematic of the joint concept selected. Placing a pin through both walls of the tubing creates an ability to support greater moment loading on the pin. Using round tubing to support bushings is very stable. The round tubing was also connected through both walls of the square tubing.

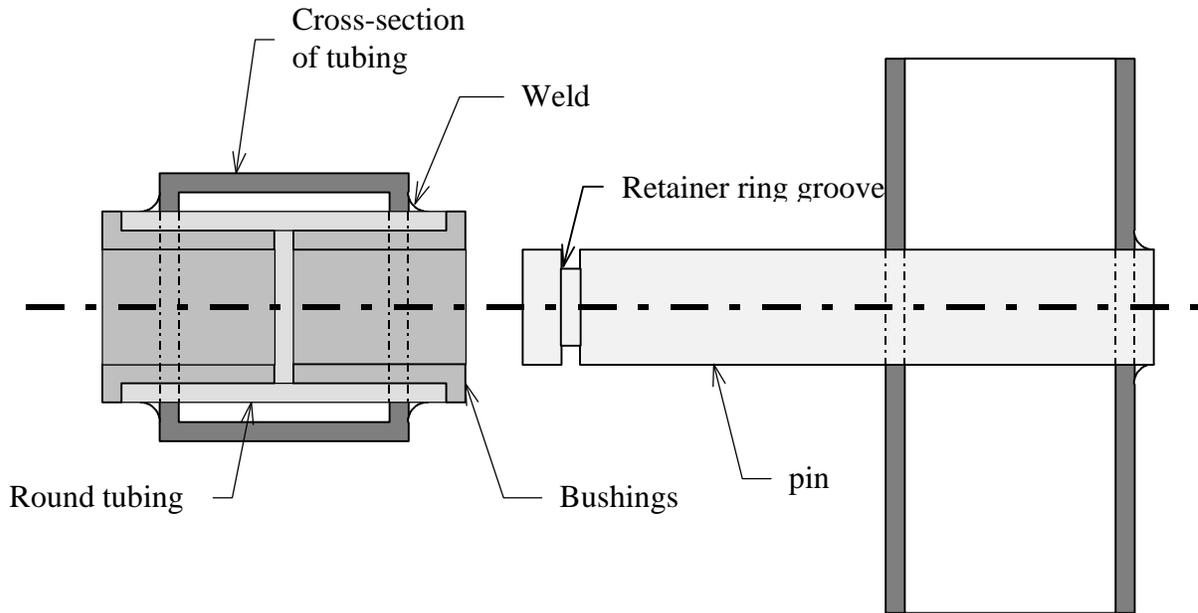


Figure 8-1, Revolute Schematic

The manufacturing operations to produce this joint include sawing the stock, turning the pin and round tubing, drilling holes in the tubing, pressing the components and welding. After these joints are fabricated to the links, the final assembly of links is simple. The pin is pushed through the bushings, and a retaining ring is placed on the end of the pin.

Due to the stiffness of the joints, only one plane of links was needed. Therefore, only 7 joints were needed for this 6 bar linkage. This joint design is an example of a stiff revolute joint for creating linkages in one plane. This idea can be applied to several types of machines using revolute joints.

8.1.2 Links

Manufacturing and assembly processes were also considered for the rest of the links. All the links were made of the same steel tubing. The seat and the base were designed with corner braces to support moments.

8.2 Detailed Drawings

The detailed drawings for this machine were created in AutoCad, version 13. They are seen in Appendix C. Each drawing has a drawing number and the name of the part appears in the drawing. The tolerances and material is also used specified.

8.3 Prototype Testing

After the prototype is assembled, the physical system was tested. The first prototype was easily assembled with only six links, five retainer rings and two pins. No screws were needed for the prototype making this a simple assembly. All pins and retainer rings were selected to be the same size. Additionally, the orientation of the two-force links were arbitrary.

The prototype assembly and testing did reveal unforeseen problems. Since the 1 inch (2.54 cm) pins are rigidly attached to the links, it is important that they are perpendicular to the links. A tolerance must be added to production drawings to maintain perpendicularity.

The first prototype was designed with holes drilled in the user input link and the upper parallel link. Holes were also drilled into the connecting link. These holes allowed adjustability in the connection location between these links. However, the outside round tubing encasing the bushings for the revolute joint, labeled A in Fig. 8-2, caused interference. The connecting link could not be pinned closer than $\frac{1}{4}$ in (3.175 cm) to either revolute joint. This problem is illustrated in Fig. 8-2. This figure shows that can only decrease until the link interferes with the joint. Even at this extreme connection point, the linkage cannot move.

Another problem was the unexpectedly large resistance curve force values. The forces increased and decrease as expected, but the magnitude was greater than expected. This is partly due to the weight of the links which was not modeled during the analysis. There is also some unmodeled friction between the bushing bearings and the pins. This is a disadvantage since the friction increases the concentric exercise resistance and decreases the eccentric exercise resistance. Rolling element bearings would reduce this friction. However, the cost of fabrication would increase. This trade-off needs to be considered. Therefore, to achieve the maximum forces in the resistance curve, the dimension L_0 did not need to vary up to 10 in (25.4 cm) as indicated in Section 7.6. The bearing stresses are low since the projected bearing surface area is $2\frac{3}{4}$ in² (12.9 cm²). The largest pin force

calculated for a value of $L_1=10$ was 1069.9 lb (4757.6 N). The bearing stress would only be 534.8 psi (3.688 Mpa).

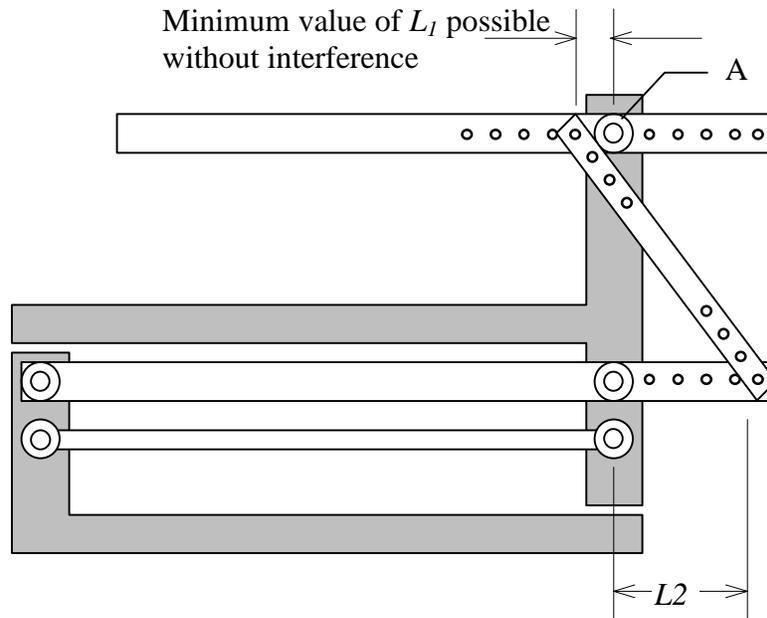


Figure 8-2, Joint Interference

8.3.1 Redesign to Eliminate Interference

Some solutions were contrived to fix these problems. Increasing manufacturing tolerances will enable a proper sliding fit for an easier assembly. Friction forces will reduce if the machine members are not deflected to force assembly.

A new concept for joining the connection link to the input link and was developed. The detailed drawings in Appendix C show the solution to this problem. A track was welded under the input link. This part was made of square tubing and a slot was milled down one side, this is called part O in Appendix C. The slot enabled a T-shaped part, called part N in Appendix C, to slide inside the track. The connection link was pinned to the flange that extends out of the slot. Part N also has a tapped hole to allow a power screw to adjust the position in the track. A schematic of this concept is shown in Fig. 8-3. This idea allows for infinitely small adjustments of L_1 . The connection link has a fixed length in this configuration, therefore as the connection position changes, the orientation of the input link will also change. A prototype may be tested to determine the effects of this connection.

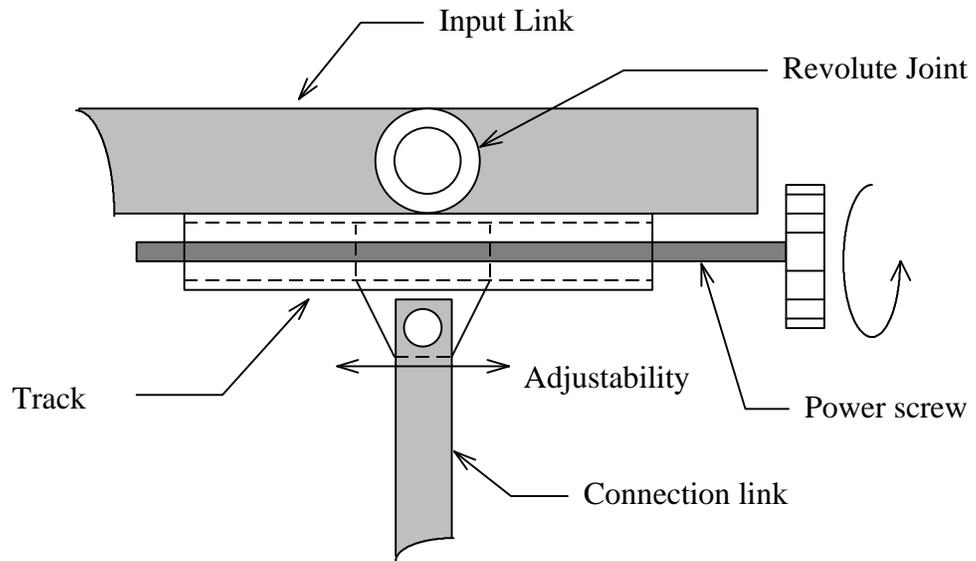


Figure 8-3, New Joint Connection

Chapter 9, Additional Considerations

This chapter is devoted to the discussion of further considerations of the prototype and the detailed design. After detailed design is completed, production must be considered. The example prototype machine developed during the course of this thesis was not brought into production. Figures 9-2 through 9-4 show this prototype. However, this step must be considered to complete the design process. This chapter will also discuss additional research ideas dealing with the design of home exercise equipment.

9.1 Production Planning and Tooling Design

Production planning starts by reviewing the design drawings to identify the machines, tooling, and forming operations required for production. The type of operations used is a function of the following parameters (Kalpakjian, 1992):

- a. Characteristics of the workpiece material
- b. Shape, size and thickness of the part
- c. Tolerance and surface finish of the parts
- d. Functional requirements for the expected service life of the component
- e. Production volume (quantity)
- f. Level of automation required to meet production volume and rate
- g. Costs involved in individual and combined aspects of the manufacturing operation.

The order of the operations depends on geometrical features such as dimensions, tolerances and materials used. The geometry of the part also determines whether the part should be fabricated as one piece or several pieces and assembled.

In general, if the tolerances become stricter, better control of processing parameters is needed. Therefore, the time and cost of manufacturing increases.

Different materials have different manufacturing characteristics. Some of these characteristics include castability, forgeability, workability, machinability and weldability.

There are many tradeoffs between these characteristics. For example, aluminum is easier to machine than steel, however, it can not be ground.

Tooling includes tools, jigs, and fixtures. This is part of production planning. The tooling must be considered for each process. This will factor into the time and cost of each process. The time to set up a process with tooling is called the lead time.

9.1.1 Manufacturing Costs

Manufacturing cost can be categorized into one of five types. The first one is the materials cost. The second is the tooling costs. This is the cost of making and setting up tools, dies, molds, patterns, jigs and fixtures. The third type is the labor costs. This is determined by the amount of labor used from the time the material is first touched until the product is finished. The fourth kind is the fixed cost. This type of cost includes power, fuel, taxes, rent and insurance. The last type of cost is the capital cost. This is the capital investment in such things as land, machines and equipment (Kalpakjian, 1992).

Cost is also affected by the scale of production. Small batch production uses general purpose machines. Versatile equipment is operated by skilled labor. Medium batch production uses the same machines with jigs and fixtures. CNC machines are also used to automate processes. There are more machines that are dedicated to one task and require less labor. The capital investment is greater for medium batch production, but over time, money is saved on manufacturing processes.

9.2 Future Research

After testing the prototype, new problems were found. New concepts were generated to solve these problems. As shown in Fig. 8-2, the connection link interfered with a revolute joint in the initial prototype. To solve this problem, a power screw was implemented to adjust the connection under the input bar as shown in Fig. 8-3. This caused a new problem. The connecting joint was never coincident with the pivot joint for the input bar. As a result, the resistance could never be adjusted to zero. The orientation of the input bar moved a considerable amount as the connection position changed along the power screw. To solve this problem, a four-bar linkage can replace the connection

link. This can be accomplished by simulating the revolute joint with a point on the coupler link of a parallelogram four-bar linkage. To maintain a constant orientation, the parallel linkage will produce a circular arc motion of that point. This concept is shown in Fig 9-1. Using this concept, the simulated revolute joint can be coincident with joint A.

