Elevation Effects on GPS Positional Accuracy

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(Abstract)

Data from a Coarse Acquisition (C/A) Global Positing System (GPS) map-grade receiver were evaluated to assess the accuracy of differentially corrected points. Many studies have focused on the accuracy of GPS units under ideal data collection conditions. Ideal conditions allow the collection of data with four satellites (3D mode), yet field data conditions are often less than ideal. Four satellites may not always be in view because of mountainous topography, heavy forest cover, or other obstructions which block satellite signals from the receiver. This study examines GPS accuracy when four satellites are not available, instead collecting data with only three satellites (2D mode).

3D GPS points compute four unknowns: x, y , z, and clock error. In comparison, 2D GPS points are less accurate as only three unknowns are calculated: x, y, and clock error. Elevation (or z) is not computed for 2D points, causing increased error in the horizontal (x, y) measurement. The effect of elevation was evaluated on 234 2D GPS data points. These points were collected and corrected at elevation intervals of true elevation, +-25 meters, +- 50 meters, and +-75 meters. These 2D points were then compared to surveyed points to measure the effect vertical error has on horizontal accuracy. In general, the more error in the vertical estimate during correction, the greater the horizontal error.

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List of Abbreviations

C/A	Coarse Acquisition
DOD	Department of Defense
DGPS	Differential GPS
DOP	Dilution of Precision
2D	2 Dimensional (3 Satellites)
3D	3 Dimensional (4 Satellites)
EDOP	East Dilution of Precision
GDOP	Geometric Dilution of Precision
GMT	Greenwich Mean Time
GPS	Global Positioning System
HDOP	Horizontal Dilution of Precision
MSL	Mean Sea Level
NAD	North American Datum
NAVSTAR	Navigation Satellite Timing and Ranging
NDOP	North Dilution of Precision
NMAS	National Map Accuracy Standard
PRN	Pseudorandom Noise
S/A	Selective Availability
TDOP	Time Dilution of Precision
VDOP	Vertical Dilution of Precision

Chapter 1: Objective

The objective of this thesis is to quantify the impact of the elevation component on the mapping accuracy of differentially corrected GPS data. GPS 2D data are compared to higher order surveyed locations. The elevation component was controlled during differential correction to simulate inaccurate elevation. The error of the 2D data will be analyzed to propose an estimated accuracy range.

Chapter 2: Introduction

2.1: Global Positioning Systems

The Navigation System with Timing and Ranging (NAVSTAR) Global Positioning System (GPS) is a satellite-based navigation system managed by the United States Department of Defense (DOD). GPS is a system of 24 satellites in asynchronous orbits that are precisely tracked from ground stations. Each ground station has a precisely known geographic location. These ground stations return updated information to each satellite. Each satellite transmits it's location to GPS receivers all over the earth. GPS was initially designed for military purposes but has since found many civilian applications.

Global Positioning System (GPS) receivers are used in two basic ways:

1. Navigation - To navigate from where you are to where you want to be.
2. Mapping - To record and map detailed routes with data about those locations stored as attributes (Oderwald & Boucher, 1997).

GPS units are used in greater quantity each day for both navigating and mapping. This study will focus on the use of GPS receivers to map features such as points, lines, and polygons. GPS is a highly accurate surveying system, but it is not flawless. There are many variables which can affect the accuracy of GPS data. One potential problem is computing positions with three satellites instead of four. There may be only three satellites in view, mainly because of signals blocked by topography or vegetation. There is a significant difference in accuracy between the computation of 3D (x, y, z) points and the less effective computation of 2D (x, y) points.

2.2: Defining the Problem

I first encountered the problem of collecting GPS 2D data points data while collecting data in Auto 2D/3D mode (collecting data in 3D when four satellites are available and in 2D when only three are available) on a forest clearcut. The clearcut was fenced to exclude deer browsing and if it was regenerating successfully, the fence would be removed. The GPS unit was used to determine the acreage, and perimeter of this polygon. An accurate measure of the perimeter was needed to calculate the cost for fence removal as a function of the length in feet. When I started to collect GPS data, the unit informed me I was collecting 2D data and prompted me to input an elevation or it would default to the last known elevation. Not knowing any better at the time, I used the last known elevation, which had been hundreds of feet lower than my current location. The unit collected for a while in 2D mode, then was able to generate a 3D point roughly 500' around the stand. I did not realize my mistake in accepting the last known elevation until returning from the field and correcting the data (Figure 2.1). The first 500' of this stand was adjacent to a forest road that was accurate on my stand map and could be compared against this polygon. The stand matched to the road very well with 3D points (errors less than 5 or 10 meters) but the 2D points along the road were off by roughly 100 meters. The 2D points had the correct shape but not the correct horizontal (x, y) location. Without the road data to compare against the stand, this error in the polygon might have been missed. This would have caused hundreds of feet to be added to the perimeter, and many dollars to the cost of fence removal.

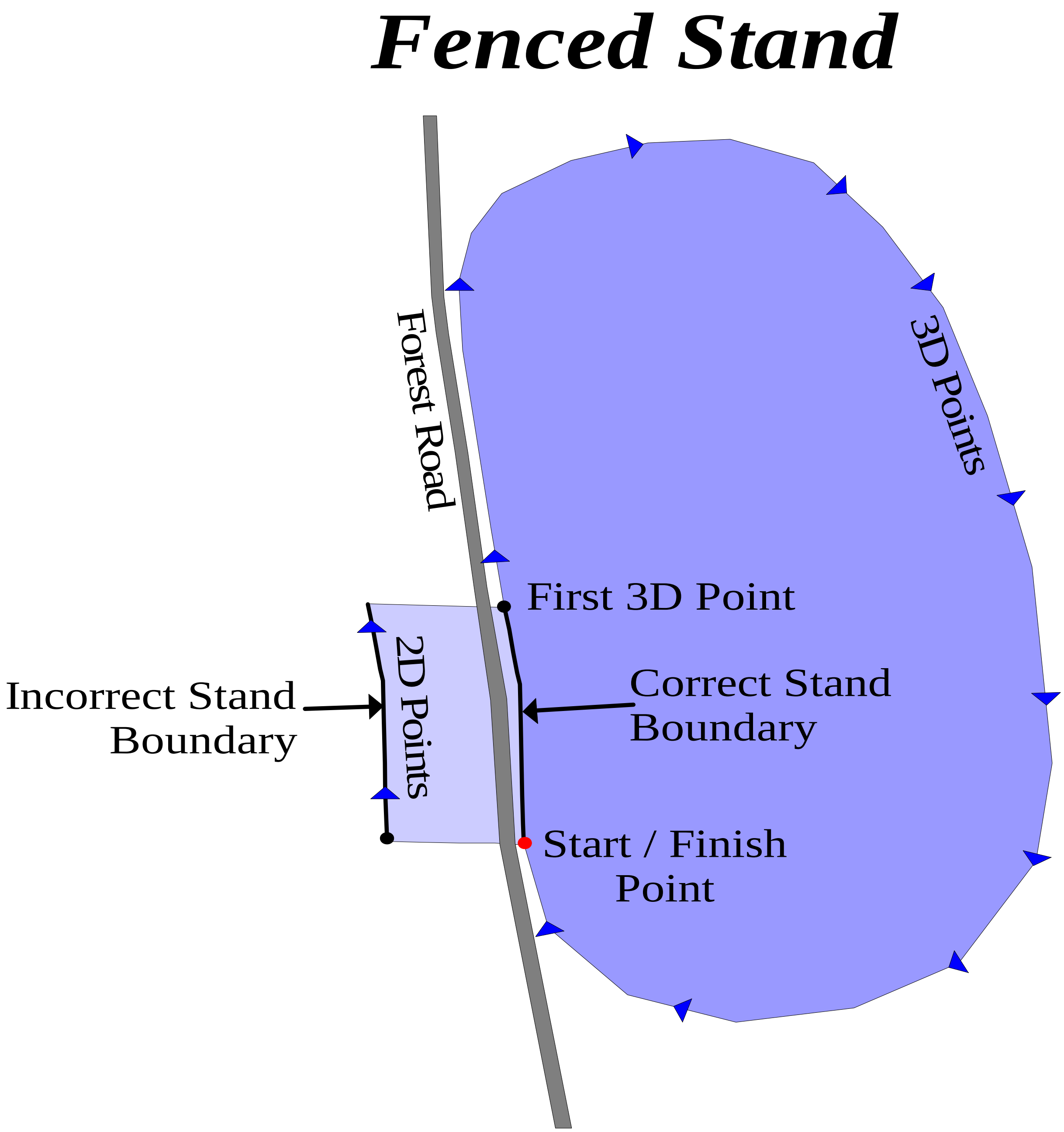


Figure 2.1 A horizontal offset from the true locations shows along this polygon as a result of 2D points collected at the wrong elevation.

In a follow-up experiment, GPS 2D data collected along lines revealed that vertical error caused a variable horizontal offset (Figure 2.2). This data was collected on a long, straight sidewalk. Elevation error was artificially introduced during differential correction. Differential correction is the comparison of GPS field data to a known location GPS base station. The horizontal offset from vertical error varied based on current satellite geometry. Small multipath and other errors on the line corrected at the true elevation were greatly amplified on the lines corrected at + or - 150 meters. The data would not correct at all if the introduced vertical error

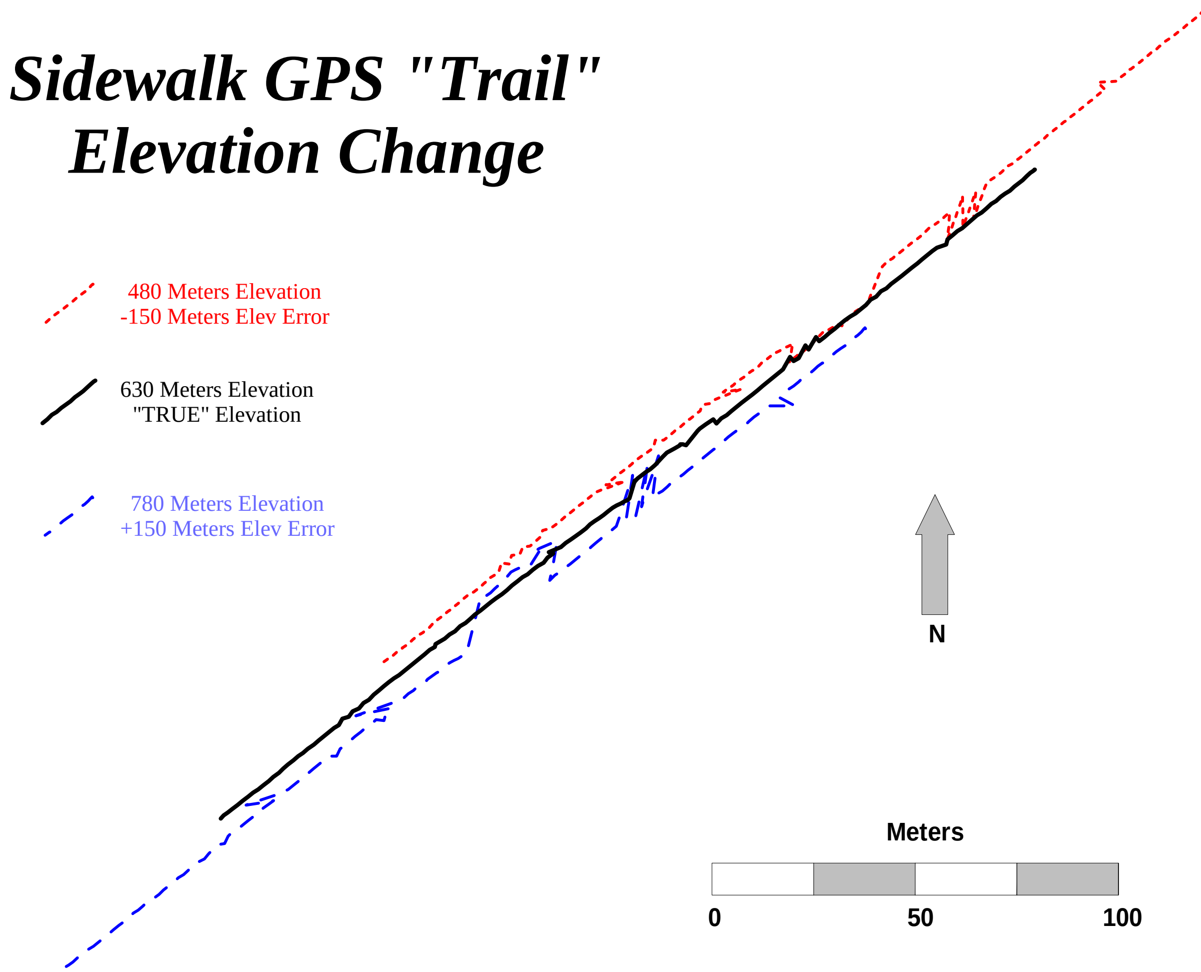


Figure 2.2 Map showing the effect changing vertical error has on a line feature. The dark line is the true elevation, the red dashed line is the line corrected at -150 meters elevation, and the blue dashed line is the line corrected at +150 meters elevation. The horizontal offset of the line is determined by the vertical error and the location of the satellites being used by the receiver.

increased more than 150 meters. Computation of a solution during correction was outside the range the algorithm found acceptable.

