5.0 Extension of Four-Unit Analysis to Five-Unit Configurations

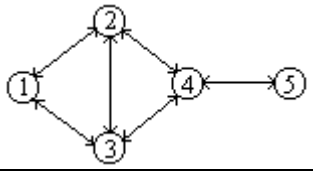
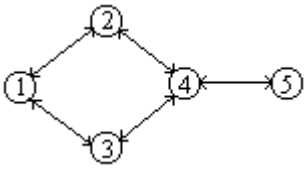
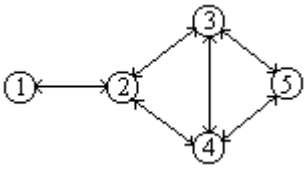
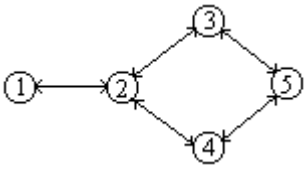
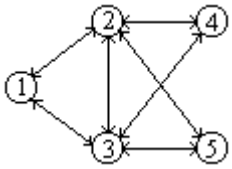
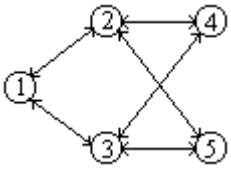
The analysis of the four-unit worst-case and intermediate configurations can be extended to configurations of five units. In fact, the results for several five-unit configurations are identical to those obtained for the four-unit configurations. These configurations are summarized in Section 5.1. However, there is a group of five-unit configurations, called the 1-3-1 configurations, which require more rigorous analysis.

The 1-3-1 configurations consist of a single alarm unit transmitting to three *middle* units, which in turn relay the message to a single fifth unit. The 1-3-1 configurations along with their analytical results are summarized in Table 5.2. Derivation of the analytical results follows Table 5.2 in Sections 5.2.1 - 5.2.4.

5.1 Five-Unit Configurations Having Results Identical to the Four-Unit Configurations

The following table summarizes those five-unit configurations which have analytical results identical to those obtained for the four-unit configurations. The results shown in the table assume that the alarm message originates from unit one. Note that in the configuration diagrams a solid arrow between two units indicates that they are within range of one another.

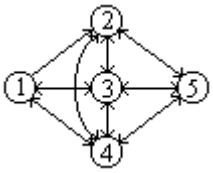
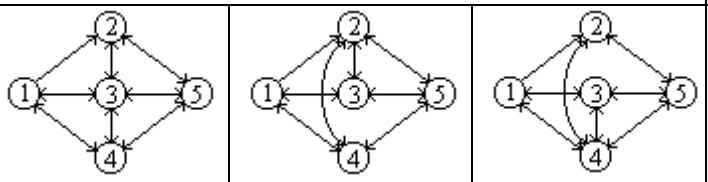
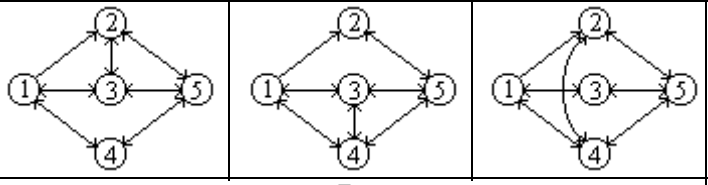
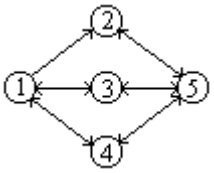
Table 5.1: Five-Unit Configurations Having Analytical Results Identical to Those of the Four-Unit Configurations

Five-Unit Configuration	Description	Pr{Link Failure(s)}
	Identical to four-unit intermediate configuration	0.0625 Link failures to units four and five
	Identical to four-unit worst-case configuration	0.6043 Link failures to units four and five
	Identical to four-unit intermediate configuration	0.0625 Link failure to unit five
	Identical to four-unit worst-case configuration	0.6043 Link failure to unit five
	Identical to four-unit intermediate configuration, but now there are two failing links	0.0625 Link failures to units four and five
	Identical to four-unit worst-case configuration, but now there are two *possible* failing links	0.6043 One link failure to either units four *or* five $(0.6043)^2 = 0.3652$ Two link failures to units four *and* five

5.2 Analysis of 1-3-1 Five-Unit Configurations

The 1-3-1 five-unit configurations are summarized in Table 5.2. Note that the analysis for these configurations assumes that the alarm message originates from unit one and then propagates throughout the system.