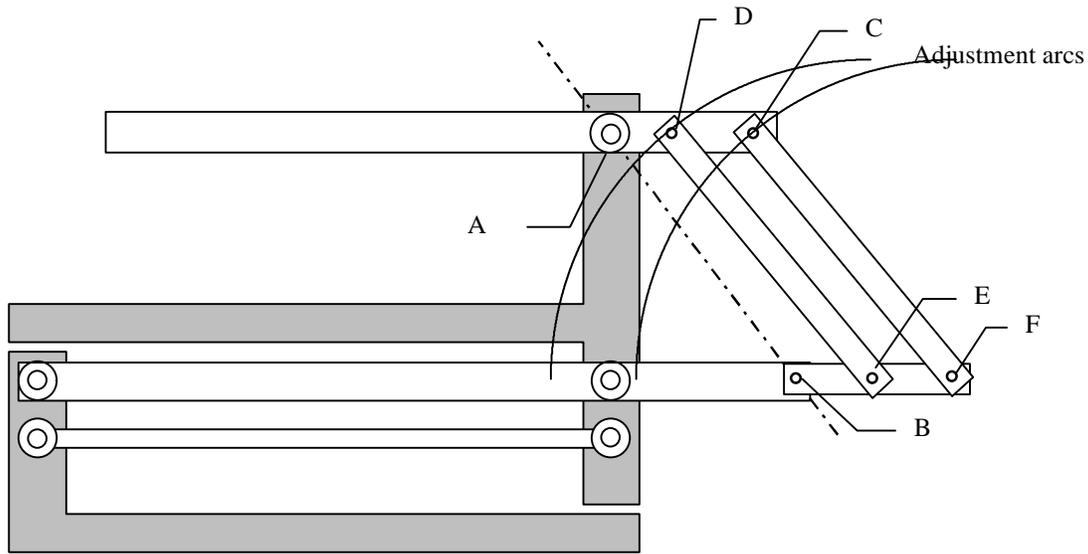


Figure 9-1, Four-Bar Replacement

Because of the parallelogram formed by the linkage CDEF, the motion of point A relative to point B is the same as if A and B were connected by a solid link. For the position shown in Fig. 9-1, the input link would not cause the machine to lift and there is zero resistance. The parallel links are pinned along the adjustment arcs to vary the resistance. Moving the parallel linkage to the right will cause a pulling exercise and moving the left will cause a pushing exercise. This concept adds two more links and three revolute joints. The kinematics of this mechanism are the same as the previous concept. Figure 9-2 shows a digital picture of this prototype. Figure 9-3 shows the use of the prototype in a pushing configuration and Fig. 9-4 shows the pulling configuration.



Figure 9-2, Prototype

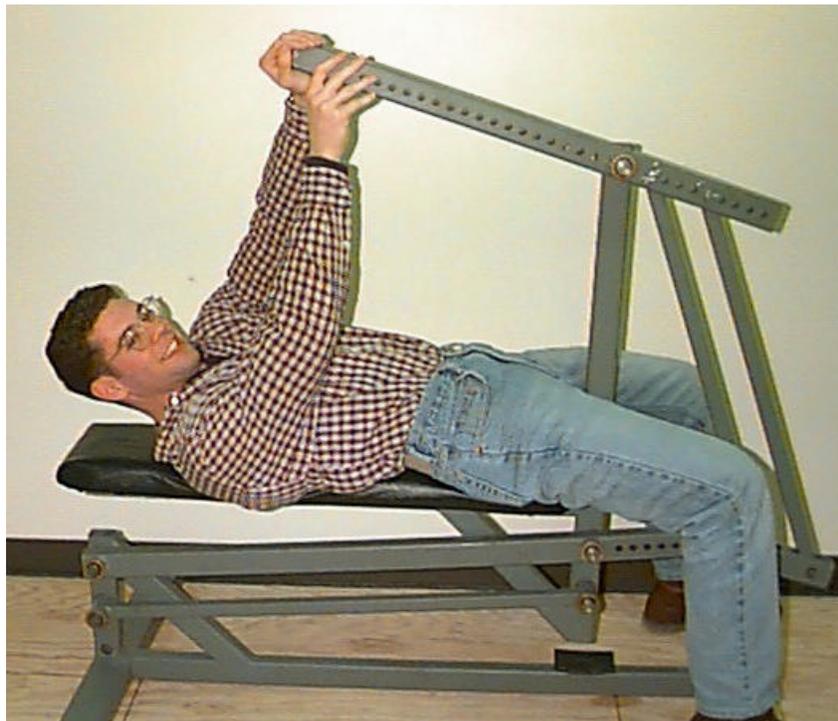


Figure 9-3, Prototype in the Pushing Configuration



Figure 9-4, Prototype in the Pulling Configuration

Experimental stress analysis is another area of continued research. Load cells and strain gages could be used to determine internal forces. These can then be compared to analytical results.

9.2.1 Force Generation

A resistance curve could be programmed into the mechanism by the design of a non-parallel four-bar linkage. Techniques for designing force-generating linkages have been documented by Soper (1995) ,Tidwell et al. (1996), and Bokelberg (1990). Force generation is the process of dimensional synthesis to generate a mechanical-advantage function. The external forces acting on the mechanism such as weights and springs are assumed to be conservative. A particular force or torque can be applied to a link in the mechanism to cause the resistance at the input. Virtual work can be used to match these forces and provide velocity information for the linkage. Integrating the velocity will

provide displacement information. The displacement information can then be used for kinematic synthesis to generate the linkage.

Scardina (1996) documented the optimal synthesis of planar four-bar linkages for force or torque generation. The numerical approach uses optimization theory to search for a possible solution which meets design specifications.

9.3 Conclusions

This thesis has demonstrated the engineering design process as applied to the design of exercise machine that uses the person's weight as a source of resistance. Synthesis and analysis have been an ongoing process through the design process. Certain design criteria which are unique to this application are addressed by this research. A mechanism was designed to allow the user to adjust the mechanical advantage without resistance during the adjustment. Also, the motion of the input link is relative to the user. This allows for correct motion for the exercise. The design also implements techniques that simplify manufacturing processes and assembly. For example, parallelogram linkage which are relatively simple to fabricate and inexpensive are used in place of linear bearings. This work shows the importance of using automated, programmed tools as an integral part of the design process. Many of the preliminary design concepts could only be rejected after kinematic or force analysis was completed.

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Appendix A, Human Factors Data (Woodson, 1992)

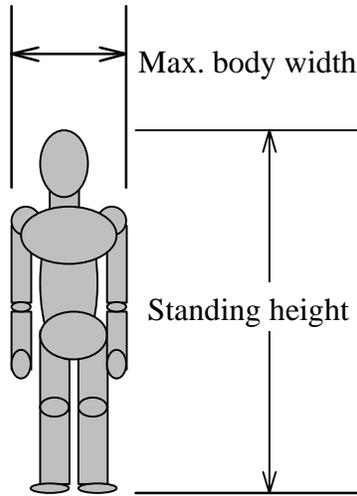


Figure A-1, Standing Dimensions

Standing Height			
<i>Adults</i>	<i>5th Percentile</i>	<i>50th Percentile</i>	<i>95th Percentile</i>
Males	63.6 in (162.3 cm)	68.3 in (173.5 cm)	72.8 in (184.9 cm)
Females	59.0 in (149.9 cm)	62.9 in (159.8 cm)	67.1 in (170.4 cm)
Maximum Body Width			
<i>Adults</i>	<i>5th Percentile</i>	<i>50th Percentile</i>	<i>95th Percentile</i>
Males	18.8 in (47.8 cm)	20.9 in (53.1 cm)	22.8 in (57.9 cm)

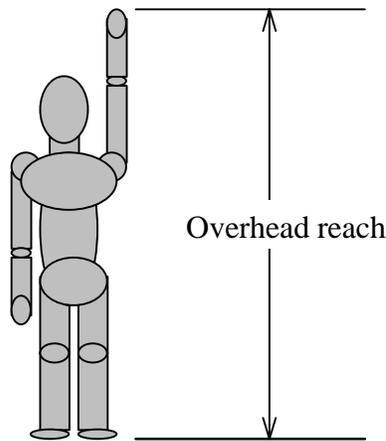


Figure A-2, Overhead Reach

<i>Adults</i>	<i>5th Percentile</i>	<i>50th Percentile</i>	<i>95th Percentile</i>
Males	82.0 in (208.3 cm)	88.0 in (223.5 cm)	94.0 in (238.8 cm)
Females	73.0 in (185.4 cm)	79.0 in (200.7 cm)	86.0 in (218.4 cm)

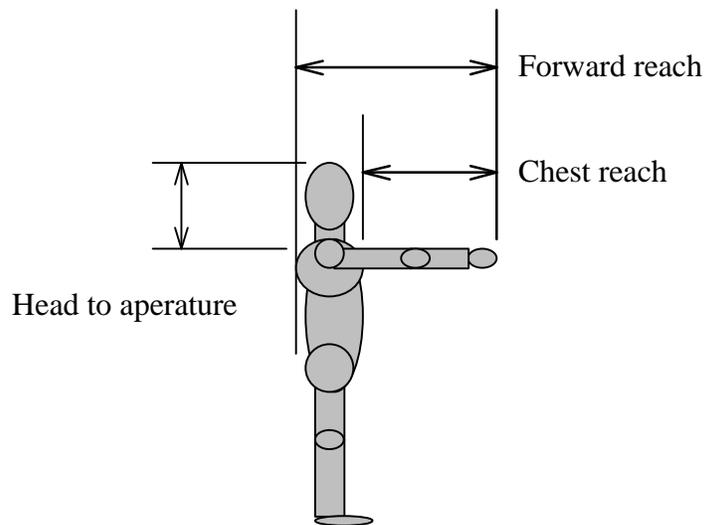


Figure A-3, Forward Reach

Forward reach			
<i>Adults</i>	<i>5th Percentile</i>	<i>50th Percentile</i>	<i>95th Percentile</i>
Males	31.9 in (81 cm)	34.6 in (87.9 cm)	37.3 in (94.7 cm)
Females	29.7 in (75.4 cm)	31.8 in (80.8 cm)	34.1 in (86.6 cm)
Chest Reach			
<i>Adults</i>	<i>5th Percentile</i>	<i>50th Percentile</i>	<i>95th Percentile</i>
Males	19.25 in (48.9 cm)	22.25 in (56.5 cm)	24.5 in (62.23 cm)
Females	-	-	-
Head to Aperture			
<i>Adults</i>	<i>5th Percentile</i>	<i>50th Percentile</i>	<i>95th Percentile</i>
Males	-	16.75 in (42.5 cm)	-
Females	-	-	-

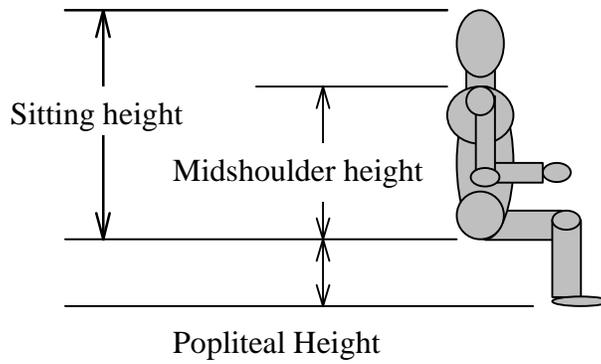


Figure A-4, Sitting Dimensions

Sitting height			
<i>Adults</i>	<i>5th Percentile</i>	<i>50th Percentile</i>	<i>95th Percentile</i>
Males	33.2 in (84.3 cm)	35.7 in (90.7 cm)	38.0 in (96.5 cm)
Females	30.9 in (78.5 cm)	33.4 in (84.8 cm)	35.7 in (90.7 cm)
Mid-shoulder height			
<i>Adults</i>	<i>5th Percentile</i>	<i>50th Percentile</i>	<i>95th Percentile</i>
Males	21.0 (53.3 cm)	-	25.0 in (63.5 cm)
Females	18.0 in (45.7 cm)	-	25.0 in (63.5 cm)
Popliteal Height			
<i>Adults</i>	<i>5th Percentile</i>	<i>50th Percentile</i>	<i>95th Percentile</i>
Males	15.5 in (39.4 cm)	17.3 in (43.9 cm)	19.3 in (49 cm)
Females	14.0 in (35.6 cm)	15.7 in (39.9 cm)	17.5 in (44.5 cm)

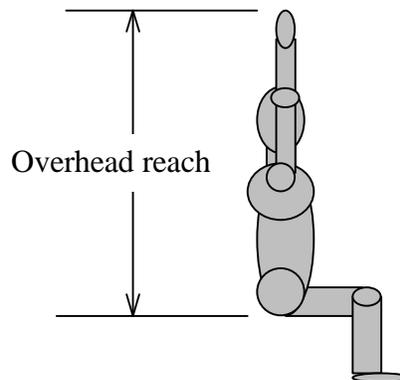


Figure A-5, Overhead Reach

<i>Adults</i>	<i>5th Percentile</i>	<i>50th Percentile</i>	<i>95th Percentile</i>
Males	50.3 in (127.8 cm)	-	57.9 in (147.1 cm)
Females	46.2 in (117.3 cm)	-	54.9 in (139.4 cm)

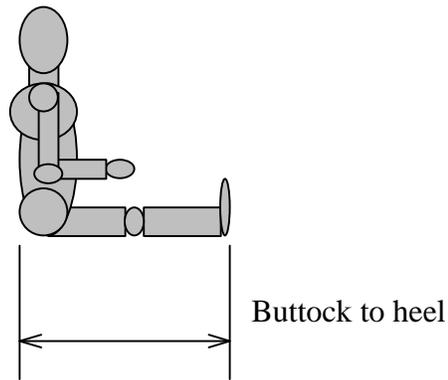


Figure A-6, Buttock to Heel

<i>Adults</i>	<i>5th Percentile</i>	<i>50th Percentile</i>	<i>95th Percentile</i>
Males	39.0 in (99.1 cm)	42.0 in (106.7 cm)	46.0 in (116.8 cm)
Females	34.0 in (86.4 cm)	37.8 in (96 cm)	41.2 in (104.6 cm)

Whole Body Weight

<i>Adults</i>	<i>5th Percentile</i>	<i>50th Percentile</i>	<i>95th Percentile</i>
Males	124 lb (27.88 N)	168 lb (37.77 N)	224 lb (50.36 N)
Females	104 lb (23.34 N)	139 lb (31.25 N)	208 lb (46.76 N)

Appendix B, Matlab M files

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%
%%FFUNC_VA.M                               %
%%Matlab M-file written by Dana J. Coombs  %
%%January 30, 1996                          %
%%This file solves for the user input force as %
%%a function of stroke.                     %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%

```