2.3: The GPS Signal

Civilian uses of GPS have outgrown even optimistic expectations. There are three signals from GPS satellites:

1. C/A (Coarse Acquisition) code - used by most civilian receivers.

2. P code - an encrypted code that can only be fully utilized by military receivers.

3. The Carrier Phase signal - transports (or carries) both the C/A and P codes.

GPS receivers use pseudo-random code to determine the difference in receiver and satellite time. The C/A code in a GPS satellite and receiver are synchronized to produce the same code at the same time (Hurn, 1989). The receiver compares its code for each satellite to the code being produced by that satellite. The time difference is computed to estimate the distance from the satellite to the receiver. The distance computed is known as a pseudo-range because it estimates the true distance between the satellite and receiver.

2.4: Using Time to Measure Distance

GPS uses time to measure the distance from each satellite to the receiver. The receiver computes the time required for the signal to travel from the satellite (a known position) to the receiver (an unknown position). The distance (pseudorange) of each satellite to the receiver is calculated by the formula: distance = rate x time. Since the rate is fixed at the speed of light (186,000 miles per second), a variation in time causes a variation in distance. "By measuring the time elapsed for a signal to propagate from a satellite to a receiver and multiplying it by the speed of light, a GPS receiver can determine the range to the satellite." (Mirsa, 1996, p.60). The receiver's location is determined by solving three (x, y, time) or preferably four (x, y, z, time) unknowns.   These unknowns are solved using three equations with three unknowns or four equations with four unknowns. Three satellites are enough to determine a location. Four satellites give a more accurate location because another unknown is calculated, elevation (z).

Data can be collected with as few as three satellites, but four are preferred for increased accuracy. "The GPS system is based on precise timing of radio signals from at least 3 satellites." (Verbyla, 1995, p.85). Collins et al. (1994) state why the fourth satellite is so important for an accurate measure of location:

> "The pseudorange is derived either from measuring the travel time of the (coded) signal and multiplying it by its velocity or by measuring the phase of the signal. In both cases, the clocks of the receiver and the satellite are employed. Since these clocks are never perfectly synchronized, instead of true ranges, "pseudoranges" are obtained where the synchronization error (denoted as clock error) is taken into account. Consequently, each equation of this type comprises four unknowns: the desired three point coordinates contained in the true range, and the clock error. Thus, four satellites are necessary to solve for the four unknowns." (Collins, Hofmann-Wellenhof, Lichtenegger, 1994, pp.13-14).

The clocks in GPS satellites are very precise and accurate because the estimate of distance requires accurate measurement of time delay. The clocks detect a small change in time (ordinarily billionths of a second) for the signal to travel from the satellite to the receiver on earth. "Effectively, the satellite signal is continually marked with its (own) transmission time so that when received the signal transit period can be measured with a synchronized receiver"(Collins, Hofmann-Wellenhof, Lichtenegger, 1994, p.13). The clocks used in GPS receivers are less expensive and less accurate than satellite clocks; this difference causes increased potential for time transfer error (Mirsa, 1996).

Chapter 3: Sources of GPS Error

GPS is an important tool for mapping features because it provides accurate data at a low cost relative to traditional surveying methods. The accuracy of digitized features is dependent on the precision of the mapping process with GPS. There are many potential sources of error in obtaining GPS measurements. Inaccurate GPS measurements may be caused by errors in the: satellite clock, satellite position (ephemeris), receiver, upper atmosphere (ionosphere), lower atmosphere (troposphere), multipath (bounced signals), and selective availability (introduced scrambling) (Kennedy, 1996).

3.1: Selective Availability - Clock Errors

The largest source of possible error in the GPS C/A code signal is selective availability (S/A). S/A is intentional signal degradation introduced for national security reasons. "Since the Department of Defense controls the Global Positioning NAVSTAR satellites and they want to protect against terrorists using accurate GPS technology, they select times to degrade the satellite signals. This mode is called selective availability. When selective availability is in effect, the military can use GPS with normal accuracy while civilians (who do not have the signal degradation parameters) suffer with reduced accuracy of GPS-derived map coordinates." (Verbyla, 1995, p.86). "S/A is essentially a method for artificially creating a significant clock error in the satellites." (Hurn, 1989, p.56). Each satellite transmits its current position with the ephemeris signal. "The goal of S/A is to degrade this navigation accuracy by dithering the satellite clock and manipulating the ephemeris. [Dithering the satellite time] is achieved by introducing varying errors into the fundamental frequency of the satellite clock. [Manipulating the ephemeris] is the truncation of the orbital information in the transmitted navigation message so that the coordinates of the satellites cannot accurately be computed. The error in satellite position roughly translates to a like position error of the receiver." (Collins, Hoffman-Wallenhof, Lichtenegger, 1994, pp.19-20).

Another source of error in the GPS signal is Dilution of Precision (DOP) or Geometric Dilution of Precision (GDOP). GDOP is the geometry between the receiver and the set of satellites in view during a location solution by the receiver (Hurn, 1989). While GDOP (geometric) is used to estimate the total geometry of the satellites to the receiver, PDOP (positional) and HDOP (horizontal) are used more often to estimate DOP for 3D and 2D positions, respectively (Carstensen, 1997). Figure 3.1 shows the relationship between the different DOP measures.

DOP measures are used as quality evaluations of the geometric arrangement of the receiver from the satellites. The wider the angle between the satellites being used for a position measure, the better the measurement. GPS receivers pick dispersed satellite arrangements (with a low DOP) based on the positions of all the satellites in view (Hurn, 1989). This study evaluates 2D points with HDOP as a geometric quality measure. A theoretical ideal (lowest error) HDOP with three satellites would be to have one satellite directly overhead and the other two satellites equally spaced 180° near the horizon (Kennedy, 1996). Figure 3.2 depicts satellite arrangement for a high and a low HDOP.

Some errors in the GPS signal, such as multipath (bounced signals) and satellite geometry (DOP) measures, cannot be removed. However, many of these errors can be reduced or virtually eliminated by comparing the data collected to a known location at the same time through a process known as differential correction.

DOP Quality Measures

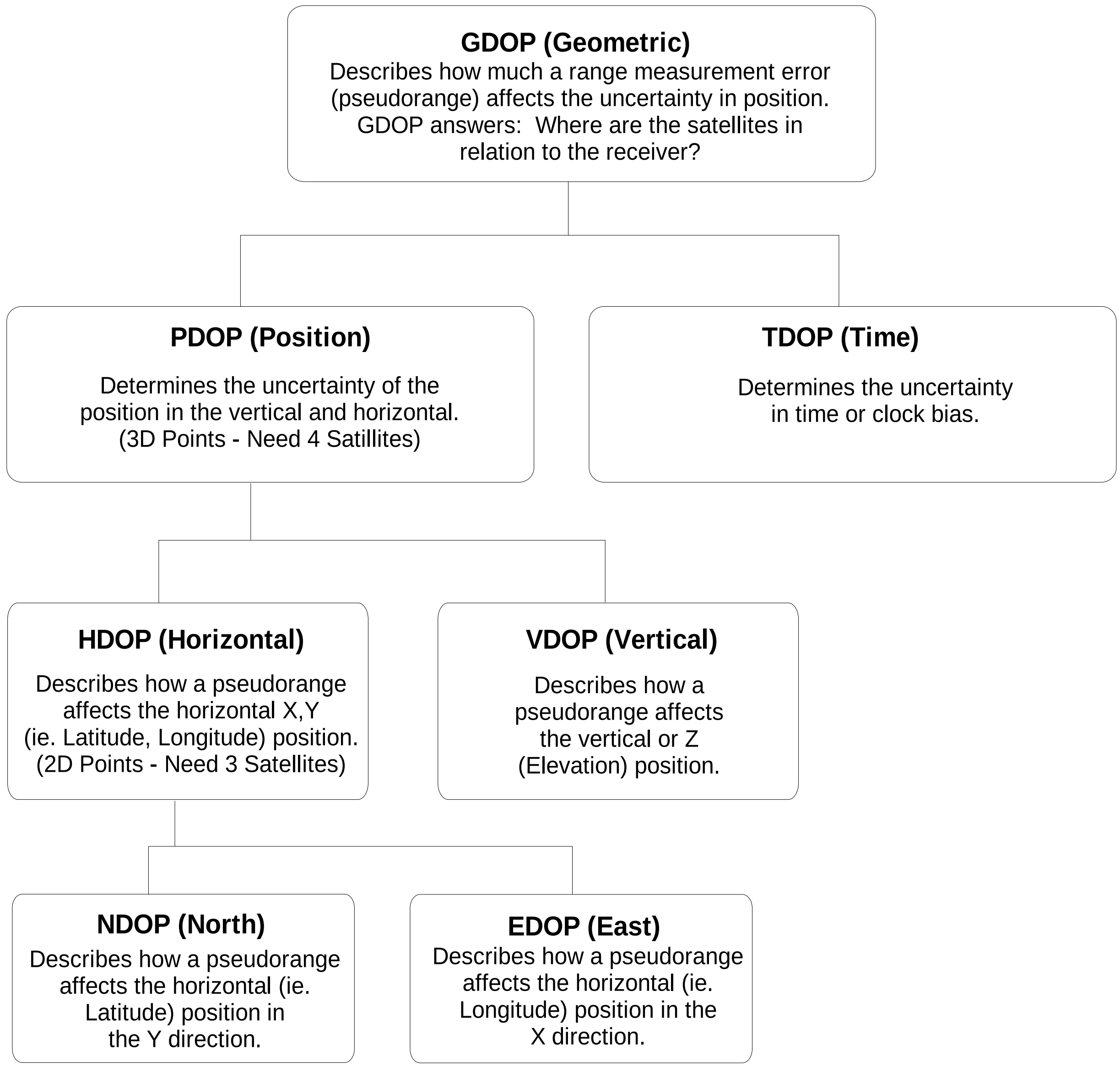


Figure 3.1 The different Dilution of Precision (DOP) quality measures and their relation to one another.

Low HDOP vs. High HDOP

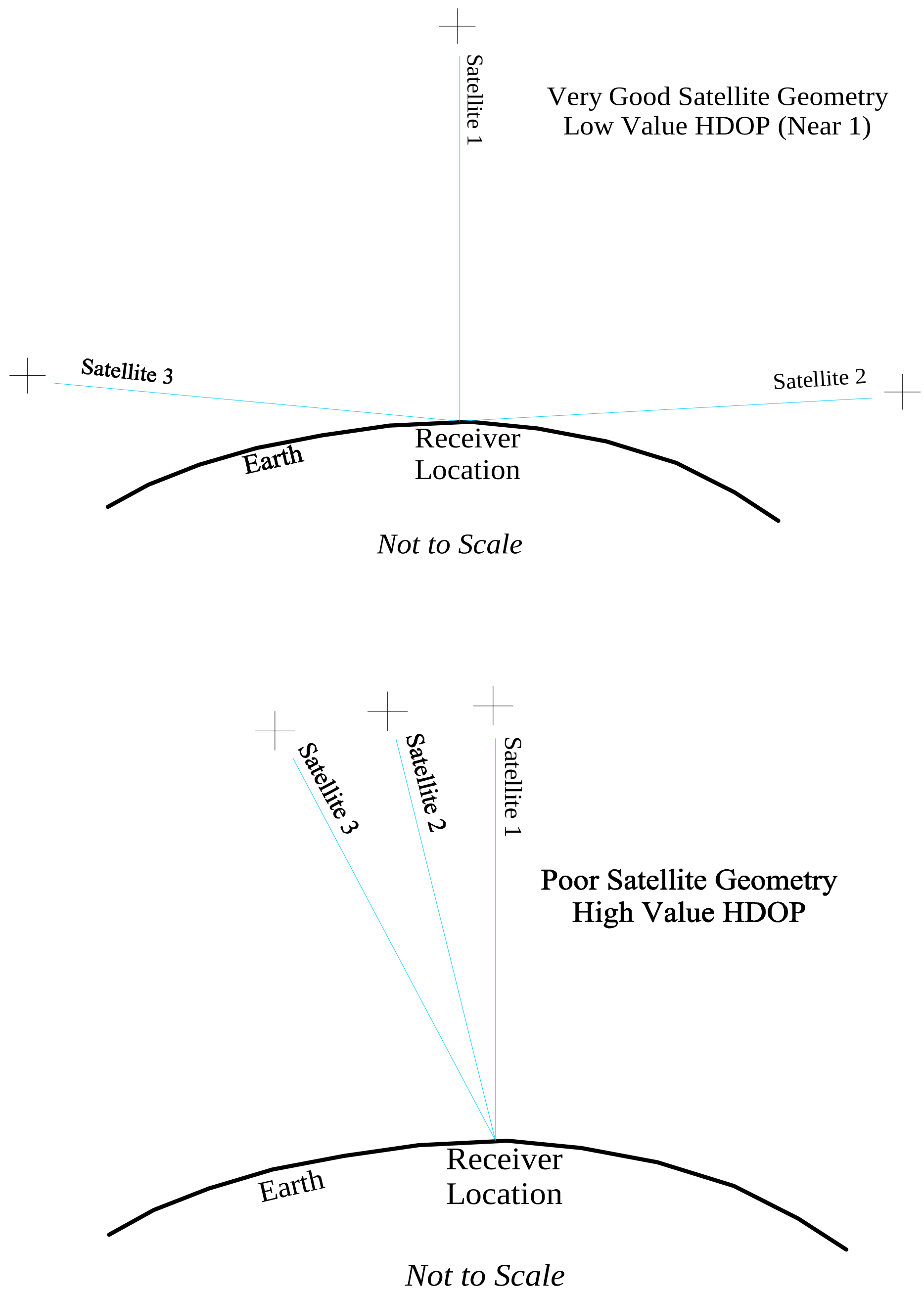


Figure 3.2 Satellite geometry is measured qualitatively by DOP value. Three satellites separated by wide angles (low HDOP) from one another are likely to give a more accurate position than those separated with narrow angles (high HDOP).