Table 5.2: Five-Unit Configurations Requiring Individual Analysis

Description	Five-Unit Configuration	Pr{Link Failure to Unit 5}
All three middle units in range of each other		0.0606
Two pairs of middle units in range		0.1257
One pair of middle units in range		0.0417
No middle units in range		0.3652

5.2.1 Analysis of 1-3-1 Five-Unit Configuration, Three Middle Units in Range

In the five-unit configuration depicted in row one of Table 5.2, units two, three, and four are in range of each other. Unit five will not receive an alarm message if any of the following four conditions occurs:

(1) The holdoff timers in units two, three, and four are synchronized with one another.
(2) The holdoff timers in units two and three are synchronized and units two and three begin transmitting before unit four. Unit four is unable to detect the simultaneous transmissions of units two and three. Thinking the channel is idle, unit four transmits, and none of its messages are received by unit five.
(3) The holdoff timers in units two and four are synchronized and units two and four begin transmitting before unit three. Unit three is unable to detect the simultaneous transmissions of units two and four. Thinking the channel is idle, unit three transmits, and none of its messages are received by unit five.
(4) The holdoff timers in units three and four are synchronized and units three and four begin transmitting before unit two. Unit two is unable to

detect the simultaneous transmissions of units three and four. Thinking the channel is idle, unit two transmits, and none of its messages are received by unit five.

The probability that unit five does not receive an alarm message in this configuration is given in equation (5.1) as the sum of the probabilities of the occurrence of each of these four conditions. Note that the probability of occurrence of conditions two through four is identical. In equation (5.1), we derive the probability of just one of these three conditions and multiply the result by three.

$$
\begin{aligned}
&Pr\{\text{Unit five does not receive an alarm message}\} \\
&= Pr\{\text{Three holdoff timers are synchronized}\} + \\
&\quad 3 \times Pr\{\text{Holdoff timers in two middle units are synchronized}\} \times \\
&\quad Pr\{\text{Those two units transmit before the third middle unit}\} \times \\
&\quad Pr\{\text{Messages from the third middle unit are not received by unit five}\} \\
&= (0.0625)(0.0625) + (3)(0.0625)(0.5)(0.6043) \\
&= 0.0606
\end{aligned}
\tag{5.1}
$$

5.2.2 Analysis of 1-3-1 Five-Unit Configuration, Two Pairs of Middle Units in Range

In the five-unit configurations depicted in row two of Table 5.2, two pairs of the three middle units are within range of each other. Consider a case in which units two and three are in range of each other, and units three and four are in range of each other. However, units two and four are not in range of one another. Unit five will not receive an alarm message if any of the following three conditions occurs:

(1) The holdoff timers in units two, three, and four are synchronized with one another.
(2) Unit two transmits first. Because unit four is out of range of unit two, it thinks the channel is idle and it begins transmitting next. Unit three cannot distinguish the simultaneous transmissions of units two and four. Thinking the channel is idle, unit three also begins transmitting. None of the transmissions from units two, three, and four are successfully received by unit five.
(3) Unit four transmits first. Because unit two is out of range of unit four, it thinks the channel is idle and begins transmitting next. Unit three cannot distinguish the simultaneous transmissions of units two and four. Thinking the channel is idle, unit three also begins transmitting. None of the transmissions from units two, three, and four are successfully received by unit five.

The probability of conditions two and three is identical. In the following analysis, we compute the probability of condition two and simply double the result. The probability that unit five does not receive an alarm message in this configuration is given by equation (5.2).