```
clear;
```

```
% User inputs
```

```
phi=input('Enter value of phi:')
```

```
phi=phi*(180/pi);
```

```
stroke=input('Enter stroke:')
```

```
% Constants
```

```
w=200;
```

```
l=36;
```

```
d=8;
```

```
%d=12;
```

```
x=linspace(1,stroke,100);
```

```
% Solve the geometry
```

```
e=x.*(tan(phi));
```

```
% Solve the forces
```

```
n1=(w.*l.*cos(atan((d-e)./x)))./(sqrt(x.^2+(d-x.*tan(phi)).^2));
```

```
n2=n1.*(cos(atan(d-e)./x))./(cos(phi));
```

```
f=n1.*(sin(atan((d-e)./x)))+n2.*(sin(phi));
```

```
% Plot the force as a function of stroke
```

```
subplot(211);
```

```
plot(x,f);
```

```
ylabel('Force (lbs)');
```

```
xlabel('Stroke (in)');
```

```
title('Force vs. Stroke');
```

```
% Plot the link displacement as a function of stroke
```

```
subplot(212);
```

```
theta=(180/pi).*(atan((d-e)./x));
```

```
plot(x,theta);
```

```
ylabel('Theta (degrees)');
```

```
xlabel('Stroke (in)');  
title('Theta vs. Stroke');
```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% GAS_CON.M %
%% Matlab M file written by Dana J. Coombs %
%% Home machine research %
%% February 29, 1996 %
%% %
%% This file solves the user input force as function %
%% of stroke. This calculation is looped through a %
%% change in the ramp angle (theta). A mesh is %
%% generated. %
%% %
%% %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```

% Constant Inputs

```

```

stroke=24;
%theta=30*(pi/180);
w=200;
l=34;
F1in=100;
F2in=135;
F1out=90;
F2out=125;
Phi1=30*(pi/180);

```

```

% Gas cylinder force as function of stroke

```

```

x=linspace(0,24,25);
Phi=atan2((l*sin(Phi1)),(l*cos(Phi1)-x));

```

```

% Force exerted on roller as function of stroke

```

```

% Looped by changing theta

```

```

for J = 1:25,
    for I = 1:50,
        theta(I)=(pi/180)*(-45+90*(I-1)/50);
        new=sqrt((l*cos(Phi1)-x(J)).^2+(l.*sin(Phi1)).^2);
        xs=l-new;
        Pin=((F2in-F1in)/stroke).*xs+F1in;
        Pout=((F2out-F1out)/stroke).*xs+F1out;
        Fin(J,I)=Pin*cos(Phi(J))+tan(theta(I))*(w+Pin*sin(Phi(J)));
        Fout(J,I)=Pout*cos(Phi(J))+tan(theta(I))*(w+Pout*sin(Phi(J)));
    end
end

```

```
    end
end

%Plot forces vs. stroke

angle=theta*180/pi;

mesh(angle,x,Fin);
title('Force vs. stroke and angle - gas spring');
xlabel('Angle');
ylabel('Stroke');
zlabel('Force (lb)');
set(gca,'XTick',[-45:10:45]);
```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% OLD_RAMP.M %
%% Matlab M file written by Dana J. Coombs %
%% Home machine research %
%% October 4, 1996 %
%% %
%% This file performs a loop closure analysis of the %
%% ramp mechanism with a roller riding under the %
%% support platform. This file also does virtual %
%% work to determine the user input forces. %
%% %
%% %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

% User inputs

```

Theta6=input('Enter ramp angle: ');
stroke=input('Enter stroke of exercise: ');

```

% Constant Inputs

```

conv=pi/180; % Convert degrees to radians
Theta6=Theta6*conv;% Convert Theta6 to radians
Theta3=90*conv; % Angles for link vectors
Theta4=90*conv;
Theta5=180*conv;
Theta7=0*conv;
Theta8=90*conv;
Theta9=90*conv;

W=150 ; % Wieght of user
r1=48.5; % Paralell link length (inches)
r2=r1 ; % Second paralell link length
r3=5.5 ; % Vertical link to platform
r9=r3 ; % Ground link for paralell 4 bar
r4=1.75; % Vertial link to bottom of platform
r5dot=1; % Speed of input cable

```

% Calculate remaining mechanism geometry

```

r5=r1-2.5-(1.5+r4+r3)/tan(Theta6); % Length of pulley under platform
r8=r4+stroke*tan(Theta6); % Vertical ground to location on adjustable ramp
r6=stroke/(cos(Theta6)); % Initial length for r6

```

```

r7=r2-r5+stroke;           %Horizontal ground to location on adjustable ramp

r5=r5-linspace(0,stroke,100);
stroke_vec=linspace(0,stroke,100);

%Calculate Theta2 from loop closure
%a, b, and c are components of a quadric equation to solve for
%the unknown r6.

a=1;
b=-2*(cos(Theta6)*(r5+r7)+(r8-r4)*sin(Theta6));
c=((r5+r7).^2+(r8-r4)^2-r2^2);

r6=(-b-sqrt(b.^2-4*a*c))/(2*a);   %Solve for unknown r6
hieght=(r8-r4)-r6*sin(Theta6);   %Solve for hieght (r2*sin(theta2))

%Plot displacement of platform vs. displacement of pulley

figure(1);
subplot(211);
plot(stroke_vec,hieght);
xlabel('stroke');
ylabel('Hieght');
title('Hieght vs. stroke');

%Determine force function

Theta2=asin(((r4+r8)-r6*sin(Theta6))/r2);
Force=W./(1/tan(Theta6)+tan(Theta2));

%Force=-((W*tan(Theta2).*(r5dot-r6.*cos(Theta6)))/r5dot);
%
%Theta2dot=-((r5dot-r6*cos(Theta6))./(r2*sin(Theta2)));
%plot(Theta2,sin(Theta2));

subplot(212);
plot(stroke_vec,Force);
xlabel('stroke');
ylabel('Force');
title('Force vs. stroke');

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% PSH_PULL.M %
%% Matlab M file written by Dana J. Coombs %
%% Home machine research %
%% October 24, 1996 %
%% %
%% This file generalizes calculations for different%
%% machine configurations. These configuration%
%% include closures for pushing or pulling exercises %
%% for the cases of having a compression or tension %
%% force in the connecting bar. %
%% This file also calculates the force that the user%
%% inputs through a range of motion. %
%% An animation is also done by plotting the vectors %
%% for each link in a loop. %
%% %
%% %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

% Constant Inputs

```

conv=pi/180; % Convert degrees to radians
Theta2=90*conv; % Angle vertical coupler to seat
Theta5=90*conv; % Angle of vertical ground link
Theta6=90*conv; % Angle of extended bench post

```

```

W=150 ; % Weight of user
r1=36 ; % Parallel link length (inches)
r3=r1 ; % Second parallel link length
r2=4 ; % Vertical link of parallel 4 bar
r5=r2 ; % Ground link for parallel 4 bar
r6=27.5 ; % Vertical post to input link
ruser=30; % Input link length for the user

```

% User inputs

```

stroke=-1;
while stroke>(ruser)|stroke<0,
    disp('Max stroke can be ruser. ');
    ruser,
    stroke=input('Enter stroke of exercise: ');
    if stroke>(ruser),

```

```

        disp('Stroke is too large, enter it again. ');
    end;
end;

L1=input('Enter the connecting distance on input link: ');
L2=input('Enter the connecting distance on the 4 bar link: ');

% Calculate input angles for ruser
% Determine if this closure is pulling or pushing
k=sign(L1*L2);
if k<0,
    disp('This will be a pushing exercise. ');
else
    disp('This will be a pulling exercise. ');
end;
ThetaU=k*linspace(0,asin(stroke/ruser),100);
ThetaUdot=k;

% Calculate remaining constant geometry
r8=sqrt((L1-L2)^2+r6^2);
r9=ruser-L1;
r4=r3-L2;

% Solve for Theta3 for displacement of output
% using the tangent half-angle identity
su=sin(ThetaU);
cu=cos(ThetaU);
A=-2*L2*L1*cu;
B=2*L2*(-L1*su+r6);
C=L2^2+L1^2+r6^2-r8^2-2*L1*r6*su;

a=C-A;
b=2*B;
c=A+C;

if L2<0,
    t=(-b-sqrt(b.^2-4*a.*c))./(2*a);
else
    t=(-b+sqrt(b.^2-4*a.*c))./(2*a);
end;

Theta3=2*atan(t);
c3=cos(Theta3);
s3=sin(Theta3);

```

```

t3=tan(Theta3);

%Solve for Theta8 using real displacement

num=L2*c3-L1*cu;
den=r8;
Theta8=acos(num./den);
s8=sin(Theta8);
c8=cos(Theta8);
t8=tan(Theta8);

%Calculate height of platform motion

hieght=sin(Theta3)*r3;

%Calculate force generation from velocity analysis

num=L1*ThetaUdot.*(-su+cu.*t8);
den=L2*c3.*t8-L2*s3;
Theta3dot=num./den;

num=W*r3*Theta3dot.*c3;
den=(ruser*ThetaUdot*cu);
F=-num./den;

figure(1);
for i=1:4:length(ThetaU),
    %Calculate all link locations
    r1s=[0,0]'; r1e=[r1*c3(i),r1*s3(i)]';
    r2s=r1e; r2e=r2s+[0,r2]';
    r5s=[0,0]'; r5e=r5s+[0,r5]';
    r3s=r5e; r3e=r2e;
    r4s=r5e; r4e=r4s+[r4*c3(i),r4*s3(i)]';
    r6s=r2e; r6e=r6s+[0,r6]';
    r8s=r4e; r8e=r8s+[r8*c8(i),r8*s8(i)]';
    ruser=r6e; rusers=ruser-[ruser*cu(i),ruser*su(i)]';
    r9s=rusers; r9e=r9s+[r9*cu(i),r9*su(i)]';
    rbs=r3e+[0,2]'; rbe=rbs+[-r3,0]';

    %Plot the link locations
    tx1=[r1s(1),r2s(1),r6s(1),r6e(1),r9s(1),r9e(1)];
    ty1=[r1s(2),r2s(2),r6s(2),r6e(2),r9s(2),r9e(2)];
    tx2=[r1s(1),r3s(1),r3e(1),r4e(1)];
    ty2=[r1s(2),r3s(2),r3e(2),r4e(2)];
    tx3=[rbs(1),rbe(1)];

```

```

        ty3=[rbs(2),rbe(2)];
        tx4=[r8s(1),r8e(1)];
        ty4=[r8s(2),r8e(2)];
        axis='square';
        axis=[-2,70,-2,70];
        plot(-10,-10, 80, -10, -10, 80, 80, 80);
        hold on;
        plot(tx1,ty1,tx2,ty2,tx3,ty3,tx4,ty4);
        hold off;
        pause(.1)
end;
title('Animation of vectors');

%Plot displacement of platform vs. stroke angle

figure(2);
subplot(311);
plot(ThetaU*180/pi,hieght);
xlabel('Theta7 input link (degrees)');
ylabel('Platform Hieght (inches)');
title('Hieght vs. ThetaU');

%Plot displacement of platform vs. stroke
%calculate inpute link vertical motion

ruvert=-(ruser)*sin(ThetaU);
subplot(312);
plot(ruvert,hieght);
xlabel('Vertical displ of input link (inches)');
ylabel('Platform Hieght (inches)');
title('Hieght vs. Stroke');

%Plot Forces vs stroke

subplot(313);
plot(ruvert,F);
xlabel('Vertical displ of input link (inches)');
ylabel('Input force (lbs)');
title('Force vs. Stroke');

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% PIN_FORC.M                               %
%% Matlab M-file written by Dana J. Coombs  %
%% December 3, 1996                         %
%% This file solves a system of linear      %
%% equations to solve the forces at each    %
%% pin in the mechanism.                    %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```

% Calculate pin forces at each position

```

```

% Length of seat
rb=abs(rbs(1)-rbe(1));

```

```

n=length(ThetaU);
A=zeros(15);
for i=1:n;

```

```

% Fill in the A matrix

```

```

A(1,1)=1 ; A(1,2)=1;
A(2,3)=1; A(2,4)=1; A(2,5)=1;
A(3,1)=L1*su(i); A(3,3)=-L1*cu(i);
A(3,5)=-ruser*cu(i);
A(4,1)=-1; A(4,6)=1;
A(5,3)=-1; A(5,7)=1;
A(6,6)=s8(i);A(6,7)=-c8(i);
A(7,2)=-1; A(7,8)=1; A(7,9)=1;
A(8,4)=-1; A(8,5)=-1; A(8,10)=1; A(8,11)=1;
A(9,2)=r6+r2; A(9,5)=rb; A(9,8)=-r2;
A(10,6)=-1; A(10,8)=-1; A(10,12)=1;
A(11,7)=-1; A(11,10)=-1; A(11,13)=1;
A(12,6)=L2*s3(i); A(12,7)=L2*c3(i);
A(12,12)=r3*s3(i); A(12,13)=-r3*c3(i);
A(13,9)=-1; A(13,14)=1;
A(14,11)-1; A(14,15)=1;
A(15,9)=s3(i); A(15,11)=-c3(i);

```

```

% Fill in the Y matrix

```

```

Y=zeros(15,1);
Y(8)=W;
Y(9)=-W*rb;

```

```

%Solve for the force vector Z

Z=inv(A)*Y;

% Assign solution to each force

Fu8x(i)=Z(1);
Fu6x(i)=Z(2);
Fu8y(i)=Z(3);
Fu6y(i)=Z(4);
Force(i)=Z(5);
F83x(i)=Z(6);
F83y(i)=Z(7);
F63x(i)=Z(8);
F61x(i)=Z(9);
F63y(i)=Z(10);
F61y(i)=Z(11);
F3gx(i)=Z(12);
F3gy(i)=Z(13);
F1gx(i)=Z(14);
F1gy(i)=Z(15);
end;

% Internal force for R8
%Determine if link is in compression or tension

if (Fu8x>=0) & (Fu8y>0),
    %Link is in compression
    k=-1;
elseif (Fu8x<0) & (Fu8y>0),
    %Link is in compression
    k=-1;
else,
    %Link is in tesnion
    k=1;
end;

F8=k*sqrt(Fu8x.^2+Fu8y.^2);

% Internal force for R1
if (F1gx>=0) & (F1gy>0),
    %Link is in compression
    k=-1;
elseif (F1gx<0) & (F1gy>0),

```

```

    %Link is in compression
    k=-1;
else,
    %Link is in tesnion
    k=1;
end;

F1=k*sqrt(F1gy.^2+F1gx.^2);

figure(3);
subplot(311);
plot(ruvert,Force);
xlabel('Vertical displ of input link (inches)');
ylabel('Input force (lbs)');
title('Force vs. Stroke');

subplot(312);
plot(ThetaU*180/pi,F8);
xlabel('ThetaU input link (degrees)');
ylabel('Internal tension of connector');
title('Connector tension vs. ThetaU');

subplot(313);
plot(ThetaU*180/pi,F1);
xlabel('ThetaU input link (degrees)');
ylabel('Internal tension of R1');
title('R1 tension vs. ThetaU');

```