3.2: Differential Correction

Most map-grade GPS receivers are capable of accuracy from 1 to 100 meters uncorrected and 1 to 10 meters using 'differential correction'. Differential correction is a technique in which GPS data is collected at a *known surveyed location* (by a 'base station') and compared to the data collected by a 'rover unit' (or field unit) at the same moment (Figure 3.3). The GPS data collected by both the base station and the rover unit record the time by the second. The field data can later be adjusted for the calculated errors for the exact same time period. The receiver placed at a known location acts as a static reference point that calculates the combined error in the satellite range data (Verbyla, 1995). The computed difference between the GPS rover location and that of the surveyed base station can be used to correct many of the signal errors (Hurn, 1989).

Differential Correction

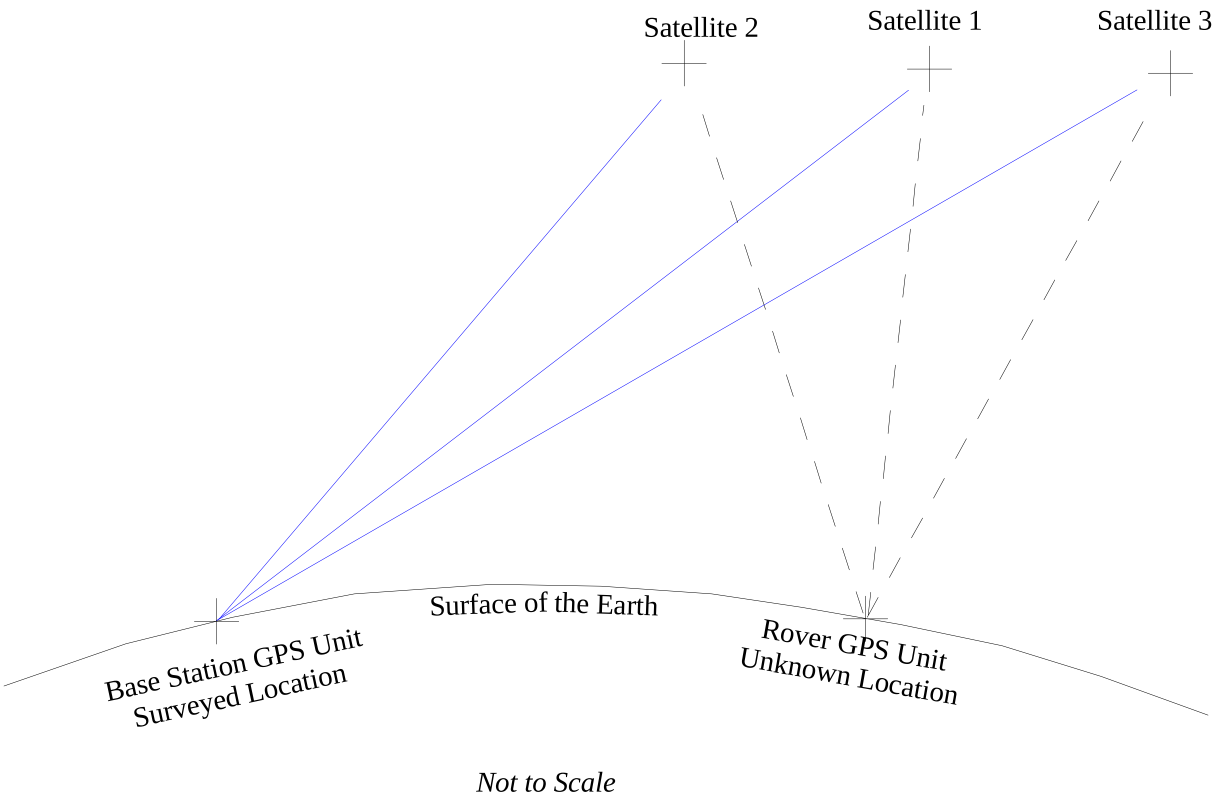


Figure 3.3 Differential Correction is the process of mathematically comparing a known location (base station) to an unknown location (rover or field unit) at the exact same moment in time to remove some of the errors from the field data. There is an effective limit of 200 to 300 miles as some satellites will not be in view by both the rover and the base station unit at the same time due to the curvature of the earth.

Chapter 4: GPS Data Collection Problems

4.1: Some Techniques Used to Aid in Collecting GPS Data

Some of the problems with collecting data in 3D mode are caused by the terrain and vegetation present at the data collection site. "Since at least 3 satellite signals must be received to estimate an X, Y position, a clear view of the satellites above the horizon is crucial. Therefore, GPS reception of satellite radio signals in narrow canyons might be limited to only a few hours per day." (Verbyla, 1995, p.86). The reason that the signal can only be viewed at limited times is because terrain and woody vegetation block the GPS signal. GPS data collection under especially difficult field conditions must be planned prior to collection. "Obtaining adequate GCPs (Ground Control Points) may be quite difficult, especially in areas like the tropics where the forest canopy or overstory is so dense that the GPS field unit has no direct line of sight to the satellite." (Demers, 1997, pp.146-147). Sometimes steep terrain, heavy forest canopy, and urban canyons can hinder the receiver from viewing satellites that would normally be in view.

Forest canopy or overstory (particularly the woody part), mountains, buildings, bridges, overpasses, valleys, gullies, and canyons can block satellite signals or bounce the signal (multipath error) (Becker, Noble & Thomas, 1996, Demers, 1997, O'Dell, Householder, & Reid, 1996, Oderwald, & Boucher, 1987, Verbyla, 1995). As previously mentioned, PDOP is a quality measure of horizontal and vertical errors caused by the position of satellites in the sky. Satellites low on the horizon can and should be 'masked out' (eliminated from consideration). "Signals from satellites low on the horizon are more susceptible to blockage because they have to come through and around the tree boles" (Oderwald, & Boucher, 1997, p.14). GPS signals from low on the horizon contribute to data collection problems in heavily forested areas as the signal becomes available one moment and is lost the next moment. Yet, these satellites are likely to be selected by the receiver for their low Position Dilution of Precision (PDOP) if they are not masked out.

Raising the antenna, especially above the head of the person collecting the data, can make a big difference in alleviating GPS data collection problems. Satellite signals may be blocked by the operator’s body and/or dense forest canopy. It is helpful to "raise the antenna three or four feet" above the operator's body and head to obtain a clearer satellite signal (Oderwald, & Boucher, 1997). Sometimes it is necessary to “[mount] the GPS antenna on a telescopic pole and [extend] the antenna above the forest canopy” (Verbyla, 1995, p.86).

High GPS accuracy can be achieved only under ideal conditions with the receiver configured to collect 3D data and a PDOP below a certain level (PDOP usually 8 or lower). Ideal conditions do not always prevail. Elevation and terrain influence the accuracy of data. One way to optimize conditions of data collection under unfavorable conditions such as mountainous topography or heavy forest cover is to acquire data using Auto2D/3D mode. This mode collects data in 3D when four satellites are available and in 2D when there are only three satellites available. The accuracy of a GPS unit in 2D mode is dependent on the elevation that is no longer computed as it would be in 3D mode. A 2D point elevation fix must be based on the elevation of the last 3D point.

The accuracy of the GPS unit in 2D mode is greatly affected by the change in elevation since the last 3D point. For example, in Virginia, if you are collecting data on the flat eastern coastal plain, collection of 2D data would not influence accuracy as much as it would in the Appalachian mountains in the western part of the state. Larger changes in elevation cause the GPS position computation less accurate when collecting in 2D mode. Conditions in forested mountainous areas are the most problematic for collecting data. Therefore, GPS users tend to use Auto 2D/3D more in the mountains -- an effect that tends to compound GPS computation errors.

Chapter 5: Methodology

5.1: GPS 2D Data Collection Methods

The following procedures were used to measure the impact of the elevation component on horizontal accuracy of GPS:

1. 39 points were established in two locations using traditional surveying methods (see surveying point accuracy methods below). These 39 points were nominally spaced 100 feet apart along two lines. These two lines consisted of one relatively flat line and one with as wide as an elevation range as possible based on available Virginia Tech campus survey points (Figure 5.1).

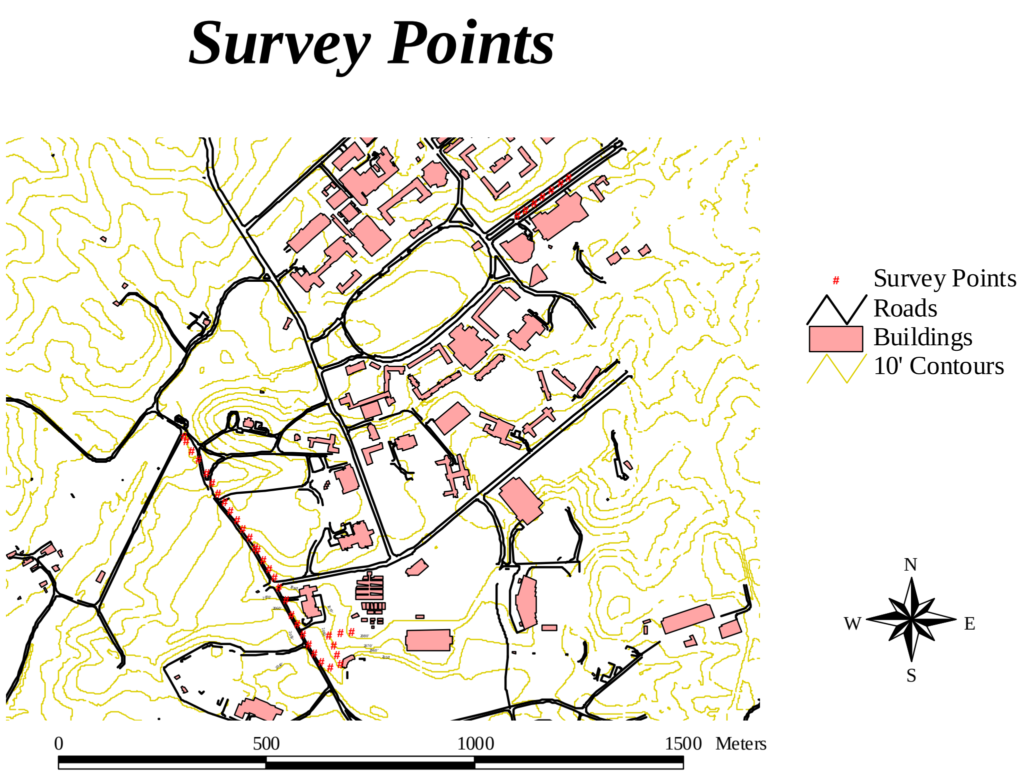


Figure 5.1 Campus survey points layout

2. 2D GPS data was collected at these 39 points during six different time periods. These six time periods were scheduled approximately two hours apart over two days. These six different collection times gave six constellations of satellites (Table 6.2), making a total of 234 GPS points.

3. Single points were collected at each location during the six time periods. Walking from point to point simulated walking a trail or forest stand boundary. A single fix was taken at each point as soon as it was reached. The GPS unit was given no settle time (Settle time allows the GPS receiver to sit in one place and compute many locations before storing a coordinate. Giving the receiver settle time will usually increase the accuracy of any points collected, but the operator must stop, then wait to collect a point). The method of no settle time was used to simulate collection of a line or polygon feature under forested conditions.

4. GPS data was differentially corrected. The data for each point was computed for seven elevations to measure the effect of the elevation component on horizontal accuracy. The seven elevations that were used are as follows:

   a) the true or correct elevation obtained from the survey.
   b) + and - 25 meters from the correct elevation.
   c) + and - 50 meters from the correct elevation.
   d) + and - 75 meters from the correct elevation.

5. A point-to-point analysis was done to compare the corrected 2D GPS data with the surveyed points.

5.2: Surveying Point Accuracy Methods

1. Each point was surveyed using traditional surveying methods. A transit was used to measure all angles between points. A distance meter was used to measure distances between points. Each point was measured with a height meter and height pole. The survey points were established from the established survey network on the Virginia Tech campus. The campus survey points are established with horizontal and vertical control as Second-Order Class I. Traverses from the campus survey points to each GPS test point had a maximum horizontal closure accuracy of 0.63 meters. Traverses from the campus survey points to each GPS point had a maximum vertical closure accuracy of 0.23 meters.

2. The survey point vertical elevations were in orthometric or mean sea level (MSL) height. These othometric heights had to be converted to ellipsoidal heights to be in common units for the GPS differential correction software. The equation for the conversion is: ellipsoidal height = orthometric height + geoid height. The conversion varies for each location (Latitude, Longitude) on earth by the height of the ellipsoid above or below the geoid. For example, point # 8 was established by the survey at 37° 13' 6.93" Latitude, -80° 25' 30.58" Longitude (NAD83). The geoid height calculated by the Geoid96 program (US Geodetic Survey) for that point location was -31.827 meters. The surveyed orthometric height for point # 8 was 627.795 meters. Ellipsoidal height = 627.795 + (-31.827) = 595.968 meters for point # 8. The height for each of the 39 points was established using this method of a geoid computation for each latitude / longitude position (Milbert & Smith, 1996).

5.3: The GPS Receiver and Data Collection

The GPS unit used for this study was a Corvalis Microtechnology L1 six channel receiver. Autonomous GPS data was collected and differentially corrected to the base station in Appomattox, Virginia. The Appomattox base station is approximately 87 miles from the data collection site in Blacksburg, Virginia. This base station is well within the 300 mile limit generally recommended in the literature.