Pr {Unit five does not receive an alarm message}
= Pr {Three holdoff timers are synchronized} +
2 ×Pr {Unit two transmits first} ×Pr {Unit four transmits next}
Pr {Since unit four cannot detect unit two's transmissions, all of its transmissions collide with those generated by unit two} ×
Pr {Thinking the channel is idle, unit three transmits, and none of its messages are successfully received by unit five}
= (0.0625)(0.0625) + (2)(0.3333)(0.5)(0.6043)(0.6043)
= 0.1257 (5.2)

5.2.3 Analysis of 1-3-1 Five-Unit Configuration, One Pair of Middle Units in Range

In the five-unit configurations shown in row three of Table 5.2, only one pair of the three middle units is in range with each other. As an example, consider the case in which neither unit two nor three can hear unit four; however, units two and three are in range of each other. Unit five will not receive an alarm message if any of the following three conditions occurs:

(1) The holdoff timers in units two, three, and four are synchronized.
(2) The holdoff timers in units two and three are synchronized. Units two and three transmit before unit four. Unit four cannot detect the transmissions generated by units two and three. As a result, unit four's transmissions are not successfully received by unit five.
(3) Unit four begins transmitting first. The holdoff timers in units two and three are synchronized with one another. Units two and three cannot detect the transmissions generated by unit four. Unit five does not successfully receive any messages from units two, three and four.

The probability of conditions two and three is identical. In the following analysis, this probability is evaluated for condition two, and the result doubled. The probability that unit five does not receive an alarm message in this configuration is shown in equation (5.3).

Pr {Unit five does not receive an alarm message}
= Pr {Three holdoff timers are synchronized} +
2 ×Pr {Holdoff timers in units two and three are synchronized} ×
Pr {Units two and three transmit before unit four} ×
Pr {Unit four's messages are not successfully received by unit five}
= (0.0625)(0.0625) + (0.0625)(0.5)(0.6043)
= 0.0417 (5.3)

5.2.4 Analysis of 1-3-1 Five-Unit Configuration, No Middle Units in Range

In the five-unit configuration shown in row four of Table 6, none of the middle units are in range with one another. In this configuration, unit five does not receive an alarm message if the following condition occurs:

(1) None of the messages generated by units two, three, and four are successfully received by unit five since units two, three, and four cannot hear one another's transmissions.

Therefore, the probability that unit five does not receive an alarm message in this configuration is given by equation (5.4).

$$\begin{aligned} &Pr\{\text{Unit five does not receive message}\} \\ &= (0.6043)(0.6043) \\ &= 0.3652 \end{aligned} \tag{5.4}$$

5.3 Summary of Five-Unit Analysis

Analytical results have been discussed for two groups of five-unit configurations. The first group is summarized in Table 5.1 and has results identical to those derived for the four-unit configurations. Of these five-unit configurations, those which resemble the four-unit worst-case configuration should be avoided.

The second group of five-unit configurations which has been analyzed is shown in Table 5.2. All of these configurations consist of a single unit transmitting to three units, which in turn transmit to a fifth unit. Of these configurations, there are two which provide relatively low probability of link failure. The first is shown in row one of Table 5.2, and represents a situation in which all three of the middle units are in range of each other. The probability of link failure in this configuration is 0.0606.

Interestingly, the second configuration which results in low probability of link failure is the scenario in which only one pair of middle units is in range of each other. This configuration is shown in row three of Table 5.2. At first glance, one might expect that as the number of middle units within range of each other decreases, the probability of link failure increases. This is not necessarily true. In fact, Table 5.2 shows that when two pairs of middle units are within range of each other, the probability of link failure is 0.1257. However, when only one pair of middle units is within range of each other, the probability of link failure is reduced to 0.0417.