```

%%%%%%%%%%
%%%%%%%%%%
%% STRESS.M %
%% Matlab M file written by Dana J. Coombs %
%% Home machine research %
%% December 10, 1996 %
%% %
%% This file calculates the internal shear and moment %
%% forces for critical machine components. It also %
%% considers buckling in compression. %
%% %
%% %
%%%%%%%%%%
%%%%%%%%%%

```

```

% Calculate the internal shear and moments

```

```

for i=1:100,
    %% For input bar
    % Ranges for shear and moments
    if sign(L1)==1,
        x1=linspace(0,ruser-L1,50);
        x2=linspace(ruser-L1,ruser,50);
    elseif sign(L1)==-1,
        x1=linspace(0,ruser,50);
        x2=linspace(ruser,ruser-L1,50);
    end;
    Xu=[x1,x2];

    % Shear
    V1=F(i)*cu(i)*ones(1,50);
    if sign(L1)==1,
        V2=(-F(i)*cu(i)+F8(i)*sin(Theta8(i)-ThetaU(i)))*ones(1,50);
    elseif sign(L1)==-1,
        V2=F8(i)*sin(Theta8(i)-ThetaU(i))*ones(1,50);
    end;
    ShearU=[V1,V2];
    % Moment
    M1=F(i)*cu(i)*x1;
    if sign(L1)==1,
        M2=F(i)*cu(i)*(ruser-L1)*(ruser-x2)/L1;
    elseif sign(L1)==-1,
        M2=-F(i)*cu(i)*(ruser)*(x2-ruser-abs(L1))/abs(L1);
    end;
    MomentU=[M1,M2];

```

```

%% For bar 3
% Ranges for shear and moments
if sign(L2)==1,
    x3=linspace(0,r3-L2,50);
    x4=linspace(r3-L2,r3,50);
elseif sign(L2)==-1,
    x3=linspace(0,r3,50);
    x4=linspace(r3,r3-L2,50);
end;
X3=[x3,x4];

% Shear
V3=(F3gy(i)*c3(i)-F3gx(i)*s3(i))*ones(1,50);
if sign(L2)==1,
    V4=(F63y(i)*c3(i)-F63x(i)*s3(i))*ones(1,50);
elseif sign(L2)==-1,
    V4=-F8(i)*sin(Theta8(i)-Theta3(i))*ones(1,50);
end;
Shear3=[V3,V4];
% Moment
M3=(F3gy(i)*c3(i)-F3gx(i)*s3(i))*x3;
if sign(L2)==1,
    M4=(F3gy(i)*c3(i)-F3gx(i)*s3(i))*(r3-x4)*(r3-L2)/L2;
elseif sign(L2)==-1,
    M4=-(F3gy(i)*c3(i)-F3gx(i)*s3(i))*r3*(x4-r3-abs(L2))/abs(L2);
end;
Moment3=[M3,M4];

% For seat

Xb=linspace(0,rb,50);           % Ranges
ShearB=(W+F(i))*ones(1,50);
MomentB=(W+F(i))*Xb;

% Determine the maximum shear and moments
% For the user input bar
MaxMU(i)=max(abs(MomentU));
MaxVU(i)=max(abs(ShearU));
% For bar 3
MaxM3(i)=max(abs(Moment3));
MaxV3(i)=max(abs(Shear3));
% For the seat
MaxMB(i)=max(abs(MomentB));
MaxVB(i)=max(abs(ShearB));

```

```

end;

%Plot the shear and moment diagrams for
%positions of maximum shear and moment

[MUval,MUpos]=max(MaxMU);
[VUval,VUpos]=max(MaxVU);
[M3val,M3pos]=max(MaxM3);
[V3val,V3pos]=max(MaxV3);
[MBval,MBpos]=max(MaxMB);
[VBval,VBpos]=max(MaxVB);

%For input bar
%Ranges for shear and moments
if sign(L1)==1,
    x1=linspace(0,ruser-L1,50);
    x2=linspace(ruser-L1,ruser,50);
elseif sign(L1)==-1,
    x1=linspace(0,ruser,50);
    x2=linspace(ruser,ruser-L1,50);
end;
Xu=[x1,x2];
%Shear
V1=F(VUpos)*cu(VUpos)*ones(1,50);
if sign(L1)==1,
    V2=(F(VUpos)*cu(VUpos)-F8(VUpos)*sin(Theta8(VUpos)-
ThetaU(VUpos)))*ones(1,50);
elseif sign(L1)==-1,
    V2=F8(VUpos)*sin(Theta8(VUpos)-ThetaU(VUpos))*ones(1,50);
end;
%%%%%% V1=F(VUpos)*cu(VUpos)*ones(1,50);
%%%%%% V2=(-F8(VUpos)*sin(Theta8(VUpos)-ThetaU(VUpos)))*ones(1,50);
ShearU=[V1,V2];
%Moment
M1=F(MUpos)*cu(MUpos)*x1;
if sign(L1)==1,
    M2=F(MUpos)*cu(MUpos)*(ruser-L1)*(ruser-x2)/L1;
elseif sign(L1)==-1,
    M2=-F(MUpos)*cu(MUpos)*(ruser)*(x2-ruser-abs(L1))/abs(L1);
end;
%%%%%%%% M2=F(MUpos)*cu(MUpos)*(ruser-L1)*(ruser-x2)/L1;
MomentU=[M1,M2];

```

```

%For bar 3

% Ranges for shear and moments
if sign(L2)==1,
    x3=linspace(0,r3-L2,50);
    x4=linspace(r3-L2,r3,50);
elseif sign(L2)==-1,
    x3=linspace(0,r3,50);
    x4=linspace(r3,r3-L2,50);
end;
X3=[x3,x4];
% Shear
V3=(F3gy(V3pos)*c3(V3pos)-F3gx(V3pos)*s3(V3pos))*ones(1,50);
if sign(L2)==1,
    V4=(F63y(V3pos)*c3(V3pos)-F63x(V3pos)*s3(V3pos))*ones(1,50);
elseif sign(L2)==-1,
    V4=-F8(V3pos)*sin(Theta8(V3pos)-Theta3(V3pos))*ones(1,50);
end;
Shear3=[V3,V4];
% Moment
M3=(F3gy(M3pos)*c3(M3pos)-F3gx(M3pos)*s3(M3pos))*x3;
if sign(L2)==1,
    M4=(F3gy(M3pos)*c3(M3pos)-F3gx(M3pos)*s3(M3pos))*(r3-x4)*(r3-L2)/L2;
elseif sign(L2)==-1,
    M4=-(F3gy(M3pos)*c3(M3pos)-F3gx(M3pos)*s3(M3pos))*r3*(x4-r3-
abs(L2))/abs(L2);
end;
Moment3=[M3,M4];

%For seat

Xb=linspace(0,rb,50);      % Ranges
ShearB=(W+F(VBpos))*ones(1,50);
MomentB=(W+F(MBpos))*Xb;

% These shear and moment diagrams do not correlate to the same
% position of the mechanism. The shear diagram is for the maximum
% shear of the member and the moment diagram is for the maximum
% moments and shears of the mechanism

figure(4);
subplot(411);
plot(Xu,ShearU);
xlabel('Length of input bar (inches)');
ylabel('Shear force (lbs)');

```

```
title('Input bar Shear Diagram');

subplot(412);
plot(Xu,MomentU);
xlabel('Length of input bar (inches)');
ylabel('Moment (in-lbs)');
title('Input bar Moment Diagram');

subplot(413);
plot(X3,Shear3);
xlabel('Length of bar 3 (inches)');
ylabel('Shear force (lbs)');
title('Bar 3 Shear Diagram');

subplot(414);
plot(X3,Moment3);
xlabel('Length of bar 3 (inches)');
ylabel('Moment (in-lbs)');
title('Bar 3 Moment Diagram');

end;
```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% DIMEN.M
%% Matlab M file written by Dana J. Coombs      %
%% Home machine research                       %
%% December 11, 1996                          %
%%                                             %
%% This file calculates the needed area moment of
%% inertia and the needed are of each member. This is %
%% done by using the shears and moments calculated for %
%% each member. This file also considers buckling. %
%%                                             %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```
%Inputs
```

```

h=input('Enter the widest cross sectional dimension of tubing: ');
maxc=h/2;          %Dimension to outside of widest side of tube
w=input('Enter the smallest cross sectional dimension of tubing: ');
t=1/8;            %tubing wall width
SF=1;            %Factor of safety

```

```

a0=h^2-h*w;      b0=0;          %Constants to determine
a1=h+w/2;        b1=6*(w*h^2+2*h^3); %thickness based on shear
a2=-1;          b2=-12*(h^2+w*h);
                b3=8*(3*h+8*w);
                b4=-16;

```

```
%Material properties of steel
```

```

material=input('Enter the material 1-steel, 2-aluminum: ');
if material==1,
    E=29e6;      %Modulus of elasticity lbs/in^2
    Ytension=36e3; %Yield strength of steel in tension lbs/in^2
    Tau=21e3;    %Yield strength of steel in shear lbs/in^2
elseif material==2,
    E=10.1e6;   %Modulus of elasticity lbs/in^2
    Ytension=14e3; %Yield strength of steel in tension lbs/in^2
    Tau=8e3;    %Yield strength of steel in shear lbs/in^2
end;

```

```
%Calculate the needed dimensions
```

```

%For the input bar
Iu=SF*MUval*maxc/Ytension; %Area moment of inertia

```

```

%Minimum wall thickness for area moment of inertia based on bending
au4=-16; %Coefficients of polynomial
au3=8*(3*h+w);
au2=-12*h*(h+w);
au1=(2*h^2)*(3*w+h);
au0=-12*Iu;
% Call function to find t by solving roots using Newton's method
[tuI]=root_nwt(au4,au3,au2,au1,au0);
ThickU(1)=tuI;
%Determine thickness based on shear
[tuShear]=quot_nwt(VUval,Tau,a2,a1,a0,b4,b3,b2,b1,b0);
ThickU(2)=tuShear;
%Determine the largest needed wall thickness for the input bar
tu=max(ThickU);

%For bar 3
I3=SF*M3val*maxc/Ytension; %Area moment of inertia
%Minimum wall thickness for area moment of inertia
a34=-16; %Coefficients of polynomial
a33=8*(3*h+w);
a32=-12*h*(h+w);
a31=(2*h^2)*(3*w+h);
a30=-12*I3;
% Call function to find t by solving roots using Newton's method
[t3I]=root_nwt(a34,a33,a32,a31,a30);
Thick3(1)=t3I;
%Determine thickness based on shear
[t3Shear]=quot_nwt(V3val,Tau,a2,a1,a0,b4,b3,b2,b1,b0);
Thick3(2)=tuShear;
%Determine the largest needed wall thickness for bar3
t3=max(Thick3);

%For bar 8
maxF8=max(abs(F8));
if F8<0,
    I8=SF*maxF8*(r8^2)/((pi^2)*E); %I determined by max length for buckling
end;
A8=SF*abs(maxF8)/Ytension; %Area of cross section
%Minimum wall thickness for needed area
Thick8(1)=((h+w)+sqrt((h+w)^2-4*A8))/4;
Thick8(2)=((h+w)-sqrt((h+w)^2-4*A8))/4;
if sign(Thick8(1)*Thick8(2))==-1,
    Thick8(1)=max(Thick8);
elseif sign(Thick8(1)*Thick8(2))==1,
    Thick8(1)=min(Thick8);

```

```

end;
%Minimum wall thickness for area moment of inertia
a84=-16; %Coefficients of polynomial
a83=8*(3*h+w);
a82=-12*h*(h+w);
a81=(2*h^2)*(3*w+h);
a80=-12*I8;
% Call function to find t by solving roots using Newton's method
[t8I]=root_nwt(a84,a83,a82,a81,a80);
Thick8(2)=t8I;
%Determine the largest needed wall thickness for bar8
t8=max(Thick8);

%For bar 1
maxF1=max(abs(F1));
if F1<0,
    I1=SF*maxf1*(r1^2)/((pi^2)*E); %I determined by max length for buckling
end;
%Determine the largest needed wall thickness for bar1
A1=SF*abs(maxF1)/Ytension; %Area of cross section
%Minimum wall thickness for needed area
Thick1(1)=((h+w)+sqrt((h+w)^2-4*A1))/4;
Thick1(2)=((h+w)-sqrt((h+w)^2-4*A1))/4;
if sign(Thick1(1)*Thick1(2))==-1,
    Thick1(1)=max(Thick1);
elseif sign(Thick1(1)*Thick1(2))==1,
    Thick1(1)=min(Thick1);
end;
%Minimum wall thickness for area moment of inertia
a14=-16; %Coefficients of polynomial
a13=8*(3*h+w);
a12=-12*h*(h+w);
a11=(2*h^2)*(3*w+h);
a10=-12*I1;
% Call function to find t by solving roots using Newton's method
[t1I]=root_nwt(a14,a13,a12,a11,a10);
Thick1(2)=t1I;

%Determine the largest needed wall thickness for bar1
t1=max(Thick1);

disp('The needed wall thickness for the input bar is: '), tu,
disp('The needed wall thickness for bar 8 is: '), t8,
disp('The needed wall thickness for bar 3 is: '), t3,
disp('The needed wall thickness for bar 1 is: '), t1,

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% ROOT_NWT.M %
%% Matlab M file written by Dana J. Coombs %
%% Home machine research %
%% December 17, 1996 %
%% %
%% This file is written as a function to solve for %
%% roots of an equation (specifically a 4th order %
%% polynomial). This function uses Newton's method %
%% for solving roots. The inputs are the polynomial %
%% coefficients. The output is the first root. The %
%% initial guess for this method is chosen to be zero. %
%% %
%% %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```
function [root] = root_nwt(a4,a3,a2,a1,a0)
```

```
% Inputs
```

```
X0=0; % Initial approximation
```

```
Tol=10^(-5); % Tolerance
```

```
N0=100; % Number of iterations
```

```
% Initialize
```

```
i=1;
```

```
% Newton-Raphson method loop
```

```
while i <= N0,
```

```
    FX=a4*X0^4+a3*X0^3+a2*X0^2+a1*X0+a0;
```

```
    dFX=4*a4*X0^3+3*a3*X0^2+2*a2*X0+a1;
```

```
    X=X0-FX/dFX;
```

```
    if abs(X-X0)<Tol,
```

```
        root=X; break
```

```
    else i=i+1;
```

```
    end;
```

```
    X0=X;
```

```
end;
```

```
if i==N0
    disp('Newton Raphson method failed after No iterations. ');
end;
```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% QUOT_NWT.M %
%% Matlab M file written by Dana J. Coombs %
%% Home machine research %
%% February 11, 1997 %
%% %
%% This file is written as a function to solve for %
%% roots of an equation (specifically a quotient of %
%% forth order). This function uses Newton's method %
%% for solving roots. The inputs are the polynomial %
%% coefficients of the numerator and denominator. %
%% The output is the first root. The initial guess %
%% for this method is chosen to be zero. %
%% %
%% %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```
function [root] = quot_nwt(V,Tau,a2,a1,a0,b4,b3,b2,b1,b0)
```