The GPS unit was forced to collect 2D data so that the elevation could be controlled. Each point was given an attribute to tie that point to its site at each different collection time. For example, point #15 was collected the first time with an attribute of 15, the second time with 15A, third time with 15B, fourth time with 15C, fifth time with 15D, and the sixth time with 15E.

5.4: GPS Accuracy for the GPS Unit Used for this Study

Accuracy of GPS data varies in part by the receiver. The accuracy of the C/A code receiver used was as follows: The error in differentially corrected points for this receiver is at most 10 meters horizontal and 28 meters vertical for single fixes with no settle time. The vertical accuracy was taken from the accuracy of the encrypted military or P-code signal.

Most literature sources state the following for 3D points for a typical GPS receiver:

1. Raw (autonomous) Coarse Acquisition (C/A) horizontal accuracy is 100 meters (95% of all points) with 50 meters typical while the vertical accuracy is 156 meters (95% of all points). Most of this potential error comes from selective availability (S/A).

2. Raw (autonomous) P-code "horizontal accuracy is 22 meters (95% of all points) with 10 meters typical while the vertical accuracy is 27.7 meters (95% of all points)." (Kline, 1997, p.31).

3. Differentially corrected C/A code horizontal positions can be improved from 100 meters to an accuracy of 5 to 10 meters or better. The accuracy gain of C/A over P-code comes mostly from virtually eliminating atmospheric errors during differential correction. P-code GPS data is normally not differentially corrected.

Chapter 6: Results

6.1: National Map Accuracy Standards

GPS data, like any other map data, is required to meet accuracy standards that are determined by the scale of the map to be produced (the earth is at a scale of 1:1). Accuracy standards are particularly important today as maps are being produced by Geographic Information Systems (GIS) that allow changing the scale at the click of a button. All federal agencies are required to comply to the National Map Accuracy Standards (NMAS). The National Map Accuracy Standards state that on all maps published at a scale of 1:20,000 or larger, 90 percent of all points tested must be accurate within 1/30th of an inch. On maps published at a scale of 1:20,000 or smaller, 90 percent of all points must be within 1/50th of an inch. The vertical accuracy standard requires that the elevation of 90 percent of all points tested must be correct within half of the map's contour interval (U.S. Geological Survey. 1998). Table 6.1 shows the required horizontal accuracy of points at common scales.

Table 6.1 National Map Accuracy Standards (NMAS) at some commonly used scales.

Scale	Standard	Accuracy (Meters)
1:2,000,000	1/50"	1,016.01
1:250,000	1/50"	127.00
1:100,000	1/50"	50.80
1:24,000 (7.5 Minute Quad)	1/50"	12.19
1:20,000	1/30"	16.93
1:15,840 (4" = 1 Mile)	1/30"	13.41
1:12,000	1/30"	10.16

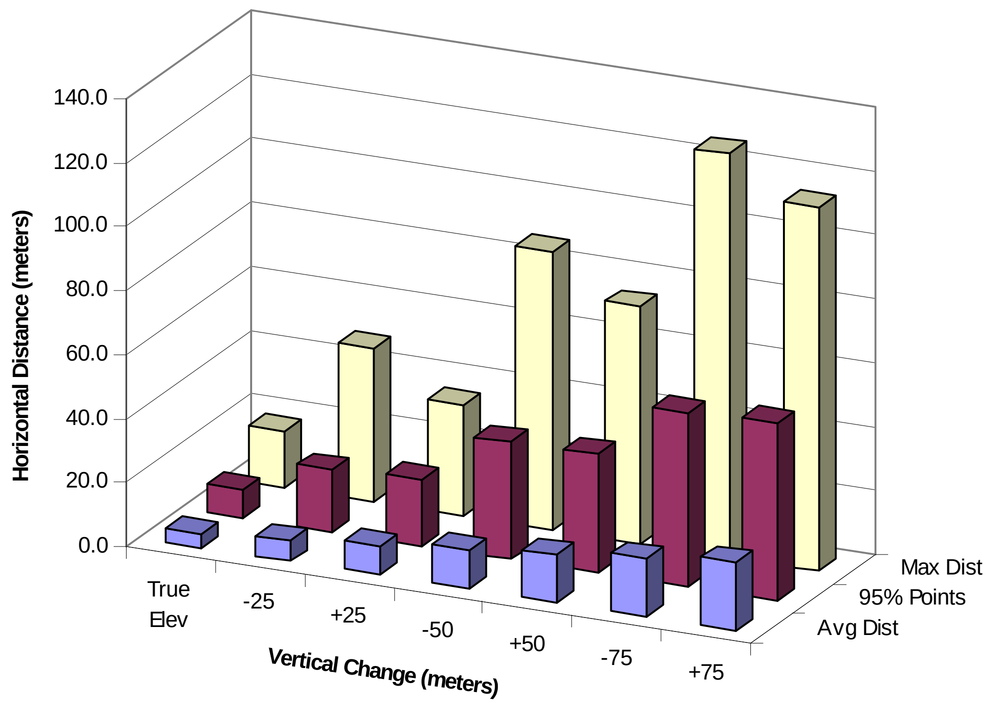
6.2: Vertical Effects on 2D Points

Single 2D points are capable of meeting the National Map Accuracy Standards (NMAS) at most of the scales shown in Table 6.1, if the elevation at which they are collected is accurately provided. However, there is not much assurance that the elevation is correct at any 2D point, because the elevation is generally extrapolated from the last known 3D point. Table 6.2 shows the introduced vertical error (meters) and the resulting horizontal change (meters) that point is from the surveyed point. For example, 90% of all 234 points corrected at the true elevation were accurate to within 8.4 meters (Table 6.1). Points within 8.4 meters (8.8 meters at GPS 95%) meet the NMAS of scales 1:12,000 and smaller and also meet the accuracy of the GPS unit of less than 10 meters for 3D points. If the elevation is off by + or - 25 meters, 90% of 468 points (234 points corrected at +25 meters and also at -25 meters) are within 19.1 meters. Points within 19.1 meters meet the NMAS of scales 1:100,000 and smaller (Table 6.1). Collecting GPS data on terrain with more than 25 meters elevation change, 2D points are not likely to meet the NMAS for maps at scales larger than 1:100,000. Figure 6.1 reveals the very linear change of horizontal distance caused by an incorrect vertical measurement.

Table 6.2 Average horizontal error (meters) for differentially corrected data as a result of vertical change (meters) (234 points at each vertical).

Vertical Error	**Horizontal Average Distance**	**Horizontal Maximum Distance**	**Horizontal Minimum Distance**	**Horizontal 95% Points**	**Horizontal 90% Points**
+75	21.1	113.6	0.1	55.5	50.9
+50	14.8	74.0	0.0	37.1	34.2
+25	9.1	34.6	0.1	20.5	19.1
0	4.3	17.9	0.0	8.8	8.4
-25	6.4	48.0	0.1	19.2	18.0
-50	12.0	86.9	0.1	36.6	34.1
-75	18.0	126.1	0.3	54.4	50.5

Horizontal Distance and Vertical Change



	True Elev	-25	+25	-50	+50	-75	+75
Avg Dist	4.3	6.4	9.1	12.0	14.8	18.0	21.1
95% Points	8.8	19.2	20.5	36.6	37.1	54.4	55.5
Max Dist	17.9	48.0	34.6	86.9	74.0	126.1	113.6

Avg Dist 95% Points Max Dist

Figure 6.1 Vertical error causing average, 95%, and maximum horizontal distances.

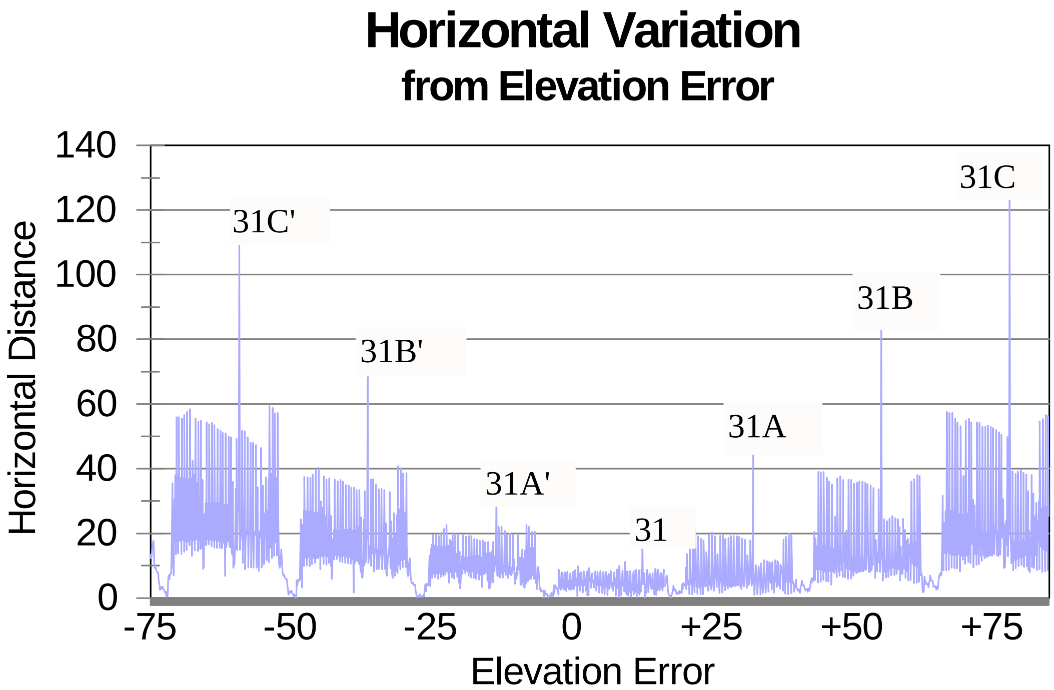


Figure 6.2 The variation of horizontal distance for each elevation error (meters). Each elevation error has data from 234 points. The horizontal variation, or difference from zero, increases as elevation error increases from 31 to 31A, 31B, 31C, 31A’, 31B’, and 31C’.

Any errors in the data at the true elevation become emphasized when the vertical errors are increased (Figure 6.2). 31, 31A, 31B, 31C, 31A', 31B', 31C' are the same point corrected at true elevation, +25, +50, +75, -25, -50, and -75 meters, respectively. 31C (+75 meters elevation error, +126 meters horizontal distance) shows the most horizontal change from elevation error. This point is probably the result of multipath or other error (the HDOP at point 31 was 5.9). Errors in GPS data are an area needing further research. Point 31 was taken right after the GPS unit had lost a satellite. A large error occurred because the receiver was given no settle time to collect a point.

Variation from Errors

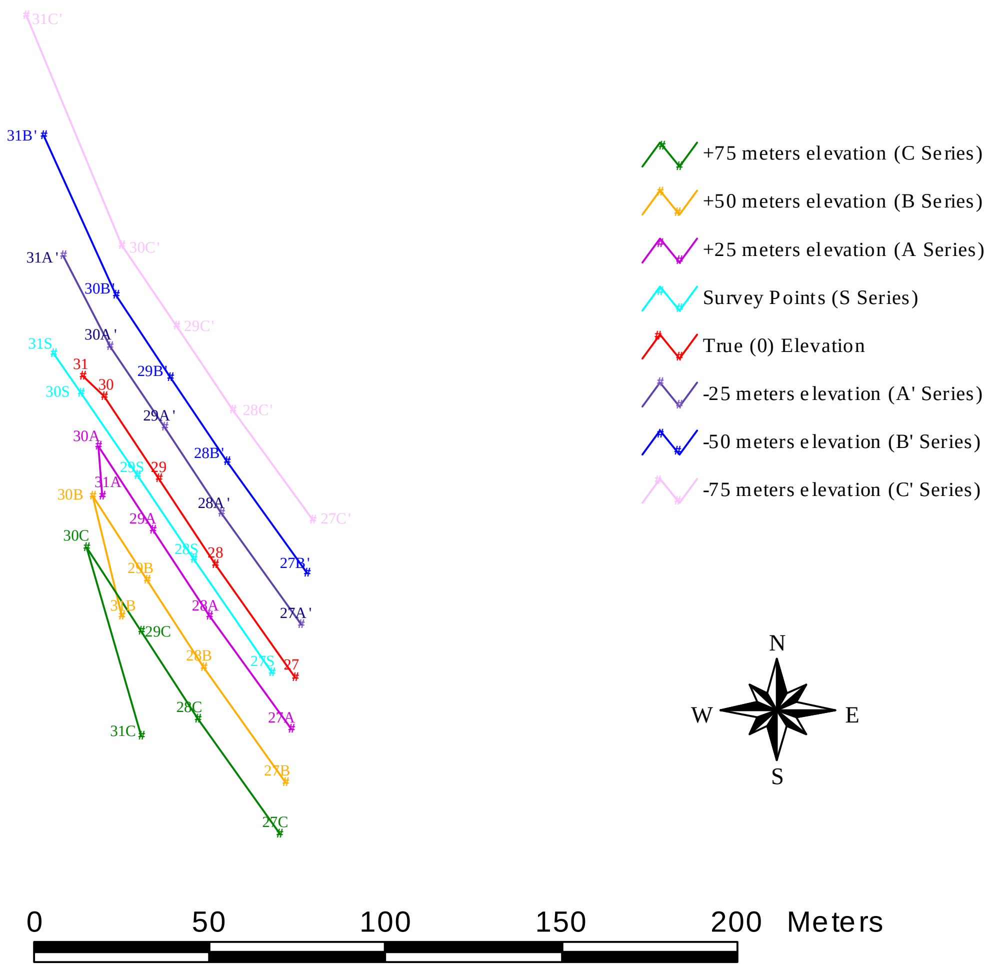


Figure 6.3 Any errors in the data get amplified in random ways. Point 31 corrected at +-75 meters, +-50 meters, +-25 meters, and true elevation is the same point in Figure 6.2.