To understand this unexpected phenomenon, refer to the first configuration shown in row two of Table 5.2. In this scenario, units two and three, and units three and four are in range of each other. However, units two and four are not in range of each other. Since units two and four are out of range of one another, they can be thought of as the four-unit worst-case configuration. Thus, there is a high probability that messages generated by units two and four will collide. In this scenario, it is actually disadvantageous for unit

three to be able to hear both units two and four, because unit three will not be able to distinguish the simultaneous transmissions of units two and four. As a result, unit three will think that the channel is idle. It too will transmit, and none of the messages generated by units two, three, and four will be successfully received by unit five.

If unit three could hear only unit two *or* unit four, then simultaneous transmissions of units two and four will not confuse unit three. Unit three will hear only one set of transmissions, and will wait for their completion before transmitting.

5.4 Five-Unit Installation Recommendations

The analytical results alone are not adequate to develop an effective installation plan for the wireless alarm system. The installation plan requires a topological analysis as well.

Recall from the five-unit analytical results that there are two sets of configurations which yield a low probability of link failure. These configurations are repeated in Figure 5.1 for reference.

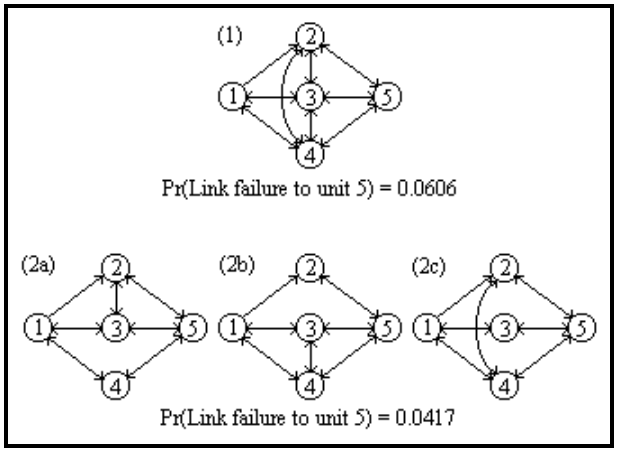


Figure 5.1: Five-Unit Configurations with Low Probability of Link Failure

Recall also that the analytical results assume that the alarm message originates from unit one and then propagates through the system. An important consideration in the installation of the wireless alarm system is the effect on system performance when the alarm message originates from a different unit.

For example, in configuration (1) shown in Figure 5.1, we can see that if the alarm originates from unit two, then the system performance is improved. Unit two is within range of all other units in the system, so message collisions are not a threat. Likewise, message collisions are not a threat if the alarm originates from either units three or four. If the alarm originates from unit five in this system, then the topology is the same as if the alarm were originating from unit one. Again, we would have a 0.0506 probability of link failure, and the system still provides a reasonable level performance. Configuration 1 in

Figure 5.1 is an effective installation, as the probability of link failure remains low regardless of which unit originates the alarm.

This is not the case when we examine configurations (2a), (2b), and (2c) shown in the second row of Figure 5.1. Since each of these three configurations are topologically equivalent, we will analyze only configuration (2a).

It has been shown that for configuration (2a), there is a 0.0417 probability of link failure when the alarm originates from unit one. However, suppose the alarm originates from unit two. Units one, three, and five will definitely receive the message, as they are all within range of unit two. Note from Figure 5.1, though, that only units one and five are within range of unit four. Furthermore, units one and five are *not* in range of each other. As a result, we essentially have the four-unit worst-case configuration, and there is a 0.6043 probability that unit four will not receive the alarm message due to collisions. This demonstrates that configurations (2a), (2b), and (2c) are not effective installations of the wireless alarm system, and should be avoided.

In summary, of the five-unit configurations which have been analyzed, there are only four which can be considered an effective installation. Three of these are those configurations in Table 5.1 which have a link failure probability of 0.0625. The other acceptable five-unit configuration is shown in row one of Table 5.2, and has a link failure probability of 0.0506. Of course, the five-unit configurations in which collisions are not even a threat are also acceptable. These include the situation in which all five units are in range of each other, and also a situation in which the five units are in a serial configuration.