```
% Inputs
```

```
X0=0; % Initial approximation
```

```
Tol=10^(-5); % Tolerance
```

```
N0=100; % Number of iterations
```

```
% Initialize
```

```
i=1;
```

```
% Newton-Raphson method loop
```

```
while i <= N0,
```

```
    High=b0+b1*X0+b2*X0^2+b3*X0^3+b4*X0^4;
```

```
    Low=a0+a1*X0+a2*X0^2;
```

```
    FX=(2/3)*(High/Low)-V/Tau;
```

```
    DeeLow=a1+2*b2*X0;
```

```
    DeeHigh=b1+2*b2*X0+3*b3*X0^2+4*b4*X0^3;
```

```
    dFX=(2/3)*(Low*DeeHigh-High*DeeLow)/Low^2;
```

```
    X=X0-FX/dFX;
```

```
    if abs(X-X0)<Tol,
```

```
        root=X; break
```

```
        else i=i+1;
        end;

        X0=X;
end;

if i==N0
    disp('Newton Raphson method failed after No iterations.');
```

```

%%%%%%%%%%
%%%%%%%%%%
%% DESING.M %
%% Matlab M file written by Dana J. Coombs %
%% Home machine research %
%% December 17, 1996 %
%% %
%% This file is the main design file. This m file %
%% other m files to calculate specific design steps%
%% The following m files are called: %
%% psh_pull.m %
%% pin_forc.m %
%% stress.m %
%% dimen.m (dimen.m calls the root_nwt.m function) %
%% %
%% %
%%%%%%%%%%
%%%%%%%%%%

```