The data in Figure 6.2 was expected to have almost perfect symmetry as this is the same data corrected at seven different elevations. The reason for this less than perfect symmetry was not determined. Figure 6.3 shows the error of point 31 (labeled in Figure 6.2 and 6.3) as lines with adjacent points. The adjacent points to 31 have almost perfect symmetry as elevation error is introduced. Point 31 does not have perfect symmetry because of its extreme error.

6.3 DOP Effect on 2D Points

The current GPS satellite constellation used by the receiver also influences the elevation effect on horizontal accuracy. Based on the error patterns found in this study, a GPS receiver looking at three satellites forms a common vector from the average of the three satellites, represented by the black line in Figures 6.4 and 6.5. This average vector appears to influence the accuracy (distance and direction) of a point based on its view angle to the horizon. The different Dilution of Precision (DOP) measurements are a quality measure to indicate whether there is good or poor satellite geometry. On the 2D points collected for this study, the measure the GPS receiver collected was the Horizontal Dilution of Precision (HDOP). Varying the vertical component (VDOP), will cause the position (PDOP) of the point to change also (Figure 3.1). The satellite geometry for PDOP is shown by the mathematical relationship of the three DOP measures where $PDOP = HDOP^2 + VDOP^2$. An increase in either HDOP or VDOP should cause an increased error in the PDOP (or position).

Satellite Constellation with 50° Elevation Angle

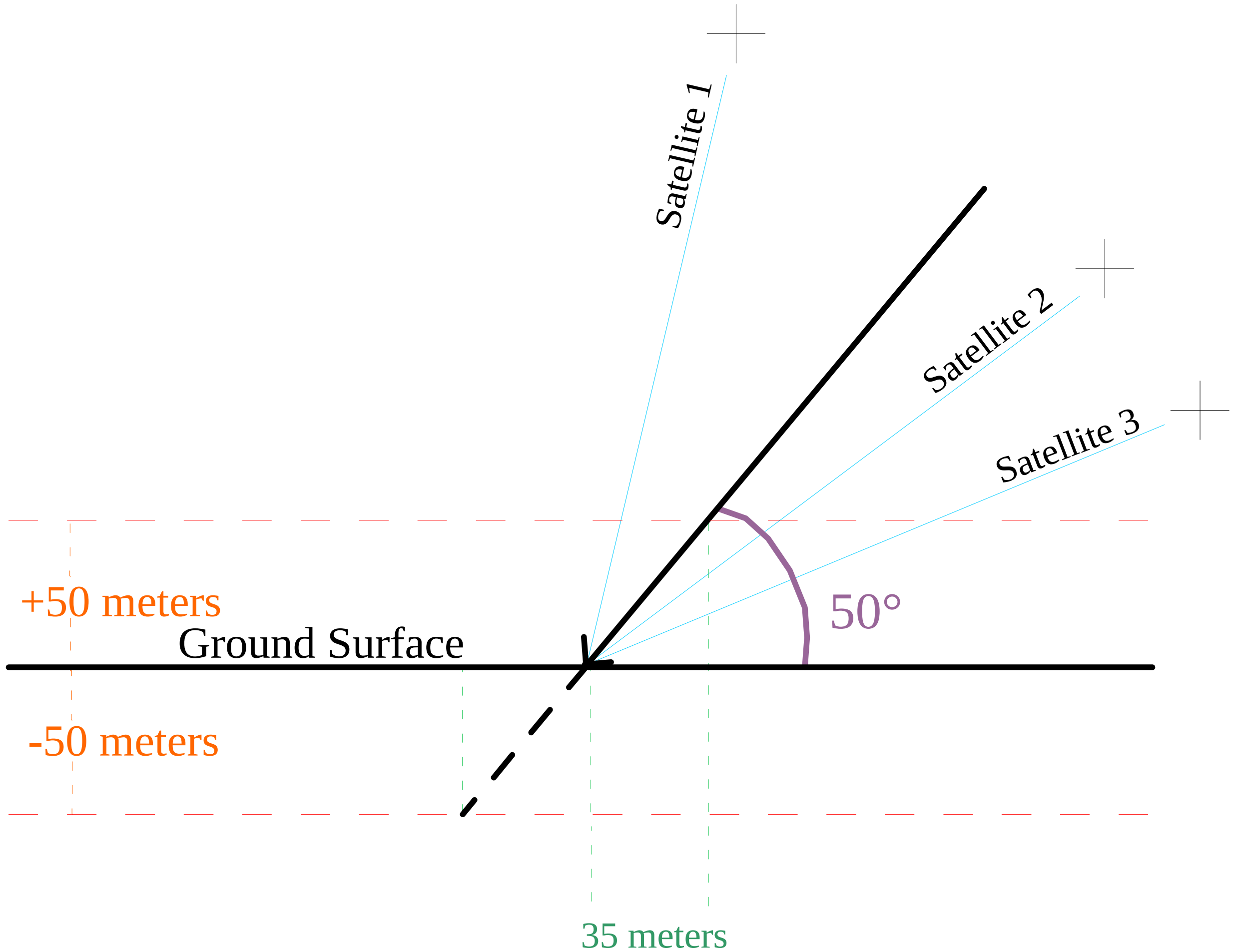


Figure 6.4 This satellite constellation averages about a 50° elevation angle above the horizon. The horizontal offset an elevation error will cause is greater than Figure 6.4, HDOP being equal.

Satellite Constellation with 75° Elevation Angle

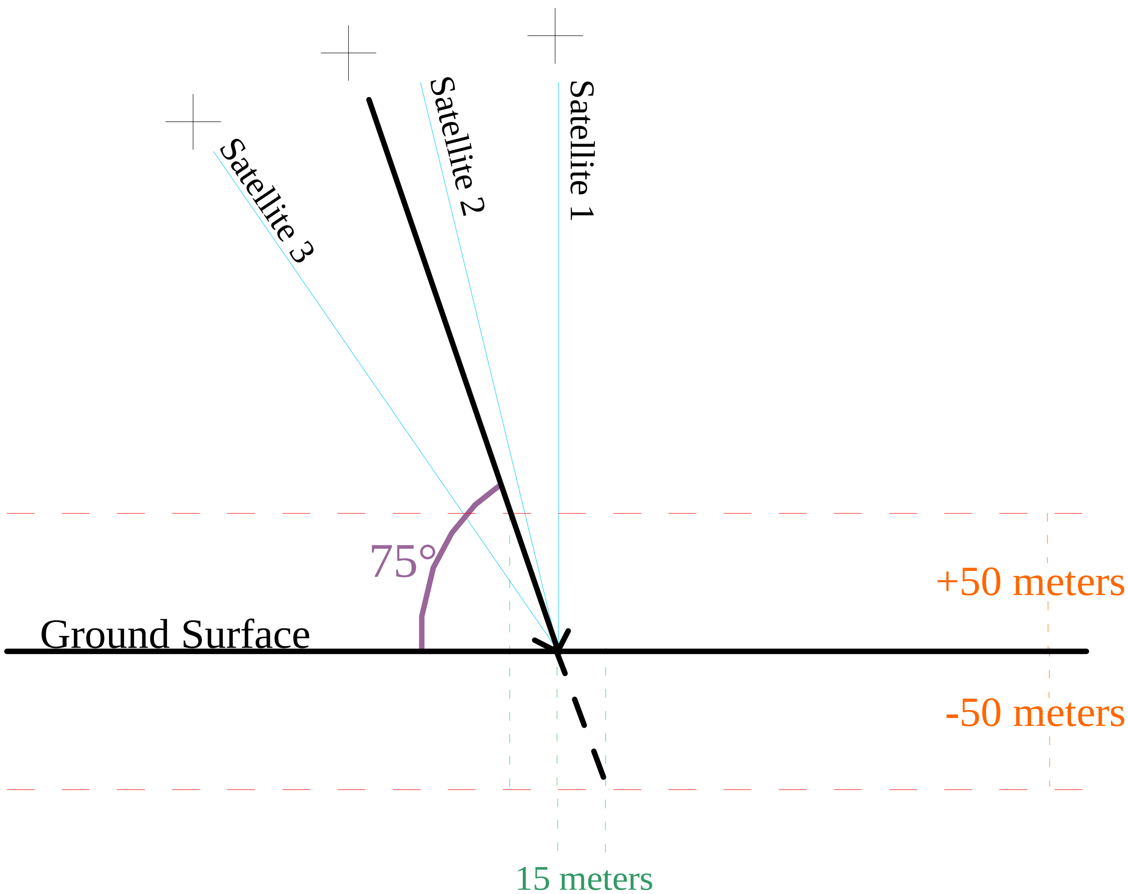


Figure 6.5 This satellite constellation averages about 75° elevation angle above the horizon. The horizontal offset an elevation error will cause is less than Figure 6.3, with HDOP being equal.

HDOP Constant at 1.7

During First Pass (1st Collection Time)

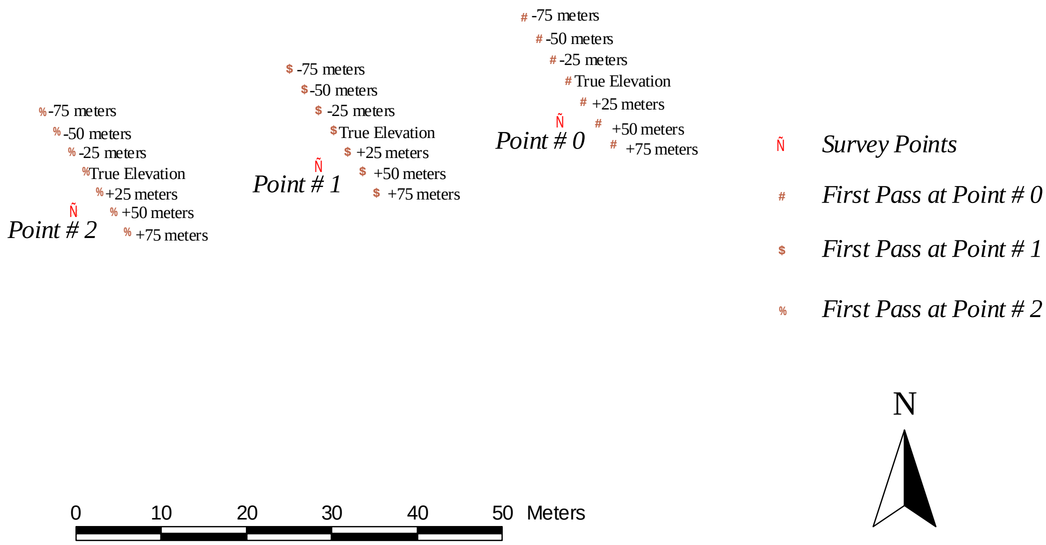


Figure 6.6 The first pass from points #0, #1, and #2 has no change in satellite geometry (HDOP). From point to point during the same pass there is almost perfect symmetry as elevation error is introduced.

Figures 6.6 and 6.7 depict the symmetry of maintaining the same satellite geometry from one point to the next. These two passes (first and third) were collected at different times during the same day.

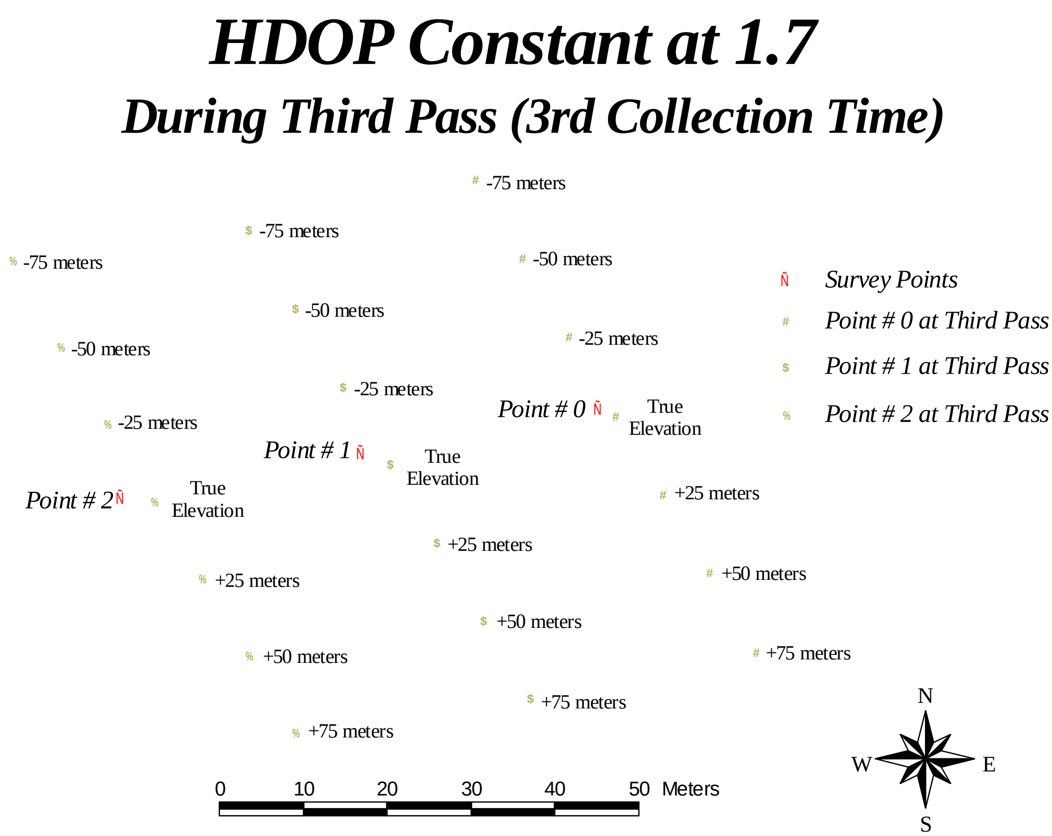


Figure 6.7 The third pass from points #0, #1, and #2 has no change in satellite geometry (HDOP). From point to point during the same pass there is almost perfect symmetry as elevation error is introduced.

