

**LIMIT CYCLE PIO ANALYSIS with SIMULTANEOUSLY
ACTING MULTIPLE ASYMMETRIC SATURATION**

by

Joel E. Lamendola

*Thesis submitted to the Faculty of the Virginia Polytechnic
Institute and State University in partial fulfillment of the
requirements for the degree of
MASTER OF SCIENCE
in
Aerospace Engineering*

Approved:
Dr. Mark Anderson, Chair
Dr. Wayne Durham
Dr. Frederick Lutze

May 14, 1998

Blacksburg, VA

Key Words: Limit Cycle, PIO, Describing Function, Dual Input Describing Function

LIMIT CYCLE PIO ANALYSIS with SIMULTANEOUSLY ACTING MULTIPLE ASYMMETRIC SATURATION

Joel E. Lamendola

(ABSTRACT)

Pilot in-the-loop oscillation (PIO) is a phenomenon which occurs due to the dynamic interaction between pilot and aircraft. This detrimental aircraft handling quality appears through a variety of flight conditions and is very difficult to predict. Due to this complex behavior, PIO is not easily eliminated. This report describes a method of PIO analysis that is capable of examining multiple asymmetric nonlinearities acting simultaneously. PIO analyses are performed on a model based on the USAF NT-33A variable stability aircraft with nonlinearities including stick position limiting, elevator deflection limiting, and elevator rate limiting. These analyses involve the use of dual input describing functions which enable the prediction of frequency, amplitude, and mean point of oscillation.

FOREWORD

This effort is sponsored by the Naval Air Warfare Center, Aircraft Division Contract N60921-89-D-A239 (053). The views and conclusions contained herein are those of the authors and should not be interpreted as necessarily representing the official policies or endorsements, either expressed or implied, of Naval Air Warfare Center or any other agency of the U.S. Government.

Thanks to Dr. Anderson, my advisor, for his clear guidance and patience. Thanks also to my committee members, Dr. Durham and Dr. Lutze. The attitude and support of the controls group was second to none. Thanks also go to Jeff Leedy, Dan Lluch and Michelle Glaze for making the work environment here a real barrel of monkeys.

TABLE OF CONTENTS

1. INTRODUCTION.....	1
1.1 Objective.....	1
2. BACKGROUND.....	2
2.1 Aircraft / Control System Design.....	2
2.2 Recent PIO Events.....	3
2.3 Possible Causes of PIO.....	5
2.4 Combating PIO.....	6
2.5 PIO Research.....	9
3. GENERAL PROBLEM SETUP.....	12
3.1 Dual Input Describing Function Method.....	12
3.2 DIDF Applied to Asymmetric limit.....	16
3.3 Definition of Problem Structure.....	17
3.4 Solution Method for Multiple Asymmetric Saturation Elements.....	21
3.5 Validation of DIDF Method / Example.....	22
3.6 Limitations of DIDF Method.....	33
4. LIMIT CYCLE PIO ANALYSIS with ASYMMETRIC SATURATION....	36
4.1 PIO Analysis with Asymmetric Stick Limits.....	37
4.2 PIO Analysis with Asymmetric Stick Limits and Asymmetric Elevator Deflection Limits.....	46
4.3 PIO Analysis with Asymmetric Stick Limits, Asymmetric Elevator Deflection Limits, and Elevator Rate Limits.....	60
5. DISCUSSION OF RESULTS AND CONCLUSIONS.....	75
5.1 Conclusions about the PIO Analysis of the NT-33A Model.....	75
5.2 Recommendations for Continuing Work.....	76
REFERENCES	
APPENDIX	

LIST OF FIGURES

3.1.1	Block Diagram of Symmetric Saturation Element.....	12
3.1.2	Symmetric Limiting of Offset Sinusoidal Function.....	13
3.2.1	Asymmetric Saturation.....	16
3.2.2	Block Diagram of Asymmetric Limiter.....	17
3.3.1	Block Diagram of Nonlinear System with Asymmetric Saturation.....	18
3.5.1	Block Diagram of Dynamic System with One Asymmetric Limit.....	28
3.5.2	Time history of Simulated and Predicted Limit Cycle Solution..... for State Variable x_2 for Gain, $k=6$	31
3.5.3	Time history of Simulated and Predicted Limit Cycle Solution..... for State Variable x_1 for Gain, $k=6$	32
3.5.4	Phase Portrait of Simulated and Predicted Limit Cycle..... Solution for Gain, $k=6$	33
4.1.1	Block Diagram of NT-33A with Active Control System..... and Asymmetric Stick Limits	37
4.1.2	Frequency of Oscillation for NT-33A with Asymmetric Stick Limit.....	42
4.1.3	Pitch Attitude Amplitude of Oscillation for NT-33A with..... Asymmetric Stick Limit	43
4.1.4	Pitch Attitude Mean Point of Oscillation for NT-33A with..... Asymmetric Stick Limit	43
4.1.5	Elevator Amplitude of Oscillation for NT-33A with..... Asymmetric Stick Limit	45
4.1.6	Elevator Mean Point of Oscillation for NT-33A with..... Asymmetric Stick Limit	45
4.2.1	Block Diagram of NT-33A with Active Control System..... and Asymmetric Stick and Elevator Deflection Limits	47