```

%Call the m files

```

```

clear;
for i=1:5,
    figure(i);
    clf reset;
end;

```

```

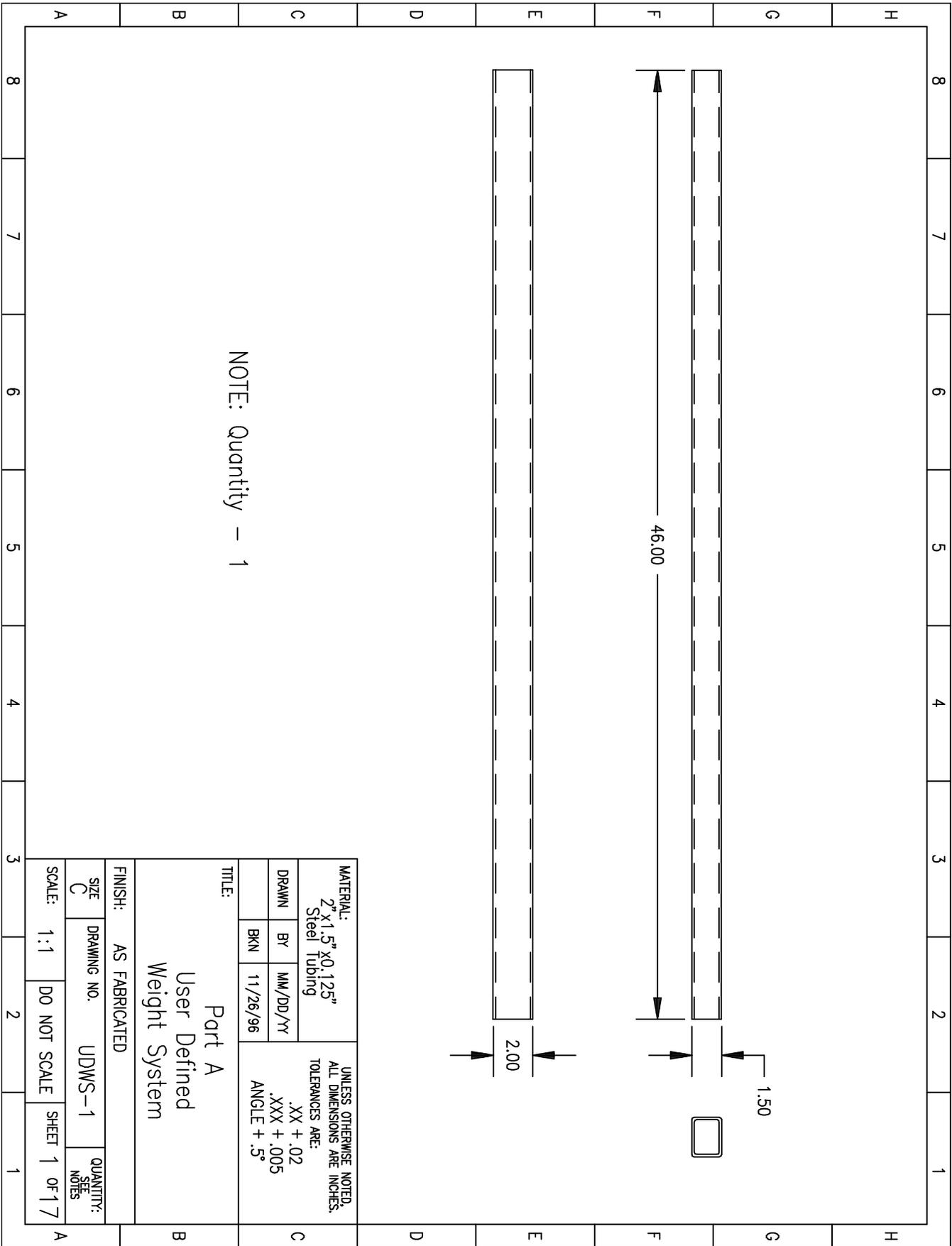
psh_pull
pin_forc
stress
dimen

```

```

end;

```

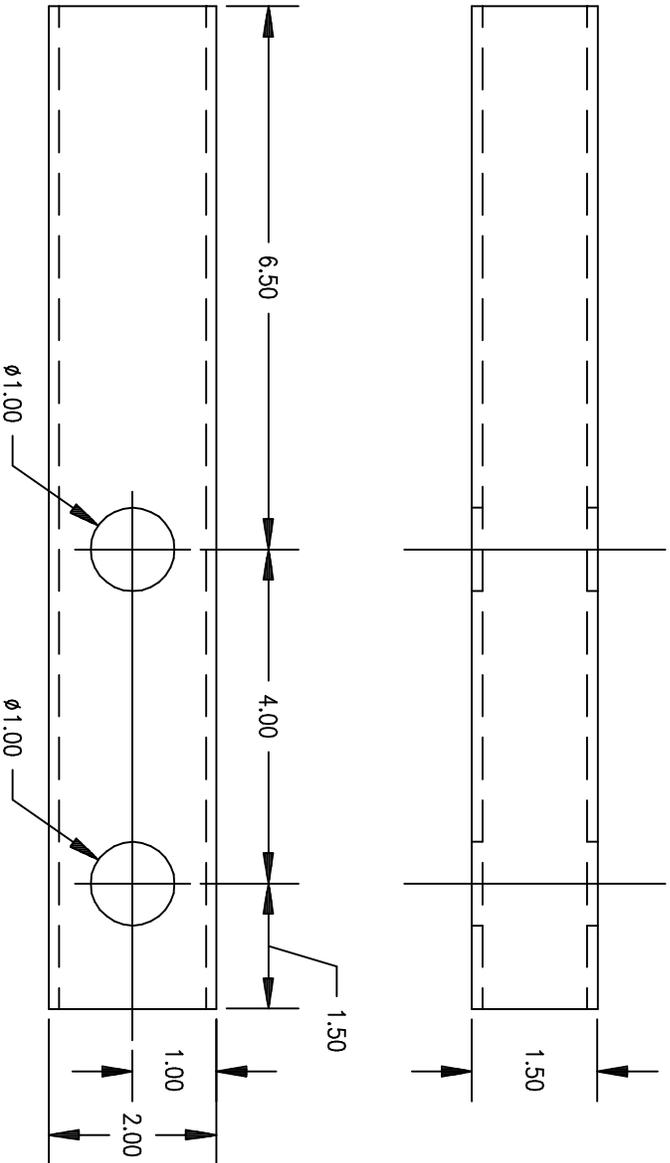


NOTE: Quantity - 1

MATERIAL: 2" X 1.5" X 0.125" Steel Tubing		UNLESS OTHERWISE NOTED, ALL DIMENSIONS ARE INCHES.	
DRAWN	BY	MM/DD/YY	TOLERANCES ARE: .XX + .02 .XXX + .005 ANGLE + .5°
	BKN	11/26/96	

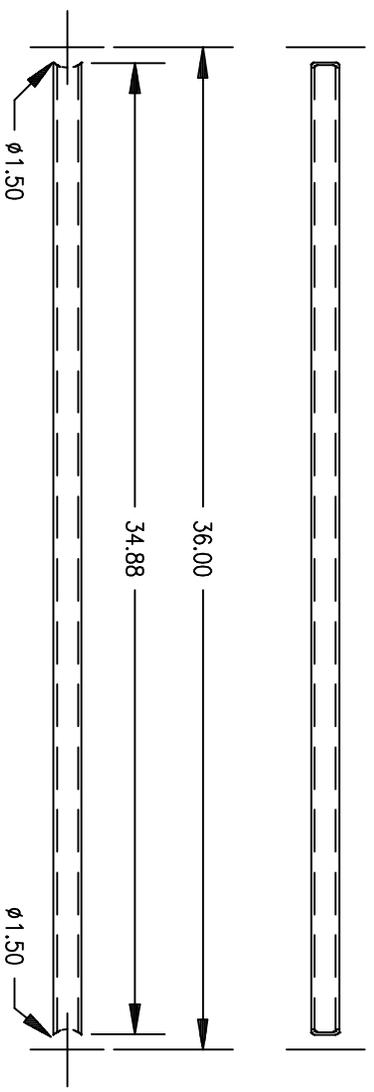
TITLE:
Part A
User Defined
Weight System

FINISH: AS FABRICATED			
SIZE C	DRAWING NO. UDWS-1	QUANTITY: SEE NOTES	
SCALE: 1:1	DO NOT SCALE	SHEET 1	OF 17



NOTE: Quantity - 1

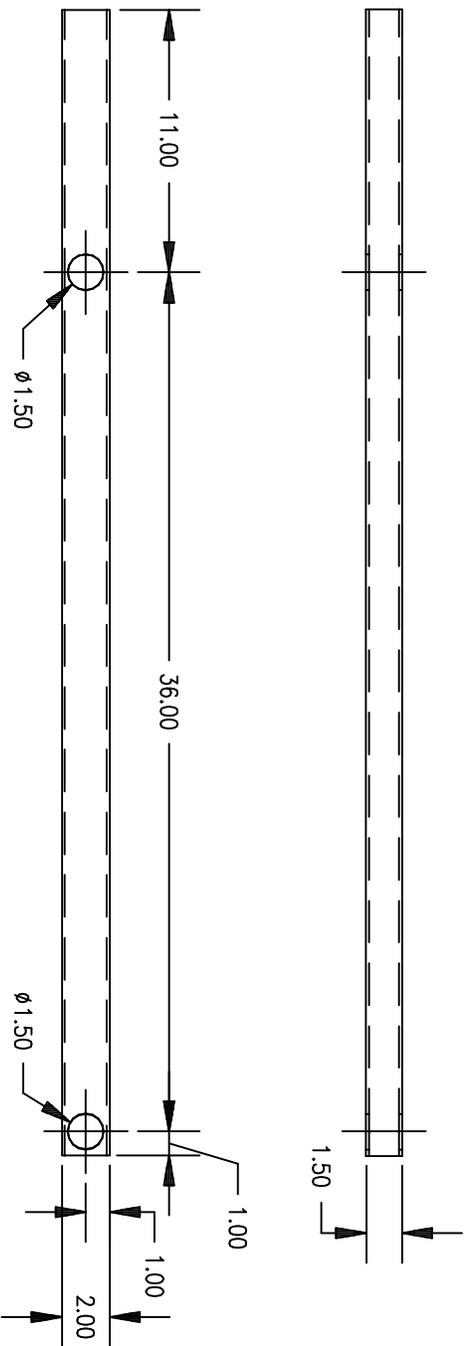
MATERIAL: 2" X 1.5" X 0.125" Steel Tubing		UNLESS OTHERWISE NOTED, ALL DIMENSIONS ARE INCHES.	
DRAWN	BY	MM/DD/YY	TOLERANCES ARE: .XX + .02 .XXX + .005 ANGLE + .5°
	BKN	12/05/96	
TITLE: Part B User Defined Weight System			
FINISH: AS FABRICATED			
SIZE C	DRAWING NO. UDWS-2	QUANTITY: SEE NOTES	
SCALE: 1:1	DO NOT SCALE	SHEET 2 OF 17	



NOTE: Quantity - 1

MATERIAL: 1" x 1" x 0.125" Steel Tubing		UNLESS OTHERWISE NOTED, ALL DIMENSIONS ARE INCHES.	
DRAWN	BY	MM/DD/YY	TOLERANCES ARE: .XX + .02 .XXX + .005 ANGLE + .5°
	BKN	12/10/96	
TITLE: Part C User Defined Weight System			
FINISH: AS FABRICATED			
SIZE C	DRAWING NO. UDWS-3	QUANTITY: SEE NOTES	
SCALE: 1:1	DO NOT SCALE	SHEET 3 OF 17	

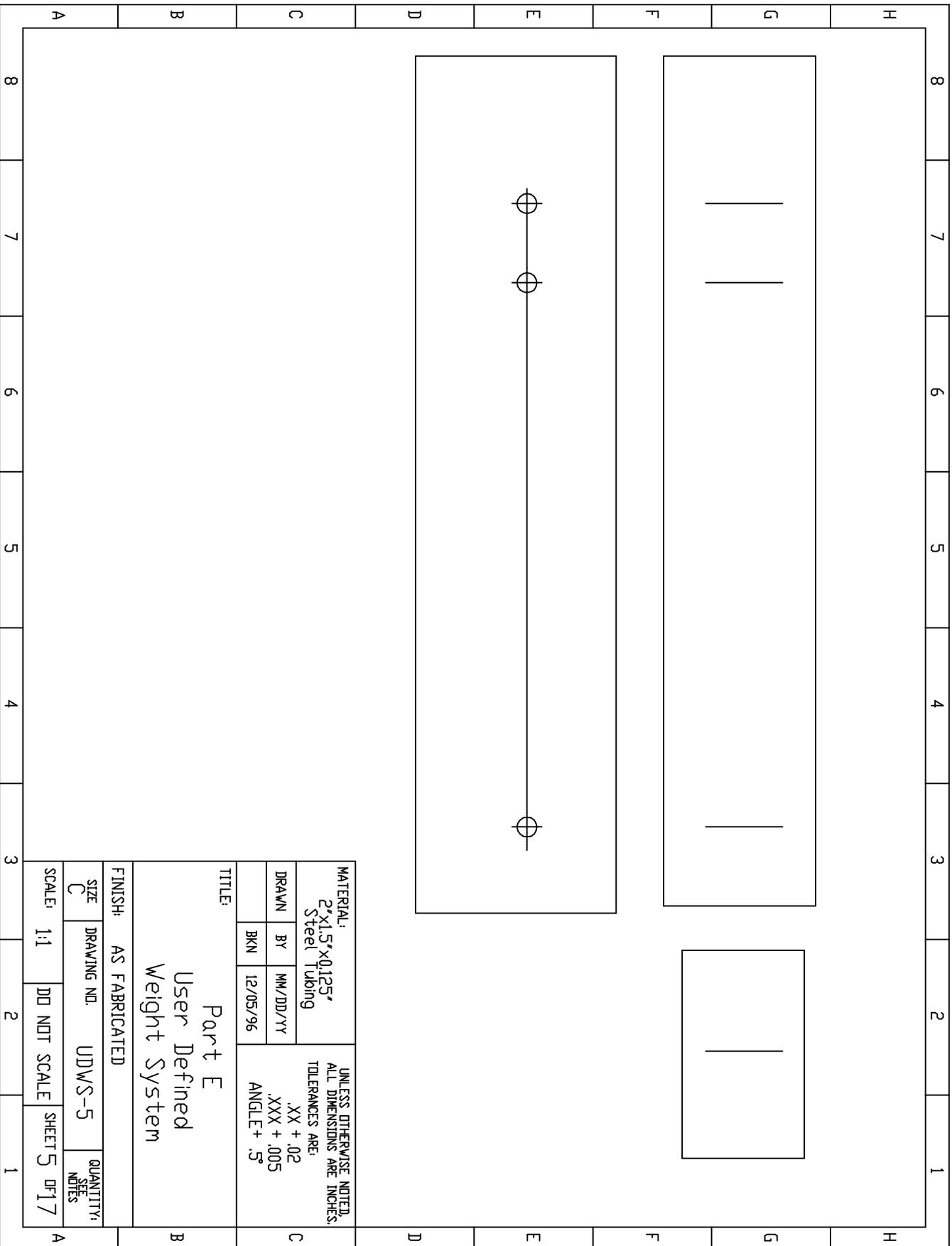
NOTE: Quantity - 1



MATERIAL: 2" X 1.5" X 0.125" Steel Tubing		UNLESS OTHERWISE NOTED, ALL DIMENSIONS ARE INCHES.	
DRAWN	BY	MM/DD/YY	TOLERANCES ARE: .XX + .02 .XXX + .005 ANGLE + .5°
	BKN	12/15/96	

TITLE:
Part D
User Defined
Weight System

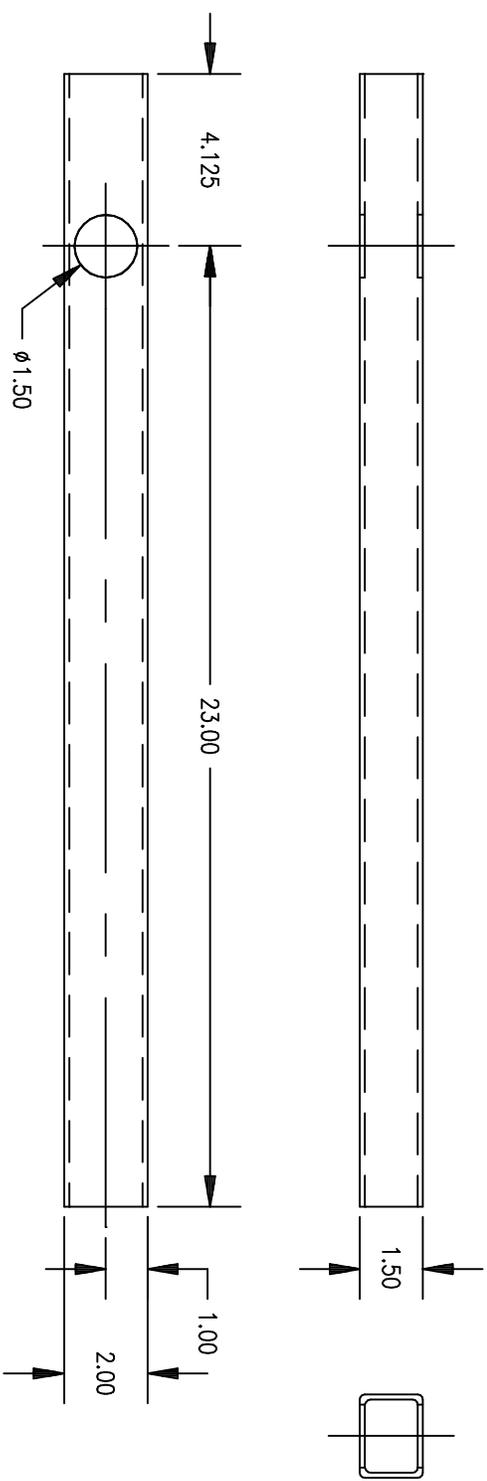
FINISH: AS FABRICATED			
SIZE C	DRAWING NO. UDWS-4	QUANTITY: SEE NOTES	
SCALE: 1:1	DO NOT SCALE	SHEET 4	OF 17



MATERIAL: 2"x1.5"x0.125" Steel Tubing		UNLESS OTHERWISE NOTED, ALL DIMENSIONS ARE INCHES. TOLERANCES ARE: .XX + .02 .XXX + .005 ANGLE + .5°	
DRAWN	BY	MM/DD/YY	
	BKN	12/05/96	

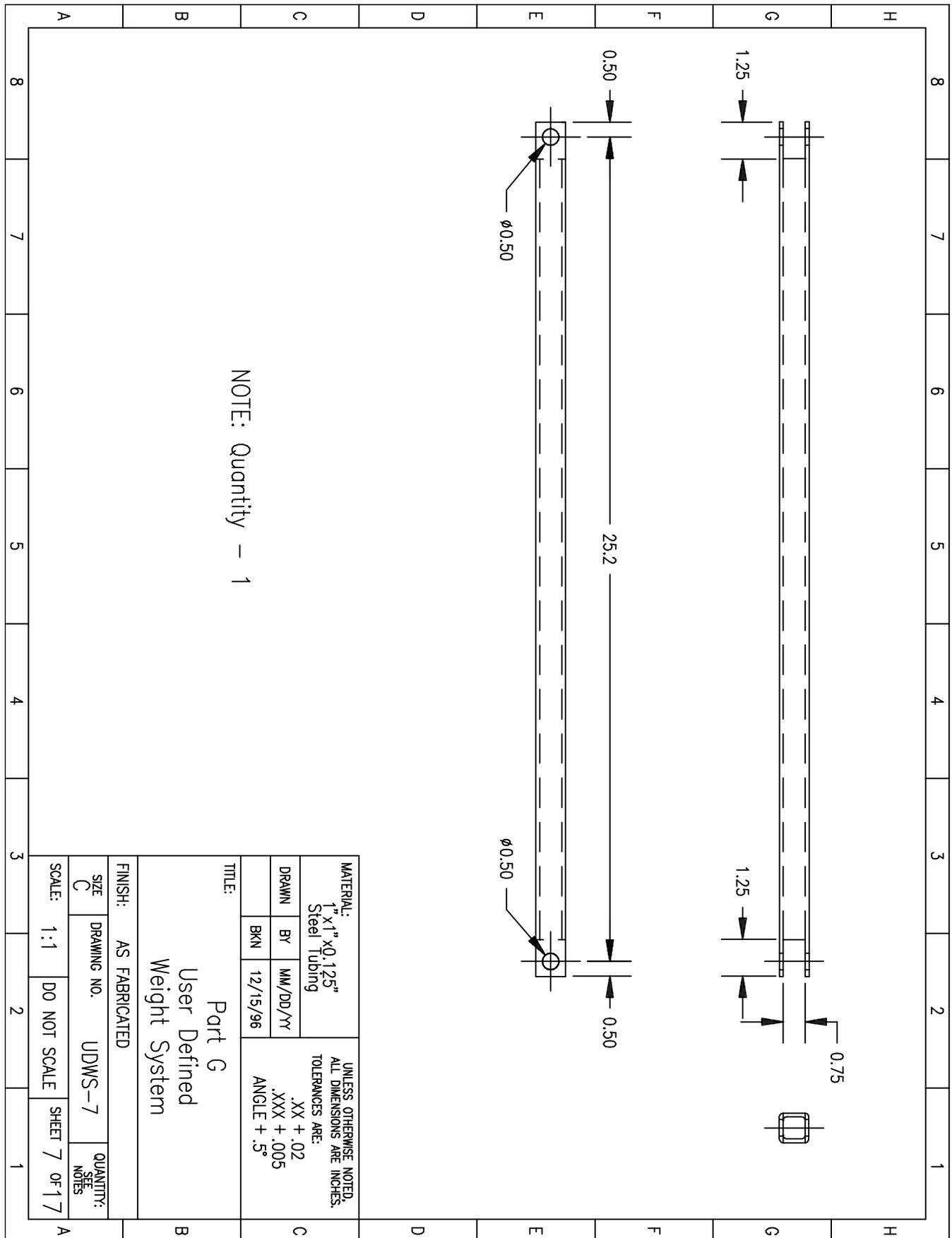
TITLE:
Part E
User Defined
Weight System

FINISH: AS FABRICATED			
SIZE	DRAWING NO.	UDWS-5	QUANTITY:
C			SEE NOTES
SCALE:	1:1	DD NOT SCALE	SHEET 5 OF 7



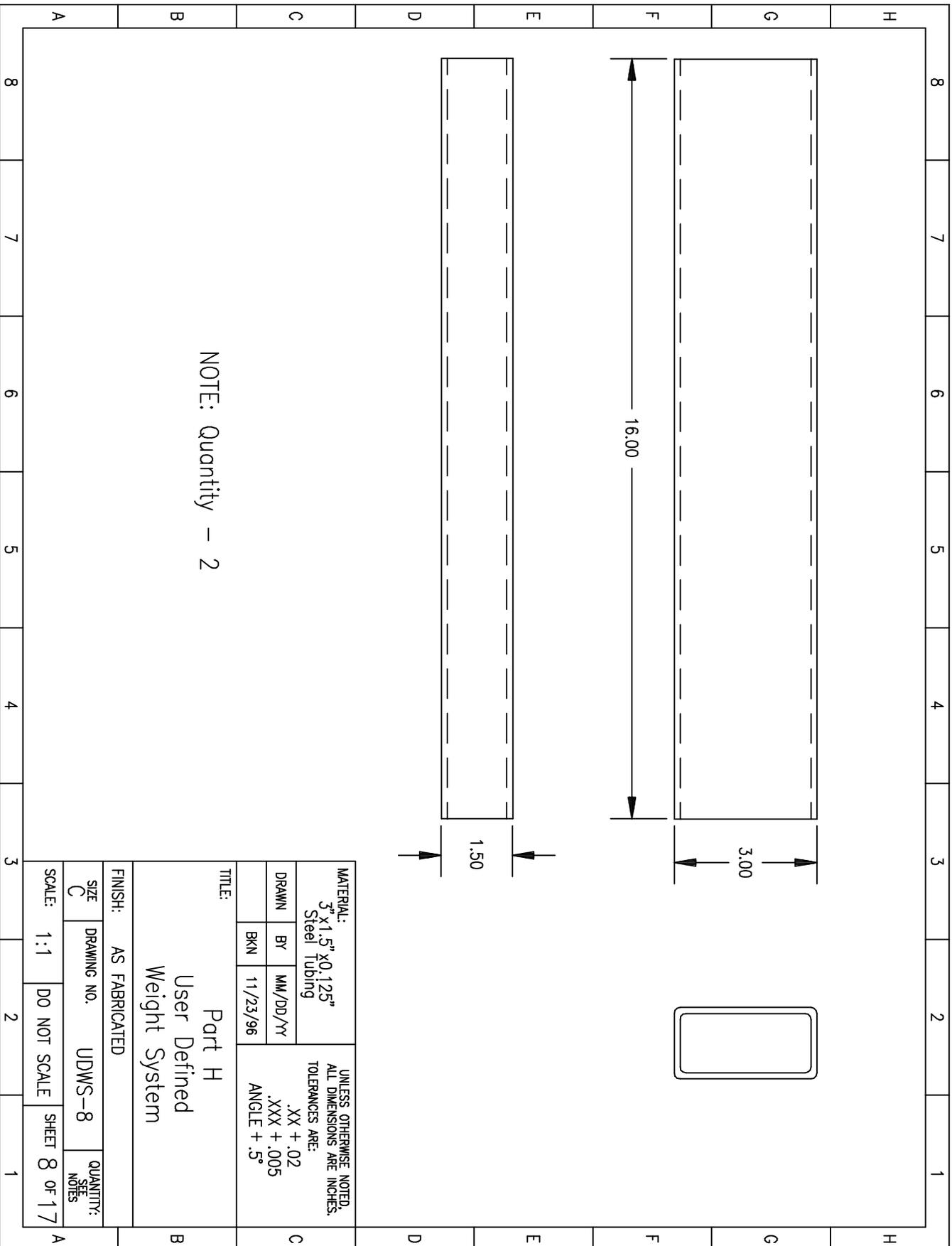
NOTE: Quantity - 1

MATERIAL:		2" X 1.5" X 0.125" Steel Tubing		UNLESS OTHERWISE NOTED, ALL DIMENSIONS ARE INCHES.	
DRAWN		BY	MM/DD/YY	TOLERANCES ARE:	
		BKN	12/15/96	.XX + .02	
				.XXX + .005	
				ANGLE + .5°	
TITLE:					
Part F					
User Defined					
Weight System					
FINISH: AS FABRICATED					
SIZE	DRAWING NO.	UDWS-6	QUANTITY:		
C			SEE NOTES		
SCALE:	DO NOT SCALE	SHEET	6	OF 17	