6.4: Calculation of Azimuth and Elevation Angles

The azimuth and elevation angles determine the amplitude and direction of horizontal error. Elevation error changes the distance between points based on the elevation angle (angle from the horizon) of the average vector. Elevation error also changes the direction the points will vary based on the direction (azimuth angle) of the average vector from the three satellites. Satellite geometry can be used to calculate the azimuth and elevation angles.

As proof of this theory, Table 6.3 indicates the satellites in use during the 6^{th} pass**.** The GPS receiver tracked six satellites at the beginning and end of data collection. Locations of specific satellites at specific times are important to calculate the azimuth and elevation angles. The receiver did not reveal the specific satellites used at a specific time. Satellite geometry was used to determine which satellites were being used.

In order to illustrate which satellites were used, all possible combinations were checked. For example, the satellites in view at the collection time for points # 0, 1, and 2 were satellite PRN numbers 18, 03, 19, 13, 22, and 31. PRN 22 was lost from view (it had set below the 10° elevation mask) and PRN 27 was acquired to replace it (Table 6.3) at the time points # 0, 1, and 2 were collected.

Table 6.3 Satellites in View at GPS data collection start and finish.

	Date	**Start Time ***	**Finish Time ***	**Weather Conditions**	**Satellites PRNs in view at Start:**	**Satellites PRNs in view at Finish:**
1st Pass	16 Mar 98	11:25 AM	12:03 PM	Overcast	31, 18, 03, 19, 13, 27	Same
2nd Pass	16 Mar 98	1:35 PM	2:10 PM	Overcast	19, 15, 02, 31, 27, 07	Same
3rd Pass	16 Mar 98	3:40 PM	4:15 PM	Overcast	07, 04, 14, 02, 27	07, 04, 14, 02, 09, 16
4th Pass	16 Mar 98	5:40 PM	6:18 PM	Overcast	07, 24, 16, 04, 02, 05	07, 24, 16, 10, 02, 05
5th Pass	19 Mar 98	10:02 AM	10:40 AM	Partly Cloudy	16, 18, 19, 03, 22, 31	13, 18, 19, 03, 22, 31
6th Pass	19 Mar 98	10:50 AM	11:25 AM	Partly Cloudy	18, 03, 19, 13, 22, 31	18, 03, 19, 13, 27, 31

* Local Time (GMT **-** 4) in Blacksburg, Virginia

Figures 6.9 and 6.10 demonstrate the effect of changing satellite geometry (HDOP) in view on points #0, #1, and #2 during the 6^{th} pass. Each pattern of points in Figure 6.10 (the seven elevations) was plotted and its azimuth angle was measured from true north. The GPS planning software was used to interpolate the azimuth angle of each satellite in view by the receiver (to the nearest degree) at the collection time. The collection times for points # 0, 1, and 2 (during the sixth pass) were from 10:50 to 10:52 AM on March 19^{th}.

All combinations of three satellites in view at the collection time of points # 0, 1, and 2 had an average azimuth angle calculated as follows:

1. Azimuth angles were determined using relative averages of X and Y coordinates using trigonometry. To convert the azimuth angles of 312°, 241°, and 68° (PRNs 13-18-22), to (X,Y) 1 or -1 was assigned to X depending on the quadrant (I, II, III, IV) in which the satellite falls (Figure 6.8). For example, satellite 22 is converted by setting X to 1, with Tan (68°) = Y / X. Solving this equation for Y gives 0.404026 Table 6.4 shows the X and Y results for each satellite.

2. Each X and Y is averaged separately and combined to form an average angle. For example, using the same three satellites above, the X average is [(-1) + (-1) + 1] / 3 = -*0.333333* and the Y average is [0.900404 + (-0.55431) + 0.404026] / 3 = *0.25004*. Table 6.5 shows, for all possible constellations, the average vectors for the specified time.

3. To determine the azimuth angle, the arc tangent is taken using the X and Y as values. The equation for this combination is: 360° - (90° + arctan (0.25004 / -0.333333) = *306.9° average azimuth angle* The equation varies depending on which quadrant it falls in. See Table 6.5 for the average calculation of all possible satellite constellations in view from 10:50 to 10:52AM for March 19, 1998. Each azimuth angle from Figure 6.10 was then compared to all combinations.

4. The average elevation angle (angle above the horizon) was approximated from the average azimuth angle (Figure 6.11). The first step is to compute the distance from the average azimuth (X,Y) from the receiver (0,0). For instance, the distance from the receiver location was computed for PRNs 13-18-22 at - 0.333333, 0.25004. This point

can be used to compute the distance (relative) directly underneath the average vector formed by the three satellites using the Pythagorean theorum ($c^2 = a^2 + b^2$).

$$c = \sqrt{-0.333333^2 + 0.25004^2} = 0.416690$$

5. The distance of 0.416690 becomes the base of a right triangle looking up at the satellite from the receiver. Trigonometry can be used to solve the angle from the formula: $\tan f = opposite / adjacent$ A relative elevation angle was calculated with all of the satellites assumed to be at the height of 1. For example: $f = \arctan(opposite / adjacent)$ or $f = \arctan(1 / 0.416690) = 67.4°$

Table 6.5 also shows all of the calculated elevation angles from the average azimuth angle for points # 0, 1 and 2. Satellites 13-18-22 for point # 2 are expected to have the highest elevation angle because of the close proximity of the points (Figure 6.10). The highest elevation angle for point # 2 was calculated at 67.4°. Satellites 18-19-31 had a calculated elevation angle of 42.8° for point # 0 and satellites 03-13-31 had a calculated elevation angle of 54° for point # 1.

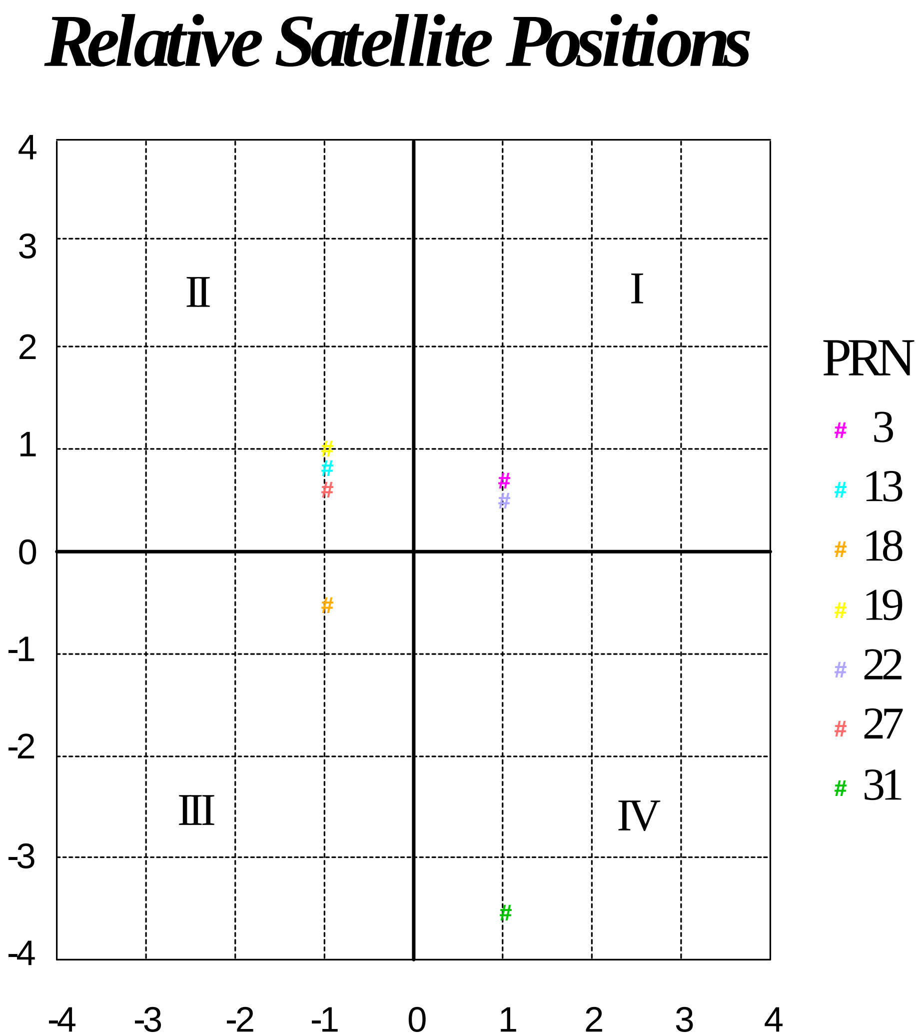


Figure 6.8 All satellites in view at the sixth collection time for points # 0, 1, and 2 were calculated using trigonometry. This calculation gives the relative positions of the satellites to the receiver.

Satellites in View During 6th Collection Time.

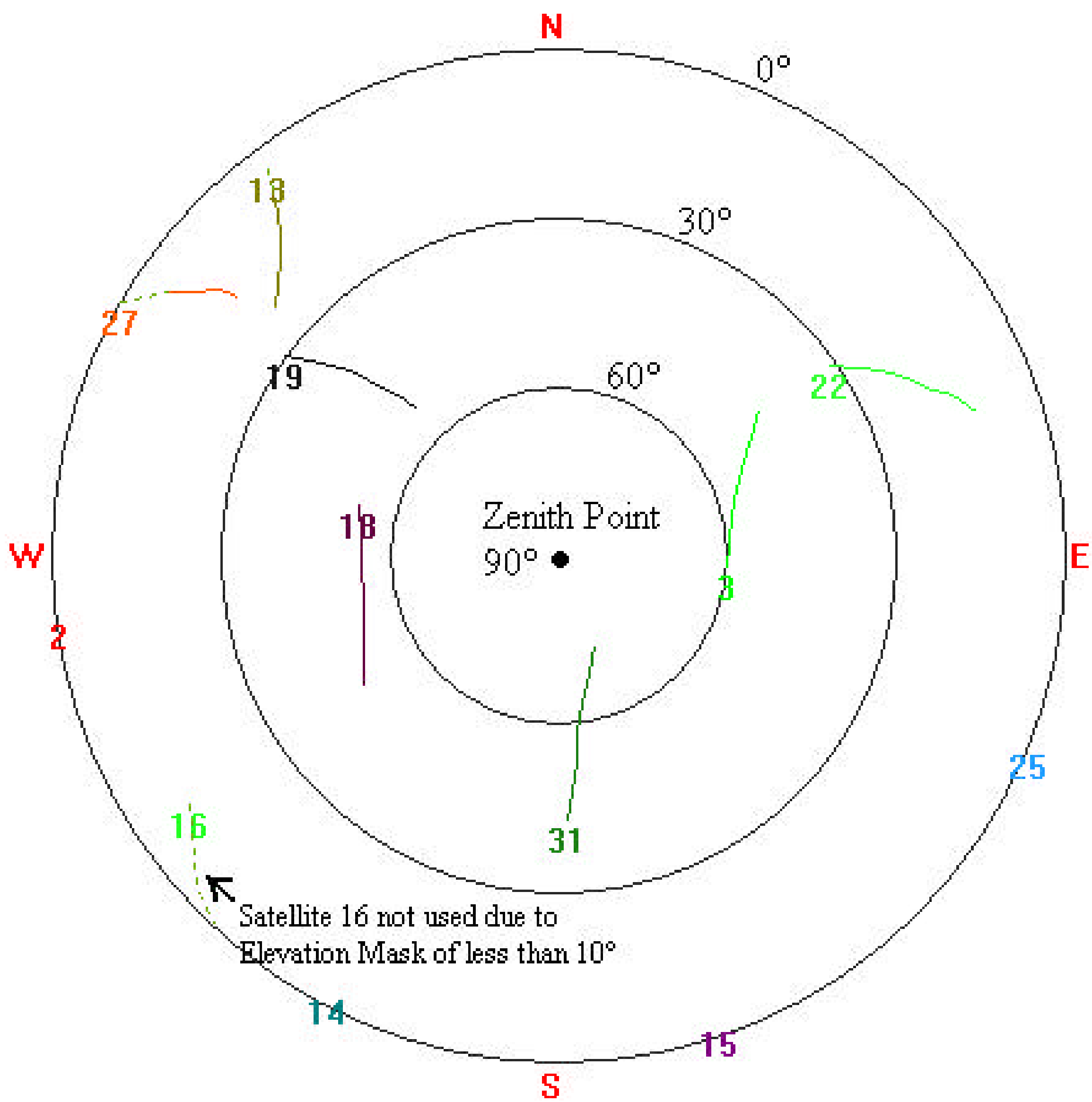


Figure 6.9 Satellites in view from 10:00AM to 11:00AM (6th collection time) on March 19, 1998 at points # 0, 1, and 2. The center of the circle indicates the GPS receiver location. The numbered arcs are the satellites in the sky above the receiver. The circles represent the elevation angle from straight above the GPS receiver (the zenith point; 90°) to the horizon. The dashed arcs (ie. Satellite 16) represent satellites that are masked out at an elevation angle of 10° or less.