4.2.1	Frequency of Oscillation for NT-33A with Asymmetric Stick Position and Asymmetric Elevator Deflection.....	53
4.2.2	Amplitude of Pitch Attitude Oscillation for NT-33A with Asymmetric Stick Position and Asymmetric Elevator Deflection.....	53
4.2.3	Mean point of Pitch Attitude Oscillation for NT-33A with Asymmetric Stick Position and Asymmetric Elevator Deflection.....	54
4.2.4	Amplitude of Stick Position Oscillation for NT-33A with Asymmetric Stick Position and Asymmetric Elevator Deflection.....	55
4.2.5	Mean point of Stick Position Oscillation for NT-33A with Asymmetric Stick Position and Asymmetric Elevator Deflection.....	56
4.2.6	Amplitude of Elevator Deflection Oscillation for NT-33A with Asymmetric Stick Position and Asymmetric Elevator Deflection.....	57
4.2.7	Mean point of Elevator Deflection Oscillation for NT-33A with Asymmetric Stick Position and Asymmetric Elevator Deflection.....	57
4.2.8	Phase Portrait of Elevator Oscillation for Pilot Gain, $k_p = 9$ lb/deg.....	58
4.2.9	Phase Portrait of Elevator Oscillation for Pilot Gain, $k_p = 14$ lb/deg.....	59
4.3.1	Block Diagram of NT-33A with Active Control System and Asymmetric Stick Position Limits, Asymmetric Elevator Deflection Limits, and Elevator Rate limits.....	60
4.3.2	Block Diagram of Actuator Dynamics with Rate and Asymmetric Deflection Limiting.....	61
4.3.3	Frequency of Oscillation for NT-33A with Asymmetric Stick Position, Asymmetric Elevator Deflection, and Elevator Rate Limiting.....	66
4.3.4	Amplitude of Pitch Attitude Oscillation for NT-33A with Asymmetric Stick Position, Asymmetric Elevator Deflection, and Elevator Rate Limiting.....	67

4.3.5	Mean Point of Pitch Attitude Oscillation for NT-33A with.....	67
	Asymmetric Stick Position, Asymmetric Elevator Deflection, and Elevator Rate Limiting	
4.3.6	Amplitude of Stick Position Oscillation for NT-33A with Asymmetric.....	69
	Stick Position, Asymmetric Elevator Deflection, and Elevator Rate Limiting	
4.3.7	Mean Point of Stick Position Oscillation for NT-33A with.....	69
	Asymmetric Stick Position, Asymmetric Elevator Deflection, and Elevator Rate Limiting	
4.3.8	Amplitude of Elevator Deflection Oscillation for NT-33A with.....	70
	Asymmetric Stick Position, Asymmetric Elevator Deflection, and Elevator Rate Limiting	
4.3.9	Mean Point of Elevator Deflection Oscillation for NT-33A with.....	71
	Asymmetric Stick Position, Asymmetric Elevator Deflection, and Elevator Rate Limiting	
4.3.10	Phase Portrait of δ_{er} vs. δ_e for a pilot gain of $k_p = 7$ lb/deg.....	72

LIST OF TABLES

3.5.1	Simulated and Predicted Solutions for the Example.....	30
4.1.1	State Space Model for NT-33A Asymmetric Stick..... Limit PIO Analysis	40
4.1.2	Simulated and Predicted Limit Cycle Solutions for..... The NT-33A with Asymmetric Stick Limiting	41
4.2.1	State Space Model for NT-33A Asymmetric Stick Position..... and Asymmetric Elevator Deflection Limit PIO Analysis	49
4.2.2	Simulated and Predicted Limit Cycle Solutions for the..... NT-33A with Asymmetric Stick Position and Asymmetric Elevator Deflection Limiting	50
4.2.3	Simulated and Predicted Limit Cycle Solutions for the..... NT-33A with Asymmetric Stick Position and Asymmetric Elevator Deflection Limiting	51
4.3.1	State Space Model for NT-33A Asymmetric Stick Position, Asymmetric Elevator Deflection, and Elevator Rate Limit PIO Analysis	63
4.3.2	Simulated and Predicted Limit Cycle Solutions for the NT-33A..... with Asymmetric Stick Position, Asymmetric Elevator Deflection and Elevator Rate Limiting	64
4.3.3	Simulated and Predicted Limit Cycle Solutions for the NT-33A..... with Asymmetric Stick Position, Asymmetric Elevator Deflection and Elevator Rate Limiting	65
4.3.4	Simulated and Predicted High Frequency Limit Cycle..... Solution for the NT-33A	73
4.3.5	Simulated and Predicted High Frequency Limit Cycle..... Solution for the NT-33A	73