1:1					



NOTE: Quantity - 1

MATERIAL: 1" x 1" x 0.125" Steel Tubing		UNLESS OTHERWISE NOTED, ALL DIMENSIONS ARE INCHES.	
DRAWN	BY	MM/DD/YY	TOLERANCES ARE: .XX + .02 .XXX + .005 ANGLE + .5°
	BKN	12/15/96	
TITLE: Part G User Defined Weight System			
FINISH: AS FABRICATED			
SIZE C	DRAWING NO.	UDWS-7	QUANTITY: SEE NOTES
SCALE: 1:1	DO NOT SCALE	SHEET 7	OF 17

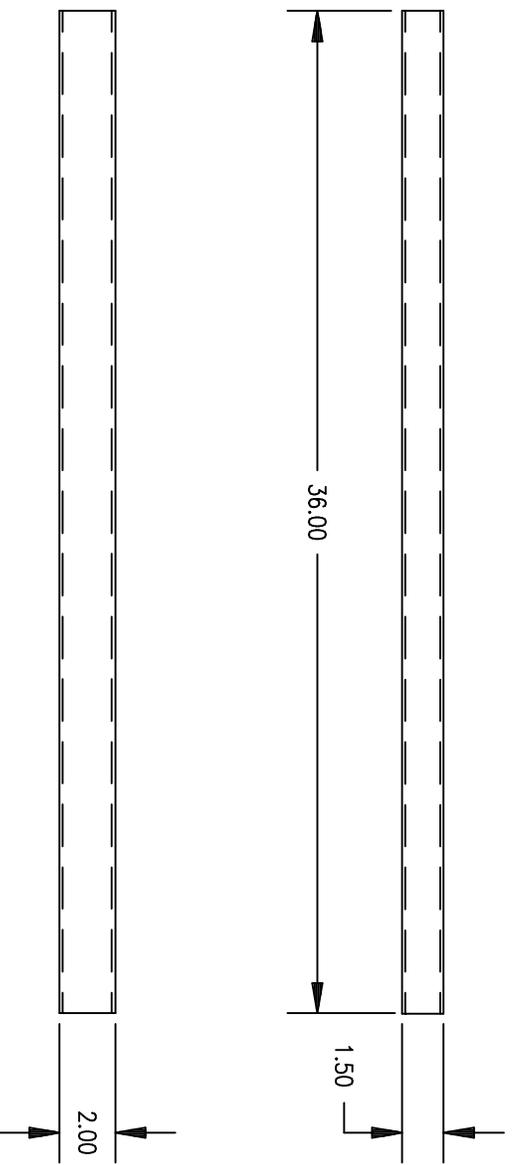


NOTE: Quantity - 2

MATERIAL: 3" X 1.5" X 0.125" Steel Tubing		UNLESS OTHERWISE NOTED, ALL DIMENSIONS ARE INCHES.	
DRAWN	BY	MM/DD/YY	TOLERANCES ARE: .XX + .02 .XXX + .005 ANGLE + .5°
	BKN	11/23/96	

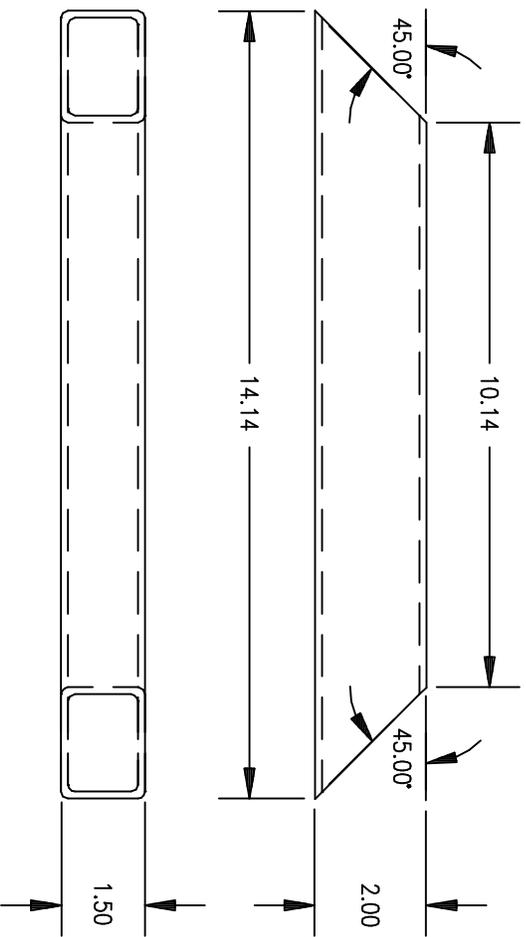
TITLE:
Part H
User Defined
Weight System

FINISH: AS FABRICATED			
SIZE C	DRAWING NO. UDWS-8	QUANTITY: SEE NOTES	
SCALE: 1:1	DO NOT SCALE	SHEET 8	OF 17



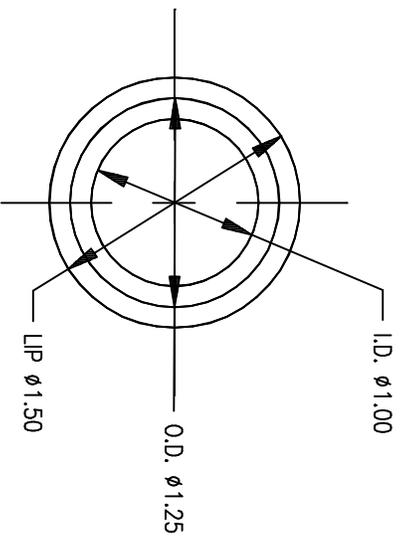
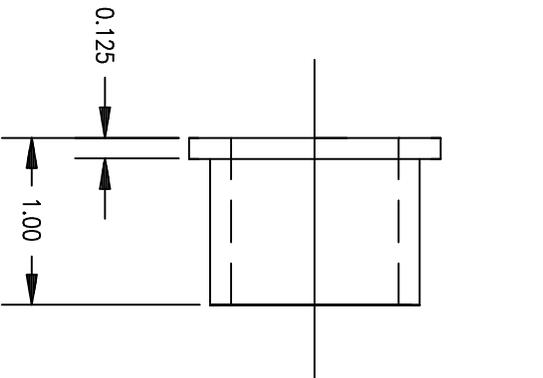
NOTE: Quantity - 1

MATERIAL: 2" X 1.5" X 0.125" Steel Tubing		UNLESS OTHERWISE NOTED, ALL DIMENSIONS ARE INCHES.	
DRAWN	BY	MM/DD/YY	TOLERANCES ARE: .XX + .02 .XXX + .005 ANGLE + .5°
	BKN	12/3/96	
TITLE: Part 1 User Defined Weight System			
FINISH: AS FABRICATED			
SIZE C	DRAWING NO. UDWS-9	QUANTITY: SEE NOTES	
SCALE: 1:1	DO NOT SCALE	SHEET 9 OF 17	



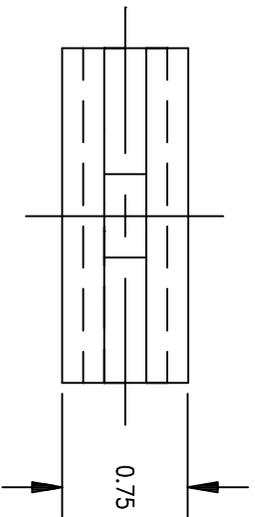
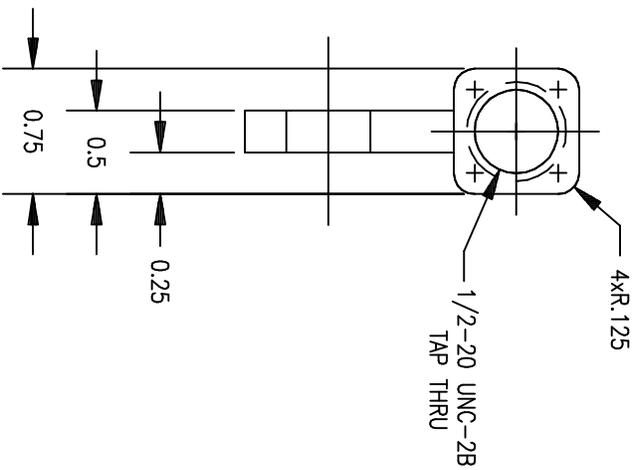
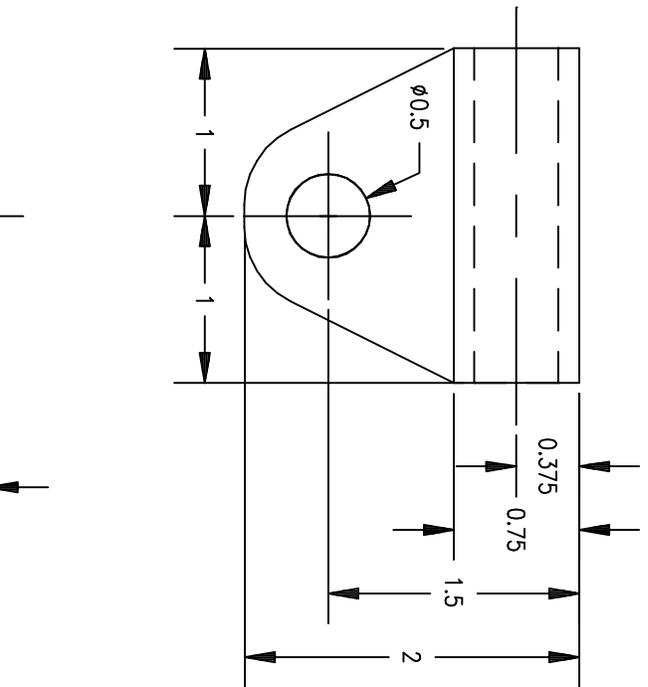
NOTE: Quantity – 2

MATERIAL: 2" X 1.5" X 0.125" Steel Tubing		UNLESS OTHERWISE NOTED, ALL DIMENSIONS ARE INCHES.	
DRAWN	BY	MM/DD/YY	TOLERANCES ARE: .XX + .02 .XXX + .005 ANGLE + .5°
	BKN	12/04/96	
TITLE: Part J User Defined Weight System			
FINISH: AS FABRICATED			
SIZE C	DRAWING NO.	UDWS-10	QUANTITY: SEE NOTES
SCALE: 1:1	DO NOT SCALE	SHEET 10	OF 17



NOTE: Quantity - 10

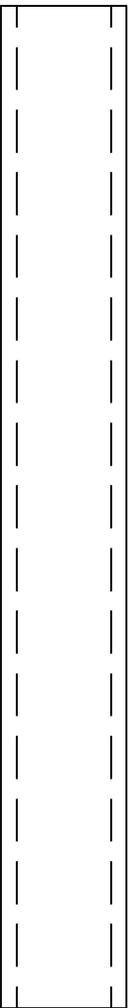
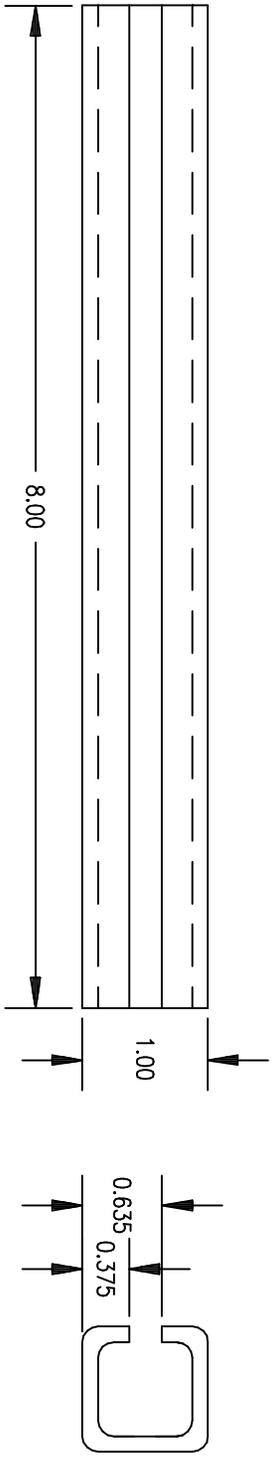
MATERIAL:		Sintered Bronze		UNLESS OTHERWISE NOTED, ALL DIMENSIONS ARE INCHES.	
FINISH:	AS FABRICATED		TOLERANCES ARE:		
SIZE	DRAWING NO.	UDWS-12	XX + .02	XXX + .005	ANGLE + .5°
C					
DRAWN	BY	MM/DD/YY			
	BKN	12/04/96			
TITLE:					
Part M					
User Defined					
Weight System					
FINISH: AS FABRICATED					
SCALE:		1:1	DO NOT SCALE	SHEET 12	OF 17



NOTE: Quantity - 1

MATERIAL: 2" x 2" x 0.75" Steel Block		UNLESS OTHERWISE NOTED, ALL DIMENSIONS ARE INCHES.	
DRAWN	BY	MM/DD/YY	TOLERANCES ARE: .XX + .02 .XXX + .005 ANGLE + .5°
	BKN	12/15/96	
TITLE: Part N User Defined Weight System			
FINISH: AS FABRICATED			
SIZE C	DRAWING NO. UDWS-13	QUANTITY: SEE NOTES	
SCALE: 1:1	DO NOT SCALE	SHEET 13 OF 17	

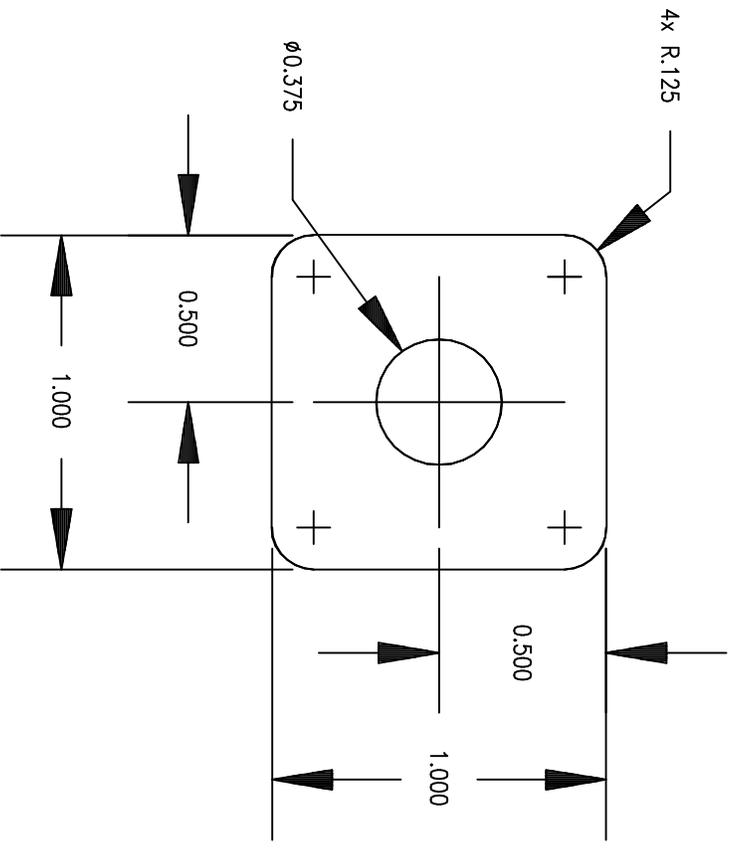
NOTE: Quantity - 1