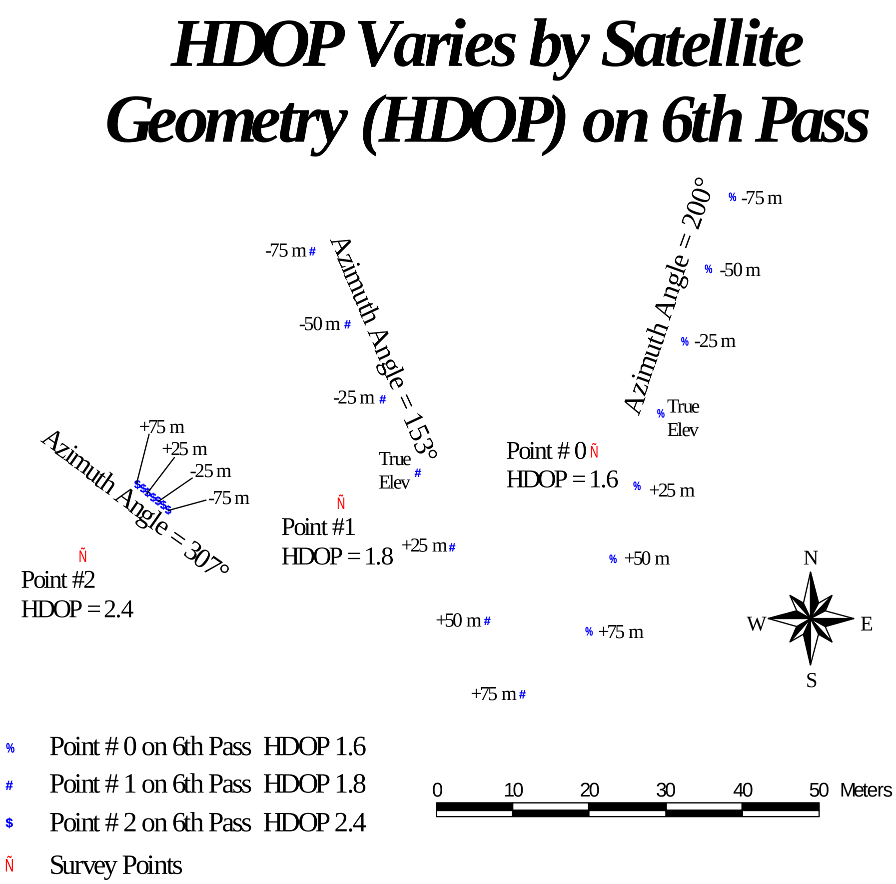


Figure 6.10 GPS points # 0, 1, and 2 collected during the 6th pass. Each point was corrected at the seven elevations shown. Satellite geometry affects the amplitude and direction of horizontal error

Table 6.4 Satellites Calculated Relative to the Receiver.

PRN	Azimuth	X	Y
3	57	1	0.649408
13	312	-1	0.900404
18	241	-1	-0.554309
19	314	-1	0.965689
22	68	1	0.404026
27	307	-1	0.753554
31	164	1	-3.487414

Figure 6.10 indicates that the common vector formed by the three satellites was from the Southwest at about a 200° azimuth angle. The angle of the corrected points is represented by the 7 blue squares around survey point #0. As the correction elevation is set too low (negative), the points shift to the Northeast, indicating that the common vector of the satellites has to be from the Southwest quadrant of the sky. As the plane of the earth moves up and down, it changes the horizontal point in an x, y direction. At point #0, the satellites being used were satellites PRNs 18, 19, and 31 (Figures 6.9 and 6.10).

The receiver uses a new satellite constellation at point #1 on the 6th pass (the 7 blue circles around survey point #1 in Figure 6.10). The three new satellites form a common vector from the Southeast (HDOP = 1.8) at 153°. The horizontal points move to the Northwest as the elevation is forced negative. The satellites being used shift to PRNs 3, 13, and 31 (Figures 6.9 and 6.10).

The receiver uses a new satellite constellation at point #2 on the 6th pass (the 7 blue triangles around survey point #2 in Figure 6.10). The three new satellites form a common vector from the Northwest (HDOP = 2.4) at 307° because the horizontal points move to the Southeast as the elevation is forced negative. The satellites being used have shifted to PRNs 3, 18, and 22 (Figures 6.9 and 6.10).

From this analysis it is shown that the geometry of the satellites to the receiver influences the accuracy of the corrected points in both direction and magnitude. Figure 6.12 helps to visualize the vertical error as a three dimensional figure beside the two dimensional map.

Table 6.5 Calculated Average Azimuth to Determine Satellite Constellation in View.

Point	PRNs	Average X	Average Y	Average Azimuth Angle	Azimuth From Figure 13	Elevation Angle
	03 18 22	0.3333	0.1664	63.5		69.6
	03 22 31	1.0000	-0.8113	129.1		37.8
	18 22 31	0.3333	-1.2126	164.6		38.5
	03 19 22	0.3333	0.6730	26.3		53.1
	18 19 22	-0.3333	0.2718	309.2		66.7
	19 22 31	0.3333	-0.7059	154.7		52.0
	03 13 22	0.3333	0.6513	27.1		53.8
2	**13 18 22**	-0.3333	0.2500	**306.9**	**307°**	**67.4**
	13 19 22	-0.3333	0.7567	336.2		50.4
	13 22 31	0.3333	-0.7277	155.4		51.3
	03 22 27	0.3333	0.6023	29.0		55.5
	13 22 27	-0.3333	0.6860	334.1		52.7
	18 22 27	-0.3333	0.2011	301.1		68.7
	19 22 27	-0.3333	0.7078	334.8		52.0
	22 27 31	0.3333	-0.7766	156.8		49.8
	03 18 31	0.3333	-1.1308	163.6		40.3
	03 18 19	-0.3333	0.3536	316.7		64.1
	03 19 31	0.3333	-0.6241	151.9		54.7
0	**18 19 31**	-0.3333	-1.0253	**198.0**	**200**	**42.8**
	03 13 18	-0.3333	0.3318	314.9		64.8
	03 13 19	-0.3333	0.8385	338.3		47.9
1	**03 13 31**	0.3333	-0.6459	152.7	**153**	**54.0**
	13 18 19	-1.0000	0.4373	293.6		42.5
	13 18 31	-0.3333	-1.0471	197.7		42.3
	13 19 31	-0.3333	-0.5404	211.7		57.6
	03 13 27	-0.3333	0.7678	336.5		50.1
	03 18 27	-0.3333	0.2829	310.3		66.4
	03 19 27	-0.3333	0.7896	337.1		49.4
	03 27 31	0.3333	-0.6948	154.4		52.4
	13 18 27	-1.0000	0.3665	290.1		43.2
	13 19 27	-1.0000	0.8732	311.1		37.0
	13 27 31	-0.3333	-0.6112	208.6		55.2
	18 19 27	-1.0000	0.3883	291.2		43.0
	18 27 31	-0.3333	-1.0961	196.9		41.1
	19 27 31	-0.3333	-0.5894	209.5		55.9

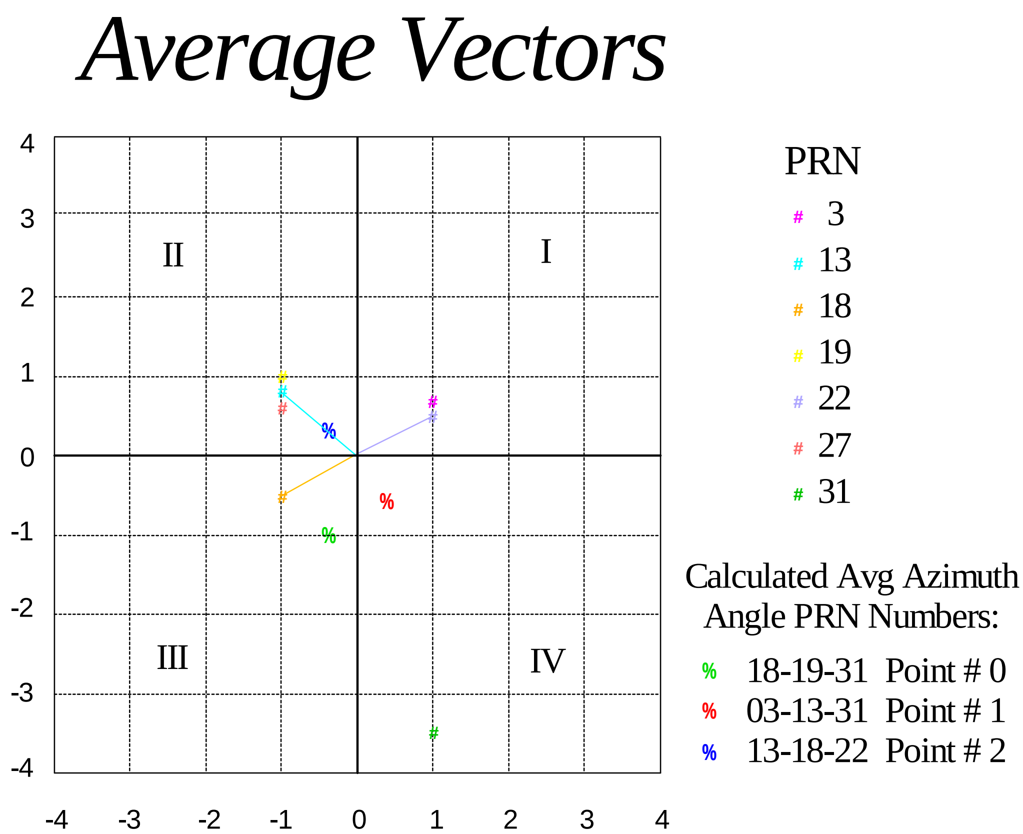


Figure 6.11 The satellites in view and the calculated average vectors at the collection time of points # 0, 1, and 2. The vectors for PRNs 13, 18, and 22 are represented by lines from each satellite to the receiver. The azimuth angle is about 307° and the average elevation angle is 67.4° (Table 6.5).

Average Vector of Three Satellites from the Northwest

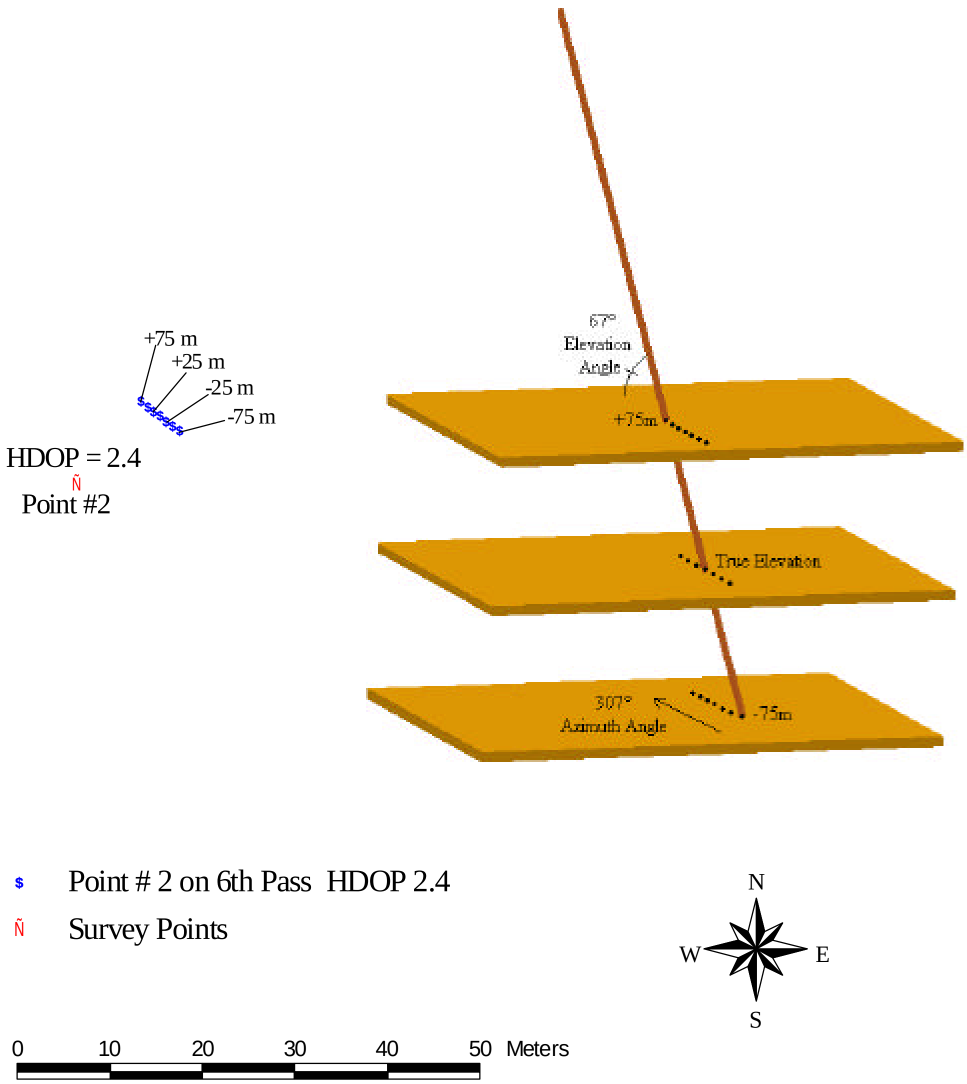


Figure 6.12 GPS average vector of three satellites. The average vector formed here was from the Northwest (about 307° on the line formed by the blue points on the left). As the elevation error becomes negative, the points go to the Northeast, depicted by -75 meters elevation on the three-dimensional map to the right.

6.5: Statistical Analysis - Regression

A linear regression was performed by HDOP to predict the horizontal distance (y - independent) change from the vertical error (x - dependant). Positive and negative vertical errors were considered separately. All HDOP values in which the number of samples was greater than seven were considered for the regression analysis (Table 6.6). 'R squared' indicates the degree to which actual GPS points conform to the regression line. Actual GPS points with HDOP values of 1.9 and 2.4 (positive and negative elevation errors) conform to the regression line very well. The regression line equation for these HDOP values is a good descriptor of these points.

Figures 6.12 and 6.13 graph these negative and positive regression equations, respectively. The expected pattern is confirmed: for increasing vertical error, the horizontal error increases. The expected pattern for HDOP is that the higher the HDOP, the less accurate the horizontal distance. This pattern was not shown for the data collected during this study. An HDOP of 1.9 has the worst horizontal error in the negative regression equations, with 1.3, then 1.6 being next. For the positive regression equations, an HDOP of 1.6 has the worst horizontal distance, followed by 1.9, then 1.2, then 1.3. The expected pattern would have greater horizontal distance as HDOP increases. I suspect there is not a high enough value of HDOP (with sufficient sample size to be included in regression) for this to show in the regression plots.

Table 6.6 Regression Equations for Greater Than Seven Samples.

HDOP	Pos or Neg Elev Error	Equation	R Squared	Std Err of X Coef.	Std Err of Y Est	Count
1.2	Negative	Y = -0.90616 * X - 8.63185	0.5558	0.0661	18.7532	38
1.3	Negative	Y = -1.04631 * X + 11.30523	0.8825	0.0589	9.8051	11
1.5	Negative	Y = -0.54793 * X - 9.54484	0.5325	0.0473	19.2732	30
1.6	Negative	Y = -1.21522 * X + 0.017072	0.6590	0.0704	16.4277	39
1.7	Negative	Y = -0.39265 * X - 23.6443	0.1987	0.0448	25.1015	78
1.9	Negative	Y = -1.27557 * X + 6.931881	0.9962	0.0155	1.7927	7
2.4	Negative	Y = -0.42788 * X + 1.718397	0.8261	0.0194	11.7691	26
1.2	Positive	Y = 1.255639 * X + 10.46958	0.5648	0.0900	18.5614	38
1.3	Positive	Y = 1.234578 * X + 6.513947	0.6074	0.1532	17.9265	11
1.5	Positive	Y = 0.90682 * X - 1.26281	0.7808	0.0442	13.1959	30
1.6	Positive	Y = 1.697471 * X - 2.95542	0.7083	0.0878	15.1945	39
1.7	Positive	Y = 0.473239 * X + 22.30832	0.2116	0.0519	24.8987	78
1.9	Positive	Y = 1.475274 * X - 2.23224	0.9653	0.0549	5.4046	7
2.4	Positive	Y = 0.431027 * X - 1.63411	0.8255	0.0196	11.7910	26
		Y = Horizontal Distance and X = Vertical Error				

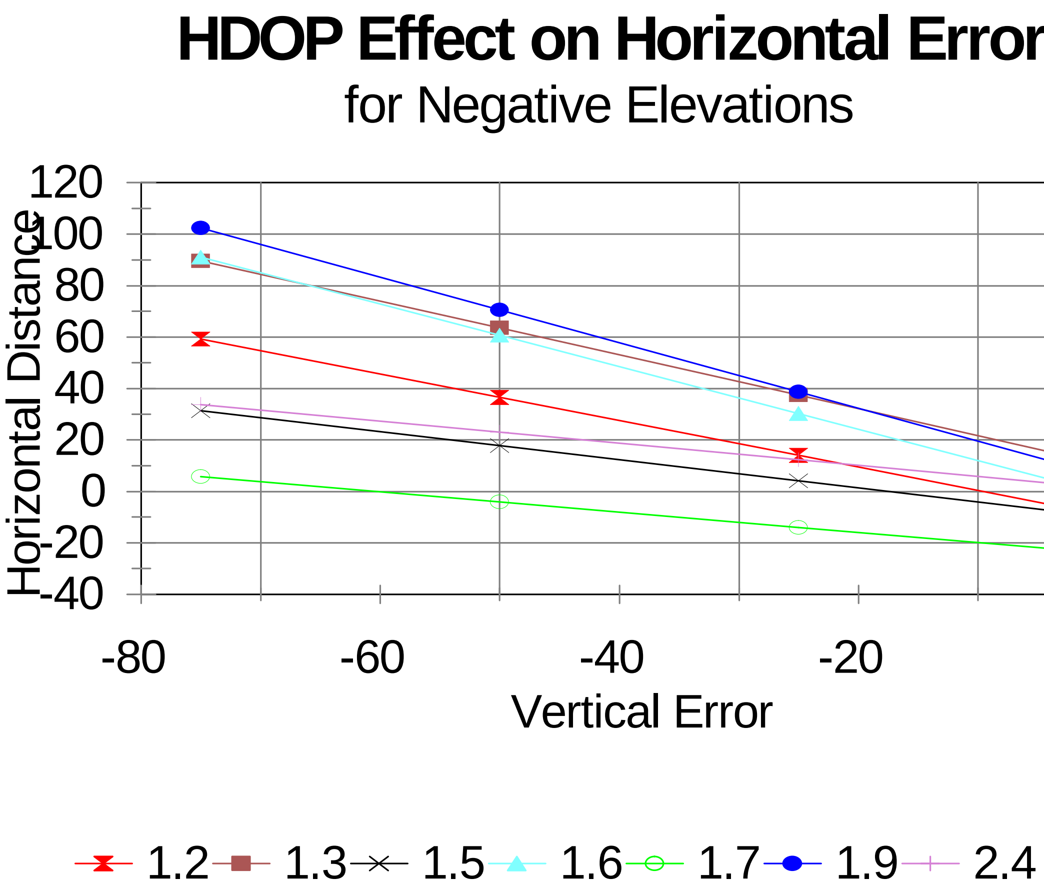


Figure 6.12 Regression equations plotted for negative vertical errors.

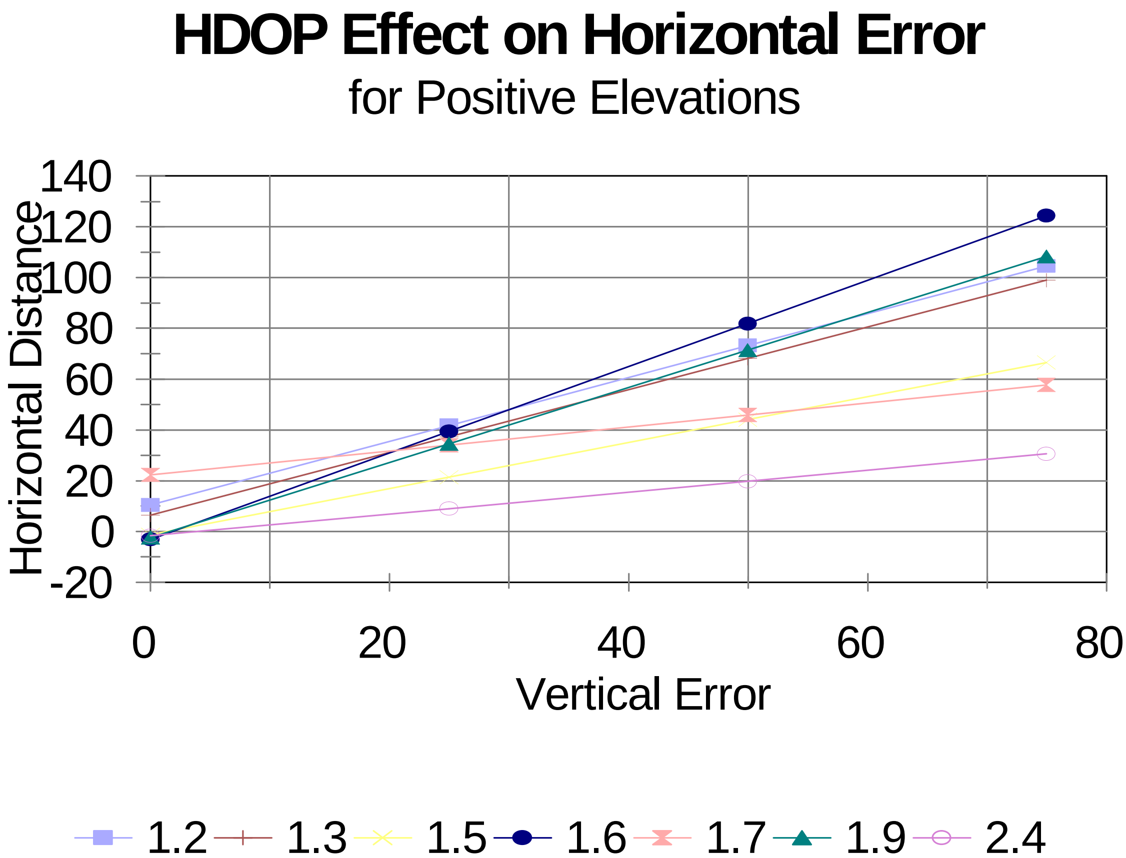


Figure 6.13 Regression lines plotted for positive vertical errors.

Chapter 7: Conclusions

Global Positioning System (GPS) use in mapping brings up the question of accuracy. How accurate are map-grade GPS units? Sometimes conditions for collecting data are less than ideal. Forest cover, mountainous topography, and urban canyons are some of the reasons data cannot be collected in the most accurate mode. When collecting data in 2D, the accuracy of the GPS unit depends on the elevation variation at the collection site.

The positional accuracy of a GPS unit is dependant on the horizontal and elevation accuracy of the unit whether collecting 2D or 3D data. The principles discussed for elevation error affecting horizontal accuracy apply equally to 3D points. Horizontal GPS accuracy is very much influenced by the vertical error of the point, line or area being collected. 3D horizontal accuracy can be degraded because of the vertical accuracy for the GPS unit.

2D GPS data can be very accurate if the elevation is known. During this study the horizontal accuracy of 2D points fulfilled the unit's accuracy for 3D points (< 10 meters using differential correction) at the *correct elevation*. The accuracy of 2D points decreased when the elevation was incorrect. Variation in horizontal accuracy is affected by the following:

1. The amount of vertical error present.

2. The direction and magnitude of the horizontal error as determined by satellite geometry.

Some of the data collected during this study would pass differential correction at elevations errors of up to + or - 150 meters. Extreme elevation error caused potential horizontal accuracy loss in excess of 100 meters. A horizontal accuracy of 100 meters is equivalent to autonomous (uncorrected) 3D GPS points. Corrected 3D points at higher PDOP values (current literature recommends less than 8) would probably be better than using 2D points. The accuracy comparison of corrected 2D data to 3D data at a higher PDOP is an area for further research.

This study did not find that HDOP was a good indicator for horizontal accuracy. The effect of the location of the average vector shows more promise as an accuracy indicator than HDOP. The findings suggest that a high HDOP with satellites clustered near the horizon will produce more horizontal error than the same cluster (with a similar HDOP) overhead. Further research is needed in this area.

Some practical considerations from this study are that heights used by GPS units are ellipsoidal heights. Current maps use mean sea level (MSL) heights. A program such as geoid96 will convert the height from MSL to ellipsoidal heights. The height above the geoid must be computed for each location. The use of topographic maps for the elevation component of 2D data must be converted from MSL to ellipsoid height as shown on page 17.

As new uses of GPS are found, both the horizontal and vertical accuracy demands by GPS users continue to increase. GPS accuracy is an important issue for not only mapping but also navigation. This study also has implications for data collected from automobiles, airplanes, and boats. In an automobile, the potential error of a 2D point due to elevation is magnified because of the speed of the vehicle. An automobile can rapidly change elevation going along a steep grade on a mountain.

Using GPS for computation of the horizontal and vertical position of airplanes is very significant. Collecting only 3D points from an airplane is usually not a problem as the view of satellites is better than on the ground. However, airplanes change elevation regularly as a matter of course while in flight, making the elevation computation of a 3D point important. As airspace around urban areas gets more crowded, positional accuracy becomes more significant. Errors involving more than one plane are compounded when the elevation errors are in opposite directions. Advances in GPS accuracy are being pushed to new levels with talk of using GPS to land planes in zero visibility. Improvements in elevation accuracy for aviation will benefit all GPS users.

Collecting 2D GPS data is inaccurate unless the receiver is positioned at a known elevation. 2D data for boat navigation is an example of an established use of GPS at a known elevation (sea or lake elevation). An interesting note is that some users prefer 2D points for boat navigation as the unit can be accurately set to collect at a fixed elevation. As found in this study, 2D points are very accurate using this technique as the earth (the fixed elevation) acts as the fourth satellite.

Horizontal accuracy of 2D points varies greatly by the elevation accuracy for each point. Each GPS user must decide whether 2D horizontal accuracy is acceptable to the individual mapping application; a scale dependant problem. The user may be willing to accept lower accuracy when the end product is a map of features at a county scale of 1:200,000. However, lower accuracy is probably not acceptable when mapping features on a detailed map at a scale of 1:12,000.

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Vita

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