MATERIAL: 1" x 1" x 0.125" Steel Tubing		UNLESS OTHERWISE NOTED, ALL DIMENSIONS ARE INCHES.	
DRAWN	BY	MM/DD/YY	TOLERANCES ARE: .XX + .02 .XXX + .005 ANGLE + .5°
	BKN	12/15/96	

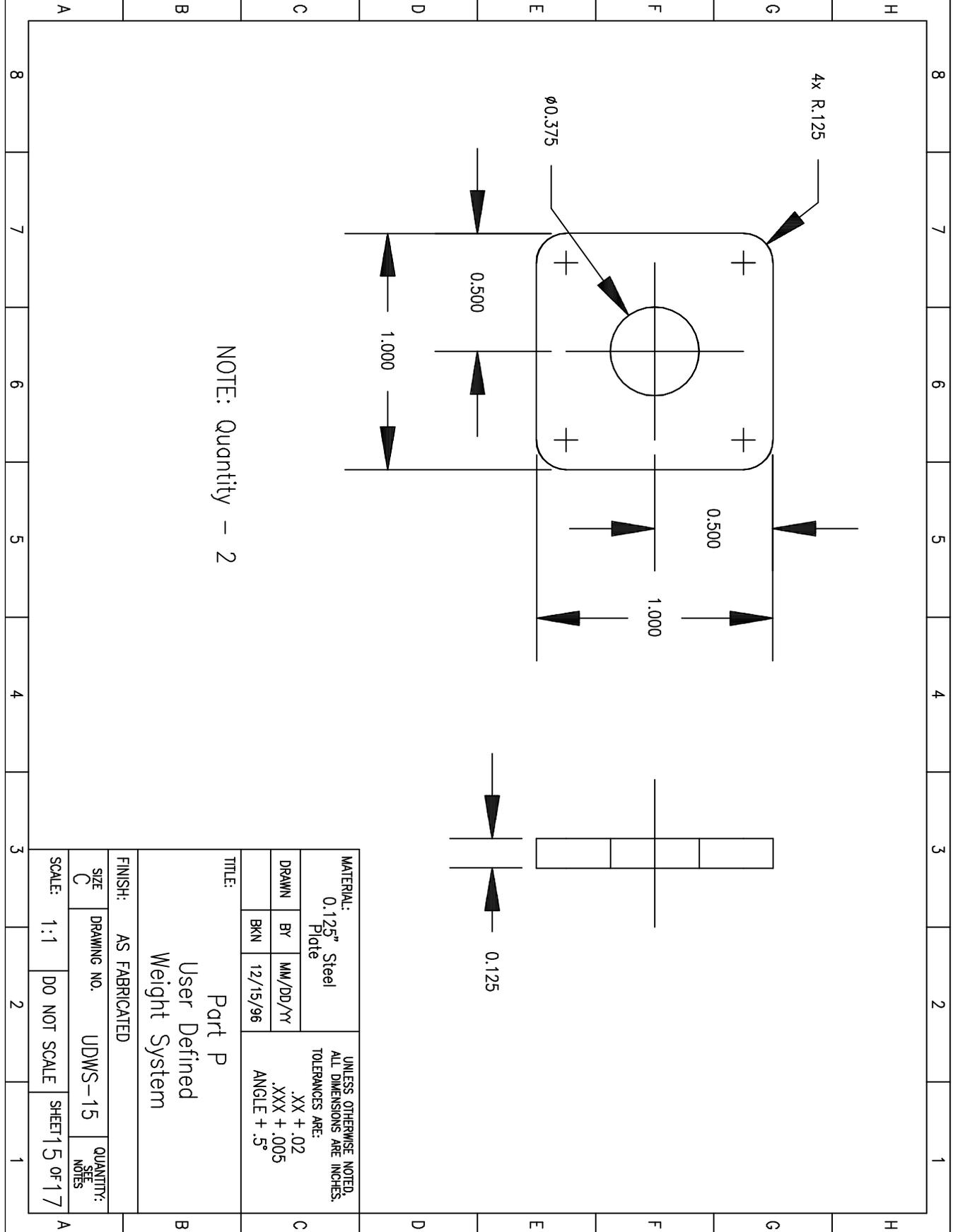
TITLE:
Part O
User Defined
Weight System

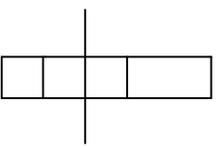
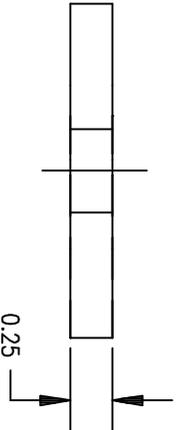
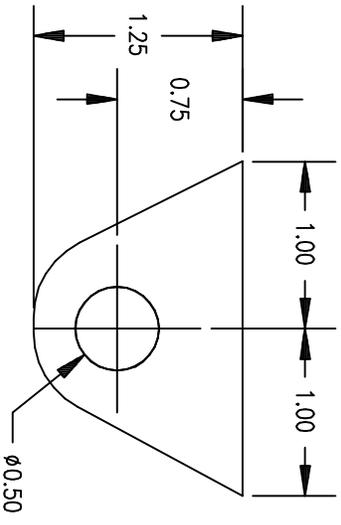
FINISH: AS FABRICATED			
SIZE C	DRAWING NO. UDWS-14	QUANTITY: SEE NOTES	
SCALE: 1:1	DO NOT SCALE	SHEET 4 OF 17	



NOTE: Quantity - 2

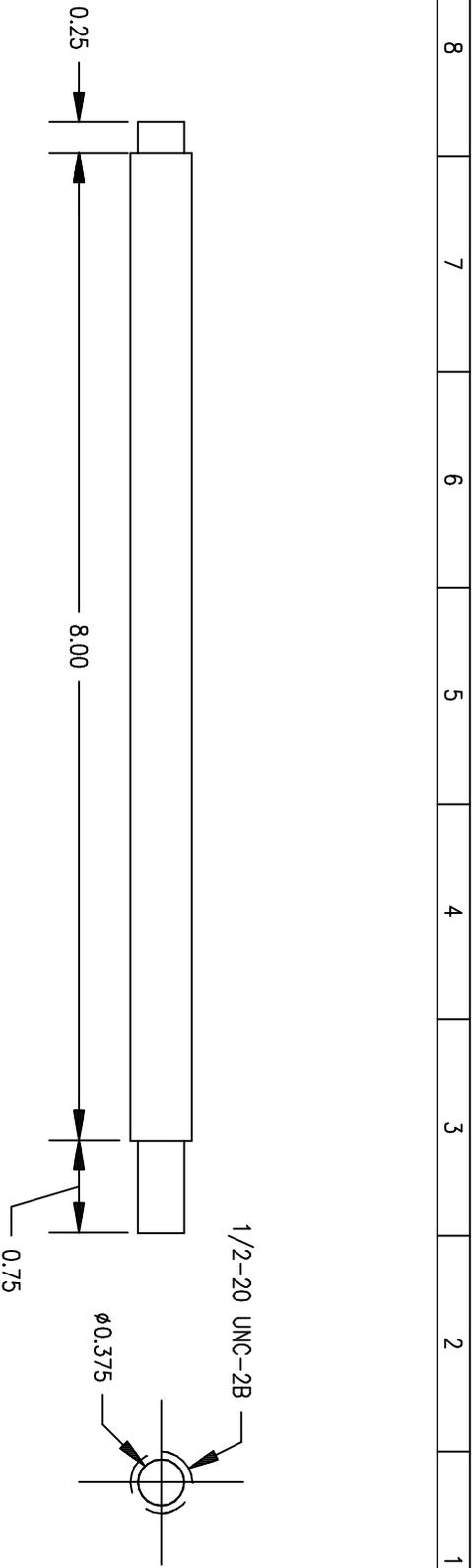
MATERIAL: 0.125" Steel Plate		UNLESS OTHERWISE NOTED, ALL DIMENSIONS ARE INCHES. TOLERANCES ARE: .XX + .02 .XXX + .005 ANGLE + .5°	
DRAWN	BY	MM/DD/YY	
	BKN	12/15/96	
TITLE: Part P User Defined Weight System			
FINISH: AS FABRICATED			
SIZE	DRAWING NO.	UDWS-15	QUANTITY: SEE NOTES
C			15 OF 17
SCALE: 1:1		DO NOT SCALE	





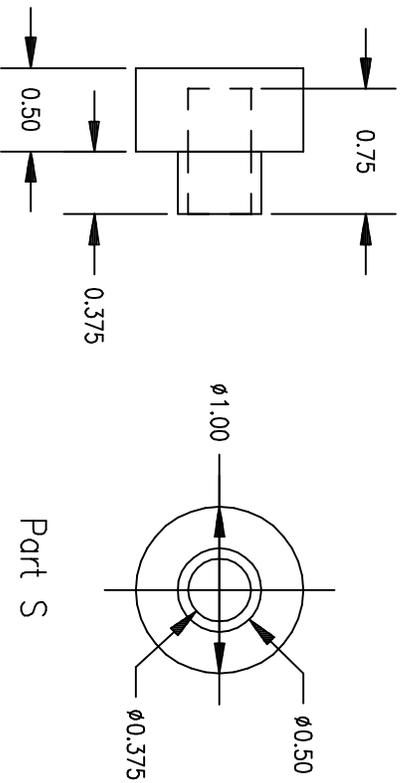
NOTE: Quantity - 1

MATERIAL: 0.25" Steel Plate		UNLESS OTHERWISE NOTED, ALL DIMENSIONS ARE INCHES.	
DRAWN	BY	MM/DD/YY	TOLERANCES ARE: .XX + .02 .XXX + .005 ANGLE + .5°
	BKN	12/15/96	
TITLE: Part Q User Defined Weight System			
FINISH: AS FABRICATED			
SIZE C	DRAWING NO.	UDWS-16	QUANTITY: SEE NOTES
SCALE: 1:1	DO NOT SCALE	SHEET 16	OF 17



Part R

NOTE: Quantity - 1



Part S

NOTE: Quantity - 1

MATERIAL:		Steel		UNLESS OTHERWISE NOTED, ALL DIMENSIONS ARE INCHES.	
DRAWN		BY	MM/DD/YY	TOLERANCES ARE:	
		BKN	12/15/96	.XX + .02	
				.XXX + .005	
				ANGLE + .5°	
TITLE:					
Part R & Part S					
User Defined					
Weight Systems					
FINISH: AS FABRICATED					
SIZE	DRAWING NO.	UDWS-17		QUANTITY:	
C				SEE NOTES	
SCALE:	1:1	DO NOT SCALE	SHEET 17	OF 17	

Vita

Dana Joseph Coombs

Dana Joseph Coombs was born in Everett, Massachusetts on September 8, 1971. He completed his bachelor's degree in Mechanical engineering from Worcester Polytechnic Institute in May of 1993. Following graduation, Dana worked for Optronics as a mechanical engineer designing plotters for printing press applications. He then worked for Parametric Technology Corporation as a quality assurance engineer in the area of computer aided manufacturing.

Dana then came to Virginia Polytechnic Institute and State University in the fall of 1995 to earn a master's degree in mechanical engineering. His focus of study was kinematics and machine design. While attending Virginia Tech, he served as a teaching assistant for a senior-level experimentation class.

Dana plans to return to Parametric Technology Corporation as an application engineer implementing CAD/CAM and kinematic and dynamic modeling solutions to industrial problems.